Question 1

In [1]:
```
#Load Libraries
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import statsmodels.api as sm
from scipy.optimize import minimize
from scipy import stats
from scipy.stats import t
```

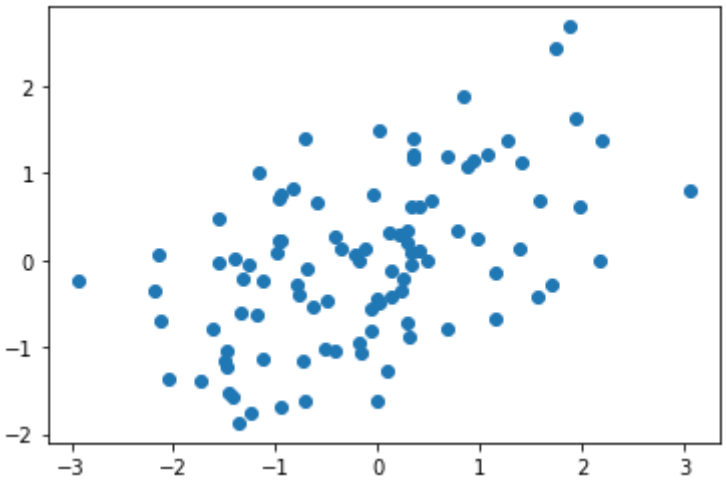
In [2]:
```
# Load data
df_1 = pd.read_csv('problem1.csv')
df_1.describe()
```

Out[2]:

	x	y
count	100.000000	100.000000
mean	-0.140546	-0.022277
std	1.146820	0.948094
min	-2.928514	-1.869396
25%	-0.972205	-0.684961
50%	-0.057900	-0.023872
75%	0.425733	0.625466
max	3.049119	2.683855

In [7]:
```
# Scatter plot visulization
plt.scatter(df_1.x,df_1.y)
```

Out[7]: <matplotlib.collections.PathCollection at 0x231d1cb49a0>



In [12]:
```
# Multivariate Normal
y_mean = df_1.y.mean()
x_mean = df_1.x.mean()
cov = np.cov(df_1.y, df_1.x)
```

```
normal_beta_0 = cov[0,1]/cov[1,1]
normal_beta_1 = y_mean - (cov[0,1]/cov[1,1] * x_mean)
```

In [16]:
```
# OLS
X = sm.add_constant(df_1.x)
model_OLS = sm.OLS(df_1.y, X)
prediction_OLS = model_OLS.fit()

OLS_beta_0 = prediction_OLS.params[1]
OLS_beta_1 = prediction_OLS.params[0]
```

In [17]:
```
print("Multivariate Normal beta_0: ", normal_beta_0)
print("Multivariate Normal beta_1: ", normal_beta_1)

print("OLS beta_0: ", OLS_beta_0)
print("OLS beta_1: ", OLS_beta_1)
```

```
Multivariate Normal beta_0:  0.4280037050563171
Multivariate Normal beta_1:  0.037876861464547947
OLS beta_0:  0.4280037050563169
OLS beta_1:  0.03787686146454787
```

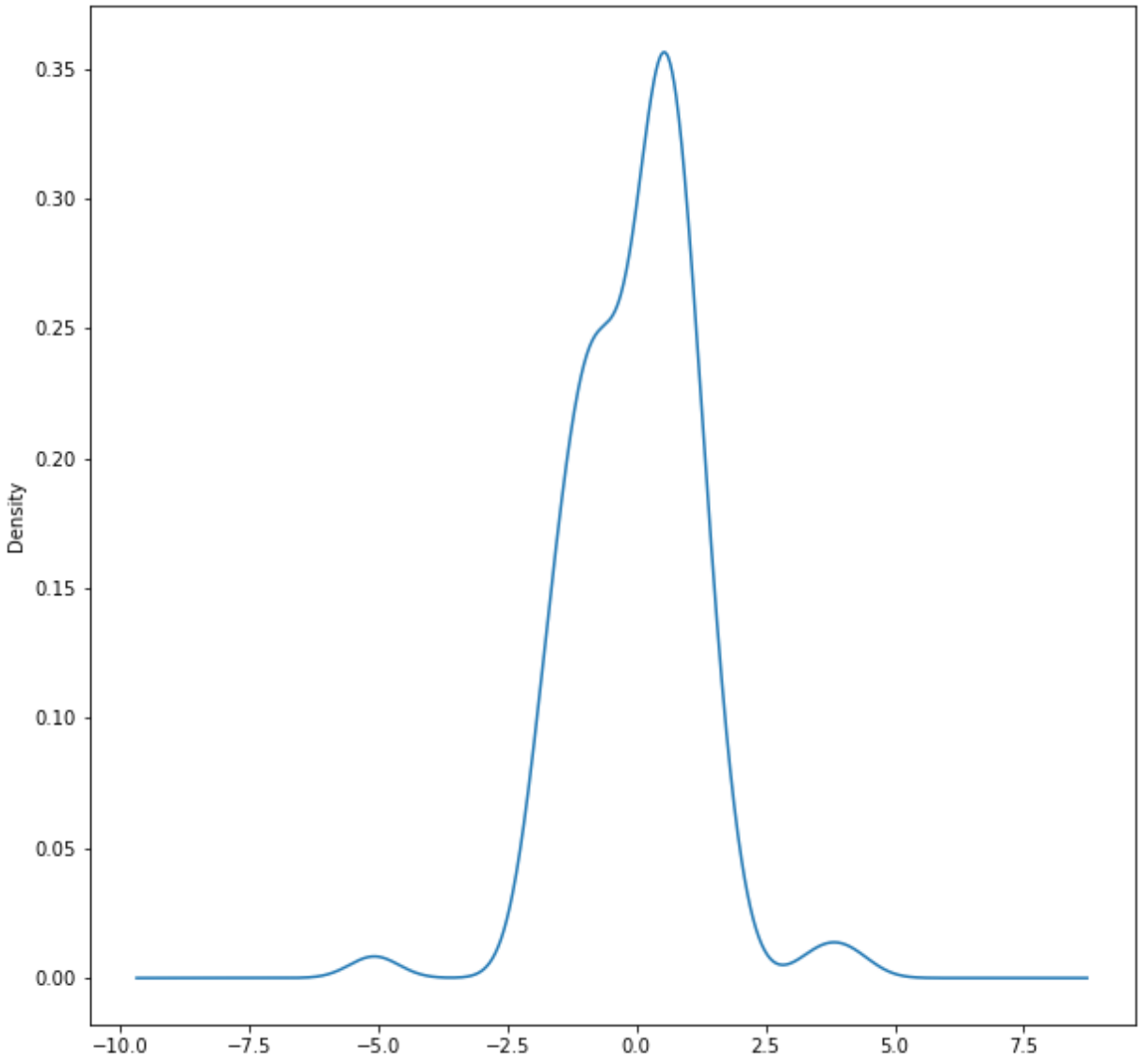
Conditional Distribution of multivariate normal gives the same result as OLS equation.

