Chapter 6: Reflection and Transmission

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In this chapter, we study the reflection and transmission of waves at a plane boundary and learn the underlying physics of various phenomena in natural life, such as phase matching, Snell's law, total reflection at critical angle and total transmission at Brewster angle.

6.1 Time-Harmonic Maxwell's Equations for Isotropic Media

Maxwell's Equations for time-harmonic fields are

$$\begin{aligned}
&\nabla\cdot\vec{D}=\rho_{free}\\
&\nabla\times\vec{E}=-j\omega\vec{B}\\
&\nabla\cdot\vec{B}=0\\
&\nabla\times\vec{H}=\vec{J}_{free}+j\omega\vec{D}
\end{aligned}\tag{6.1}$$

where all field quantities are space dependent and complex.

For a boundary surface separating regions 1 and 2, and with a surface normal $\hat{n}$ point from region 2 to region 1. The boundary conditions as developed in Chapter 4 are as follows:

$$\begin{aligned}
&n\cdot\left(\vec{D}_1-\vec{D}_2\right)=\rho_s\\
&\hat{n}\times\left(\vec{E}_1-\vec{E}_2\right)=0\\
&n\cdot\left(\vec{B}_1-\vec{B}_2\right)=0\\
&\hat{n}\times\left(\vec{H}_1-\vec{H}_2\right)=\vec{J}_s
\end{aligned}\tag{6.2}$$

where ρ_s is the surface charge density and $\vec{J}_s$ is the surface current density.

In source-free regions of isotropic media, where $\rho=\vec{J}=0$, $\vec{D}=\varepsilon\vec{E}$ and $\vec{B}=\mu\vec{H}$, consider plane wave solutions where all field vectors have spatial dependence

$$\exp\left(-j\vec{k}\cdot\vec{r}\right)=\exp\left[-j\left(k_x x+k_y y+k_z z\right)\right]\tag{6.3}$$

characterized by the wave vector

$$\vec{k}=\hat{x}k_x+\hat{y}k_y+\hat{z}k_z\tag{6.4}$$

with the dispersion relation

$$k_x^2+k_y^2+k_z^2=\omega^2\mu\varepsilon\tag{6.5}$$

Maxwell's Equations become

$$\boxed{\begin{aligned}&\vec{k}\cdot\vec{E}=0\\&\vec{k}\times\vec{E}=\omega\mu\vec{H}\\&\vec{k}\cdot\vec{H}=0\\&\vec{k}\times\vec{H}=-\omega\varepsilon\vec{E}\end{aligned}} \tag{6.6}$$

We shall assume homogeneous media in each region, and the various regions are separated by boundary surfaces subject to the boundary conditions in Eq. (6.2).

6.2 Reflection and Transmission of TE waves

Consider a TE plane wave incident from an isotropic medium with permittivity ε_1 and permeability μ_1 upon another isotropic medium with permittivity ε_2 and permeability μ_2 (Fig. 6.1). We assume the plane of incidence to be parallel to the *xz* plane, which contains the incident wave vector and the surface normal. An incident wave with any polarization can be decomposed into TE (transverse electric) and TM (transverse magnetic) wave components. The TE wave is linearly polarized with the electric field vector perpendicular to the plane of incidence and is also called perpendicularly polarized, horizontally polarized, or simply an $\vec{E}$ wave or *s* wave. The TM wave is linearly polarized with the electric vector parallel to the plane of incidence and is also called paralleled polarized, vertically polarized, or simply an $\vec{H}$ wave or *p* wave.

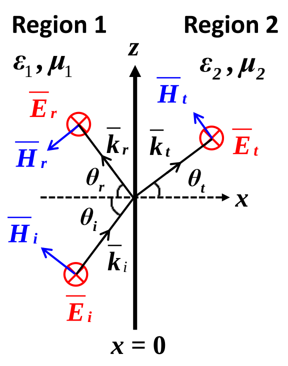

Figure 6.1 Reflection and transmission of TE waves at a plane boundary separating regions 1 and 2.

