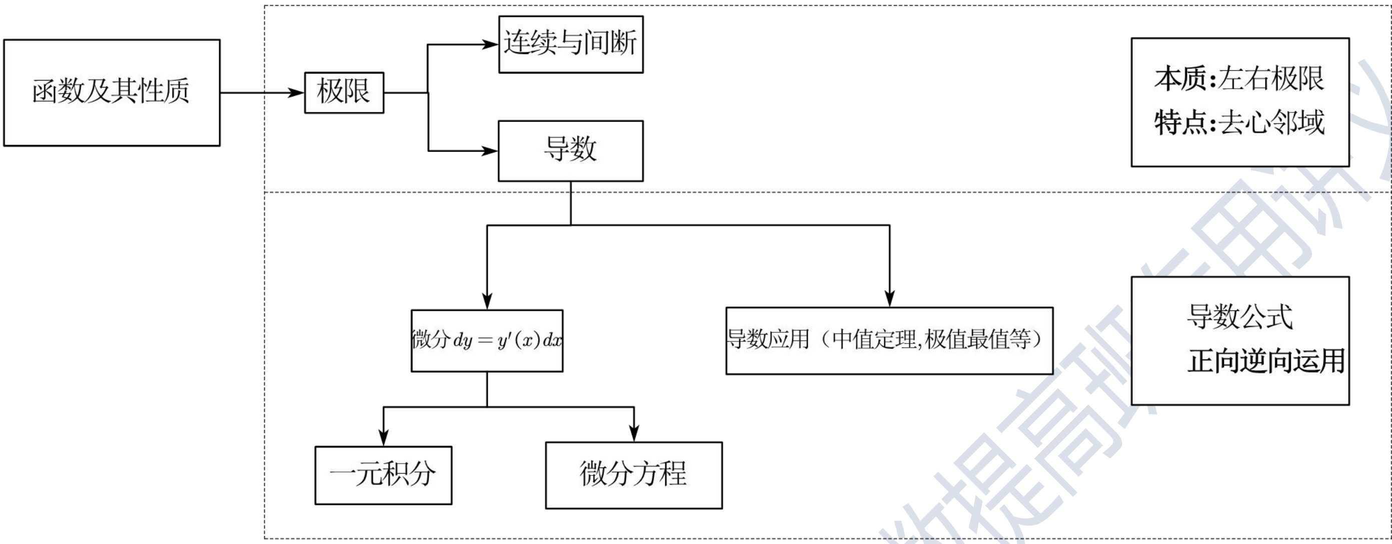
# 一元函数学习架构




高等数学(或微积分)源头：**极限、导数、积分**

学好高数请务必掌握：**极限、导数、积分基本公式和方法，达到倒背如流！**

基本等价无穷小: $x\to 0$ ,$x\sim\sin x\sim\arcsin x\sim\tan x\sim\arctan x\sim\ln(1+x)\sim e^{x}-1$

$$1-\cos x\sim\frac{1}{2}x^{2}\ ;\text{注}\cos x-1\sim-\frac{1}{2}x^{2}$$

$$(1+x)^{a}-1\sim ax$$

$$a^{x}-1\sim x\ln a$$

推广等价无穷小: $\square\to 0$ ,$\square\sim\sin\square\sim\arcsin\square\sim\tan\square\sim\arctan\square\sim\ln(1+\square)\sim e^{\square}-1$

$$1-\cos\square\sim\frac{1}{2}\square^{2}$$

$$(1+\square)^{a}-1\sim a\square$$

$$a^{\square}-1\sim\square\ln a$$

简单应用:极限下列极限

1.求 $\lim\limits_{x\to 0}\dfrac{x\sin x}{(e^{x}-1)\arctan x}$

2.求 $\lim\limits_{x\to 0}\dfrac{x\sin x}{e^{x^{2}}-1}$

3.求 $\lim\limits_{x\to 0}\dfrac{\sin x}{\ln(1+x)}$

4.求 $\lim\limits_{x\to 0}\dfrac{\ln(1+x)}{\ln\left(x+\sqrt{1+x^{2}}\right)}$

5.求 $\lim\limits_{x\to 0}\dfrac{1-\cos x}{x^{2}}$

6.求 $\lim\limits_{x\to 0}\dfrac{\cos^{a}x-1}{x^{2}}$

7.求 $\lim\limits_{x\to 0}\dfrac{\sqrt[4]{\cos x}-1}{x^{2}}$

8.求 $\lim\limits_{x\to 0}\dfrac{\ln\cos x}{x^{2}}$

9.求 $\lim\limits_{x\to 0}\dfrac{1-\cos x\cos 2x\cos 3x}{x^{2}}$

10.求 $\lim\limits_{x\to 1}\dfrac{\left(1-x^{\frac{1}{2}}\right)\left(1-x^{\frac{1}{4}}\right)\cdots\left(1-x^{\frac{1}{2n}}\right)}{(1-x)^{n}}$

11.求$\lim\limits_{x\to\infty}x^2\left(\arctan\dfrac{1}{1+x}-\arctan\dfrac{1}{2+x}\right)$

12.求$\lim\limits_{n\to\infty}\sin\left(\pi\sqrt{(n!)^2-n!}\right)$

13.求$\lim\limits_{x\to0^+}\left(1+e^{\frac{1}{x}}\right)^{\ln(1+x)}$

14.求$\lim\limits_{x\to0}\dfrac{\sin^x x-x^x}{x^3}$

15.设$y+\cos xy+e^{y\sin x}-2=x$确定$y=y(x)$.

计算$\lim\limits_{n\to\infty}n^2\cdot\ln\left[\cos f\left(\dfrac{1}{n}\right)\right]$

16.设$f(x)=\lim\limits_{n\to\infty}\dfrac{2x^{2n-1}+x-2}{x^{2n}+x^2+1}$,求$f(x)$并判断其连续性.

17.设$\alpha>0,\beta\neq0$为常数,且$\lim\limits_{x\to+\infty}\left[(x^{2\alpha}+x^{\alpha})^{\frac{1}{\alpha}}-x^2\right]=\beta$,求$\alpha,\beta$

18.求$\lim\limits_{x\to0}\dfrac{520!\ \ x^{520}-\sin x\cdot\sin(2x)\cdots\sin(520x)}{x^{522}}$