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ScienceDirect

Journal of the Franklin Institute 357 (2020) 12700–12723

www.elsevier.com/locate/jfranklin

Distributed coordination of multi-agent systems for neutralizing unknown threats based on a mixed coverage-tracking metric☆

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Received 8 January 2020; received in revised form 27 June 2020; accepted 28 August 2020

Available online 10 September 2020

Abstract

This paper presents a coordination control method for neutralizing multiple threats by a team of mobile agents in a bounded area. Threat here refers to an unexpected target intruding into the area. Without knowing the number of threats, how they maneuver and when to appear a priori, agents equipped with active sensing and actuating devices are driven to detect, track and intercept those threats as many as possible. In order to increase the probability of detecting new threats, a metric (called the mixed coverage-tracking metric) with dynamic task assignment mechanism is introduced. More specifically, the metric is a weighted sum of the travel cost for area coverage and threat tracking respectively. The control objective is to find optimal trajectories and task assignment values of agents that can minimize the expected mixed metric. Based on Voronoi partition of the mission area, a gradient based control law is designed to drive each agent towards its locally optimum configuration. Meanwhile, a task assignment control law is employed to smoothly switch between area coverage and threat tracking depending on the density of detected threats within each agent's Voronoi cell. Resorting to optimal control and Lyapunov stability theory, the proposed control method can guarantee that each agent asymptotically converges

☆ This work was partially supported by the National Natural Science Foundation of China under Grants 61633011, 61672245, 61772086, 61672244, 61873287 and 61976099.

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https://doi.org/10.1016/j.jfranklin.2020.08.039

to its centroid of Voronoi cell. Simulation examples are provided to illustrate the effectiveness of the theoretical results.

1. Introduction

In recent years, multi-agent systems have received extensive attentions from the scientific community [1–3]. For such systems, one common feature is that coordination among agents can realize some collective behaviors just using simple rules through interactions subjected to limited environmental information. Some typical coordination problems, such as rendezvous [4], formation [5], flocking [6] and containment [7], are the hot topics in this research field, which resort to consensus-based control algorithms [8]. In these problems, the state of each agent finally converges to a common target state. Different from above-mentioned problems, coverage control can be seen as in opposite to consensus-type problems. Using Voronoi-based area partition method, a team of agents is assigned to some non-overlapping subareas for implementing active sensing task [9]. Based on the distribution of information of interest in an area (called the density function), mobile agents are driven gradually to their optimal positions respectively such that the sensing coverage performance is maximized. Coverage control problem not only involves agent-agent interactions, but also agent-environment connections. Besides, the final states of agents are needed to be found and optimized, instead of giving in advance like consensus-type problems. At present, related methods of coverage control have been widely applied to accomplish various practical tasks [10,11], such as environmental monitoring, space exploration, pollution neutralization, etc.

In this paper, we focus on distributed coordination of a team of mobile agents for neutralizing unknown threats based on a coverage control method. Threat refers to an unexpected target with motion capability intruding into a pre-defined area. Threat neutralization is a practical problem with engineering background which is widely occurred both in military and civilian field [12,13]. Consider a scenario where uninvited drones operated by remote users wander as threats into some restricted or sensitive space, such as airport, harbour or military camp. In order to mitigate the potential risk, a platoon of autonomous anti-drones with weapons can be deployed in the protected space to detect, track and intercept such threats [14].

Neutralizing moving threats involves three basic subtasks: environment monitoring, target tracking and task assignment. For a bounded area of interest, these subtasks need to be solved simultaneously in defending against threats. Some works focus on optimal deployment of sensor networks to detect the emergence of threats [15,16], some works propose optimal threat tracking strategies considering costs of energy or resource [17,18], and others develop task assignment mechanisms to switch between sensor deployment and target tracking [19,20]. However, related studies often assume that the number and dynamics of threats are known in advance or the information of each threat is available with dense deployment of sensors. The main challenge of this problem is to design a control algorithm for each agent considering both three subtasks in a distributed manner when the threat information is not known a priori.

Different from above-mentioned methods, some researchers also deal with this problem based on a coverage control framework. The advantage of coverage control is to reformulate three subtasks of neutralizing unknown threats as a distributed optimal deployment problem.

In [21], the authors firstly proposes a Voronoi-based coverage control scenario to deal with moving targets tracking under the assumption that the state information of each target can be estimated by active sensors. This work is extended in [22] to address the problem of simultaneously integrating three fundamental subtasks, i.e., covering an environment, assigning and tracking dynamic targets. Experiments on real robots were provided to validate guaranteed convergence of the proposed control algorithm. However, theoretical analysis of singularities which may occur in the controller was absent. Recently, by defining a risk density function associated to the motion of targets, the authors [23] propose a decentralized control law deploying a team of mobile agents to optimize a generalized non-autonomous coverage metric. This method can be used to deal with the problem of multiple moving targets tracking. Similar results of dynamic coverage control can be also found in [24,25].

Motivated by the works [22,23], our solution to neutralizing unknown threats is based on a dynamic coverage control method and two density functions (one is uniform and static, the other is non-uniform and time-variant) are introduced respectively. In contrast to [22,23], we employ a mixed coverage-tracking metric with dynamic task assignment mechanism to balance area coverage task and threat tracking task. The advantages of the proposed coverage control method are threefold: First, it can avoid over-clustering of mobile agents around the detected moving threats. Second, potential collision can also be avoided which may occur among mobile agents and moving threats. Third, the probability of detecting new moving threats throughout the mission area can be increased noticeably.

The contributions of this paper are summarized as follows.

- We propose a new formulation for the solution to neutralizing unknown threats. In order to detect and intercept threats timely, a mixed coverage-tracking metric with task assignment variable is introduced. Based on the metric, the dynamic coverage control problem is transformed into a multi-objective optimization problem with respect to the total travel cost of all agents.
- We derive the condition that the objective function takes the minimum value using the Leibniz differential rule of integral, and then the local optimal configuration set of objective function is determined.
- We design the motion control law and task assignment control law of each agent respectively in a distributed manner using the state-switching method, and the stability of the closed-loop system under two control laws is proved.

The remainder of this paper is organized as follows. Section 2 defines the threat neutralization problem. In Section 3, a dynamic coverage control law and a dynamic task assignment control law are proposed respectively. In Section 4, numerical examples of two cases are given to validate the effectiveness of the control algorithm. Section 5 draws the conclusions of this research.

Notations: $\mathbb{R}^n$ denotes the n-dimensional Euclidean space. $\mathbb{R}_{\geq 0}$ and $\mathbb{R}^+$ represents the set of nonnegative and positive real numbers respectively. The symbol "$\cdot$" stands for the inner product between two vectors and $\|\cdot\|$ denotes the Euclidean norm of a vector. The symbol δ_{ij} denotes the Kronecker delta function of two variables i and j, which is 1 if the variables are equal, and 0 otherwise. $\mathbf{1}_B$ is defined as an indicator function, i.e., $\mathbf{1}_B = 1$ if the condition B is true and $\mathbf{1}_B = 0$ otherwise.

2. Problem formulation

Consider a team of mobile agents equipped with active sensors to monitor a bounded and convex area $Q \in \mathbb{R}^2$. Let $q \in \mathbb{R}^2$ be an arbitrary point of Q. N mobile agents start from distinct initial positions $p_{i_0}, i \in \mathcal{I}$, where the index set $\mathcal{I} = 1, \dots, N$. The mission area is large compared to the physical size of agents, whose dynamics are given by

$$\dot{p}_i(t) = v_i(t), \quad p_i(0) = p_{i_0}, \quad i \in \mathcal{I} \tag{2.1}$$

where $p_i(t), v_i(t) \in \mathbb{R}^2$ are the position and velocity control input of the ith mobile agent at time $t \in \mathbb{R}_{\geq 0}$ respectively. Define $P = \{p_1^T, \dots, p_N^T\}^T \in \mathbb{R}^{2N}$ as the collective position configuration of mobile agents.

Remark 2.1. In this paper, the bounded mission area assumes to be convex. This assumption ensures that a Voronoi centroid mentioned in the following always remains within its own Vornnoi cell. For a non-convex environment, the Voronoi centroid may lie outside the environment or inside an obstacle, which cannot be reached by any mobile agent. A constrained centroidal Voronoi tessellation (CCVT) method [26] can be used to overcome the non-convex problem.

Multiple unexpected threats may intrude into the mission area at any instant. Without knowing the number of threats, how they maneuver and when to appear a priori, mobile agents are driven to detect, track and intercept those threats as many as possible. Let R_d be the detection radius of each agent. Using active sensing devices, the states (position and velocity) of any threat within R_d can be measured without noise. Define $s_k, \dot{s}_k \in \mathbb{R}^2$ as the position and velocity vector of the kth detected threat respectively, $k \in \mathcal{L}$, where the index set $\mathcal{L} = \{1, \dots, l(t)\}$. $l(t)$ is a time-varying value denoting the number of threats already detected at time t. Obviously, if no threats have been detected at time $t \in \mathbb{R}^+$, then $l(t) = 0$ and $\mathcal{L} = \emptyset$.

Mobile agents' goal is to intercept all moving threats in the mission area. If the agent successfully intercept a threat at time $t_c \in \mathbb{R}^+$, it remains intercepted for all $t > t_c$, and needn't to be considered any longer. The kth threat is intercepted at t_c when

$$\min_{i \in \mathcal{I}} \|p_i(t_c) - s_k(t_c)\| \leq R_c, \quad k \in \mathcal{L} \tag{2.2}$$

where R_c is the interception radius.

