

# Physics I: Introduction to Wave Theory SDU Course Number: sd01232810 (Fall 2024)

# **Lecture 7: Wave Guidance**

#### **Outline**

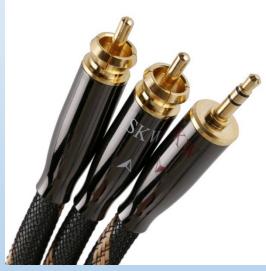
- Guidance by Conducting Parallel Plates
- Rectangular Waveguide
- TEM Mode and Coaxial Cable
- Generic Form of Guided Waves
- Slab Dielectric Waveguide

### **Wave Guidance Devices**

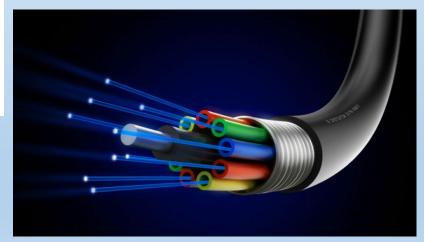


Microwave Waveguides

### **Digital Audio Coaxial cable**



**Optical fiber** 

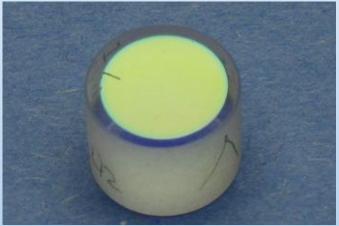


# **Reflection by Mirrors**

#### **METAL REFLECTION**



#### MULTILAYER REFLECTION



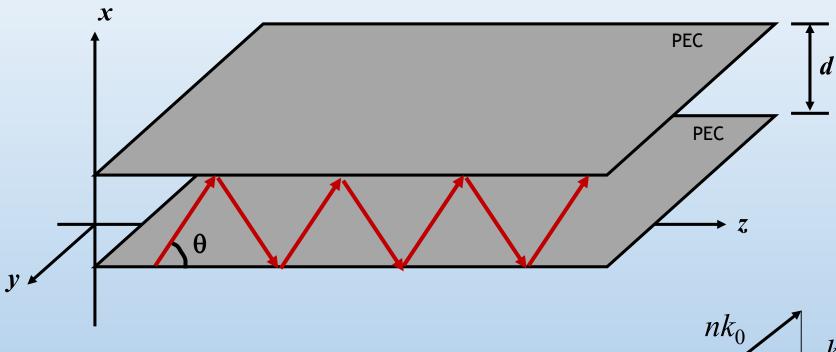
http://en.wikipedia.org/wiki/File:Dielectric \_laser\_mirror\_from\_a\_dye\_laser.JPG

# TOTAL INTERNAL REFLECTION



# **Guidance by Conducting Parallel Plates**

We can transport light along the z-direction by bouncing it between two PECs

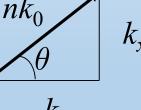


.. the ray moves along both y- and z-axes...

$$E = (A_1 e^{-jk_{\mathcal{X}}x} + A_2 e^{jk_{\mathcal{X}}x})e^{-jk_{\mathcal{Z}}z}$$

...where

$$k_x = nk_0 \sin \theta$$
  $k_z = nk_0 \cos \theta$ 



$$k_z$$

$$\nabla \times \vec{E} = -j\omega \mu \vec{H}$$

$$abla imes \vec{H} = j\omega \varepsilon \vec{E}$$

$$\left\{ \begin{array}{l} \frac{\partial}{\partial y} E_z - \frac{\partial}{\partial z} E_y = -j\omega\mu H_x \\ \frac{\partial}{\partial z} E_x - \frac{\partial}{\partial x} E_z = -j\omega\mu H_y \\ \frac{\partial}{\partial x} E_y - \frac{\partial}{\partial y} E_x = -j\omega\mu H_z \end{array} \right. \quad \left\{ \begin{array}{l} \frac{\partial}{\partial y} H_z - \frac{\partial}{\partial z} H_y = j\omega\varepsilon E_x \\ \frac{\partial}{\partial z} H_x - \frac{\partial}{\partial x} H_z = j\omega\varepsilon E_y \\ \frac{\partial}{\partial x} H_y - \frac{\partial}{\partial y} H_z = j\omega\varepsilon E_z \end{array} \right.$$

$$\frac{\partial}{\partial y} H_z - \frac{\partial}{\partial z} H_y = j\omega \varepsilon E_x$$

$$\frac{\partial}{\partial z} H_x - \frac{\partial}{\partial x} H_z = j\omega \varepsilon E_y$$

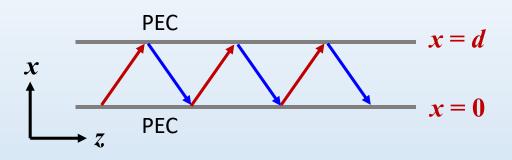
$$\frac{\partial}{\partial x} H_y - \frac{\partial}{\partial y} H_x = j\omega \varepsilon E_z$$

$$\begin{cases} \frac{\partial}{\partial z} E_y = j\omega \mu H_x \\ \frac{\partial}{\partial x} E_y = -j\omega \mu H_z \end{cases} E_y, H_x, H_z$$
 **TE**  $\xi$ 

$$\frac{\partial}{\partial z} H_x + \frac{\partial}{\partial x} H_z = j\omega \varepsilon E_y$$

 $\begin{cases}
\frac{\partial}{\partial z} H_y = -j\omega \varepsilon E_x \\
\frac{\partial}{\partial x} H_y = j\omega \varepsilon E_z
\end{cases} \qquad \mathbf{H}_y, \quad \mathbf{E}_x, \quad \mathbf{E}_z$   $\frac{\partial}{\partial z} E_x + \frac{\partial}{\partial x} E_z = -j\omega \mu H_y$ 