Question 2

In [3]:
```
# Load Data
df_2 = pd.read_csv('problem2.csv')
```

In [4]:
```
# OLS
X = sm.add_constant(df_2.x)
model_OLS = sm.OLS(df_2.y, X).fit()
prediction_OLS = model_OLS.predict(X)

plt.figure(figsize = (10,10))
error = df_2.y - prediction_OLS
OLS_dist = error.plot.kde(label = "Distribution of OLS")
```



In [5]:
```
def MLE_Norm(parameters):
  const, beta, std_dev = parameters
  pred = const + beta*df_2.x

  LL = np.sum(stats.norm.logpdf(df_2.y, pred, std_dev))
  neg_LL = -1*LL
  return neg_LL
```

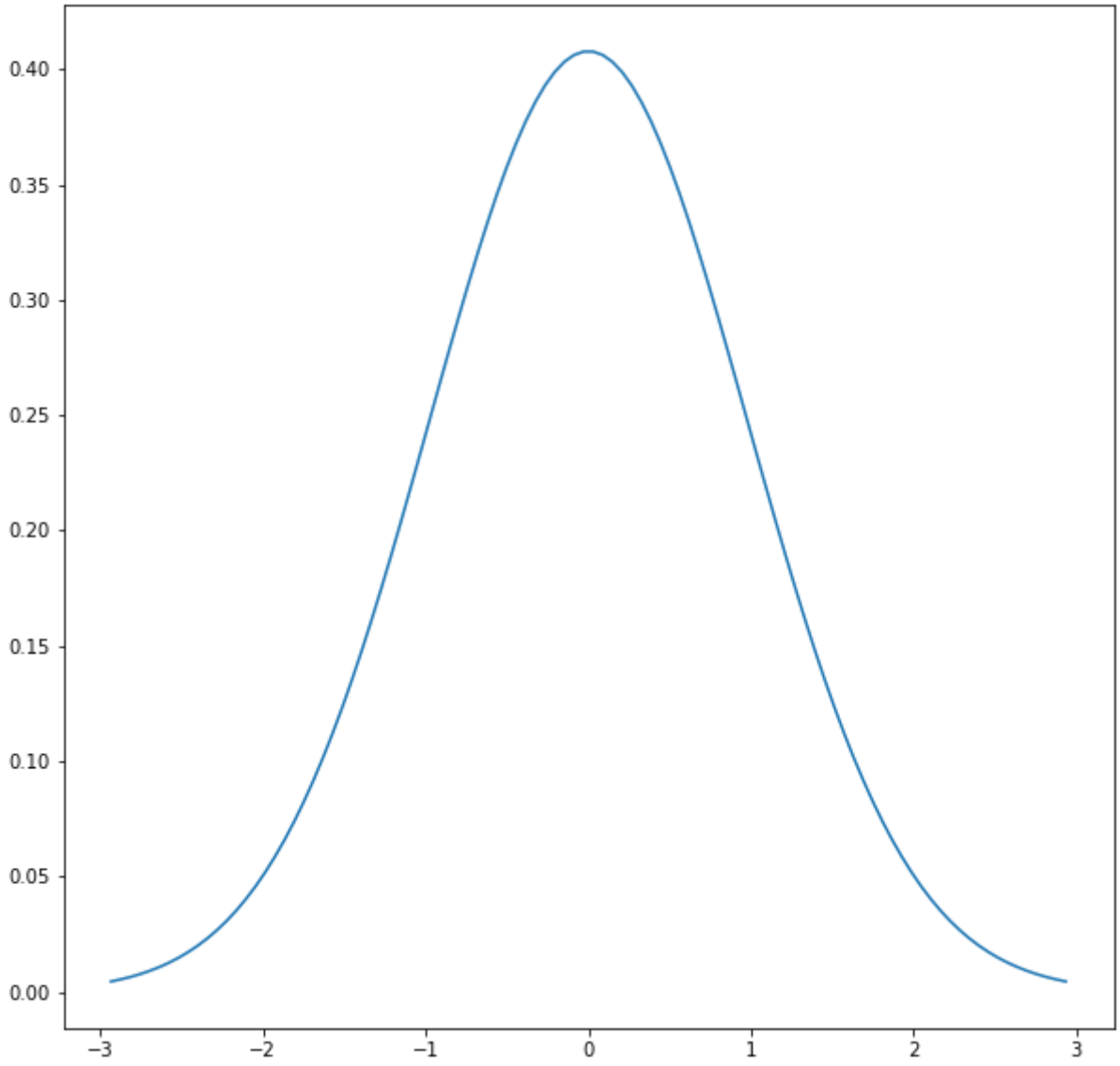
In [6]:
```
model_mle = minimize(MLE_Norm, np.array([1,1,1]), method='L-BFGS-B')
model_mle
```