In order to evaluate the threat neutralization performance of each agent, a non-decreasing and differentiable function $f(\|q - p_i\|) : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is defined as a simplified measure of the travel cost

$$f(\|q - p_i\|) = \alpha \|q - p_i\|^2, \tag{2.3}$$

where $\alpha > 0$ is a constant parameter. Note that the travel cost function f is zero as $\|q - p_i\| = 0$ and tends to infinity as $\|q - p_i\| \to +\infty$.

Although it is unknown when the threats may appear, we can adopt a coverage control method to deploy mobile agents over the mission area for neutralizing threats. In order to increase the probability of detecting new threats, one can define a uniform density function $\psi_a : Q \to \mathbb{R}^+$ over the mission area Q, where $\psi_a(q) = (\int_Q dq)^{-1}, \forall q \in Q$. Intuitively, this density function captures the equal chance of any point in Q to find threats. Based on this density function, mobile agents moves apart to be evenly distributed over Q for implementing an area coverage task. Analogously to [9], a coverage control metric denoting the total travel cost of all agents over the mission area is given by

$$H_1(P) = \int_Q \min_{i\in\mathcal{I}}[f(\|q - p_i\|)\psi_a(q)]dq, \tag{2.4}$$

which is to determine an optimal solution to the area coverage task of each agent. This uniform area coverage will allow mobile agents to quickly and efficiently detect a new threat.

Once a new threat has been detected, it is necessary for the agents to move towards it as fast as possible. One direct choice is using optimal-time tracking control methods to intercept the threat, such as Bang-Bang control. However, it will inevitably result in an "over-centralized" deployment of mobile agents around the detected threats, which is adverse to finding new threats further in the mission area. One possible solution is to employ a threat assignment scheme, where the newly detected threat is only assigned to the nearest agent to track it. Unfortunately, it needs a global supervisor to coordinate the assignment procedures.

Motivated by the work [22], in this paper, a time-varying density function $\psi_t : Q \times \mathbb{R}^+ \to \mathbb{R}^+$ depending on the positions of detected threats in Q is defined as

$$\psi_t(q, t) = \lambda(t)\sum_{k=1}^{l(t)} \exp\Big(-\frac{\|q - s_k\|^2}{2\sigma^2}\Big), \tag{2.5}$$

where $l(t)$ is the number of detected threats in Q at time instant $t \in \mathbb{R}^+$, s_k are the position of the kth detected threats and the radius $\sigma > 0$ determines how narrow is the distribution of ψ_t around s_k. $\lambda(t)$ is a normalizing term to ensure that $\int_Q \psi_t(q, t)dq = 1$.

It should be noted that the setting of parameter σ is related to the detection radius R_d. Let $\sigma = \mu R_d$, where $\mu \in (0, 1]$ is a positive constant such that the values of $\psi_t(q, t)$ at points out of this detection range R_d can be almost neglected. Based on this density function, mobile agents are driven towards those detected threats for implementing a threat tracking task.

In order to overcome the "over-centralized" problem, one can put the density functions ψ_a and ψ_t together for implementing area coverage task and threat tracking task simultaneously. Similar to what has been done in multi-objective optimization, a mixed coverage-tracking metric of mobile agents is introduced

$$\begin{aligned} H(P, \rho, t) &= (1 - \varrho_i)\int_Q \min_{i\in\mathcal{I}}[f(\|q - p_i\|)\psi_a(q)]dq + \varrho_i\int_Q \min_{i\in\mathcal{I}}[f(\|q - p_i\|)\psi_t(q, t)]dq \\ &= \int_Q \min_{i\in\mathcal{I}}[(1 - \varrho_i)\psi_a(q) + \varrho_i\psi_t(q, t)]f(\|q - p_i\|)dq \end{aligned} \tag{2.6}$$

to balance the total travel cost between area coverage and threat tracking to serve a location $q \in Q$, where $\varrho_i \in (0, 1)$ is defined as a time-varying task assignment variable which needs to be determined, and $\rho = \{\varrho_1, \dots, \varrho_N\}^T$ denotes a task assignment configuration of all agents with respect to the mixed metric. Let $\varphi_i(\varrho_i, q, t) = (1 - \varrho_i)\psi_a(q) + \varrho_i\psi_t(q, t)$ be the density function of each agent for neutralizing unknown threats. Since the density $\psi_t(q, t)$ is time-varying in terms of detected moving threats, each agent has to design a feedback control law of task assignment variable to automatically adjust the weight between area coverage and threat tracking, i.e.,

$$\dot{\varrho}_i(t) = u_i(t), \quad \varrho_i(0) = \frac{1}{2}, \quad i \in \mathcal{I} \tag{2.7}$$

where $u_i(t) \in \mathbb{R}$ is the control input of ith agent for dynamic task assignment at time $t \in \mathbb{R}_{\geq 0}$. In order to obtain an optimal solution to (2.6), a position configuration P of agents and a task assignment configuration ρ are to be found that minimize the mixed metric.

Remark 2.2. In our scenario, we define a density function putting threat-tracking and uniform-coverage tasks together. Intuitively, the uniform-coverage task is important to keep an adjustable force term of dispersion even if all agents move to the detected threats. It is beneficial to alleviate over-clustering of mobile agents around certain detected threats and increase the probability of detecting new threats.

It should be noted that the mixed metric implies that each point in Q is assigned to the agent which has the minimum travel cost towards that point. Obviously, it requires each agent to communicate with other agents in the team to realize such assignment, which contradicts the distributed nature of mobile agents. Hence, we need to convert this mixed metric into an equivalent form such that each agent is capable of optimizing its relevant part of the metric in a distributed manner.

To this end, one can partition the mission area Q into N non-empty disjoint cells $W_1, \dots, W_N$ using the agent positions as the generating points. The union of these cells suffices the condition $Q = \cup_{i=1}^{N} W_i$. The ith agent is only responsible for monitoring over its dominance cell W_i. In this way, the mixed metric (2.6) is recast into

$$H(P, W, \rho, t) = \sum_{i=1}^{N} \int_{W_i} f(\|q - p_i\|)\varphi_i(\varrho_i, q, t)dq. \tag{2.8}$$

Notice that the metric (2.8) has an additional degree of freedom, called the partition $W = \{W_1, \dots, W_N\}$ of Q. Each agent can only optimize one part of the metric within its own cell. For a fixed agent position configuration P, $H(P,\ W,\ \rho,\ t)$ is bounded below by

$$H(P, W, \rho, t) \geq \int_{Q} \min_{i \in \mathcal{I}} f(\|q - p_i\|)\varphi_i(\varrho_i, q, t)dq, \tag{2.9}$$

which means an optimal partition of Q has to be found if the metric (2.8) is used to replace (2.6).

As mentioned in [29], the optimal partition of Q in terms of the Euclidean distance is Voronoi diagram based partition. Define the Voronoi partition $V := \{V_1, \dots, V_N\}$ of Q as

$$V_i := \{q \in Q \mid f(\|q - p_i\|) \leq f(\|q - p_j\|), \forall i, j \in \mathcal{I}, i \neq j\}, \tag{2.10}$$

where V_i is the ith Voronoi cell generated by the point p_i of the ith mobile agent. Different Voronoi cells V_i and V_j which share a boundary are called Voronoi neighbors. In order to construct the Voronoi cell of each agent in a distributed way, the following assumption is needed.

Assumption 1. Each agent has completed knowledge of its own states and the ability to communicate with other agents in its neighboring Voronoi cells.

According to the definition, the Voronoi cell V_i is the locus of all points that are closer to the ith agent than other agents in Q. Hence, the metric (2.8) is reformulated to

$$H(P, V, \rho, t) = H(P, \rho, t) = \sum_{i=1}^{N} \int_{V_i} f(\|q - p_i\|)\varphi_i(\varrho_i, q, t)dq = \sum_{i=1}^{N} H_i(P, \rho, t). \tag{2.11}$$

The metric above encodes how much effort of mobile agents makes in travelling to neutralize moving threats. In contrast to the metric (2.8), the optimal solution to Eq. (2.11) can be easily computed distributively since $H(P, V, \rho, t)$ is a composition of N summands which can be optimized in parallel.

Based on the mixed coverage-tracking metric (2.11), unknown threats neutralization problem can be converted into a coverage control problem. The objective of this paper can be stated as follows: Given multiple moving threats and a team of mobile agents, we seek to spatially find an optimal motion control law and a task assignment control law of each agent to minimize the mixed coverage-tracking metric for neutralizing unknown threats, i.e.,

$$\min_{\substack{P\in\mathbb{R}^{2N}\\ \rho\in[0,1]^N}} H(P, \rho, t), \quad \text{subject to (2.1) and (2.7) as } t \to \infty \tag{2.12}$$

Note that the mixed metric (2.11) has fundamental differences to the metric (2.4) which is common adopted in the existing literature related to coverage control problem. First, the density function $\varphi_i(\varrho_i, q, t)$ of each agent is computed in a distributed way due to the task assignment variable ϱ_i. As a consequence, the density distributions on each side of the boundary between two Voronoi neighbors are different. Second, the density function $\varphi_i(\varrho_i, q, t)$ of each agent is time-varying due to the detected moving threats, a dynamic coverage control law should be designed to address this problem.

Remark 2.3. The coverage control problem considered here is under fully connected communication network. However, for the communication network subject to noises, attacks or using special protocols, such as stochastic intermittent communication, deception attacks or Round-Robin scheduling protocols for networked control systems, it remains a challenge for mobile agents to perform the coordinated task with unreliable communications and frequent packet drops.

3. Optimal coverage control for neutralizing unknown threats

In this section, the gradient of $H(P, \rho, t)$ with respect to p_i and ϱ_i is firstly derived, then a dynamic coverage control law considering time-varying density function and a dynamic task assignment law are proposed respectively. Finally, the optimality and asymptotic convergence of the proposed control laws are analyzed.

