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Stability of general class of hairy black holes in $D = 2 + 1$

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Abstract: We present an exact.

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1 Theory

We are interested in asymptotically AdS hairy black hole solutions with a spherical horizon. The action is [1]

$$I[g_{\mu\nu},\phi] = \frac{1}{2\kappa}\int_{\mathcal{M}} d^3x\sqrt{-g}\left[R - \frac{(\partial\phi)^2}{2} - V(\phi)\right] + \frac{1}{\kappa}\int_{\partial\mathcal{M}} d^2xK\sqrt{-h} \tag{1.1}$$

where $V(\phi)$ is the scalar potential, $\kappa = 8\pi G_N$, and the last term is the Gibbons-Hawking boundary term. Here, h_{ab} is the boundary metric and K is the trace of the extrinsic curvature.

$$I[g_{\mu\nu},\phi] = \frac{1}{2\kappa}\int_{\mathcal{M}} d^3x\sqrt{-g}\,[R + L_M] \tag{1.2}$$

$$G_{\alpha\beta} = \frac{1}{2}T_{\alpha\beta} = g_{\alpha\beta}L_M - 2\frac{\partial L_M}{\partial g^{\alpha\beta}}, \qquad L_M = -\frac{(\partial\phi)^2}{2} - V(\phi) \tag{1.3}$$

Them the equation of motion for the metric are

$$G_{\alpha\beta} = \frac{1}{2}T_{\alpha\beta} = \partial_\alpha\phi\partial_\beta\phi - g_{\alpha\beta}\left[\frac{(\partial\phi)^2}{2} + V(\phi)\right] \tag{1.4}$$

And the variation of the action respect to scalar field we have

$$\delta I = \frac{1}{2\kappa}\int d^3x\sqrt{-g}\left[\frac{1}{\sqrt{-g}}\partial_\alpha\left(\sqrt{-g}g^{\alpha\beta}\partial_\beta\phi\right) - \frac{\partial V}{\partial\phi}\right]\delta\phi - \frac{1}{2\kappa}\int d^2x\sqrt{-h}\; n^\alpha\partial_\alpha\phi\;\delta\phi \tag{1.5}$$

$$\frac{1}{\sqrt{-g}}\partial_\alpha\left(\sqrt{-g}g^{\alpha\beta}\partial_\beta\phi\right) - \frac{\partial V}{\partial\phi} = 0 \tag{1.6}$$

2 Solution

$$ds^2 = \Omega(x)\left[-f(x)dt^2 + \frac{\eta^2dx^2}{f(x)} + d\varphi^2\right] \tag{2.1}$$

$$E_t{}^t - E_x{}^x = 0 \quad \Rightarrow \quad (\phi')^2 = \frac{3\Omega'^2 - 2\Omega\Omega''}{2\Omega^2} \tag{2.2}$$

$$E_t{}^t - E_\phi{}^\phi = 0 \quad \Rightarrow \quad (\Omega^{1/2} f')' = 0$$

$$E_t{}^t + E_\phi{}^\phi = 0 \quad \Rightarrow \quad V = -\frac{1}{4\eta^2\Omega(x)^3}\left[2\Omega'' f\Omega + 2f''\Omega^2 - f\Omega'^2 + 3\Omega\Omega' f'\right]$$

$$\Omega(x) = \frac{\nu^2 x^{\nu-1}}{\eta^2(x^\nu - 1)^2} \tag{2.3}$$

$$f(x) = \frac{1}{L^2} + \frac{\alpha}{2\nu}\left[2\nu + (3-\nu)\, x^{\frac{3+\nu}{2}} - (3+\nu)\, x^{\frac{3-\nu}{2}}\right] \tag{2.4}$$

$$\phi(x) = \ell^{-1}\ln x, \qquad \ell^{-1} = \frac{\sqrt{2}}{2}\sqrt{\nu^2 - 1} \tag{2.5}$$

And the on-shell potential is

$$V(\phi) = -\frac{3(\alpha L^2 + 1)\exp(-\phi\ell)}{4\nu^2 L^2}\left[(1+\nu)\left(1 + \frac{\nu}{3}\right)\exp(\phi\ell\nu) + (1-\nu)\left(1 - \frac{\nu}{3}\right)\exp(-\phi\ell\nu)\right. \tag{2.6}$$

$$\left. - 2(1-\nu^2)\right] - \frac{\alpha\exp(\phi\ell/2)}{\nu}\left[(1-\nu)\exp\left(\frac{\phi\ell\nu}{2}\right) - (1+\nu)\exp\left(-\frac{\phi\ell\nu}{2}\right)\right] \tag{2.7}$$

$$V(\nu) = V(-\nu), \quad V(\phi) = V(-\phi), \quad V(\phi,\nu) = V(-\phi,-\nu) \tag{2.8}$$

The non-hair limit is well defined in

$$\nu = 1 \ \Rightarrow \ V = -\frac{2}{L^2}, \quad \Omega(x) = \frac{1}{\eta^2(x-1)^2}, \quad f(x) = \frac{1}{L^2} + \alpha(x-1)^2 \tag{2.9}$$

$$V(\phi) = -\frac{2}{L^2} - \frac{\ell^2(\nu^2-1)}{4L^2}\phi^2 - \frac{\ell^3(\nu^2-1)}{12L^2}\phi^3 - \frac{\phi^4\ell^4}{192}(\nu^2-1)\left[3\alpha(\nu^2-9) + \frac{4(\nu^2-6)}{L^2}\right] + \mathcal{O}(\phi^4) \tag{2.10}$$

$$x = 1 \pm \frac{1}{\eta r} - \frac{\nu^2-1}{24\eta^3 r^3} + \frac{\nu^2-1}{24\eta^4 r^4} + \mathcal{O}(r^{-5}) \tag{2.11}$$

For the negative and positive branch respectively

$$\phi(r) = \pm\frac{1}{\eta\ell r} - \frac{1}{2\eta^2\ell r^2} + \mathcal{O}(r^{-3}) \tag{2.12}$$

3 Thermodynamic stability

4 Mechanics stability

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