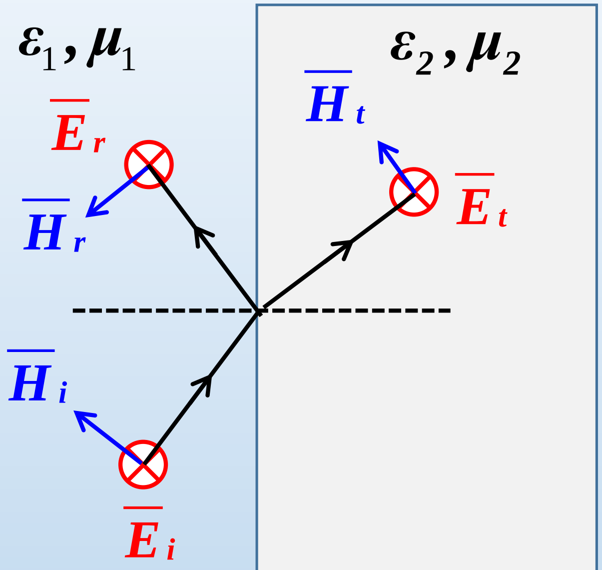
Course Review (II)

<u>Outline</u>

- L6. Reflection and Transmission
- L7. Wave Guidance

L6. Reflection and Transmission

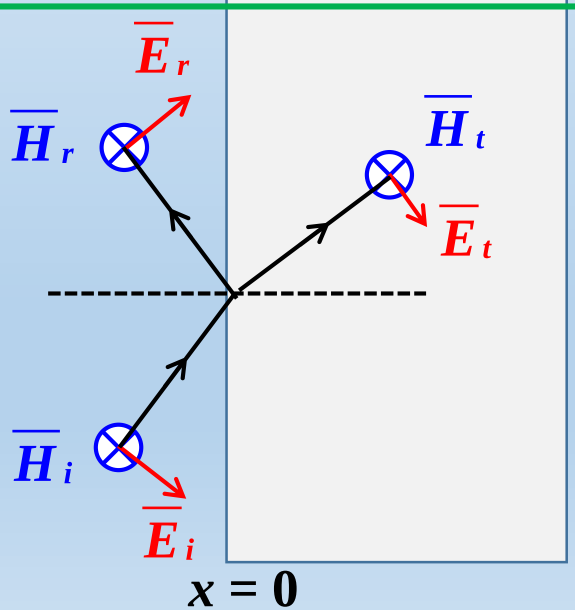
Fresnel Equations - Summary



TE-polarization

$$R^{TE} = \frac{\mu_2 k_{ix} - \mu_1 k_{tx}}{\mu_2 k_{ix} + \mu_1 k_{tx}} = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$$

$$T^{TE} = \frac{2\mu_2 k_{ix}}{\mu_2 k_{ix} + \mu_1 k_{tx}} = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$$



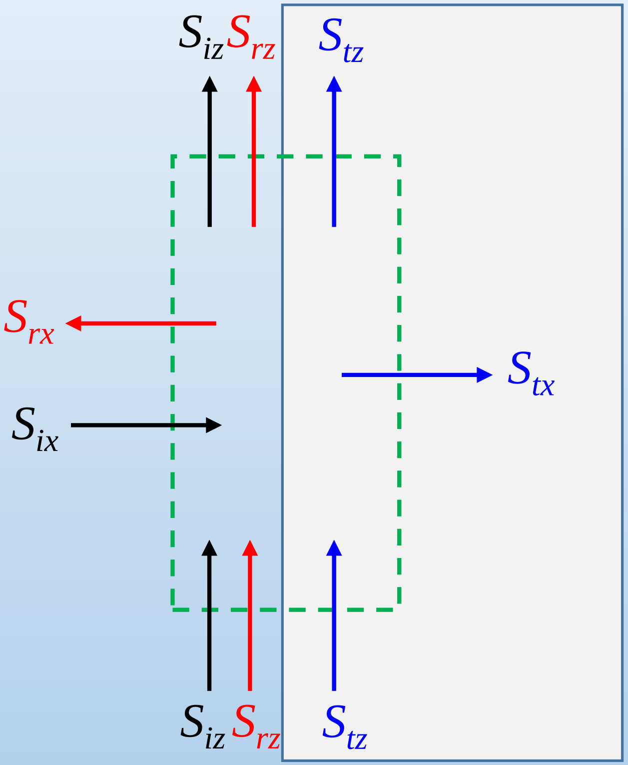
TM-polarization

$$R^{TM} = \frac{\varepsilon_2 k_{ix} - \varepsilon_1 k_{tx}}{\varepsilon_2 k_{ix} + \varepsilon_1 k_{tx}} = \frac{\eta_1 \cos\theta_i - \eta_2 \cos\theta_t}{\eta_1 \cos\theta_i + \eta_2 \cos\theta_t}$$

$$T^{TM} = \frac{2\varepsilon_2 k_{ix}}{\varepsilon_2 k_{ix} + \varepsilon_1 k_{tx}} = \frac{2\eta_1 \cos\theta_i}{\eta_1 \cos\theta_i + \eta_2 \cos\theta_t}$$