We write, for an incident TE wave with unit amplitude,

$$\begin{aligned}&\vec{E}_i=\hat{y}\exp\left(-j\vec{k}_i\cdot\vec{r}\right)\\&\vec{H}_i=\frac{1}{\omega\mu_1}\vec{k}_i\times\vec{E}_i\\&\vec{S}_i=\vec{E}_i\times\vec{H}_i^*=\vec{k}_i\frac{1}{\omega\mu_1}\left|\vec{E}_i\right|^2\end{aligned} \tag{6.7}$$

The reflected field components for the incident TE wave are

$$\vec{E}_r = \hat{y}R^{TE}\exp\left(-j\vec{k}_r\cdot\vec{r}\right)$$
$$\vec{H}_r = \frac{1}{\omega\mu_1}\vec{k}_r\times\vec{E}_r \tag{6.8}$$
$$\vec{S}_r = \vec{E}_r\times\vec{H}_r^* = \vec{k}_r\frac{1}{\omega\mu_1}\left|\vec{E}_r\right|^2$$

where R^{TE} is the *reflection coefficient* for the electric field component E_{iy}.

In region 2, the transmitted TE field components are

$$\vec{E}_t = \hat{y}T^{TE}\exp\left(-j\vec{k}_t\cdot\vec{r}\right)$$
$$\vec{H}_t = \frac{1}{\omega\mu_2}\vec{k}_t\times\vec{E}_t \tag{6.9}$$
$$\vec{S}_t = \vec{E}_t\times\vec{H}_t^* = \vec{k}_t\frac{1}{\omega\mu_2}\left|\vec{E}_t\right|^2$$

where T^{TE} is the *transmission coefficient* for the electric field component E_{iy}.

The above vectors and the corresponding dispersion relations are

$$\begin{aligned}
\vec{k}_i &= \hat{x}k_{ix} + \hat{z}k_{iz} \\
\vec{k}_r &= \hat{x}k_{rx} + \hat{z}k_{rz} \\
\vec{k}_t &= \hat{x}k_{tx} + \hat{z}k_{tz} \\
k_{ix}^2 + k_{iz}^2 &= \omega^2\mu_1\varepsilon_1 \\
k_{rx}^2 + k_{rz}^2 &= \omega^2\mu_1\varepsilon_1 \\
k_{tx}^2 + k_{tz}^2 &= \omega^2\mu_2\varepsilon_2
\end{aligned} \tag{6.10}$$

First we determine the wave vector components by phase matching conditions as derived below.

Let the boundary surface be at $x = 0$ where the tangential components of $\vec{E}$ and $\vec{H}$ are continuous. From continuity of E_y, we obtain,

$$\exp\left(-jk_{iz}z\right) + R^{TE}\exp\left(-jk_{rz}z\right) = T^{TE}\exp\left(-jk_{tz}z\right) \tag{6.11}$$

This equation must be true for ALL z, and as a consequence, we obtain the *phase matching condition*

$$\boxed{k_{iz} = k_{rz} = k_{tz}} \tag{6.12}$$

Then we replace k_{iz}, k_{rz} and k_{tz} by k_z. From the dispersion relation of the region 1, we also find that the x-component wave vectors for the incident and reflected fields yield to k_{ix} = -k_{rx}. The x-component wave vector for the transmitted field can be obtain by $k_{tx}^2 = \omega^2\mu_2\varepsilon_2 - k_z^2$. Then the boundary conditions of continuity of tangential $\vec{E}$ and $\vec{H}$ give

$$\begin{aligned}
&1+R^{TE}=T^{TE}\\
&\frac{k_{ix}}{\mu_1}\left(1-R^{TE}\right)=\frac{k_{tx}}{\mu_2}T^{TE}
\end{aligned} \tag{6.13}$$

Note that we did not use the boundary conditions that normal $\vec{D}$ and normal $\vec{B}$ components are continuous at z = 0, because these two conditions are not independent of the two tangential $\vec{E}$ and $\vec{H}$ conditions, just as Gauss' two laws are not independent of Faraday's and Ampere's laws.