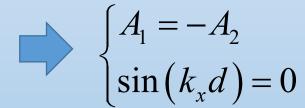
### **TE Waveguide Modes**



$$\vec{E} = \hat{y} \left( A_1 e^{-jk_x x} + A_2 e^{jk_x x} \right) e^{-jk_z z}$$

m = 2 m = 1 m = 3 m = 3 m = 3

Boundary Conditions: 
$$E_y(0,z) = E_y(d,z) = 0$$



$$k_{x}d = m\pi$$

$$E_{y}(x,z) = E_{0} \sin(k_{x}x) e^{-jk_{z}z}$$

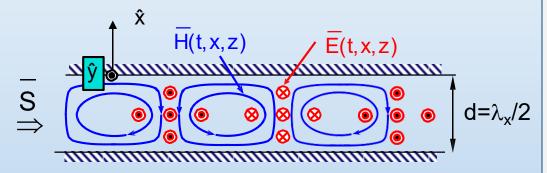
$$H_{x}(x,z) = -\frac{k_{z}}{\omega\mu} E_{0} \sin(k_{x}x) e^{-jk_{z}z}$$

$$H_{z}(x,z) = -\frac{k_{x}}{j\omega\mu} E_{0} \cos(k_{x}x) e^{-jk_{z}z}$$

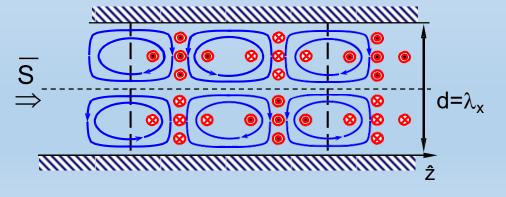
### **TE Mode Patterns**

TE modes:  $E_y(x,z) = E_0 \sin(k_x x) e^{-jk_z z}$ 

#### TE<sub>1</sub> mode:



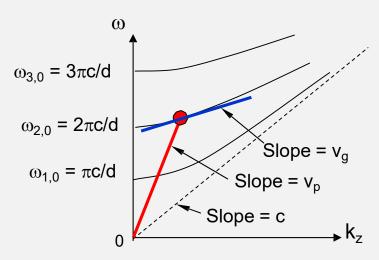
#### TE<sub>2</sub> mode:



$$k_x d = m\pi \implies m\lambda_x/2 = d$$

$$k_z = \sqrt{\left(\frac{\omega}{c}\right)^2 - \left(\frac{m\pi}{d}\right)^2}$$

#### (Dispersion Relation)

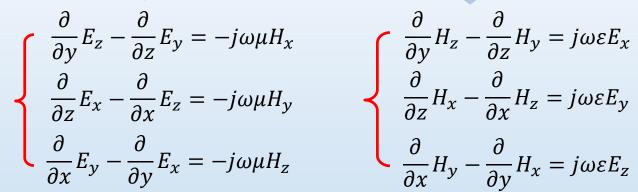


$$v_{phase} = \omega/k_z$$

$$v_{group} = d\omega/dk_z$$

$$\nabla \times \vec{E} = -j\omega \mu \vec{H}$$

$$abla imes \vec{H} = j\omega \varepsilon \vec{E}$$



$$\frac{\partial}{\partial y} H_z - \frac{\partial}{\partial z} H_y = j\omega \varepsilon E_x$$

$$\frac{\partial}{\partial z} H_x - \frac{\partial}{\partial x} H_z = j\omega \varepsilon E_y$$

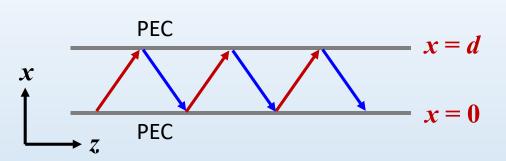
$$\frac{\partial}{\partial x} H_y - \frac{\partial}{\partial y} H_x = j\omega \varepsilon E_z$$

$$\begin{cases} \frac{\partial}{\partial z} E_y = j\omega \mu H_x \\ \frac{\partial}{\partial x} E_y = -j\omega \mu H_z \end{cases} E_y, H_x, H_z$$

$$\frac{\partial}{\partial z} H_x + \frac{\partial}{\partial x} H_z = j\omega \varepsilon E_y$$

$$\begin{cases} \frac{\partial}{\partial z} H_y = -j\omega \varepsilon E_x \\ \frac{\partial}{\partial x} H_y = j\omega \varepsilon E_z \end{cases} \qquad \begin{array}{c} \boldsymbol{H_y}, \quad \boldsymbol{E_x}, \quad \boldsymbol{E_z} \\ \frac{\partial}{\partial z} E_x + \frac{\partial}{\partial x} E_z = -j\omega \mu H_y \end{cases}$$

### **TM Waveguide Modes**

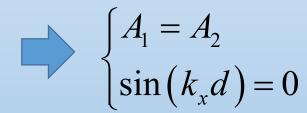


$$\nabla \times \overrightarrow{H} = j\omega\varepsilon \overrightarrow{E}$$

$$\nabla \times \overrightarrow{E} = -j\omega\mu \overrightarrow{H}$$

$$\overrightarrow{H} = \hat{y} \left( A_1 e^{-jk_x x} + A_2 e^{jk_x x} \right) e^{-jk_z z}$$