Out[6]:
```
      fun: 159.99209669026132
hess_inv: <3x3 LbfgsInvHessProduct with dtype=float64>
      jac: array([-1.13686838e-05, -3.58113537e-04,  1.87583283e-04])
  message: 'CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH'
     nfev: 36
      nit: 7
     njev: 9
   status: 0
  success: True
        x: array([0.11983604, 0.6051994 , 1.19839549])
```

In [7]:
```
t.fit(t.rvs(df=99, size=100, loc=0, scale=1))
```

Out[7]: (5375597.558719911, 0.056390734742719925, 1.1855841703943142)

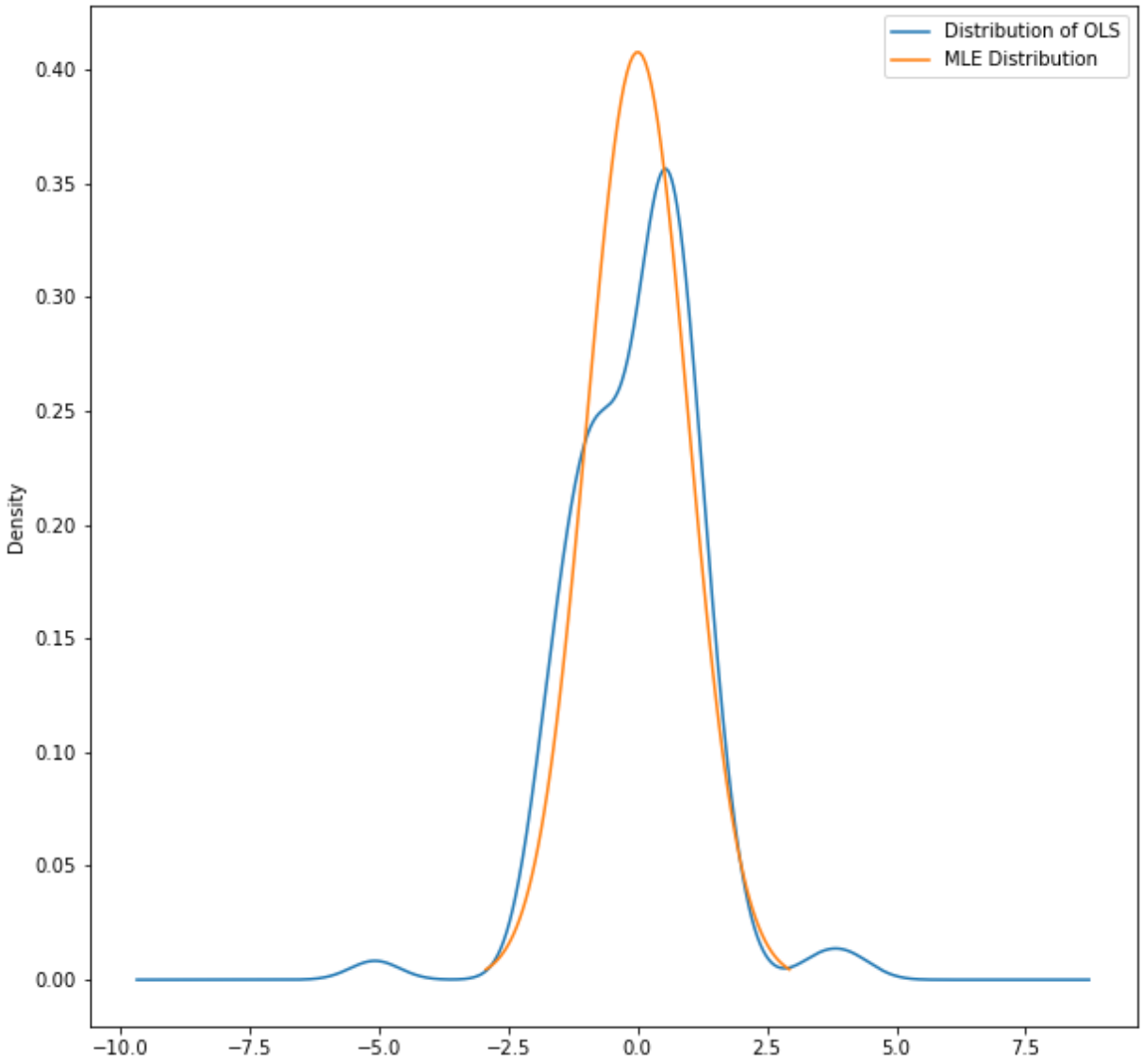
In [9]:
```
plt.figure(figsize = (10,10))
prediction_mu = df_2.x.mean()
prediction_sigma = df_2.x.std()
x_dom = np.linspace(prediction_mu - 3*prediction_sigma, prediction_mu + 3*prediction_
MLE_dist = plt.plot(x_dom, stats.norm.pdf(x_dom, prediction_mu, prediction_sigma), la
```



