Question 1

In [1]:

```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt

from numpy import linalg as LA
import time
```

In [2]:

```
# Load the data
df1 = pd.read_csv('DailyReturn.csv')

df1.describe()
```

Out[2]:

	SPY	AAPL	MSFT	AMZN	TSLA	GOOGL	GOOG	FB	I
count	60.000000	60.000000	60.000000	60.000000	60.000000	60.000000	60.000000	60.000000	60.0(
mean	0.000545	0.002621	0.000307	-0.000735	0.004213	-0.000149	-0.000193	-0.000239	0.0(
std	0.008858	0.016045	0.015962	0.016150	0.045304	0.015734	0.015441	0.020170	0.0:
min	-0.022303	-0.039264	-0.042323	-0.028955	-0.119903	-0.045876	-0.046830	-0.050515	-0.0(
25%	-0.003467	-0.005858	-0.007011	-0.012032	-0.028789	-0.006252	-0.006575	-0.011258	-0.0:
50%	0.000877	0.001231	0.000594	-0.001731	0.006190	-0.000209	0.000146	-0.001346	0.0(
75%	0.005470	0.015612	0.009943	0.007864	0.033007	0.008939	0.009319	0.015175	0.0:
max	0.020685	0.035446	0.042114	0.041437	0.135317	0.049595	0.048367	0.040123	0.1:

8 rows × 101 columns

In [3]:

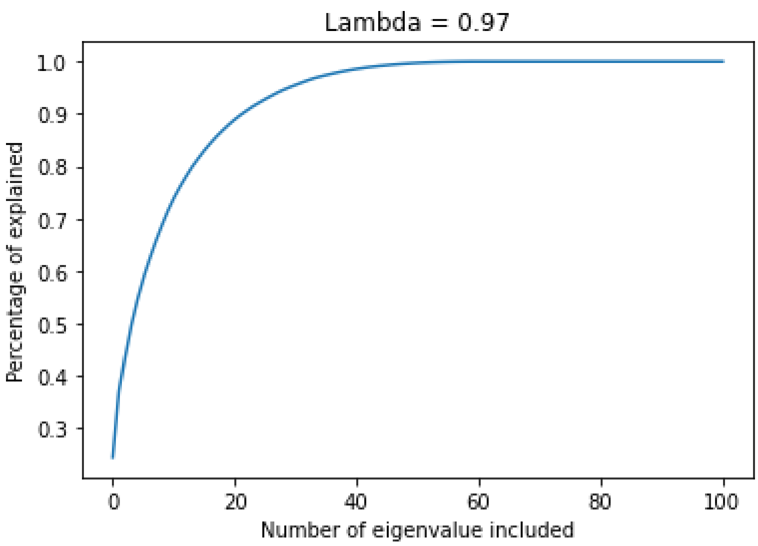
```
# Write the function to get the exponentially weighted covariance matrix

def ewm (dataFrame, expect_weight, lamda):
    weight = []
    sum_weight = 0
    for i in range(1, 61):
        weight.append((1-lamda)*lamda**(i-1))
        sum_weight = sum_weight + weight[i-1]
    for i in range(0,60):
        expect_weight.append(weight[i] / sum_weight)


    cov_matrix = np.zeros([101,101])
    for i in range (0,60):
        for j in range (1,102):
            dataFrame.iloc[i,j] = dataFrame.iloc[i,j] - np.mean(dataFrame.iloc[:,j])


    for i in range (0,101):
        for j in range (0,101):
            #cov_matrix.iloc[i,j] = sum( np.dot( np.dot(expect_weight, dataFrame.iloc[
            temp = expect_weight * dataFrame.iloc[:,i+1]
            cov_matrix[i,j] = np.dot(temp, dataFrame.iloc[:, j+1])
```

```
    return cov_matrix
```