Boundary Conditions: 
$$E_z(0,z) = E_z(d,z) = 0$$



$$k_x d = m\pi$$

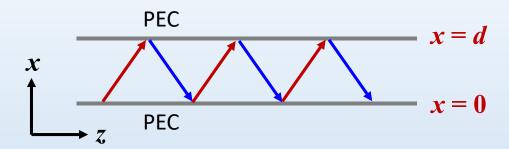
(Guidance Condition)

$$H_{y}(x,z) = H_{0}\cos(k_{x}x)e^{-jk_{z}z}$$

$$E_{x}(x,z) = \frac{k_{z}}{\omega\varepsilon}H_{0}\cos(k_{x}x)e^{-jk_{z}z}$$

$$E_{z}(x,z) = -\frac{k_{x}}{j\omega\varepsilon}H_{0}\sin(k_{x}x)e^{-jk_{z}z}$$

### **Cutoff Frequency**



$$k_x d = m\pi$$
  $\Rightarrow$   $k_z = \sqrt{\left(\frac{\omega}{c}\right)^2 - \left(\frac{m\pi}{d}\right)^2}$  (Dispersion Relation)

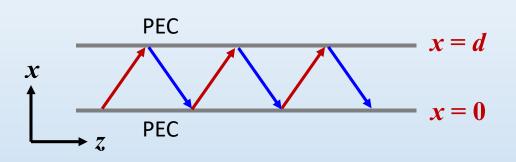
$$\omega = \frac{m\pi c}{d}$$

- Cutoff Frequencies of the TE<sub>m</sub> and TM<sub>m</sub> modes (m>0)
- $\omega = \frac{m\pi c}{d}$  > No cutoff frequency and TM<sub>0</sub> (TEM mode) > TF, mode does not exist.

**TEM mode:** 
$$\overrightarrow{H} = \hat{y}H_0 \exp(-jkz)$$
  
 $\overrightarrow{E} = \hat{x}E_0 \exp(-jkz)$ 

# Rectangular Waveguide

#### (a) Parallel-plate waveguide (Two PEC boundaries)



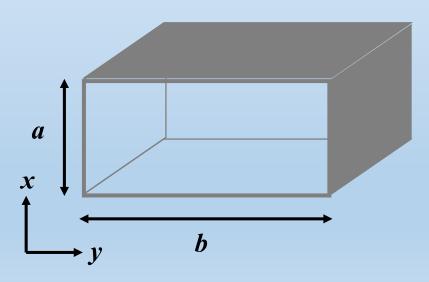
#### TE mode:

$$H_z(x,z) = H_0 \cos(k_x x) e^{-jk_z z}$$

#### TM mode:

$$E_z(x,z) = E_0 \sin(k_x x) e^{-jk_z z}$$

#### (a) Rectangular waveguide (Four PEC boundaries)



#### TE mode:

$$H_z(x, y, z) = H_0 \cos(k_x x) \cos(k_y y) e^{-jk_z z}$$

#### TM mode:

$$E_z(x, y, z) = E_0 \sin(k_x x) \sin(k_y y) e^{-jk_z z}$$

### **Transverse Components**

$$\nabla \times \overrightarrow{E} = -j\omega\mu \overrightarrow{H} \qquad \nabla \times \overrightarrow{H} = j\omega\varepsilon \overrightarrow{E} \qquad \nabla \rightarrow \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} - \hat{z}jk_z$$

$$\frac{\partial}{\partial y}E_z + jk_zE_y = -j\omega\mu H_x \qquad \frac{\partial}{\partial y}H_z + jk_zH_y = j\omega\varepsilon E_x$$

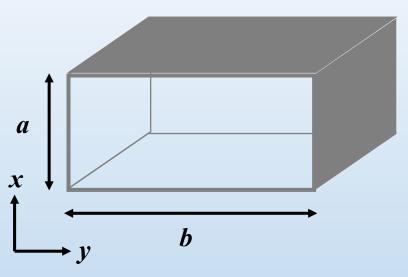
$$-\frac{\partial}{\partial x}E_z - jk_zE_x = -j\omega\mu H_y \qquad -\frac{\partial}{\partial x}H_z - jk_zH_x = j\omega\varepsilon E_y$$

$$\frac{\partial}{\partial x}E_y - \frac{\partial}{\partial y}E_x = -j\omega\mu H_z \qquad \frac{\partial}{\partial x}H_y - \frac{\partial}{\partial y}H_x = j\omega\varepsilon E_z$$

$$H_{x} = \frac{1}{k^{2} - k_{z}^{2}} \left( j\omega\varepsilon \frac{\partial}{\partial y} E_{z} - jk_{z} \frac{\partial}{\partial x} H_{z} \right) \qquad E_{x} = \frac{-1}{k^{2} - k_{z}^{2}} \left( jk_{z} \frac{\partial}{\partial x} E_{z} + j\omega\mu \frac{\partial}{\partial y} H_{z} \right)$$

$$H_{y} = \frac{-1}{k^{2} - k_{z}^{2}} \left( j\omega\varepsilon \frac{\partial}{\partial x} E_{z} + jk_{z} \frac{\partial}{\partial y} H_{z} \right) \qquad E_{y} = \frac{-1}{k^{2} - k_{z}^{2}} \left( jk_{z} \frac{\partial}{\partial y} E_{z} - j\omega\mu \frac{\partial}{\partial x} H_{z} \right)$$

# **Rectangular Waveguide - TE<sub>mn</sub> Mode**



#### **Boundary Conditions:**

- (1) Ex = 0 at y = 0 and b
- (2) Ey = 0 at x = 0 and a

$$k_{x}a = m\pi$$
$$k_{y}b = n\pi$$

(Guidance Condition)

$$H_z = \cos(k_x x)\cos(k_y y)e^{-jk_z z}$$

$$H_{x} = \frac{jk_{x}k_{z}}{\omega^{2}\mu\varepsilon - k_{z}^{2}}\sin(k_{x}x)\cos(k_{y}y)e^{-jk_{z}z}$$