In [11]:
```
# Combine OLS and MLE
plt.figure(figsize = (10,10))

OLS_dist = error.plot.kde(label = "Distribution of OLS")
MLE_dist = plt.plot(x_dom, stats.norm.pdf(x_dom, prediction_mu, prediction_sigma), la
plt.legend()
```

Out[11]: <matplotlib.legend.Legend at 0x233cd1d96a0>



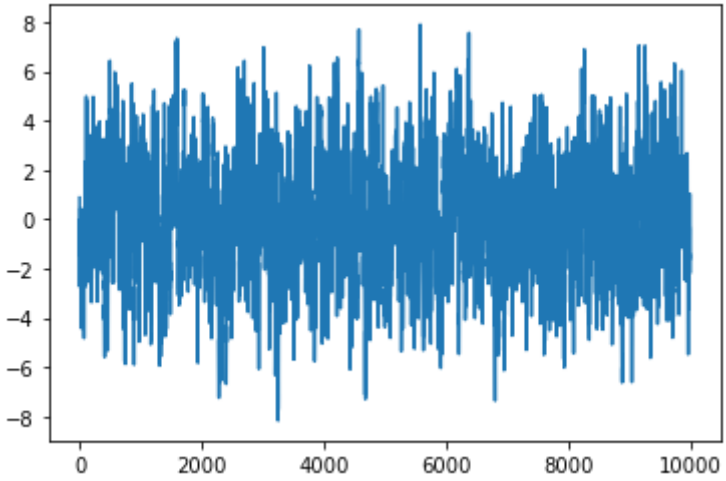
MLE gives the same result as OLS.

Question 3

In [12]:
```
from statsmodels.tsa.arima_process import ArmaProcess
from statsmodels.graphics.tsaplots import plot_acf
from statsmodels.graphics.tsaplots import plot_pacf
```

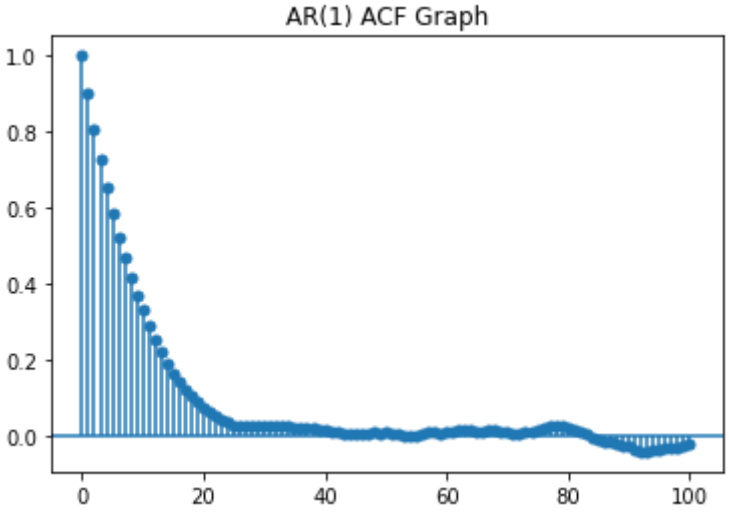
In [25]:
```
#AR(1)
ar1 = np.array([1, -0.9])
ma1 = np.array([1])
AR_1 = ArmaProcess(ar1, ma1)
simulate_1 = AR_1.generate_sample(nsample=10000)
plt.plot(simulate_1)
```

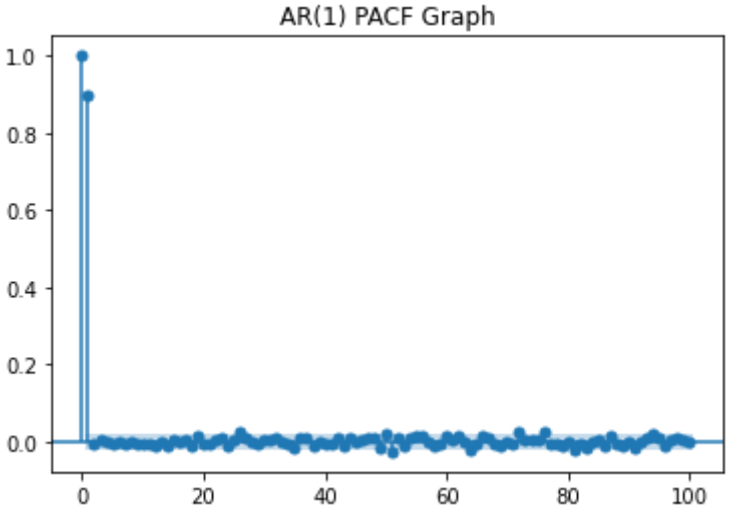
Out[25]: [<matplotlib.lines.Line2D at 0x233cecf2f10>]



In [26]:

```
# Under AR(1), compare ACF and PACF graph.
plot_acf(simulate_1, alpha=1, lags=100)
plt.title("AR(1) ACF Graph")
plt.show()
plot_pacf(simulate_1, lags=100)
plt.title("AR(1) PACF Graph")
plt.show()
```