In [4]:

```
# Lambda = 0.97
expect_weight_1 = []
cov_matrix_1 = ewm(df1, expect_weight_1, 0.97)

value_1, vector_1 = LA.eigh(cov_matrix_1)
value_1 = value_1[::-1]

total_1 = 0
for i in range(len(value_1)):
    total_1 = total_1 + value_1[i]

cumu_var_1 = []
k = 1
for i in range(len(value_1)):
    sum_1 = 0
    for j in range(k):
        sum_1 = sum_1 + value_1[j]
    cumu_var_1.append(sum_1 / total_1)
    k = k + 1

fig, ax = plt.subplots()
ax.set_title("Lambda = 0.97")
ax.set_xlabel('Number of eigenvalue included')
ax.set_ylabel('Percentage of explained')
ax.plot(cumu_var_1)
```

Out[4]: [<matplotlib.lines.Line2D at 0x17c07f8d1f0>]

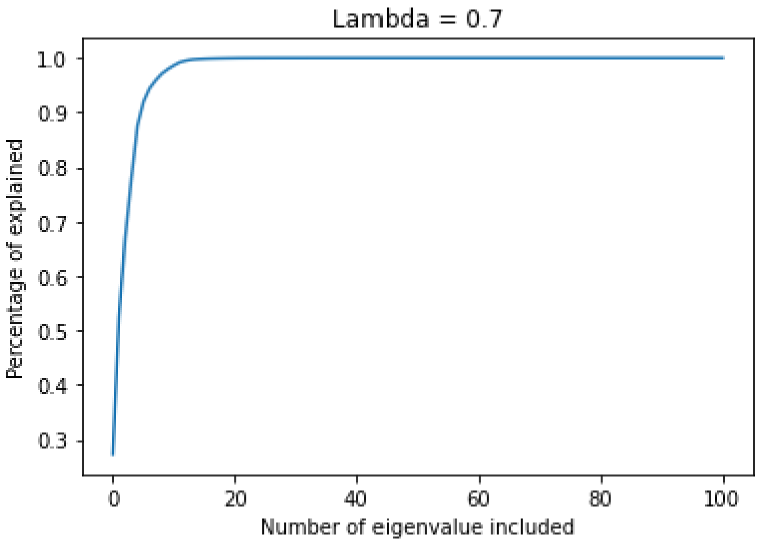


In [5]:

```
# Lambda = 0.7
expect_weight_2 = []
cov_matrix_2 = ewm(df1, expect_weight_2, 0.7)

value_2, vector_2 = LA.eigh(cov_matrix_2)
value_2 = value_2[::-1]

total_2 = 0
for i in range(len(value_2)):
    total_2 = total_2 + value_2[i]

cumu_var_2 = []
k = 1
```

```
for i in range(len(value_2)):
    sum_2 = 0
    for j in range(k):
        sum_2 = sum_2 + value_2[j]
    cumu_var_2.append(sum_2 / total_2)
    k = k + 1

fig, ax = plt.subplots()
ax.set_title("Lambda = 0.7")
ax.set_xlabel('Number of eigenvalue included')
ax.set_ylabel('Percentage of explained')
ax.plot(cumu_var_2)
```

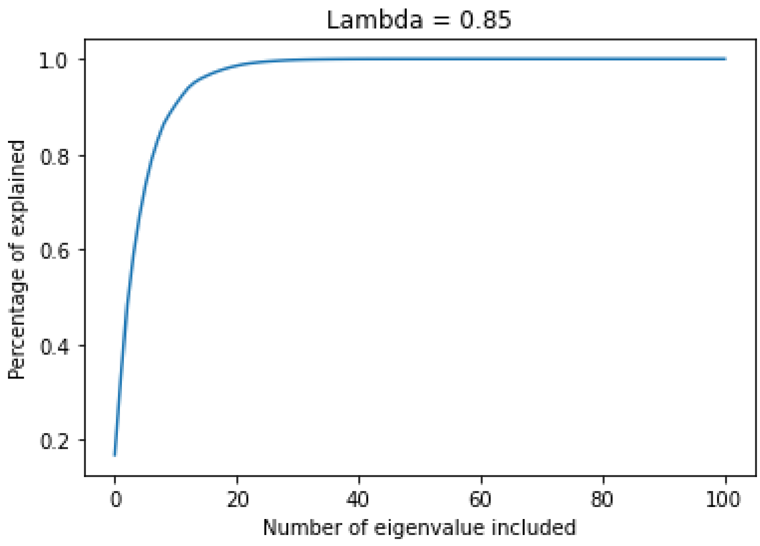
Out[5]: [<matplotlib.lines.Line2D at 0x17c0a8b6550>]