Energy Transport

$$\vec{S}_r(t) = \vec{E}(t) \times \vec{H}(t)$$



TE-polarization

$$r = \frac{-\hat{x} \cdot \langle \vec{S}_r \rangle}{\hat{x} \cdot \langle \vec{S}_i \rangle} = \left| R^{TE} \right|^2$$

$$t = \frac{\hat{x} \cdot \langle \vec{S}_t \rangle}{\hat{x} \cdot \langle \vec{S}_i \rangle} = \frac{\eta_1 \cos\theta_t}{\eta_2 \cos\theta_i} \left| T^{TE} \right|^2$$

$$\boxed{\begin{aligned} R^{TM} &= H_r / H_i \\ T^{TM} &= H_t / H_i \end{aligned}}$$

TM-polarization

$$r = \frac{-\hat{x} \cdot \langle \vec{S}_r \rangle}{\hat{x} \cdot \langle \vec{S}_i \rangle} = \left| R^{TM} \right|^2$$

$$t = \frac{\hat{x} \cdot \langle \vec{S}_t \rangle}{\hat{x} \cdot \langle \vec{S}_i \rangle} = \frac{\eta_2 \cos\theta_t}{\eta_1 \cos\theta_i} \left| T^{TM} \right|^2$$

$$\boxed{\begin{aligned} R^{TE} &= E_r / E_i \\ T^{TE} &= E_t / E_i \end{aligned}}$$

***r*: reflectivity**

***t*: transmission**

Energy Conservation

TE-polarization

$$r+t=\left|\frac{\eta_2\cos\theta_i-\eta_1\cos\theta_t}{\eta_2\cos\theta_i+\eta_1\cos\theta_t}\right|^2+\frac{\eta_1\cos\theta_t}{\eta_2\cos\theta_i}\left|\frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i+\eta_1\cos\theta_t}\right|^2=1$$