3.1. Gradient of the mixed coverage-tracking metric

To determine optimal configurations of P and ρ for the distributed optimization problem (2.12) indicates the necessary condition

$$\frac{\partial H}{\partial p_i}(P, \rho, t) = 0 \text{ and } \frac{\partial H}{\partial \varrho_i}(P, \rho, t) = 0, \quad i \in \mathcal{I}. \tag{3.13}$$

Hence, the gradient of H with respect to the agent position p_i and task assignment variable ϱ_i are next derived in detail.

Since the construction of Voronoi cells V_i depends on the positions of agent p_i and its Voronoi neighbors, an extended form of the Leibniz integral rule for differentiating under the integral sign has to be applied.

Lemma 3.1 (Leibniz's Rule [27]). *Let $\mathcal{Q}(p)$ be a region whose boundary depends on p smoothly, such that the unit outward normal vector $n(p)$ is uniquely defined almost everywhere* *on the boundary $\partial\mathcal{Q}(p)$. Define $\phi(p, x)$ as a smooth function of p, $x \in \mathcal{Q}(p)$, and let $\Phi = \int_{\mathcal{Q}(p)} \phi(p, x)dx$. Then,*

$$\frac{\partial \Phi}{\partial p} = \int_{\mathcal{Q}(p)} \frac{\partial \phi(p, x)}{\partial p} dx + \int_{\partial\mathcal{Q}(p)} \phi(p, x) \frac{\partial x}{\partial p} n(x)dx,$$

where $\frac{\partial x}{\partial p}$ denotes the gradient of boundary point x with respect to p and $\int_{\partial\mathcal{Q}(p)}$ is the line integral over the boundary of $\mathcal{Q}(p)$.

Proof. The proof can be done similarly as in [10] Proposition 2.23, and is omitted here. □

Let ∂Q, ∂V_i be the boundary of Q and V_i respectively, $q_{\partial V_i}(P)$ is a point on ∂V_i and $n_{\partial V_i}$ is the outward facing unit normal of ∂V_i. For the ith agent, define $\mathcal{N}_i$ as the index set of its Voronoi neighbors which share Voronoi boundary with V_i, i.e., $\mathcal{N}_i = \{j \mid \partial V_i \cap \partial V_j \neq \emptyset, i \neq j,\ i, j \in \mathcal{I}\}$. Denote the set of points on the Voronoi boundary shared by agents i and j as $\ell_{ij} = \partial V_i \cap \partial V_j$, then $q_{\ell_{ij}}(p_i, p_j)$ is a point on that shared boundary, and $n_{\ell_{ij}}$ is the unit normal of ℓ_{ij} from p_i to p_j. According to Leibniz integral rule, it has

$$\frac{\partial H}{\partial p_i} = \sum_{i=1}^{N} \int_{V_i} \frac{\partial f(\|q - p_i\|)}{\partial p_i} \varphi_i(\varrho_i, q, t)dq + \sum_{i=1}^{N} \int_{\partial V_i} f(\|q - p_i\|)\varphi_i(\varrho_i, q, t) \frac{\partial q_{\partial V_i}(P)}{\partial p_i} n_{\partial V_i} dq. \tag{3.14}$$

For the first term of Eq. (3.14), since all summands $j \neq i$ are zero with respect to differential movements in p_i, it can be rewritten as

$$\begin{aligned}
\frac{\partial H}{\partial p_i} &= \int_{V_i} \frac{\partial f(\|q - p_i\|)}{\partial p_i} \varphi_i(\varrho_i, q, t)dq + \int_{\partial V_i} f(\|q - p_i\|)\varphi_i(\varrho_i, q, t) \frac{\partial q_{\partial V_i}(P)}{\partial p_i} n_{\partial V_i} dq \\
&\quad + \sum_{j \in \mathcal{N}_i} \int_{\ell_{ij}} f(\|q - p_j\|)\varphi_j(\varrho_j, q, t) \frac{\partial q_{\ell_{ij}}(p_i, p_j)}{\partial p_i} n_{ji} dq \\
&= \int_{V_i} \frac{\partial f(\|q - p_i\|)}{\partial p_i} \varphi_i(\varrho_i, q, t)dq + \int_{\partial V_i \cap \partial Q} f(\|q - p_i\|)\varphi_i(\varrho_i, q, t) \frac{\partial q_{\partial V_i}(P)}{\partial p_i} n_{\partial V_i} dq \\
&\quad + \sum_{j \in \mathcal{N}_i} \int_{\ell_{ij}} \frac{\partial q_{\ell_{ij}}(p_i, p_j)}{\partial p_i} (f(\|q - p_i\|)\varphi_i(\varrho_i, q, t)n_{ij} + f(\|q - p_j\|)\varphi_j(\varrho_j, q, t)n_{ji})dq
\end{aligned} \tag{3.15}$$

where the derivative $\frac{\partial q}{\partial p_i}, q \in \partial V_i$ of boundary points with respect to p_i is a 2×2 matrix.

Since the boundary of mission area does not change as agents move, it has $\frac{\partial q_{\partial V_i}(P)}{\partial p_i} = O, \forall q \in \partial V_i \cap \partial Q$, where is O is the 2×2 zero matrix. For any Voronoi neighbor $j \in \mathcal{N}_i$ of the ith agent, it has $f(\|q - p_i\|) = f(\|q - p_j\|), q \in \ell_{ij}$ due to the property of Voronoi cells. Beside, the outward facing unit normals point in opposite directions, i.e., $n_{ij} = -n_{ji}$. Hence, the gradient of H can be recast into

$$\frac{\partial H}{\partial p_i} = \int_{V_i} \frac{\partial f(\|q-p_i\|)}{\partial p_i}\varphi_i(\varrho_i, q, t)dq + \sum_{j\in\mathcal{N}_i}\int_{\ell_{ij}}(\varphi_i(\varrho_i, q, t) - \varphi_j(\varrho_j, q, t))f(\|q-p_i\|) \times \frac{\partial q_{\ell_{ij}}(p_i, p_j)}{\partial p_i} n_{ij} dq. \tag{3.16}$$

Since the density distributions of ψ_i and ψ_j are different on each side of the Voronoi boundary ℓ_{ij}, the second term of moving boundaries in Eq. (3.16) cannot be canceled directly. In order to compute the boundary term, the following lemma is needed.

Lemma 3.2. *Consider a pair of agents $i, j \in \mathcal{I}$ which are Voronoi neighbors. For any point q on the shared boundary ℓ_{ij}, let n_{ij}, n_{ji} be the outward unit normals on ℓ_{ij} and define $\frac{\partial q_{\ell_{ij}}}{\partial p_i}$ as the derivative of q with respect to p_i, the following equations hold:*

$$n_{ij} = -n_{ji} = \frac{p_j - p_i}{\|p_j - p_i\|}, \tag{3.17a}$$

$$\frac{\partial q_{\ell_{ij}}}{\partial p_i} n_{ij} = \frac{q - p_i}{\|p_j - p_i\|}. \tag{3.17b}$$

Proof. Based on the property of Voronoi partition, the boundary ℓ_{ij} is just the perpendicular bisector of the line segment between p_i and p_j. Therefore, the outward facing unit normal vector n_{ij} or n_{ji} of any point $q \in \ell_{ij}$ is in parallel with the unit vector of $p_j - p_i$ or $p_i - p_j$, then one can get Eq. (3.17a), i.e, $n_{ij} = -n_{ji} = \frac{p_j - p_i}{\|p_j - p_i\|}$. By the definition of the Voronoi cell, it has $\|q - p_i\|^2 = \|q - p_j\|^2$. Take the gradient of this equation with respect to p_i at both sides, it has

$$\frac{\partial q_{\ell_{ij}}}{\partial p_i}(p_j - p_i) = q - p_i$$

and then divide both sides by $\|p_j - p_i\|$, one has the Eq. (3.17b). □

Then the following lemma about the gradient of H with respect to p_i and ϱ_i can be obtained.

Lemma 3.3. *For the ith mobile agent with Voronoi cell V_i and its Voronoi neighbors $j \in \mathcal{N}_i$, $i, j \in \mathcal{I}$, define $m_a^i, c_a^i, m_t^i, c_t^i$ as the mass and centroid of Voronoi cell V_i based on the density function $\psi_a(q)$ and density function $\psi_t(q, t)$ respectively. Based on the density function $\varphi_i(\varrho_i, q, t)$, define $m_+^i, c_+^i, m_-^i, c_-^i$ as the mass and centroid of the boundary ℓ_{ij} shared with the ith agent's Voronoi neighbors $\mathcal{N}_i$ pointing outward and inward respectively. The gradients of $H(P, \rho, t)$ with respect to p_i and ϱ_i are given by*

$$\frac{\partial H}{\partial p_i} = 2\alpha \tilde{m}_i(p_i - \tilde{c}_i), \tag{3.18a}$$

$$\frac{\partial H}{\partial \varrho_i} = \alpha(m_t^i - m_a^i), \tag{3.18b}$$

and $\tilde{m}_i$ and $\tilde{c}$ are the generalized mass and centroid of Voronoi cell V_i defined as

$$\tilde{m}_i = \varrho_i m_t^i + (1 - \varrho_i)m_a^i + \frac{1}{2}(m_+^i - m_-^i), \tag{3.19a}$$

$$\tilde{c}_i = \frac{1}{\tilde{m}_i}\left[\varrho_i m_t^i c_t^i + (1 - \varrho_i)m_a^i c_a^i + \frac{1}{2}(m_+^i c_+^i - m_-^i c_-^i)\right], \tag{3.19b}$$

where

$$m_a^i = \int_{V_i} \psi_a(q)dq, \quad c_a^i = \frac{1}{m_a^i}\int_{V_i} q\psi_a(q)dq, \quad m_t^i = \int_{V_i} \psi_t(q,t)dq, c_t^i = \frac{1}{m_t^i}\int_{V_i} q\psi_t(q,t)dq,$$

$$m_+^i = \sum_{j=1}^{N}\int_{\ell_{ij}} \frac{\|q-p_i\|^2}{\|p_j-p_i\|}\varphi_i(\varrho_i,q,t)dq, \quad c_+^i = \frac{1}{m_+^i}\sum_{j=1}^{N}\int_{\ell_{ij}} \frac{\|q-p_i\|^2}{\|p_j-p_i\|}q\varphi_i(\varrho_i,q,t)dq,$$

$$m_-^i = \sum_{j=1}^{N}\int_{\ell_{ij}} \frac{\|q-p_i\|^2}{\|p_j-p_i\|}\varphi_j(\varrho_j,q,t)dq, \quad c_-^i = \frac{1}{m_+^i}\sum_{j=1}^{N}\int_{\ell_{ij}} \frac{\|q-p_i\|^2}{\|p_j-p_i\|}q\varphi_j(\varrho_j,q,t)dq.$$

