山东大学
SHANDONG UNIVERSITY

Physics I: Introduction to Wave Theory
SDU Course Number: sd01232810 (Fall 2024)

Lecture 1: Electrostatics

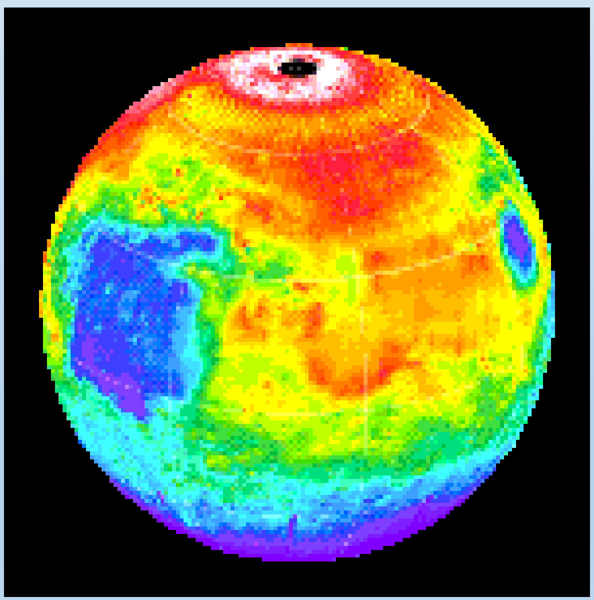
Outline

- Scalar and Vector Fields
- Electric Fields and Forces
- Coulomb's Law
- Electric Dipoles and Torque
- Electric Potential
- Gauss's Law and Electric Flux

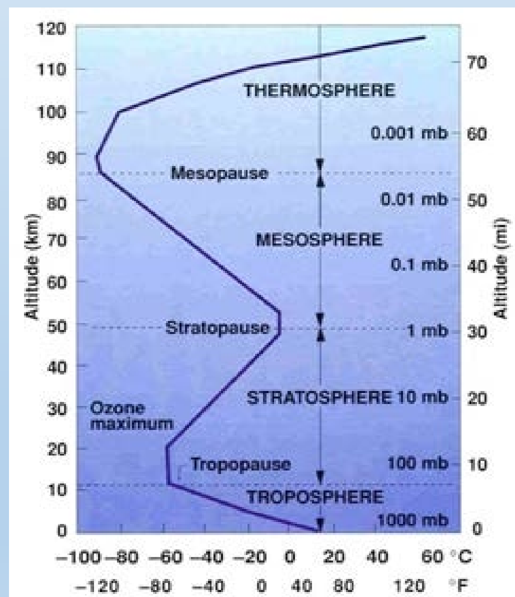
Scalar Fields

- A ***scalar field*** **is a function that gives us a single value of some variable for every point in space.**

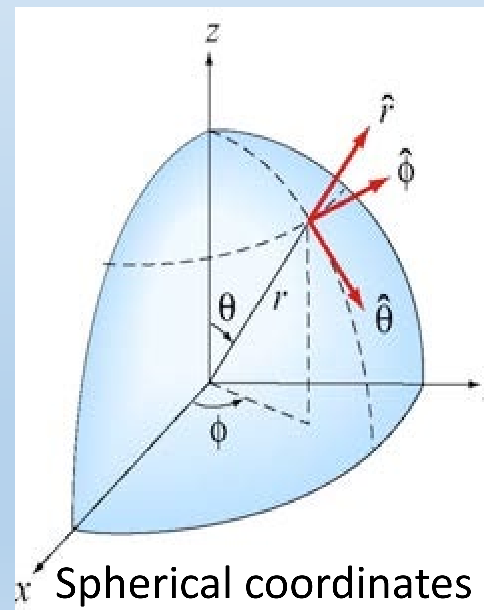
- **The temperature function T (r,θ,ϕ) is an example of a "scalar field." The term "scalar" implies that temperature at any point is a number rather than a vector.**



Nighttime temperature map for Mars



Atmospheric temperature Vs. altitude above the Earth's surface

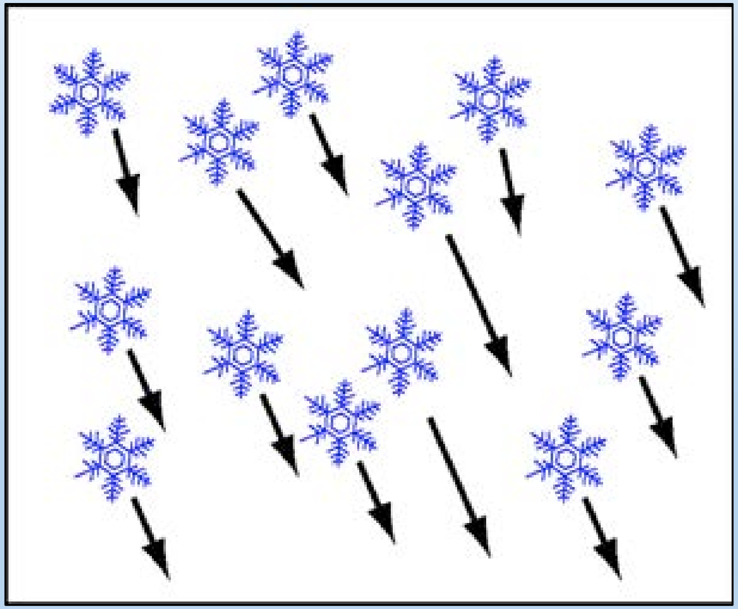


Spherical coordinates

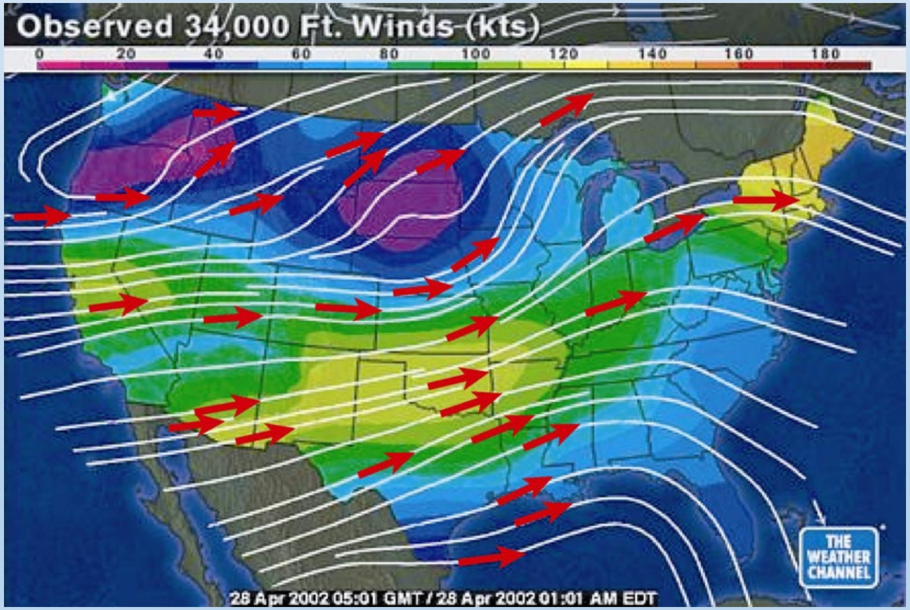
Vector Fields

- **A vector is a quantity which has both a magnitude and a direction in space. The collection of all the velocity vectors is called the velocity vector field.**

$$\boxed{\vec{F}(x,y,z) = \hat{x}F_x(x,y,z) + \hat{y}F_y(x,y,z) + \hat{z}F_z(x,y,z)}$$



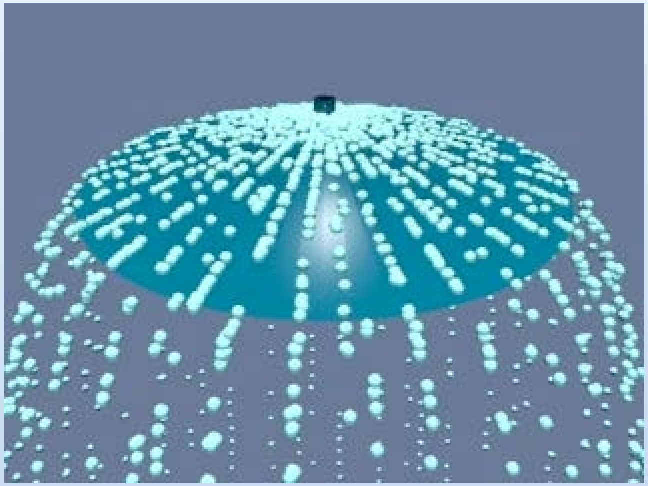
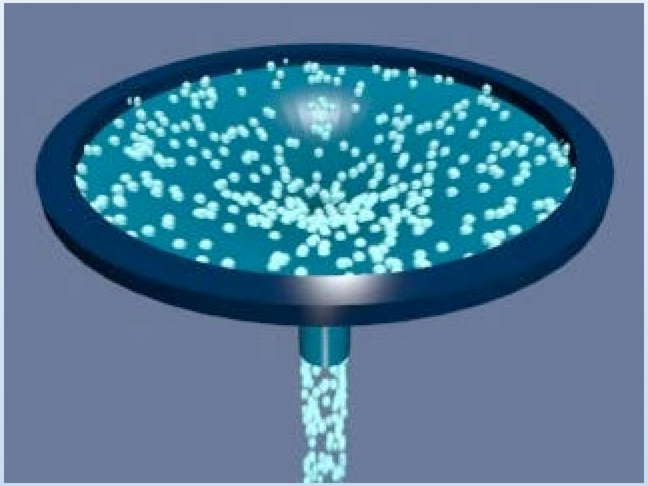
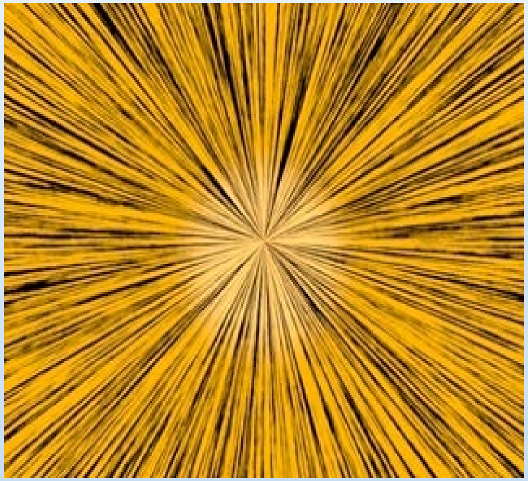
Falling snow



