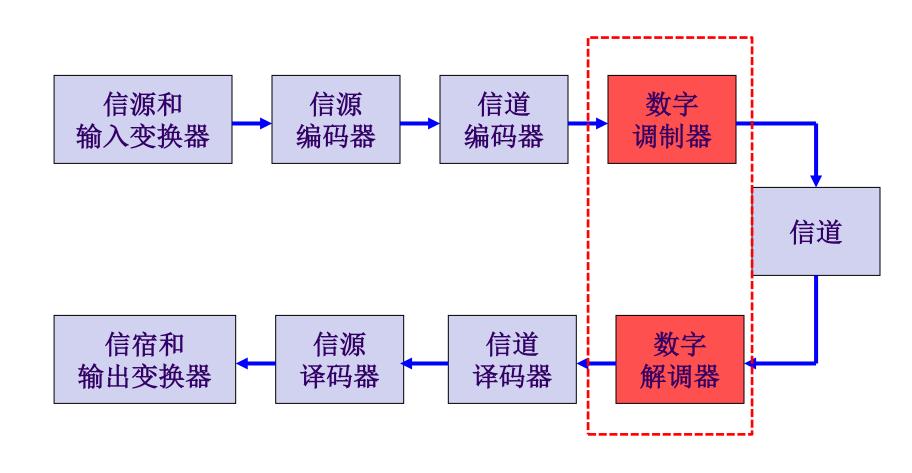


FUNDAMENTALS OF INFORMATION SCIENCE

PART 4 INFORMATION TRANSMISSION V
—— DIGITAL MODULATION

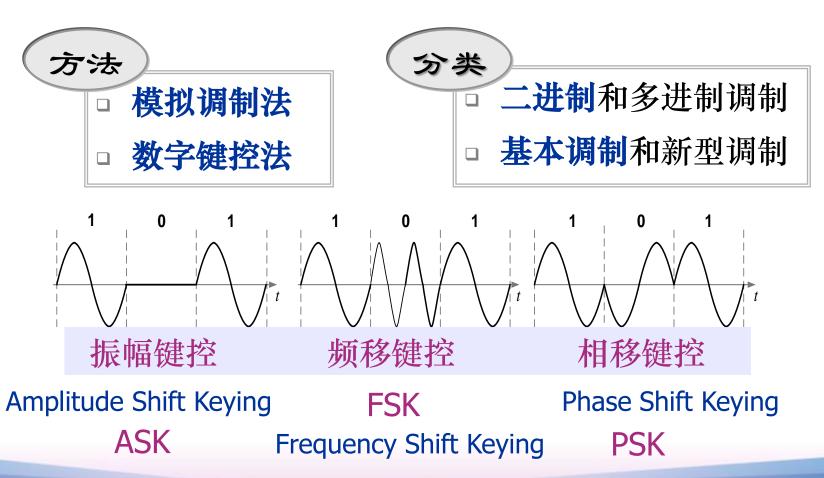
Shandong University 2025 Spring

数字信息传输系统



§ 20 数字调制

- ◆ 数字调制:用数字基带信号控制载波某个参数的过程。
- ◆ 数字带通传输系统:包括调制/解调过程的数字传输系统。



§ 20.1

二进制数字调制原理

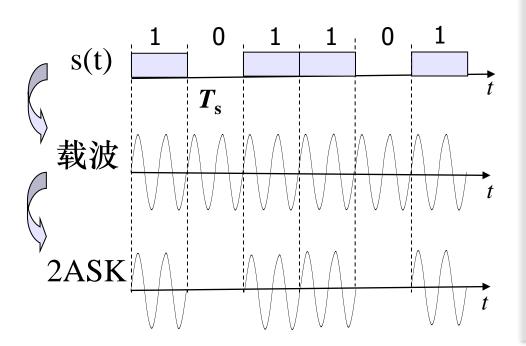
§ 20.1.1 二进制振幅键控 (2ASK)

■ 原理: s(t)→载波幅度

■ 表达式:

单极性

■ 波形:



$$e_{2ASK}(t) = s(t) \cos \omega_c t$$

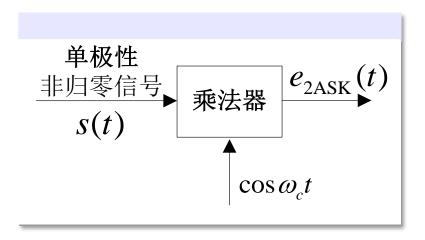
$$s(t) = \sum_{n} a_{n} g(t - nT_{s})$$

$$\begin{vmatrix} a_n = \begin{cases} +1, & P \\ 0, & (1-P) \end{cases}$$

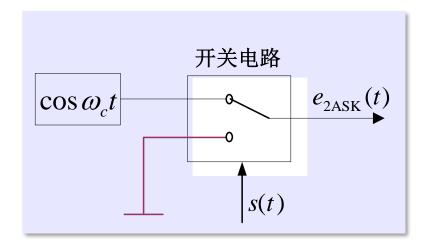
2ASK 也称 OOK

■ 2ASK 产生

◆ 模拟调制法

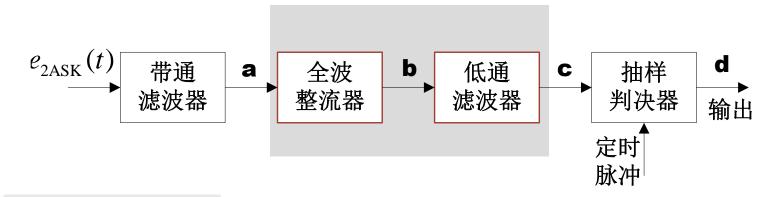


* 键控法

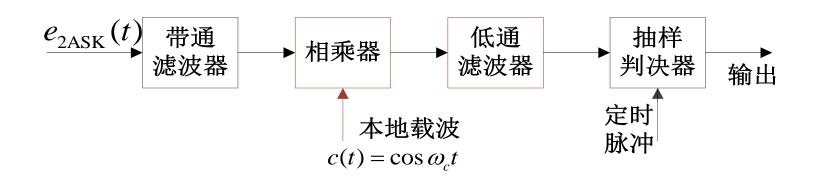


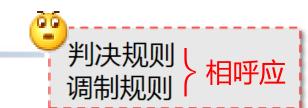
■ 2ASK 解调

◆ 包络检波法

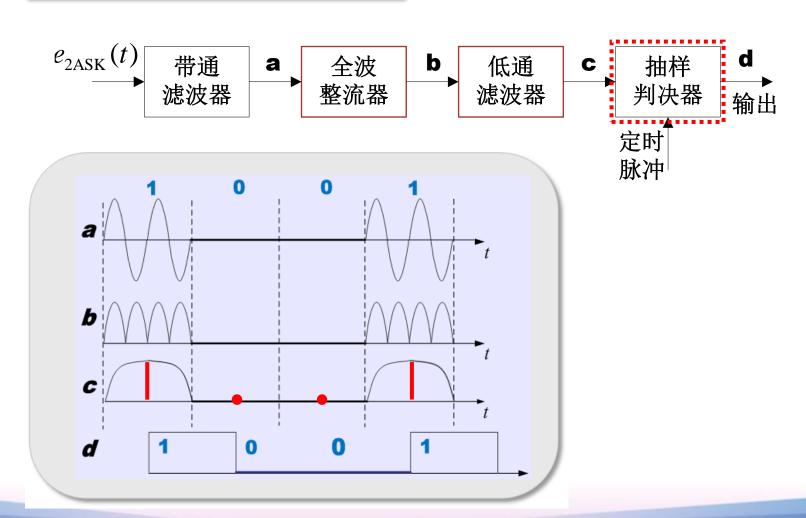


◆ 相干解调法





◆ 包络检波法的各点波形



§ 20.1.2 二进制频移键控 (2FSK)

- 原理: s(t)→载波频率
- 波形:

2FSK可视为 两个不同载频的 2ASK的叠加

表达式:

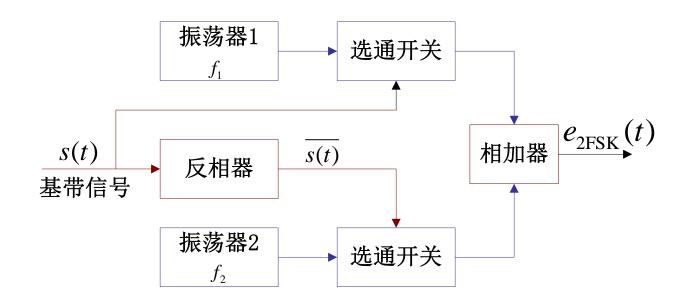
$$e_{2\text{FSK}}(t) = s_1(t)\cos\omega_1 t + s_2(t)\cos\omega_2 t$$

$$s_1(t) = \sum_n a_n g(t - nT_s) \qquad s_2(t) = \sum_n \overline{a}_n g(t - nT_s)$$

$$S_2(t) = \sum_{n=0}^{\infty} a_n g(t - nT_s)$$

■ 2FSK 产生

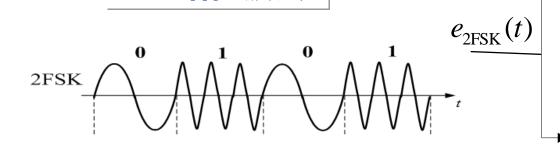
- ◆ 模拟调频法:
- 键控法:

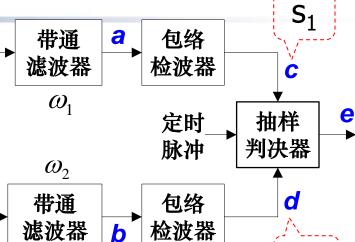


特点:转换速度快、电路简单、

产生的波形好、频率稳定度高。

2FSK 解调包络检波法



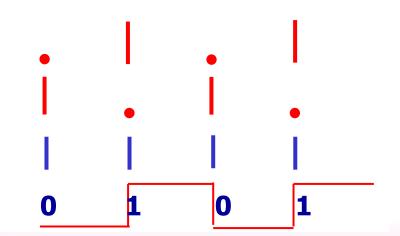


 S_2

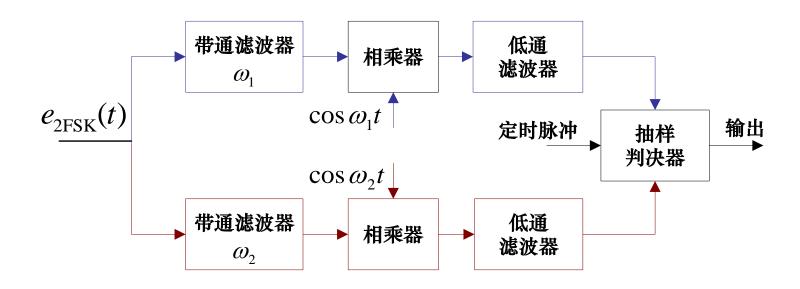
调制规则:

 ω_2 —"0"

判决规则:

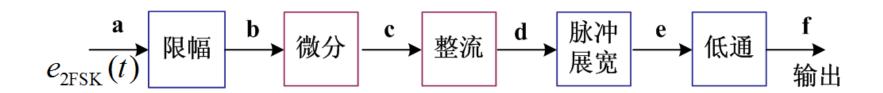


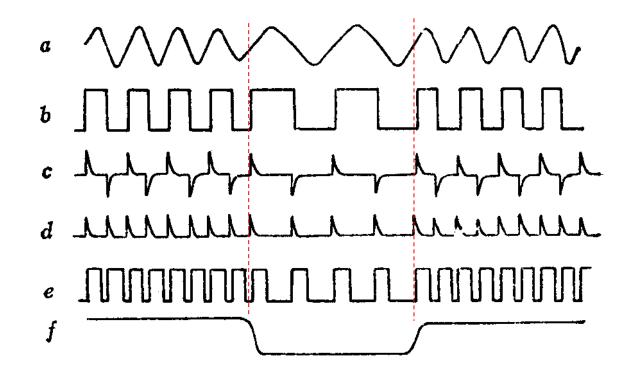
◆ 相干解调法



其他解调方法: 鉴频法、差分检测法、过零检测法等。

◆ 过零检测法





■ FSK应用:

在数字通信中应用较为广泛,尤其适用于衰落信道 (如短波无线电信道)的场合。

国际电信联盟(ITU)建议在数据率低于1200 b/s 的场合采用2FSK体制。

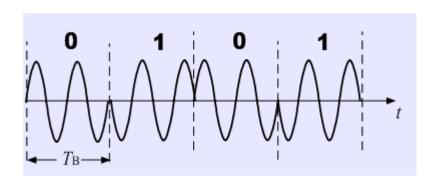
§ 20.1.3 二进制相移键控 (2PSK)

- **原理**: s(t)→载波相位
- 波形:
- 表达式: 双极性

$$e_{2\text{PSK}}(t) = s(t) \cos \omega_c t$$

$$s(t) = \sum_{n} a_{n} g(t - nT_{s})$$

$$a_n = \begin{cases} 1, & P \\ -1, & 1-P \end{cases}$$



$$e_{2PSK}(t) = A\cos(\omega_c t + \varphi_n)$$

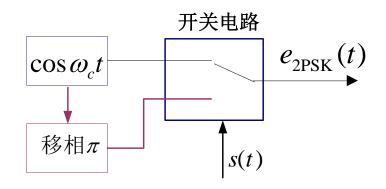
$$\varphi_n = \begin{cases} 0, & \text{发 "0" 时} \\ \pi, & \text{发 "1" 时} \end{cases}$$

$$e_{2\text{PSK}}(t) = \begin{cases} A\cos\omega_c t, & P \\ -A\cos\omega_c t, & 1-P \end{cases}$$

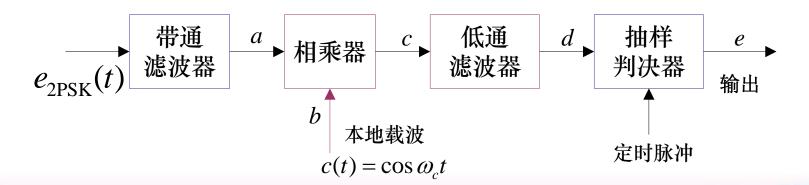
■ 2PSK 产生

◆ 模拟调制法

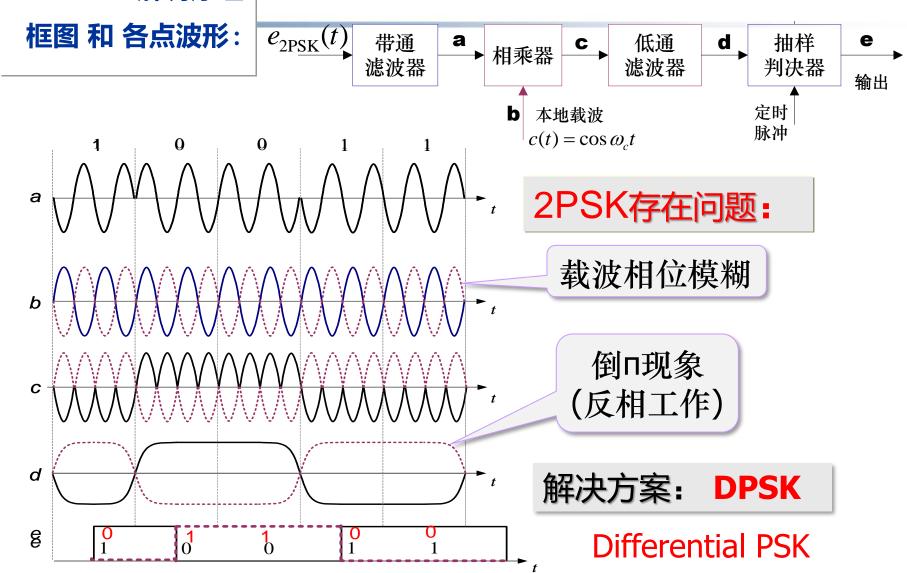
◆ 键控法



■ 2PSK 解调



2PSK 解调原理

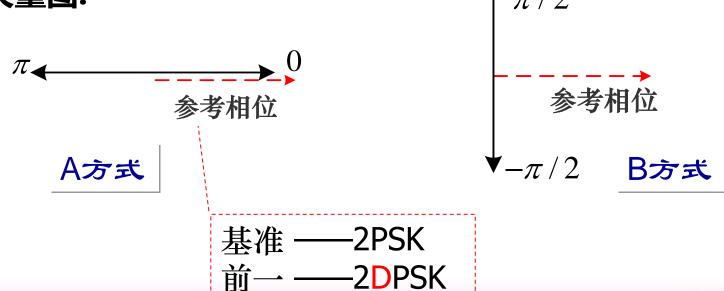


§ 20.1.4 二进制差分相移键控 (2DPSK)

■ 原理: 利用前后相邻码元的载波相对相位表示信息。

$$\Delta \varphi = \varphi_n - \varphi_{n-1} = \begin{cases} 0 \to \text{``0''} \\ \pi \to \text{``1''} \end{cases}$$

■ 矢量图:



示例:

$$\Delta \varphi = \varphi_n - \varphi_{n-1} = \begin{cases} 0 \to 0 \\ \pi \to 1 \end{cases}$$

二进制信码 1 1 0 1 0 2DPSK (0) π 0 0 π π 信号相位 (π) 0 π π 0 0

■ 波形:

绝对码{an}

1

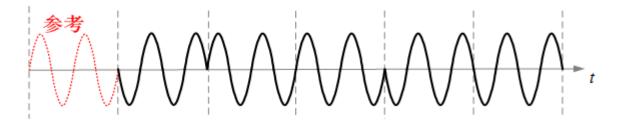
1

O

1

A

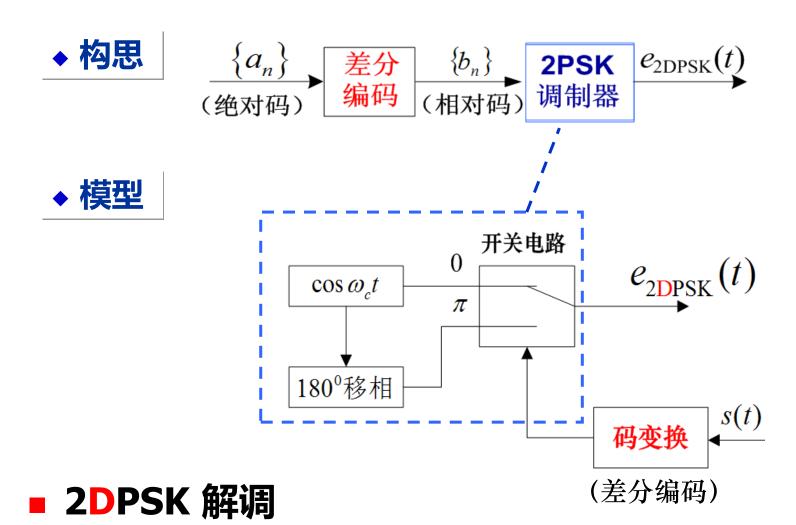
2DPSK



■ 差分编码规则:

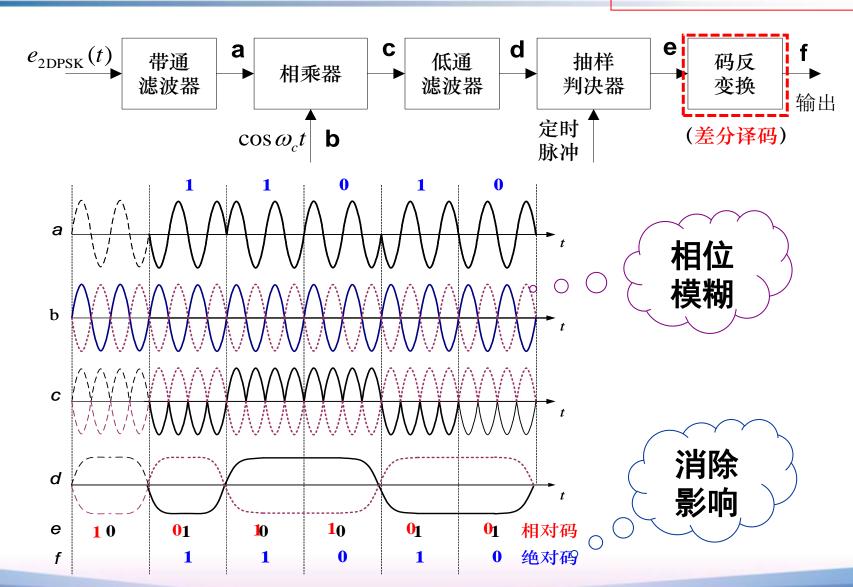
$$b_{n} = a_{n} \oplus b_{n-1}$$

■ 2DPSK 产生

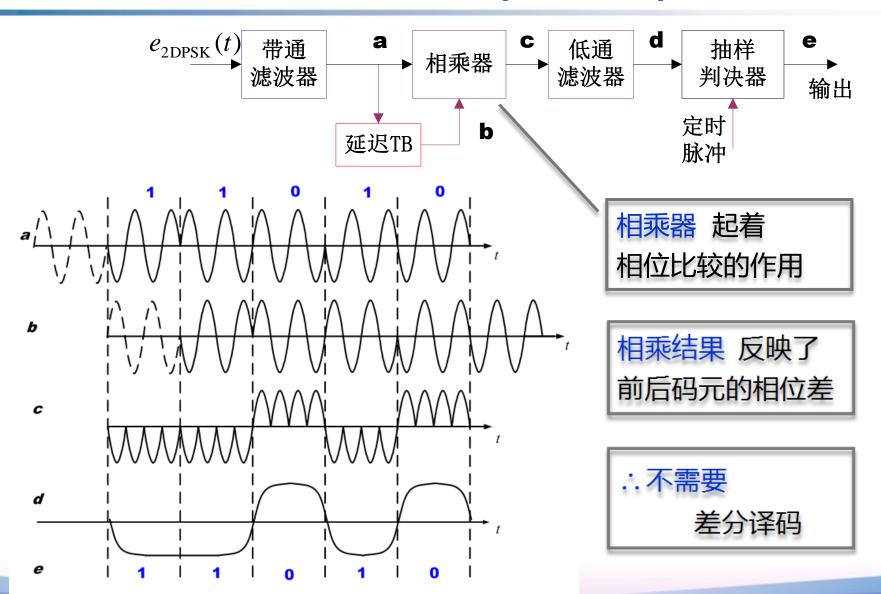


(1) 2DPSK 相干解调 + 码反变换法

 $a_n = b_n \oplus b_{n-1}$



(2) **2DPSK 差分相干解调 (相位比较)法**



§ 20.2

二进制数字调制系统 抗噪声性能

概述

■ 性能指标:系统的误码率 Pe

■ 分析方法: 借用数字基带系统的方法和结论

■ 分析条件: 恒参信道(传输系数取为 K)

信道噪声是加性高斯白噪声

■ 背景知识:

窄带噪声

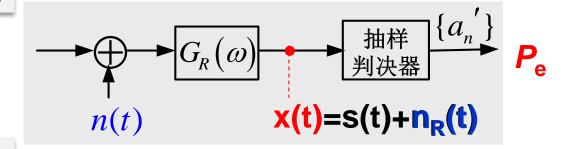
正弦波+窄带噪声

基带传输系统的抗噪声性能

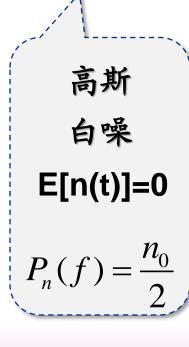
研究: 在无ISI条件下,噪声n(t)引起的误码率 P_e

§20.2.0.1 二进制双极性基带系统的Pe

■分析模型



■ n_R(t)特性



高斯
$$E[n_{R}(t)]=0$$

$$P_{R}(f) = \frac{n_{0}}{2} |G_{R}(f)|^{2}$$

$$\sigma_{n}^{2} = \int_{-\infty}^{\infty} \frac{n_{0}}{2} |G_{R}(f)|^{2} df$$

:. n_R(t) 的一维概率密度函数为

$$f(v) = \frac{1}{\sqrt{2\pi}\sigma_n} \exp\left(-\frac{v^2}{2\sigma_n^2}\right) \quad \text{可简记为:}$$

$$n_R \sim N(0, \sigma_n^2)$$

■ $\mathbf{x}(\mathbf{t})$ 特性 --高斯 $x \sim N(\pm A, \sigma_n^2)$

对于双极性基带信号,其抽样值为(+A,-A),则合成波 $x(t)=s(t)+n_{R}(t)$ 在抽样时刻的取值为:

$$x(kT_{\rm B}) = \begin{cases} A + n_{R}(kT_{\rm B}), & \text{"1"} \\ -A + n_{R}(kT_{\rm B}), & \text{"0"} \end{cases}$$

$$f_0(x) = \frac{1}{\sqrt{2\pi}\sigma_n} \exp\left(-\frac{(x+A)^2}{2\sigma_n^2}\right) \qquad f_1(x) = \frac{1}{\sqrt{2\pi}\sigma_n} \exp\left(-\frac{(x-A)^2}{2\sigma_n^2}\right)$$
P(1/0) = P(x > V_d)
P(0/1) = P(x \le V_d)

设判决门限为 V_d, 判决规则:

"1"

◆ P(0/1) ——发 1 错判为 0 的概率:

$$= P(x \le V_d) = \int_{-\infty}^{V_d} f_1(x) dx$$

$$= \int_{-\infty}^{A} f_1(x) dx - \int_{V_d}^{A} f_1(x) dx$$

$$= \int_{-\infty}^{A} f_1(x) dx - \int_{V_d}^{A} f_1(x) dx$$

$$= \int_{-\infty}^{A} f_1(x) dx - \int_{V_d}^{A} f_1(x) dx$$

$$= \frac{1}{2} - \int_{V_d}^{A} \frac{1}{\sqrt{2\pi\sigma_n}} \exp\left(-\frac{(x-A)^2}{2\sigma_n^2}\right) dx \qquad \Leftrightarrow \frac{x-A}{\sqrt{2}\sigma_n} = t$$

$$= \frac{1}{2} - \int_{\frac{V_d - A}{\sqrt{2}\sigma_n}}^{0} \frac{1}{\sqrt{\pi}} e^{-t^2} dt = \frac{1}{2} + \frac{1}{2} erf(\frac{V_d - A}{\sqrt{2}\sigma_n})$$

◆ P(1/0) ——发 0 错判为 1 的概率:

$$= P(x > V_d) = \int_{V_d}^{\infty} f_0(x) dx$$

$$= \int_{-A}^{\infty} f_0(x) dx - \int_{-A}^{V_d} f_0(x) dx$$

$$= \int_{-A}^{\infty} f_0(x) dx - \int_{-A}^{V_d} f_0(x) dx$$

$$= \frac{1}{2} - \int_{-A}^{V_d} \frac{1}{\sqrt{2\pi\sigma_n}} \exp\left(-\frac{(x+A)^2}{2\sigma_n^2}\right) dx \qquad \left[\Leftrightarrow : \frac{x+A}{\sqrt{2\sigma_n}} = t \right]$$

$$= \frac{1}{2} - \int_{0}^{\frac{V_d + A}{\sqrt{2}\sigma_n}} \frac{1}{\sqrt{\pi}} e^{-t^2} dt = \frac{1}{2} - \frac{1}{2} erf(\frac{V_d + A}{\sqrt{2}\sigma_n})$$

◆ 双极性基带系统的总误码率:

$$P_{e} = P(1)P(0/1) + P(0)P(1/0)$$

$$= P(1) \int_{-\infty}^{V_{d}} f_{1}(x) dx + P(0) \int_{V_{d}}^{\infty} f_{0}(x) dx$$

$$= P(1) \left[\frac{1}{2} + \frac{1}{2} \operatorname{erf}(\frac{V_{d} - A}{\sqrt{2}\sigma_{n}}) \right] + P(0) \left[\frac{1}{2} - \frac{1}{2} \operatorname{erf}(\frac{V_{d} + A}{\sqrt{2}\sigma_{n}}) \right]$$

可见

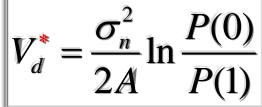
Pe的值取决于

—— P(1)、P(0)、A、 σ_n^2 和 V_d



■最佳门限电平

一使尸。最小的判决门限电平

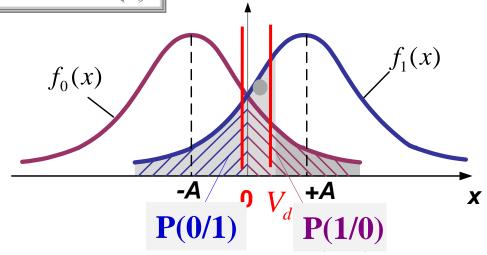


P(1)=P(0)时:

$$V_d^* = 0$$

$$P_{e} = P(0/1) = P(1/0)$$

$$= \frac{1}{2} \pm \frac{1}{2} erf(\frac{0 \mp A}{\sqrt{2}\sigma_n})$$



$$P_e = \frac{1}{2} \operatorname{erfc}(\frac{A}{\sqrt{2}\sigma_n})$$

§ 20.2.0.2 二进制单极性基带系统的Pe

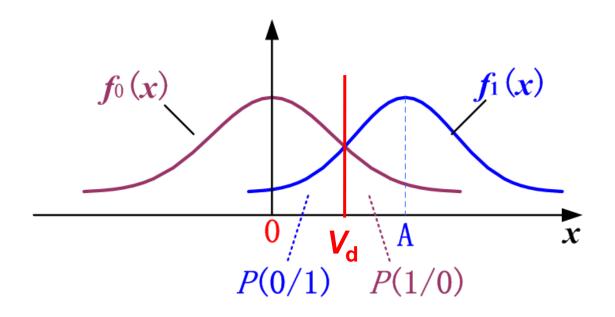
对于单极性基带信号,其抽样值为(+A,0),则合成波 $x(t)=s(t)+n_R(t)$ 在抽样时刻的取值为:

$$x(kT_B) = \begin{cases} A + n_R(kT_B), & \text{"1"} \\ 0 + n_R(kT_B), & \text{"0"} \end{cases}$$

对比: 双极性基带信号, 其抽样值为(+A,-A)

$$x(kT_B) = \begin{cases} A + n_R(kT_B), & \text{"1"} \\ -A + n_R(kT_B), & \text{"0"} \end{cases}$$

二只需将 $f_0(x)$ 的分布中心由-A 移到0即可:



- 推导过程与双极性系统类同;
- 推导结果也可借助双极性的结果进行变通。

■ 归纳 对比:

双极性系统

$$V_d^* = \frac{\sigma_n^2}{2A} \ln \frac{P(0)}{P(1)}$$

等概时:
$$V_d^*=0$$

$$P_e = \frac{1}{2} \operatorname{erfc}(\frac{A}{\sqrt{2}\sigma_n})$$

单极性系统

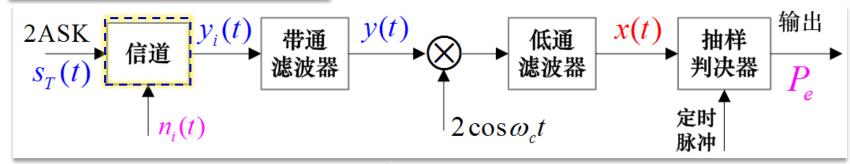
$$V_d^* = \frac{\frac{A}{2}}{2} + \frac{\sigma_n^2}{A} \ln \frac{P(0)}{P(1)}$$

等概时:
$$V_d^* = A/2$$

$$P_e = \frac{1}{2} \operatorname{erfc}(\frac{A}{2\sqrt{2}\sigma_n})$$

§ 20.2.1 2ASK系统的抗噪声性能

■ 2ASK----相干解调



$$\mathbf{s}_{T}(t) = \begin{cases} A\cos\omega_{c}t, & \text{发 "1" 时} \\ 0, & \text{发 "0" 时} \end{cases}$$

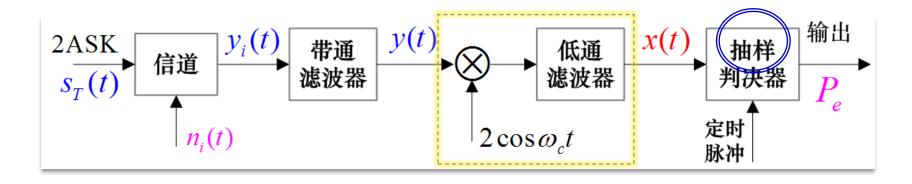
$$\mathbf{a} = \mathbf{k}\mathbf{A}$$

$$\begin{array}{c|c} K & \longrightarrow \\ n_i(t) & \end{array}$$

$$y_i(t) = \begin{cases} a \cos \omega_c t + n_i(t) & \text{发 "1" 时} \\ 0 & + n_i(t) & \text{发 "0" 时} \end{cases}$$

$$y(t) = \begin{cases} a\cos\omega_c t + n(t) & \text{发 "1" 时} \\ 0 & + n(t) & \text{发 "0" 时} \end{cases}$$

$$y(t) = \begin{cases} a\cos\omega_c t + n(t) & \text{发 "1" 时} \\ 0 & + n(t) & \text{发 "0" 时} \end{cases}$$



$$n(t) = n_c(t) \cos \omega_c t - n_s(t) \sin \omega_c t$$

$$y(t) = \begin{cases} [a + n_c(t)]\cos \omega_c t - n_s(t)\sin \omega_c t \\ n_c(t)\cos \omega_c t - n_s(t)\sin \omega_c t \end{cases}$$

$$x(t) = \begin{cases} a + n_c(t), & \text{发 "1"} 时 \\ 0 + n_c(t), & \text{发 "0"} 时 \end{cases}$$

 $n_{c}(t)$ 是高斯过程 (0, σ_{n}^{2})

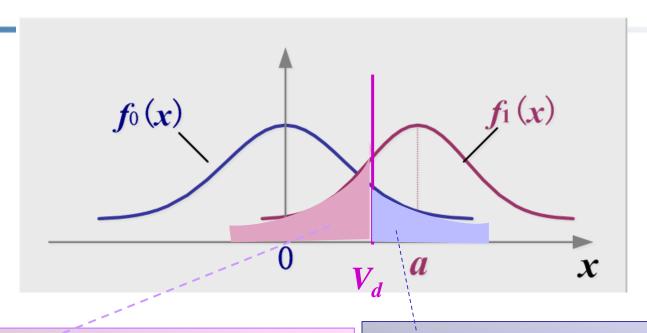
抽样:

$$\mathbf{x} = x(kT_s) = \begin{cases} a + n_c(kT_s) & \text{发 "1" 时} \\ 0 + n_c(kT_s) & \text{发 "0" 时} \end{cases}$$

 $x \sim$ 高斯 (均值 a或 0,方差 σ_n^2),一维概率密度函数:

$$f_1(x) = \frac{1}{\sqrt{2\pi\sigma_n}} \exp\left[-\frac{(x-a)^2}{2\sigma_n^2}\right]$$
 发送"1" 时

$$f_0(x) = \frac{1}{\sqrt{2\pi}\sigma_n} \exp[-\frac{x^2}{2\sigma_n^2}]$$
 发送"0"时



$$P(0/1) = P(x \le V_d) = \int_{-\infty}^{V_d} f_1(x) dx$$

$$P(0/1) = P(x \le V_d) = \int_{-\infty}^{V_d} f_1(x) dx \qquad P(1/0) = P(x > V_d) = \int_{V_d}^{\infty} f_0(x) dx$$

设判决门限为 **V**_d,则判决规则为:

◆ 总误码率:

$$P_{e} = P(1)P(0/1) + P(0)P(1/0)$$

$$= P(1)\int_{-\infty}^{V_{d}} f_{1}(x)dx + P(0)\int_{V_{d}}^{\infty} f_{0}(x)dx$$

$$V_d^* = \frac{\frac{a}{2} + \frac{\sigma_n^2}{a} \ln \frac{P(0)}{P(1)}$$



$$V_d^* = a/2$$

$$V_d^* = \frac{a}{2} + \frac{\sigma_n^2}{a} \ln \frac{P(0)}{P(1)}$$
 等概时 $V_d^* = a/2$ $P_e = \frac{1}{2} erfc(\frac{a}{2\sqrt{2}\sigma_n})$

双极性系统

$$V_d^* = \frac{\sigma_n^2}{2A} \ln \frac{P(0)}{P(1)}$$

等概时

1

$$V_d^* = 0$$

$$P_e = \frac{1}{2} \operatorname{erfc}(\frac{A}{\sqrt{2}\sigma_n})$$

单极性系统

$$V_d^* = \frac{A}{2} + \frac{\sigma_n^2}{A} \ln \frac{P(0)}{P(1)}$$

等概时

3

$$V_d^* = A/2$$

$$P_e = \frac{1}{2} \operatorname{erfc}(\frac{A}{2\sqrt{2}\sigma_n})$$

借用:

$$x(t) = \begin{cases} A + n_R(t) \\ 0 + n_R(t) \end{cases}$$

因此,借助单极性基带系统的分析结果:

$$V_d^* = \frac{A}{2} + \frac{\sigma_n^2}{A} \ln \frac{P(0)}{P(1)}$$
 等概时 $V_d^* = A/2$ $P_e = \frac{1}{2} erfc(\frac{A}{2\sqrt{2}\sigma_n})$

$$V_d^* = A/2$$

$$P_e = \frac{1}{2} erfc(\frac{A}{2\sqrt{2}\sigma_n})$$

可方便地得到2ASK-相干系统的分析结果:

$$V_d^* = \frac{a}{2} + \frac{\sigma_n^2}{a} \ln \frac{P(0)}{P(1)}$$

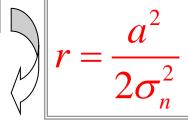
$$V_d^* = a/2$$

$$V_d^* = \frac{a}{2} + \frac{\sigma_n^2}{a} \ln \frac{P(0)}{P(1)}$$
 等概时 $V_d^* = a/2$ $P_e = \frac{1}{2} erfc(\frac{a}{2\sqrt{2}\sigma_n})$

2ASK信号相干解调时系统的总误码率为

$$P_e = \frac{1}{2} \operatorname{erfc}(\frac{a}{2\sqrt{2}\sigma_n})$$

$$P_e = \frac{1}{2} \operatorname{erfc}\left(\sqrt{r/4}\right)$$



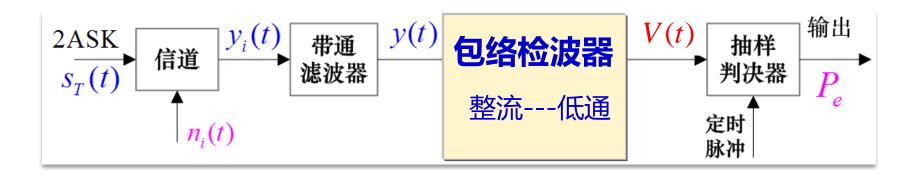
解调器 输入端 信噪比





$$P_e \approx \frac{1}{\sqrt{\pi r}} e^{-r/4}$$

■ 2ASK----包络检波



$$y(t) = \begin{cases} a\cos\omega_c t + n(t) & \text{发 "1" 时~正弦波+窄带高斯噪声} \\ 0 & + n(t) & \text{发 "0" 时~窄带高斯噪声} \end{cases}$$

$$n(t) = n_c(t)\cos\omega_c t - n_s(t)\sin\omega_c t$$

• 当发送"1"符号时,包络检波器的输出为

$$V(t) = \sqrt{[a + n_c(t)]^2 + n_s^2(t)}$$

$$f_1(V) = \frac{V}{\sigma_n^2} I_0 \left(\frac{aV}{\sigma_n^2}\right) e^{-(V^2 + a^2)/2\sigma_n^2} \sim$$
 个义瑞利分布

• 当发送"0"符号时,包络检波器的输出为

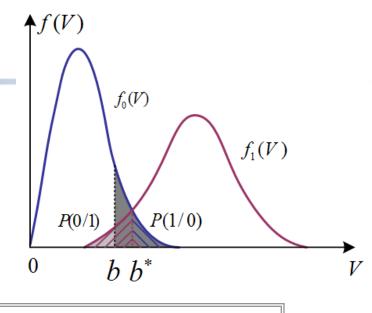
$$V(t) = \sqrt{n_c^2(t) + n_s^2(t)}$$

$$f_{\mathbf{0}}(V) = \frac{V}{\sigma_n^2} e^{-V^2/2\sigma_n^2}$$
 ~ 端利分布

式中, σ_n^2 为窄带高斯噪声n(t)的方差。

设判决门限为₺,判决规则为:

抽样值 V > b 时, 判为"1" 抽样值 V ≤ b 时, 判为"0"



• 发 "1" 错判为 "0" 的概率为

$$P(0/1) = P(V \le b) = \int_0^b f_1(V)dV = 1 - \int_b^\infty f_1(V)dV$$

$$= \mathbf{1} - \int_{b}^{\infty} \frac{V}{\sigma_{n}^{2}} I_{0} \left(\frac{aV}{\sigma_{n}^{2}} \right) e^{-(V^{2} + a^{2})/2\sigma_{n}^{2}} dV$$

利用Marcum Q函数:

$$Q(\alpha,\beta) = \int_{\beta}^{\infty} t I_0(\alpha t) e^{-(t^2 + \alpha^2)/2} dt$$

$$\alpha = \frac{a}{\sigma_{n}}$$

$$\beta = \frac{b}{\sigma_{n}}$$

$$t = \frac{V}{\sigma_{n}}$$

则 P(0/1) 可借助Marcum Q函数表示为

$$P(0/1) = 1 - Q(\frac{a}{\sigma_n}, \frac{b}{\sigma_n}) = 1 - Q(\sqrt{2r}, b_0)$$

$$r = a^2/2\sigma_n^2$$
 信号噪声功率比 $b_0 = b/\sigma_n$ 归一化门限值

• 发 "0" 错判为 "1" 的概率为

$$P(1/0) = P(V > b) = \int_{b}^{\infty} f_{0}(V) dV$$

$$= \int_{b}^{\infty} \frac{V}{\sigma^{2}} e^{-V^{2}/2\sigma_{n}^{2}} dV = e^{-b^{2}/2\sigma_{n}^{2}} = e^{-b_{0}^{2}/2}$$

• 系统的总误码率为

$$P_e = P(1)P(0/1) + P(0)P(1/0)$$

$$= P(1)\left[1 - Q(\sqrt{2r}, b_0)\right] + P(0)e^{-b_0^2/2}$$

当 P(1) = P(0) 时,有

$$P_e = \frac{1}{2} \left[1 - Q(\sqrt{2r}, b_0) \right] + \frac{1}{2} e^{-b_0^2/2}$$

包检系统的P。取决于信噪比r和归一化门限值bo

• 求最佳判决门限,令:

$$\frac{\partial P_e}{\partial b} = \mathbf{0}$$

$$P(1)f_1(b^*) = P(0)f_0(b^*)$$

当*P*(1)=*P*(0)时,有

$$f_1(b^*) = f_0(b^*)$$

最佳判决门限:

$$b^* = \begin{cases} a/2, & r >> 1 时 \\ \sqrt{2}\sigma_n, & r << 1 时 \end{cases}$$

归一化最佳判决门限:
$$b_0^* = \frac{b^*}{\sigma_n} = \begin{cases} \sqrt{r/2}, & r >> 1 \text{ of } t >> 1 \end{cases}$$

实际情况

——系统工作在大信噪比情况下, : 最佳门限应取:

$$\boldsymbol{b}^* = \frac{\boldsymbol{a}}{2}$$

$$b^* = \begin{cases} a/2, & r >> 1 \text{ 时} \\ \sqrt{2}\sigma_n, & r << 1 \text{ 时} \end{cases}$$

此时,系统的总误码率为:

$$P_e = \frac{1}{4} \operatorname{erfc}\left(\sqrt{\frac{r}{4}}\right) + \frac{1}{2} e^{-\frac{r}{4}}$$

当 r → ∞ 时,上式的下界为:

$$P_e = \frac{1}{2}e^{-\frac{r_4}{4}}$$

-2ASK系统抗噪声性能

相干解调:
$$P_e = \frac{1}{2} \operatorname{erfc} \left(\sqrt{\frac{r}{4}} \right) \xrightarrow{r >> 1} \frac{1}{\sqrt{\pi r}} e^{-r/4}$$

包络检波:
$$P_e = \frac{1}{2}e^{-r/4}$$

$$r = \frac{a^2}{2\sigma_n^2}$$

条件:
$$P(0) = P(1)$$
, $b^* = \frac{a}{2}$

$$\sigma_n^2 = n_0 B$$

- - \rightarrow 当 r >> 1时,两者性能相差不大。

【7-1】 **2ASK**系统, $R_B = 4.8 \times 10^6$ 波特,1、0等概,接收机输入信号幅度 a = 1 mV,信道加性高斯白噪声的单边PSD为 $n_0 = 2 \times 10^{-15}$ W/Hz。试求:

(1) 相干解调时系统的 P_e ; (2) 包络检波时系统的 P_e 。

解

接收端带通滤波器带宽为:

$$B = 2R_B = 9.6 \times 10^6 \text{ Hz}$$

带通滤波器输出噪声平均功率为:

$$\sigma_n^2 = n_0 B = 1.92 \times 10^{-8} \text{ W}$$

信噪比为:

$$r = \frac{a^2}{2\sigma_n^2} = \frac{1 \times 10^{-6}}{2 \times 1.92 \times 10^{-8}} \approx 26 >> 1$$

(1) 同步检测法解调时系统的误码率为

$$P_e \approx \frac{1}{\sqrt{\pi r}} e^{-r/4} = \frac{1}{\sqrt{3.1416 \times 26}} \times e^{-6.5} = 1.66 \times 10^{-4}$$

(2) 包络检波法解调时系统的误码率为

$$P_e = \frac{1}{2}e^{-\frac{r}{4}} = \frac{1}{2}e^{-6.5} = 7.5 \times 10^{-4}$$

评注

- \rightarrow 当 Γ 相同时, $P_{e相}$ < $P_{e_{0}}$ \wedge
- \rightarrow 当 $\Gamma >>1$ 时,两者的性能相差不大。

谢谢!