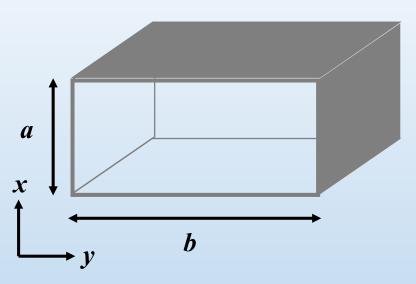
$$H_{y} = \frac{jk_{y}k_{z}}{\omega^{2}\mu\varepsilon - k_{z}^{2}}\cos(k_{x}x)\sin(k_{y}y)e^{-jk_{z}z}$$

$$E_{x} = \frac{j\omega\mu k_{y}}{\omega^{2}\mu\varepsilon - k_{z}^{2}}\cos(k_{x}x)\sin(k_{y}y)e^{-jk_{z}z}$$

$$E_{y} = \frac{-j\omega\mu k_{x}}{\omega^{2}\mu\varepsilon - k_{z}^{2}}\sin(k_{x}x)\cos(k_{y}y)e^{-jk_{z}z}$$

$$k_z = \sqrt{\left(\frac{\omega}{c}\right)^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}$$

# Rectangular Waveguide - TM<sub>mn</sub> Mode



#### **Boundary Conditions:**

- (1) Ex = 0 at y = 0 and b
- (2) Ey = 0 at x = 0 and a

$$k_{x}a = m\pi$$
$$k_{y}b = n\pi$$

(Guidance Condition)

$$E_z(x, y, z) = \sin(k_x x) \sin(k_y y) e^{-jk_z z}$$

$$E_{x} = \frac{-jk_{x}k_{z}}{\omega^{2}\mu\varepsilon - k_{z}^{2}}\cos(k_{x}x)\sin(k_{y}y)e^{-jk_{z}z}$$

$$E_{y} = \frac{-jk_{y}k_{z}}{\omega^{2}\mu\varepsilon - k_{z}^{2}}\sin(k_{x}x)\cos(k_{y}y)e^{-jk_{z}z}$$

$$H_{x} = \frac{j\omega\varepsilon k_{y}}{\omega^{2}\mu\varepsilon - k_{z}^{2}}\sin(k_{x}x)\cos(k_{y}y)e^{-jk_{z}z}$$

$$H_{y} = \frac{-j\omega\varepsilon k_{x}}{\omega^{2}\mu\varepsilon - k_{z}^{2}}\cos(k_{x}x)\sin(k_{y}y)e^{-jk_{z}z}$$

$$k_z = \sqrt{\left(\frac{\omega}{c}\right)^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}$$

### **Cut-off characteristics**

$$k_c = \sqrt{k_x^2 + k_y^2} = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

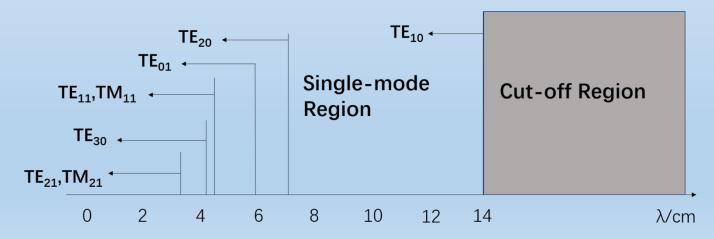
#### **Cut-off frequency**

$$f_c = \frac{1}{2\sqrt{\varepsilon\mu}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

#### **Cut-off wavelength**

$$\lambda_c = \frac{2\pi}{k_c} = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}}$$

#### For a=7cm, b=3cm



# **TE<sub>10</sub> Waveguide Mode**

 $k_{x}a = m\pi$  $k_{y}b = n\pi$ 

**TE**<sub>10</sub> Mode m=1, n=0

$$H_{z} = \cos(k_{x}x)\cos(k_{y}y)e^{-jk_{z}z} \longrightarrow H_{z} = \cos\left(\frac{\pi}{a}x\right)e^{-jk_{z}z}$$

$$H_{x} = \frac{jk_{x}k_{z}}{\omega^{2}\mu\varepsilon - k_{z}^{2}}\sin(k_{x}x)\cos(k_{y}y)e^{-jk_{z}z} \longrightarrow H_{x} = \frac{jk_{z}\pi}{k_{c}^{2}a}\sin(\frac{\pi}{a}x)e^{-jk_{z}z}$$

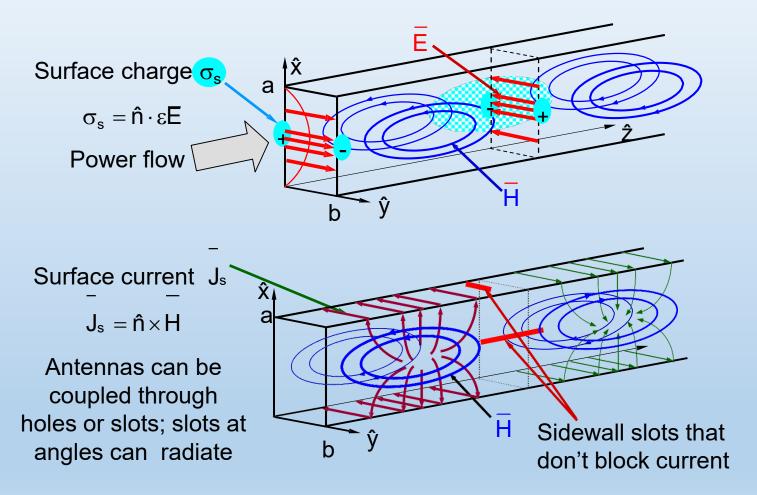
$$H_{y} = \frac{jk_{y}k_{z}}{\omega^{2}\mu\varepsilon - k_{z}^{2}}\cos(k_{x}x)\sin(k_{y}y)e^{-jk_{z}z} \longrightarrow H_{y} = 0$$