The reflection and transmission coefficients R^{TE} and T^{TE} are determined from Eq. (6.13) as

$$\begin{aligned}
R^{TE}&=\frac{\mu_2 k_{ix}-\mu_1 k_{tx}}{\mu_2 k_{ix}+\mu_1 k_{tx}}\\
T^{TE}&=\frac{2\mu_2 k_{ix}}{\mu_2 k_{ix}+\mu_1 k_{tx}}
\end{aligned} \tag{6.14}$$

Note that the reflection coefficient R^{TE} for TE waves represents the ratio of the reflected and incident *electric* fields.

The time-averaged Poynting power vectors for the incident, the reflected, and the transmitted waves are calculated to be

$$\begin{aligned}
\langle \vec{S}_i\rangle&=\frac{1}{2}\mathrm{Re}\left\{\vec{k}_i\frac{1}{\omega\mu_1}\right\}\\
\langle \vec{S}_r\rangle&=\frac{1}{2}\mathrm{Re}\left\{\vec{k}_r\frac{1}{\omega\mu_1}\left|R^{TE}\right|^2\right\}\\
\langle \vec{S}_t\rangle&=\frac{1}{2}\mathrm{Re}\left\{\vec{k}_t^*\frac{1}{\omega\mu_2^*}\left|T^{TE}\right|^2\exp\left[-j\left(k_{tx}-k_{tx}^*\right)x\right]\right\}
\end{aligned} \tag{6.15}$$

where we assume that k_{tx} and μ_2 may be complex.

6.3 Reflection and Transmission of TM waves

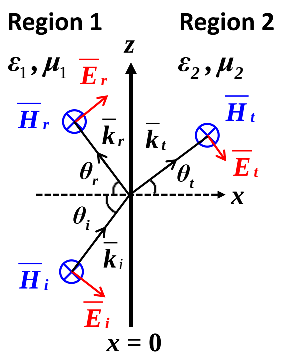

Figure 6.2 Reflection and transmission of TM waves at a plane boundary separating regions 1 and 2.

Consider a TM plane wave incidence with (Fig. 6.2)

$$\begin{aligned}\vec{H}_i &= \hat{y}\exp\left(-j\vec{k}_i\cdot\vec{r}\right)\\ \vec{E}_i &= -\frac{1}{\omega\varepsilon_1}\vec{k}_i\times\vec{H}_i\\ \vec{S}_i &= \vec{E}_i\times\vec{H}_i^* = \vec{k}_i\frac{1}{\omega\varepsilon_1}\left|\vec{H}_i\right|^2\end{aligned} \tag{6.16}$$

The reflected wave takes the form

$$\begin{aligned}\vec{H}_r &= \hat{y}R^{TM}\exp\left(-j\vec{k}_r\cdot\vec{r}\right)\\ \vec{E}_r &= -\frac{1}{\omega\varepsilon_1}\vec{k}_r\times\vec{H}_r\\ \vec{S}_r &= \vec{E}_r\times\vec{H}_r^* = \vec{k}_r\frac{1}{\omega\varepsilon_1}\left|\vec{H}_r\right|^2\end{aligned} \tag{6.17}$$

and the transmitted field

$$\begin{aligned}\vec{H}_t &= \hat{y}T^{TM}\exp\left(-j\vec{k}_t\cdot\vec{r}\right)\\ \vec{E}_t &= -\frac{1}{\omega\varepsilon_2}\vec{k}_t\times\vec{H}_r\\ \vec{S}_t &= \vec{E}_t\times\vec{H}_t^* = \vec{k}_t\frac{1}{\omega\varepsilon_2}\left|\vec{H}_t\right|^2\end{aligned} \tag{6.18}$$

The reflection and transmission coefficients R^{TM} and T^{TM} are

$$\begin{aligned}R^{TM} &= \frac{\varepsilon_2 k_{ix}-\varepsilon_1 k_{tx}}{\varepsilon_2 k_{ix}+\varepsilon_1 k_{tx}}\\ T^{TM} &= \frac{2\varepsilon_2 k_{ix}}{\varepsilon_2 k_{ix}+\varepsilon_1 k_{tx}}\end{aligned} \tag{6.19}$$