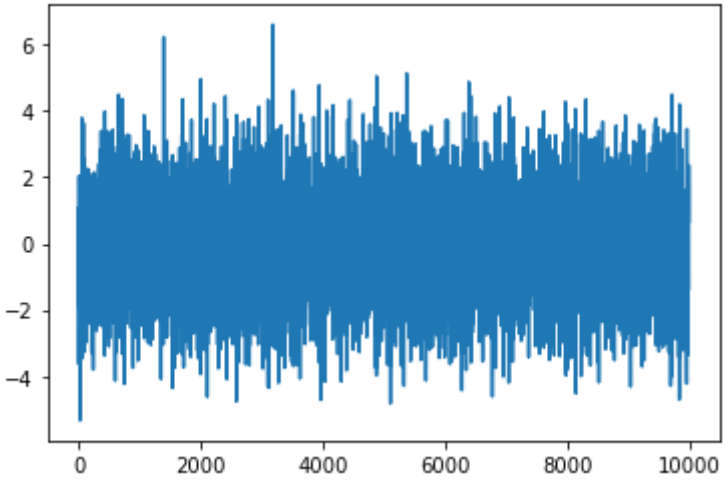


In [27]:

```
#AR(2)
ar2 = np.array([1, -0.9, 0.5])
ma2 = np.array([1])
AR_2 = ArmaProcess(ar2, ma2)
```

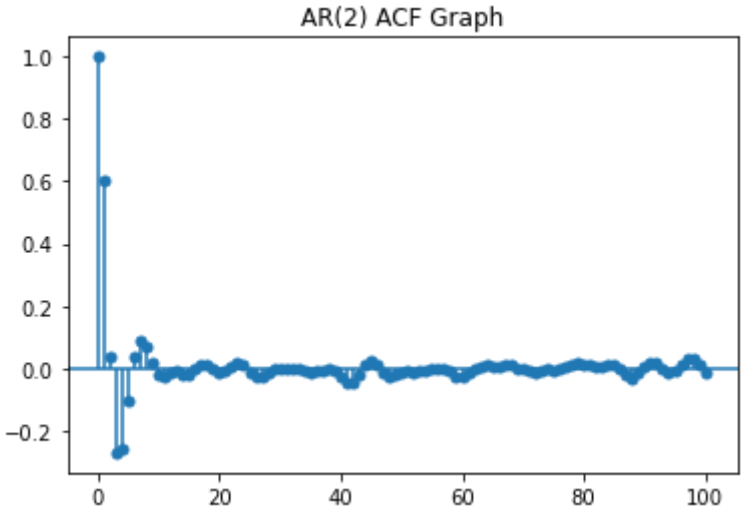
```
simulate_2 = AR_2.generate_sample(nsample=10000)
plt.plot(simulate_2)
```

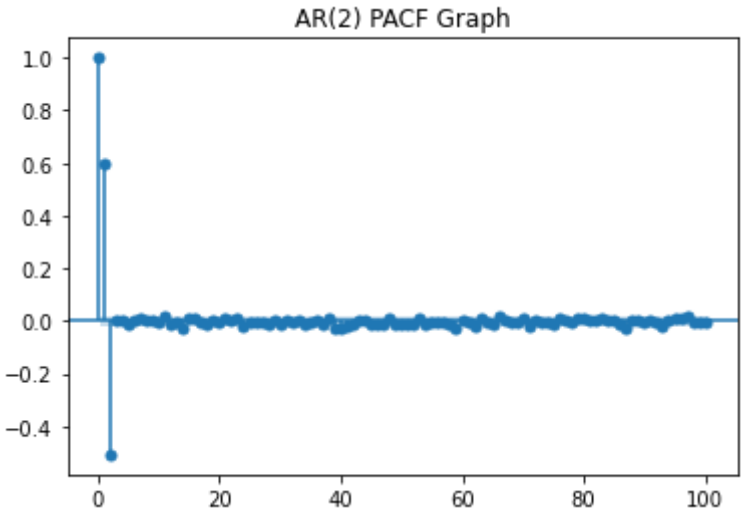
Out[27]: [<matplotlib.lines.Line2D at 0x233cf423580>]



In [28]:

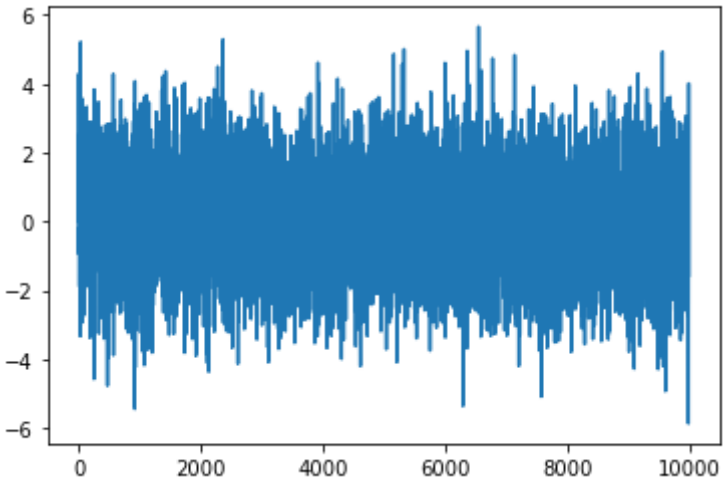
```
# Under AR(2), compare ACF and PACF graph.
plot_acf(simulate_2, alpha=1, lags=100)
plt.title("AR(2) ACF Graph")
plt.show()
plot_pacf(simulate_2, lags=100)
plt.title("AR(2) PACF Graph")
plt.show()
```