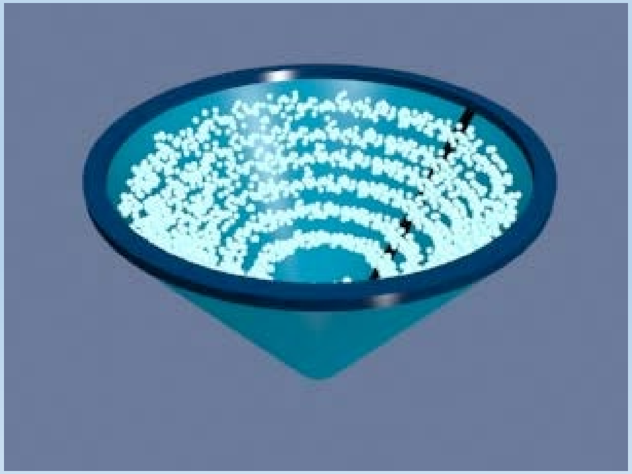
Jet stream with arrows indicating flow velocity

Example Of Vector Field: Fluid

- **Sources and Sinks**

- **Circulations**



Example Of Vector Field: Gravitation

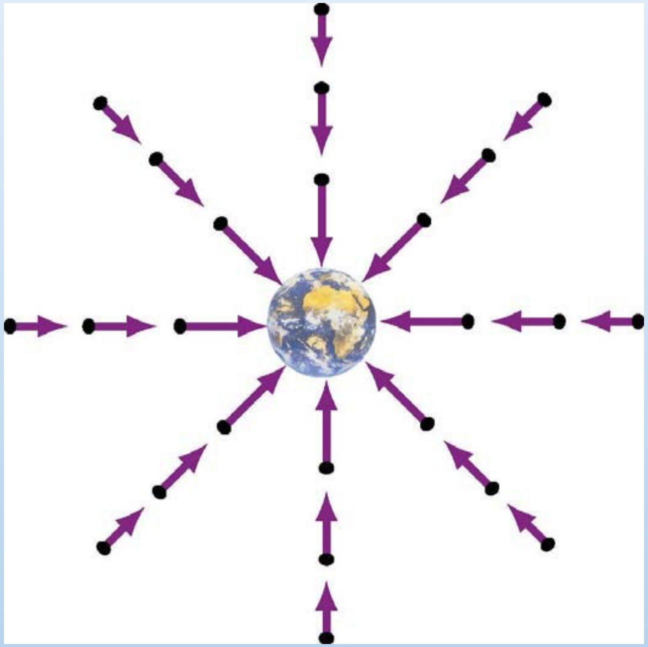
Gravitational Force:

$$\vec{F}_g = -G\frac{Mm}{r^2}\hat{r}$$

Gravitational Field:

$$\vec{g} = \frac{\vec{F}_g}{m} = -G\frac{M}{r^2}\hat{r}$$ **(Created by M)**

$$\vec{F}_g = m\vec{g}$$ **(Felt by m)**



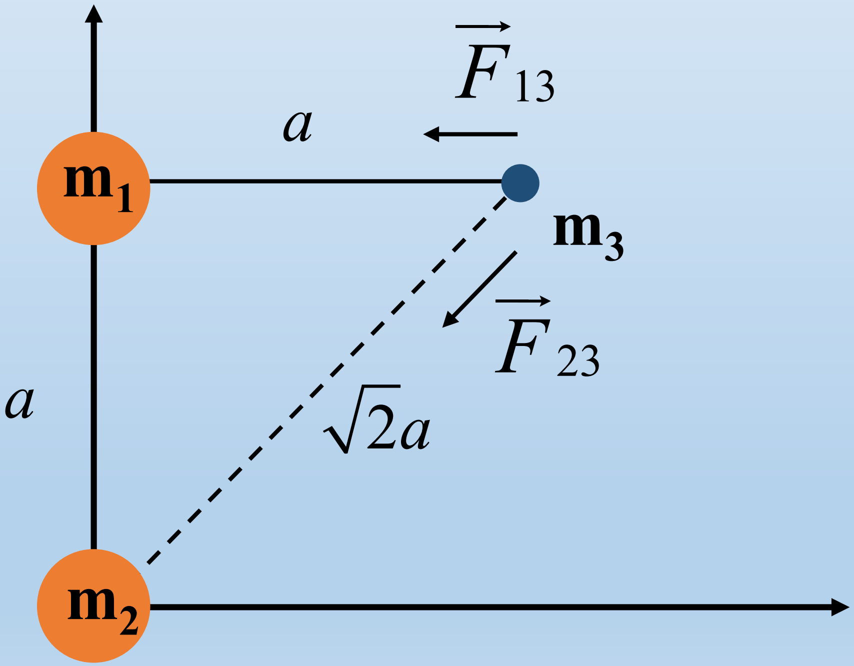
M: Mass of Earth

r: unit vector from M to m $\hat{r} = \frac{\vec{r}}{r}$

The Superposition Principle

Net force/field is vector sum of forces/fields

Example:



$$\vec{F} = \vec{F}_{13} + \vec{F}_{23}$$

In general:

$$\boxed{\vec{F}_j = \sum_{i=1}^{N} \vec{F}_{ij}}$$

From gravitational to electric fields

Electric Charge (~Mass)

Two types of electric charge: positive and negative Unit of charge is the coulomb [C]

Charge of electron (negative) or proton (positive)is

$$\pm e \qquad\qquad e = 1.602\times 10^{-19}\,C$$

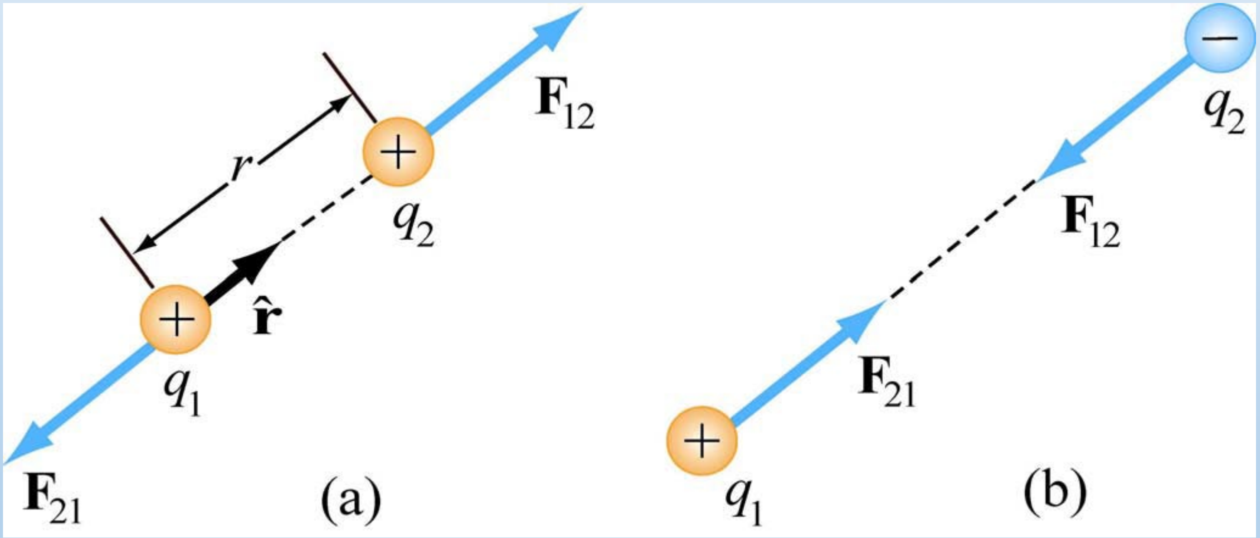
Charge is quantized

$$Q = Ne$$

An electron carries one unit of negative charge, −e , while a proton carries one unit of positive charge, +e . In a closed system, the total amount of charge is conserved since charge can neither be created nor destroyed. A charge can, however, be transferred from one body to another.

Electric Force (~Gravity)

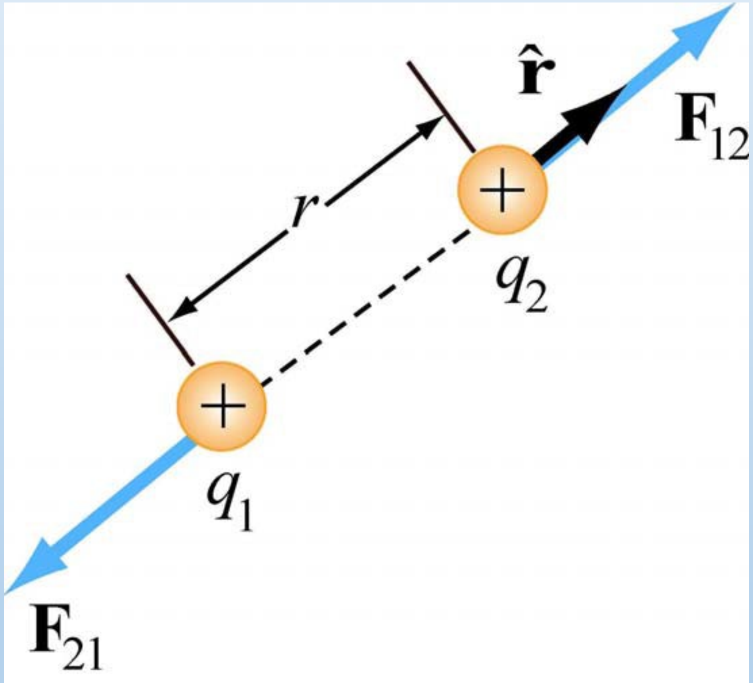
The electric force between charges q_1 and q_2 is
(a) repulsive if charges have same signs
(b) attractive if charges have opposite signs



Like charges repel and opposites attract !!