The time-averaged Poynting power vectors for the incident, the reflected, and the transmitted waves are calculated to be

$$\begin{aligned}\left\langle\vec{S}_i\right\rangle &= \frac{1}{2}\mathrm{Re}\left\{\vec{k}_i\frac{1}{\omega\varepsilon_1}\right\}\\ \left\langle\vec{S}_r\right\rangle &= \frac{1}{2}\mathrm{Re}\left\{\vec{k}_r\frac{1}{\omega\mu_1}\left|R^{TM}\right|^2\right\}\\ \left\langle\vec{S}_t\right\rangle &= \frac{1}{2}\mathrm{Re}\left\{\vec{k}_t\frac{1}{\omega\varepsilon_2}\left|T^{TM}\right|^2\exp\left[-j\left(k_{tx}-k_{tx}^*\right)x\right]\right\}\end{aligned} \tag{6.20}$$

where we assume that k_{tx} and ε_2 may be complex.

The above results for the reflection and transmission of TM waves is easily obtained by using the duality property of Maxwell's Equations with the replacements $\vec{E}\to\vec{H}$, $\vec{H}\to-\vec{E}$, and $\varepsilon\rightleftharpoons\mu$.

Let θ_i denote the angle of incidence, θ_r the angle of reflection, and θ_t the angle of transmission, with θ_i, θ_r and θ_t all less than $\pi/2$. It follows that $k_{ix}/k_i = \cos\theta_i$, $k_{tx}/k_i = \cos\theta_t$. The Fresnel reflection and transmission equations can be written by

$$\boxed{\begin{aligned} R^{TE} &= \frac{\mu_2 k_{ix} - \mu_1 k_{tx}}{\mu_2 k_{ix} + \mu_1 k_{tx}} = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t} \\ T^{TE} &= \frac{2\mu_2 k_{ix}}{\mu_2 k_{ix} + \mu_1 k_{tx}} = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t} \end{aligned}} \tag{6.21}$$

for TE waves and

$$\boxed{\begin{aligned} R^{TM} &= \frac{\varepsilon_2 k_{ix} - \varepsilon_1 k_{tx}}{\varepsilon_2 k_{ix} + \varepsilon_1 k_{tx}} = \frac{\eta_1 \cos\theta_i - \eta_2 \cos\theta_t}{\eta_1 \cos\theta_i + \eta_2 \cos\theta_t} \\ T^{TM} &= \frac{2\varepsilon_2 k_{ix}}{\varepsilon_2 k_{ix} + \varepsilon_1 k_{tx}} = \frac{2\eta_1 \cos\theta_i}{\eta_1 \cos\theta_i + \eta_2 \cos\theta_t} \end{aligned}} \tag{6.22}$$

for TM waves. Here $\eta_1 = \sqrt{\mu_1/\varepsilon_1}$ and $\eta_2 = \sqrt{\mu_2/\varepsilon_2}$ are the *characteristic impedances* for two regions.

The power conservation is observed by considering a control volume across the boundary surface (Fig. 6.3). We must prove that the x components of all Poynting vectors entering and exiting the control volume are equal. We define the power reflection or the *reflectivity* to be

$$\boxed{\begin{aligned} r^{TE} &= \frac{-\hat{x}\cdot\langle\vec{S}_r\rangle}{\hat{x}\cdot\langle\vec{S}_i\rangle} = \left|R^{TE}\right|^2 \\ t^{TE} &= \frac{\hat{x}\cdot\langle\vec{S}_t\rangle}{\hat{x}\cdot\langle\vec{S}_i\rangle} = \frac{\eta_1 \cos\theta_t}{\eta_2 \cos\theta_i}\left|T^{TE}\right|^2 \end{aligned}} \tag{6.23}$$

for TE waves and

$$\boxed{\begin{aligned} r^{TE} &= \frac{-\hat{x}\cdot\langle\vec{S}_r\rangle}{\hat{x}\cdot\langle\vec{S}_i\rangle} = \left|R^{TM}\right|^2 \\ t^{TE} &= \frac{\hat{x}\cdot\langle\vec{S}_t\rangle}{\hat{x}\cdot\langle\vec{S}_i\rangle} = \frac{\eta_2 \cos\theta_t}{\eta_1 \cos\theta_i}\left|T^{TM}\right|^2 \end{aligned}} \tag{6.24}$$

for TM waves. For both polarizations, we can easily find that

$$r^{TE} + t^{TE} = 1 \quad \text{and} \quad r^{TM} + t^{TM} = 1 \tag{6.25}$$

This demonstrates that power conservation for reflection and transmission at a plane boundary surface.

