# Fundamentals of Information Science: Homework 1

February 18, 2025

**Problem 1.**
Proof the following statements are true for a 0-1 Boolean algebra.

$$(1) \quad a \cdot (a + (b \cdot c)) = a.$$

$$(2) \quad (a \cdot b) + (\bar{a} + \bar{b}) = 1.$$

**Problem 2.**
The idea of proving DeMorgan's Theorem is to show that $\overline{a} + \overline{b}$ is the complement of $(a \cdot b)$. Namely, by axiom A2 we need to prove the following two statements: (I). $(a \cdot b) + (\overline{a} + \overline{b}) = 1$. and (II) $(a \cdot b) \cdot (\overline{a} + \overline{b}) = 0$.

Here you will prove statement (I). In your proofs, you cannot use the Duality Theorem.

(a) Prove statement (I), please use the Associativity Theorem in your proof.

(b) Prove statement (I) without using the Associativity Theorem in your proof.

(c) Use DeMorgan's Theorem and other axioms and theorems to find the complements of: (i) $(a \cdot \overline{b}) + (\overline{a} \cdot b)$, (ii) $(a + b + c + d)$, and (iii) $a + (\overline{a} \cdot b \cdot c)$. Please justify every step in your derivations using the axioms, lemmas and theorems from class.

**Problem 3.**
A generalization of the switching (relay) circuit model is a circuit with multiple terminals. The Boolean function $X_{ab}$ is 1 if there is a closed path between terminals $a$ and $b$, and 0 otherwise. With multiple terminals, $a, b, c, d, ...$, Boolean functions exist between any pair of terminals.

(a) Construct a circuit with 3 relays that implements the functions

$$f_1 = x \cdot y$$

$$f_2 = \overline{x} \cdot y$$

(b) Construct a circuit with 4 relays that implements the functions

$$f_1 = x \cdot y + \overline{x} \cdot \overline{y}$$

$$f_2 = \overline{x} \cdot y + x \cdot \overline{y}$$

(c) Construct a circuit with 6 relays that implements the functions:

$$f1 = x \cdot (y + z)$$

$$f2 = y \cdot (x + z)$$
$$f3 = z \cdot (x + y)$$
$$f4 = x + y \cdot z$$
$$f5 = y + x \cdot z$$
$$f6 = z + x \cdot y$$