Coulomb's Law

The force exerted by q_1 on q_2 is given by Coulomb's law:



$$\boxed{\vec{F}_{12} = k_e \frac{q_1 q_2}{r^2} \hat{r}}$$

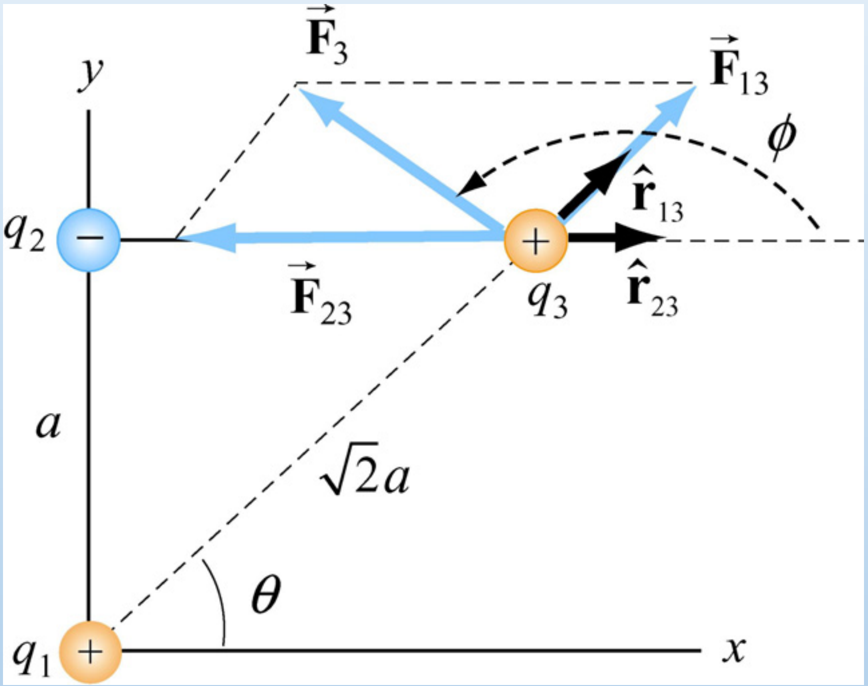
$$k_e = \frac{1}{4\pi\varepsilon_0} = 8.9875 \times 10^9 \ \mathrm{Nm^2/C^2}$$

$$\varepsilon_0 = 8.85 \times 10^{-12} \ \mathrm{C^2/N \cdot m^2}$$

$\hat{r}$: **unit vector from q_1 to q_2**

Principle of Superposition

$$q_1 = -q_2$$



$$\vec{F} = \vec{F}_{13} + \vec{F}_{23}$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_3}{r_{13}^2}\hat{r}_{13} + \frac{q_2 q_3}{r_{23}^2}\hat{r}_{23}\right)$$

$$= \frac{q_1 q_3}{4\pi\varepsilon_0 a^2}\left[\left(\frac{\sqrt{2}}{4} - 1\right)\hat{x} + \frac{\sqrt{2}}{4}\hat{y}\right]$$

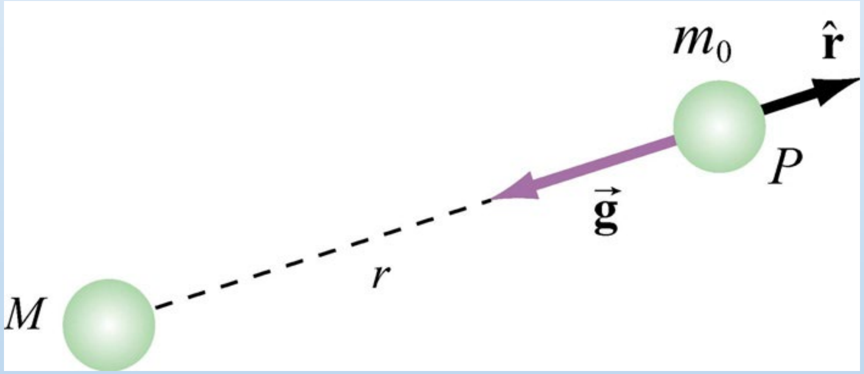
$$\hat{r}_{13} = \hat{x}\cos\theta + \hat{y}\sin\theta = \frac{\sqrt{2}}{2}\left(\hat{x} + \hat{y}\right)$$

$$\hat{r}_{23} = \hat{x}$$

Electric Field (~g)

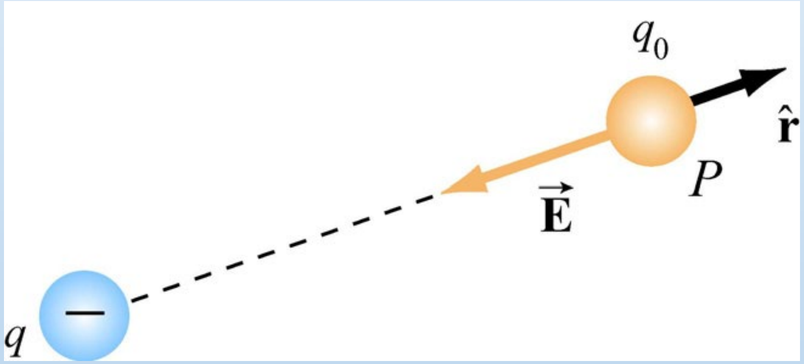
The electric field at a point is the force acting on a test charge q_0 at that point, divided by the charge q_0:

Gravitational field



$$\vec{g} = \frac{\vec{F}_g}{m} = -G\frac{M}{r^2}\hat{r}$$

Electric field

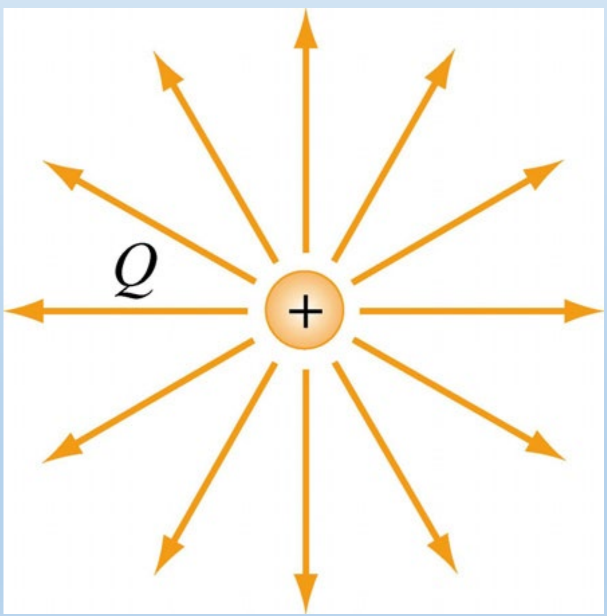


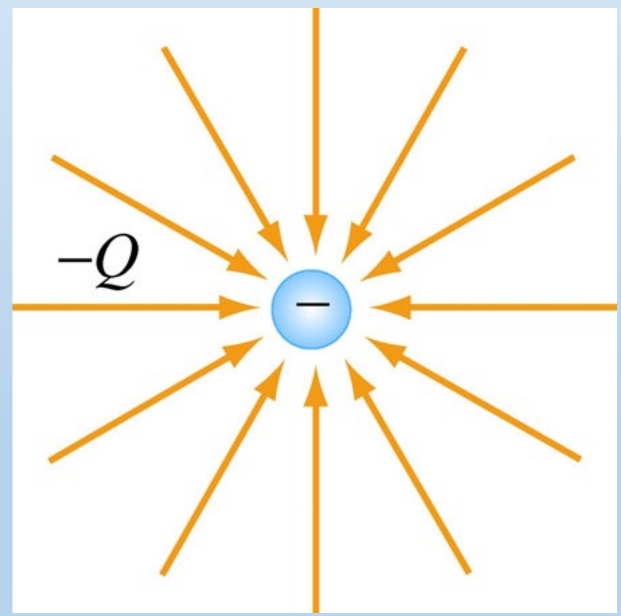
$$\boxed{\vec{E} = \frac{\vec{F}}{q_0} = k_e\frac{q}{r^2}\hat{r}}$$

Superposition of electric fields: $$\vec{E}_{total} = \vec{E}_1 + \vec{E}_2 + \cdots = \sum_{i=1}^{N}\vec{E}_i$$

Electric Field Lines

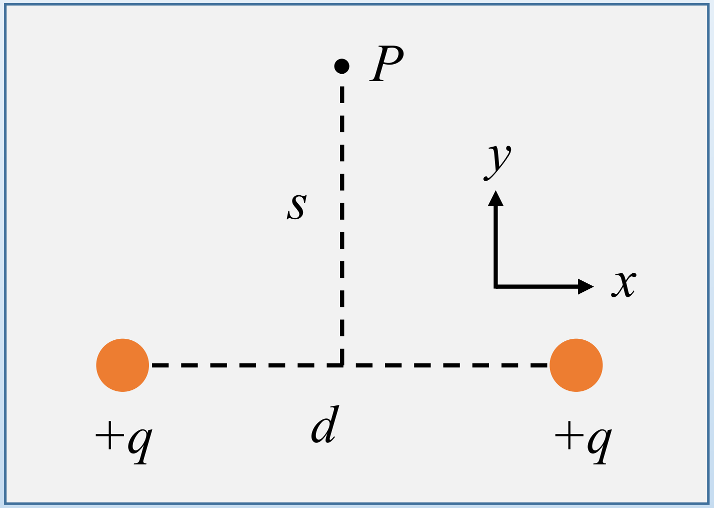
1. Direction of field at any point is tangent to field line at that point.
2. Field lines point away from positive charges and terminate on negative charges.
3. Field lines never cross each other.