$$E_{x} = \frac{j\omega\mu k_{y}}{\omega^{2}\mu\varepsilon - k_{z}^{2}}\cos(k_{x}x)\sin(k_{y}y)e^{-jk_{z}z} \longrightarrow E_{x} = 0$$

$$E_{y} = \frac{-j\omega\mu k_{x}}{\omega^{2}\mu\varepsilon - k_{z}^{2}}\sin(k_{x}x)\cos(k_{y}y)e^{-jk_{z}z} \longrightarrow E_{y} = \frac{-j\omega\mu\pi}{k_{c}^{2}a}\sin(\frac{\pi}{a}x)e^{-jk_{z}z}$$

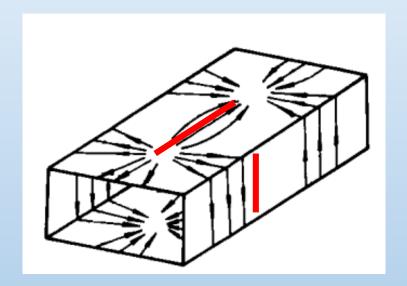
# **TE<sub>10</sub> Waveguide Mode**

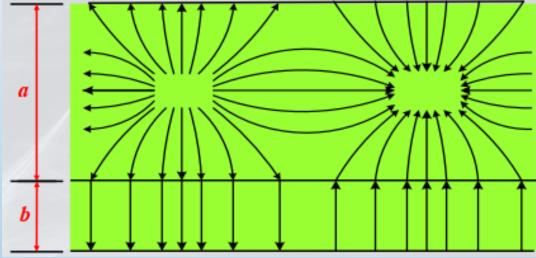
### Add Sidewalls to $TE_1$ Parallel Plate Waveguide $\Rightarrow TE_{10}$ :



# **TE<sub>10</sub> Waveguide Mode**

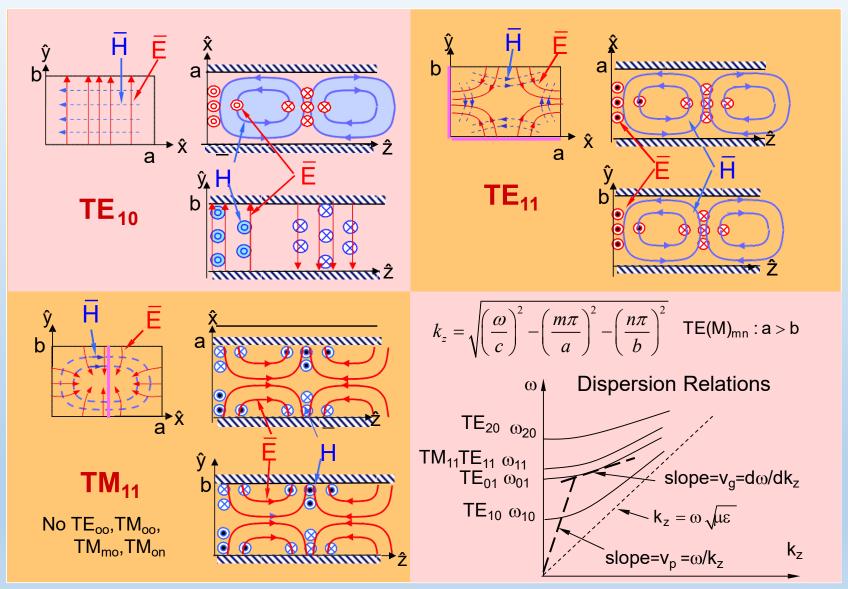
### **TE<sub>10</sub> Mode: Surface current distribution**





Sidewall slots that don't block current

# **Rectangular Waveguide Modes**



### **Example**

For regular metal rectangular waveguides BJ-100(a=22.86 mm, b=10.16mm), the waveguide is filled with a uniform medium  $\varepsilon_r$ =2.1.

- (a). Find the cutoff frequencies of the top 5 modes with longer Cut-off wavelengths.
- (b). If the operating frequency is 9GHz, 11GHz, what modes might exist in waveguides?

#### **Solution**

$$\lambda_c = \frac{2\pi}{k_c} = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}} \qquad f_c = \frac{v}{\lambda_c} = \frac{c}{\sqrt{\varepsilon_r}\lambda_c}$$

(a) For TE<sub>10</sub> mode,  $\lambda_c = 2a = 4.57cm$   $f_c = 4.53GHz$ 

For TE<sub>20</sub> mode,  $\lambda_c = a = 2.29cm$   $f_c = 9.06GHz$ 

For TE<sub>01</sub> mode,  $\lambda_c = 2b = 2.03cm$   $f_c = 10.19GHz$ 

For TE<sub>11</sub> & TM<sub>11</sub> mode,  $\lambda_c = \frac{2ab}{\sqrt{a^2 + b^2}} = 1.86cm$   $f_c = 11.15GHz$ 

(b). If the operating frequency is 9GHz, 11GHz, what modes might exist in waveguides?