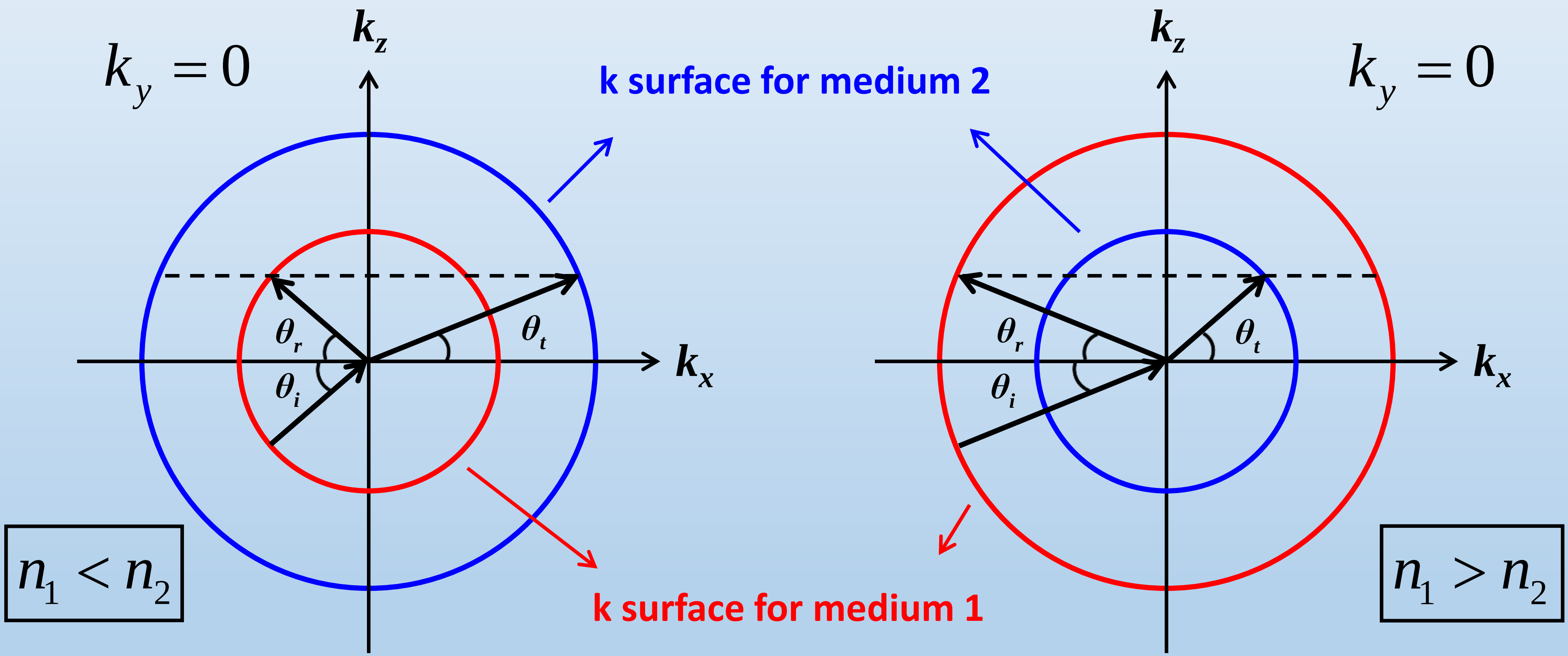
TM-polarization

$$r+t=\left|\frac{\eta_1\cos\theta_i-\eta_2\cos\theta_t}{\eta_1\cos\theta_i+\eta_2\cos\theta_t}\right|^2+\frac{\eta_2\cos\theta_t}{\eta_1\cos\theta_i}\left|\frac{2\eta_1\cos\theta_i}{\eta_1\cos\theta_i+\eta_2\cos\theta_t}\right|^2=1$$

Phase Matching

Phase matching condition: $\boxed{k_{iz} = k_{rz} = k_{tz}}$

k surface: $k_x^2 + k_y^2 + k_z^2 = \omega^2 \mu \varepsilon = n^2 k_0^2$



Snell's law: $\begin{cases} k_{iz} = k_{rz} \\ k_{ix} = -k_{rx} \end{cases} \Rightarrow \theta_i = \theta_r$

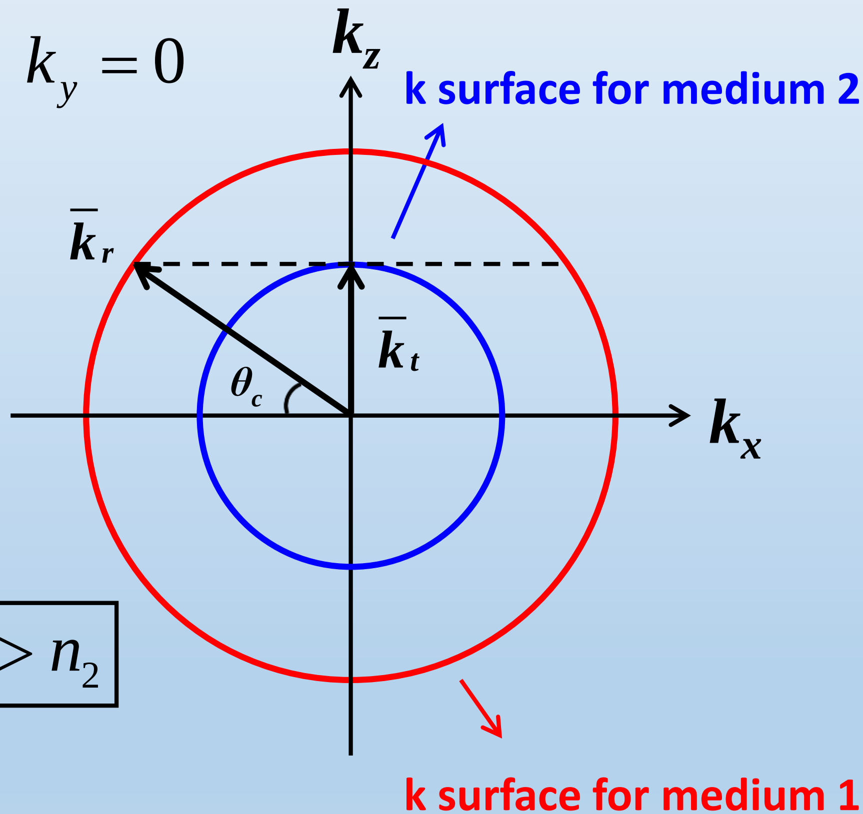
$$\frac{\sin\theta_i}{\sin\theta_t} = \frac{k_{iz}/k_i}{k_{tz}/k_t} = \frac{k_t}{k_i} = \frac{n_2}{n_1}$$