In-Class Concept Question

Electric field at P is:



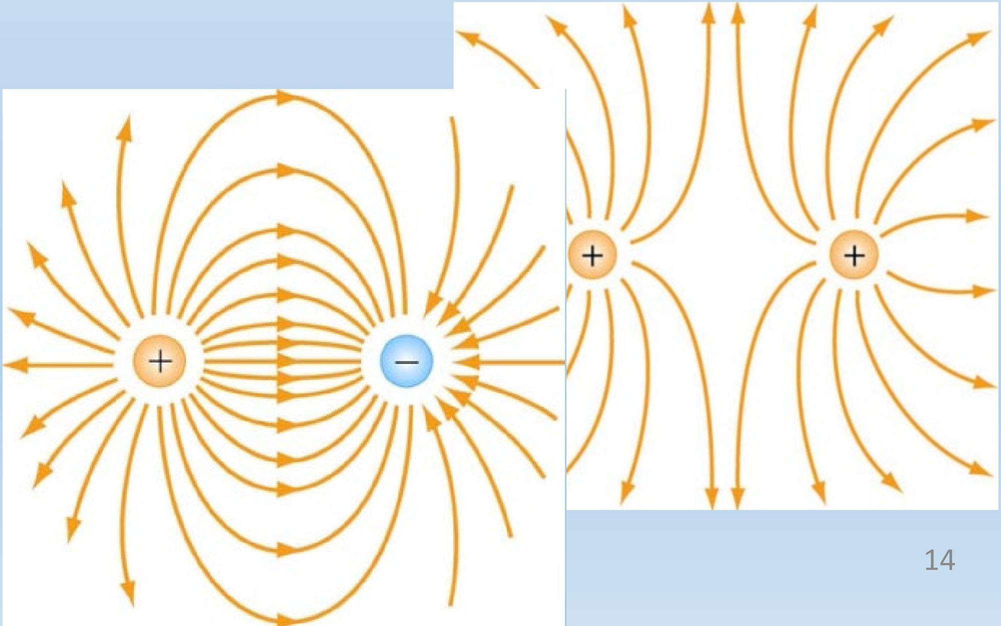
(1) $\vec{E} = \dfrac{2k_e q s}{\left[s^2 + d^2/4\right]^{3/2}}\hat{y}$

(2) $\vec{E} = -\dfrac{2k_e q d}{\left[s^2 + d^2/4\right]^{3/2}}\hat{x}$

(3) $\vec{E} = \dfrac{2k_e q d}{\left[s^2 + d^2/4\right]^{3/2}}\hat{y}$

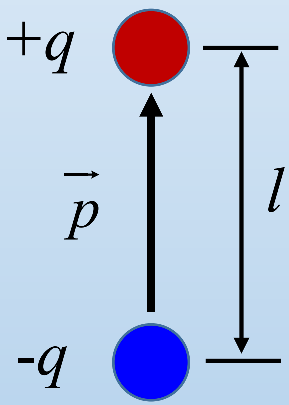
(4) $\vec{E} = -\dfrac{2k_e q s}{\left[s^2 + d^2/4\right]^{3/2}}\hat{x}$

(5) I don't know.



Electric Dipole

An electric dipole consists of two equal but opposite charges, +q and -q, separated by a distance l.

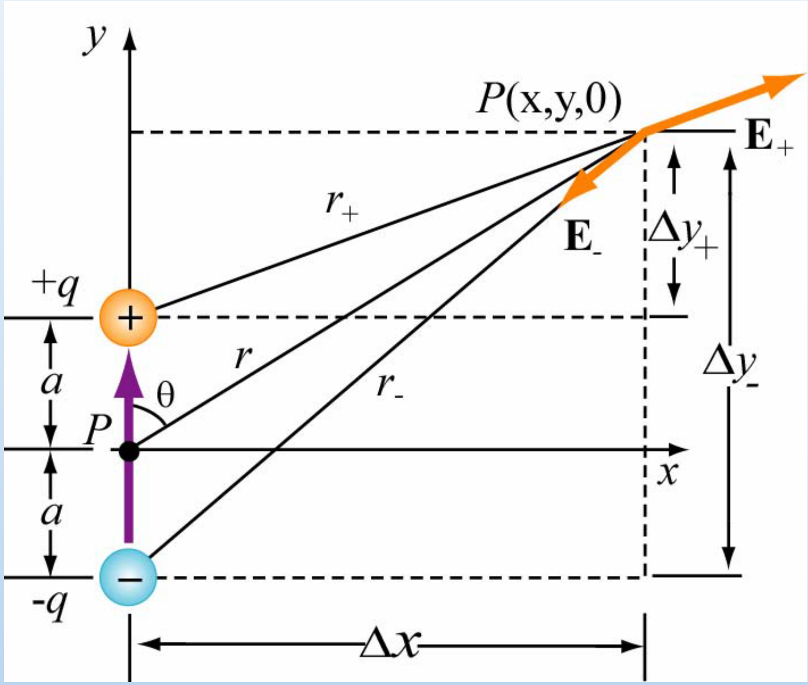


Dipole Moment

$$\vec{p} = charge \times displacement$$
$$= q\vec{l} = \hat{y}ql$$

$\vec{p}$ points from negative to positive charge.

The Electric Field of a Dipole



$$r_{\pm} = r^2 + a^2 \mp 2ra\cos\theta = x^2 + \left(y \mp a\right)^2$$

$$E_x = \frac{q}{4\pi\varepsilon_0}\left(\frac{\cos\theta_+}{r_+^2} - \frac{\cos\theta_-}{r_-^2}\right)$$

$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{x}{\left[x^2 + \left(y-a\right)^2\right]^{3/2}} - \frac{x}{\left[x^2 + \left(y+a\right)^2\right]^{3/2}}\right)$$

$$E_y = \frac{q}{4\pi\varepsilon_0}\left(\frac{\sin\theta_+}{r_+^2} - \frac{\sin\theta_-}{r_-^2}\right) = \frac{q}{4\pi\varepsilon_0}\left(\frac{y-a}{\left[x^2 + \left(y-a\right)^2\right]^{3/2}} - \frac{y+a}{\left[x^2 + \left(y+a\right)^2\right]^{3/2}}\right)$$

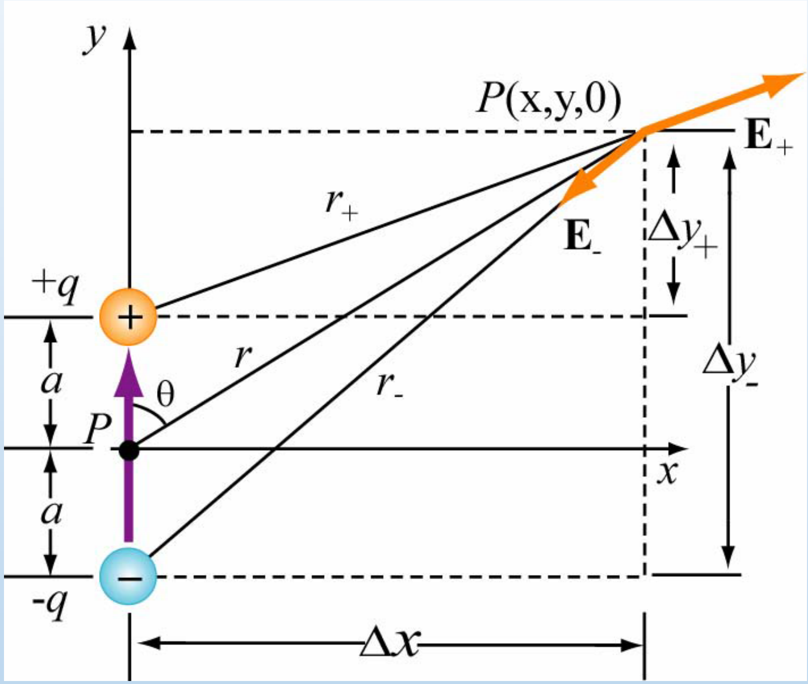
Point-Dipole Limit

For $r \gg a$

$$E_x = \frac{3p}{4\pi\varepsilon_0 r^3}\sin\theta\cos\theta$$

$$E_y = \frac{p}{4\pi\varepsilon_0 r^3}\left(3\cos^2\theta - 1\right)$$

$$\vec{E}\left(\vec{r}\right) = \frac{1}{4\pi\varepsilon_0}\left(-\frac{\vec{p}}{r^3} + \frac{3\left(\vec{p}\cdot\vec{r}\right)\vec{r}}{r^5}\right)$$



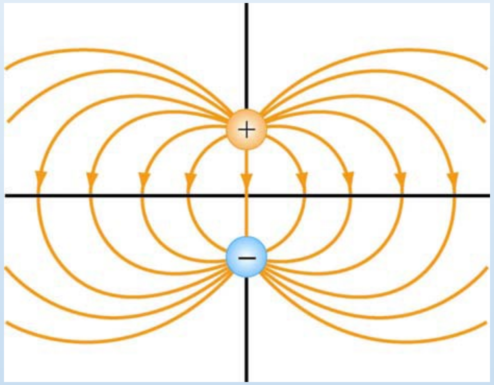
$$\sin\theta = x/r$$

$$\cos\theta = y/r$$

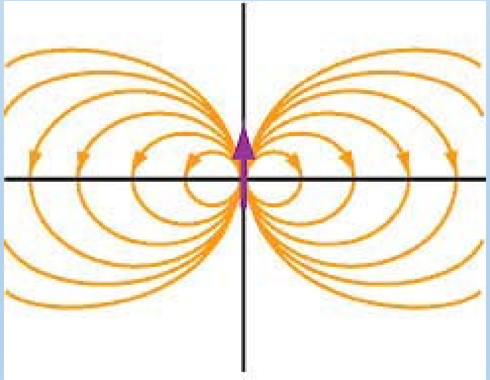
$$3pr\cos\theta = 3\vec{p}\cdot\vec{r}$$

