Physics I: Introduction to Wave Theory
SDU Course Number: sd01232810 (Fall 2024)

Lecture 5: Waves in Media

Outline

- Time-Harmonic Fields
- Lorentz Oscillator Model of an Atom
- Complex Refractive Index
- Plasma in Ionosphere
- Penetration Depth in Conducting Media
- Optical Anisotropy and Birefringence
- Circular Polarization
- Chiral Media

Instantaneous Form of Maxwell's Equations

$$\nabla \cdot \vec{D} = \rho_{free}$$ **(Gauss's Law)**

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$ **(Faraday's Law)**

$$\nabla \cdot \vec{B} = 0$$ **(Magnetic Gauss's Law)**

$$\nabla \times \vec{H} = \vec{J}_{free} + \frac{\partial \vec{D}}{\partial t}$$ **(Ampere's Law)**

$$\nabla \cdot \vec{J} = -\partial \rho / \partial t$$ **(The continuity equation)**

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P} = \varepsilon_0 \left(1 + \chi_e\right) \vec{E} = \varepsilon \vec{E}$$ **ε : permittivity**

$$\vec{B} = \mu_0 \left(\vec{H} + \vec{M}\right) = \mu_0 \left(1 + \chi_m\right) \vec{H} = \mu \vec{H}$$ **μ : permeability**

(Constitutive Relations)

Time-Harmonic Fields

In many practical systems involving electromagnetic waves, the time variations are of cosinusoidal form and are referred to as *time-harmonic.*

For time-harmonic fields, we can relate the instantaneous fields, current density and charge (represented by script letters) to their complex forms (represented by roman letters) by

$$\vec{\mathcal{E}}\left(\vec{r},t\right)=\mathrm{Re}\left\{\vec{E}\left(\vec{r}\right)e^{j\omega t}\right\} \qquad \vec{\mathcal{D}}\left(\vec{r},t\right)=\mathrm{Re}\left\{\vec{D}\left(\vec{r}\right)e^{j\omega t}\right\}$$

$$\vec{\mathcal{B}}\left(\vec{r},t\right)=\mathrm{Re}\left\{\vec{B}\left(\vec{r}\right)e^{j\omega t}\right\} \qquad \vec{\mathcal{H}}\left(\vec{r},t\right)=\mathrm{Re}\left\{\vec{H}\left(\vec{r}\right)e^{j\omega t}\right\}$$

$$\vec{\mathcal{J}}\left(\vec{r},t\right)=\mathrm{Re}\left\{\vec{J}\left(\vec{r}\right)e^{j\omega t}\right\} \qquad \rho\left(\vec{r},t\right)=\mathrm{Re}\left\{\rho\left(\vec{r}\right)e^{j\omega t}\right\}$$

Example: $\vec{E}\left(\vec{r}\right)=e^{-jkz} \longrightarrow \vec{\mathcal{E}}\left(\vec{r},t\right)=\mathrm{Re}\left\{e^{-jkz}e^{j\omega t}\right\}=\cos\left(kz-\omega t\right)$

Time-Harmonic Form of Maxwell's Equations

$$\nabla \cdot \vec{D} = \rho_{free}$$ **(Gauss's Law)**

$$\nabla \times \vec{E} = -j\omega \vec{B}$$ **(Faraday's Law)**

$$\nabla \cdot \vec{B} = 0$$ **(Magnetic Gauss's Law)**

$$\nabla \times \vec{H} = \vec{J}_{free} + j\omega \vec{D}$$ **(Ampere's Law)**

$$\partial/\partial t \rightarrow j\omega \qquad \partial^2/\partial t^2 \rightarrow (j\omega)^2$$

Helmholtz wave equation (Source-Free):

$$\left(\nabla^2 + \omega^2 \mu\varepsilon\right)\vec{E} = 0$$

$$\vec{k} = \hat{x}k_x + \hat{y}k_y + \hat{z}k_z$$

Plane wave solution:

$$k_x^2 + k_y^2 + k_y^2 = \omega^2 \mu\varepsilon = k^2$$

$$\vec{E}\left(\vec{r}\right) = \vec{E}_0 \exp\left(-j\vec{k}\cdot\vec{r}\right)$$

$$\vec{k}\cdot\vec{r} = k_x x + k_y y + k_z z$$

Time-Averaged Poynting Power Vector

$$\vec{S}(t) = \vec{E}(t) \times \vec{H}(t) = \mathrm{Re}\left\{\vec{E}e^{j\omega t}\right\} \times \mathrm{Re}\left\{\vec{H}e^{j\omega t}\right\}$$

$$= \frac{1}{2}\left\{\vec{E}e^{j\omega t} + \vec{E}^{*} e^{-j\omega t}\right\} \times \frac{1}{2}\left\{\vec{H}e^{j\omega t} + \vec{H}^{*} e^{-j\omega t}\right\}$$

$$= \frac{1}{4}\left\{\vec{E} \times \vec{H}e^{j2\omega t} + \vec{E} \times \vec{H}^{*} + \vec{E}^{*} \times \vec{H} + \vec{E}^{*} \times \vec{H}^{*} e^{-j2\omega t}\right\}$$

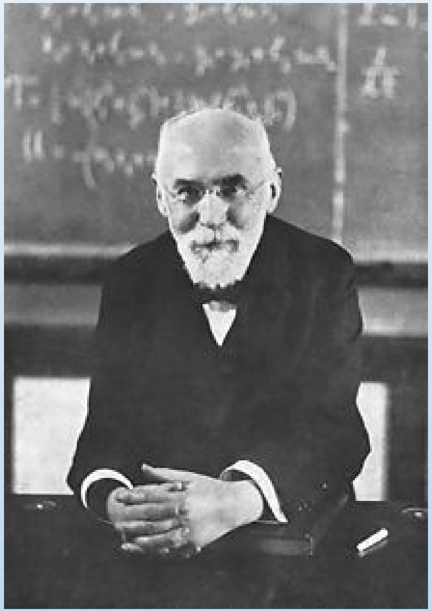
$$= \frac{1}{2}\mathrm{Re}\left\{\vec{E} \times \vec{H}^{*}\right\} + \frac{1}{2}\mathrm{Re}\left\{\vec{E} \times \vec{H}e^{j2\omega t}\right\}$$

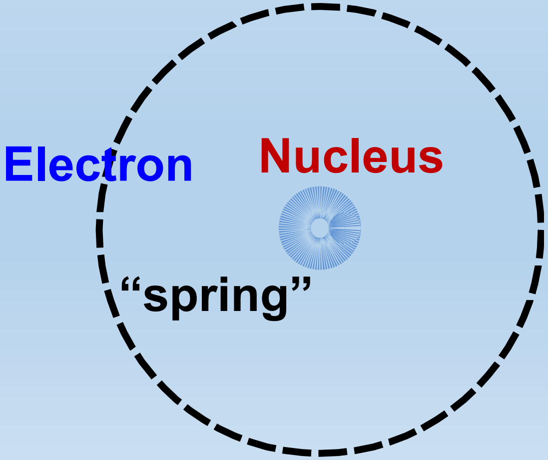
$$\left\langle \vec{S}(t) \right\rangle = \frac{1}{T}\int_0^T \vec{S}(t)\,dt = \frac{1}{2}\mathrm{Re}\left(\vec{E} \times \vec{H}^{*}\right)$$

$$\boxed{\vec{S} = \vec{E} \times \vec{H}^{*}}$$ **(Complex Poynting Vector)**

Lorentz Oscillator

Hendrik Lorentz
(1853-1928)

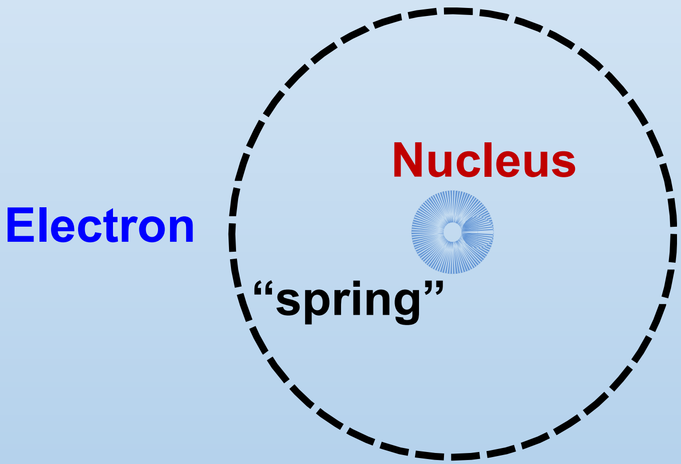




Lorentz was a late nineteenth century physicist, and quantum mechanics had not yet been discovered. However, he did understand the results of classical mechanics and electromagnetic theory. Therefore, he described the problem of atom-field interactions in these terms. Lorentz thought of an atom as a mass (the nucleus) connected to another smaller mass (the electron) by a spring. The spring would be set into motion by an electric field interacting with the charge of the electron. The field would either repel or attract the electron which would result in either compressing or stretching the spring.

Atomic Oscillators

Classical model of an atom. Electrons are bound to the nucleus by springs (due to the Coulomb force), which determine the natural frequencies



Hooke's Law: $$F\left(y\right) = -ky$$

Newton's 2nd Law: $$F\left(y\right) = m\frac{d^2 y}{dt^2}$$

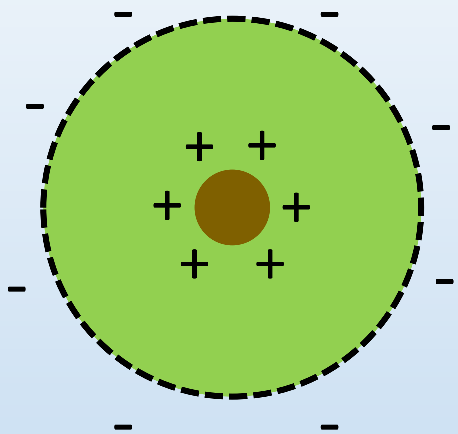
$$\frac{d^2 y}{dt^2} = -\frac{k}{m}y \quad \Rightarrow \quad y = \cos\left(\omega_0 t + \phi\right)$$

$$\omega_0 = \sqrt{k/m}$$

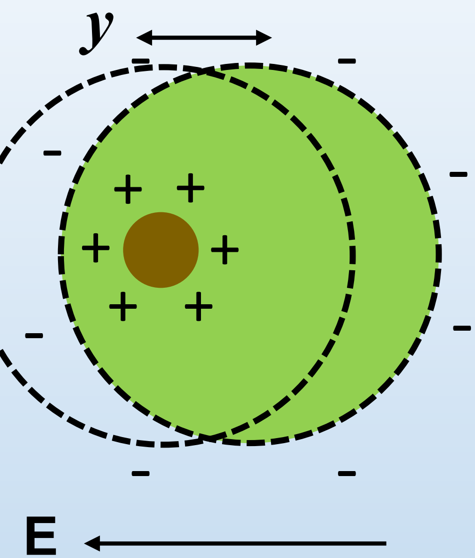
Resonant frequency (or natural frequency)