Total Reflection and Critical angle

Phase matching condition: $\boxed{k_{iz} = k_{rz} = k_{tz}}$

k surface: $k_x^2 + k_y^2 + k_z^2 = \omega^2 \mu\varepsilon = n^2 k_0^2$

$k_y = 0$



$\boxed{n_1 > n_2}$

$n_1 > n_2, k_{ix} > k_t \ \left(\theta_i > \theta_c\right)$

$$k_{tx} = \sqrt{k_t^2 - k_z^2} = -jk''_{tx}$$

(purely imaginary)

$$\langle \bar{S}_t \rangle = \hat{z}\frac{k_z}{2\omega\varepsilon_t}\left|T^{TM}\right|^2 e^{-2k''_{tx}x} \quad \textbf{(TM waves)}$$

$$\langle \bar{S}_t \rangle = \hat{z}\frac{k_z}{2\omega\mu_t}\left|T^{TE}\right|^2 e^{-2k''_{tx}x} \quad \textbf{(TE waves)}$$

No power transmitted in the x direction into the region t

Critical angle: $\theta_c = \sin^{-1}\frac{k_t}{k_i} = \sin^{-1}\frac{n_2}{n_1}$

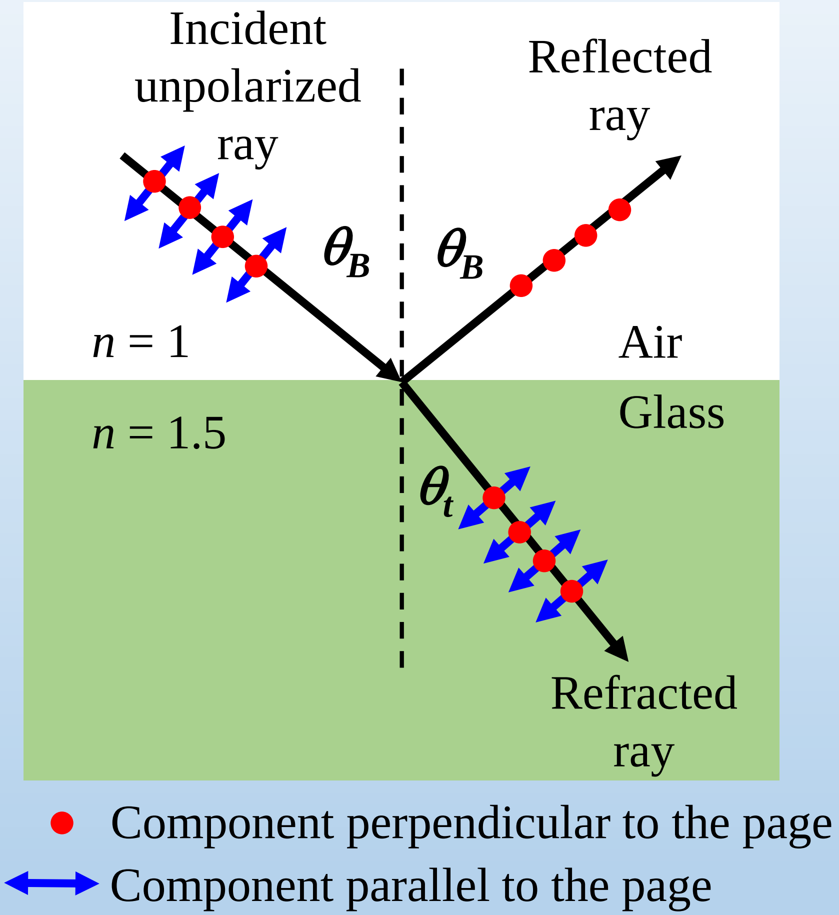
Polarization by reflection

Different polarization of light get reflected and refracted with different amplitudes.