Point Dipole Approximation

Take the limit $r \gg a$



Finite Dipole



Point Dipole

$$E_x = \frac{3p}{4\pi\varepsilon_0 r^3}\sin\theta\cos\theta$$

$$E_y = \frac{p}{4\pi\varepsilon_0 r^3}\left(3\cos^2\theta - 1\right)$$

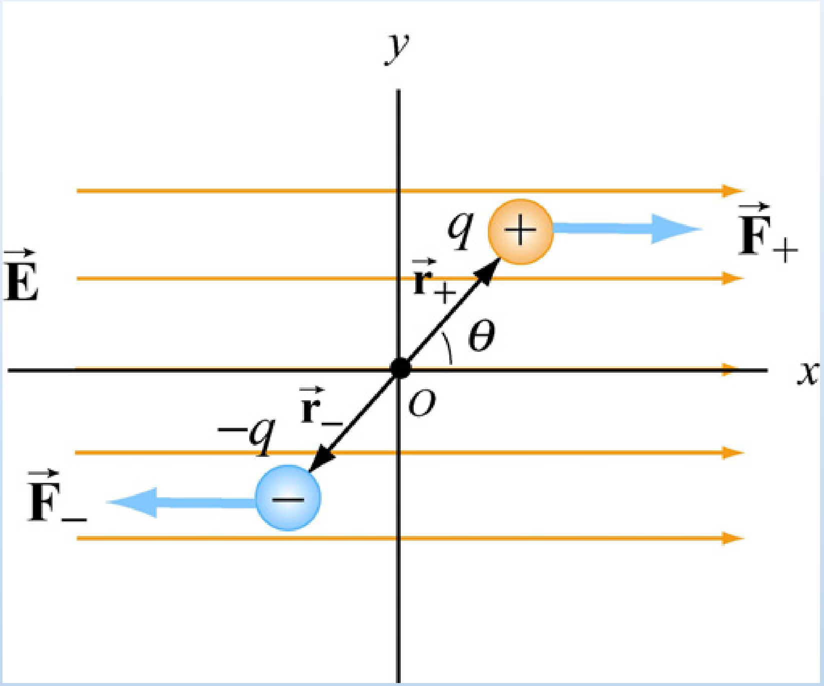
$$\boxed{\vec{E}\left(\vec{r}\right) = \frac{1}{4\pi\varepsilon_0}\left(-\frac{\vec{p}}{r^3} + \frac{3\left(\vec{p}\cdot\vec{r}\right)\vec{r}}{r^5}\right)}$$

$$\sin\theta = x/r$$

$$\cos\theta = y/r$$

$$3pr\cos\theta = 3\vec{p}\cdot\vec{r}$$

Dipole in Electric Field



$$\vec{E} = \hat{x}E$$

$$\vec{p} = 2qa\left(\hat{x}\cos\theta + \hat{y}\sin\theta\right)$$

Total Net Force:

$$\vec{F}_{net} = \vec{F}_+ + \vec{F}_- = 0$$

Torque on Dipole:

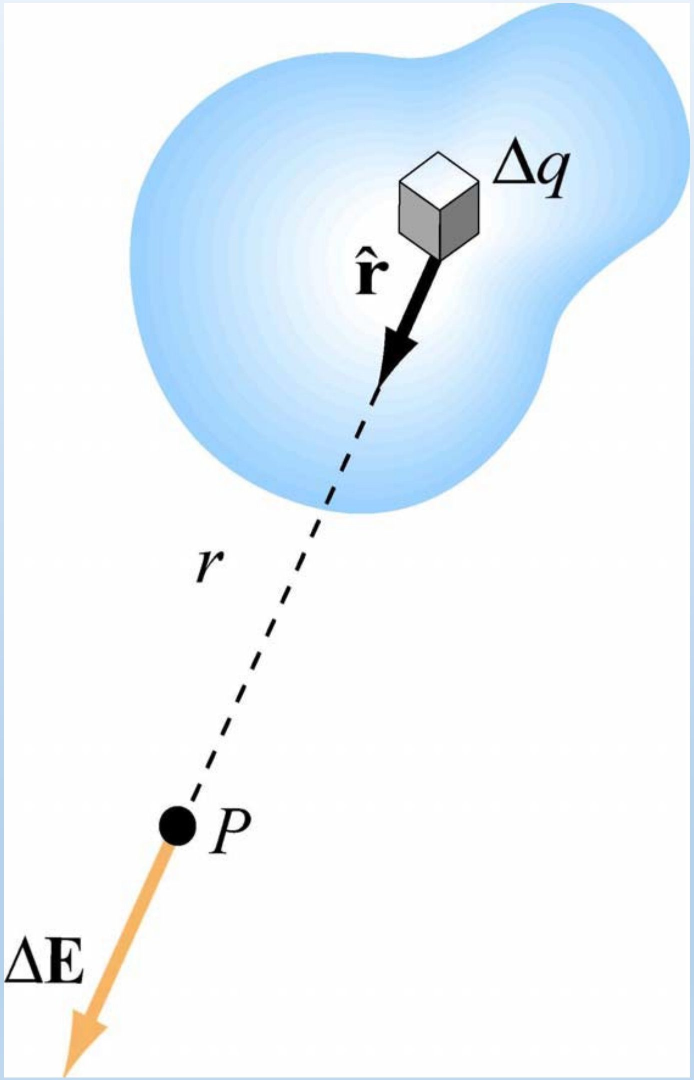
$$\boxed{\vec{\tau} = \vec{r}\times\vec{F} = \vec{p}\times\vec{E}}$$

$$\vec{\tau} = \vec{r}_+ \times \vec{F}_+ + \vec{r}_- \times \vec{F}_-$$

$$= \left(\hat{x}a\cos\theta + \hat{y}a\sin\theta\right)\times\vec{F}_+ + \left(-\hat{x}a\cos\theta - \hat{y}a\sin\theta\right)\times\vec{F}_-$$

$$= -\hat{z}2aF\sin\theta$$

Continuous Charge Distributions



Break distribution into parts:

$$Q = \sum_i \Delta q_i \rightarrow \int_V dq$$

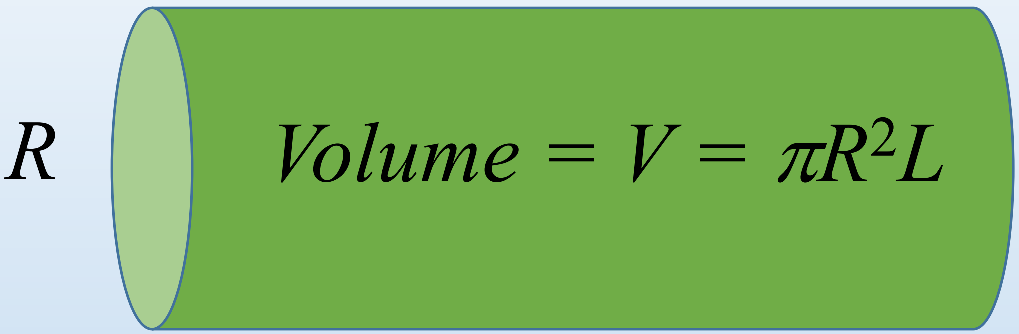
E field at P due to Δq

$$\Delta \vec{E} = k_e \frac{\Delta q}{r^2}\hat{r} \rightarrow d\vec{E} = k_e \frac{dq}{r^2}\hat{r}$$

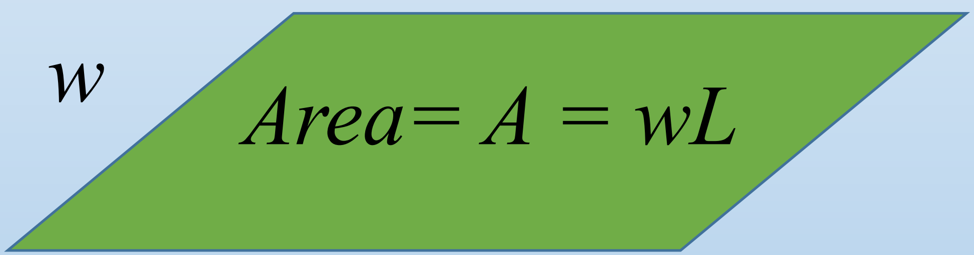
Superposition:

$$\boxed{\vec{E} = \sum \Delta \vec{E} \rightarrow \int d\vec{E}}$$

Continuous Sources: Charge Density

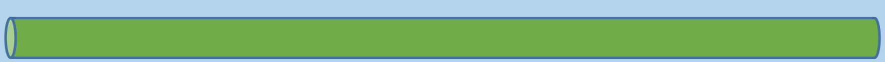


$$dQ = \rho dV$$
$$\rho = dQ/dV$$



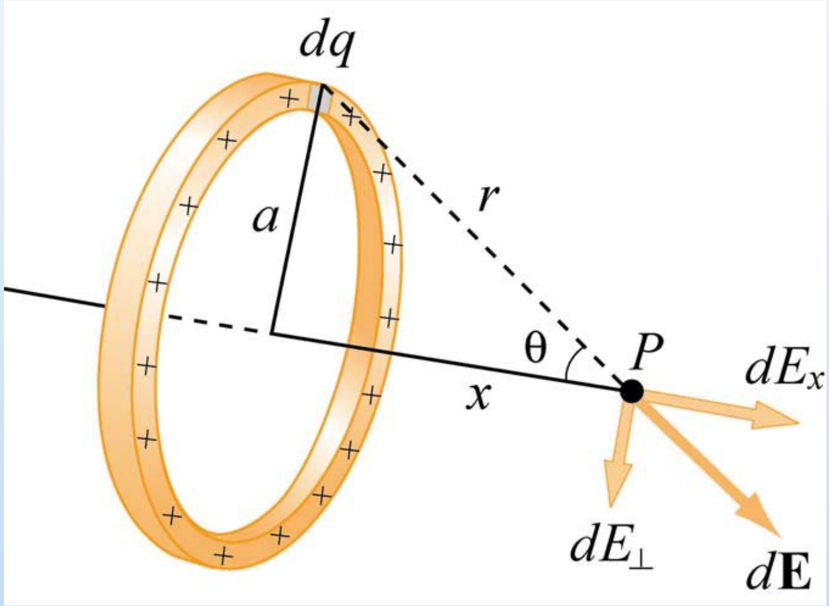
$$dQ = \sigma dA$$
$$\sigma = dQ/dA$$

$Length = L$



$$dQ = \lambda dL$$
$$\lambda = dQ/dL$$

Example: Ring of Charge



$$d\vec{E} = k_e \frac{dq}{r^2}\hat{r} = k_e dq \frac{\vec{r}}{r^3}$$

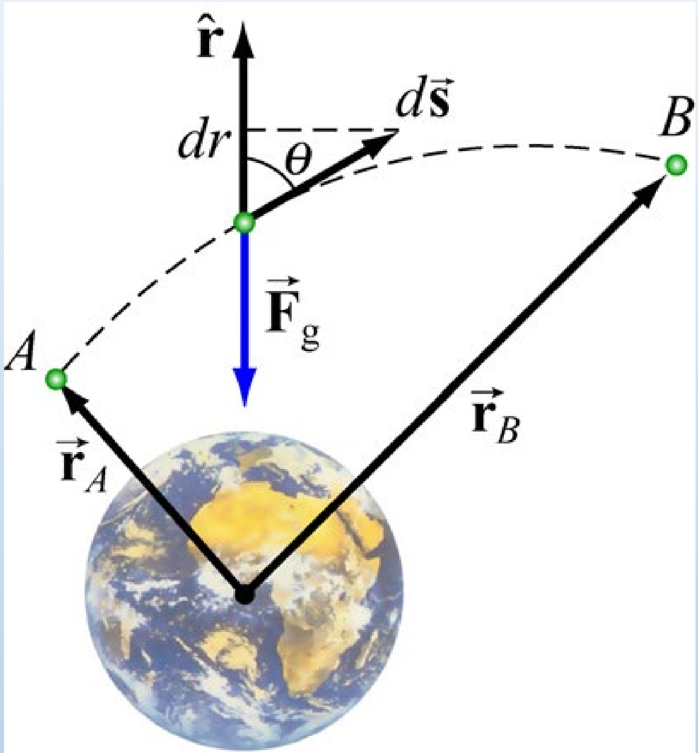
$$dE_x = k_e dq \frac{x}{r^3}$$

$$\int dq = \int_0^{2\pi} \lambda a d\varphi = 2\pi\lambda a = Q$$

Integrate: $$E_x = \int dE_x = \int k_e dq \frac{x}{r^3} = k_e \frac{x}{r^3}\int dq$$

$$\boxed{\vec{E} = \hat{x} k_e \frac{x}{r^3} Q = \hat{x} k_e Q \frac{x}{\left(a^2 + x^2\right)^{3/2}}}$$

Potential and Potential Energy (Gravity)



The work done by gravity in moving m from A to B is

$$W_g = \int \vec{F}_g \cdot d\vec{s} = \int_{r_A}^{r_B} \left(-\frac{GMm}{r^2} \right) dr$$

$$= GMm \left(\frac{1}{r_B} - \frac{1}{r_A} \right)$$

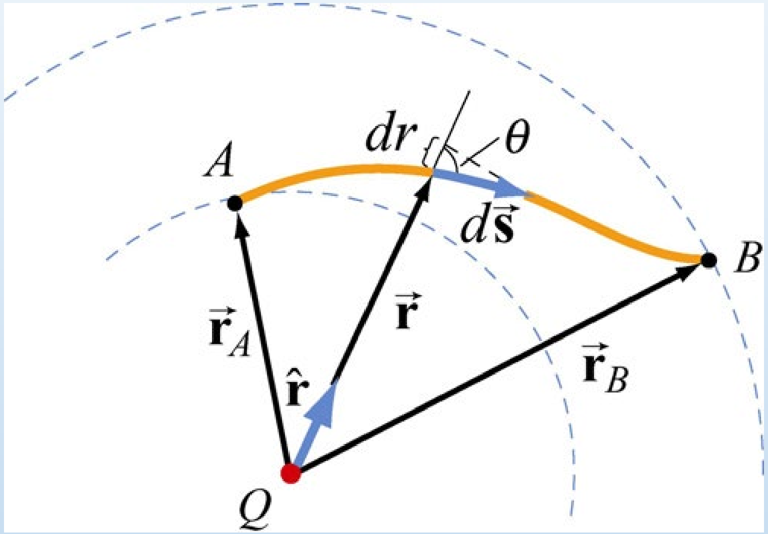
PATH INTEGRAL

W is the work done by the force on the object

Potential Energy: $$\boxed{\Delta U = U_B - U_A = -\int_A^B \vec{F}_g \cdot d\vec{s} = -W}$$

Gravitational potential: $$\boxed{\Delta V = \frac{\Delta U}{m} = -\int_A^B \frac{\vec{F}_g}{m} \cdot d\vec{s} = -\int_A^B \vec{g} \cdot d\vec{s}}$$

Electric Potential and Energy



Electric Potential difference between two points A and B

$$\boxed{\Delta V = -\int_A^B \frac{\vec{F}_e}{q_0} \cdot d\vec{s} = -\int_A^B \vec{E} \cdot d\vec{s}}$$

Units: Joules/Coulomb = Volts

Work done to move q from A to B:

$$W_{ext} = \Delta U = U_B - U_A = q\Delta V$$

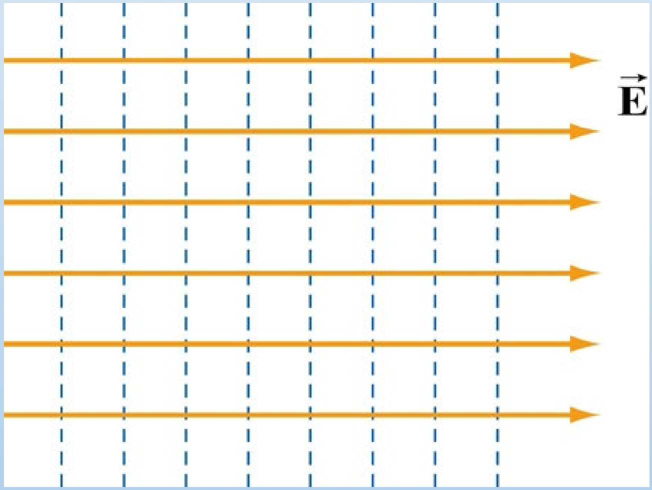
The potential created by point charge:

$$\boxed{\Delta V = -\int_A^B k_e Q \frac{\hat{r}}{r^2} \cdot d\vec{s} = -k_e Q \int_A^B \frac{dr}{r^2} = k_e Q \left(\frac{1}{r_B} - \frac{1}{r_A} \right)}$$

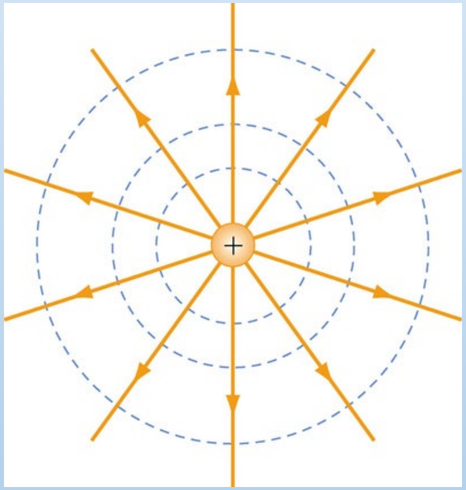
Equipotential curves

The curves characterized by constant V(x,y,z) are called equipotential curves.

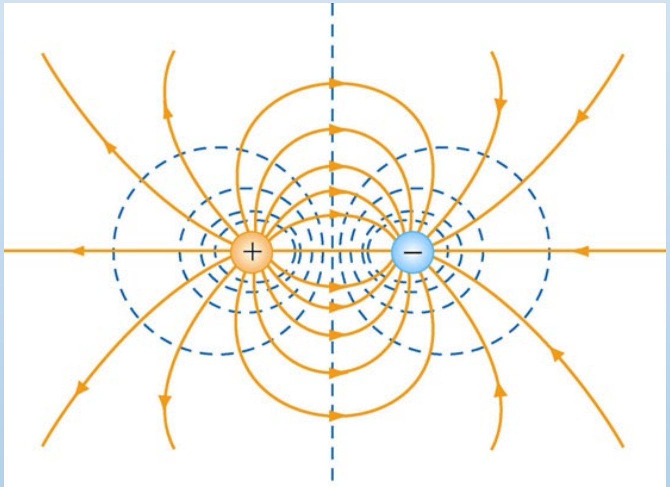
E is perpendicular to all equipotentials