Proof. According to the Eq. (3.16), the gradient of $H(P, \rho, t)$ with respect to p_i is

$$\frac{\partial H}{\partial p_i} = \int_{V_i} 2\alpha(p_i-q)\varphi_i(\varrho_i,q,t)dq + \sum_{j\in\mathcal{N}_i}\int_{\ell_{ij}} \alpha\|q-p_i\|^2\varphi_i(\varrho_i,q,t)\frac{\partial q_{\ell_{ij}}}{\partial p_i}n_{ij}dq$$
$$-\sum_{j\in\mathcal{N}_i}\int_{\ell_{ij}} \alpha\|q-p_i\|^2\varphi_j(\varrho_j,q,t)\frac{\partial q_{\ell_{ij}}}{\partial p_i}n_{ij}dq, \tag{3.20}$$

Substitute (3.17b) into (3.20), reformulate it with the centroidal definition of Voronoi cell, then it has

$$\frac{\partial H}{\partial p_i} = 2\alpha[\varrho_i m_t^i(p_i-c_t^i) + (1-\varrho_i)m_a^i(p_i-c_a^i)] + \alpha[m_+^i(p_i-c_+^i) - m_-^i(p_i-c_-^i)]. \tag{3.21}$$

Take the derivative of H with respect to ϱ_i, one can get Eq. (3.18b)

$$\frac{\partial H}{\partial \varrho_i} = \int_{V_i} \alpha\|q-p_i\|^2\psi_t(q,t)dq - \int_{V_i} \alpha\|q-p_i\|^2\psi_a(q)dq = \alpha(m_t^i - m_a^i). \tag{3.22}$$

According to the Eq. (3.21), the direction of gradient $\frac{\partial H}{\partial p_i}$ is determined by four weighted vectors imposing on p_i within Voronoi cell V_i: three forces pointing to centroidal position of area coverage, centroidal position of threat tracking, outward centroidal position of the boundary ℓ_{ij}, and one force pointing in the opposite way to inward centroidal position of boundary ℓ_{ij}. Using the centroidal definition of Voronoi cell, it can be combined as follow

$$\tilde{m}_i = \varrho_i m_t^i + (1-\varrho_i)m_a^i + \frac{1}{2}(m_+^i - m_-^i),$$

$$\tilde{c}_i = \frac{1}{\tilde{m}_i}\left[\varrho_i m_t^i c_t^i + (1-\varrho_i)m_a^i c_a^i + \frac{1}{2}(m_+^i c_+^i - m_-^i c_-^i)\right].$$

where $\tilde{m}_i$ and $\tilde{c}_i$ are the generlized mass and centroid of Voronoi cell V_i respectively. Then the gradient $\frac{\partial H}{\partial p_i}$ can be recast into Eq. (3.18a). □

According to Eqs. (3.18a) and (3.18b) in Lemma 3.3, the mixed metric is minimized when

$$p_i = \tilde{c}_i \ \text{ and } \ m_t^i = m_a^i. \tag{3.24}$$

Unfortunately, the solution (3.24) is only local minimum due to the non-convex property of the mixed metric. Global optimization of H is known as a NP-hard problem without polynomial time algorithm [30]. Hence, in this paper, a dynamic coverage control law and a dynamic task assignment control law are designed simultaneously to ensure that each mobile agent can converge to its local minimum configuration.

3.2. Dynamic coverage control law

Due to the time-varying feature of the density function $\varphi_i(\varrho_i, q, t)$, the gradient-based control law

$$v_i = -\frac{\partial H}{\partial p_i}(P, \rho, t) \tag{3.25}$$

under static density function can not guarantee the stability of multi-agent systems (2.1). Taking the time derivative of the mixed metric along the trajectories generated by the control law (3.25), it has

$$\begin{aligned} \frac{dH}{dt} &= \sum_{i=1}^{N}\left(\frac{\partial H}{\partial p_i}\cdot \dot{p}_i + \frac{\partial H_i}{\partial t}\right) \\ &= -\sum_{i=1}^{N} \tilde{m}_i^2 \|\tilde{c}_i - p_i\|^2 + \sum_{i=1}^{N}\int_{V_i} f(\|q-p_i\|)\frac{\partial \varphi_i}{\partial t}(\varrho_i, q, t)dq. \end{aligned} \tag{3.26}$$

When $p_i \to \tilde{c}_i$, the first term of Eq. (3.26) converges to zero eventually. However, it is not the case for the second term due to the fact that the partial derivative $\frac{\partial \varphi_i}{\partial t}(\varrho_i, q, t)$ is dependent on the positions of moving threats. Hence, a modified gradient-based control law should be designed to drive mobile agents to their generalized Voronoi centroids. Intuitively, it can be considered as a time-varying trajectories tracking problem. The objective is to design a control law for agent i, $\forall i \in \mathcal{I}$ to track its optimal trajectory $\tilde{c}_i(t)$ such that

$$\lim_{t\to\infty} \|p_i(t) - \tilde{c}_i(t)\| = 0. \tag{3.27}$$

Define the move-to-centroid error as $e_i(t) = p_i(t) - \tilde{c}_i(t)$. Motivated by the coverage control law proposed in [23] to optimize a time-varying coverage metric, the dynamic coverage control law of each agent i, $\forall i \in \mathcal{I}$ is given by

$$v_i = -\frac{H_{\partial p_i}}{\|H_{\partial p_i}\|^2}(\kappa_c \|e_i(t)\|^2 + H^i_{\partial t}) \tag{3.28}$$

where κ_c is a positive control gain and

$$H_{\partial p_i} = \frac{\partial H}{\partial p_i}(P, \rho, t) = 2\alpha \tilde{m}_i e_i(t), \tag{3.29a}$$

$$H^i_{\partial t} = \frac{\partial H_i}{\partial t}(P, \rho, t) = \int_{V_i} f(\|q-p_i\|)\frac{\partial \varphi_i}{\partial t}(\varrho_i, q, t)dq, \tag{3.29b}$$

It should be noticed that there exist two non-trivial problems when using the above motion control law. One is the singularity problem if $\|H_{\partial p_i}\|$ is sufficiently approaching to zero, and the other is how to compute $H^i_{\partial t}$ in a distributed way.

To avoid the singularity problem, a switching control strategy is given by

$$v_i = -\frac{H_{\partial p_i}}{\|H_{\partial p_i}\|^2}(\kappa_c \|e_i(t)\|^2 + H^i_{\partial t}\mathbf{1}_{\|H_{\partial p_i}\|\geq d}), \tag{3.30}$$

where $\mathbf{1}_{(\cdot)}$ is defined as an indicator function and $d > 0$ is a fixed threshold related to the bound of control input, i.e., $\sup_{t\geq 0}\|v_i\|$. When $\sup_{t\geq 0}\|v_i\|$ is large, decrease threshold d accordingly, and vice versa.

Based on the density function $\psi_t(q, t)$, the term $H^i_{\partial t}$ can be rewritten as

$$H^i_{\partial t} = \alpha \varrho_i \int_{V_i} \|q - p_i\|^2 \frac{\partial \psi_t}{\partial t}(q, t) dq,$$

where

$$\frac{\partial \psi_t}{\partial t} = \sum_{k=1}^{l(t)} [\lambda(t) \dot{s}_k^T (q - s_k) + \dot{\lambda}(t)] \exp\left(-\frac{\|q - s_k\|^2}{2\sigma^2} \right) \tag{3.31}$$

According to the above Eq. (3.31), it is shown that $H^i_{\partial t}$ is determined by the positions and velocities of detected moving threats, which has the following property.

Lemma 3.4. *If the velocities of $l(t)$ detected threats are bounded, then the term $H^i_{\partial t}$ and $\frac{\partial \psi_t}{\partial t}$ are all bounded and will attenuate to zero in a finite time.*

Proof. Since mission area Q is bounded, for each q, $s_k \in Q$, $q - s_k$ and $\exp(-\frac{\|q-s_k\|^2}{2\sigma^2})$ are all bounded. If the velocities $\dot{s}_k$ of moving threats are finite, then $H^i_{\partial t}$ and $\frac{\partial \psi_t}{\partial t}$ are all bounded. In addition, according to the sufficient condition (2.2) of intercepting threats, once a detected threat is intercepted, it will not be considered any longer. Hence, as time goes by, there must exist a certain time instant $t = t_L$ such that all moving threats over the mission area would have been interpreted successfully. It means that the density function $\psi_t(q, t) \equiv 0$ as $t \geq t_L$. As a result, the term $H^i_{\partial t}$ will attenuate to zero in a finite time. Meanwhile, the coverage control law (3.30) will turn into $v_i = -\frac{H_{\partial p_i}}{\|H_{\partial p_i}\|^2} k_c \|e_i(t)\|^2$ as $t \geq t_L$. □

In order to compute the term $H^i_{\partial t}$ within Voronoi cell V_i, each mobile agent should know the number, positions and velocities of detected moving threats in the mission area at time t. This will contradict the distributed feature of mobile agents. The following Algorithm 1 using a flooding communication protocol is given to help mobile agents obtain these values of detected threats in a distributed manner.