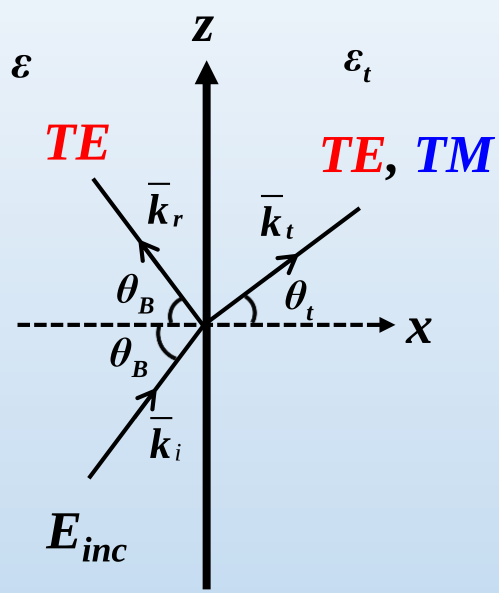
At one particular angle, the parallel polarization is NOT reflected at all! This is the "Brewster angle" θ_B, and $\theta_B + \theta_t = 90^o$.

$$n_1 \sin\theta_B = n_2 \sin(90^o - \theta_B) = n_2 \cos\theta_B$$

$$\boxed{\tan\theta_B = \frac{n_2}{n_1}}$$



Total Transmission and Brewster Angle



(TM waves)

If $\eta_1 \cos\theta_i = \eta_2 \cos\theta_t$

$$R^{TM} = \frac{\eta_1 \cos\theta_i - \eta_2 \cos\theta_t}{\eta_1 \cos\theta_i + \eta_2 \cos\theta_t} = 0$$

$$T^{TM} = \frac{2\eta_1 \cos\theta_i}{\eta_1 \cos\theta_i + \eta_2 \cos\theta_t} = 1$$

$$\frac{\sin\theta_i}{\sin\theta_t} = \frac{n_2}{n_1}$$

(Snell Law)

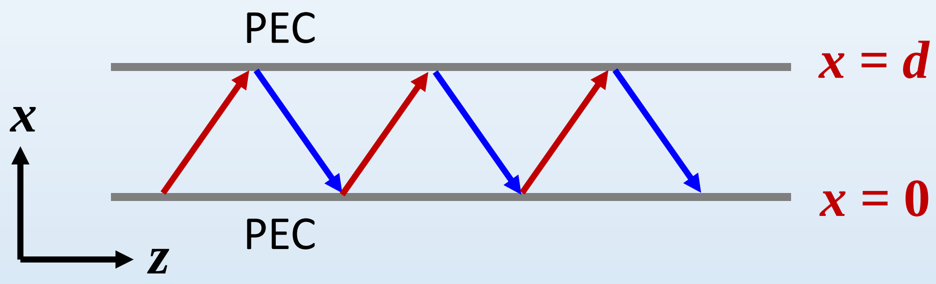
$$\Rightarrow \quad \frac{\cos\theta_i}{\cos\theta_t} = \frac{n_1}{n_2}$$

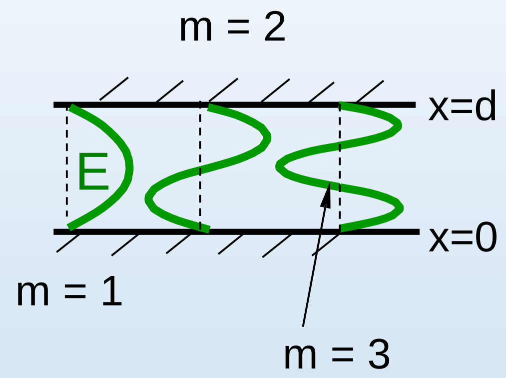
$$\Rightarrow \quad \boxed{\theta_i + \theta_t = \frac{\pi}{2}}$$

Brewster Angle: θ_B

L7. Wave Guidance

TE Waveguide Modes





$$\vec{E} = \hat{y}\left(Ae^{-jk_x x} + Be^{jk_x x}\right)e^{-jk_z z}$$

$$m\lambda_x/2 = d$$

Boundary Conditions: $E_y(0,z) = E_y(d,z) = 0$

$$\Rightarrow \begin{cases} A = -B \\ \sin(k_x d) = 0 \end{cases}$$

$$\boxed{k_x d = m\pi}$$

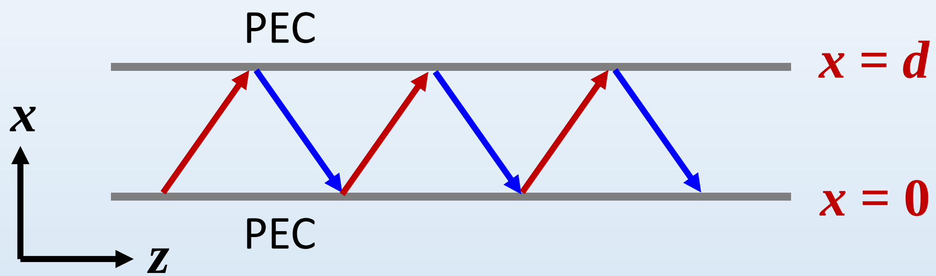
(Guidance Condition)

$$E_y(x,z) = E_0 \sin(k_x x)e^{-jk_z z}$$

$$H_x(x,z) = -\frac{k_z}{\omega\mu}E_0 \sin(k_x x)e^{-jk_z z}$$

$$H_z(x,z) = -\frac{k_x}{j\omega\mu}E_0 \cos(k_x x)e^{-jk_z z}$$

TM Waveguide Modes



$$\nabla \times \vec{H} = j\omega\varepsilon\vec{E}$$

$$\nabla \times \vec{E} = -j\omega\mu\vec{H}$$

$$\vec{H} = \hat{y}\left(Ae^{-jk_x x} + Be^{jk_x x}\right)e^{-jk_z z}$$

Boundary Conditions: $E_z(0,z) = E_z(d,z) = 0$

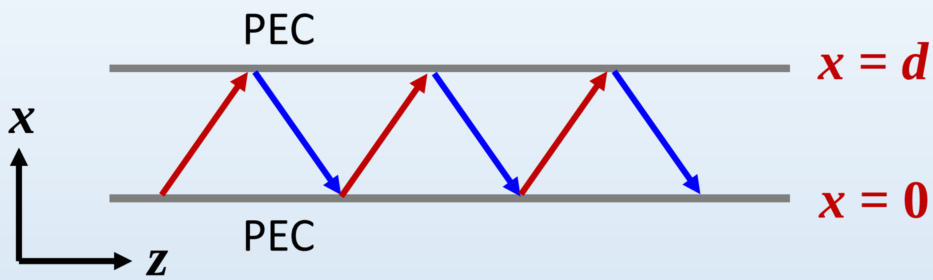
$$\begin{cases} A = B \\ \sin(k_x d) = 0 \end{cases}$$

$$\boxed{k_x d = m\pi}$$

(Guidance Condition)

$$\boxed{\begin{aligned} H_y(x,z) &= H_0\cos(k_x x)e^{-jk_z z} \\ E_x(x,z) &= \frac{k_z}{\omega\varepsilon}H_0\cos(k_x x)e^{-jk_z z} \\ E_z(x,z) &= -\frac{k_x}{j\omega\varepsilon}H_0\sin(k_x x)e^{-jk_z z} \end{aligned}}$$

Cutoff Frequency



$$k_x d = m\pi \Rightarrow k_z = \sqrt{\left(\frac{\omega}{c}\right)^2 - \left(\frac{m\pi}{d}\right)^2}$$ **(Dispersion Relation)**

$$\boxed{\omega = \frac{m\pi c}{d}}$$

- **Cutoff Frequencies of the TE_m and TM_m modes (m>0)**
- **No cutoff frequency and TM_0 (TEM mode)**
- **TE_0 mode does not exist.**

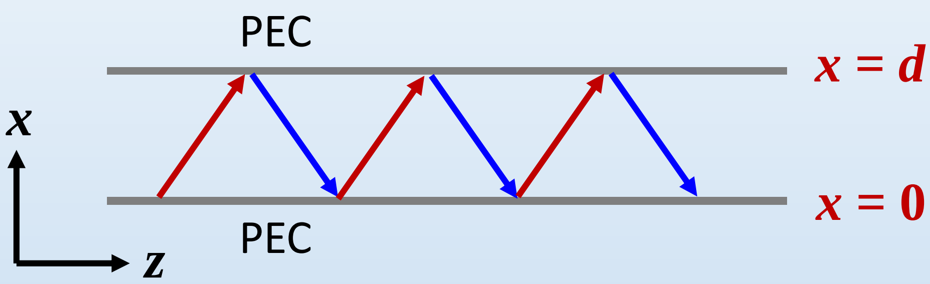
TEM mode:

$$\vec{H} = \hat{y} H_0 \exp(-jkz)$$

$$\vec{E} = \hat{x} E_0 \exp(-jkz)$$

Rectangular Waveguide

(a) Parallel-plate waveguide (Two PEC boundaries)



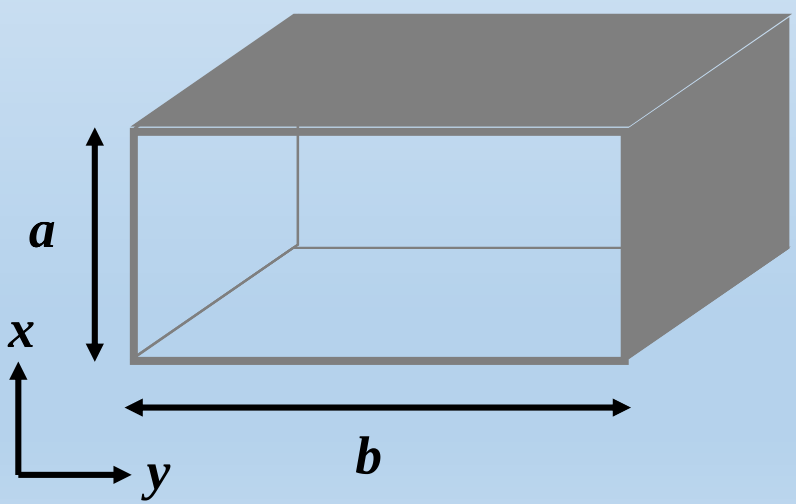
TE mode:

$$H_z(x,z) = H_0 \cos(k_x x) e^{-jk_z z}$$

TM mode:

$$E_z(x,z) = E_0 \sin(k_x x) e^{-jk_z z}$$

(a) Rectangular waveguide (Four PEC boundaries)



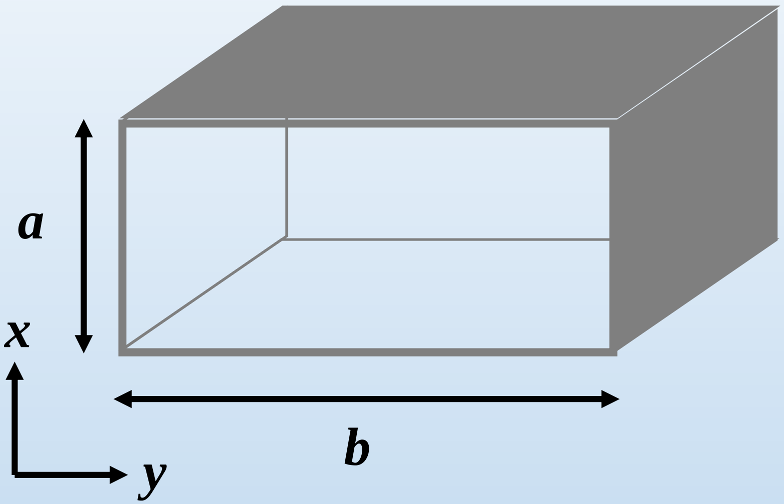
TE mode:

$$H_z(x,y,z) = H_0 \cos(k_x x) \cos(k_y y) e^{-jk_z z}$$

TM mode:

$$E_z(x,y,z) = E_0 \sin(k_x x) \sin(k_y y) e^{-jk_z z}$$

Rectangular Waveguide - TE_{mn} Mode



Boundary Conditions:

(1) Ex = 0 at y = 0 and b

(2) Ey = 0 at x = 0 and a

$$\boxed{\begin{aligned} k_x a &= m\pi \\ k_y b &= n\pi \end{aligned}}$$

(Guidance Condition)

TE_{01}, TE_{10}, TE_{11}, TE_{02}, …

$$H_z = \cos(k_x x)\cos(k_y y)e^{-jk_z z}$$

$$H_x = \frac{jk_x k_z}{\omega^2\mu\varepsilon - k_z^2}\sin(k_x x)\cos(k_y y)e^{-jk_z z}$$

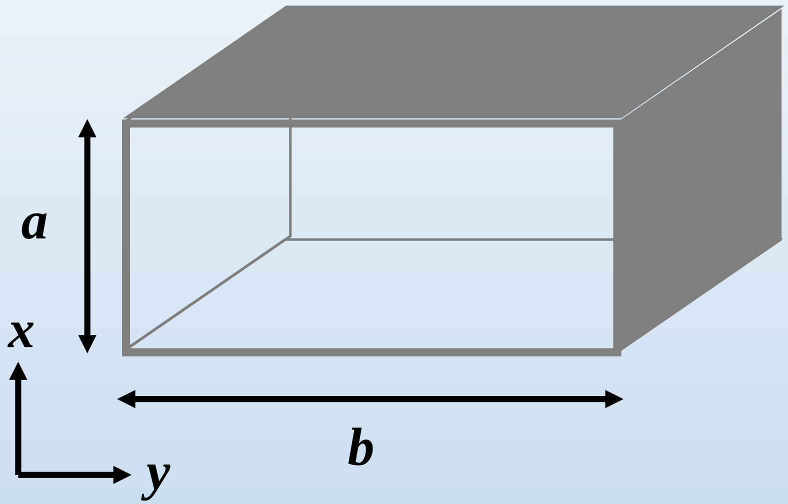
$$H_y = \frac{jk_y k_z}{\omega^2\mu\varepsilon - k_z^2}\cos(k_x x)\sin(k_y y)e^{-jk_z z}$$