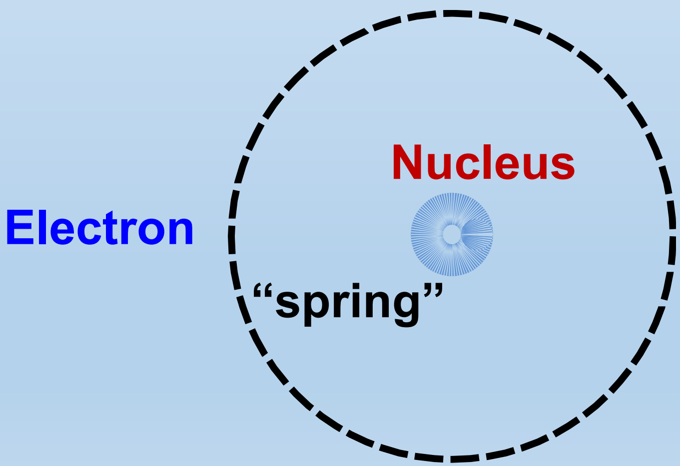
$$k = m\omega_0^2$$

Atomic Oscillators under Electric Field



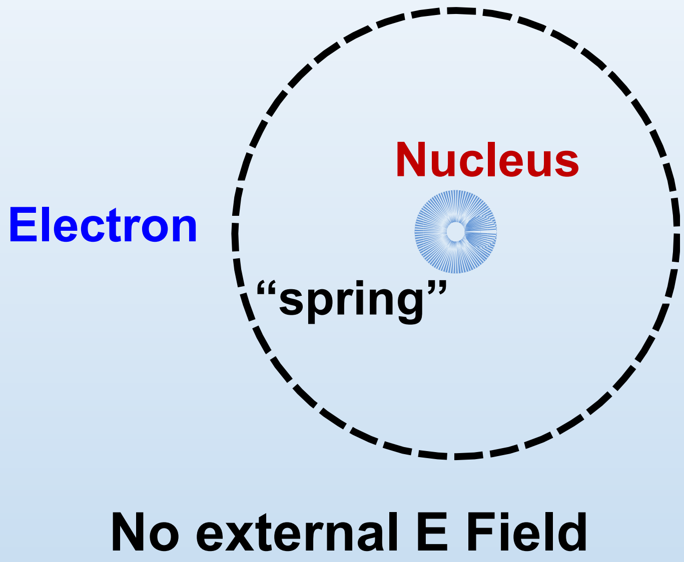
No external E Field

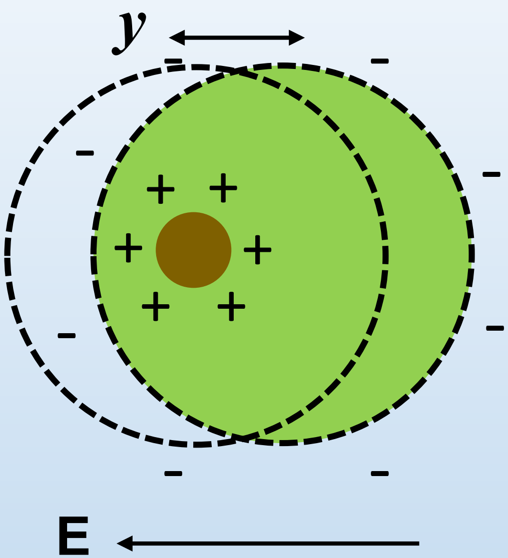




$$\boxed{m\frac{d^2y}{dt^2} = \overbrace{-eE}^{\text{electric force}} - \underbrace{\gamma m v}_{\text{damping force}} - \overbrace{ky}^{\text{restoring force}}}$$

Lorentz Oscillator Model





$$P = -Ney$$

$$m\frac{d^2y}{dt^2} = -eE - m\gamma\frac{dy}{dt} - m\omega_0^2 y \quad \Rightarrow \quad m\left(\omega^2 - \gamma j\omega - \omega_0^2\right)y = eE$$

$$\boxed{\varepsilon(\omega) = \varepsilon_0\left(1 + \frac{P}{\varepsilon_0 E}\right) = \varepsilon_0\left(1 - \frac{\omega_p^2}{\omega^2 - \omega_0^2 - j\gamma\omega}\right)}$$

$$\omega_P = \sqrt{Ne^2/m\varepsilon_0}$$ **(Plasma frequency)**

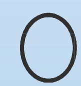
Oscillator Resonance

$$y = \frac{e}{m} \frac{1}{\left(\omega^2 - \omega_0^2\right) - j\gamma\omega} E_y$$

$$E_y(t) = Re\{E_y e^{j\omega t}\}$$

$$y(t) = Re\{y e^{j\omega t}\}$$

Driven harmonic oscillator: **Amplitude** and **Phase** depend on frequency



Low frequency

medium amplitude

Displacement, y
in phase with E_y

At resonance

large amplitude

Displacement, y
90º out of phase with E_y

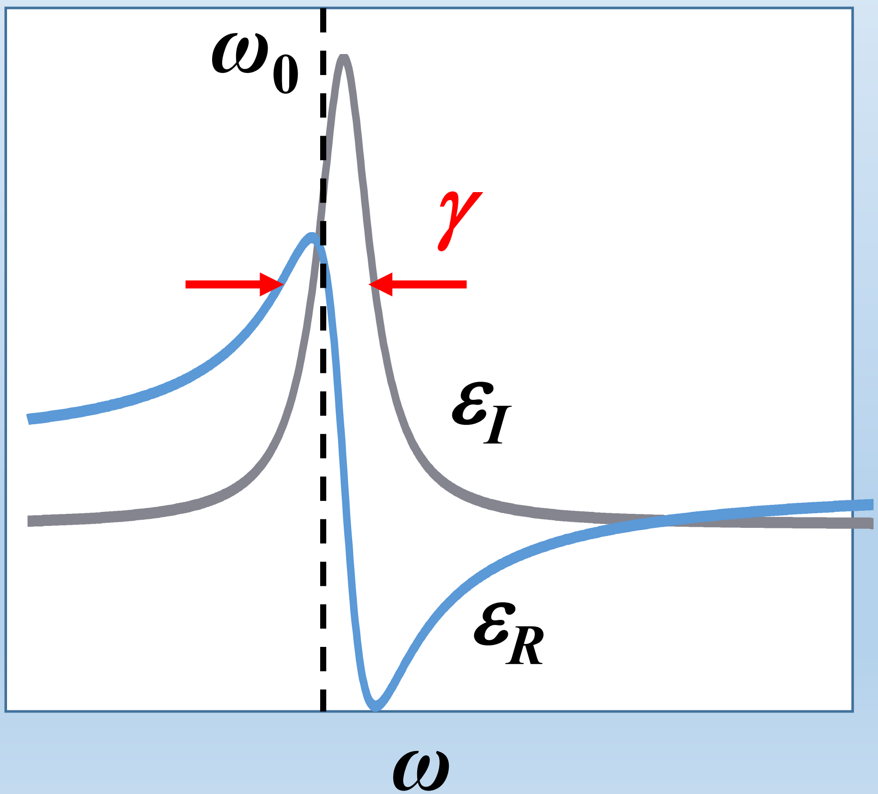
High frequency

vanishing amplitude

Displacement y and E_y
in antiphase

Complex permittivity

$$\varepsilon(\omega) = \varepsilon_0\left(1 - \frac{\omega_p^2}{\omega^2 - \omega_0^2 - j\gamma\omega}\right) = \varepsilon_0\ (\varepsilon_R - j\varepsilon_I)$$



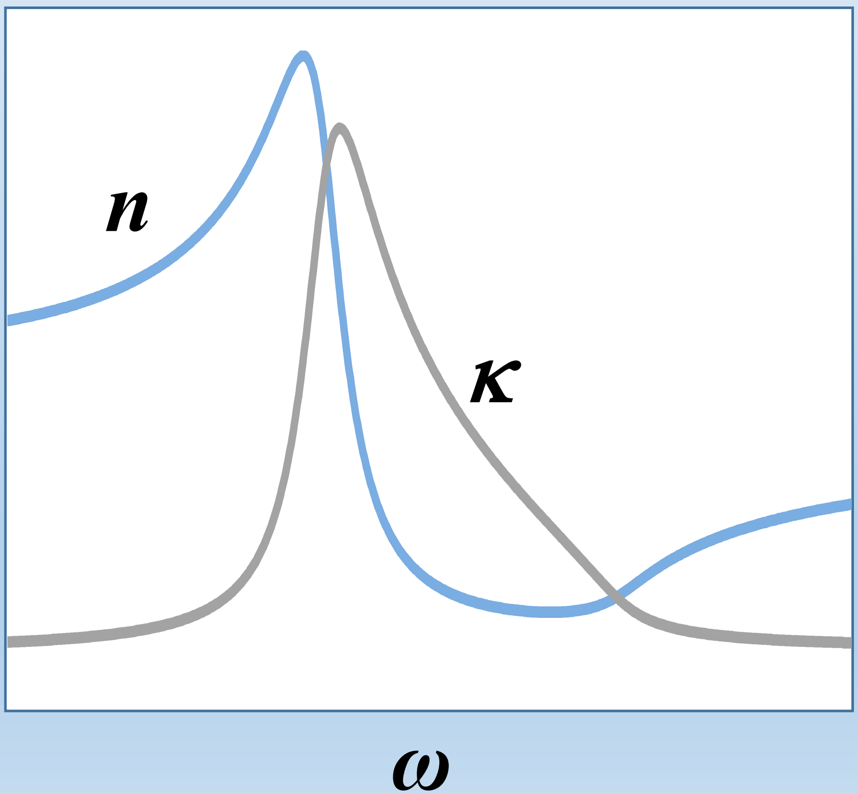
$$\varepsilon_R = 1 + \frac{\omega_p^2\left(\omega_0^2 - \omega^2\right)}{\left(\omega^2 - \omega_0^2\right)^2 + (\gamma\omega)^2}$$

$$\varepsilon_I = \frac{\omega_p^2\gamma\omega}{\left(\omega^2 - \omega_0^2\right)^2 + (\gamma\omega)^2}$$

Around the resonance frequency ω_0, the magnitude of ε_R has a drastic change and ε_I has the maximum value.