Remark 3.1. In practice, Algorithm 1 always implements in a sampled-data method. If the communication delay of broadcasting is much smaller than the sampling period, then the number of detected threats $l_i(t) = l(t), \forall i \in \mathcal{I}$ during a sampling period, due to the non-overlapping property of Voronoi partition.

Meanwhile, computation of $\lambda(t)$ also needs the global information $\int_Q \psi_t(q, t) dq$. Fortunately, using similar flooding broadcasting mechanism for each agent as proposed in Algorithm 1, the normalized term can be given as

$$\lambda_i(t) = \lambda(t) = \left(\sum_{i=1}^N m_t^i \right)^{-1}, \quad \forall i \in \mathcal{I}. \tag{3.32}$$

3.3. Dynamic task assignment control law

According to the optimal condition mentioned in Eq. (3.24), the objective of controlling the task assignment variable ϱ_i is to balance area coverage task and threat tracking task of mobile agents such that $m_a^i = m_t^i, \forall i \in \mathcal{I}$. Hence, the dynamic task assignment control law of each agent is designed as

$$\dot{\varrho}_i = \begin{cases} -\kappa_m \varrho_i (m_a^i - m_t^i), & m_a^i \geq m_t^i; \\ -\kappa_m (1 - \varrho_i)(m_a^i - m_t^i), & m_a^i < m_t^i; \end{cases} \quad \forall i \in \mathcal{I}. \tag{3.33}$$

Algorithm 1 Number and states of detected threats flooding (executed by the ith agent).

Require: The network of mobile agents is connected
Require: Agents have synchronized clocks with which they broadcast during a pre-defined time slot
Require: Communication with Voronoi neighbors $\mathcal{N}_i$

while 1 **do**
  Initialize $l_i(t) = 0$ for the ith agent
  if the ith agent has detected $a_i(t)$ $(a_i(t) \geq 1)$ moving threats within R_d **then**
    for $k = 1$ to $a_i(t)$ **do**
      if $s_k(t) \in V_i$ **then**
        Store the position $s_k(t)$ and velocity $\dot{s}_k(t)$ of the detected threat
        if $s_k(t) \in \partial V_i \cap \partial V_j,\ j \in \mathcal{N}_i$ **then**
          $l_i(t) + \frac{1}{2}$
        else
          Broadcast the states $s_k(t), \dot{s}_k(t)$ of detected threats, and $l_i(t) + 1$
        end if
      end if
    end for
  else
    Broadcast the number $l_i(t)$
    Broadcast messages received from other agents in $\mathcal{N}_i$
    Store the states $s_k(t), \dot{s}_k(t)$ of detected threats from each agent $j \in \mathcal{N}_i$
    $l_i(t) + \sum_{j \in \mathcal{N}_i} l_j(t)$
  end if
end while

Since task assignment variable $\varrho_i \in (0,\ 1)$, the proposed control law (3.33) can guarantee that the value of ϱ_i remains in (0,1). When $m_a^i \geq m_t^i$, it has $\varrho_i \to 0$, $\varphi_i \to \psi_a$. The ith agent tends to move further away from other agents. Conversely, when $m_a^i < m_t^i$, it has $\varrho_i \to 1$, $\varphi_i \to \psi_t$. The ith agent tends to move more closer to detected moving threats. Note that ϱ_i will be approaching to either 0 or 1 finally although its transient state may frequently shift between these two values.

Combining two control laws of dynamic coverage and task assignment, a move-to-centroid algorithm based on Voronoi partition is implemented to address unknown threats neutralization problem. The procedures of Algorithm 2 are given in detail as follow.

Remark 3.2. In the requirement of Algorithm 2, computing Voronoi tessellation in a distributed way is needed. To do so, each agent needs to know at least the relative locations of its Voronoi neighbors to determine the boundaries of its Voronoi cell. Some algorithms have been proposed for the local computation of Voronoi cells. One typical algorithm with distributed asynchronous computation can be found in [9].

3.4. Optimality and asymptotic convergence analysis

In this section, under dynamic coverage control law (3.30) and task assignment control law (3.33), it is shown that local optimality of the mixed metric and asymptotic convergence of

Algorithm 2 Move-to-centroid algorithm for neutralizing unknown threats.

Require: Set of mobile agents $i \in \mathcal{I} = \{1, \ldots, N\}$ in mission area Q provide with:

- Distributed Voronoi tessellation computation
- Knowledge of threat's states flooding algorithm (Algorithm 1) with update period T_s

1: Set initial position p_i of the ith agent, initial density distribution $\psi_a(q) = (\int_Q dq)^{-1}$ and $\psi_t(q,t) = 0$ of mission area Q and initial task assignment variable $\varrho_i = \frac{1}{2}$ at time $t = 0$, $i \in \mathcal{I}$. Give the switching threshold d.
2: Construct the Voronoi partition $V(P)$ generated by mobile agents.
3: Compute the mass m_a^i and m_t^i of Voronoi cell V_i.
4: If $m_a^i \geq m_t^i$, the ith agent adjusts task assignment variable ϱ_i via the control law $\dot{\varrho}_i = -\kappa_m \varrho_i (m_a^i - m_t^i)$.
5: Otherwise, the ith agent adjusts according to the control law $\dot{\varrho}_i = -\kappa_m (1 - \varrho_i)(m_a^i - m_t^i)$.
6: Compute the values of $\|e_i(t)\|$, $\|H_{\partial p_i}\|$ and $H_{\partial t}^i$.
7: If $\|H_{\partial p_i}\| \geq d$, drive the ith agent to its generalized centroid $\tilde{c}_i$ using the control law (3.28).
8: Otherwise, move the ith agent to its generalized centroid $\tilde{c}_i$ via the control law $v_i = -\frac{H_{\partial p_i}}{\|H_{\partial p_i}\|^2} k_c \|e_i(t)\|^2$.
9: Repeat step 2 to 8 until no threats have been detected over mission area Q in a pre-defined period.

mobile agents to their generalized Voronoi centroids can be guaranteed. Before moving on, a key lemma for convergence analysis is introduced.

Lemma 3.5. *The following functions are uniformly continuous:*

$$\eta_1(t) = -\kappa_c \sum_{i=1}^{N} \|p_i - \tilde{c}_i\|^2, \eta_2(t) = -\kappa_m \sum_{i=1}^{N} (m_a^i - m_t^i)^2 \big[\varrho_i \mathbf{1}_{m_a^i \geq m_t^i} + (1 - \varrho_i)\mathbf{1}_{m_a^i < m_t^i}\big],$$

where $\mathbf{1}_{(\cdot)}$ is defined as an indicator function.

Proof. Firstly, $\tilde{c}_i, m_a^i, m_t^i$ are continuous in that Voronoi cells are continuous functions of p_i, which results in η_1 and η_2 are all continuous functions. Since the bounded derivative of a function is sufficient for uniform continuity, one has to prove the derivatives of η_1 and η_2 are bounded.

Note that $p_i, \tilde{c}_i \in V_i \subset Q$, so p_i, $\tilde{m}_i$ and $\tilde{c}_i$ are bounded. Besides, $m_a^i, m_t^i, \varrho_i \in (0, 1)$ are also bounded. Take the derivative of η_1, it has

$$\dot{\eta}_1 = 2\kappa_c \sum_{i=1}^{N} \sum_{j=1}^{N} (p_i - \tilde{c}_i)^T \left(\mathbf{E}\delta_{ij} - \frac{\partial \tilde{c}_i}{\partial p_i}\right) v_j + 2\kappa_c \sum_{i=1}^{N} (p_i - \tilde{c}_i)^T \frac{\partial \tilde{c}_i}{\partial t} \tag{3.34}$$

Where δ_{ij} denotes the Kronecker delta and $\mathbf{E}$ is a 2×2 identity matrix and

$$\begin{aligned} \frac{\partial \tilde{c}_i}{\partial t} = \frac{1}{\tilde{m}_i} \Bigg[& \int_{V_i} \varrho_i \frac{\partial \psi_t}{\partial t} q dq + \sum_{j=1}^{N} \int_{\ell_{ij}} \frac{\|q - p_i\|^2}{\|p_j - p_i\|} \varrho_i \frac{\partial \psi_t}{\partial t} q dq \\ & - \tilde{c}_i \left(\int_{V_i} \varrho_i \frac{\partial \psi_t}{\partial t} dq + \sum_{j=1}^{N} \int_{\ell_{ij}} \frac{\|q - p_i\|^2}{\|p_j - p_i\|} \varrho_i \frac{\partial \psi_t}{\partial t} dq \right) \Bigg]. \end{aligned} \tag{3.35}$$

Since Q is bounded, the term $\mathbf{E}\delta_{ij} - \frac{\partial \tilde{c}_i}{\partial p_i}$ is also bounded, see [24] for more details. According to Eq. (3.28), v_j is bounded as long as $H^j_{\partial t}$ is finite. This has been proved by Lemma 3.4. Hence, the term $\dot{\eta}_1$ is bounded as long as $\frac{\partial \tilde{c}_i}{\partial t}$ is finite. According to Lemma 3.4, $\frac{\partial \tilde{c}_i}{\partial t}$ Eq. (3.35) is finite due to the fact that the term $\frac{\partial \psi_t}{\partial t}$ is bounded and attenuates to zero as $t \to \infty$. Then it concludes that the function η_1 is uniformly continuous.