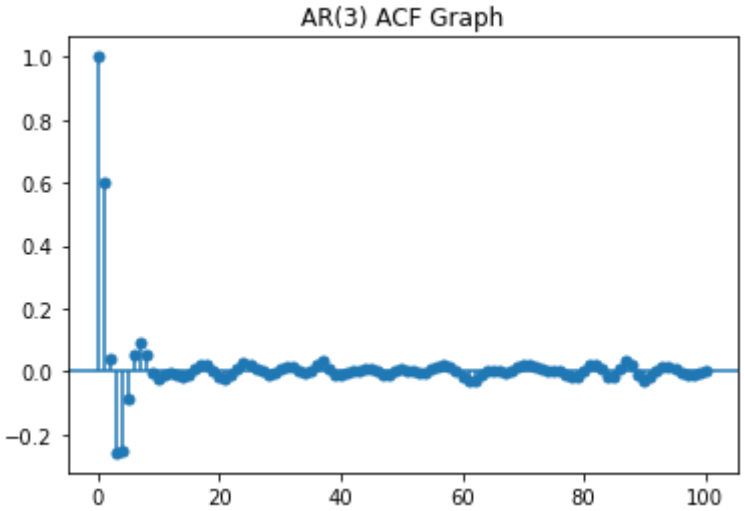


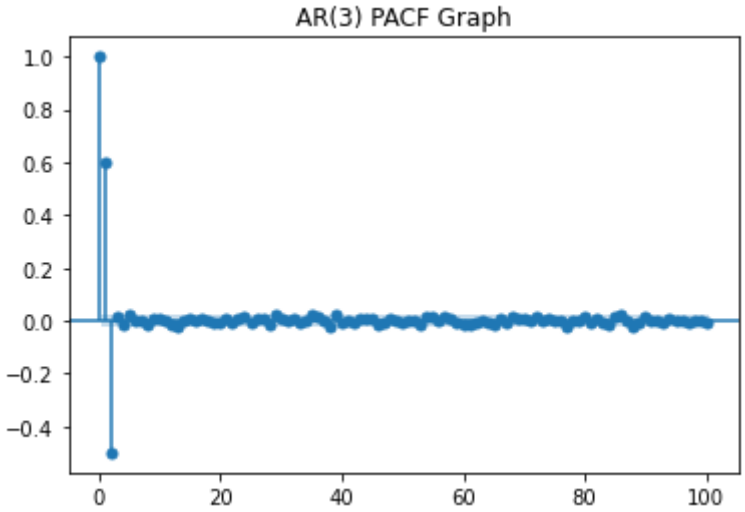
In [29]:
```
#AR(3)
ar3 = np.array([1, -0.9, 0.5,0])
ma3 = np.array([1])
AR_3 = ArmaProcess(ar3, ma3)
simulate_3 = AR_3.generate_sample(nsample=10000)
plt.plot(simulate_3)
```

Out[29]: [<matplotlib.lines.Line2D at 0x233ceebfb50>]



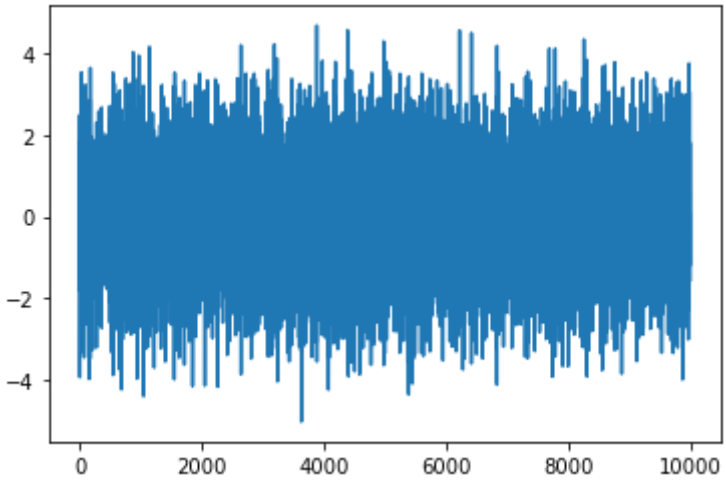
In [30]:
```
# Under AR(3), compare ACF and PACF graph.
plot_acf(simulate_3, alpha=1, lags=100)
plt.title("AR(3) ACF Graph")
plt.show()
plot_pacf(simulate_3, lags=100)
plt.title("AR(3) PACF Graph")
plt.show()
```