Complex refractive index

Refractive index is defined as the ratio between the propagation speed of light in vacuum and the propagation speed of light in the medium.



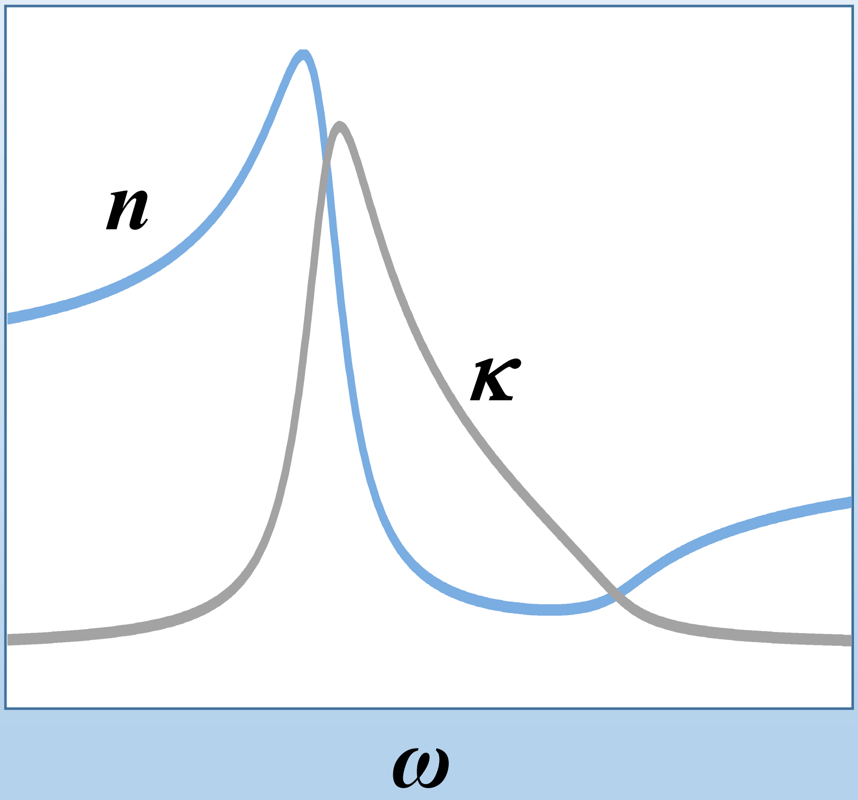
$$\tilde{n} \equiv \frac{c}{v_p} = \frac{\sqrt{\varepsilon\mu}}{\sqrt{\varepsilon_0\mu_0}}$$

$$\tilde{n} = n - j\kappa = \sqrt{\varepsilon_R - j\varepsilon_I}$$

$$\tilde{n} = n^2 - 2jn\kappa - \kappa^2 = \varepsilon_R - j\varepsilon_I$$

$$\boxed{\begin{aligned} \varepsilon_R &= n^2 - \kappa^2 \\ \varepsilon_I &= 2n\kappa \end{aligned}}$$

Absorption



$$n = \frac{1}{\sqrt{2}}\sqrt{\varepsilon_R + \sqrt{\varepsilon_R^2 + \varepsilon_I^2}}$$

$$\kappa = \frac{1}{\sqrt{2}}\sqrt{-\varepsilon_R + \sqrt{\varepsilon_R^2 + \varepsilon_I^2}}$$

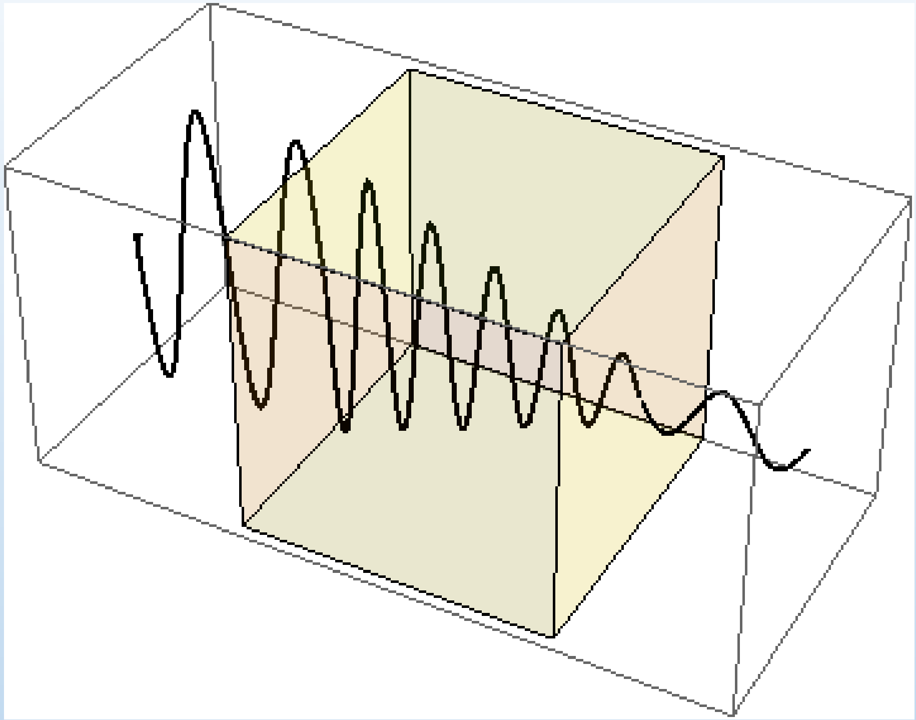
$$E = \exp\left(-jnk_0 z\right)$$

$$= \exp\left[-j\left(n - j\kappa\right)k_0 z\right]$$

$$= \exp\left(\underline{-\alpha z/2} - jnk_0 z\right)$$

Absorption

$$\boxed{\alpha = 2k_o\kappa = 2\frac{2\pi}{\lambda_o}\kappa \quad [\text{cm}^{-1}]}$$



$$E(t,z) = Re\{\tilde{E}_0 e^{-\alpha z/2} e^{j(\omega t - k_0 n z)}\}$$

Absorption coefficient

Refractive index

$$I(z) = I_o e^{-\alpha z}$$ Beer-Lambert Law or Beer's Law

Dispersion

$$\varepsilon(\omega) = \varepsilon_0\left(1 - \frac{\omega_p^2}{\omega^2 - \omega_0^2 - j\gamma\omega}\right)$$

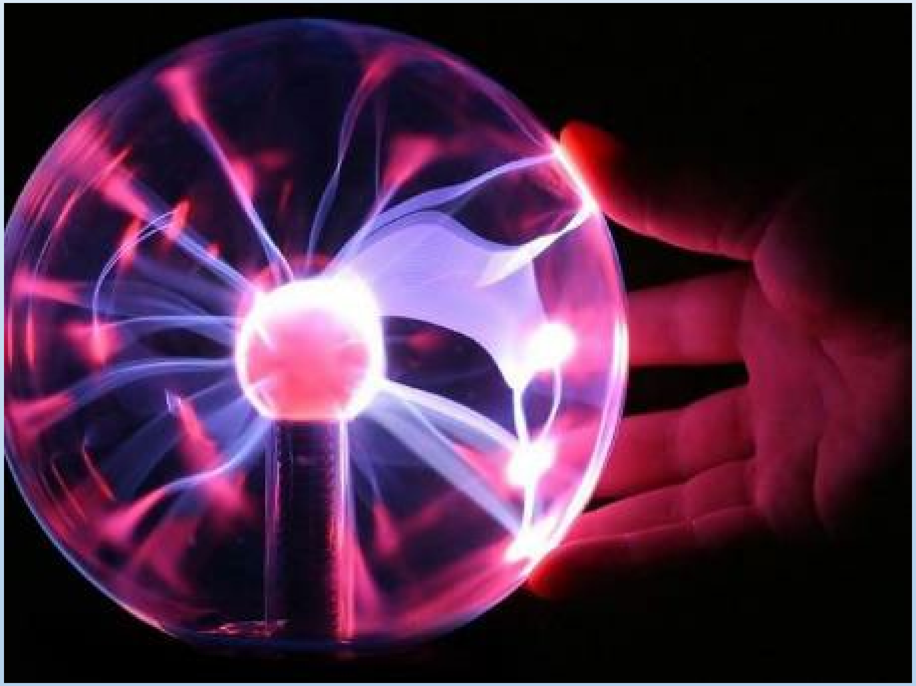
(Lorentz Oscillator Model)

Prism

The above equation shows that permittivity depends on the frequency, besides the plasma frequency and damping (which are properties of the medium). A medium displaying such behavior (that is, whose permittivity depends on the frequency of the wave) is called dispersive, named after ”dispersion”, which is the phenomenon exhibited in a prism or raindrop that causes white light to be spread out into a rainbow of colors (white light is a mixture of beams of many different colors {all traveling at the same speed, but having different frequencies and wavelengths).

Plasma in Ionosphere

Plasma is an ionized gas consisting of positively charged molecules (ions) and negatively charged electrons that are free to move. Plasma exists naturally in what we call ionosphere (80 km ~ 120 km above the surface of the Earth).