In [6]:

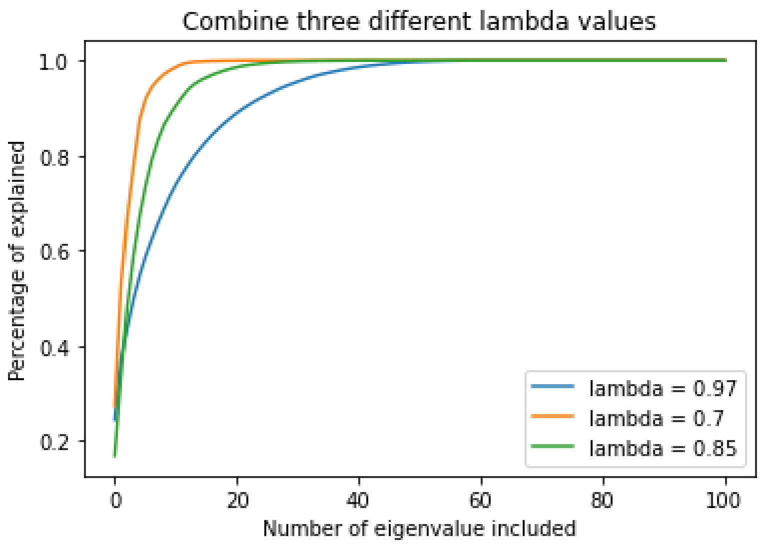
```
# Lambda = 0.85
expect_weight_3 = []
cov_matrix_3 = ewm(df1, expect_weight_3, 0.85)

value_3, vector_3 = LA.eigh(cov_matrix_3)
value_3 = value_3[::-1]

total_3 = 0
for i in range(len(value_3)):
    total_3 = total_3 + value_3[i]

cumu_var_3 = []
k = 1
for i in range(len(value_3)):
    sum_3 = 0
    for j in range(k):
        sum_3 = sum_3 + value_3[j]
    cumu_var_3.append(sum_3 / total_3)
    k = k + 1

fig, ax = plt.subplots()
ax.set_title("Lambda = 0.85")
ax.set_xlabel('Number of eigenvalue included')
ax.set_ylabel('Percentage of explained')
ax.plot(cumu_var_3)
```

Out[6]: [<matplotlib.lines.Line2D at 0x17c0a915ca0>]



In [7]:
```
fig, ax = plt.subplots()
ax.set_title("Combine three different lambda values")
ax.set_xlabel('Number of eigenvalue included')
ax.set_ylabel('Percentage of explained')

ax.plot(cumu_var_1, label = 'lambda = 0.97')
ax.plot(cumu_var_2, label = 'lambda = 0.7')
ax.plot(cumu_var_3, label = 'lambda = 0.85')

ax.legend()
```

Out[7]: <matplotlib.legend.Legend at 0x17c0a8e68e0>



Question 2

In [8]:
```
# non-psd correlation matrix that is 500x500
n = 500
sigma = np.full((n,n), 0.9)
for i in range(n):
    sigma[i,i] = 1.0
sigma[1,2] = 0.7357
sigma[2,1] = 0.7357
```

In [9]:

```
# determined whether the matix is psd
def is_psd(x):
    return np.all(np.linalg.eigvals(x) >= -1e-8)
```