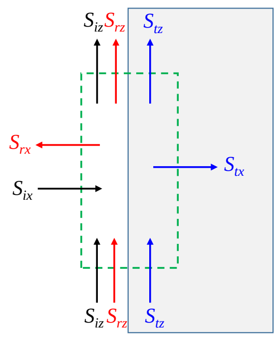

Figure 6.3 Power conservation at a plane boundary

6.4 Phase Matching

According to the phase-matching condition of Eq. (6.12),

$$\boxed{k_{iz} = k_{rz} = k_{tz}} \tag{6.26}$$

the incident, reflected and transmitted wave vectors must all lie in the same plane called the *plane of incidence* determined by the incident $\vec{k}_i$ vector and the normal to the boundary surface. Although Eq. (6.26) is derived for isotropic media, it holds for general homogeneous media with the plane wave solutions.

The phase matching condition states that the tangential components of the incident, the reflected, and the transmitted wave vectors are continuous. We illustrate the phase matching condition with Fig. 6.4 with *xz* plane as the plane of incidence. We find that the angle of incidence is equal to the angle of reflection, $\theta_i = \theta_r$, and

$$\frac{\sin\theta_i}{\sin\theta_t} = \frac{k_t}{k_i} = \frac{n_2}{n_1} \tag{6.27}$$

where $n_2 = c\sqrt{\mu_2\varepsilon_2}$ and $n_1 = c\sqrt{\mu_1\varepsilon_1}$ are the refractive indices. Eq. (6.27) is known as Snell's law.

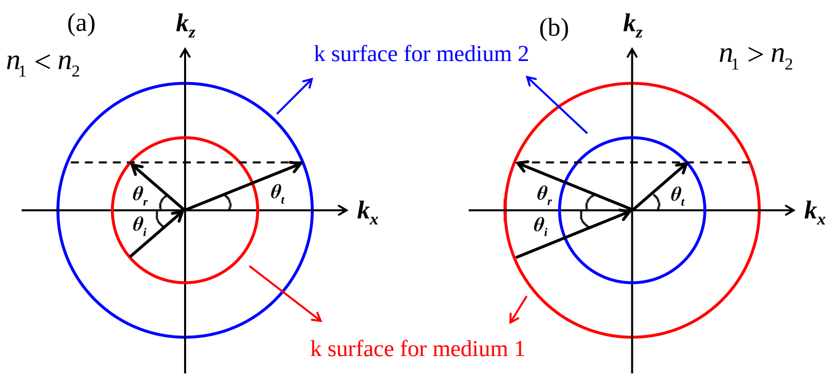

Figure 6.4 Phase matching by using *k* surfaces. (a) $n_1 < n_2$. (b) $n_1 > n_2$.

In Fig. 6.4, we let the magnitudes of the wave vectors represented by circles with radii $k_i = k_r = \omega\sqrt{\mu_1\varepsilon_1}$ and $k_t = \omega\sqrt{\mu_2\varepsilon_2}$ on the k_x-k_z plane. The circles are called k *surfaces*. In three dimensional k space when $k_y \neq 0$, the k surfaces are spheres. When $n_1 < n_2$, the radius of the k surface in region 1 is shorter than that in region 2, by the phase matching condition, the angle of transmission is smaller than that of incidence, $\theta_t < \theta_i$. In comparison, when $n_1 > n_2$, then $\theta_t > \theta_i$.

6.5 Total Reflection and Critical angle

Suppose that the medium in region 1 has larger refractive index than the medium does in region 2 such that $n_1 > n_2$. Then the radius of the k surface in region 2 is shorter than that in region 1 (Fig. 6.5). By the phase matching conditions, we see that as k_z of the incident wave becomes larger that k_t, there is no intersection with the small circle because this amounts to requiring that one component of a vector be greater than its magnitude — an impossibility unless the vector is complex.