The effect of a conducting object (a hand) touching the plasma globe

An aurora seen above Bear Lake, Alaska, USA

For Plasma, we will assume $\gamma = 0$ (lossless) and $\omega_0 = 0$

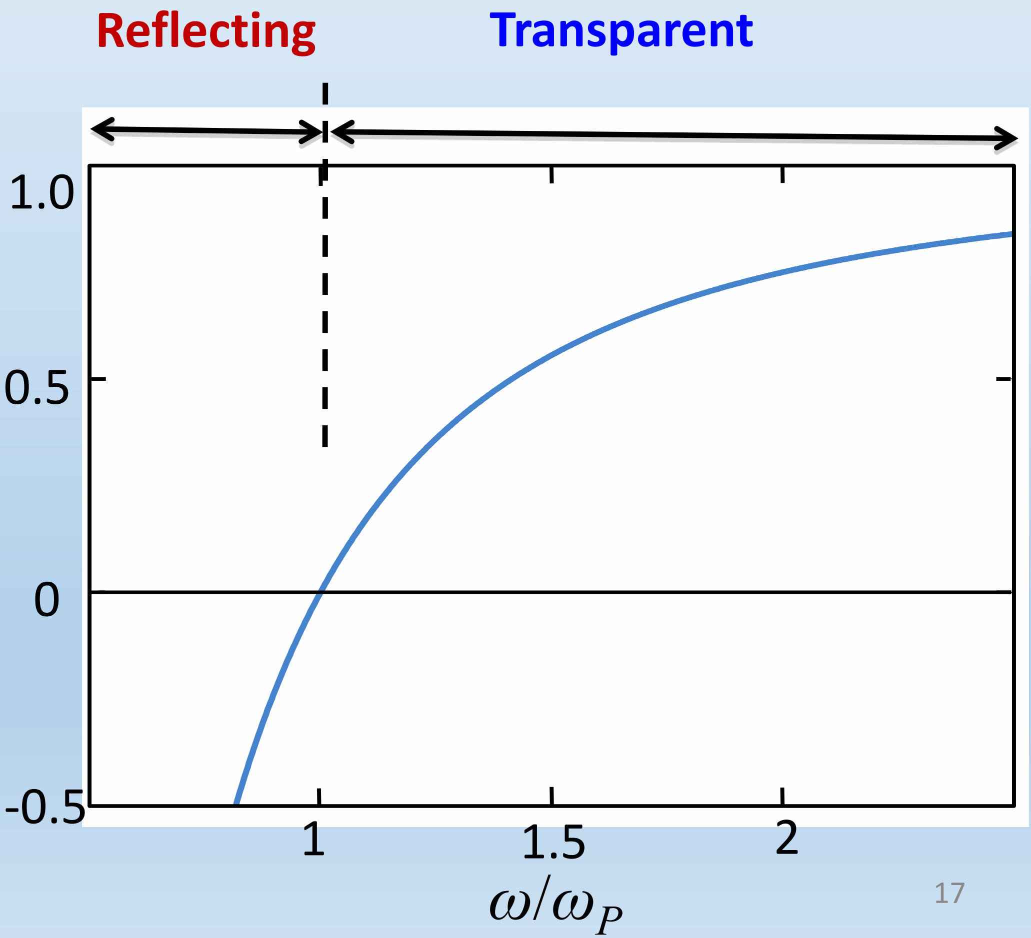
$$\varepsilon(\omega) = \varepsilon_0\left(1 - \frac{\omega_p^2}{\omega^2 - \omega_0^2 - j\gamma\omega}\right) = \varepsilon_0\left(1 - \frac{\omega_p^2}{\omega^2}\right)$$

$$\omega_P = \sqrt{Ne^2/m\varepsilon_0}$$

(Plasma frequency)

$$\omega < \omega_P \longrightarrow \varepsilon < 0$$
$$\omega > \omega_P \longrightarrow \varepsilon > 0$$

What happens when the dielectric constant is negative?



Plasma frequency

$$\omega_P = \sqrt{Ne^2 / m\varepsilon_0}$$ **(In the ionosphere)**

$$= \sqrt{\frac{\left(10^{12} m^{-3}\right)\left(1.602 \times 10^{-19} C\right)^2}{\left(9.1 \times 10^{-31} kg\right)\left(8.85 \times 10^{-12} F/m\right)}}$$

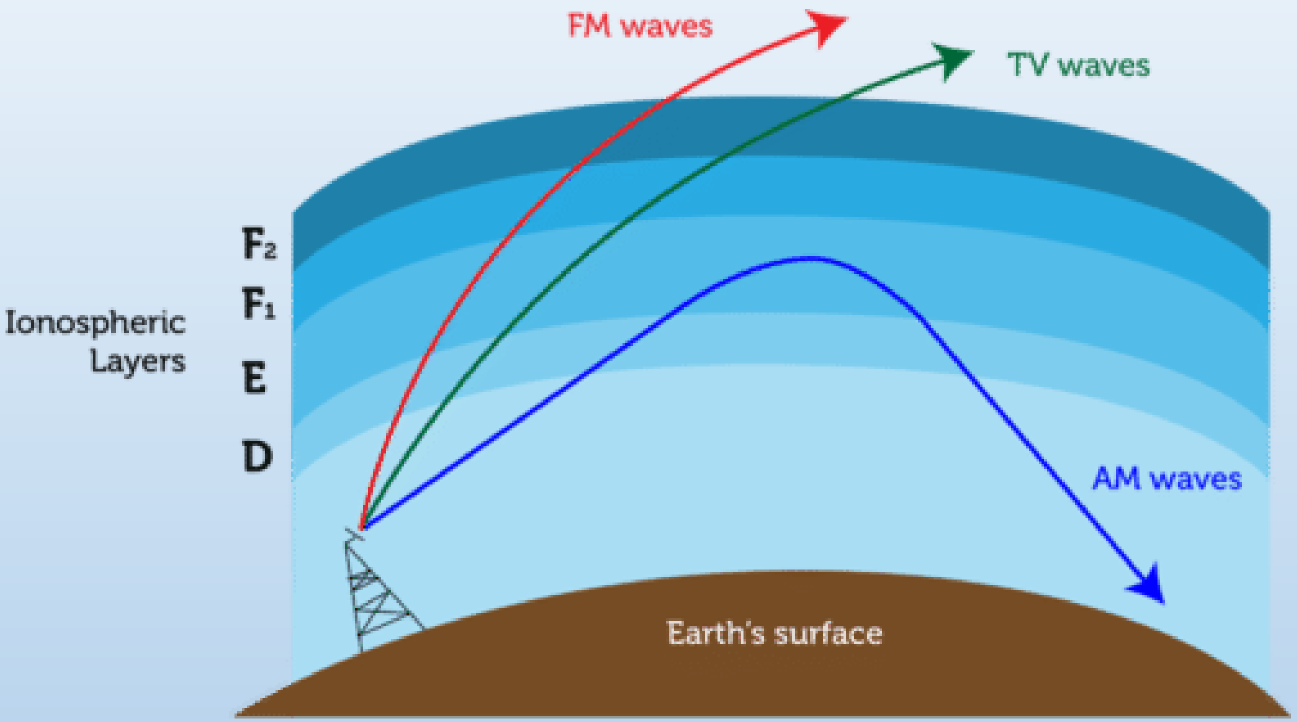
$$= 5.64 \times 10^7 \ rad/s$$

$$= 2\pi \times \left(8.98 MHz\right)$$

AM radio is in the range 520-1610 kHz **Reflected**

FM radio in in the range 87.5 to 108 MHz **Transmitted**

Radio Waves in Atmosphere



AM radio is in the range 520-1610 kHz **Reflected**

FM radio in in the range 87.5 to 108 MHz **Transmitted**

Behavior of Metals

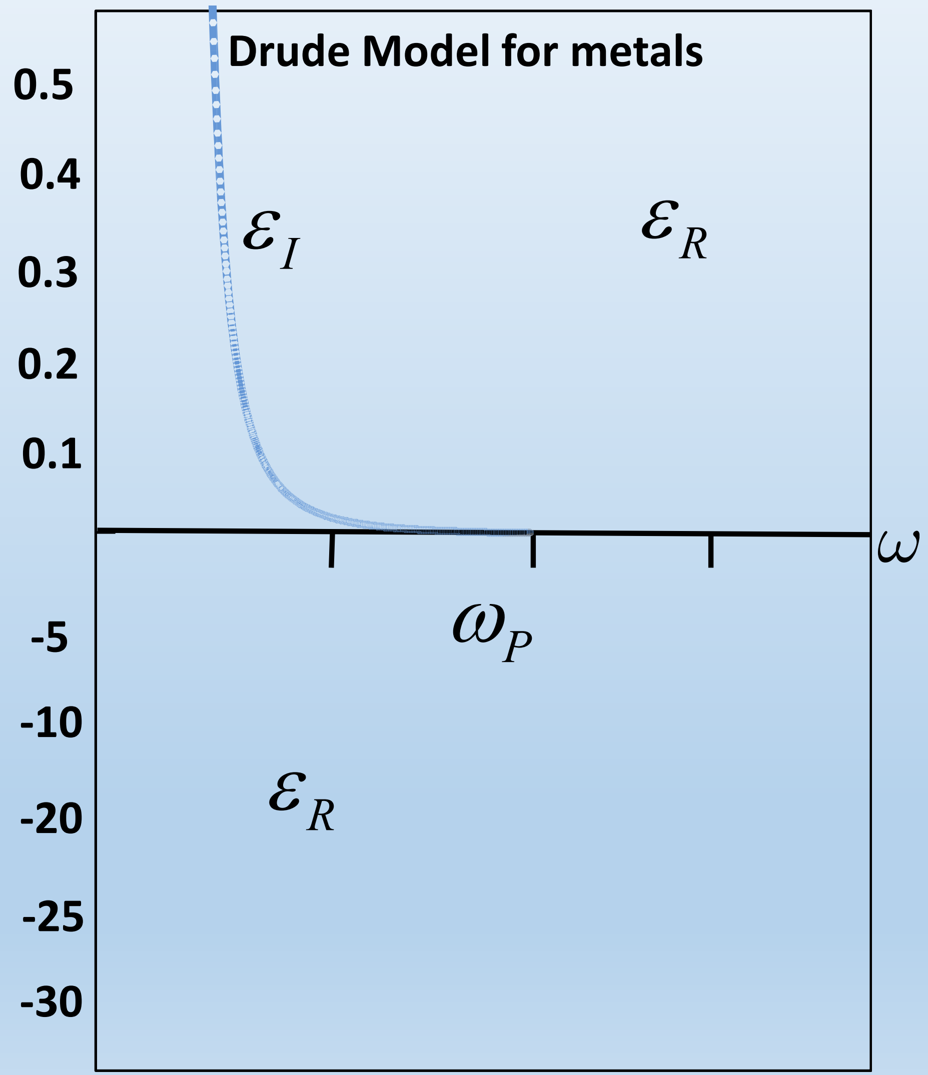
For metal, we have $\gamma \neq 0$ (lossy) and $\omega_0 = 0$

$$\varepsilon(\omega) = \varepsilon_0 \left(1 - \frac{\omega_p^2}{\omega^2 - j\gamma\omega} \right)$$