In [10]:

```
# Near PSD function
def near_psd(A):
    n = A.shape[0]
    eigval, eigvec = LA.eigh(A)
    val = np.matrix(np.maximum(eigval, 0))
    vec = np.matrix(eigvec)

    T = 1/(np.multiply(vec,vec) * val.T)
    T = np.matrix(np.sqrt(np.diag(np.array(T).reshape((n)) ))) #already take the squa
    B = T * vec * np.diag(np.array(np.sqrt(val)).reshape((n)))
    out = B * B.T

    return out
```

In [11]:

```
# Higham PSD function

def _getAplus(A):
    eigval, eigvec = LA.eigh(A)
    Q = np.matrix(eigvec)
    xdiag = np.matrix(np.diag(np.maximum(eigval, 0)))
    return eigvec @ xdiag @ eigvec.T

def _getPs(A, W=None):
    W05 = np.matrix(W**.5)
    iW = W05.I
    return  iW @ _getAplus(W05 @ A @ W05) @ iW

def _getPu(A, W=None):
    Aret = A.copy()
    for i in range(0, A.shape[0]):
        Aret[i,i] = 1
    return Aret

def _wgtNorm(A, W = None):
    W05 = np.sqrt(W)
    W05 = W05 @ A @ W05
    return (W05 * W05).sum()

def higham_nearestPSD(pc, W = None, epsilon = 1e-9, maxIter = 100, tol = 1e-9):
    n = pc.shape[0]
    if W == None:
        W = np.identity(n)

    deltaS = np.zeros((n,n))

    Yk = pc.copy()
    norml = 9999999
    i = 1

    while i <= maxIter:
        Rk = Yk - deltaS
        Xk = _getPs(Rk, W)
        deltaS = Xk - Rk
        Yk = _getPu(Xk, W)
        norm = _wgtNorm(Yk - pc, W)

        w, v = LA.eigh(Yk)
```

```python
        minEigVal = np.min(w)

        if ((norm - norml) < tol) and (minEigVal > -epsilon):
            break

        norml = norm
        i = i + 1

    if i < maxIter:
        print("Converged in %d iterations.\n" % i)
    else:
        print("Converged failed after %d iterations.\n" % (i-1))

    return Yk
```

In [12]:

```python
# Cholesky PSD Function
def chol_psd(root, a):
    n = a.shape[0]

    for j in range(n):
        s = 0.0
        if j>0:
            s = root[j, :j] @ root[j, :j].T

        temp = a[j, j] - s

        if -1e-8 <= temp <= 0:
            temp = 0.0

        root[j,j] = np.sqrt(temp);

        if root[j,j] == 0.0:
            continue

        ir = 1.0/root[j,j]

        for i in range(j+1, n):
                s = root[i, :j] @ root[j, :j].T
                root[i,j] = (a[i,j] - s) * ir
    return root
```

In [13]:

```python
# Test whether the covariance is PSD after using near_psd function
out_near_psd = near_psd(sigma)
test_near_psd = is_psd(out_near_psd)
if test_near_psd == True:
    print("The covariance martix is PSD by using near PSD method.")
```

```
The covariance martix is PSD by using near PSD method.
```

In [14]:

```python
# Test whether the covariance is PSD after using higham_nearestPSD function
out_higham_nearestPSD = higham_nearestPSD(sigma, W = None, epsilon = 1e-9, maxIter =
test_higham_nearestPSD = is_psd(out_higham_nearestPSD)
if test_higham_nearestPSD == True:
    print("The covariance martix is PSD by using Higham method.")
```

```
Converged in 26 iterations.

The covariance martix is PSD by using Higham method.
```

In [15]:

```python
# Frobenius norm function
```

```python
def F_norm (cov, cov_psd):
    temp = cov - cov_psd
    return LA.norm(temp, 'fro')
```

In [16]:

```python
# Calculate the Frobenius norm for both methods.
near_psd_Fnorm = F_norm(sigma, out_near_psd)
higham_psd_Fnorm = F_norm(sigma, out_higham_nearestPSD)

print("Frobenius Norm of near PSD method is %f" % near_psd_Fnorm)
print("Frobenius Norm of Higham PSD method is %f" %higham_psd_Fnorm)
```

```
Frobenius Norm of near PSD method is 0.627523
Frobenius Norm of Higham PSD method is 0.089648
```

In [17]:

```python
# Calculate time consuming
start1 = time.time()
near_psd(sigma)
end1 = time.time()
near_psd_time = end1 - start1

start2 = time.time()
higham_nearestPSD(sigma, W = None, epsilon = 1e-9, maxIter = 100, tol = 1e-9)
end2 = time.time()
higham_psd_time = end2 - start2

print("Near PSD method time consuming is %f" % near_psd_time)
print("Near Higham method time consuming is %f" % higham_psd_time)
```

```
Converged in 26 iterations.

Near PSD method time consuming is 0.028581
Near Higham method time consuming is 0.938344
```

In [18]:

```python
name = ['nearPSD', 'Higham2002']

norm = []
norm.append(near_psd_Fnorm)
norm.append(higham_psd_Fnorm)

runtime = []
runtime.append(near_psd_time)
runtime.append(higham_psd_time)

resultTable = pd.DataFrame(list(zip(name,norm,runtime)), columns = ['Name','Norm', 'R
resultTable
```

Out[18]:

	Name	Norm	RunTime
0	nearPSD	0.627523	0.028581
1	Higham2002	0.089648	0.938344

Question 3

In [19]:

```python
# Cholesky PSD Function
def chol_psd(root, a):
    n = a.shape[0]

    for j in range(n):
```

```
        s = 0.0
        if j>0:
            s = root[j, :j] @ root[j, :j].T

        temp = a[j, j] - s

        if -1e-8 <= temp <= 0:
            temp = 0.0

        root[j,j] = np.sqrt(temp);

        if root[j,j] == 0.0:
            continue

        ir = 1.0/root[j,j]

        for i in range(j+1, n):
                s = root[i, :j] @ root[j, :j].T
                root[i,j] = (a[i,j] - s) * ir
    return root

# Frobenius norm function
def F_norm (cov, cov_psd):
    temp = cov - cov_psd
    return LA.norm(temp, 'fro')
```

In [20]:

```
def getCor(cov):
    cov_diag = np.diag(cov)
    invSD = np.diag(np.divide(1, np.sqrt(cov_diag)))
    cor = invSD * cov * invSD
    return cor
```

In [21]:

```
# Standard Pearson correlation/variance
sp_corr = np.matrix(df1.corr())
sp_var = df1.var()
```

In [22]:

```
# Expoentially weighted lambda = 0.97
ew_var = np.diag(cov_matrix_1)

cov_matrix_1 = np.matrix(cov_matrix_1)
ew_corr = getCor(cov_matrix_1)
```

Combine these variance and correlation to form 4 different covariance matrix

In [23]:

```
def combineVar(Var, Corr):
    std = np.sqrt(Var)
    n = len(Var)
    new_cov = np.matrix(np.zeros((n,n)))
    for i in range(n):
        for j in range(n):
            new_cov[i,j] = Corr[i,j] * std[i] * std[j]

    return new_cov
```

In [24]:

```
# Combine Standard Peason's Variables and Standard Peason's Correlation
PEARSON = combineVar(sp_var, sp_corr)
```

In [25]:
```
# Combine Standard Peason's Variables and Exponentially Weighted's Correlation
EWMA_COR_PEARSON_STD = combineVar(sp_var, ew_corr)
```

In [26]:
```
# Combine Exponentially Weighted's Variables and Exponentially Weighted's Correlation
EWMA = combineVar(ew_var, ew_corr)
```

In [27]:
```
# Combine Exponentially Weighted's Variables and Standard Peason's Correlation
PEARSON_COR_EWMA_STD = combineVar(ew_var, sp_corr)
```