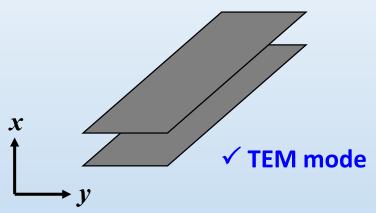
Transmission condition:  $f > f_c$ ,

For 9GHz, only TE<sub>10</sub> mode

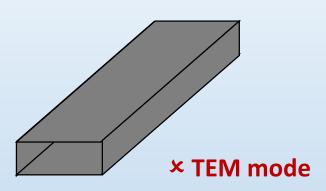
For 11GHz, three modes can exist in waveguides : $TE_{10}TE_{20}TE_{01}$ 

# **TEM Mode Analysis**

#### **Parallel Metallic Plates**



#### **Rectangular Metallic Waveguide**

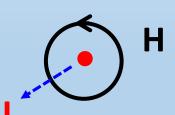


#### What's the reason?

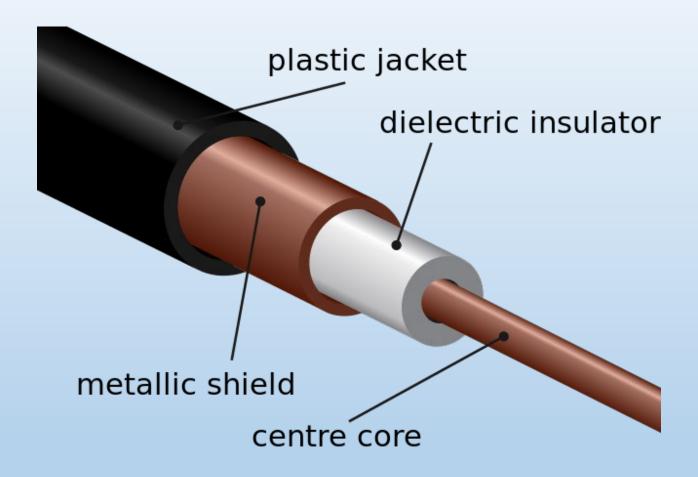
### TEM mode requires $E_z = 0$ and $H_z = 0$

Single-conductor waveguides cannot support TEM waves. Because:

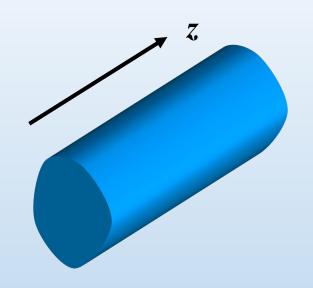
- > H forms a loop
- Absence of magnetic charges
- > There must be a longitudinal electric current/field



### **Coaxial Cable**



### **Generic Form of Guided Waves**



$$\overrightarrow{E}(x,y,z) = \overrightarrow{E}_0(x,y) \exp(-jkz)$$

$$\overrightarrow{H}(x,y,z) = \overrightarrow{H}_0(x,y) \exp(-jkz)$$



$$\vec{E}_0(x,y), \vec{H}_0(x,y)$$

$$\vec{E}_0(x,y) = \hat{x}E_{0x}(x,y) + \hat{y}E_{0y}(x,y) + \hat{z}E_{0z}(x,y)$$

$$\vec{H}_0(x,y) = \hat{x}H_{0x}(x,y) + \hat{y}H_{0y}(x,y) + \hat{z}H_{0z}(x,y)$$

$$\nabla^2 \vec{E} + k^2 \vec{E} = 0$$
$$\nabla^2 \vec{H} + k^2 \vec{H} = 0$$

# (Helmholtz wave equation)

$$\nabla \to \hat{x} \frac{\partial}{\partial x} + \hat{y} \frac{\partial}{\partial y} - \hat{z}jk_z$$

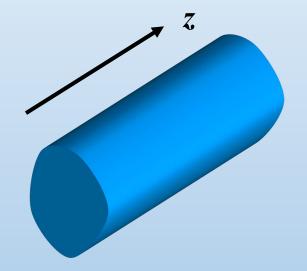
$$\nabla^2 \overrightarrow{H} + k^2 \overrightarrow{H} = 0$$

$$\nabla^2 \to \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} - k_z^2$$

### Uncoupled Equations for $E_{07}$ and $H_{07}$

$$\begin{cases}
\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k^2 - k_z^2\right) E_{0z}(x, y) = 0 \\
\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k^2 - k_z^2\right) H_{0z}(x, y) = 0
\end{cases}$$

$$\left[ \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k^2 - k_z^2 \right) H_{0z}(x, y) = 0 \right]$$



TE mode:  $E_{07} = 0$ 

TM mode:  $H_{0z} = 0$ 

TEM mode:  $E_{0z} = 0$  and  $H_{0z} = 0$ 

### **Transverse Components**

$$\nabla \times \overrightarrow{E} = -j\omega\mu \overrightarrow{H} \qquad \nabla \times \overrightarrow{H} = j\omega\varepsilon \overrightarrow{E} \qquad \nabla \rightarrow \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} - \hat{z}jk_z$$

$$\frac{\partial}{\partial y}E_z + jk_zE_y = -j\omega\mu H_x \qquad \frac{\partial}{\partial y}H_z + jk_zH_y = j\omega\varepsilon E_x$$