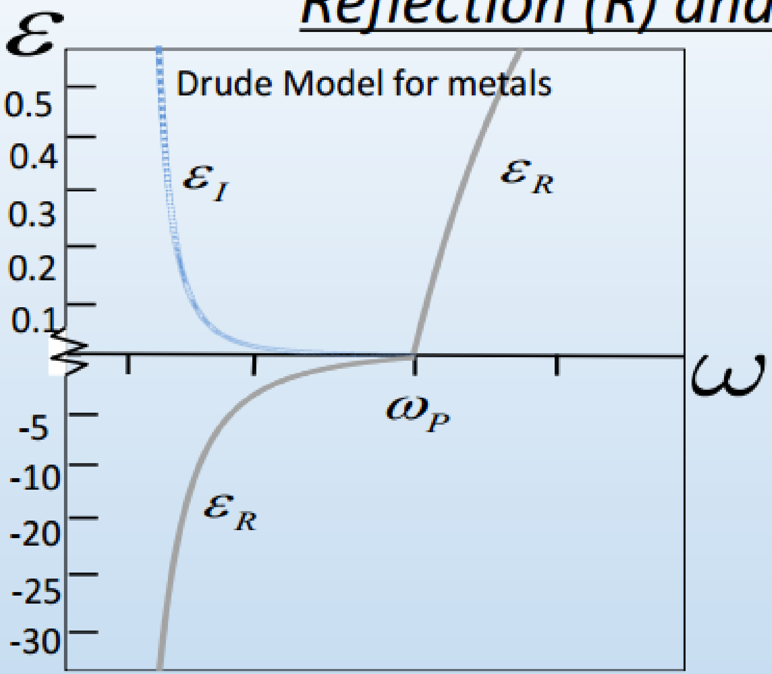
$$= \varepsilon_R - j\varepsilon_I$$

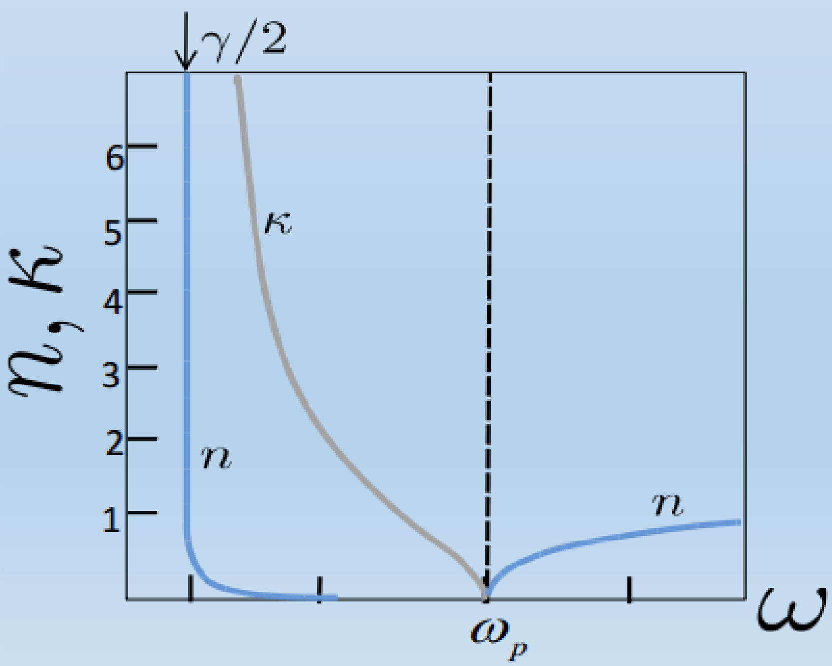
$$\varepsilon_R = \varepsilon_0 \left(1 - \frac{\omega_p^2}{\omega^2 + \gamma^2} \right)$$

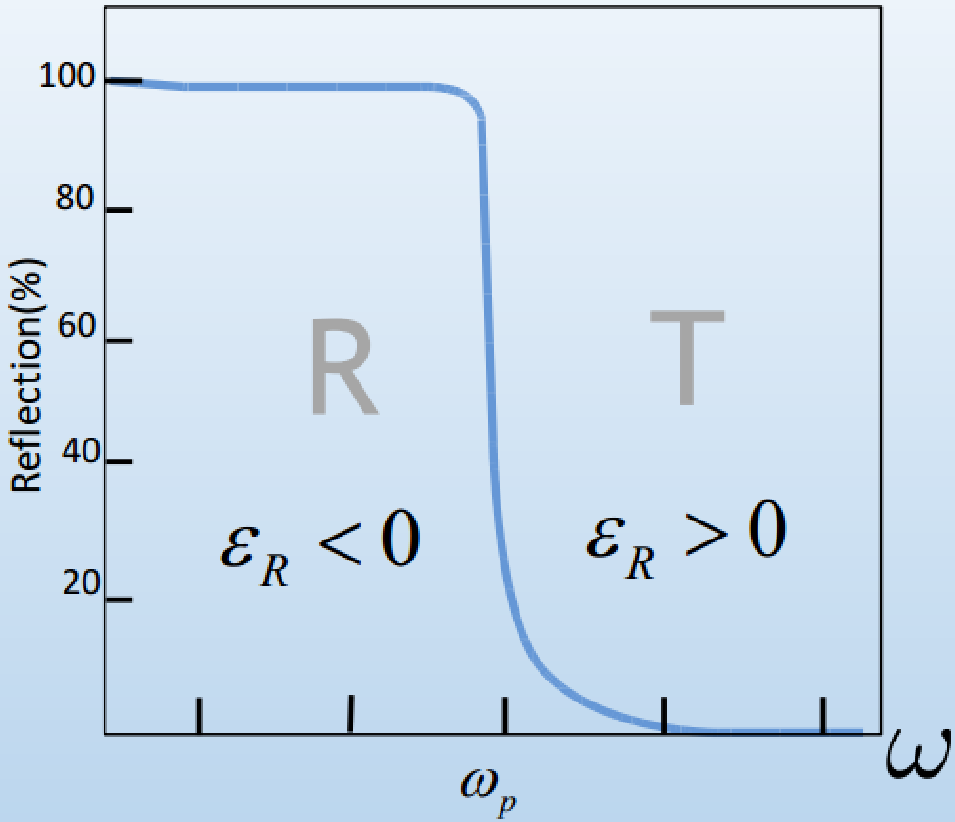
$$\varepsilon_I = \varepsilon_0 \left(\frac{\gamma\omega_p^2}{\omega\left(\omega^2 + \gamma^2\right)} \right)$$



Reflection (R) and transmission (T) of Metals







$$n = \frac{1}{\sqrt{2}} \sqrt{\varepsilon_R + \sqrt{\varepsilon_R^2 + \varepsilon_I^2}}$$

$$\kappa = \frac{1}{\sqrt{2}} \sqrt{-\varepsilon_R + \sqrt{\varepsilon_R^2 + \varepsilon_I^2}}$$

Conducting Media

Consider a conducting medium governed by Ohm's law

$$\vec{J}_c = \sigma \vec{E} \qquad \nabla \times \vec{H} = j\omega \vec{D} + \vec{J}_{free} + \vec{J}_c$$

$$\Downarrow$$

$$\nabla \times \vec{H} = j\omega \left(\varepsilon + \frac{\sigma}{j\omega} \right) \vec{E} + \vec{J}_{free}$$

We can define a new permittivity for conducting media

$$\boxed{\varepsilon_c = \varepsilon + \frac{\sigma}{j\omega} = \varepsilon - j\frac{\sigma}{\omega}}$$

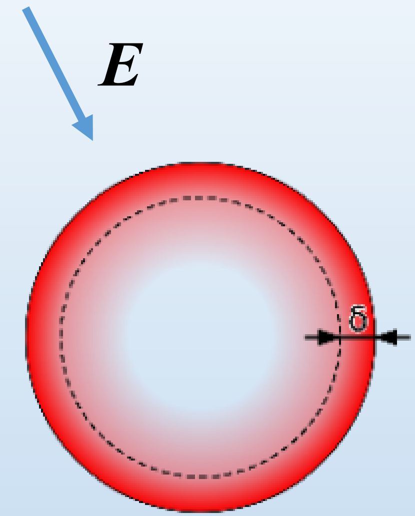
$$\varepsilon(\omega) = \varepsilon_0 \left(1 - \frac{\omega_p^2}{\omega^2 - j\gamma\omega} \right)$$

Penetration Depth

$$\varepsilon_c = \varepsilon + \frac{\sigma}{j\omega} = \varepsilon - j\frac{\sigma}{\omega}$$

$$k = \omega\sqrt{\mu\varepsilon}\left[1 - j\frac{\sigma}{\omega\varepsilon}\right]^{1/2} = k_R - jk_I$$

$$E = \exp(-jkz) = \exp(-k_I z - jk_R z)$$



Penetration depth is defined as

$$\boxed{d_P = \frac{1}{k_I}}$$

- **The wave amplitude attenuates by a factor of e^{-1} in a distance d_p**
- **d_p of Copper at 10 GHz is about 0.65 um**

For a highly conducting medium with $1 \ll \sigma/\omega\varepsilon$

$$k = \omega\sqrt{\mu\varepsilon}\left[1 - j\frac{\sigma}{\omega\varepsilon}\right]^{1/2} \approx \omega\sqrt{\mu\varepsilon}\left[-j\frac{\sigma}{\omega\varepsilon}\right]^{1/2} = \sqrt{\frac{\omega\mu\sigma}{2}}[1 - j]$$

