SHANDONG UNIVERSITY

Physics I: Introduction to Wave Theory
SDU Course Number: sd01232810 (Fall 2021)

Lecture 11: Photon: quantum of energy

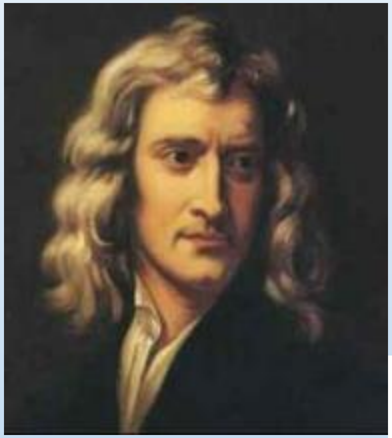
Outline

- Light as Waves
- Light as Particles
    . Photoelectric Effect - photon energy
    . Compton Effect - photon momentum
- Wavepackets
- Heisenberg's Uncertainty Principle
- Wave Particle Duality
- Matter Waves

Are Photons Particles or Waves ?

Newton believed that light was particles:

- light travels in straight lines !



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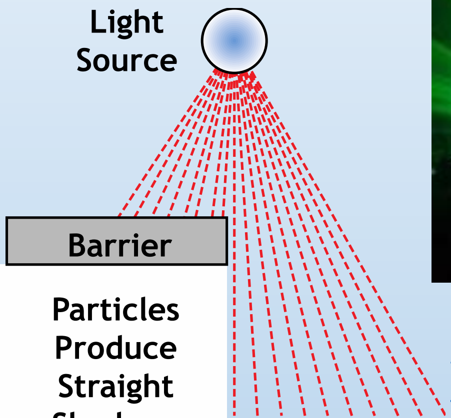

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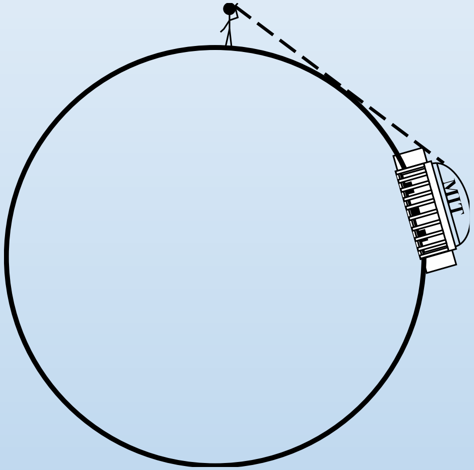

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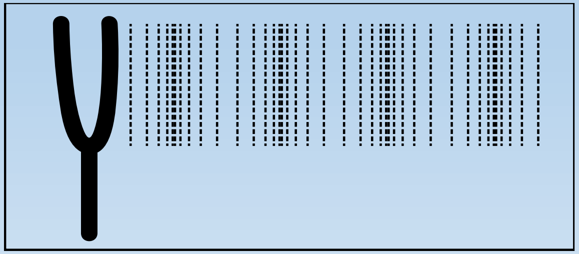
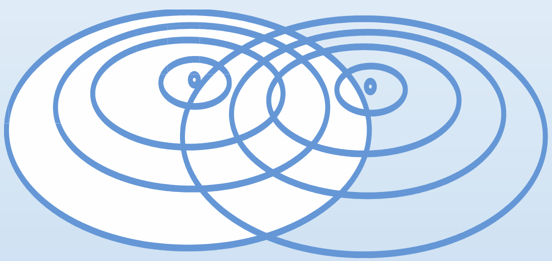
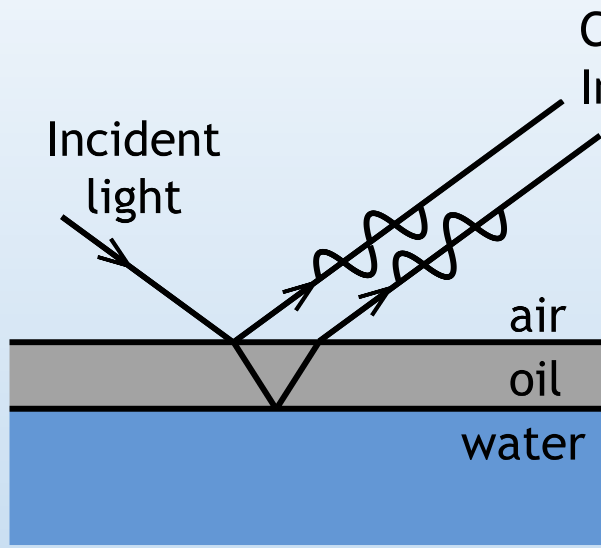
- what is ‘waving’ in an EM wave ?

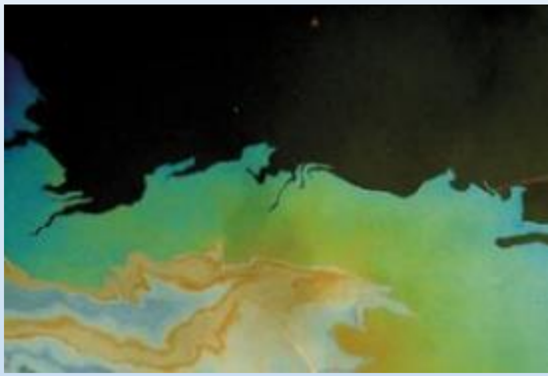
  A wave is a vibration of some medium through which it propagates, e.g., water waves, waves propagating on a string



We described them as WAVES up till now







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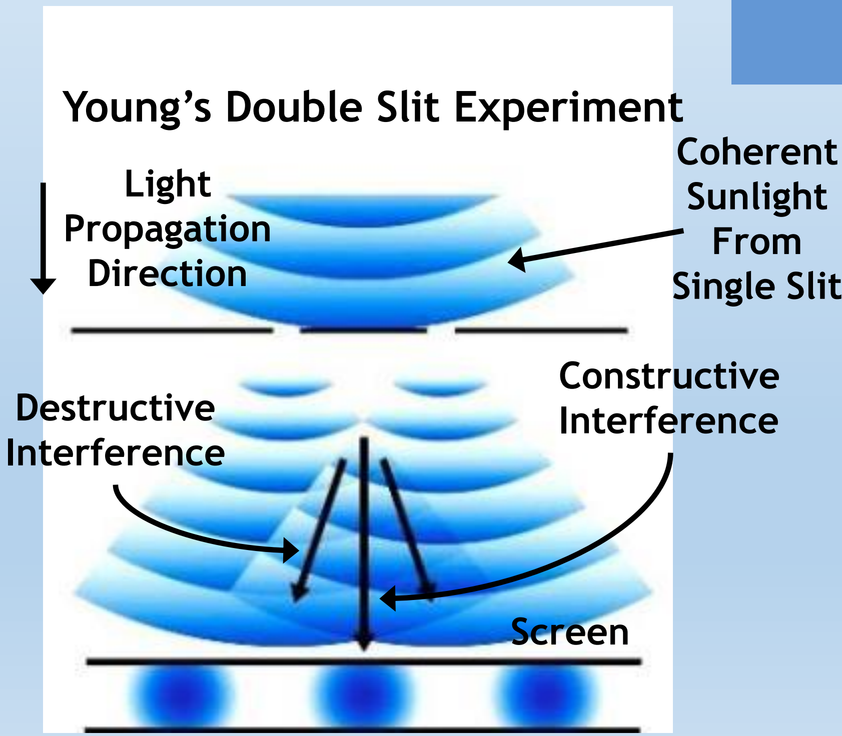

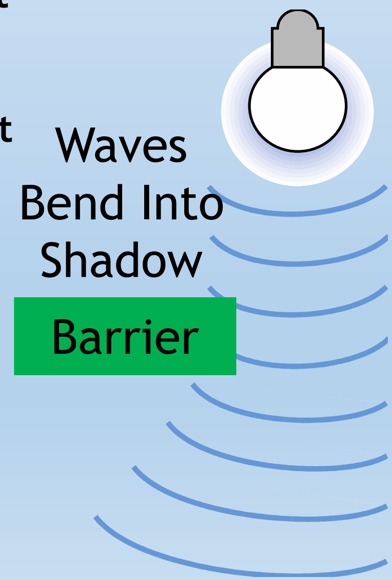

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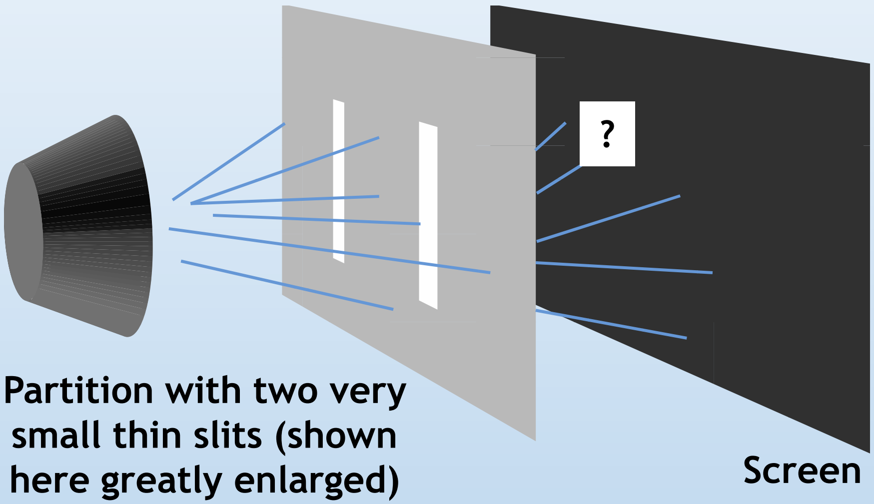
Thomas Young's Double Slit Experiment



Partition with two very small thin slits (shown here greatly enlarged) to let light through.

Interference is the defining characteristic of <u>waves</u>

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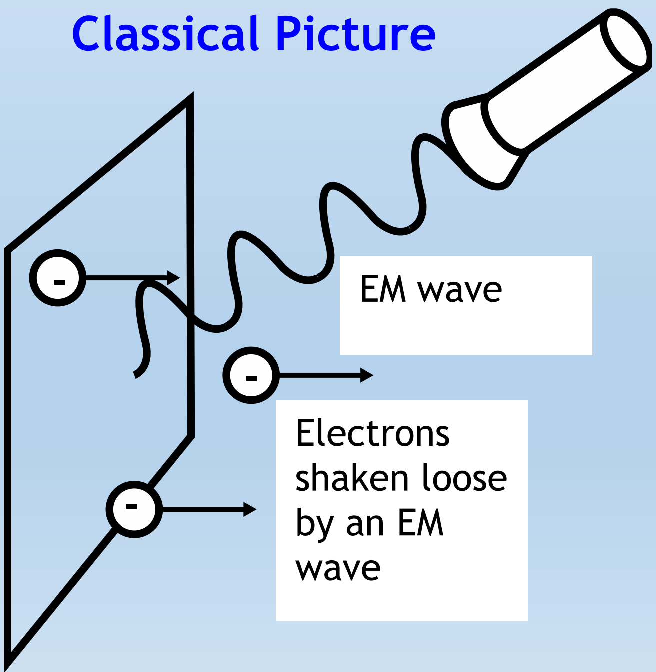
But what happens when we <u>reduce the intensity</u> of incident light … everything should just get dimmer … Right ?

Photoelectric Effect

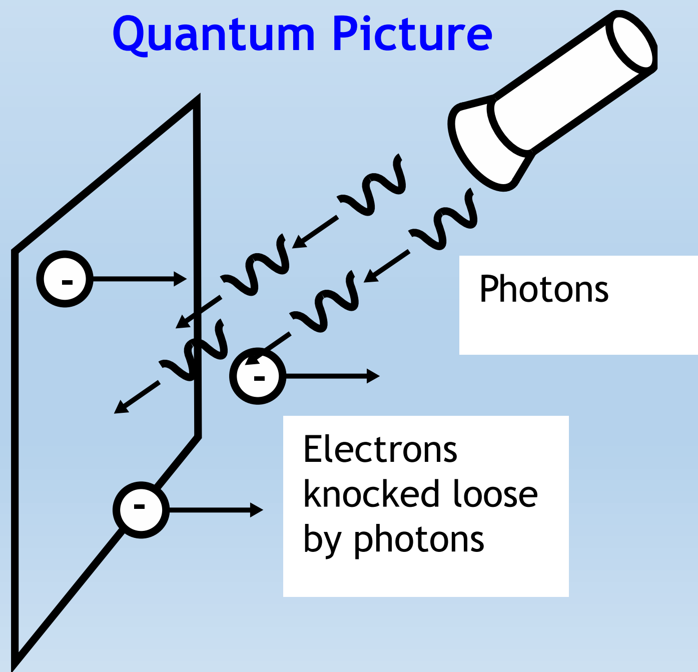
- When light is incident on certain metallic surfaces, electrons are emitted from the surface
  - This is called the *photoelectric effect*
  - The emitted electrons are called *photoelectrons*

- The effect was first discovered by Hertz
- The successful explanation of the effect was given by Einstein in 1905
  - Received Nobel Prize in 1921 for paper on electromagnetic radiation, of which the photoelectric effect was a part

Classical Picture

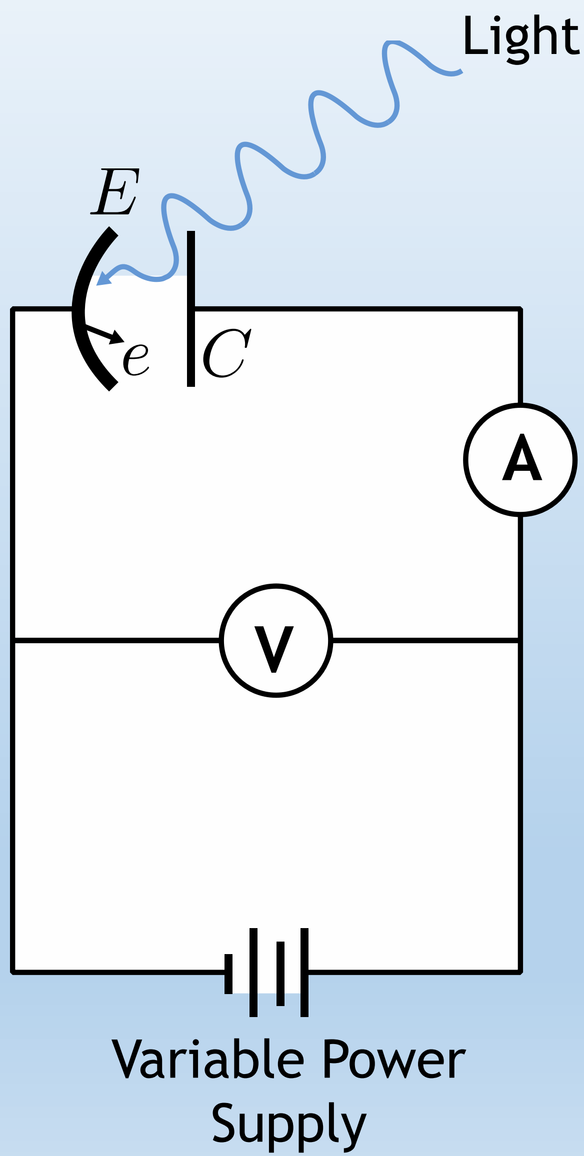


Quantum Picture



Photoelectric Effect Schematic

- When light strikes E, photoelectrons are emitted
- Electrons collected at C and passing through the ammeter are a current in the circuit
- C is maintained at a positive potential by the power supply



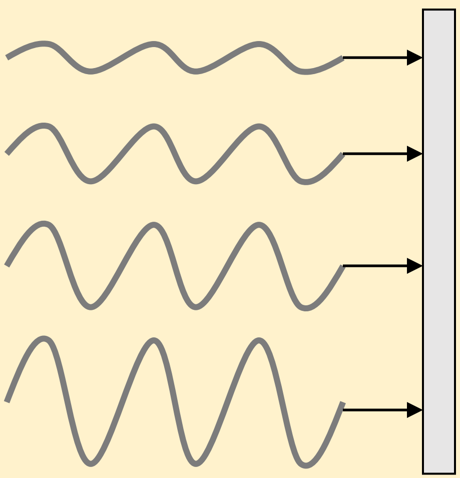
Observation of the Photoelectric Effect

… a Quantum Phenomenon

"Classical" Method

Increase energy by increasing amplitude

electrons emitted ?



No

No

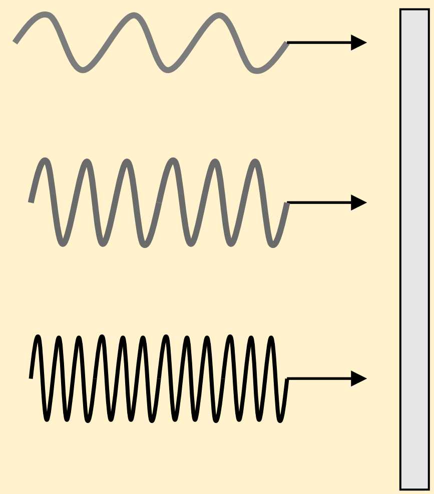
No

No

What if we try this ?

Vary wavelength, fixed amplitude

electrons emitted ?



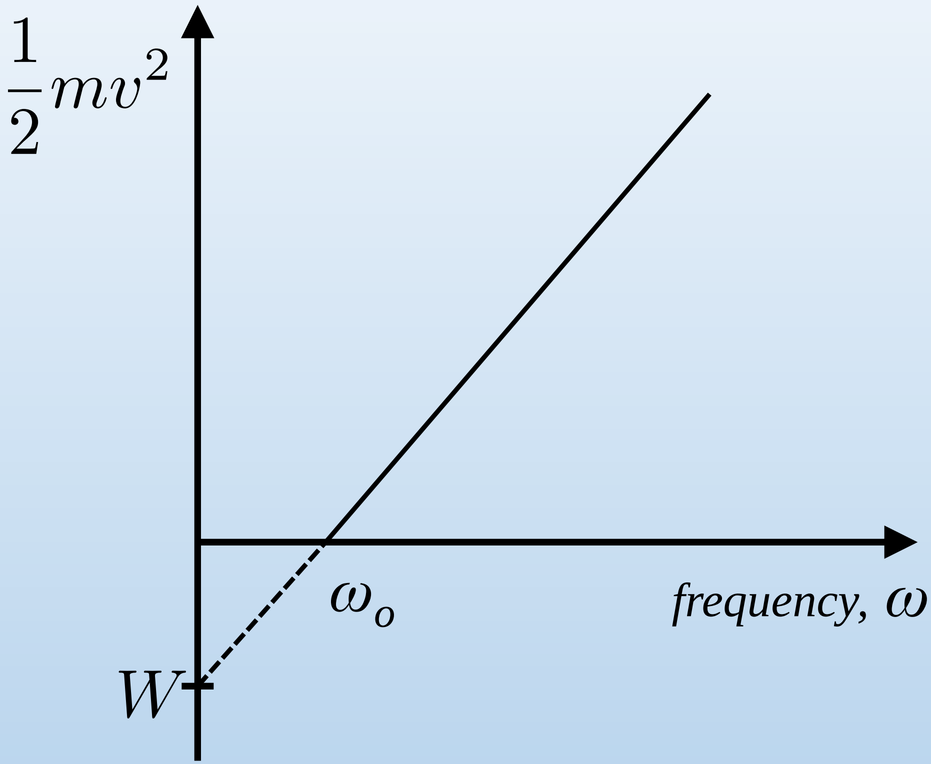
No

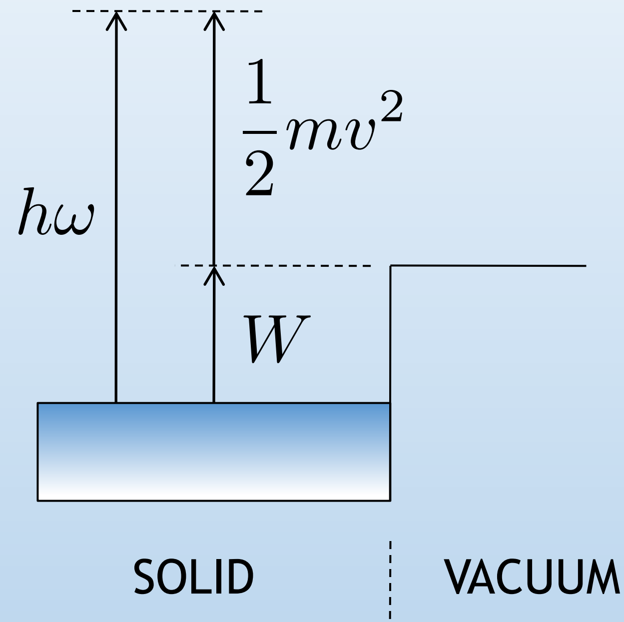
Yes, with low KE

Yes, with high KE

No electrons were emitted until the frequency of the light exceeded a critical frequency, at which point electrons were emitted from the surface ! (Recall: small $\lambda \rightarrow$ large ν)

Electron Energy as a Function of Frequency





$$\hbar\omega = W + \frac{1}{2}mv^2$$

PHOTON ENERGY

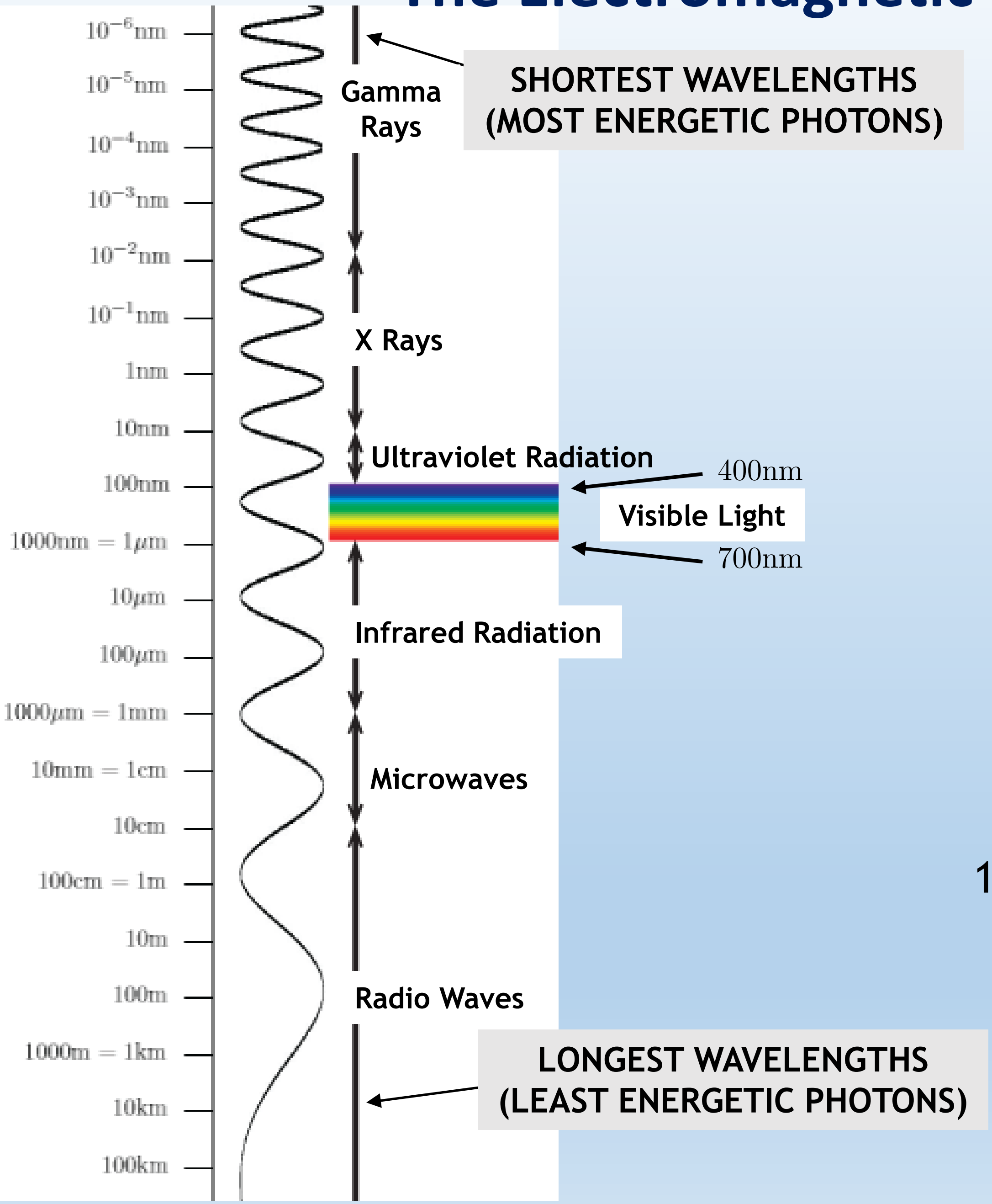
BINDING ENERGY OF ELECTRON

ELECTRON KINETIC ENERGY

$$h = 6.626 \times 10^{-34}\,[J \cdot s]$$

$$\hbar = \frac{h}{2\pi} = 1.055 \times 10^{-34}\,[J \cdot s]$$

The Electromagnetic Spectrum



According to **quantum theory**, a **photon** has an **energy** given by

$$E = h\nu = \frac{hc}{\lambda} = \hbar\omega$$

$$h = 6.6 \times 10^{-34}\,[\mathrm{J \cdot s}]$$

(Planck's constant)

$$\hbar = 1.05 \times 10^{-34}\,[\mathrm{J \cdot s}]$$

10 photons have an energy equal to ten times that of a single photon

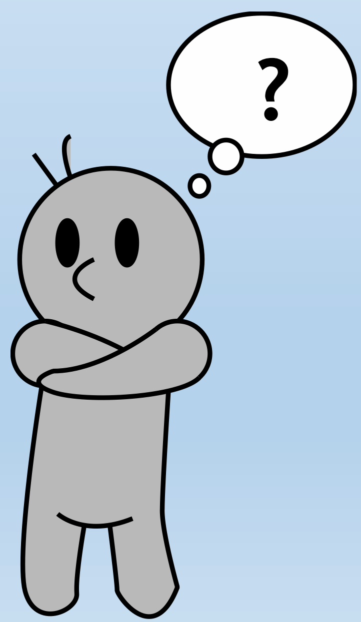
$$E[eV] = \frac{1239.84}{\lambda[nm]}$$

So how do I reconcile wave and particle pictures?

I thought that Maxwell's equations described light …

What is the connection between
Maxwell's equations and photons ?

When to use classical Maxwell's equations ?



Intensity

Classical Intensity

$$\vec{S} = \vec{E} \times \vec{H} \Longrightarrow \frac{\text{Watts}}{\text{cm}^2}$$

Intensity in terms of Photons

$$\left|\vec{S}\right| = \frac{n\hbar\omega}{\tau A} \Longrightarrow \frac{\text{photons}}{\text{sec cm}^2}\frac{\text{J}}{\text{photon}} = \frac{\text{Watts}}{\text{cm}^2}$$

Do Photons Have Momentum ?

What is momentum ?

$$E = \frac{1}{2}mv^2 = \frac{1}{2}(mv) \cdot v = \frac{1}{2}p \cdot v$$

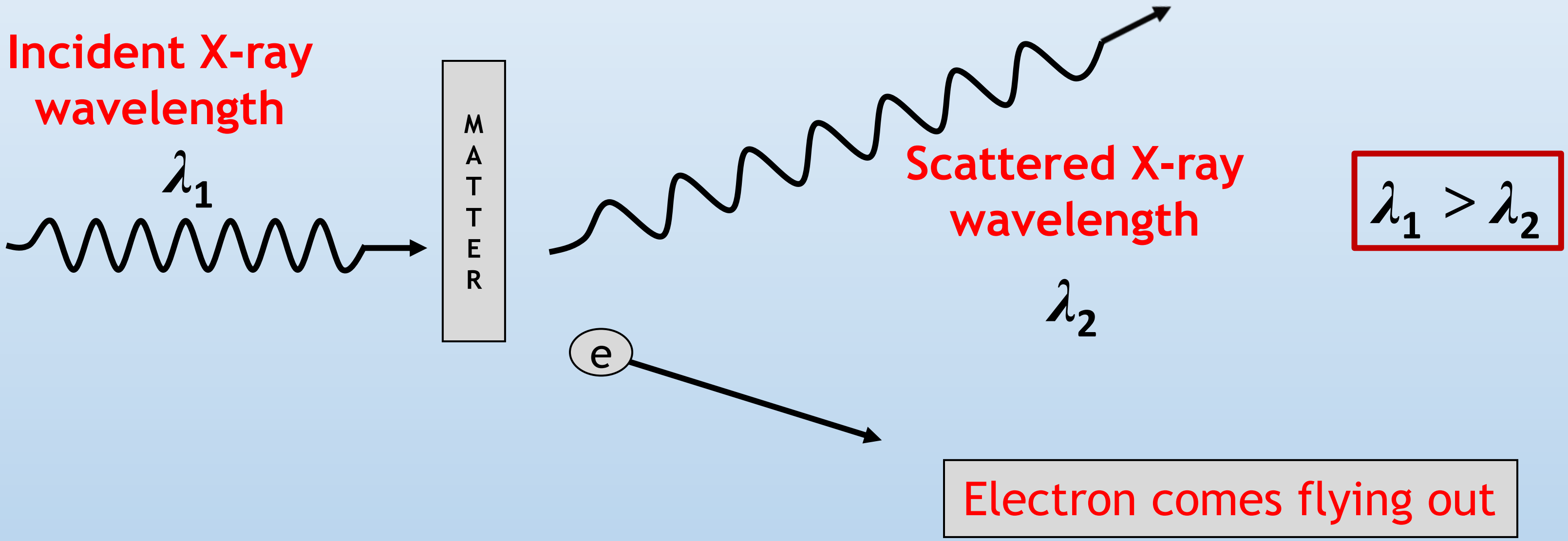
Just like Energy,
TOTAL MOMENTUM IS ALWAYS CONSERVED

Photons have energy and a finite velocity so there must be some momentum associated with photons !

$$p = \frac{E}{c} = \frac{h\nu}{c}$$

The Compton Effect

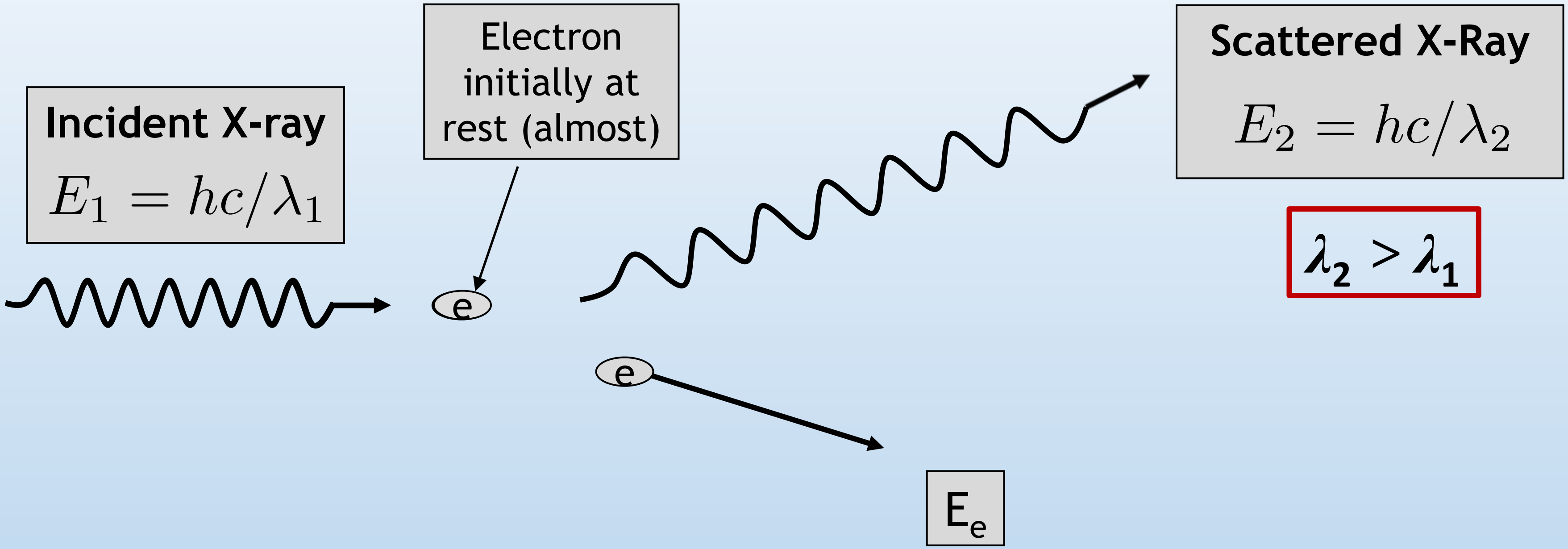
In 1924, A. H. Compton performed an experiment where X-rays impinged on matter, and he measured the scattered radiation.



Problem: According to the wave picture of light, the incident X-ray should give up some of its energy to the electron, and emerge with a lower energy (*i.e.*, the amplitude is lower), but should have $\lambda_1 = \lambda_2$.

It was found that the scattered X-ray did not have the same wavelength !

Quantum Picture to the Rescue



Compton found that if you treat the photons as if they were particles of zero mass, with energy $E = hc/\lambda$ and momentum $p = \lambda/h$.

➔ The collision behaves just as if it were two billiard balls colliding !

Photon behaves like a particle with energy & momentum as given above!

Photon Momentum

IN FREE SPACE:

$$E = cp \quad \Rightarrow \quad p = \frac{E}{c} = \frac{\hbar\omega}{c} = \hbar k$$

IN OPTICAL MATERIALS:

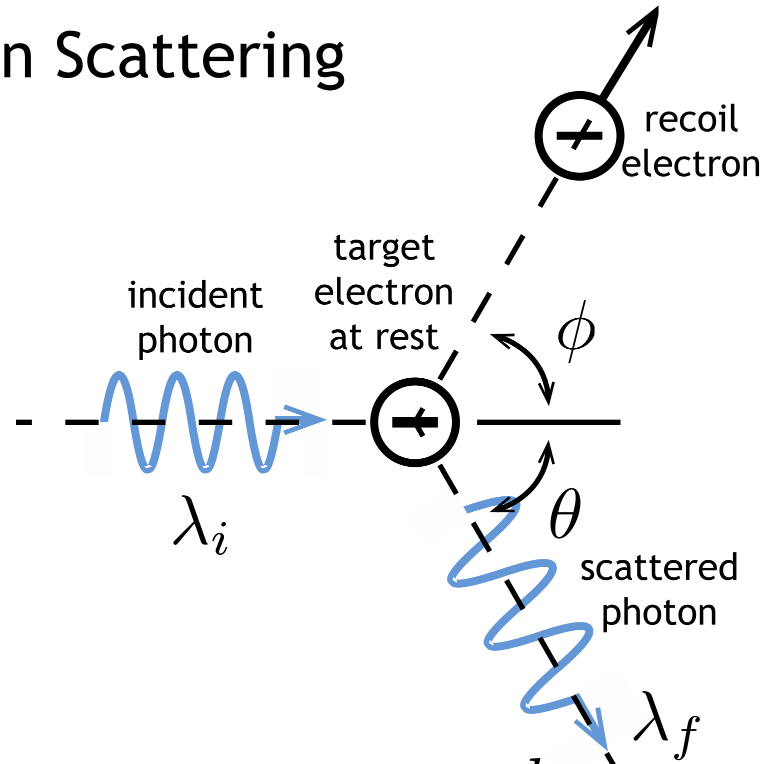
$$E = v_p p \quad \Rightarrow \quad p = \frac{E}{v_p} = \frac{\hbar\omega}{v_p} = \hbar k_{vac} n$$

Image by GFHund http://commons.wikimedia.org/wiki/File:Compton,Arthur_1929_Chicago.jpg Wikimedia Commons.

It was found that the scattered X-ray did not have the same wavelength !

In 1924, A. H. Compton performed an experiment where X-rays impinged on matter, and he measured the scattered radiation.

Compton Scattering



$$\lambda_f - \lambda_i = \Delta\lambda = \frac{h}{m_o c}(1 - \cos\theta)$$

Compton found that if you treat the photons as if they were particles of zero mass, with energy *$E=hc/\lambda$* and momentum *$p=h/\lambda$* .

the collision behaves just as if it were two billiard balls colliding !
(with total momentum always conserved)

Photon Momentum - Moves Solar Sails

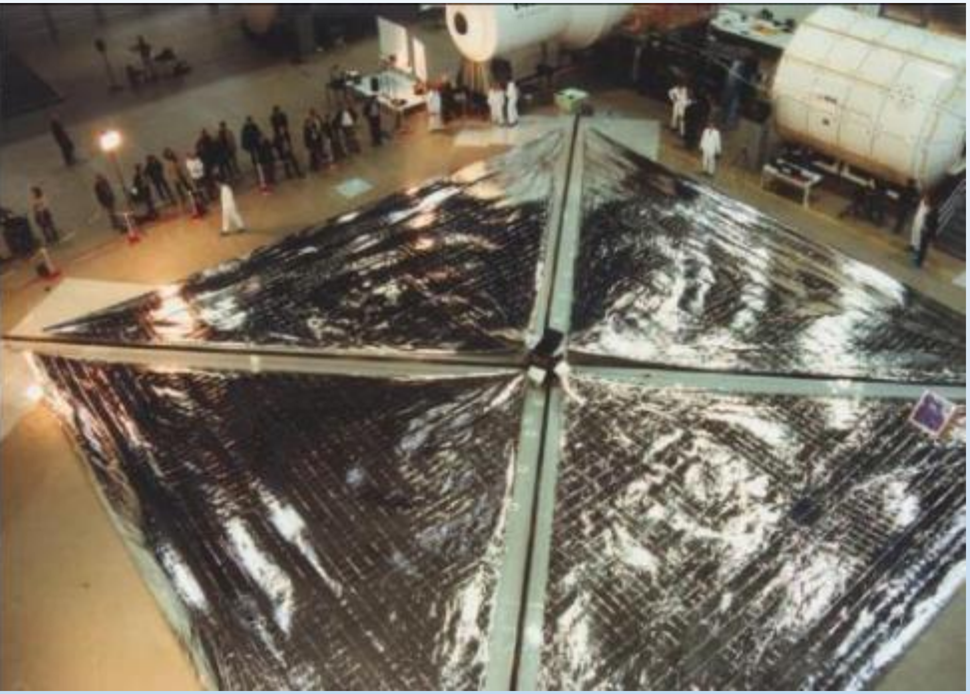
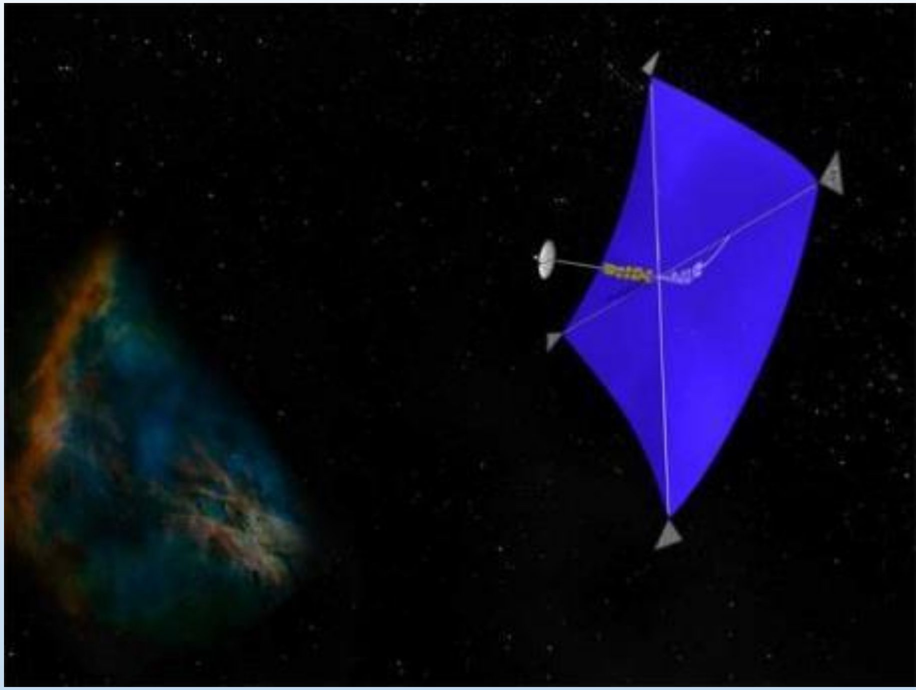


Image by D. Kassing http://en.wikipedia.org/wiki/File:SolarSail-DLR-ESA.jpg on Wikipedia



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INCOMING PHOTONS
1000 W/m² →

SOLAR SAIL
at rest

every second
photons with momentum
+ (1000 J/m²)/c impact the sail

REFLECTED PHOTONS
1000 W/m² ←

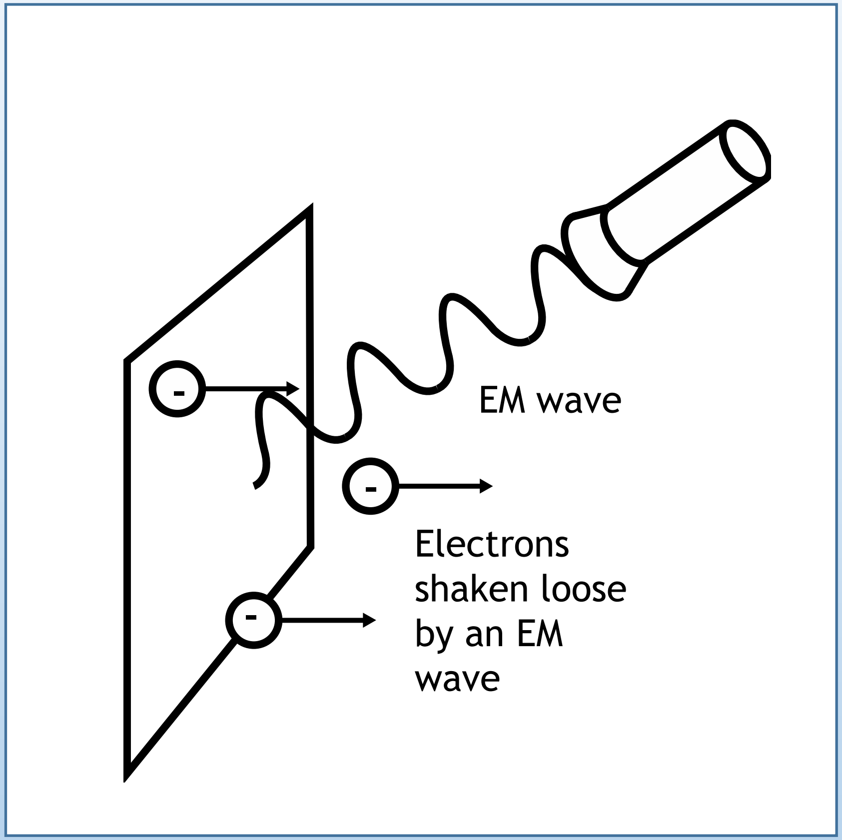
SOLAR SAIL
moves →

every second
photons with momentum
- (1000 J/m²)/c leave the sail

with
momentum
+ (2000 J/m²)/c
… and gets that
much more
momentum
every second …

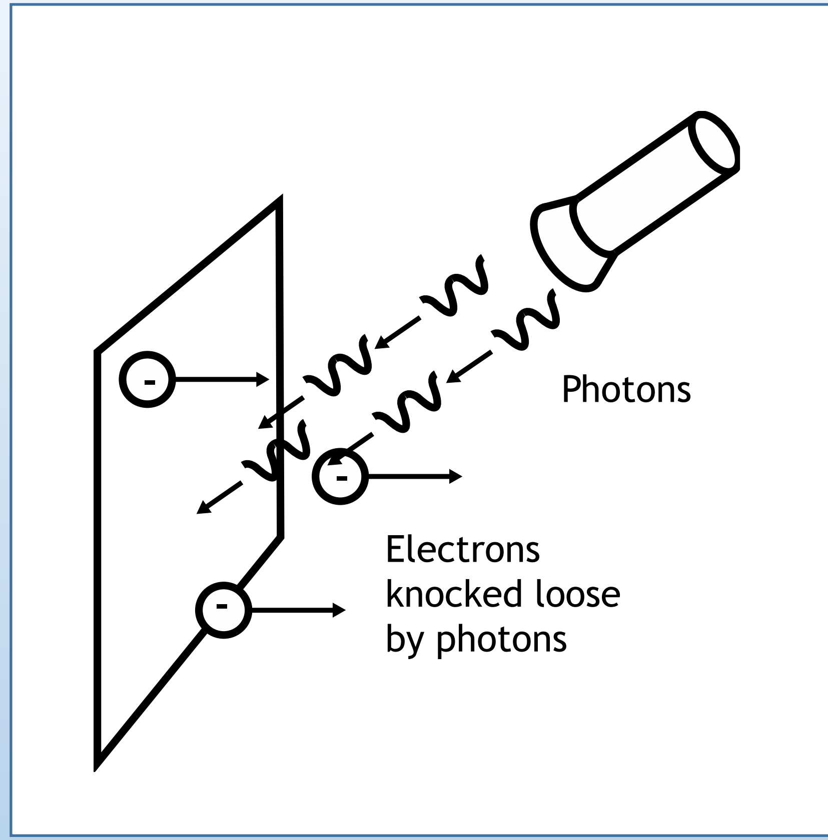
Pressure acting on the sail = (2000 J/m²) /c /second = 6.7 Newtons/km²

Classical Picture



Energy of EM wave ~ (Amplitude)^2

Quantum Picture



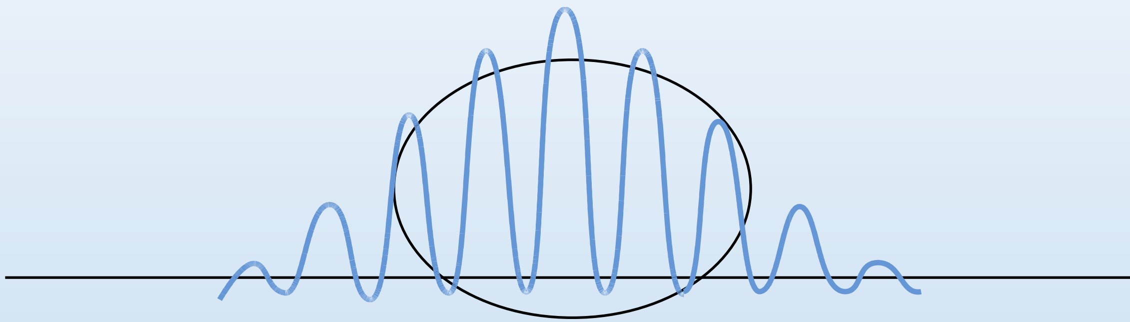
Energy per photon $= hc/\lambda$

photon momentum $= E/c$

$$= (hc/\lambda)/c = h/\lambda$$

So is Light a Wave or a Particle ?

Light is always both Wave and Particle !



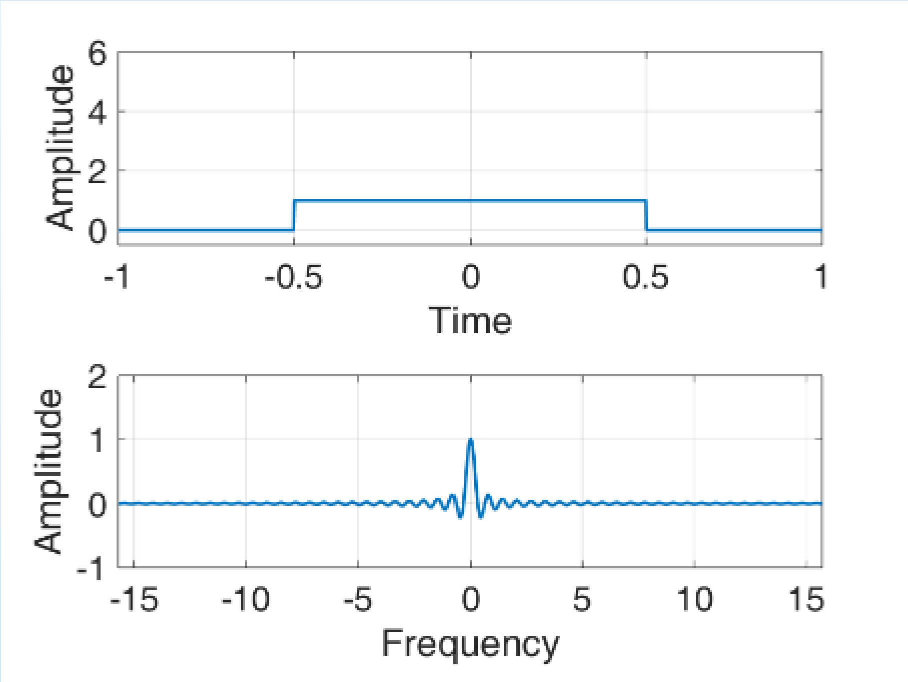
On macroscopic scales, large number of photons look like they exhibit only wave phenomena.

A single photon is still a wave, but your act of trying to measure it makes it look like a localized particle.

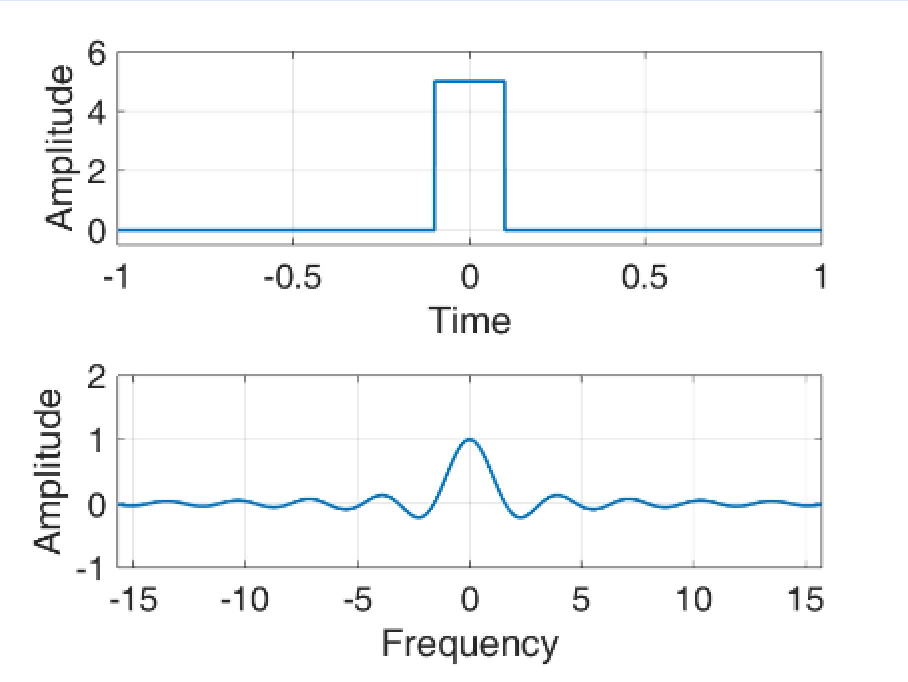
Review of Fourier transform

$$f(t)=\begin{cases} 1/\tau & -\tau/2<t<\tau/2 \\ 0 & others \end{cases} \Rightarrow F(\omega)=\int_{-\infty}^{+\infty} f(t)e^{-i\omega t}dt=\frac{\sin(\omega\tau/2)}{\omega\tau/2}$$

$\tau = 1$



$\tau = 0.2$



$$\boxed{\Delta t \uparrow \;\Leftrightarrow\; \Delta\omega \downarrow}$$

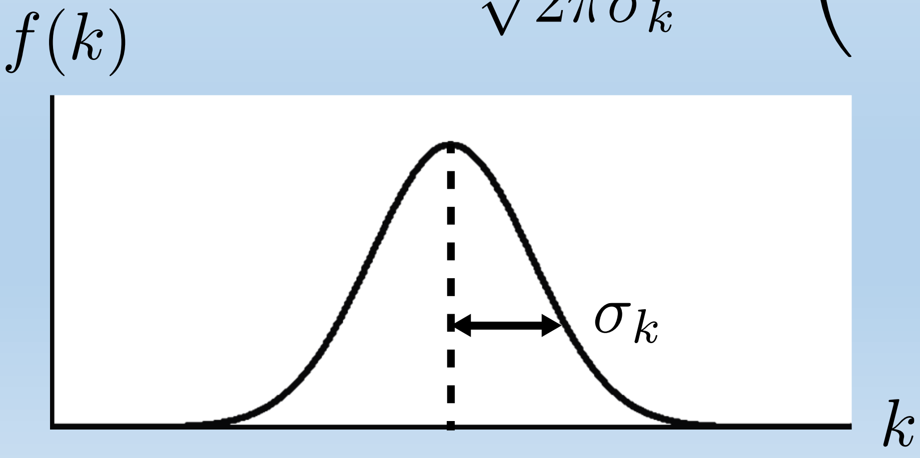
Review of Wavepackets

What would we get if we superimposed waves of many different frequencies ?

$$\vec{E} = \vec{E}_o \int_{-\infty}^{+\infty} f\left(k\right) e^{+j(\omega t - kz)} dk$$

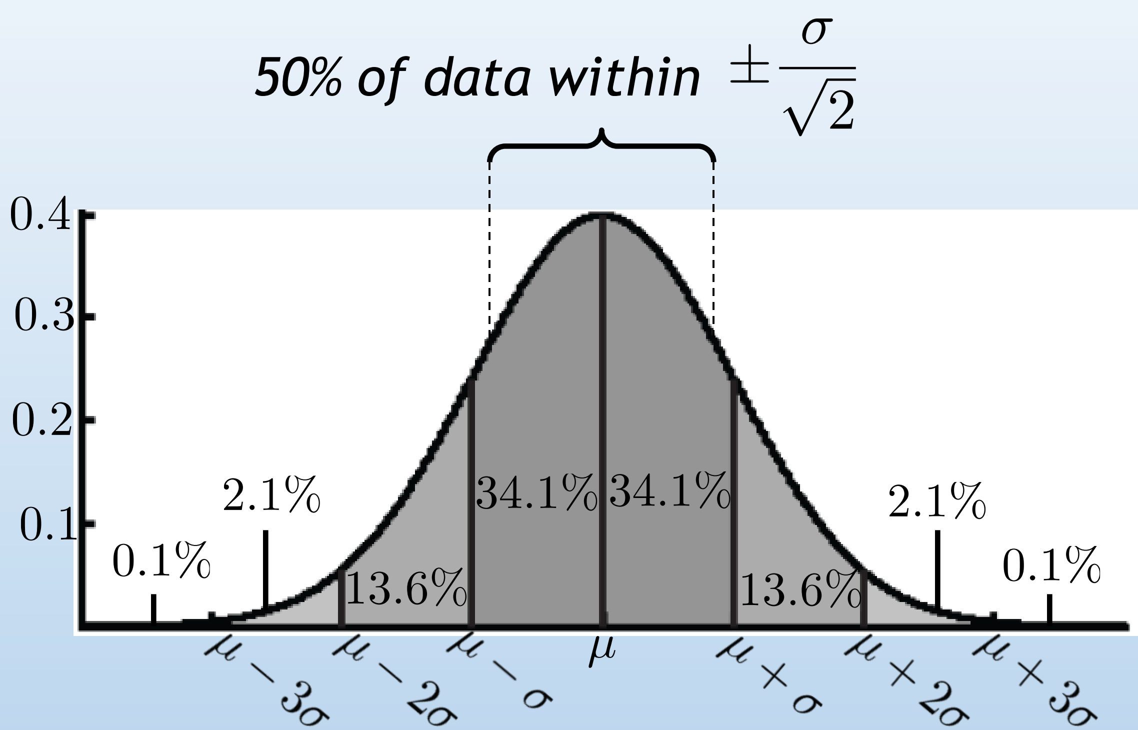
Let's set the frequency distribution as Gaussian

$$f\left(k\right) = \frac{1}{\sqrt{2\pi\sigma_k}} exp\left(-\frac{\left(k-k_o\right)^2}{2\sigma_k^2}\right)$$



$$\Delta k = \frac{\sigma_k}{\sqrt{2}}$$

Reminder: Gaussian Distribution



$$f\left(x\right)=\frac{1}{\sqrt{2\pi\sigma}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

- **μ specifies the position of the bell curve's central peak**
- **σ specifies the half-distance between inflection points**

Fourier transform of Gaussian Function

$$f\left(x\right)=e^{-ax^{2}}$$ **(Gaussian Function)**

$$\Delta x=\frac{\sigma_x}{\sqrt{2}}=\frac{1}{2\sqrt{a}}$$

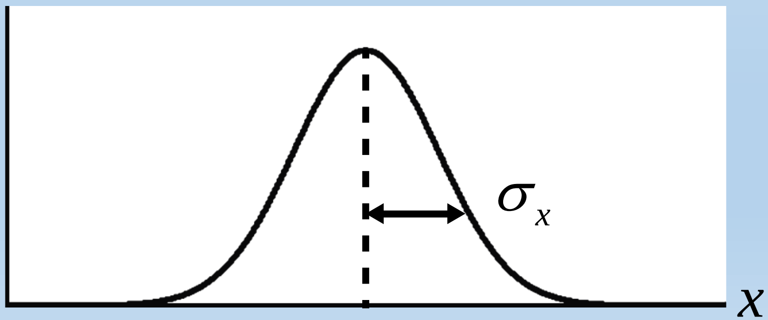
Fourier transform

$$F\left(k\right)=\sqrt{\frac{\pi}{a}}e^{-\frac{k^{2}}{4a}}$$ **(Gaussian Function)**

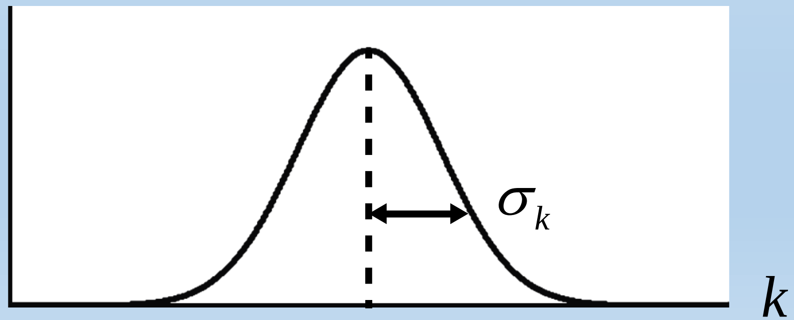
$$\Delta k=\frac{\sigma_k}{\sqrt{2}}=\sqrt{a}$$

$$\boxed{\Delta x\Delta k=\frac{1}{2}}$$

$f\left(x\right)$



$f\left(k\right)$

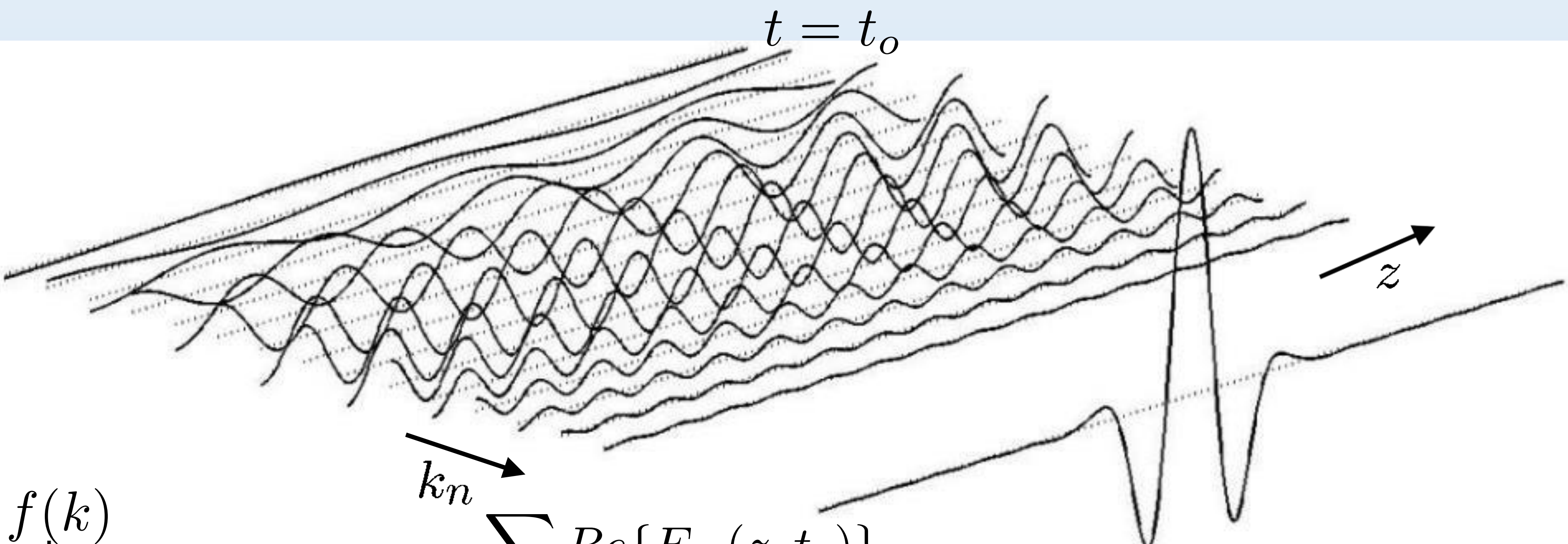


REMEMBER:

$$k = \frac{n}{c}\omega$$

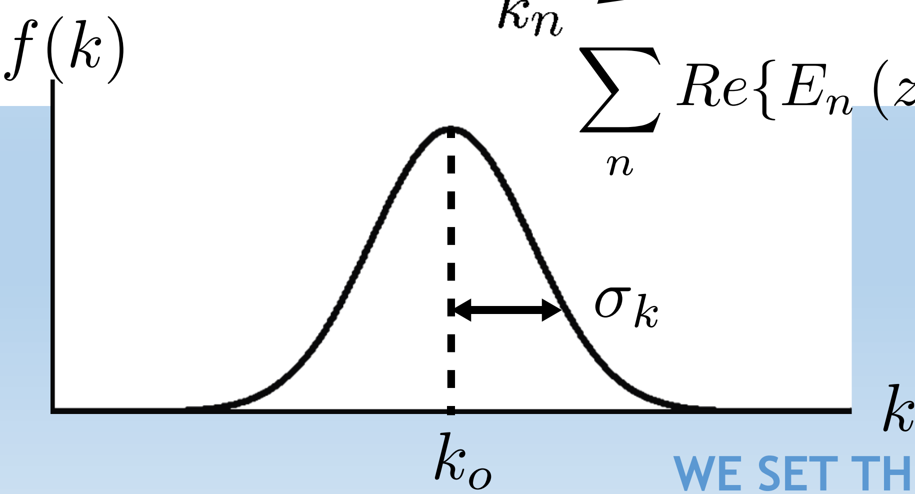
Gaussian Wavepacket in Space

$$Re\{E_n(z, t_o)\}$$



$$\sum_n Re\{E_n(z, t_o)\}$$

SUM OF SINUSOIDS = WAVEPACKET

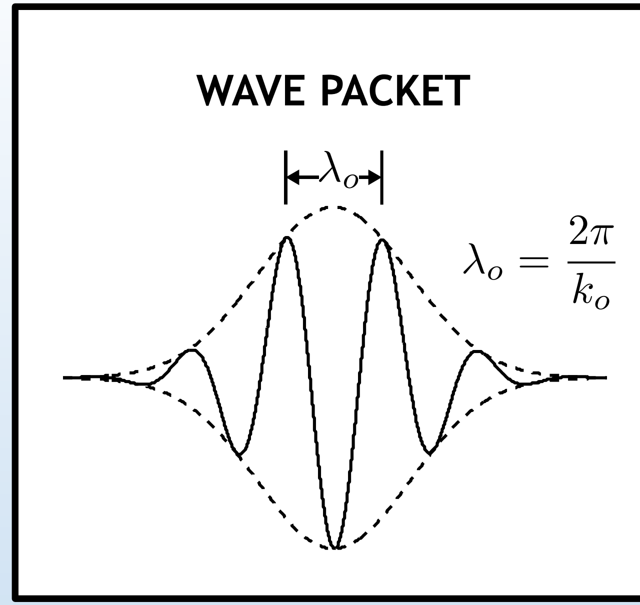


$$f(k) = \frac{1}{\sqrt{2\pi\sigma_k}} exp\left(-\frac{(k-k_o)^2}{2\sigma_k^2}\right)$$

WE SET THE FREQUENCY DISTRIBUTION as GAUSSIAN

Gaussian Wavepacket in Space

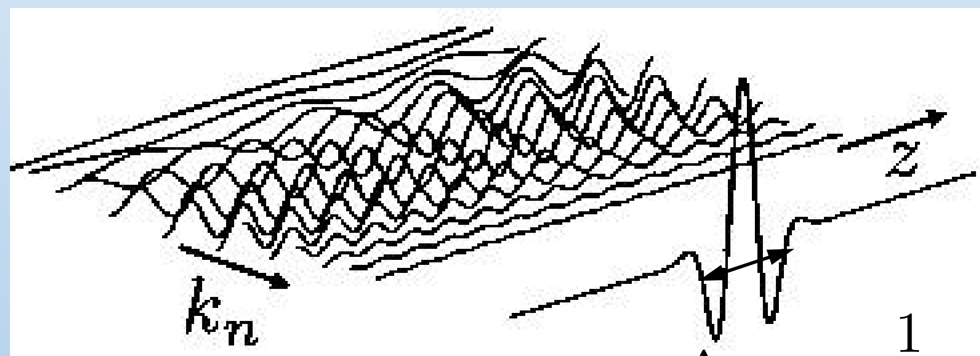
$$E(z,t) = E_o \underbrace{exp\left(-\frac{\sigma_k^2}{2}(ct-z)^2\right)}_{\text{GAUSSIAN ENVELOPE}} \cos\left(\omega_o t - k_o z\right)$$



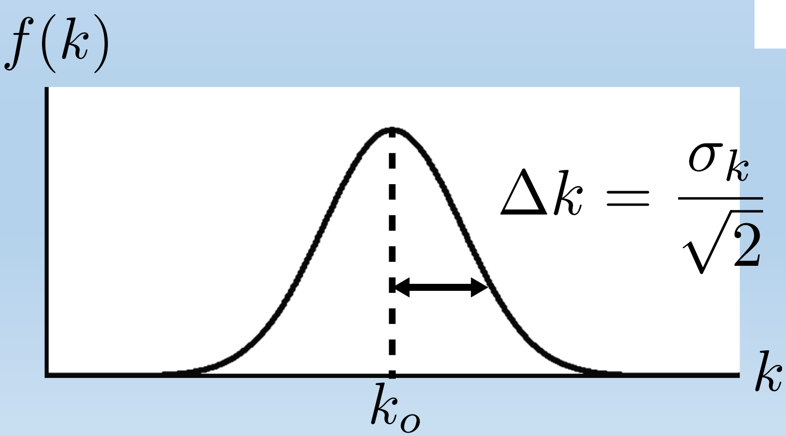
In free space ...

$$k = \frac{\omega}{c} = \frac{2\pi E}{hc}$$

... this plot then shows the PROBABILITY OF WHICH k (or ENERGY) EM WAVES are MOST LIKELY TO BE IN THE WAVEPACKET



$$\Delta z = \frac{1}{\sqrt{2}\sigma_k}$$

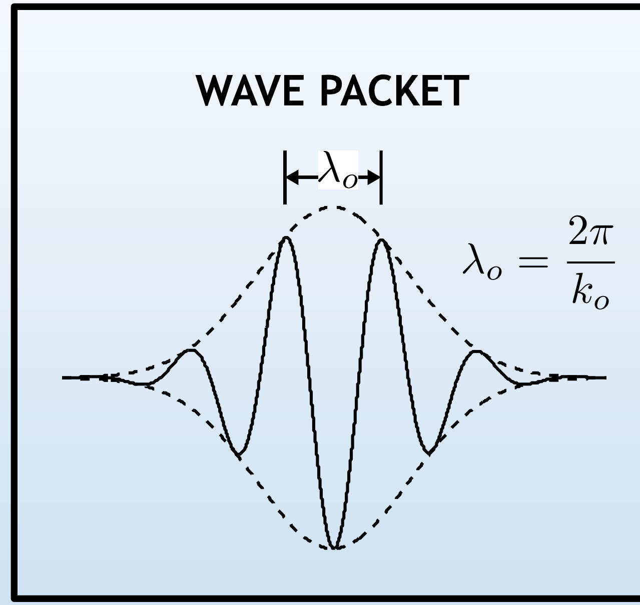


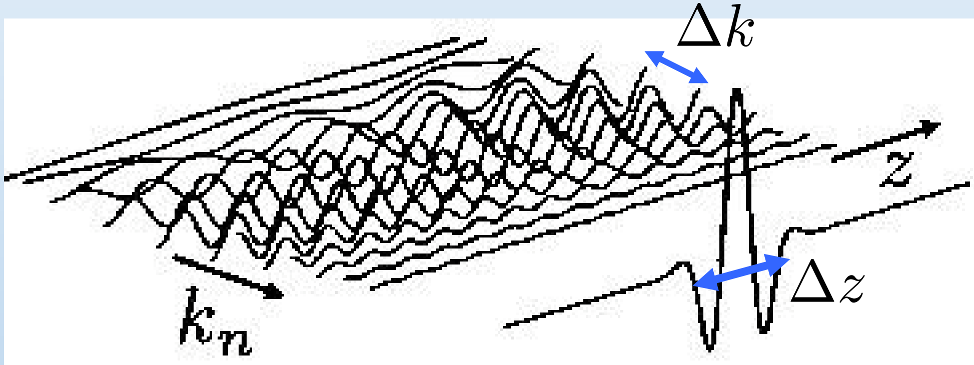
$$\Delta k = \frac{\sigma_k}{\sqrt{2}}$$

$$\Rightarrow \boxed{\Delta k \Delta z = 1/2}$$

Gaussian Wavepacket in Time

$$E(z,t) = E_o exp\left(-\frac{\sigma_k^2}{2}(ct-z)^2\right)\cos\left(\omega_o t - k_o z\right)$$





UNCERTAINTY RELATIONS

$$\Delta z = \frac{c}{n}\Delta t$$

$$\Delta k = \frac{n}{c}\Delta\omega$$

$$\Delta k \Delta z = 1/2$$

$$\Delta\omega\Delta t = 1/2$$

$$\boxed{\Delta p \Delta z = \hbar/2}$$

$$\boxed{\Delta E \Delta t = \hbar/2}$$

Heisenberg realized that ...

- In the world of very small particles, one cannot measure any property of a particle without interacting with it in some way
- This introduces an unavoidable uncertainty into the result
- One can never measure all the properties exactly

Werner Heisenberg (1901-1976)

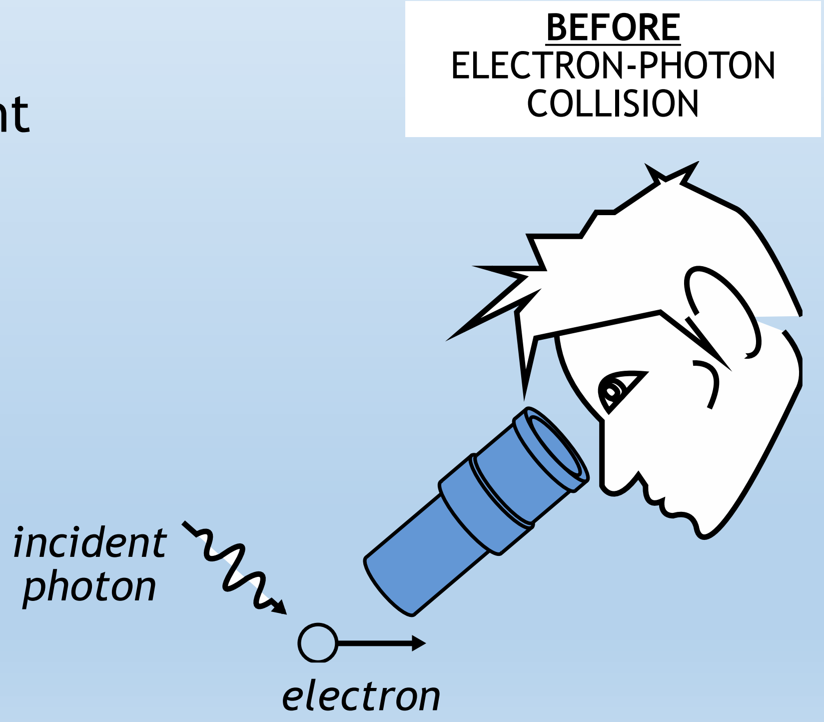
Image in the Public Domain

Measuring Position and Momentum of an Electron

- Shine light on electron and detect reflected light using a microscope

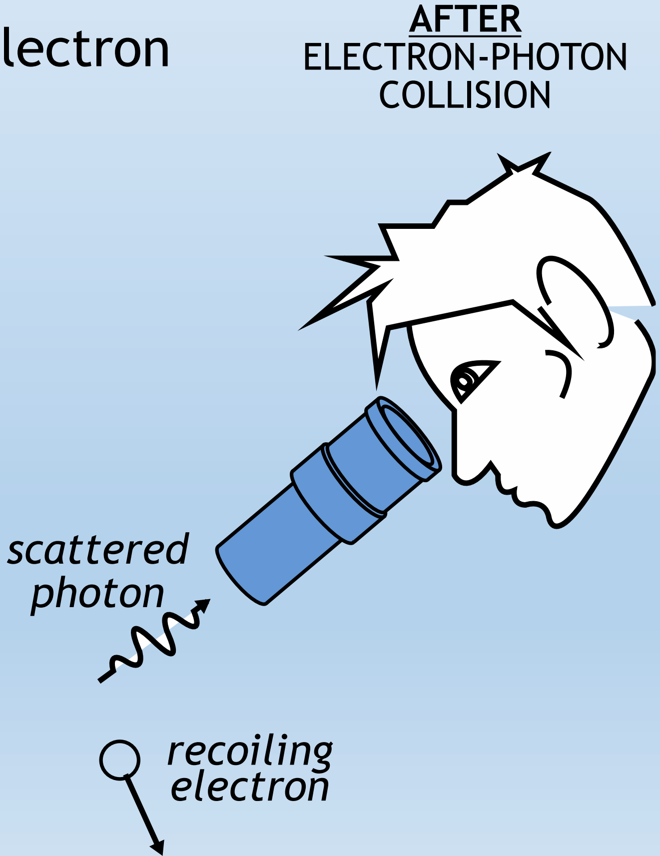
- Minimum uncertainty in position is given by the wavelength of the light

- So to determine the position accurately, it is necessary to use light with a short wavelength



Measuring Position and Momentum of an Electron

- By Planck's law $E = hc/\lambda$, a photon with a short wavelength has a large energy
- Thus, it would impart a large 'kick' to the electron
- But to determine its momentum accurately, electron must only be given a small kick
- This means using light of long wavelength !



Implications

- It is impossible to know *both* the position and momentum exactly, i.e., $\Delta x=0$ and $\Delta p=0$

- These uncertainties are inherent in the physical world and have nothing to do with the skill of the observer

- Because *h* is so small, these uncertainties are not observable in normal everyday situations

$$\hbar = 1.054 \times 10^{-34}\,[\mathrm{J} \cdot \mathrm{s}]$$

Example of Baseball

- A pitcher throws a 0.1-kg baseball at 40 m/s
- So momentum is 0.1 x 40 = 4 kg m/s
- Suppose the momentum is measured to an accuracy
  of 1 percent , i.e.,

$$\Delta p = 0.01\, p = 4 \times 10^{-2} \text{ kg m/s}$$

Example of Baseball *(cont'd)*

- The uncertainty in position is then

$$\Delta x \geq \frac{h}{4\pi \Delta p} = 1.3 \times 10^{-33}\mathrm{m}$$

- No wonder one does not observe the effects of the uncertainty principle in everyday life!

Example of Electron

- Same situation, but baseball replaced by an electron which has mass 9.11×10^{-31} kg traveling at 40 m/s

- So momentum = 3.6×10^{-29} kg m/s
  and its uncertainty = 3.6×10^{-31} kg m/s

- The uncertainty in position is then

$$\Delta x \geq \frac{h}{4\pi \Delta p} = 1.4 \times 10^{-4}\mathrm{m}$$

One might ask:

"If light can behave like a particle,
might <u>particles act like waves</u>"?

YES !

Particles, like photons, also have a wavelength given by:

$$\boxed{\lambda = h/p = h/mv}$$

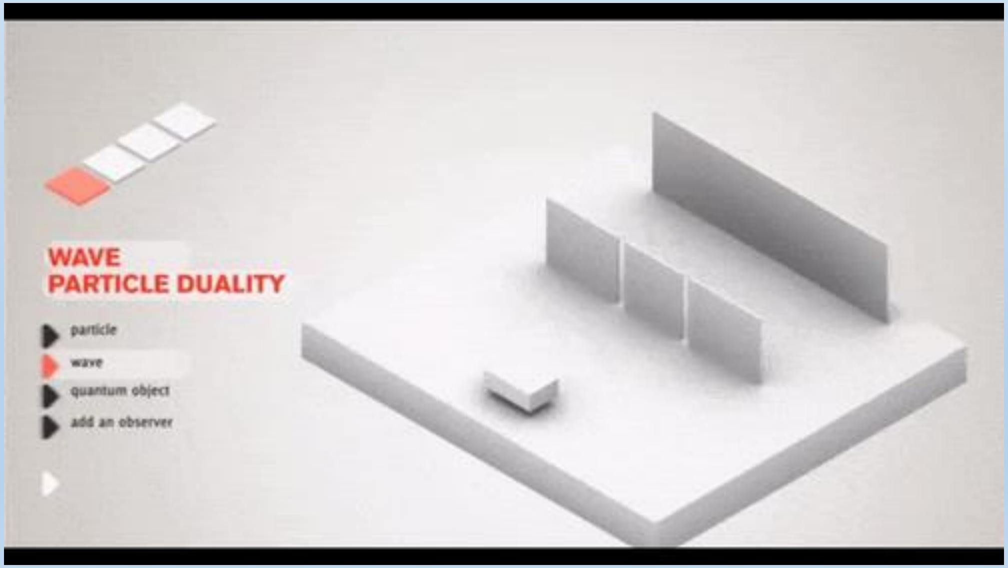
The wavelength of a particle depends on its momentum,
just like a photon!

The main difference is that matter particles have mass,
and photons don't!

Double-Slit Experiment

Video #1:
https://www.youtube.com/watch?v=76GM8goImEA&t=12s

Video #2:
https://www.youtube.com/watch?v=bb3mQAvceZU



Matter Waves

Compute the wavelength of a 10 [g] bullet moving at 1000 [m/s].

$\lambda = h/mv = 6.6\times10^{-34}$ [J s] / (0.01 [kg])(1000 [m/s])

= **6.6×10^{-35} [m]**

This is **immeasureably small**

For ordinary "everyday objects," we don't experience that

MATTER CAN BEHAVE AS A WAVE

But, what about small particles ?

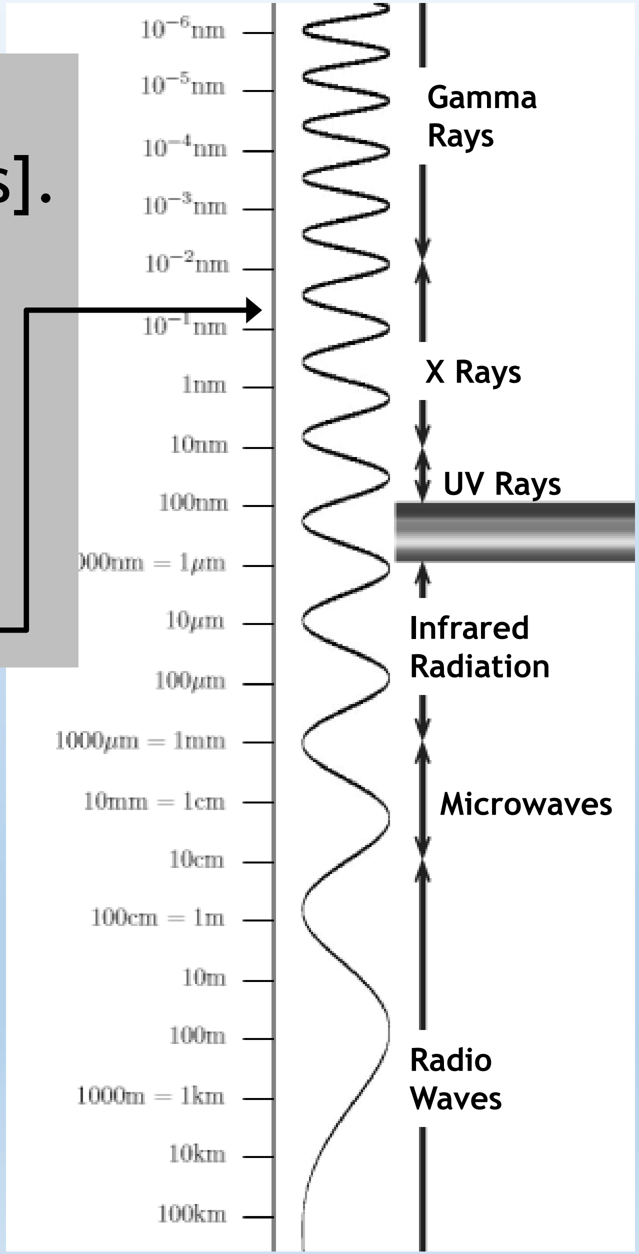
Compute the wavelength of an electron ($m = 9.1 \times 10^{-31}$ [kg]) moving at 1×10^{7} [m/s].

$\lambda = h/mv$

$= 6.6 \times 10^{-34}$ [J s]/(9.1×10^{-31} [kg])(1×10^{7} [m/s])

= 7.3×10^{-11} [m].

= 0.073 [nm]



These electrons have a wavelength in the region of X-rays

Wavelength versus Size

With a visible light microscope, we are limited to being able to resolve objects which are at least about <u>$0.5*10^{-6}$ m</u> = <u>0.5 µm</u> = <u>500 nm</u> in size.

This is because visible light, with a wavelength of ~500 nm cannot resolve objects whose size is smaller than it's wavelength.

Image is in the public domain

Bacteria, as viewed using visible light

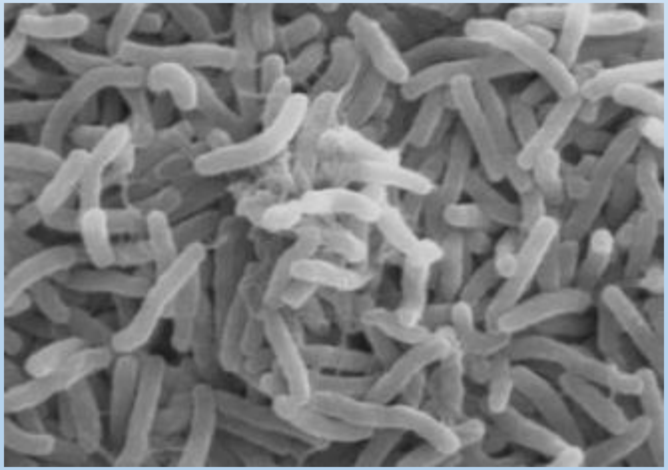


Image is in the public domain

Bacteria, as viewed using electrons!

Electron Microscope

The electron microscope is a device which uses the wave behavior of electrons to make images which are otherwise too small for visible light!

This image was taken with a Scanning Electron Microscope (SEM).

SEM can resolve features as small as 5 nm. This is about 100 times better than can be done with visible light microscopes!

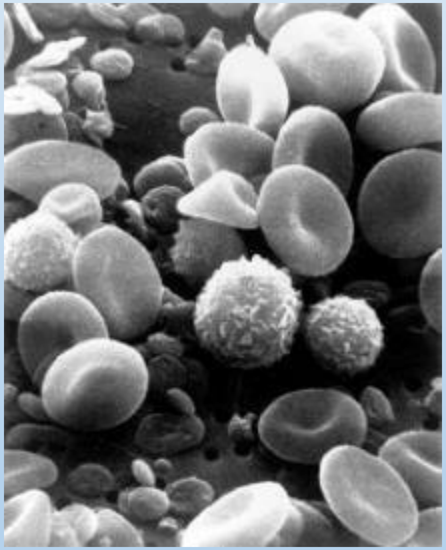
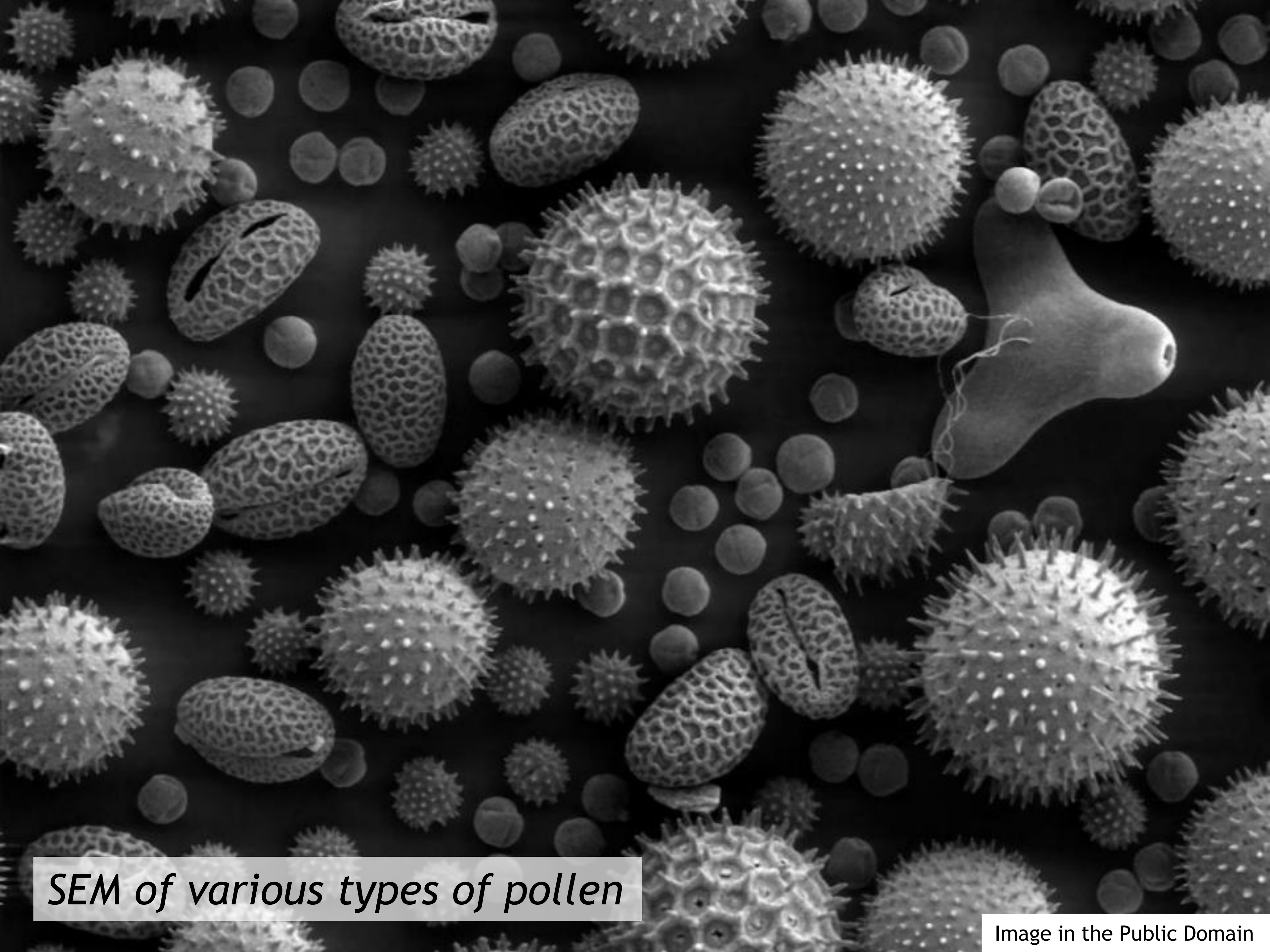


Image in the Public Domain

IMPORTANT POINT:

High energy particles can be used to reveal the structure of matter !



SEM of various types of pollen

SEM of an ant head

Summary

- Light is made up of photons, but in macroscopic situations it is often fine to treat it as a wave.

- Photons carry both energy & momentum.

$$E = hc/\lambda \qquad p = E/c = h/\lambda$$

- Matter also exhibits wave properties. For an object of mass m, and velocity, v, the object has a wavelength, $\lambda = h\ /\ m\mathrm{v}$

- One can probe 'see' the fine details of matter by using high energy particles (they have a small wavelength !)

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- Heisenberg's uncertainty principle:

Uncertainty in momentum

$$\Delta x \Delta p \geq \frac{h}{4\pi} = \frac{\hbar}{2}$$

Uncertainty in position

Key Takeaways

Photoelectric effect:

$$\hbar\omega = W + \frac{1}{2}mv^2$$

Photon Energy | **Binding energy of electron** | **Electron kinetic energy**

$$E = h\nu = hc/\lambda = \hbar\omega$$

$$E\,[eV] = \frac{1240}{\lambda\,[nm]}$$

(Planck's constant)

$$h = 6.626 \times 10^{-34}\,[J\cdot s]$$

$$\hbar = \frac{h}{2\pi} = 1.055 \times 10^{-34}\,[J\cdot s]$$

Light intensity in terms of photons:

$$|\vec{S}| = \frac{n\hbar\omega}{\tau A}$$

Photon momentum:

$$p = \frac{E}{c} = \frac{h\nu}{c}$$

Appendix

Fourier transform of Gaussian Function

$$f\left(x\right)=e^{-ax^2}$$ **(Gaussian Function)**

Fourier transform

$$F\left(k\right)=\int_{-\infty}^{+\infty} f\left(x\right)e^{-jkx}dx=\int_{-\infty}^{+\infty} e^{-\left(ax^2+jkx\right)}dx=\int_{-\infty}^{+\infty} e^{-a\left(x+\frac{jk}{2a}\right)^2-\frac{k^2}{4a}}dx$$

$$=e^{-\frac{k^2}{4a}}\int_{-\infty}^{+\infty} e^{-a\left(x+\frac{jk}{2a}\right)^2}dx=e^{-\frac{k^2}{4a}}\int_{-\infty}^{+\infty} e^{-ax^2}dx=\boxed{\sqrt{\frac{\pi}{a}}e^{-\frac{k^2}{4a}}}$$

$$\int_{-\infty}^{+\infty} e^{-ax^2}dx=\sqrt{2\pi}\sigma\int_{-\infty}^{+\infty}\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{x^2}{2\sigma^2}}dx=\sqrt{2\pi}\sigma=\sqrt{\frac{\pi}{a}} \qquad \sigma=\frac{1}{\sqrt{2a}}$$