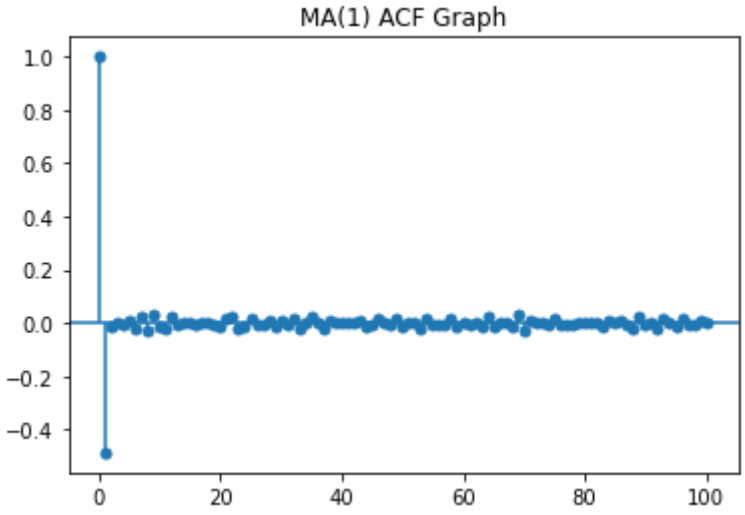


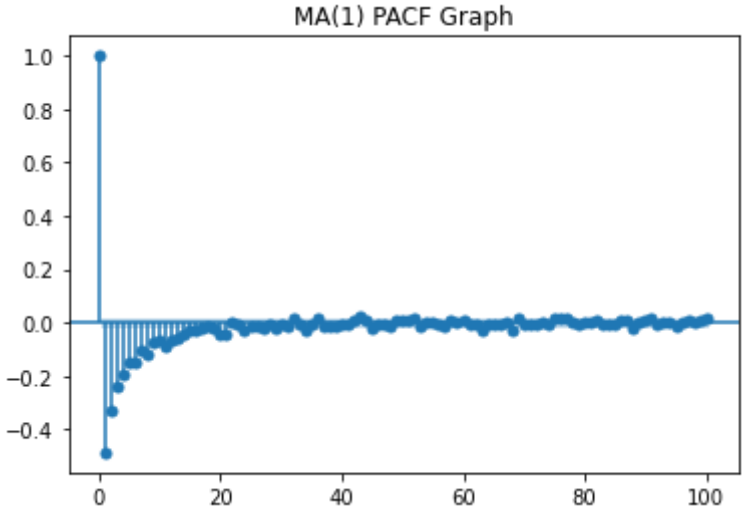
In [31]:
```
#MA(1)
ar1 = np.array([1])
ma1 = np.array([1, -0.9])
MA_1 = ArmaProcess(ar1, ma1)
simulate_ma_1 = MA_1.generate_sample(nsample=10000)
plt.plot(simulate_ma_1)
```

Out[31]: [<matplotlib.lines.Line2D at 0x233cd070130>]



In [32]:
```
# Under MA(1), compare ACF and PACF graph.
plot_acf(simulate_ma_1, alpha=1, lags=100)
plt.title("MA(1) ACF Graph")
plt.show()
plot_pacf(simulate_ma_1, lags=100)
plt.title("MA(1) PACF Graph")
plt.show()
```

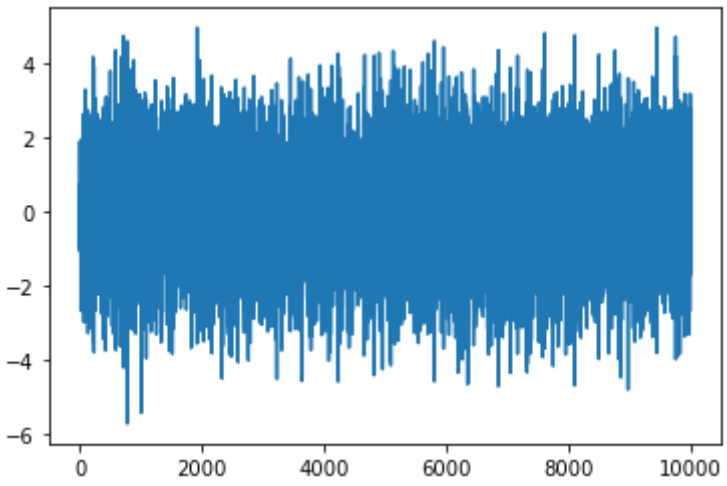




In [35]:

```
#MA(2)
ar2 = np.array([1])
ma2 = np.array([1, -0.9, 0.5])
MA_2 = ArmaProcess(ar2, ma2)
simulate_ma_2 = MA_2.generate_sample(nsample=10000)
plt.plot(simulate_ma_2)
```

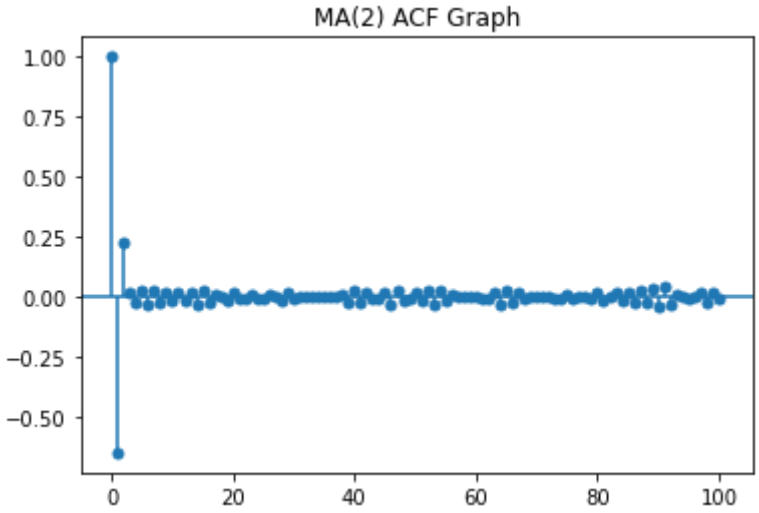
Out[35]: [<matplotlib.lines.Line2D at 0x233d3364f70>]