$$\boxed{d_P = \sqrt{\frac{2}{\omega\mu\sigma}}}$$

Skin depth

For a slightly conducting medium with $1 \gg \sigma/\omega\varepsilon$

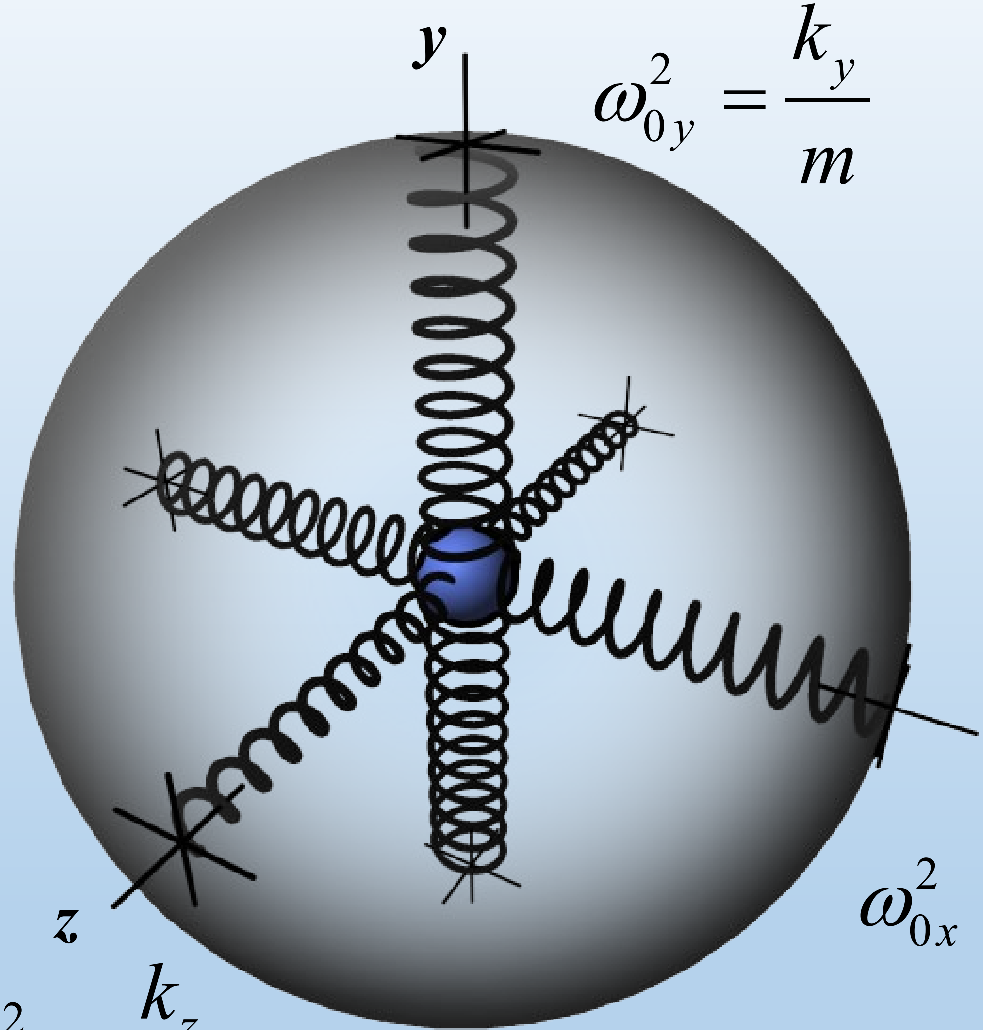
$$k = \omega\sqrt{\mu\varepsilon}\left[1 - j\frac{\sigma}{\omega\varepsilon}\right]^{1/2} \approx \omega\sqrt{\mu\varepsilon}\left[1 - j\frac{\sigma}{2\omega\varepsilon}\right] = \omega\sqrt{\mu\varepsilon} - j\frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}$$

$$\boxed{d_P = \frac{2}{\sigma}\sqrt{\frac{\varepsilon}{\mu}}}$$

Anisotropic Material

The molecular "spring constant" can be different for different directions

If $\omega_{0x} = \omega_{0z}$,
then the material has a single optics axis and is called
uniaxial crystal



$$\omega_{0y}^2 = \frac{k_y}{m}$$

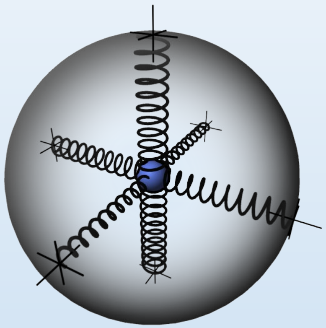
$$\omega_{0x}^2 = \frac{k_x}{m}$$

$$\omega_{0z}^2 = \frac{k_z}{m}$$

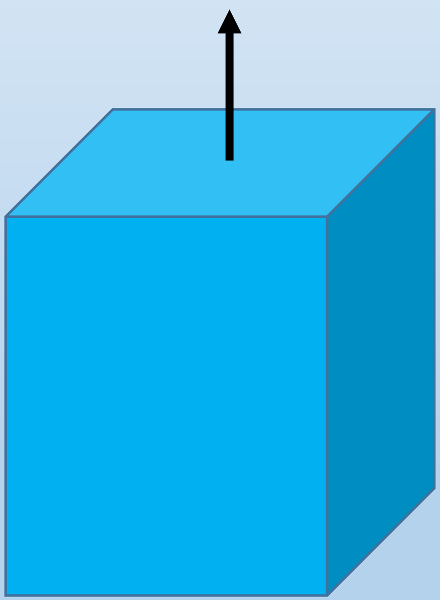
$$\varepsilon_x \neq \varepsilon_y \neq \varepsilon_z$$

$$n_x \neq n_y \neq n_z$$

Uniaxial Crystal



Optic axis



Uniaxial crystals have one refractive index for light polarized along the **optic axis** (n_e)

and another for light polarized in either of the two directions perpendicular to it (n_o).

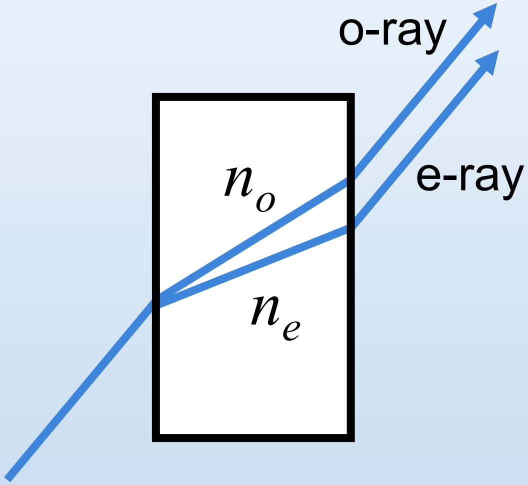
Light polarized along the optic axis is called the **extraordinary** ray,

and light polarized perpendicular to it is called the **ordinary** ray.

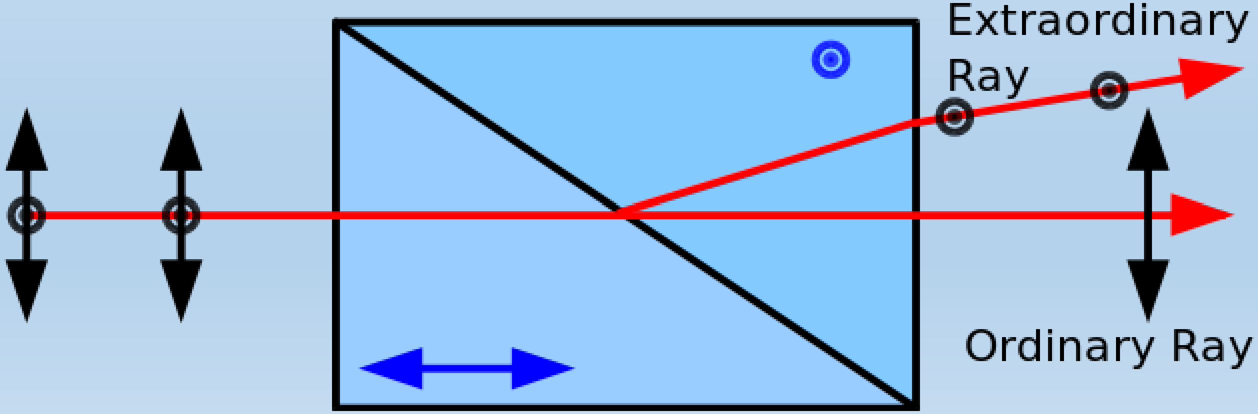
These polarization directions are the **crystal principal axes**.

Birefringence

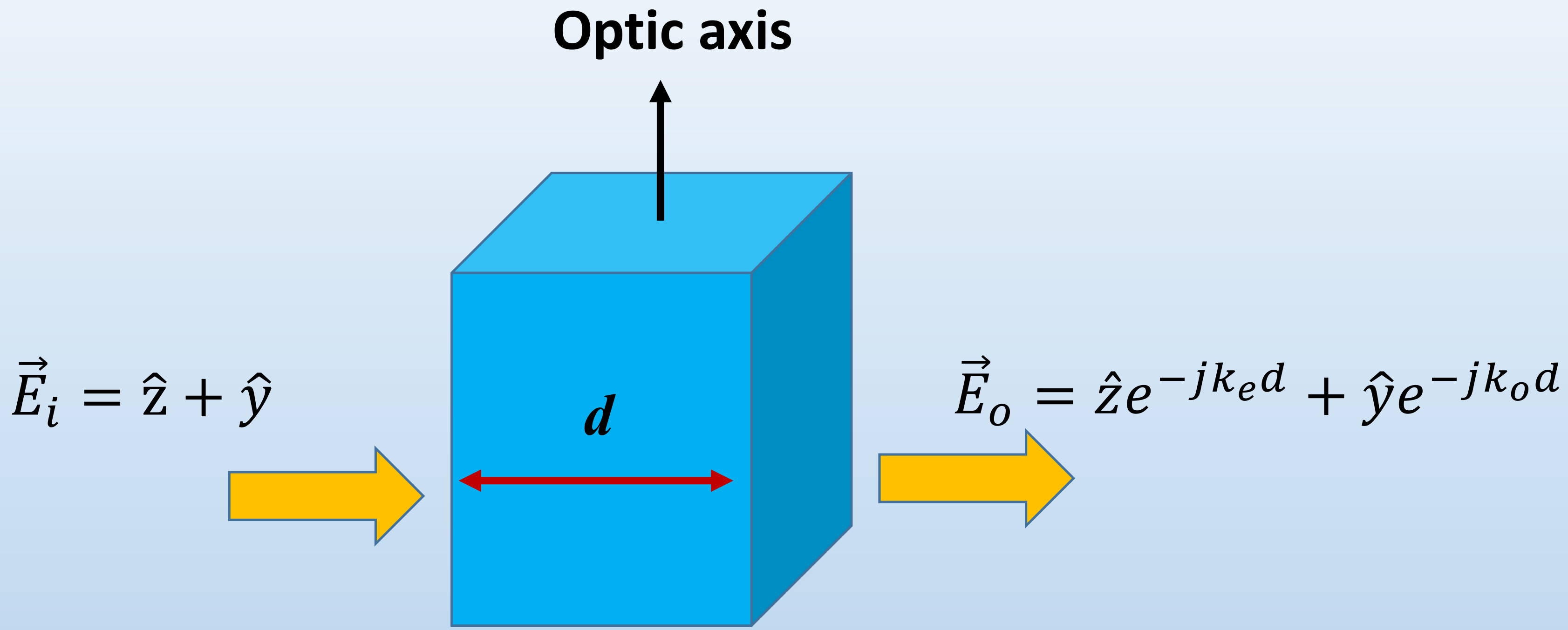
A calcite crystal



Rochon prism



Polarization Conversion



Polarization of output wave is determined by

$$\frac{E_y}{E_z} = \frac{e^{-jk_o d}}{e^{-jk_e d}} = e^{-j(k_o - k_e)d}$$

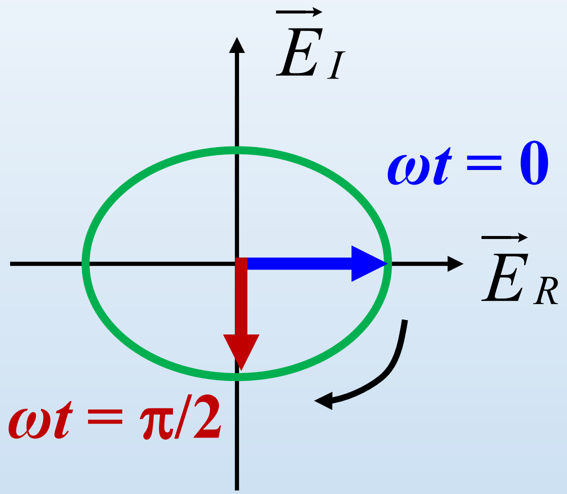
Polarization of Monochromatic Waves

At plane z = 0:

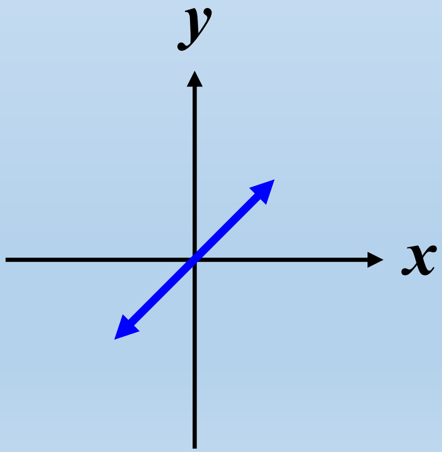
$$\vec{E} = \vec{E}_R + j\vec{E}_I$$

$$\vec{E}(t) = \mathrm{Re}\left\{\left(\vec{E}_R + j\vec{E}_I\right)e^{j\omega t}\right\}$$

$$= \vec{E}_R \cos\omega t - \vec{E}_I \sin\omega t$$

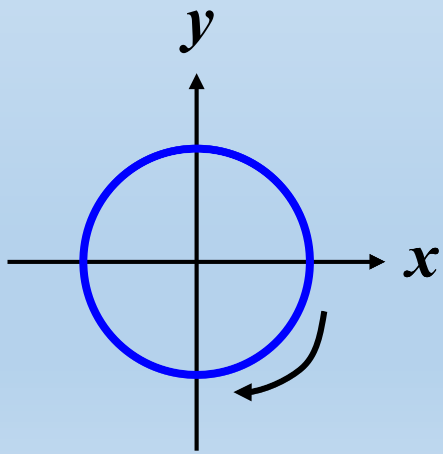


(a) Linear Polarization



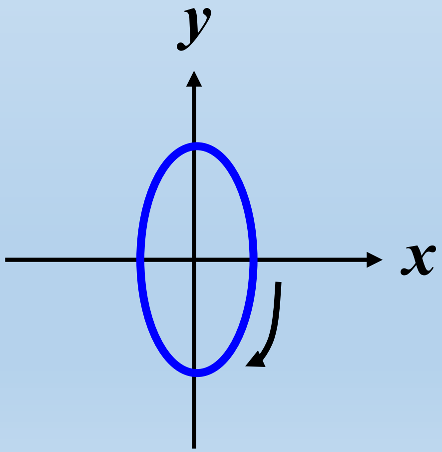
$$\vec{E} = \hat{x} + \hat{y}$$

(b) Circular Polarization



$$\vec{E} = \hat{x} + j\hat{y}$$

(c) Elliptical Polarization

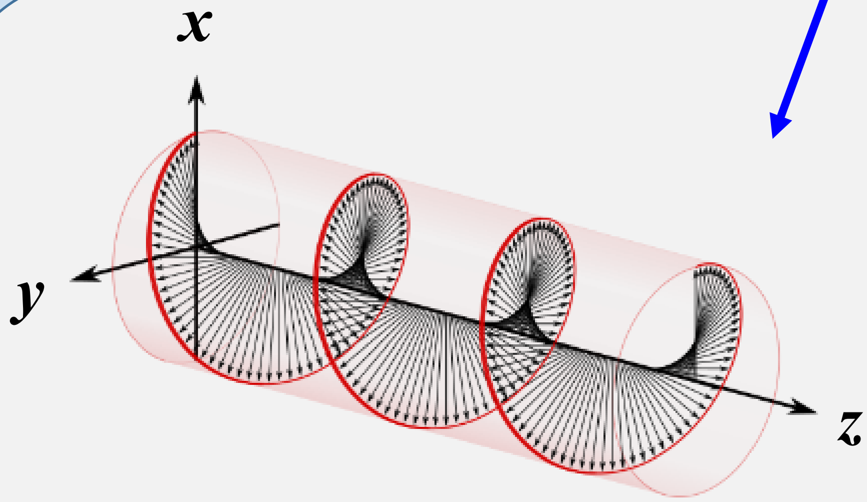


$$\vec{E} = 0.5\hat{x} + j\hat{y}$$

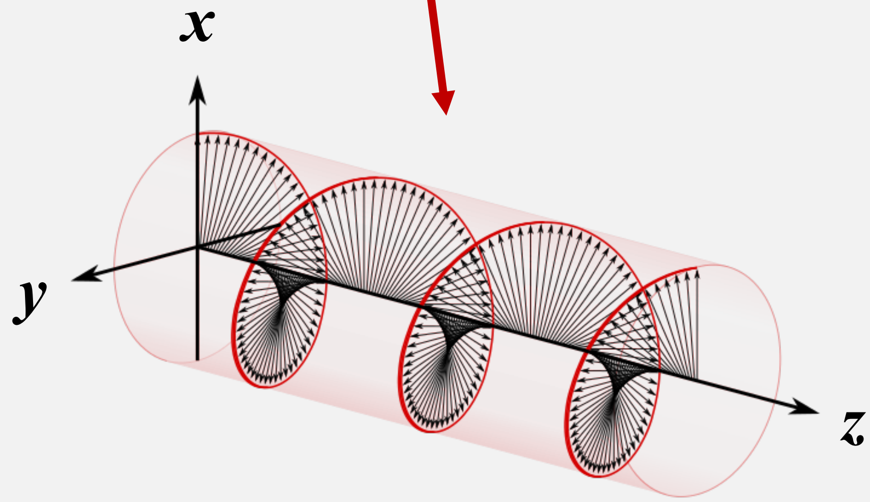
Circular Polarization

For any plane wave propagating along +z direction, the electric field can be decomposed into LCP and RCP components.

$$\vec{E} = \vec{E}_R + j\vec{E}_I = E_{LCP}\frac{\hat{x} + j\hat{y}}{\sqrt{2}} + E_{RCP}\frac{\hat{x} - j\hat{y}}{\sqrt{2}}$$



Left-handed Circularly Polarized Waves (LCP waves)



Right-handed Circularly Polarized Waves (RCP waves)

Question

What is the polarization of the wave?

$$\vec{E}(z,t) = \hat{x}E_0 \sin\left(kz - \omega t - \frac{\pi}{4}\right) + \vec{y}E_0 \cos\left(kz - \omega t + \frac{\pi}{4}\right)$$

$$\vec{E}(z,t) = \hat{x}E_0 \cos(kz - \omega t) + \vec{y}E_0 \sin\left(kz - \omega t - \frac{\pi}{4}\right)$$

3D Movies Technology

Polarizer A

Left eye source

Film or digital projector

Right eye source

Film or digital projector

Polarizer B

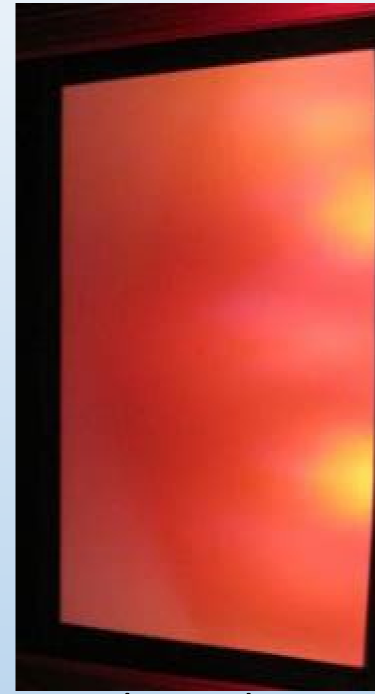
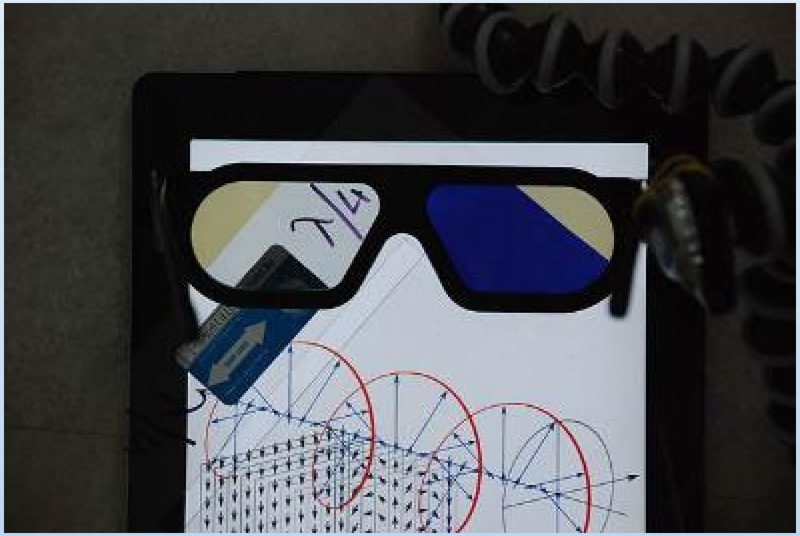


Image by comedy_nose http://www.flickr.com/photos/comedynose/4482682966/ on flickr

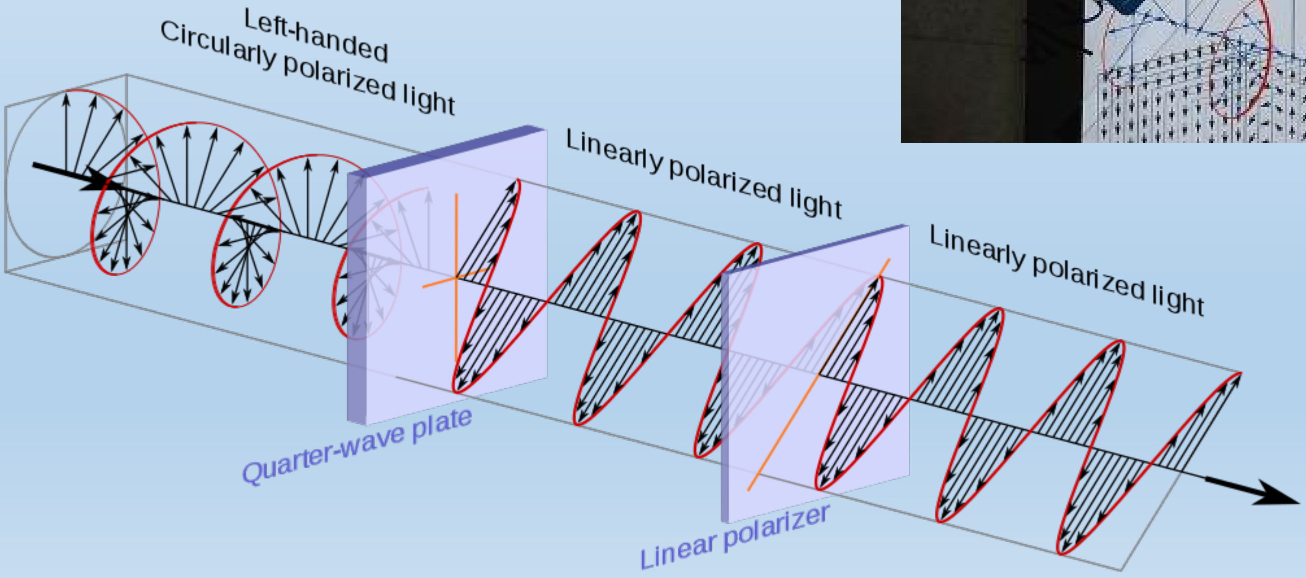
Which approach is better ? Linear or circular polarization ?

Polarized 3D glasses

Circularly polarized 3D glasses in front of an LCD tablet with a quarter-wave retarder on top of it; the λ/4 plate at 45° produces a definite handedness, which is transmitted by the left filter but blocked by the right filter.

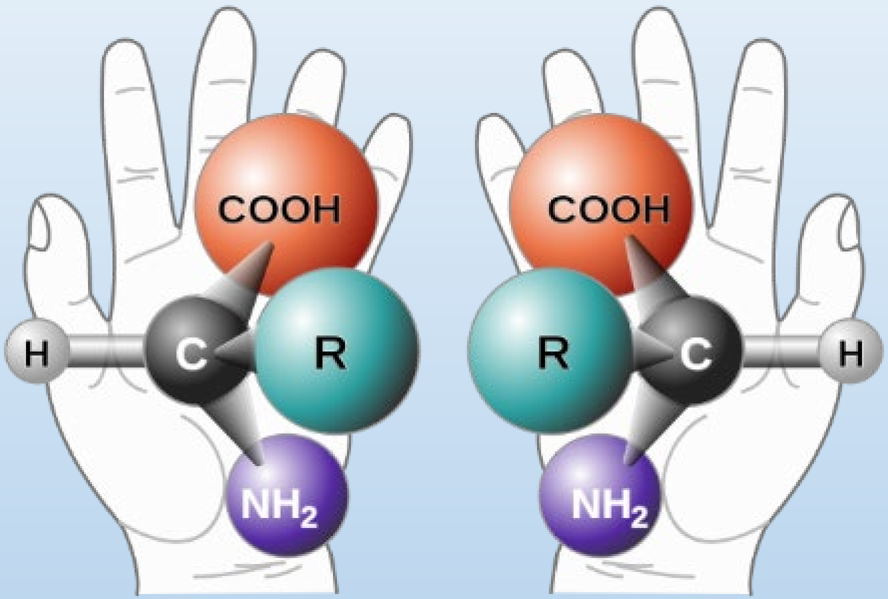


Circular polarizer

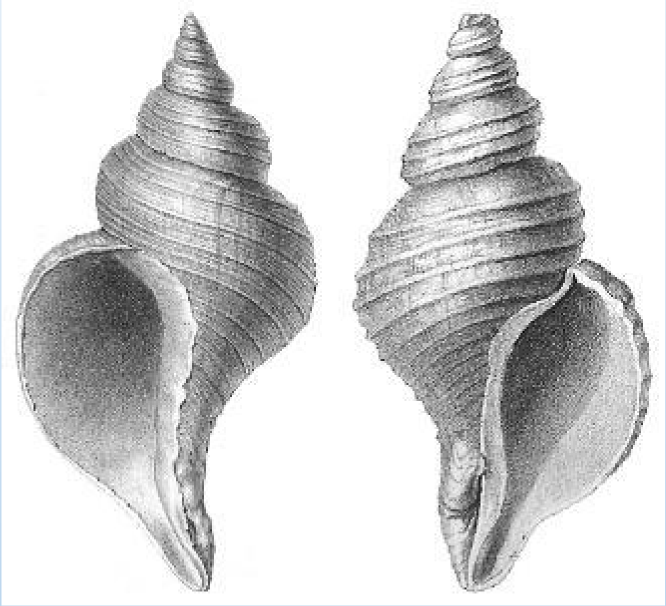


Chiral Materials

An object or a system is chiral if it is distinguishable from its mirror image.



Two enantiomers of a generic amino acid that is chiral



Shells of two different species of sea snail: on the left is the normally sinistral (left-handed) shell of *Neptunea angulata*, on the right is the normally dextral (right-handed) shell of *Neptunea despecta*

Chiral materials…
…different interactions with left- and right-circular polarizations

Optical Properties of Chiral Media

Constitutive relation for chiral media

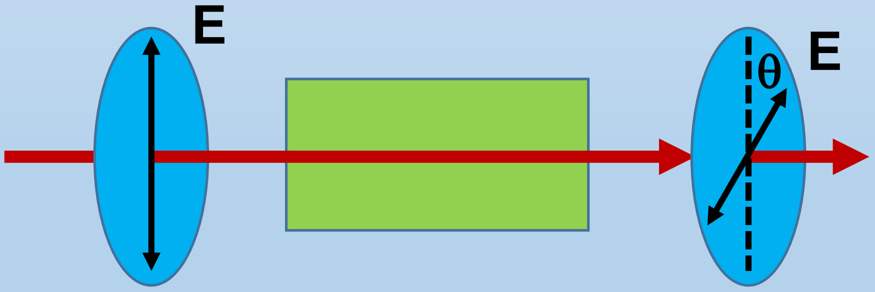
$$\vec{D} = \varepsilon_0 \varepsilon_r \vec{E} - j\kappa\sqrt{\varepsilon_0 \mu_0}\,\vec{H}$$

$$\vec{B} = \mu_0 \mu_r \vec{H} + j\kappa\sqrt{\varepsilon_0 \mu_0}\,\vec{E}$$

Refractive index

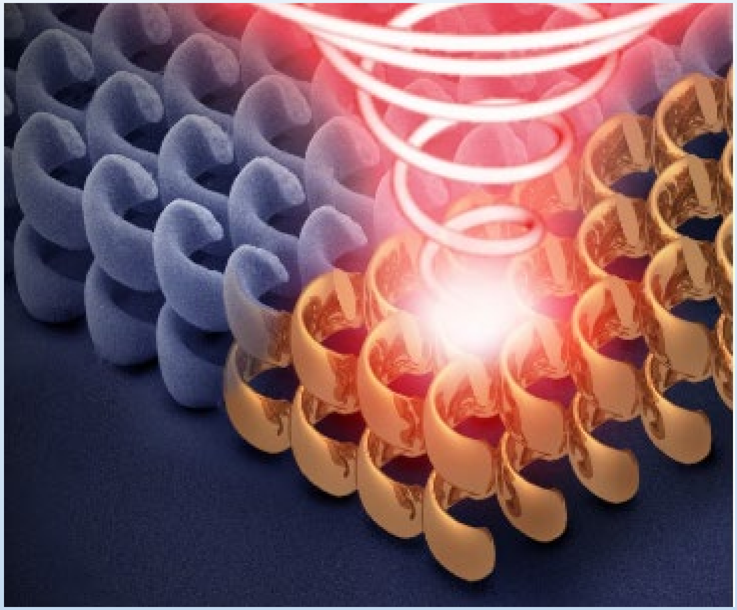
$$n_{R/L} = \sqrt{\varepsilon_r \mu_r} \pm \kappa$$

Optical rotation



$$\theta = \frac{\pi d}{\lambda}\left(n_R - n_L\right)$$

Natural and Artificial Chiral Materials



- How does biological handedness arise? (1 of 125 big questions posed by *Science* Magazine)
- Chiral metamaterials are artificially designed media whose chiral responses are much stronger than natural materials.

习题5.1

(1) 在微波炉的工作频率（2.5 GHz）下，圆形牛排的介电常数约为 $40\varepsilon_0$，电导率 $\sigma = 2$ mho/m。求牛排的趋肤深度？将此趋肤深度与聚苯乙烯泡沫进行比较（介电常数为 $1.03\ \varepsilon_0$，电导率 $\sigma = 4\times10^{-6}$ mho/m）

(2) 求海水在 100Hz 和 5MHz 频率处的趋肤深度。设海水的电导率为 $\sigma = 4$ mho/m，介电常数为 $80\ \varepsilon_0$，磁导率为 μ_0。

(3) 一艘轮船想要与水下 100 米处的潜艇通信，如果用 1kHz 的电磁波通信，有多少电磁波能量可以到达水下潜艇。海水的电磁参数同（2）。

习题5.2　如图所示，考虑一个圆极化波垂直入射到单轴媒质中，入射波的电场为

$$\vec{E}_{in} = \hat{y}E_0 \exp(-jk_x x) + \hat{z}\alpha E_0 \exp(-jk_x x - j\beta)$$

这里我们忽视界面处的电磁波反射。试求：

（1）　如果入射波为右旋圆极化波，且α和β均为正实数，求α和β的取值。

（2）　设单轴媒质的介电常数为$\bar{\bar{\varepsilon}} = \begin{bmatrix} \varepsilon_x & & \\ & \varepsilon_y & \\ & & \varepsilon_z \end{bmatrix}$，其中$\varepsilon_x = \varepsilon_y = 4\varepsilon_0$，$\varepsilon_z = 9\varepsilon_0$，磁导率为$\mu_0$。求单轴媒质里，电场沿$y$方向极化的电磁波波数$k_x$。

（3）　在（2）中，令入射波为右旋圆极化波，求单轴媒质的最小厚度d，使得电磁波透过该媒质后转化为左旋圆极化波。

实验作业

通过**MATLAB**、 **COMSOL**等软件来仿真如下的实例。

第五章 介质中的波:

仿真一维单频率电磁波通过带损耗的介质的情况;

画出左旋和右旋偏振光在三维空间传播的具体形状。