Take the derivative of η_2, it has

$$\dot{\eta}_2 = 2\kappa_m \sum_{i=1}^{N} (m_a^i - m_t^i)\left[\varrho_i \mathbf{1}_{m_a^i \geq m_t^i} + (1-\varrho_i)\mathbf{1}_{m_a^i < m_t^i}\right] \int_{V_i} \frac{\partial \psi_t}{\partial t} dq. \tag{3.36}$$

Note that $\dot{\eta}_2(t)$ is not differentiable when the condition $m_a^i(t) \geq m_t^i(t)$ switches to $m_a^i(t) < m_t^i(t)$. However, its non-differentiability occurs only at some isolated points in time. Hence, according to Eq. (3.36), the term $\dot{\eta}_2$ is bounded if $\frac{\partial \psi_t}{\partial t}$ is finite. This is the fact due to the proof of Lemma 3.4. Then it concludes that the function η_2 is uniformly continuous. □

Using Barbalat's lemma [28], the optimality of the mixed metric and asymptotic convergence of such control system is guaranteed by the following theorem.

Theorem 3.1. *For a team of mobile agents with dynamical model (2.1), suppose that Assumption 1 holds. If dynamic coverage control law (3.30) and task assignment control law (3.33) are adopted, then*

$$\lim_{t\to\infty} \|p_i(t) - \tilde{c}_i(t)\| = 0, \quad \lim_{t\to\infty} |m_a^i(t) - m_t^i(t)| = 0, \quad \forall i \in \mathcal{I} \tag{3.37}$$

and the mixed area-threat metric $H(P, \rho, t)$ is locally minimized simultaneously.

Proof. Consider a Lyapunov-like function $\mathcal{V} = H(P, \rho, t)$. Obviously, the proposed Lyapunov function $\mathcal{V}$ is lower bounded by zero. Taking the time derivative of $\mathcal{V}$ along the trajectories of the system (2.1) and (2.7) gives

$$\dot{\mathcal{V}} = \sum_{i=1}^{N} \left(\frac{\partial H}{\partial p_i} \cdot \dot{p}_i + \frac{\partial H}{\partial \varrho_i} \dot{\varrho}_i \right) + \frac{\partial H}{\partial t},$$

When the norm of $H_{\partial p_i}$ is larger than the fixed threshold d, under the control law (3.30) and (3.33), it has

$$\begin{aligned}\dot{\mathcal{V}} = \sum_{i=1}^{N} \Bigg(&-\frac{H_{\partial p_i}}{\|H_{\partial p_i}\|^2} (\kappa_c \|p_i - \tilde{c}_i\|^2 + H^i_{\partial t} \mathbf{1}_{\|H_{\partial p_i}\| \geq d}) \cdot \frac{\partial H}{\partial p_i} \\ &- \kappa_m (m_a^i - m_t^i)(\varrho_i \mathbf{1}_{m_a^i \geq m_t^i} + (1-\varrho_i)\mathbf{1}_{m_a^i < m_t^i}) \frac{\partial H}{\partial \varrho_i} \Bigg) + \frac{\partial H}{\partial t}.\end{aligned} \tag{3.38}$$

Substituting (3.18a) and (3.18b) into the Eq. (3.38), one can get

$$\dot{\mathcal{V}} = -\sum_{i=1}^{N} [\kappa_c \|p_i - \tilde{c}_i\|^2 + \kappa_m (m_a^i - m_t^i)^2 \left(\varrho_i \mathbf{1}_{m_a^i \geq m_t^i} + (1-\varrho_i)\mathbf{1}_{m_a^i < m_t^i}\right)], \tag{3.39}$$

where $\mathbf{1}_{(\cdot)}$ is defined as an indicator function.

Since $\mathcal{V}$ is lower bounded and $\dot{\mathcal{V}} \leq 0$, then $\mathcal{V}$ approaches a limit. Define $\eta_1(t) := -\kappa_c \sum_{i=1}^{N} \|p_i - \tilde{c}_i\|^2$ and $\eta_2(t) := -\kappa_m \sum_{i=1}^{N} (m_a^i - m_t^i)^2 [\varrho_i \mathbf{1}_{m_a^i \geq m_t^i} + (1-\varrho_i)\mathbf{1}_{m_a^i < m_t^i}]$, which gives $\dot{\mathcal{V}} = \eta_1(t) + \eta_2(t)$. In light of Lemma 3.5, $\dot{\mathcal{V}}$ is uniformly continuous. Hence, by Barbalat's lemma, it has

$$\lim_{t\to\infty} \dot{\mathcal{V}} = 0. \tag{3.40}$$

To avoid singularity problem, if the norm of $H_{\partial p_i}$ is smaller than the threshold d, the coverage control law (3.30) will switch into

$$v_i = -\frac{H_{\partial p_i}}{\|H_{\partial p_i}\|^2}\kappa_c\|p_i - \tilde{c}_i\|^2.$$

Substituting the above control law and Eq. (3.18b) into the Eq. (3.38), then it has the same equation with Eq. (3.39) except for the term $\frac{\partial H}{\partial t}$. According to Lemma 3.4, $\frac{\partial H}{\partial t} \to 0$ as $t \to 0$. Then one can get

$$\dot{\mathcal{V}} \to -\sum_{i=1}^{N}[\kappa_c\|p_i - \tilde{c}_i\|^2 + \kappa_m(m_a^i - m_t^i)^2\big(\varrho_i\mathbf{1}_{m_a^i\geq m_t^i} + (1-\varrho_i)\mathbf{1}_{m_a^i<m_t^i}\big)] \text{ as } t \to 0,$$

which means the tendency of mobile agents converging to their Voronoi centroids will not be affected even if the second term of control law (3.30) disappears.

From the Eq. (3.39), the result (3.40) indicates $\lim_{t\to\infty}\|p_i(t) - \tilde{c}_i(t)\| = 0, \forall i \in \mathcal{I}$ for all mobile agents from the first term in the sum. That is, mobile agents converge to their generalized Voronoi centroids. For the second term of Eq. (3.39), the result (3.40) implies

$$\lim_{t\to\infty}(m_a^i - m_t^i)^2\big[\varrho_i\mathbf{1}_{m_a^i\geq m_t^i} + (1-\varrho_i)\mathbf{1}_{m_a^i<m_t^i}\big] = 0. \tag{3.41}$$

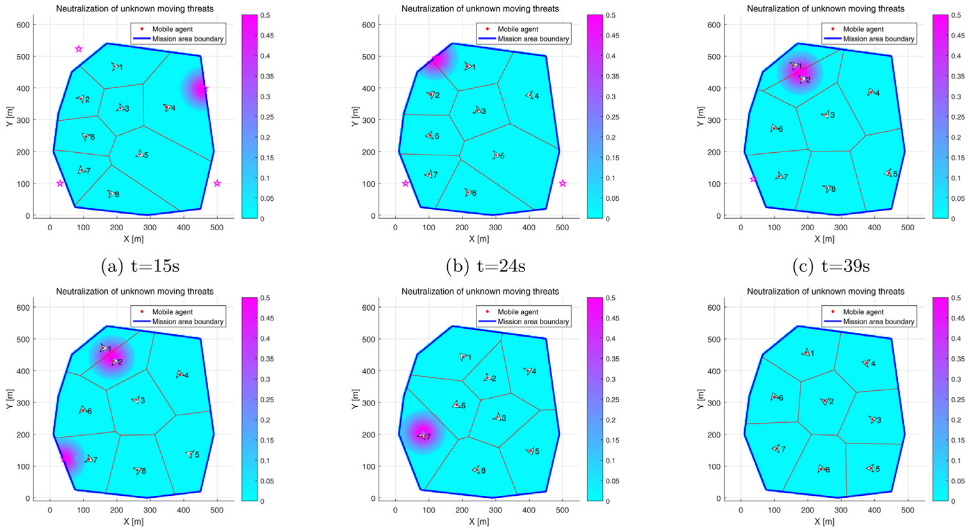
Note that the term $[\varrho_i\mathbf{1}_{m_a^i\geq m_t^i} + (1-\varrho_i)\mathbf{1}_{m_a^i<m_t^i}] \in (0, 1)$ is strictly positive and $(m_a^i - m_t^i)^2$ is non-negative. Therefore, one can get $\lim_{t\to\infty}|m_a^i(t) - m_t^i(t)| = 0, \forall i \in \mathcal{I}$. $\square$

4. Simulation results

In this section, numerical simulation examples are given to validate the effectiveness of our dynamic coverage control method. The area Q is a convex polygonal with 9 boundary vertices at (75, 25), (290, 0), (450, 20), (490, 200), (450, 500), (170, 540), (65, 450), (25, 320) and (10, 200)m and the discretization step of this irregular region is set as $D_x = D_y = 3$m. Consider a scenario with 8 mobile robots and 4 threats during the whole simulation time $t = 150s$. Mobile agents are initially placed at distinct positions determined by $P_0 = \{(90, 440)^T, (130, 392)^T, (170, 343)^T, (210, 295)^T, (190, 246)^T, (164, 198)^T, (137, 149)^T, (110, 100)^T\}$. The parameter of travel cost function $f(\cdot)$ is $\alpha = 0.0001$. Adopting Voronoi partition method, the area Q is divided into N non-empty disjoint cells $V_1, \ldots, V_N$ using the agent positions as the generating points. The configuration of agents and corresponding Voronoi cells within Q are updated at each sampling period $t_s = 1s$. The initial positions of threats lie outside the area Q. The density function $\psi_a(q)$ and $\psi_t(q, t)$ are given as follows.