$$E_x = \frac{j\omega\mu k_y}{\omega^2\mu\varepsilon - k_z^2}\cos(k_x x)\sin(k_y y)e^{-jk_z z}$$

$$E_y = \frac{-j\omega\mu k_x}{\omega^2\mu\varepsilon - k_z^2}\sin(k_x x)\cos(k_y y)e^{-jk_z z}$$

$$\boxed{k_z = \sqrt{\left(\frac{\omega}{c}\right)^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}}$$

Rectangular Waveguide - TM_{mn} Mode



Boundary Conditions:

(1) Ex = 0 at y = 0 and b

(2) Ey = 0 at x = 0 and a

$$\boxed{\begin{aligned} k_x a &= m\pi \\ k_y b &= n\pi \end{aligned}}$$

(Guidance Condition)

TM_{11}, TM_{12}, TM_{21}, TM_{22}, …

$$E_z(x,y,z) = \sin(k_x x)\sin(k_y y)e^{-jk_z z}$$

$$E_x = \frac{-jk_x k_z}{\omega^2\mu\varepsilon - k_z^2}\cos(k_x x)\sin(k_y y)e^{-jk_z z}$$

$$E_y = \frac{-jk_y k_z}{\omega^2\mu\varepsilon - k_z^2}\sin(k_x x)\cos(k_y y)e^{-jk_z z}$$

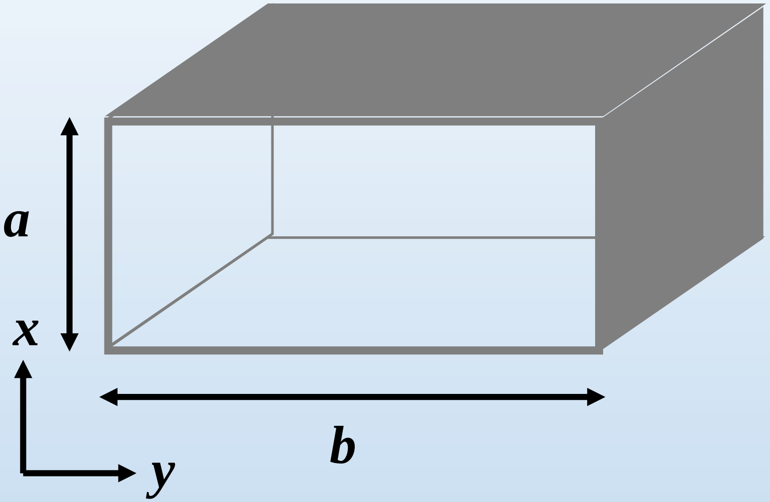
$$H_x = \frac{j\omega\varepsilon k_y}{\omega^2\mu\varepsilon - k_z^2}\sin(k_x x)\cos(k_y y)e^{-jk_z z}$$

$$H_y = \frac{-j\omega\varepsilon k_x}{\omega^2\mu\varepsilon - k_z^2}\cos(k_x x)\sin(k_y y)e^{-jk_z z}$$

$$\boxed{k_z = \sqrt{\left(\frac{\omega}{c}\right)^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}}$$

TE_{mn} Mode vs. TM_{mn} Mode

No TEM mode



TE_{mn} Mode (TE_{01}, TE_{10}, TE_{11}, TE_{02}, …)

$$H_z = \cos\left(k_x x\right)\cos\left(k_y y\right)e^{-jk_z z}$$

TM_{mn} Mode (TM_{11}, TM_{12}, TM_{21}, TM_{22}, …)

$$E_z = \sin\left(k_x x\right)\sin\left(k_y y\right)e^{-jk_z z}$$

(Guidance Condition)

$$\boxed{\begin{aligned} k_x a &= m\pi \\ k_y b &= n\pi \end{aligned}}$$

(propagation constant)

$$\boxed{k_z = \sqrt{\left(\frac{\omega}{c}\right)^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}}$$

(the cutoff frequency)

$$\boxed{k_{cmn} = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}}$$

$$\boxed{f_{cmn} = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}}$$