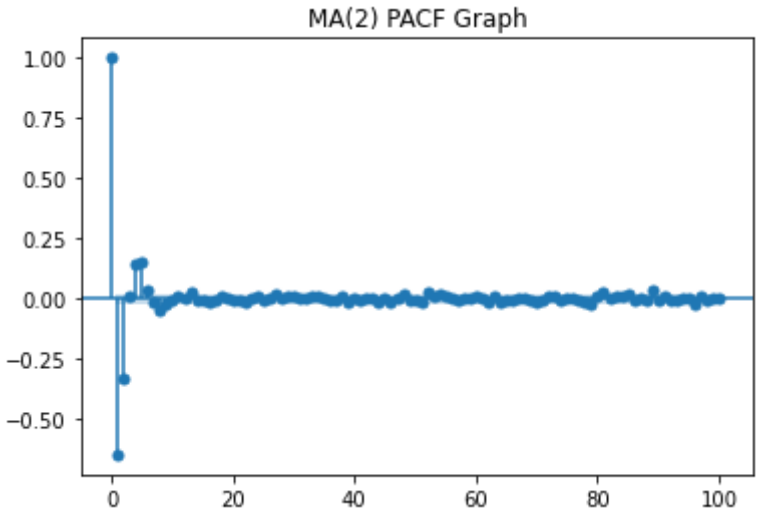


In [36]:

```
# Under MA(2), compare ACF and PACF graph.
plot_acf(simulate_ma_2, alpha=1, lags=100)
plt.title("MA(2) ACF Graph")
```

```
plt.show()
plot_pacf(simulate_ma_2, lags=100)
plt.title("MA(2) PACF Graph")
plt.show()
```

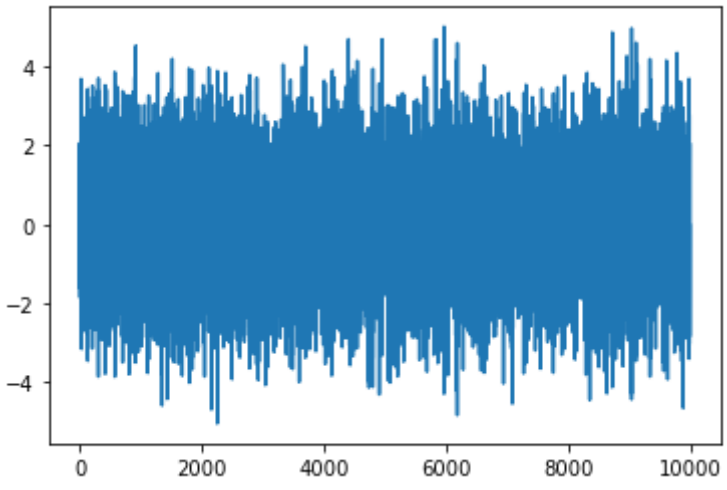




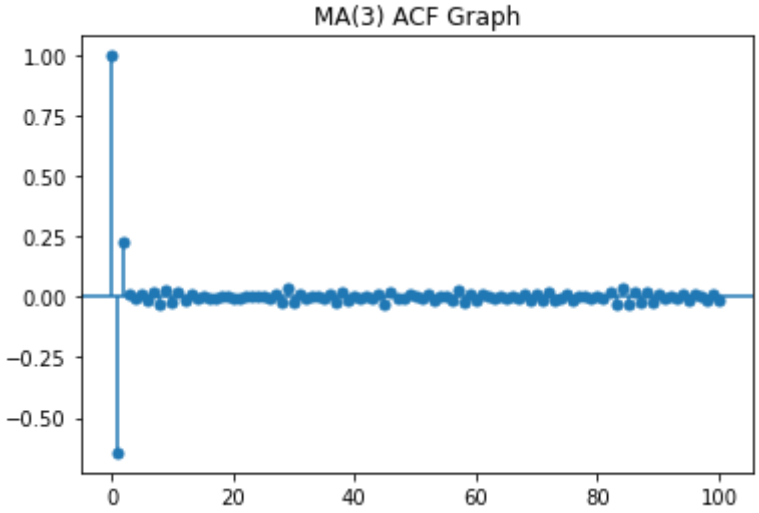
In [37]:

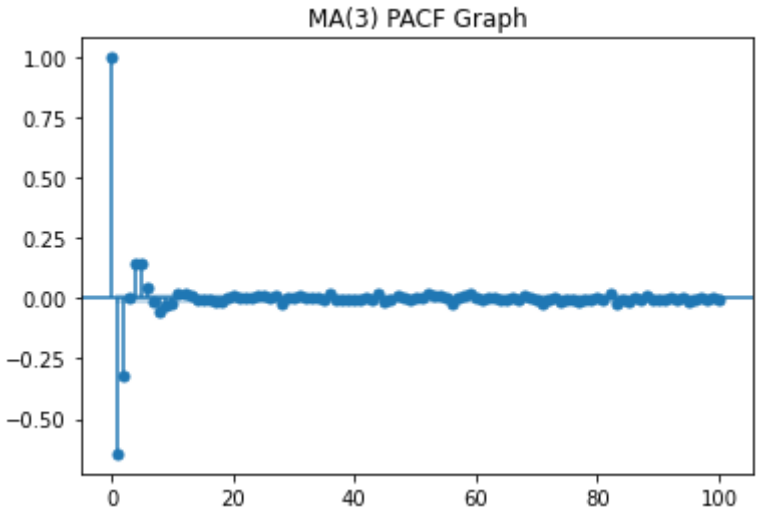
```
#MA(3)
ar3 = np.array([1])
ma3 = np.array([1, -0.9, 0.5, 0])
MA_3 = ArmaProcess(ar3, ma3)
simulate_ma_3 = MA_3.generate_sample(nsample=10000)
plt.plot(simulate_ma_3)
```

Out[37]: [<matplotlib.lines.Line2D at 0x233d48f1b80>]



In [38]:
```
# Under MA(3), compare ACF and PACF graph.
plot_acf(simulate_ma_3, alpha=1, lags=100)
plt.title("MA(3) ACF Graph")
plt.show()
plot_pacf(simulate_ma_3, lags=100)
plt.title("MA(3) PACF Graph")
plt.show()
```





In [ ]: