第十周信科提高班讲义

1.设$f(x)$二阶可偏导, 且$z=f\left(xy, x+y^2\right)$, 则$\dfrac{\partial^2 z}{\partial x \partial y}=$_____.

2.设$f(x, y)$连续, 且$f(x, y)=3x+4y+6+O(\rho)$, 其中$\rho=\sqrt{(x-1)^2+y^2}$, 则$dz\big|_{(1,0)}=$_____.

3.设$z=f(x, y)$在点$(1,1)$处可微, $f(1,1)=1, f'_1(1,1)=a, f'_2(1,1)=b$, 又
$u=f\left[x, f(x, x)\right]$, 求$\left.\dfrac{du}{dx}\right|_{x=1}$.

4.设$y=y(x), z=z(x)$由$\begin{cases}x^2+y^2=2z\\ x+y-z+1=0\end{cases}$确定, 求$\dfrac{dz}{dx}$.

5.设$z=xyf\left(\dfrac{y}{x}\right)$, 其中$f(u)$可导, 则$x\dfrac{\partial z}{\partial x}+y\dfrac{\partial z}{\partial y}=$_____.

6.设$z=yf\left(x^2-y^2\right)$, 其中f可导, 证明: $\dfrac{1}{x}\dfrac{\partial z}{\partial x}+\dfrac{1}{y}\dfrac{\partial z}{\partial y}=\dfrac{z}{y^2}$.

7.设变换$\begin{cases}u=x-2y\\ v=x+ay\end{cases}$, 可把方程$6\dfrac{\partial^2 z}{\partial x^2}+\dfrac{\partial^2 z}{\partial x \partial y}-\dfrac{\partial^2 z}{\partial y^2}=0$简化为$\dfrac{\partial^2 z}{\partial u \partial v}=0$, 求常数$a$

8.求$u=x^2+y^2+z^2$在$\dfrac{x}{a}+\dfrac{y}{b}+\dfrac{z}{c}=1$上的最小值.