Constant E field



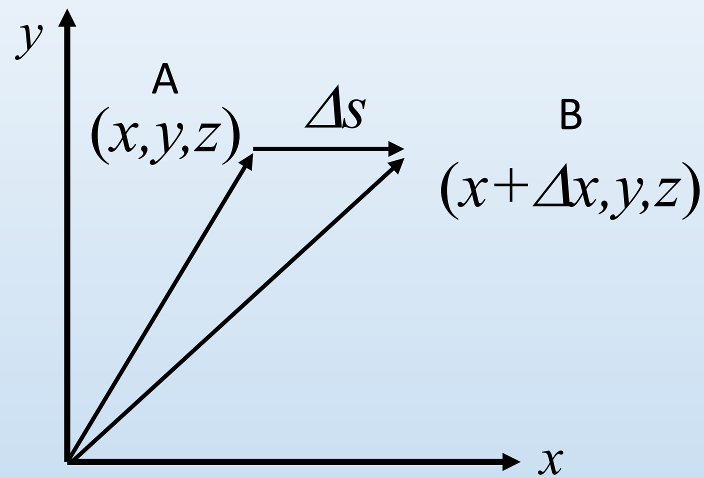
Point Charge



Electric dipole

Deriving Electric Field from the Electric Potential

$$\Delta V = -\int_A^B \vec{E} \cdot d\vec{s}$$



$$A = (x,y,z)$$

$$B = (x+\Delta x,y,z)$$

$$\Delta\vec{s} = \hat{x}\Delta x$$

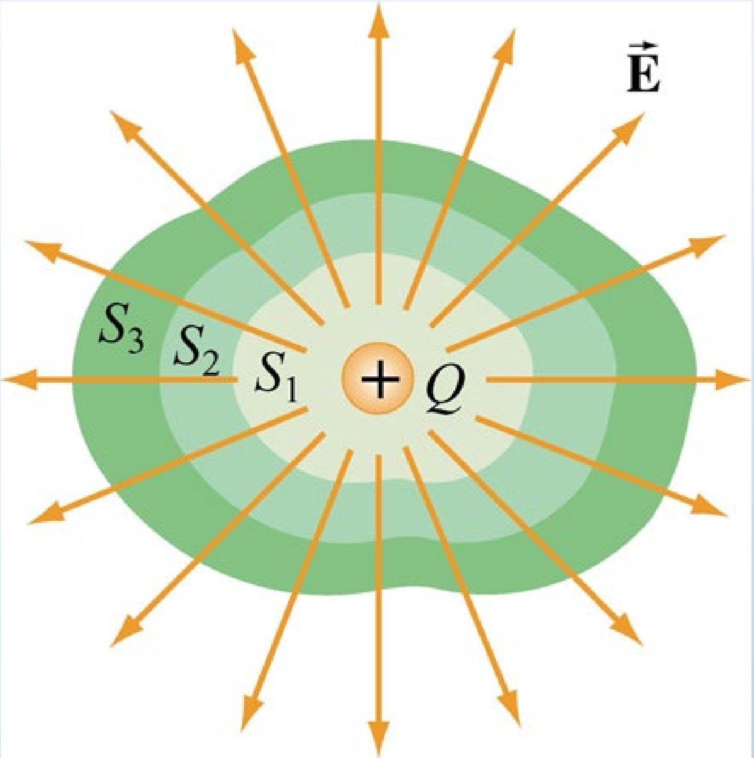
$$\Delta V = -\int_A^B \vec{E} \cdot d\vec{s} = -\vec{E} \cdot \Delta\vec{s} = -E_x \Delta x$$

$$E_x = -\frac{\Delta V}{\Delta x} = -\frac{\partial V}{\partial x}$$

$$\boxed{\vec{E} = -\nabla V}$$

$$\nabla = \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}$$

Gauss's Law



$$\Phi_E = \oiint_S \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\varepsilon_0}$$

Φ_E : **Electric flux**

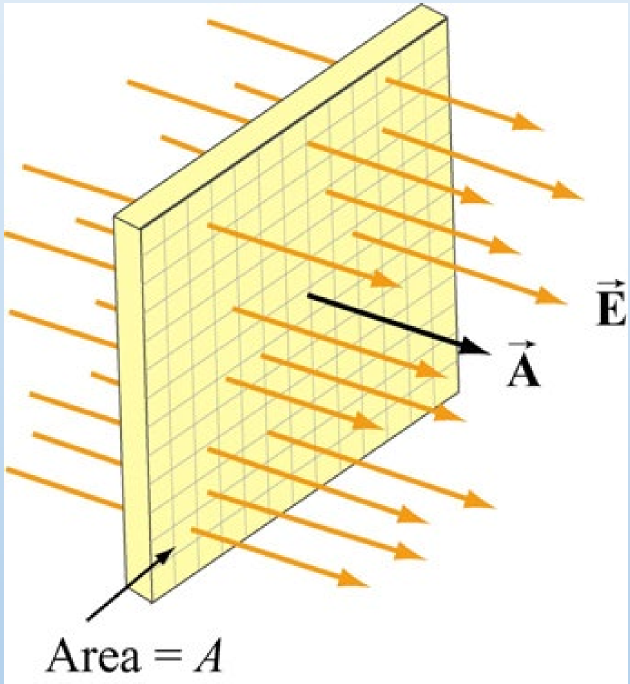
S : **Arbitrary closed surface**

The first Maxwell's equation!

The total "flux" of field lines penetrating any of these closed surfaces is the same and depends only on the amount of charge inside

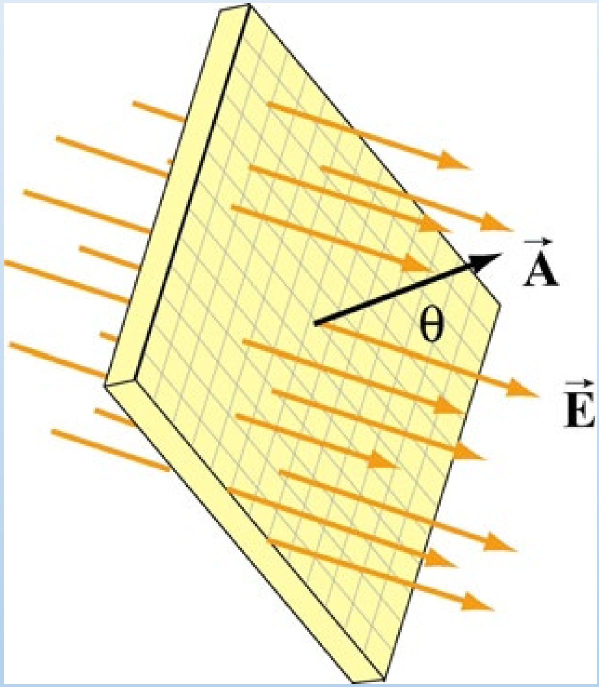
Electric Flux Φ_E

Case I: E is constant vector field perpendicular to planar surface S of area A



$$\Phi_E = \oiint_S \vec{E} \cdot d\vec{A} = +EA$$

Case II: E is constant vector field directed at angle θ to planar surface S of area A

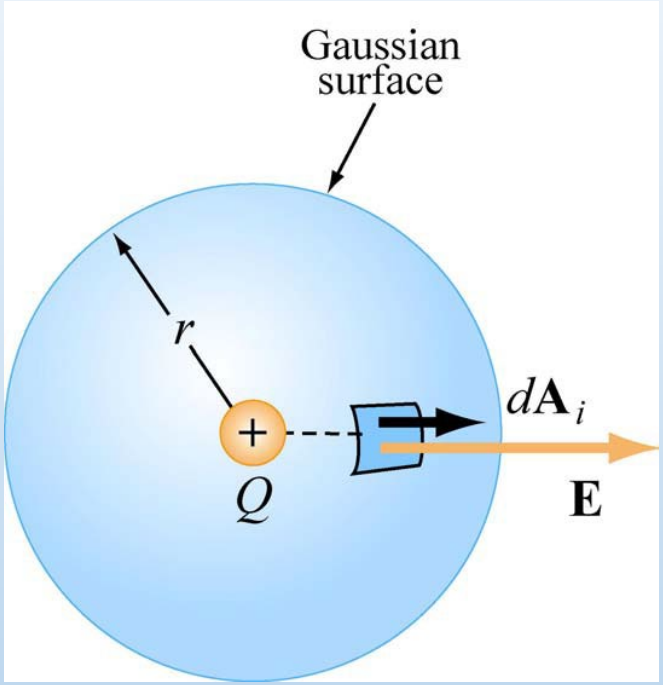


$$\Phi_E = \oiint_S \vec{E} \cdot d\vec{A} = +EA\cos\theta$$

Electric Flux: Sphere

E field at surface:

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$$



Electric flux through sphere:

$$\Phi_E = \oiint_S \vec{E}\cdot d\vec{A} = \oiint_S \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}\cdot\hat{r}dA$$

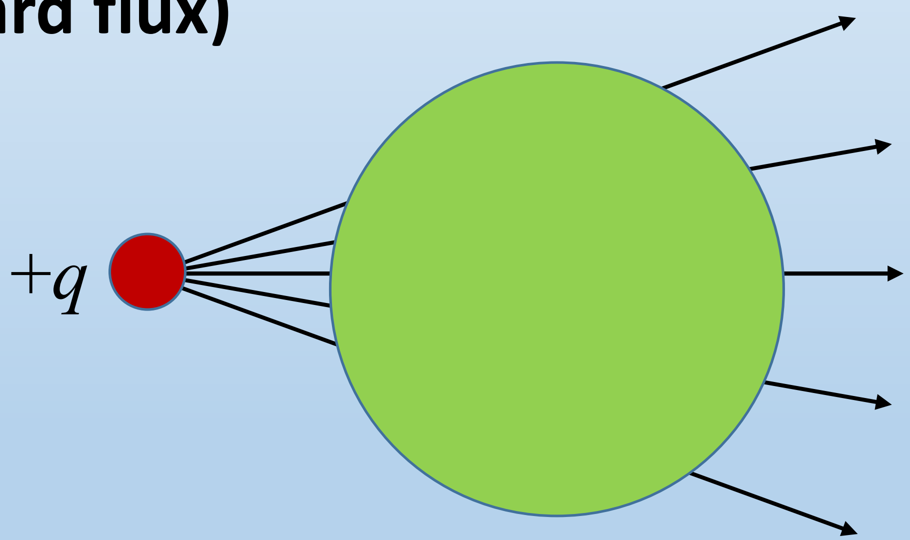
$$= \frac{Q}{4\pi\varepsilon_0 r^2}\oiint_S dA = \frac{Q}{4\pi\varepsilon_0 r^2}4\pi r^2 = \frac{Q}{\varepsilon_0}$$

$$\boxed{d\vec{A} = \hat{r}dA}$$

In-Class Concept Question: Flux through Sphere

The total flux through the below spherical surface is

(1) positive (net outward flux)
(2) negative (net inward flux)
(3) zero
(4) I don't know



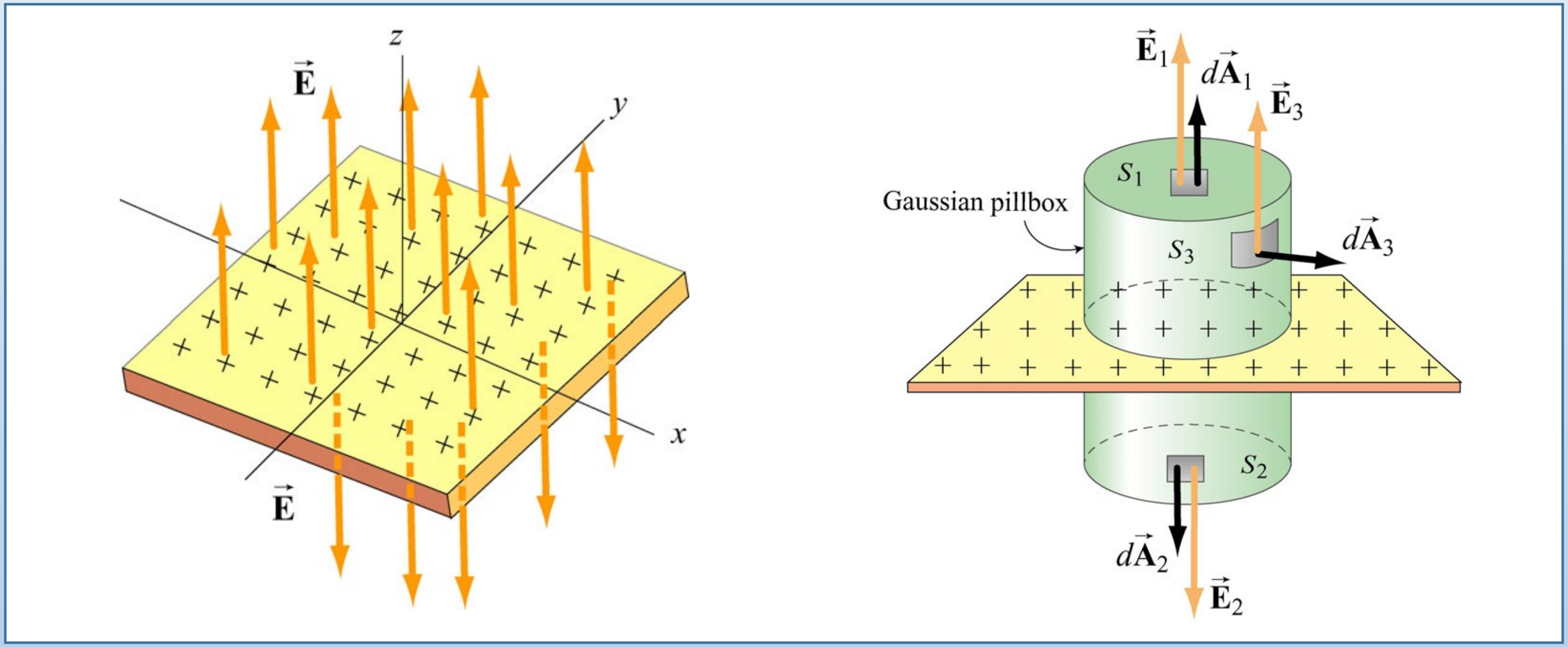
Applying Gauss's Law

1. Based on the source, identify regions in which to calculate E field
2. Choose Gaussian surfaces S: Symmetry
3. Calculate electric flux Φ_E
4. Calculate q_{in}, charge enclosed by surface S
5. Apply Guass's Law to calculate E

$$\boxed{\Phi_E = \oiint_S \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\varepsilon_0}}$$

Gauss: Planar Symmetry

Consider an infinitely large non-conducting plane in the xy-plane with uniform surface charge density σ . Determine the electric field everywhere in space.

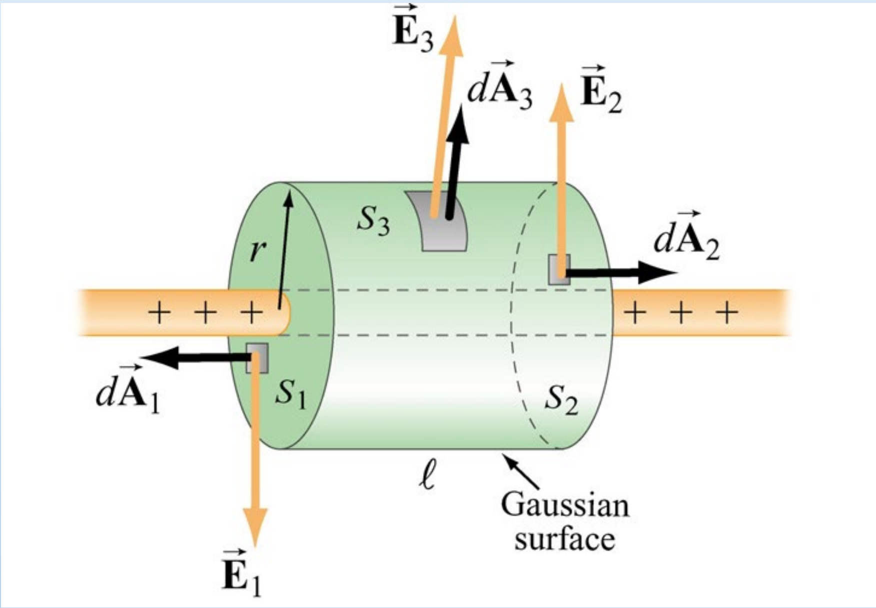


The total flux through the Gaussian pillbox flux is

$$\Phi_E = \oiint_S \vec{E} \cdot d\vec{A} = 2EA = \frac{\sigma A}{\varepsilon_0} \quad \Rightarrow \quad E = \frac{\sigma}{2\varepsilon_0}$$

Gauss: Cylindrical Symmetry

An infinitely long rod of negligible radius has a uniform charge density λ . Calculate the electric field at a distance r from the wire.



Charge enclosed by the Gaussian surface

$$q_{enc} = \lambda l$$

The total flux through the Gaussian pillbox

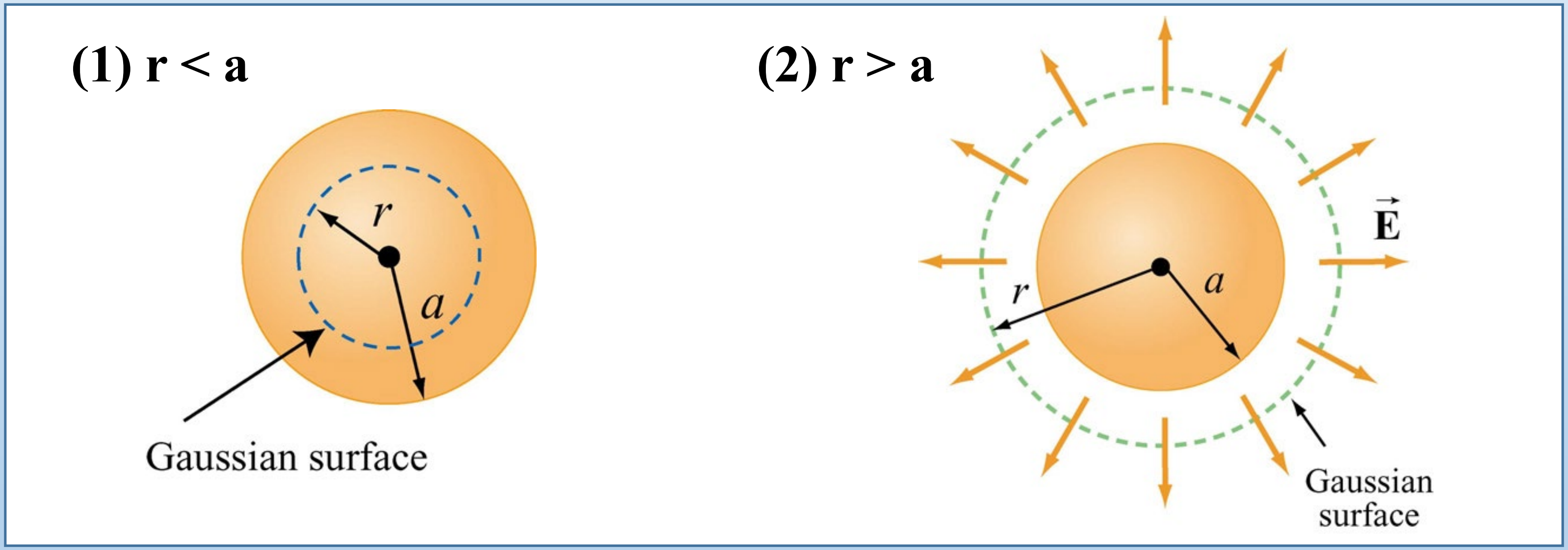
$$\Phi_E = \oint\!\!\oint_S \vec{E} \cdot d\vec{A} = E 2\pi r l = \frac{\lambda l}{\varepsilon_0}$$

$$\Rightarrow \quad \vec{E} = \frac{\lambda}{2\pi r \varepsilon_0} \hat{r}$$

Gauss: Spherical Symmetry

An electric charge +Q is uniformly distributed throughout a non-conducting solid sphere of radius a . Determine the electric field everywhere inside and outside the sphere.

Charge distribution: $\rho = Q/V = 3Q/\left(4\pi a^3\right)$



$$\Phi_E = 4\pi r^2 E = \frac{r^3}{a^3}\frac{Q}{\varepsilon_0}$$

$$\Phi_E = 4\pi r^2 E = \frac{Q}{\varepsilon_0}$$

Key points- Electrostatics

1. **Coulomb's Law.**

2. To look at the meaning of linear, area, and volume charge densities.

3. To calculate the electric field and the electric potential from Continuous Charge Distributions.

4. To calculate the electric field from Gauss’s law.

习题

习题 **1.1**： 两个固定的点电荷，电荷量分别是 q 和 $4q$，相距为 l。
试问在什么地方放一个什么样的点电荷，可以使这三个电荷都达到平衡（即每个电荷受另外两个电荷的库仑力之和都等于零）？

习题 **1.2**： 电荷量 q 均匀地分布在半径为 R 的半圆环上，
试求环心的电场强度。

习题 **1.3**： 电荷量分别为 q 和 $-q$ 的两个点电荷，相距为 l，对于离它们很远的区域来说（ $r >> l$ ），这两个电荷构成一个电偶极子。
试求电偶极子在远区的电势分布和等势面方程。

习题 **1.4**： 一球壳体的内外半径分别是 a 和 b，壳体中均匀分布着电荷，电荷量密度为 ρ。
试求离球心为 r 处的电场强度 E，并做出 E-r 曲线（以 r 为横坐标、 E 的大小为纵坐标）

实验作业

通过MATLAB、 COMSOL等软件来仿真课程相关的实例。

第一章静电场:

点电荷周围电场、电势、通量的计算和展示;

换成电偶极子的上述情况 。