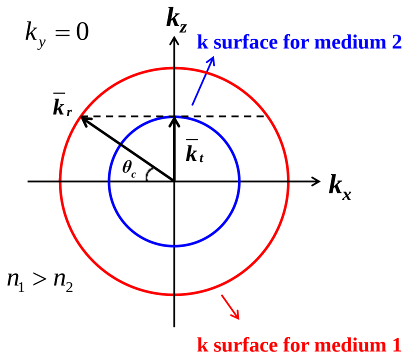

Figure 6.5 At critical angle of reflection when the k surface for medium 1 is larger than that for medium 2.

The k surface in region 2 is described by

$$k_{tx}^2 + k_z^2 = k_t^2 \tag{6.28}$$

Since $k_z > k_t$, k_{tx} must be purely imaginary,

$$k_{tx} = \sqrt{k_t^2 - k_z^2} = -jk_{tx}'' \tag{6.29}$$

Remember that the wave in region 2 is characterized by $\exp(-jk_{tx}x - jk_z z)$. For $\underline{k_z} > k_t$, it becomes $\exp(-k_{tx}''x - jk_z z)$. The transmitted wave thus decays exponentially in the x direction and propagates along the z direction with the phase velocity ω/k_z. This can be regarded as a plane wave with constant phase fronts perpendicular to the boundary surface. Its amplitude is maximum at the surface and decays exponentially away from the surface. The wave is known as a *surface wave*. The surface wave is evanescent in the x direction. Since evanescence in the transmitted wave begins when $k_t = k_z = k\sin\theta_c$, with

$$\boxed{\theta_c = \sin^{-1}\frac{k_t}{k_i} = \sin^{-1}\frac{n_2}{n_1}} \tag{6.30}$$

the angle θ_c is the *critical angle* of incidence. In Fig. 6.5, we illustrate the phase matching at $k_z = k_t$.

6.6 Total Transmission and Brewster Angle

Consider total transmission by setting the reflection coefficient equal to zero. For TM wave, $R^{TM} = 0$ yields

$$k_{tx} = \frac{\varepsilon_2 k_{ix}}{\varepsilon_1} \tag{6.31}$$

From the dispersion relations

$$\begin{aligned} k_i^2 &= k_{ix}^2 + k_z^2 \\ k_t^2 &= k_{tx}^2 + k_z^2 = \frac{\varepsilon_2^2}{\varepsilon_1^2}k_{ix}^2 + k_z^2 \end{aligned} \tag{6.32}$$

we find

$$\begin{aligned} k_{ix}^2 &= \frac{k_t^2 - k_i^2}{\left(\varepsilon_2/\varepsilon_1\right)^2 - 1} \\ k_z^2 &= \frac{\left(\varepsilon_2/\varepsilon_1\right)^2 k_i^2 - k_t^2}{\left(\varepsilon_2/\varepsilon_1\right)^2 - 1} \end{aligned} \tag{6.33}$$

Brewster angle is defined for the case of $\mu_2 = \mu_1$, namely for reflection and transmission at a dielectric interface, which is determined to be

$$\boxed{\tan\theta_B^{TM} = \frac{k_z}{k_{ix}} = \frac{\sqrt{\left(\varepsilon_2/\varepsilon_1\right)^2 k_i^2 - k_t^2}}{\sqrt{k_t^2 - k_i^2}} = \frac{k_t}{k_i} = \frac{\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1}}} \tag{6.34}$$

From the phase matching condition, we find $k_t \sin\theta_t = k_i \sin\theta_B^{TM} = k_t \cos\theta_B^{TM}$. It follows that

$$\boxed{\theta_B^{TM} + \theta_t = \frac{\pi}{2}} \tag{6.35}$$

The angle of reflection is equal to the angle of incidence, thus the reflected and transmitted vectors are perpendicular to each other.

