Chapter 10: Interference and Diffraction

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Chapter 10: Interference and Diffraction

10.1 Superposition of Waves

Consider a region in space where two or more waves pass through at the same time. According to the superposition principle, the net displacement is simply given by the vector or the algebraic sum of the individual displacements. Interference is the vector or the algebraic sum of the individual displacements. *Interference* is the combination of two or more waves to form a composite wave, based on such principle. The idea of the superposition principle is illustrated in Figure 10.1

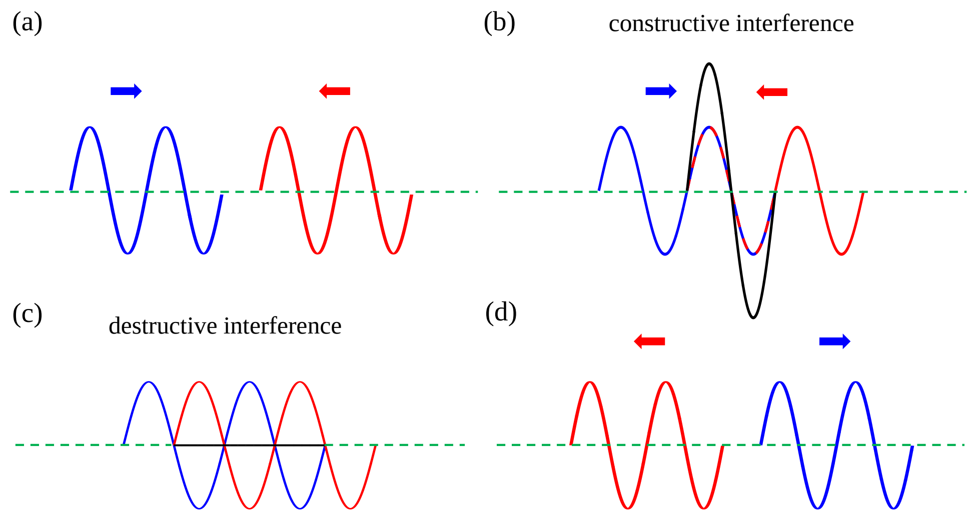

Figure 10.1 Superposition of waves. (a) before interference, (b) constructive interference, (c) destructive interference and (d) after interference.

Suppose we are given two waves,

$$E_1(x,t) = E_{10}\sin(k_1x \pm \omega_1 t + \alpha_1), \quad E_2(x,t) = E_{20}\sin(k_2x \pm \omega_2 t + \alpha_2) \tag{10.1}$$

the resulting wave is simply

$$E(x,t) = E_{10}\sin(k_1x \pm \omega_1 t + \alpha_1) + E_{20}\sin(k_2x \pm \omega_2 t + \alpha_2) \tag{10.2}$$

The interference is constructive if the amplitude of $E(x,t)$ is greater than the individual ones (Figure 10.1b), and destructive if smaller (Figure 10.1c).

As an example, consider the superposition of the following two waves at $t = 0$:

$$E_1(x,t) = \sin(x), \quad E_2(x,t) = 2\sin\left(x + \frac{\pi}{4}\right) \tag{10.3}$$

The resultant wave is given by

$$E(x) = E_1(x) + E_2(x) = \sin(x) + 2\sin\left(x + \frac{\pi}{4}\right) = \left(1+\sqrt{2}\right)\sin(x) + \sqrt{2}\cos(x) \tag{10.4}$$

where we have used

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta \tag{10.5}$$

and $\sin(\pi/4) = \cos(\pi/4) = \sqrt{2}/2$. Further use of the identity

$$\begin{aligned} a\sin x + b\cos x &= \sqrt{a^2+b^2}\left[\frac{a}{\sqrt{a^2+b^2}}\sin x + \frac{b}{\sqrt{a^2+b^2}}\cos x\right] \\ &= \sqrt{a^2+b^2}\left[\cos\gamma\sin x + \sin\gamma\cos x\right] \\ &= \sqrt{a^2+b^2}\sin(x+\gamma) \end{aligned} \tag{10.6}$$

with

$$\gamma = \tan^{-1}\frac{b}{a} \tag{10.7}$$

then leads to

$$E(x) = \sqrt{5+2\sqrt{2}}\sin(x+\gamma) \tag{10.8}$$

where $\gamma = \tan^{-1}\left(\sqrt{2}/\left(1+\sqrt{2}\right)\right) = 30.4° = 0.53\,\text{rad}$. The superposition of the waves is depicted in Figure 10.2.

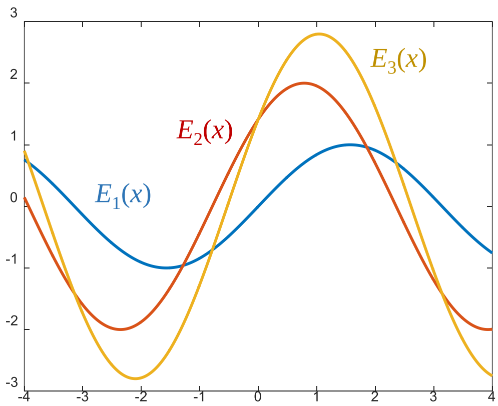

Figure 10.2 Superposition of two sinusoidal waves.

We see that the wave has a maximum amplitude when $\sin(x+\gamma) = 1$, or $x = \pi/2 - \gamma$. The interference there is constructive. On the other hand, destructive interference occurs at $x = \pi - \gamma$, where $\sin(\pi) = 0$.

In order to form an interference pattern, the incident light must satisfy two conditions:

(i) The light sources must be *coherent*. This means that the plane waves from the sources must maintains a constant phase relation. For example, if two waves are completely out of phase with $\gamma = \pi$, this phase difference must not change with time.

(ii) The light must be *monochromatic*. This means that the light consists of just one wavelength $\lambda = 2\pi / k$.

Light emitted from an incandescent lightbulb (Figure 10.3) is *incoherent* because the light consists of waves of different wavelengths and they do not maintain a constant phase relationship. Thus, no interference pattern is observed.

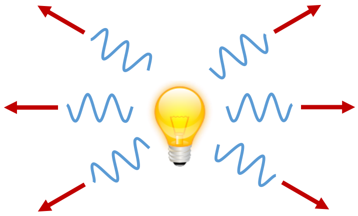

Figure 10.3 Incoherent light source.

10.2 Young's Double-Slit Experiment

In 1801 Thomas Young carried out an experiment in which the wave nature of light was demonstrated. The schematic diagram of the double-slit experiment is shown in Figure 10.4.

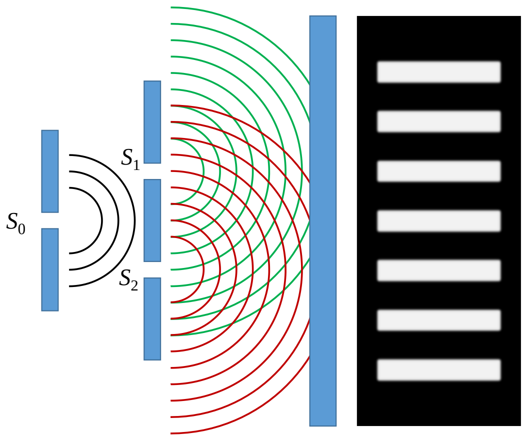

Figure 10.4 Young's double-slit experiment.

A monochromatic light source is incident on the first screen which contains a slit S_0. The emerging light then arrives at the second screen which has two parallel slits S_1 and S_2. which serve as the sources of coherent light. The light waves emerging from the two slits then interfere and form an interference pattern on the viewing screen. The bright bands (fringes) correspond to interference maxima, and the dark band interference minima.

The geometry of the double-slit interference is shown in the Figure 10.5

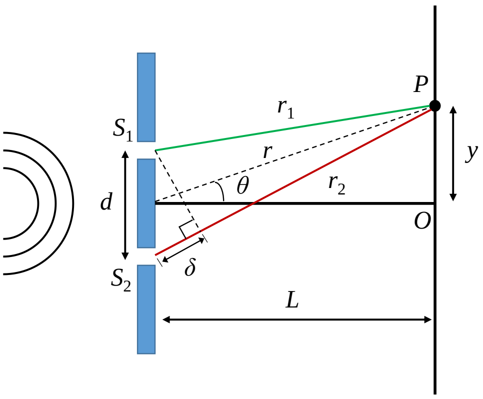

Figure 10.5 Double-slit experiment.

Consider light that falls on the screen at a point P a distance from the point O that lies on the screen a perpendicular distance L from the double-slit system. The two slits are separated by a distance d. The light from slit 2 will travel an extra distance $\delta = r_2 - r_1$ to the point P than the light from slit 1. This extra distance is called the path difference. From Figure 10.5, we have, using the law of cosines,

$$r_1^2 = r^2 + \left(\frac{d}{2}\right)^2 - dr\cos\left(\frac{\pi}{2} - \theta\right) = r^2 + \left(\frac{d}{2}\right)^2 - dr\sin\theta \tag{10.9}$$

and

$$r_2^2 = r^2 + \left(\frac{d}{2}\right)^2 - dr\cos\left(\frac{\pi}{2} + \theta\right) = r^2 + \left(\frac{d}{2}\right)^2 + dr\sin\theta \tag{10.10}$$

Subtracting Eq. (10.9) from Eq. (10.10) yields

$$r_2^2 - r_1^2 = (r_2 + r_1)(r_2 - r_1) = 2dr\sin\theta \tag{10.11}$$

In the limit $L >> d$, i.e., the distance to the screen is much greater than the distance between the slits, the sum of r_1 and r_2 may be approximated by $r_1 + r_2 \approx 2r$, and the path difference becomes

$$\delta = r_2 - r_1 \approx d\sin\theta \tag{10.12}$$

In this limit, the two rays and are essentially treated as being parallel (see Figure 10.6)

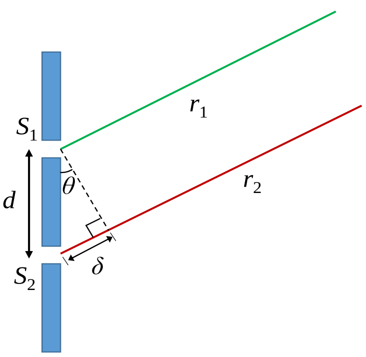

Figure 10.6 Path difference between the two rays, assuming $L >> d$

Whether the two waves are in phase or out of phase is determined by the value of δ. Constructive interference occurs when δ is zero or an integer multiple of the wavelength λ:

$$\boxed{\delta = d\sin\theta = m\lambda, \quad m = 0, \pm 1, \pm 2, \cdots \ (\text{constructive inteference})} \tag{10.13}$$

where m is called the order number. The zeroth-order (m = 0) maximum corresponds to the central bright fringe at θ = 0, and the first-order maxima (m= ±1) are the bright fringes on either side of the central fringe.

On the other hand, when δ is equal to an odd integer multiple of $\lambda/2$, the waves will be 180° out of phase at P, resulting in destructive interference with a dark fringe on the screen. The condition for destructive interference is given by

$$\boxed{\delta = d\sin\theta = \left(m + \frac{1}{2}\right)\lambda, \quad m = 0, \pm 1, \pm 2, \cdots \ (\text{constructive inteference})} \tag{10.14}$$

To locate the positions of the fringes as measured vertically from the central point O, in addition to $L >> d$, we shall also assume that the distance between the slits is much greater than the wavelength of the monochromatic light, $d >> \lambda$. The conditions imply that the angle θ is very small, so that

$$\sin\theta \approx \tan\theta = \frac{y}{L} \tag{10.15}$$

Substituting the above expression into the constructive and destructive interference conditions given in Eqs. (10.13) and (10.14), the positions of the bright and dark fringes are, respectively,

$$y_b = m\frac{\lambda L}{d} \tag{10.16}$$

and

$$y_d = \left(m + \frac{1}{2}\right)\frac{\lambda L}{d} \tag{10.17}$$

Example 10.1 Double-Slit Experiment

Suppose in the double-slit arrangement, d = 0.150 mm, L =120 cm, λ = 833 nm, and y = 2.00 cm.

(a) What is the path difference δ for the rays from the two slits arriving at point P?

(b) Express this path difference in terms of λ.

(c) Does point P correspond to a maximum, a minimum, or an intermediate condition?

Solutions:

(a) The path difference is given by $\delta = d \sin\theta$. When $L >> y$, θ is small and we can make the approximation $\sin\theta \approx \tan\theta = y / L$. Thus,

$$\delta \approx d\,\frac{y}{L} = \left(1.50\times10^{-4}\,\mathrm{m}\right)\frac{2.00\times10^{-2}\,\mathrm{m}}{1.20\mathrm{m}} = 2.50\times10^{-6}\,\mathrm{m}$$

(b) From the answer in part (a), we have

$$\frac{\delta}{\lambda} = \frac{2.50\times10^{-6}\,\mathrm{m}}{8.33\times10^{-7}\,\mathrm{m}} \approx 3.00$$

or $\delta = 3.00\,\lambda$.

(c) Since the path difference is an integer multiple of the wavelength, the intensity at point P is a maximum.

10.3 Intensity Distribution

Consider the double-slit experiment shown in Figure 10.5. The total instantaneous electric field $\vec{E}$ at the point P on the screen is equal to the vector sum of the two sources: $\vec{E} = \vec{E}_1 + \vec{E}_2$. On the other hand, the Poynting flux S is proportional to the square of the total field:

$$S \propto E^2 = \left(\vec{E}_1 + \vec{E}_2\right)^2 = E_1^2 + E_2^2 + 2\vec{E}_1\cdot\vec{E}_2 \qquad (10.18)$$

Taking the time average of S, the intensity I of the light at P may be obtained as:

$$I = \langle S\rangle \propto \left\langle E_1^2\right\rangle + \left\langle E_2^2\right\rangle + 2\left\langle \vec{E}_1\cdot\vec{E}_2\right\rangle \qquad (10.19)$$

The cross term $2\left\langle \vec{E}_1\cdot\vec{E}_2\right\rangle$ represents the correlation between the two light waves. For incoherent light sources, since there is no definite phase relation between $\vec{E}_1$ and $\vec{E}_2$, the cross term vanishes, and the intensity due to the incoherent source is simply the sum of the two individual intensities:

$$I_{inc} = I_1 + I_2 \qquad (10.20)$$

For coherent sources, the cross term is non-zero. In fact, for constructive interference, $\vec{E}_1 = \vec{E}_2$, and the resulting intensity is

$$I = 4I_1 \qquad (10.21)$$

which is four times greater than the intensity due to a single source. On the other hand, when destructive interference takes place, $\vec{E}_1 = -\vec{E}_2$, and $\left\langle \vec{E}_1\cdot\vec{E}_2\right\rangle \propto -I_1$, and the total intensity becomes

$$I = I_1 - 2I_1 + I_1 = 0 \qquad (10.22)$$

as expected.

Suppose that the waves emerged from the slits are coherent sinusoidal plane waves. Let the electric field components of the wave from slits 1 and 2 at P be given by

$$E_1 = E_0 \sin\omega t \qquad (10.23)$$

and

$$E_2 = E_0 \sin\left(\omega t + \alpha\right) \tag{10.24}$$

respectively, where the waves from both slits are assumed have the same amplitude E_0. For simplicity, we have chosen the point P to be the origin, so that the k_x dependence in the wave function is eliminated. Since the wave from slit 2 has traveled an extra distance δ to P, E_2 has an extra phase shift α relative to E_1 from slit 1.

For constructive interference, a path difference of $\delta = \lambda$ would correspond to a phase shift of $\alpha = 2\pi$. This then implies

$$\frac{\delta}{\lambda} = \frac{\alpha}{2\pi} \tag{10.25}$$

or

$$\alpha = \frac{2\pi}{\lambda}\delta = \frac{2\pi}{\lambda} d \sin\theta \tag{10.26}$$

Assuming that both fields point in the same direction, the total electric field may be obtained by using the superposition principle

$$E = E_1 + E_2 = E_0\left[\sin\omega t + \sin\left(\omega t + \alpha\right)\right] = 2E_0 \cos\left(\frac{\alpha}{2}\right)\sin\left(\omega t + \frac{\alpha}{2}\right) \tag{10.27}$$

where we have used the trigonometric identity

$$\sin\alpha + \sin\beta = 2\cos\left(\frac{\alpha-\beta}{2}\right)\sin\left(\frac{\alpha+\beta}{2}\right) \tag{10.28}$$

The intensity I is proportional to the time average of the square of the total electric field:

$$I \propto \left\langle E^2 \right\rangle = 4E_0^2 \cos^2\left(\frac{\alpha}{2}\right)\left\langle \sin^2\left(\omega t + \frac{\alpha}{2}\right)\right\rangle = 2E_0^2 \cos^2\left(\frac{\alpha}{2}\right) \tag{10.29}$$

or

$$I = I_0 \cos^2\left(\frac{\alpha}{2}\right) \tag{10.30}$$

where I_0 is the maximum intensity on the screen. Upon substituting Eq. (10.26), the above expression becomes

$$\boxed{I = I_0 \cos^2\left(\frac{\pi d \sin\theta}{\lambda}\right)} \tag{10.31}$$

For small angle θ, the intensity can be rewritten as

$$I = I_0 \cos^2\left(\frac{\pi d}{\lambda L} y\right) \tag{10.32}$$

10.4 Diffraction

In addition to interference, waves also exhibit another property – *diffraction*, which is the bending of waves as they pass by some objects or through an aperture. The phenomenon of diffraction can be understood using Huygens's principle which states that:

Every unobstructed point on a wavefront will act a source of secondary spherical waves. The new wavefront is the surface tangent to all the secondary spherical waves.

Figure 10.8 illustrates the propagation of the wave based on Huygens's principle.

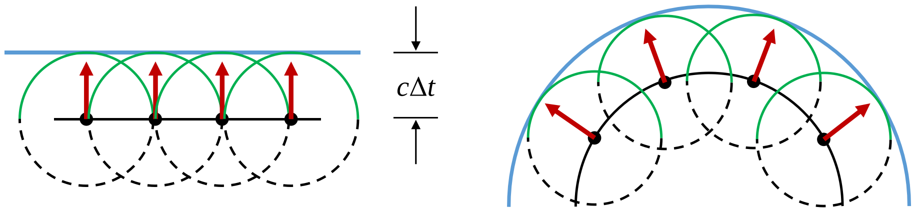

Figure 10.8 Propagation of wave based on Huygens's principle

According to Huygens's principle, light waves incident on two slits will spread out and exhibit an interference pattern in the region beyond (Figure 10.9a). The pattern is called a diffraction pattern. On the other hand, if no bending occurs and the light wave continue to travel in straight lines, then no diffraction pattern would be observed (Figure 10.9b).

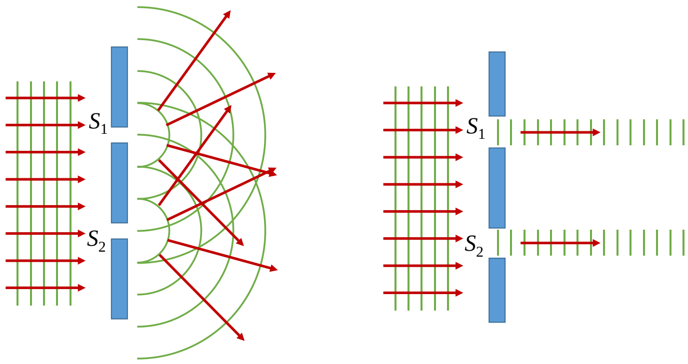

Figure 10.9 (a) Spreading of light leading to a diffraction pattern. (b) Absence of diffraction pattern if the paths of the light wave are straight lines.

We shall restrict ourselves to a special case of diffraction called the *Fraunhofer diffraction*. In this case, all light rays that emerge from the slit are approximately parallel to each other. For a diffraction pattern to appear on the screen, a convex lens is placed between the slit and screen to provide convergence of the light rays.

10.5 Single-Slit Diffraction

In our consideration of the Young's double-slit experiments, we have assumed the width of the slits to be so small that each slit is a point source. In this section we shall take the width of slit to be finite and see how Fraunhofer diffraction arises.

Let a source of monochromatic light be incident on a slit of finite width a, as shown in Figure 10.10.

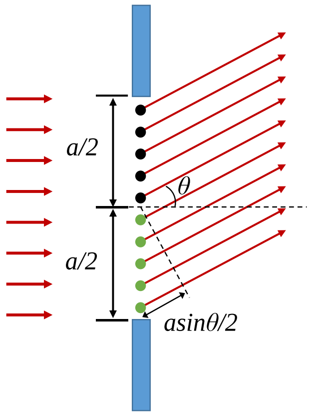

Figure 10.10 Diffraction of light by a slit of width *a*.

In diffraction of Fraunhofer type, all rays passing through the slit are approximately parallel. In addition, each portion of the slit will act as a source of light waves according to Huygens's principle. For simplicity we divide the slit into two halves. At the first minimum, each ray from the upper half will be exactly 180° out of phase with a corresponding ray form the lower half. For example, suppose there are 100 point sources, with the first 50 in the lower half, and 51 to 100 in the upper half. Source 1 and source 51 are separated by a distance $a/2$ and are out of phase with a path difference $\delta = \lambda/2$. Similar observation applies to source 2 and source 52, as well as any pair that are a distance $a/2$ apart. Thus, the condition for the first minimum is

$$\frac{a}{2}\sin\theta = \frac{\lambda}{2} \tag{10.33}$$

or

$$\sin\theta = \frac{\lambda}{a} \tag{10.34}$$

Applying the same reasoning to the wavefronts from four equally spaced points a distance $a/4$ apart, the path difference would be $\delta = a \sin\theta / 4$, and the condition for destructive interference is

$$\sin\theta = \frac{2\lambda}{a} \tag{10.35}$$

The argument can be generalized to show that destructive interference will occur when

$$\boxed{a\sin\theta = m\lambda, \quad m = \pm1, \pm2, \pm3, \cdots \quad (\text{destructive inteference})} \tag{10.36}$$

Figure 10.11 illustrates the intensity distribution for a single-slit diffraction. Note that $\theta = 0$ is a maximum

By comparing Eq. (10.36) with Eq.(10.13), we see that the condition for minima of a single-slit diffraction becomes the condition for maxima of a double-slit interference when the width of a single slit *a* is replaced by the separation between the two slits *d*. The reason is that in the double-slit case, the slits are taken to be so small that each one is considered as a single light source, and the interference of waves originating within the same slit can be neglected. On the other hand, the minimum condition for the single-slit diffraction is obtained precisely by taking into consideration the interference of waves that originate within the same slit.

Example 10.2: Single-Slit Diffraction

A monochromatic light with a wavelength of λ = 600 nm passes through a single slit which has a width of 0.800 mm.

(a) What is the distance between the slit and the screen be located if the first minimum in the diffraction pattern is at a distance 1.00 mm from the center of the screen?

(b) Calculate the width of the central maximum.

Solutions:

(a) The general condition for destructive interference is

$$a\sin\theta = m\lambda, \quad m = \pm1, \pm2, \pm3, \cdots$$

For small θ, we employ the approximation $\sin\theta \approx \tan\theta = y / L$, which yields

$$\frac{y}{L} \approx m\frac{\lambda}{a}$$

The first minimum corresponds to m = 1. If y_1 =1.00 mm, then

$$L = \frac{ay_1}{m\lambda} = \frac{\left(8.00\times10^{-4}\text{ m}\right)\left(1.00\times10^{-3}\text{ m}\right)}{1\left(600\times10^{-9}\text{ m}\right)} = 1.33\,\text{m}$$

(b) The general condition for destructive interference is

$$w = 2y_1 = 2\left(1.00\times10^{-3}\text{ m}\right) = 2.00\,\text{m}$$

10.6 Intensity of Single-Slit Diffraction

How do we determine the intensity distribution for the pattern produced by a single-slit diffraction? To calculate this, we must find the total electric field by adding the field contributions from each point.

Let's divide the single slit into N small zones each of width $\Delta y = a / N$, as shown in Figure 10.11. The convex lens is used to bring parallel light rays to a focal point P on the screen. We shall assume that $\Delta y \ll \lambda$ so that all the light from a given zone is in phase. Two adjacent zones have a relative path length $\delta = \Delta y \sin\theta$. The relative phase shift $\Delta\beta$ is given by the ratio

$$\frac{\Delta\beta}{2\pi}=\frac{\delta}{\lambda}=\frac{\Delta y\sin\theta}{\lambda} \quad => \quad \Delta\beta=\frac{2\pi}{\lambda}\Delta y\sin\theta \tag{10.37}$$

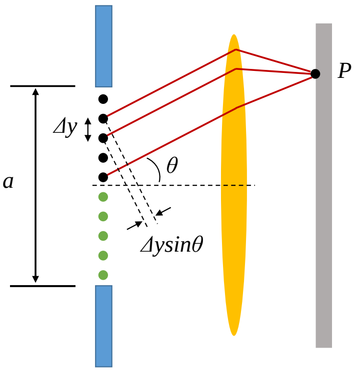

Figure 10.11 Single-slit Fraunhofer diffraction

Suppose the wavefront from the first point (counting from the top) arrives at the point P on the screen with an electric field given by

$$E_1=E_{10}\sin(\omega t) \tag{10.38}$$

The electric field from point 2 adjacent to point 1 will have a phase shift $\Delta\beta$, and the field is

$$E_2=E_{10}\sin(\omega t+\Delta\beta) \tag{10.39}$$

Since each successive component has the same phase shift relative the previous one, the electric field from point N is

$$E_N=E_{10}\sin(\omega t+(N-1)\Delta\beta) \tag{10.40}$$

The total electric field is the sum of each individual contribution:

$$E=E_1+E_2+\cdots+E_N=E_{10}\left[\sin(\omega t)+\sin(\omega t+\Delta\beta)\cdots+\sin(\omega t+(N-1)\Delta\beta)\right] \tag{10.41}$$

Adding the terms, we find

$$E=E_{10}\left[\frac{\sin(N\Delta\beta/2)}{\sin(\Delta\beta/2)}\right]\sin\left[\omega t+(N-1)\Delta\beta/2\right] \tag{10.42}$$

The intensity I is proportional to the time average of E^2:

$$I\propto\langle E^2\rangle=E_{10}^2\left[\frac{\sin(N\Delta\beta/2)}{\sin(\Delta\beta/2)}\right]^2\left\langle\sin\left[\omega t+(N-1)\Delta\beta/2\right]\right\rangle^2=\frac{1}{2}E_{10}^2\left[\frac{\sin(N\Delta\beta/2)}{\sin(\Delta\beta/2)}\right]^2 \tag{10.43}$$

and we express I as

$$I = \frac{I_0}{N}\left[\frac{\sin(N\Delta\beta/2)}{\sin(\Delta\beta/2)}\right]^2 \tag{10.44}$$

where the extra factor N^2 has been inserted to ensure that I_0 corresponds to the intensity at the central maximum $\beta = 0$ ($\theta = 0$). In the limit where $\Delta\beta \to 0$

$$I = \frac{I_0}{N}\left[\frac{\sin(N\Delta\beta/2)}{\sin(\Delta\beta/2)}\right]^2 \quad N\sin(\Delta\beta/2) \approx N\Delta\beta/2 = \beta/2 \tag{10.45}$$

and the intensity becomes

$$\boxed{I = I_0\left[\frac{\sin(\beta/2)}{\beta/2}\right]^2 = I_0\left[\frac{\sin(\pi a\sin\theta/\lambda)}{\pi a\sin\theta/\lambda}\right]^2} \tag{10.46}$$

In Figure 10.12, we plot the ratio of the intensity I / I_0 as a function of $\beta / 2$.

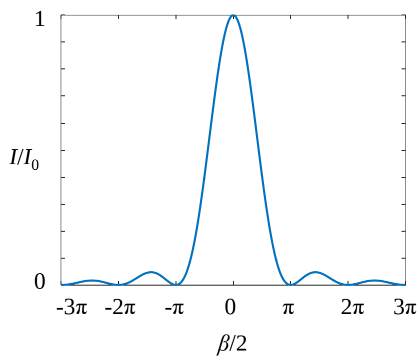

Figure 10.12 Intensity of the single-slit Fraunhofer diffraction pattern.

From Eq. (10.46), we readily see that the condition for minimum intensity is

$$\frac{\pi a\sin\theta}{\lambda} = m\pi, \quad m = \pm1, \pm2, \pm3, \cdots \tag{10.47}$$

or

$$\boxed{\sin\theta = m\frac{\lambda}{a}, \quad m = \pm1, \pm2, \pm3, \cdots} \tag{10.48}$$

In Figure 10.13 the intensity is plotted as a function of the angle θ, for $a = \lambda$ and $a = 2\lambda$. We see that as the ratio a / λ grows, the peak becomes narrower, and more light is concentrated in the central peak. In this case, the variation of I_0 with the width a is not shown.

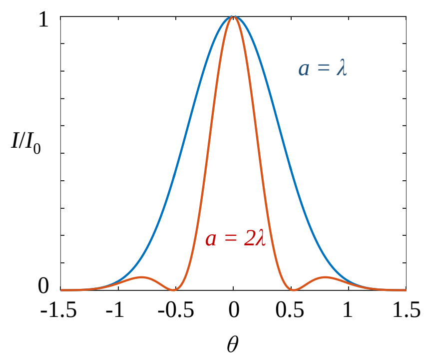

Figure 10.13 Intensity of single-slit diffraction as a function of θ for $a = \lambda$ and $a = 2\lambda$.

10.7 Intensity of Double-Slit Diffraction Patterns

In the previous sections, we have seen that the intensities of the single-slit diffraction and the double-slit interference are given by:

$$I = I_0 \left[\frac{\sin(\pi a \sin\theta/\lambda)}{\pi a \sin\theta/\lambda} \right]^2 \qquad \text{(single-slit diffraction)} \qquad (10.49)$$

$$I = I_0 \cos^2\left(\frac{\pi d \sin\theta}{\lambda} \right) \qquad \text{(double-slit interference)} \qquad (10.50)$$

Suppose we now have two slits, each having a width a, and separated by a distance d. The resulting interference pattern for the double-slit will also include a diffraction pattern due to the individual slit. The intensity of the total pattern is simply the product of the two functions:

$$\boxed{I = I_0 \cos^2\left(\frac{\pi d \sin\theta}{\lambda} \right) \left[\frac{\sin(\pi a \sin\theta/\lambda)}{\pi a \sin\theta/\lambda} \right]^2} \qquad (10.51)$$

The first and the second terms in the above equation are referred to as the "interference factor" and the "diffraction factor," respectively. While the former yields the interference substructure, the latter acts as an envelope which sets limits on the number of the interference peaks (see Figure 10.14).

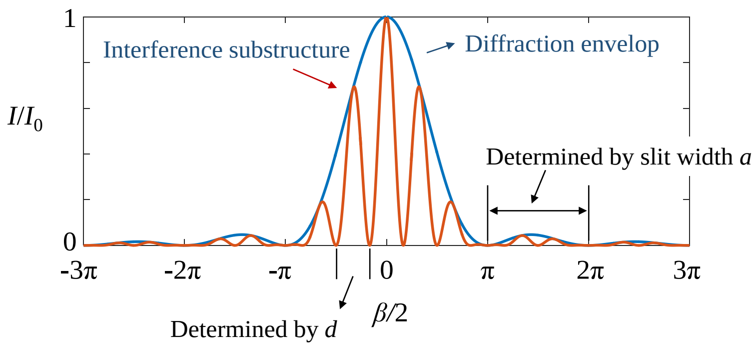

Figure 10.14 Double-slit interference with diffraction

We have seen that the interference maxima occur when $d\ \sin\theta = m\lambda$. On the other hand, the condition for the first diffraction minimum is $a\ \sin\theta = \lambda$. Thus, a particular interference maximum with order number m may coincide with the first diffraction minimum. The value of m may be obtained as:

$$\frac{d\sin\theta}{a\sin\theta} = \frac{m\lambda}{\lambda} \tag{10.52}$$

or

$$\boxed{m = \frac{d}{a}} \tag{10.53}$$

Since the m-th fringe is not seen, the number of fringes on each side of the central fringe is $m - 1$. Thus, the total number of fringes in the central diffraction maximum is

$$N = 2(m+1)+1 = 2m-1 \tag{10.54}$$

10.8 Diffraction Grating

A diffraction grating consists of a large number of slits each of width and separated from the next by a distance , as shown in Figure 10.15.

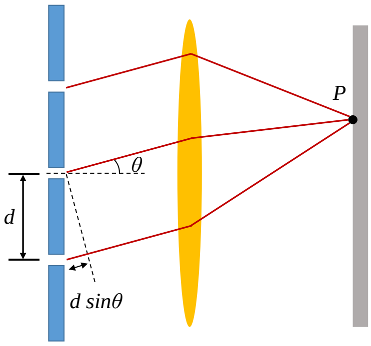

Figure 10.15 Diffraction grating

If we assume that the incident light is planar and diffraction spreads the light from each slit over a wide angle so that the light from all the slits will interfere with each other. The relative path difference between each pair of adjacent slits is $\delta = d\ \sin\theta$, similar to the calculation we made for the double-slit case. If this path difference is equal to an integral multiple of wavelengths then all the slits will constructively interfere with each other and a bright spot will appear on the screen at an angle θ. Thus, the condition for the principal maxima is given by

$$\boxed{d\sin\theta = m\lambda, \quad m = \pm1, \pm2, \pm3, \cdots} \tag{10.55}$$

If the wavelength of the light and the location of the m-order maximum are known, the distance d between slits may be readily deduced.

The location of the maxima does not depend on the number of slits, N. However, the maxima become sharper and more intense as N is increased. The width of the maxima can be shown to be inversely proportional to N. In Figure 10.16, we show the intensity distribution as a function of $d\sin\theta/\lambda$ for diffraction grating with N = 5 and N = 10. Notice that the principal maxima become sharper and narrower as increases.

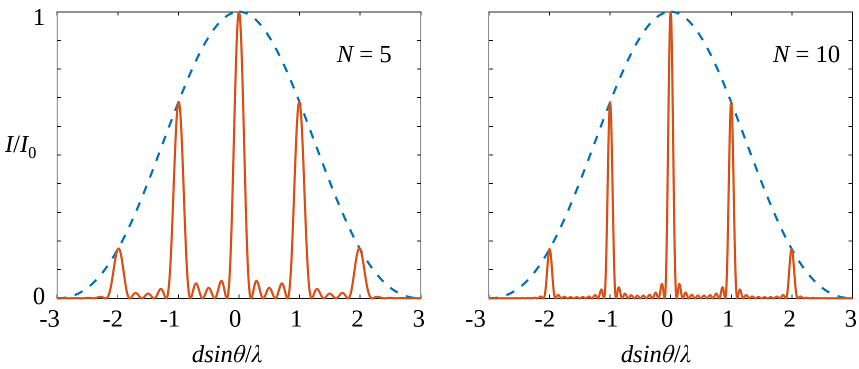

Figure 10.16 Intensity distribution for a diffraction grating for (a) N = 5 and (b) N = 10.

The observation can be explained as follows: suppose an angle θ (recall that $\beta = 2\pi a \sin\theta / \lambda$) which initially gives a principal maximum is increased slightly, if there were only two slits, then the two waves will still be nearly in phase and produce maxima which are broad. However, in grating with a large number of slits, even though θ may only be slightly deviated from the value that produces a maximum, it could be exactly out of phase with light wave from another slit far away. Since grating produces peaks that are much sharper than the two-slit system, it gives a more precise measurement of the wavelength.

10.9 Additional Problems

See References [2-7].

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