山东大学 SHANDONG UNIVERSITY

Physics I: Introduction to Wave Theory
SDU Course Number: sd01232810 (Fall 2024)

Lecture 4: Boundary Conditions

Outline

- Dielectrics and Polarization
- Magnetization and Bound Currents
- Maxwell's Equations in Matter
- Electric/Magnetic Boundary Conditions
- Scalar and Vector potentials in Static Fields
- Uniqueness Theorems
- The Method of Images

Maxwell's Equations in Vacuum

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$ (Gauss's Law)

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$ (Faraday's Law)

$$\nabla \cdot \vec{B} = 0$$ (Magnetic Gauss's Law)

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$ (Ampere's Law)

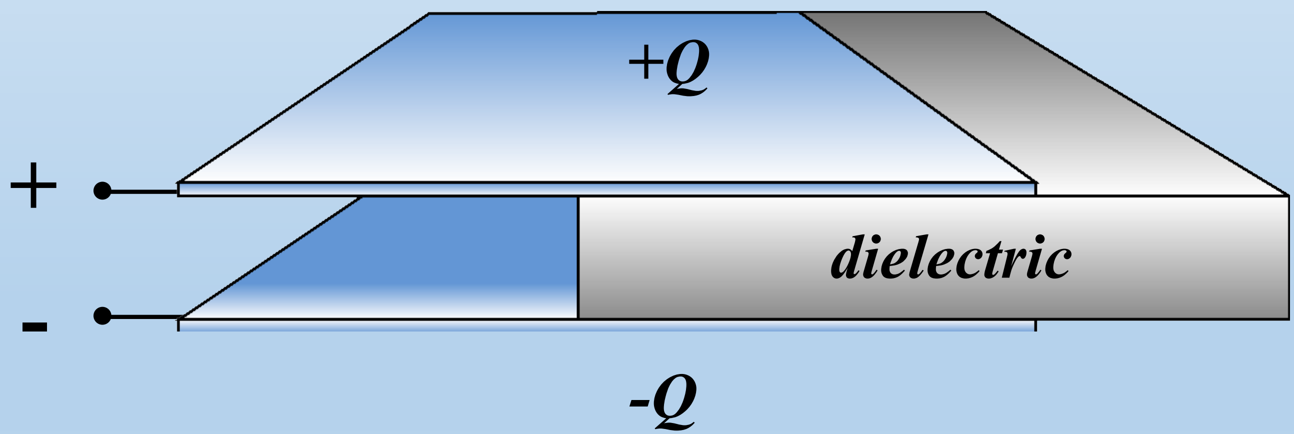
E: electric field [volts/meter, V/m]
B: Magnetic flux density [weber/m^2, Wb/m^2]
J: Electric current density [amperes/m^2, A/m^2]
ρ: Electric change density [coulombs/m^2,C/m^3]

Dielectrics

A dielectric is a non-conductor or insulator.

Examples: rubber, glass, waxed paper

When placed in a charged capacitor, the dielectric reduces the potential difference between the two plates