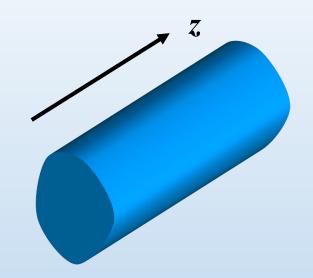
$$-\frac{\partial}{\partial x}E_z - jk_zE_x = -j\omega\mu H_y \qquad -\frac{\partial}{\partial x}H_z - jk_zH_x = j\omega\varepsilon E_y$$

$$\frac{\partial}{\partial x}E_y - \frac{\partial}{\partial y}E_x = -j\omega\mu H_z \qquad \frac{\partial}{\partial x}H_y - \frac{\partial}{\partial y}H_x = j\omega\varepsilon E_z$$

$$H_{x} = \frac{1}{k^{2} - k_{z}^{2}} \left( j\omega\varepsilon \frac{\partial}{\partial y} E_{z} - jk_{z} \frac{\partial}{\partial x} H_{z} \right) \qquad E_{x} = \frac{-1}{k^{2} - k_{z}^{2}} \left( jk_{z} \frac{\partial}{\partial x} E_{z} + j\omega\mu \frac{\partial}{\partial y} H_{z} \right)$$

$$H_{y} = \frac{-1}{k^{2} - k_{z}^{2}} \left( j\omega\varepsilon \frac{\partial}{\partial x} E_{z} + jk_{z} \frac{\partial}{\partial y} H_{z} \right) \qquad E_{y} = \frac{-1}{k^{2} - k_{z}^{2}} \left( jk_{z} \frac{\partial}{\partial y} E_{z} - j\omega\mu \frac{\partial}{\partial x} H_{z} \right)$$

# **General Method for Waveguide Problems**



### (1) Solve for $E_{0z}$ and $H_{0z}$ with boundary conditions

$$\begin{cases} \left(\frac{\partial^{2}}{\partial x^{2}} + \frac{\partial^{2}}{\partial y^{2}} + k^{2} - k_{z}^{2}\right) E_{0z}(x, y) = 0 \\ \left(\frac{\partial^{2}}{\partial x^{2}} + \frac{\partial^{2}}{\partial y^{2}} + k^{2} - k_{z}^{2}\right) H_{0z}(x, y) = 0 \end{cases}$$

#### (2) Calculate the transverse components

$$H_{x} = \frac{1}{k^{2} - k_{z}^{2}} \left( j\omega\varepsilon \frac{\partial}{\partial y} E_{z} - jk_{z} \frac{\partial}{\partial x} H_{z} \right) \qquad E_{x} = \frac{-1}{k^{2} - k_{z}^{2}} \left( jk_{z} \frac{\partial}{\partial x} E_{z} + j\omega\mu \frac{\partial}{\partial y} H_{z} \right)$$

$$H_{y} = \frac{-1}{k^{2} - k_{z}^{2}} \left( j\omega\varepsilon \frac{\partial}{\partial x} E_{z} + jk_{z} \frac{\partial}{\partial y} H_{z} \right)$$

$$E_{x} = \frac{-1}{k^{2} - k_{z}^{2}} \left( jk_{z} \frac{\partial}{\partial x} E_{z} + j\omega\mu \frac{\partial}{\partial y} H_{z} \right)$$

$$H_{y} = \frac{-1}{k^{2} - k_{z}^{2}} \left( j\omega\varepsilon \frac{\partial}{\partial x} E_{z} + jk_{z} \frac{\partial}{\partial y} H_{z} \right) \qquad E_{y} = \frac{-1}{k^{2} - k_{z}^{2}} \left( jk_{z} \frac{\partial}{\partial y} E_{z} - j\omega\mu \frac{\partial}{\partial x} H_{z} \right)$$

# **Slab Dielectric Waveguide**

**Region 3** 

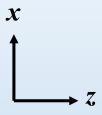
 $n_2$ 

**Region 2** 

 $n_1$ 

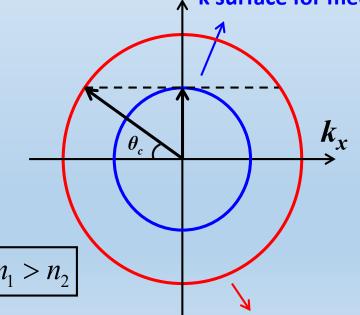
x = d

x = -d



Region 1

 $\mathbf{A}_{z}$  k surface for medium 2



$$\vec{E}_1 = \hat{y}E_1 \exp(\alpha x - jk_z z)$$
,  $x < -d$ 

$$\mathbf{k}_{x}$$
  $\overrightarrow{E}_{2} = \widehat{y}(Asin(\mathbf{k}_{x}x) + Bcos(\mathbf{k}_{x}x)) exp(-j\mathbf{k}_{z}z), -d < x < d$ 

$$\vec{E}_3 = \hat{y}E_3 \exp(-\alpha x - jk_z z)$$
,  $x > d$ 

Critical angle: 
$$\theta_c = \sin^{-1} \frac{n_2}{n_1}$$

k surface for medium 1

# Slab Dielectric Waveguide – TE mode

#### **Dispersion Relation**

$$k_x^2 + k_z^2 = n_1^2 k_0^2$$
$$-\alpha^2 + k_z^2 = n_2^2 k_0^2$$



$$k_x^2 + k_z^2 = n_1^2 k_0^2 \qquad (k_x d)^2 + (\alpha d)^2 = (n_1^2 - n_2^2)(k_0 d)^2$$

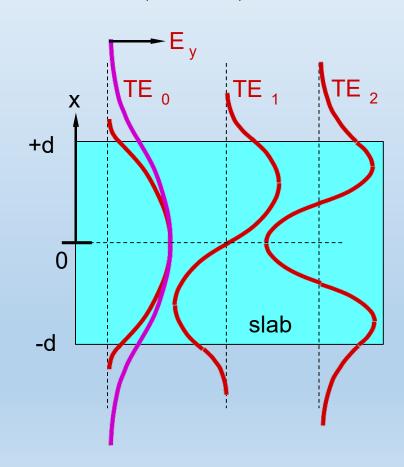


#### **Guidance Condition**

$$\alpha d = k_x d \tan \left( k_x d - \frac{m\pi}{2} \right)$$

$$\alpha d = k_x d \tan(k_x d)$$
 (TE even)

$$\alpha d = -k_x d \cot(k_x d)$$
 (TE odd)