$$\begin{cases} \psi_a(q) = (\int_Q dq)^{-1}, \ \forall q \in Q; \\ \psi_t^i(q,t) = \frac{1}{\sum_{i=1}^N m_t^i}\sum_{k=1}^{l(t)}\exp\big(-\frac{1}{2\times 35^2}\|q - s_k\|^2\big), \ \forall q \in Q \times \mathbb{R}^+. \end{cases}$$

Without loss of generality, we assume that each threat intrudes into the area from different orientations and moves in a straight line with the same constant velocity $\|\dot{s}_k\| = 5$m/s. Before



(a) t=15s (b) t=24s (c) t=39s

(d) t=41s (e) t=60s (f) t=150s

Fig. 1. Optimal deployment of mobile agents at different time instants with task assignment controller.

the simulation time is over, all threats move out of the mission area. At each sampling time instant, two distinctive colors (pink for lower and green for higher) are used to reflect the density value in different subareas. All simulation examples are implemented in a time period $T = 150s$. The threshold of switching controller (3.30) is $d = 6$ and the control gain is $\kappa_c = 1$. The initial value ϱ_i of switching controller (3.33) is $\varrho_i(0) = 0.5$ and the control gain is $\kappa_m = 0.01$. Set the interception radius of threats as $R_c = 6m, \forall k \in \mathcal{L}$. For each agent, it has two main tasks. One is to distribute over Q for implementing an area coverage task, and the other is to move towards detected threats for implementing a threat tracking task. When the distance between one detected threat and any agent is less than radius R_c, the threat is successfully intercepted.

In the first example, we set the arrival time of moving threats as $t = 15s, 24s, 33s, 41s$ respectively, which means sequentially entering the mission area. Deployment of mobile agents at different time instants under coverage control law (3.30) and task assignment control law (3.33) is shown in Fig. 1. From the figure, it can be observed that the moving threat is intercepted very quickly after it is detected by any agent within Q. When the simulation time is over, all unknown threats are neutralized successfully. Consequently, the density function $\varphi_i(\varrho_i, q, t)$ turn into static density function $\psi_a(q)$ and task assignment variable ϱ_i approach zero on the right side. The trajectories of mobile agents are shown in Fig. 2, from which we can observe that mobile robots (white solid triangles) will ultimately converge to generalized centroidal Voronoi configuration (green diamonds). The error distance between each mobile robot and its generalized Voronoi centroid is presented in Fig. 3, which further verify the convergence of proposed control algorithms.

To further verify the performance of our control method, we use the coverage control algorithms developed in [22,23] with the same parameters to simulate again. The deployment of mobile agents at different time instants under algorithms [22,23] is shown in Fig. 4. From the figure, it can be noticed that two threats are still moving within the mission area without

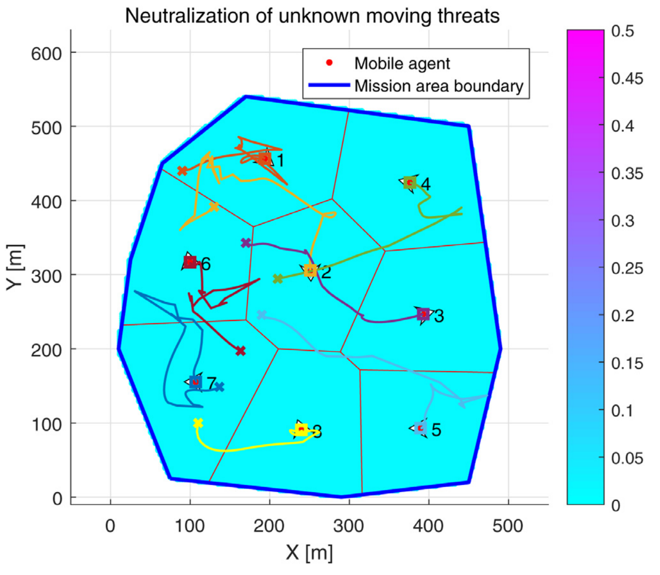


Fig. 2. Trajectories of 8 mobile agents within the mission area from $t = 1s$ to $t = 150s$. The paths of mobile agents begin at crosses and end at squares.

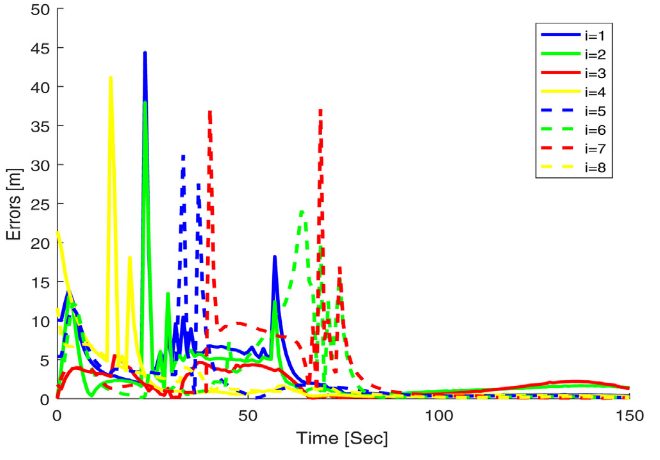
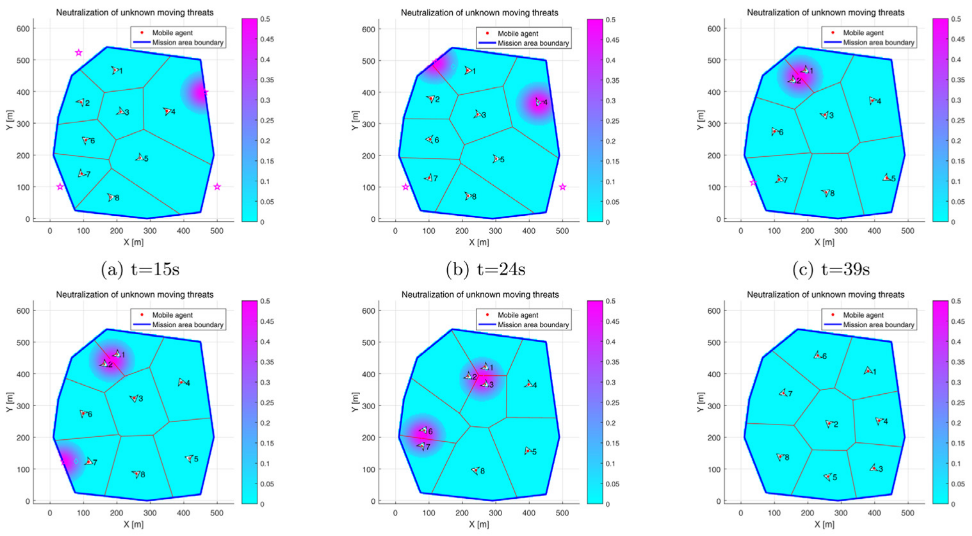


Fig. 3. Norm of position error between the ith mobile agent and its generalized Voronoi centroid.

being neutralized at $t = 60s$. Meanwhile, Table 1 also verifies this observation. Moreover, compared with control algorithms [22,23] without task assignment mechanism, our control method can take much less total travel cost $H(t)$ and total position error $E(t)$ of mobile agents during the simulation time. This result can be verified obviously in Fig. 5. It shows the effectiveness of our proposed coverage control method.

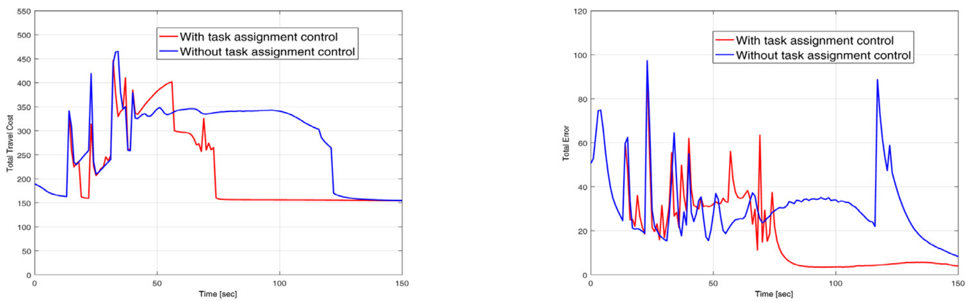
In the second example, we set the arrival time of moving threats as $t = 15s, 16s, 16s, 17s$ respectively which means all threats enter the mission area at very close time and adjacent locations. Deployment of mobile agents at different time instants under coverage control law (3.30) and task assignment control law (3.33) is shown in Fig. 6. From the figure, it can be



(a) t=15s (b) t=24s (c) t=39s

(d) t=41s (e) t=60s (f) t=150s

Fig. 4. Deployment of mobile agents at different time instants without task assignment controller.



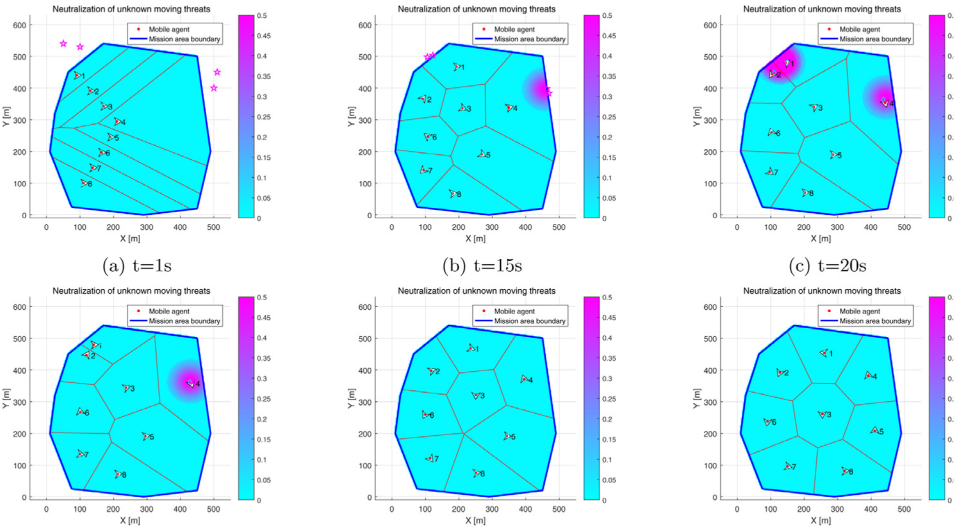
(a) Total travel cost $H(t)$ (b) Total tracking error $E(t)$

Fig. 5. Time evolution of $H(t)$ and $E(t) = \sum_{i=1}^{N} e_i(t)$ for two cases, where red line: with task assignment controller; blue line: without task assignment controller.