For TE waves, when $\mu_2 = \mu_1$, $R^{TE} = 0$ yields $k_{tx} = \mu_2 k_{ix}/\mu_1 = k_{ix}$, which gives $k_t = k_i$. We see that there is no zero reflection unless the two media are identical. When an unpolarized wave is incident upon a dielectric medium at the Brewster angle, the reflected wave becomes linearly polarized perpendicular to the plane of incidence (Fig. 6.6). For this reason, the Brewster angle is also called the polarization angle.

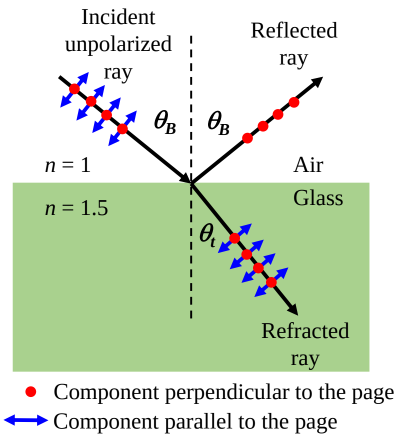

Figure 6.6 At Brewster angle of reflection.

6.7 Reflection and Transmission by a Layered Medium

In Fig. 6.7, consider a plane wave incident on a n-layer stratified isotropic medium with n+2 regions (0, 1, 2, …, n+1). For a TM plane wave, $H_y = H_0 \exp(-jk_x x - jk_z z)$, incident on the stratified medium, the total field in region l can be written as

$$\begin{aligned}
\vec{k}_l &= \hat{x}k_{lx} + \hat{z}k_z \\
\vec{E}_l &= \frac{1}{j\omega\varepsilon_l}\nabla\times\vec{H}_l \\
\vec{H}_l &= \hat{y}\left(A_l e^{-jk_{lx}x} + B_l e^{jk_{lx}x}\right)e^{-jk_z z}
\end{aligned} \tag{6.36}$$

and

$$k_{lx}^2 + k_z^2 = \omega^2\mu_l\varepsilon_l \tag{6.37}$$

is the dispersion relation for region l. We do not write a subscript l for the k_z as a consequence of the phase matching conditions. The amplitude A_l represents all wave components that have a propagating velocity component along the positive x direction, and B_l represents those with a velocity component along the negative x direction.

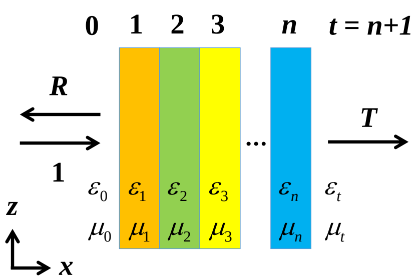

Figure 6.7 Layered medium.

We let, in region 0, $A_0 = 1$, $B_0 = R$, and in region t, $A_t = T$, $B_t = 0$. Notice that region t is semi-infinite and there is no wave propagating with a velocity component in the negative x direction. We denote the transmitted amplitude by T.

The wave amplitudes A_l and B_l are related to wave amplitudes in neighboring regions by the boundary conditions

$$\begin{aligned} \hat{n}\times\left(\vec{E}_l - \vec{E}_{l+1}\right) = 0 \\ \hat{n}\times\left(\vec{H}_l - \vec{H}_{l+1}\right) = 0 \end{aligned} \tag{6.38}$$

In Eq. (6.36), there are $2n+2$ unknown parameters

$$\begin{cases} A_l,\ l = 1, 2, \cdots, n+1 \\ B_l,\ l = 0, 1, 2, \cdots, n \end{cases} \tag{6.39}$$

From the boundary conditions of Eq. (6.38), we also have $2n+2$ equations from $n+1$ boundaries. Therefore, we can completely solve all the unknown parameters A_l and B_l. To solve for the $2n+2$ unknowns from the $2n+2$ linear equations, we can arrange the equations in matrix form with the unknowns forming a $2n+2$ column matrix and the coefficients forming a $(2n+2)\times(2n+2)$ square matrix. The solution is then obtained by inverting the square matrix. This procedure is straightforward but tedious. A simpler way to solve this problem is the use of propagation matrices (Ref.[3]).

6.8 Additional Problems

See References [2-6].

Reference:

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