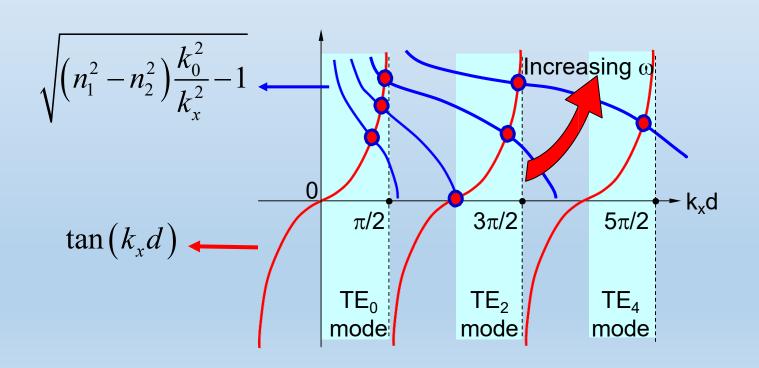


# Slab Dielectric Waveguide – TE<sub>even,n</sub>

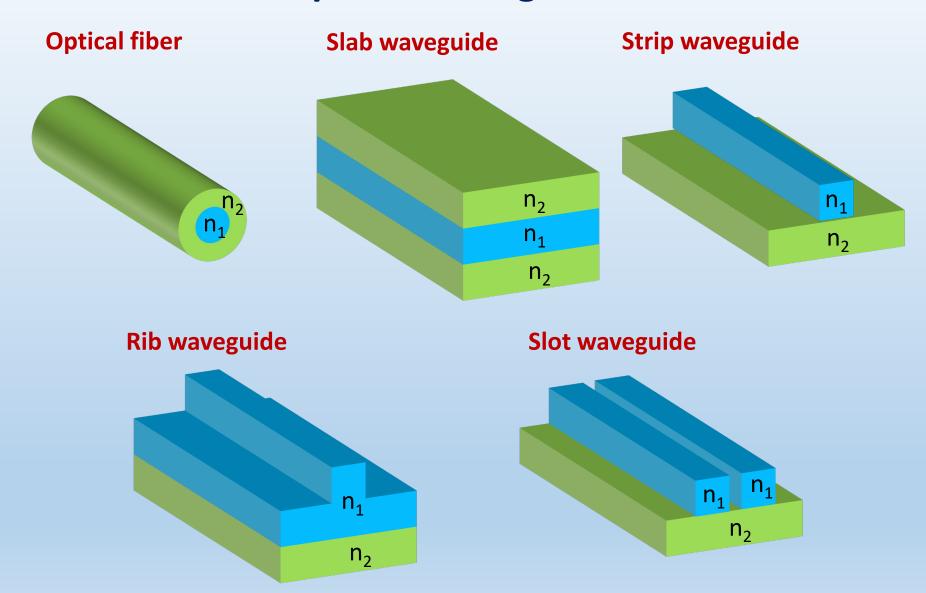
$$k_{x}^{2} + \alpha_{x}^{2} = \left(n_{1}^{2} - n_{2}^{2}\right)k_{0}^{2}$$

$$\tan\left(k_{x}d - \frac{m\pi}{2}\right) = \sqrt{(n_{1}^{2} - n_{2}^{2})\frac{k_{0}^{2}}{k_{x}^{2}} - 1}$$

$$\alpha d = k_{x}d\tan\left(k_{x}d - \frac{m\pi}{2}\right)$$

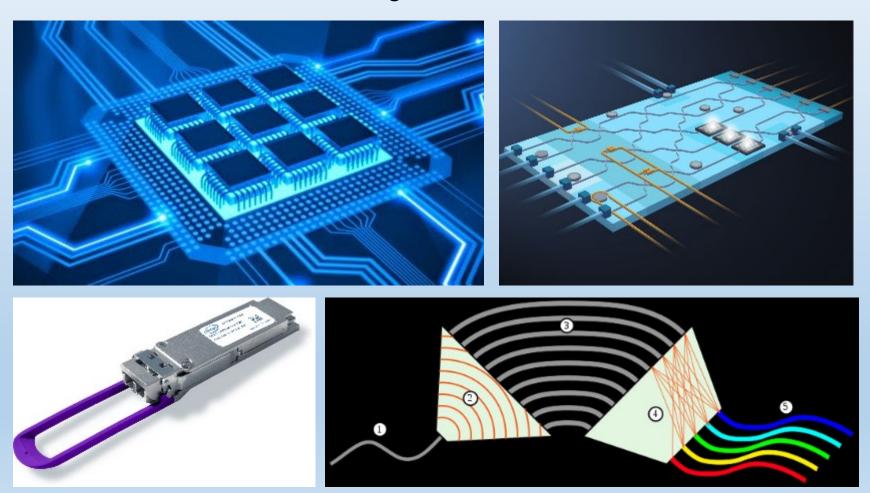


# **Optical Waveguides**



### **Photonic Integrated Circuits**

A photonic integrated circuit (PIC) or integrated optical circuit is a device that integrates multiple (at least two) photonic functions and as such is similar to an electronic integrated circuit.



### **Electromagnetic spectrum**