Table 1
Performance comparison by two methods for neutralizing unknown threats.

Control method	Detected moving threats (in time sequence of appearance)			
	1st threat	2nd threat	3rd threat	4th threat
Algorithm [22,23]	25s	failed	39s	failed
Proposed Algorithm 2	20s	58s	39s	75s

noticed that the detected threats are intercepted more quickly by mobile agents than in the first example. When the simulation time is over, all unknown threats are neutralized successfully. The error distance between each mobile robot and its generalized Voronoi centroid is presented in Fig. 7, which further verify the convergence of proposed control algorithms.



(a) t=1s (b) t=15s (c) t=20s

(d) t=23s (e) t=40s (f) t=150s

Fig. 6. Optimal deployment of mobile agents at different time instants with task assignment controller.

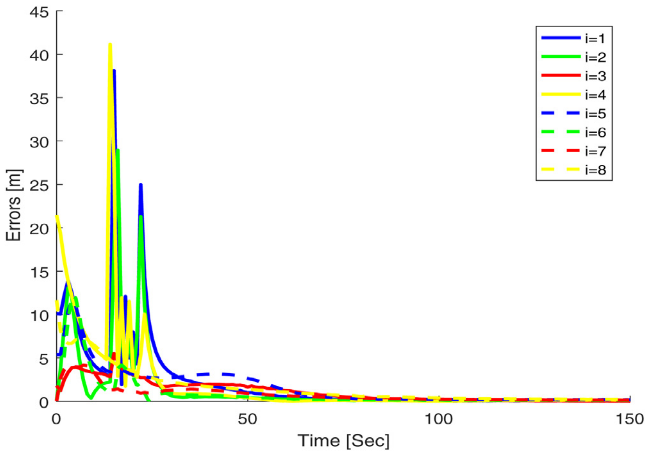
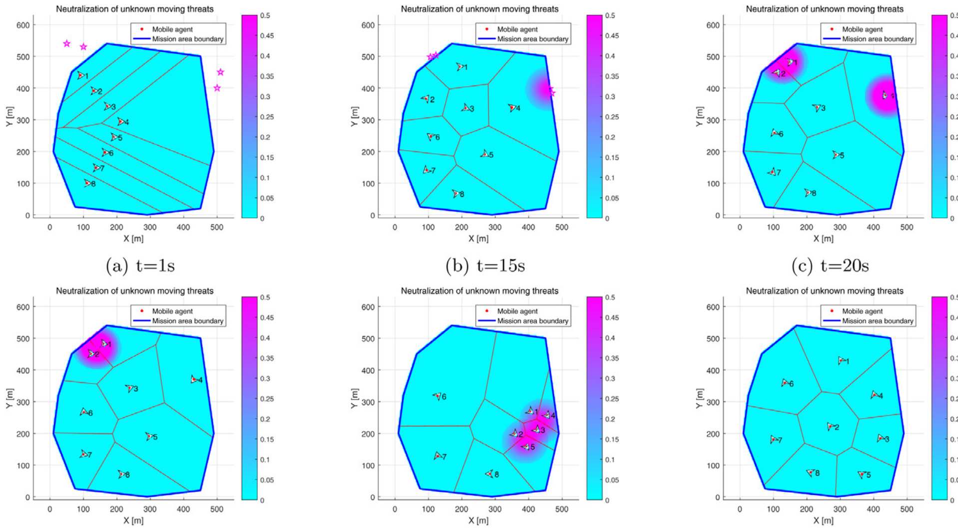


Fig. 7. Norm of position error between the ith mobile agent and its generalized Voronoi centroid.

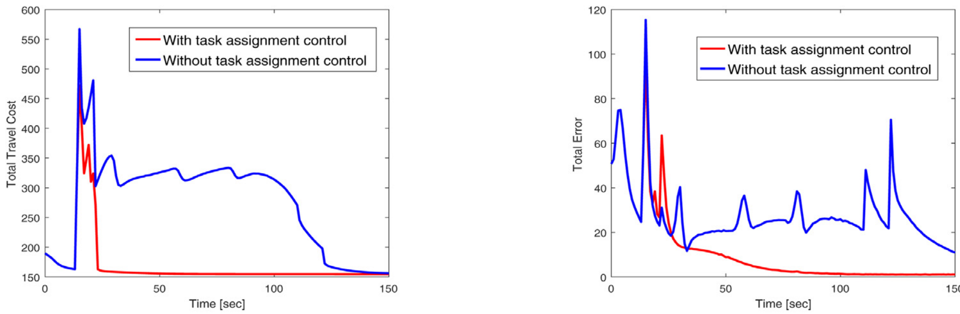
Then we use the coverage control algorithms in [22,23] with the same parameters to simulate again. The deployment of mobile agents at different time instants under algorithms [22,23] is shown in Fig. 8. From the figure, it can be noticed that two threats are still moving within the mission area without being neutralized at $t = 70s$. Meanwhile, Table 2 also verifies this observation. Moreover, from Fig. 9, it shows that our control algorithm takes much less total travel cost $H(t)$ and total position error $E(t)$ of mobile agents during the simulation time and the advantage over other methods is more obvious than in the first example. These results further demonstrate how well the proposed algorithm performs when more than one threat show up in the Voronoi cell of one agent.



(a) t=1s (b) t=15s (c) t=20s

(d) t=23s (e) t=100s (f) t=150s

Fig. 8. Deployment of mobile agents at different time instants without task assignment controller.



(a) Total travel cost $H(t)$ (b) Total tracking error $E(t)$

Fig. 9. Time evolution of $H(t)$ and $E(t)$ for two cases, where red line: with task assignment controller; blue line: without task assignment controller.

Table 2
Performance comparison by two methods for neutralizing unknown threats.

Control method	Detected moving threats (in time sequence of appearance)			
	1st threat	2nd threat	3rd threat	4th threat
Algorithm [22,23]	22s	failed	22s	failed
Proposed Algorithm 2	17s	22s	23s	20s

5. Further discussion and future extensions

In this section, a comprehensive discussion for further real-world applications of the proposed method is presented.

From the results of simulation it can be verified that, our algorithm provides an effective approach to dealing with unknown threats neutralization problem, where a team of pursuit agents can neutralize the invading threats as many as possible. It is equally applicable to neutralizing rogue watercraft in harbors, as well as suspicious persons or vehicles appearing on the protected zone. As mentioned in Section 2, the problem can be divided into three subtasks: environment monitoring, threat tracking and task assignment. Based on Voronoi partition of mission area computed distributively, each agent is just responsible for the region in its own Voronoi cell. Then Voronoi-based coverage control algorithm with dynamic task assignment mechanism is designed to balance threat tracking and environment monitoring tasks in the area. As a consequence, the success ratio of neutralizing unknown moving threats increase noticeably. However, there still exist some limitations and practical difficulties when extending the proposed control algorithms to real-world situations. Open problems to be addressed include the following

- Under our control policy, mobile agents attempt to simultaneously approach all detected threats. For the single threat case, this control policy can guarantee neutralization of the threat as long as its velocity is not too fast. However, in the multi-threat case, this policy cannot guarantee neutralization and has the probability of failing to neutralize the threat. Especially, when two detected threats are just symmetrically around the agent in adjacent locations, then the agent will be caught in a symmetry trap and towards which threat to move becomes a compromised decision. Hence, how to increase the probability of neutralization in the multi-threat case, is one of the open problems that needs to be solved in future research.
- In this paper, with the help of Algorithm 1, where a flooding communication protocol is used for synchronously broadcasting the number and states of detected threats to mobile agents in a distributed manner. As a result, at each time instant, the data of threats already detected are shared among all agents. However, without this sharing mechanism, i.e., each mobile agent computes the number of its detected threats and response asynchronously, it becomes a non-trivial problem. In this case, a global or local assignment mechanism of neutralizing threats needs to be considered.
- The threat considered in this paper cannot adjust its dynamic behavior against the neutralization of mobile agents. A future research avenue for this work is to take into account the role of threat as an intelligent attacker. In this scenario, the threat neutralization problem will be extended to a defense-attack game between mobile agents and unknown threats. This is currently under investigation.

6. Conclusions

In this paper, a distributed coordination scenario of multi-agent systems for neutralizing unknown threats has been investigated. In order to increase the possibility of intercepting threats successfully, a variable of task assignment for each agent is introduced. Based on the variable, the dynamic coverage control problem is reformulated into a multi-objective optimization problem with respect to the total travel cost of all agents. Using the Leibniz differential rule of integral, the condition that the objective function takes the minimum value is obtained, and then the local optimal configuration set of objective function is determined. The motion controller and task assignment controller of the agent are designed respectively by using the state-switching control method, and the stability of the closed loop system is proved. While ensuring that the objective function converges to the minimum value, the designed control algorithm can significantly improve the success ratio of neutralizing unknown threats.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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