In [28]:
```
# Direct Simulation Function
def simulateNormal(cov, nsim):
    if(cov.shape[0] != len(cov)):
        raise exception("covariance matrix is not square")

    n = cov.shape[0]
    root = np.zeros(cov.shape)
    root = chol_psd(root, cov)
    np.random.seed(1998)
    z = np.random.normal(size=(n, nsim))
    ans = root @ z
    return ans
```

In [29]:
```
# PCA Simulation Function
def simulate_pca(cov, nsim, target):
    val, vec = LA.eigh(cov)

    tot = sum(val)
    val = val[::-1]
    vec = vec[::-1]

    cumm_val_explained = np.cumsum(val) / tot
    i=0
    for i in range(len(val)):
        if cumm_val_explained[i] < target:
            i += 1
        else:
            break

    val = val[0:i+1]
    vec = vec[:, :i+1]

    B = vec @ np.diag(np.sqrt(val))
    np.random.seed(1998)
    z = np.random.normal(size=(len(val), nsim))
    return B @ z
```

In [30]:
```
# Print Function
matrixType = ["PEARSON", "EWMA_COR_PEARSON_STD", "EWMA", "PEARSON_COR_EWMA_STD"]
simType = ["Full", "PCA=1", "PCA=0.75", "PCA=0.5"]

matrix = []
simulation = []
runtimes = []
norms = []

i = 0
nsim = 25000
```

```
for sim in simType:
    for mat in matrixType:
        global i
        matrix.append(mat)
        simulation.append(sim)
        elapse = 0

        if mat == "PEARSON":
            c = PEARSON
        elif mat == "EWMA_COR_PEARSON_STD":
            c = EWMA_COR_PEARSON_STD
        elif mat == "EWMA":
            c = EWMA
        elif mat == "PEARSON_COR_EWMA_STD":
            c = PEARSON_COR_EWMA_STD

        if sim == 'Full':
            start = time.time()
            s = simulateNormal(c, nsim)
            end = time.time()
            elapse = end - start
        elif sim == 'PCA=1':
            start = time.time()
            s = simulate_pca(c, nsim, 1)
            end = time.time()
            elapse = end - start
        elif sim == 'PCA=0.75':
            start = time.time()
            s = simulate_pca(c, nsim, 0.75)
            end = time.time()
            elapse = end - start
        elif sim == 'PCA=0.5':
            start = time.time()
            s = simulate_pca(c, nsim, 0.5)
            end = time.time()
            elapse = end - start

        covar = np.cov(s)
        runtimes.append(elapse)
        norms.append(F_norm(covar, c))
        i = i + 1


outTable = pd.DataFrame(list(zip(matrix, simulation, norms, runtimes,)), columns = ['
outTable
```

Out[30]:

	Name	Simulation	Norm	RunTime
0	PEARSON	Full	0.000188	0.065420
1	EWMA_COR_PEARSON_STD	Full	0.000194	0.086290
2	EWMA	Full	0.000185	0.086526
3	PEARSON_COR_EWMA_STD	Full	0.000180	0.084338
4	PEARSON	PCA=1	0.016373	0.044749
5	EWMA_COR_PEARSON_STD	PCA=1	0.014891	0.044611
6	EWMA	PCA=1	0.014138	0.046169
7	PEARSON_COR_EWMA_STD	PCA=1	0.015429	0.040044
8	PEARSON	PCA=0.75	0.016434	0.010963

	Name	Simulation	Norm	RunTime
9	EWMA_COR_PEARSON_STD	PCA=0.75	0.014960	0.010965
10	EWMA	PCA=0.75	0.014188	0.012957
11	PEARSON_COR_EWMA_STD	PCA=0.75	0.015492	0.012956
12	PEARSON	PCA=0.5	0.016318	0.007499
13	EWMA_COR_PEARSON_STD	PCA=0.5	0.014772	0.007973
14	EWMA	PCA=0.5	0.014050	0.008970
15	PEARSON_COR_EWMA_STD	PCA=0.5	0.015358	0.007549