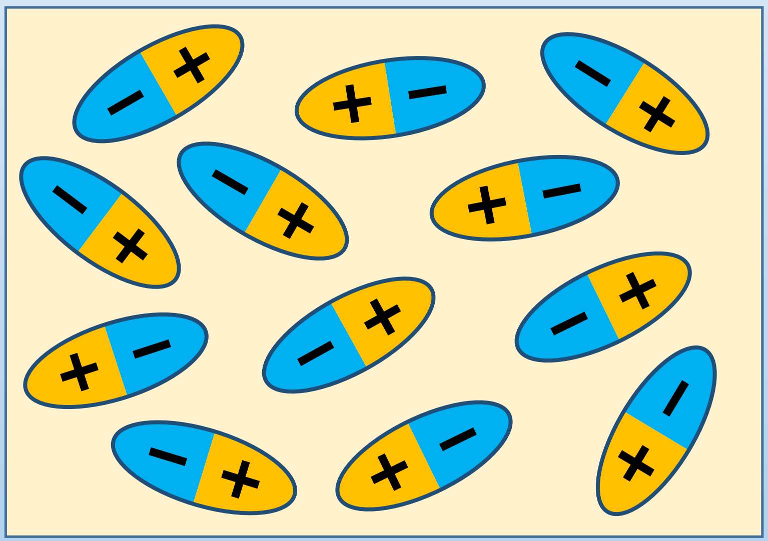


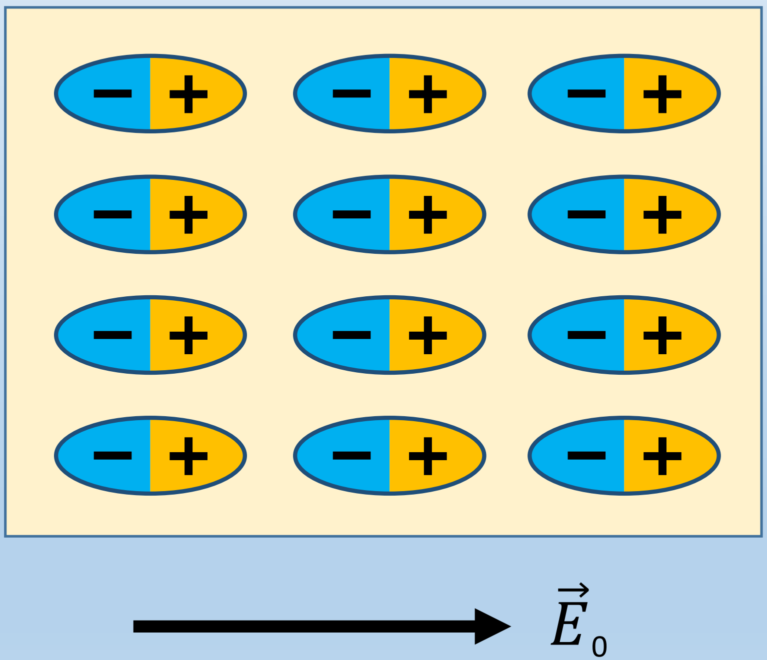
HOW???

Molecular View of Dielectrics

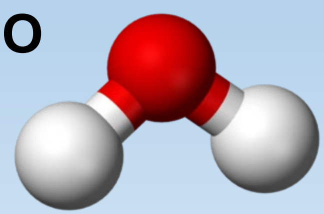
Polar Dielectrics:
Dielectrics permanent electric dipole moments with electric dipole moments.

<u>Example: Water</u>





H_2O

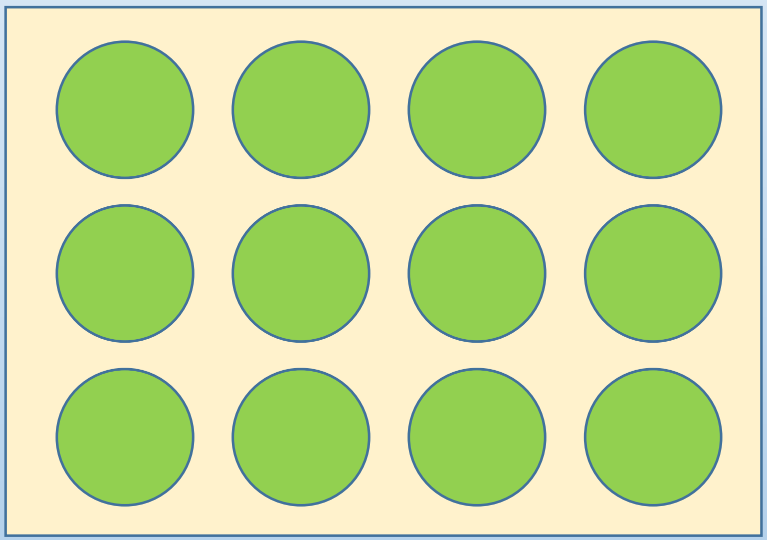


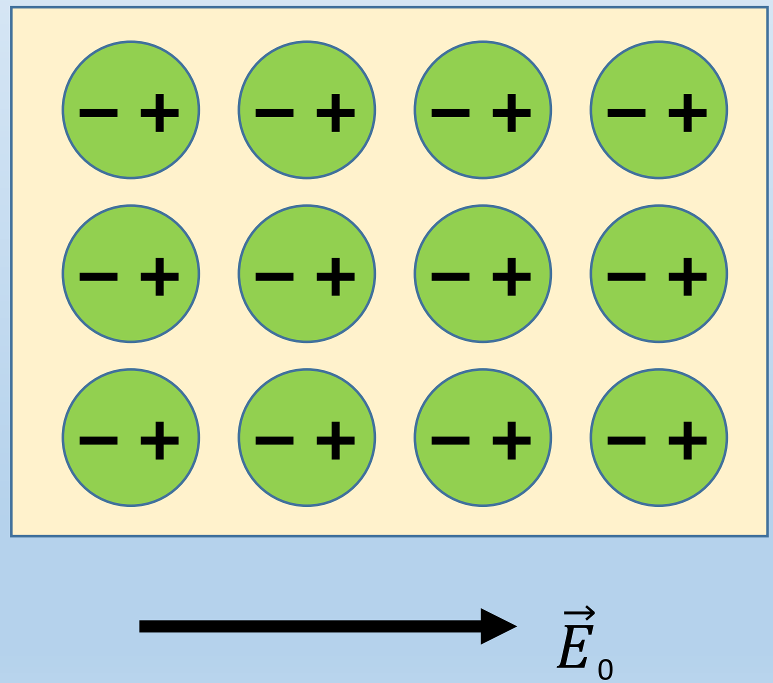
Molecular View of Dielectrics

Non-Polar Dielectrics

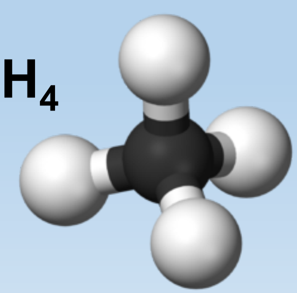
Dielectrics with induced electric dipole moments

<u>Example: CH_4</u>





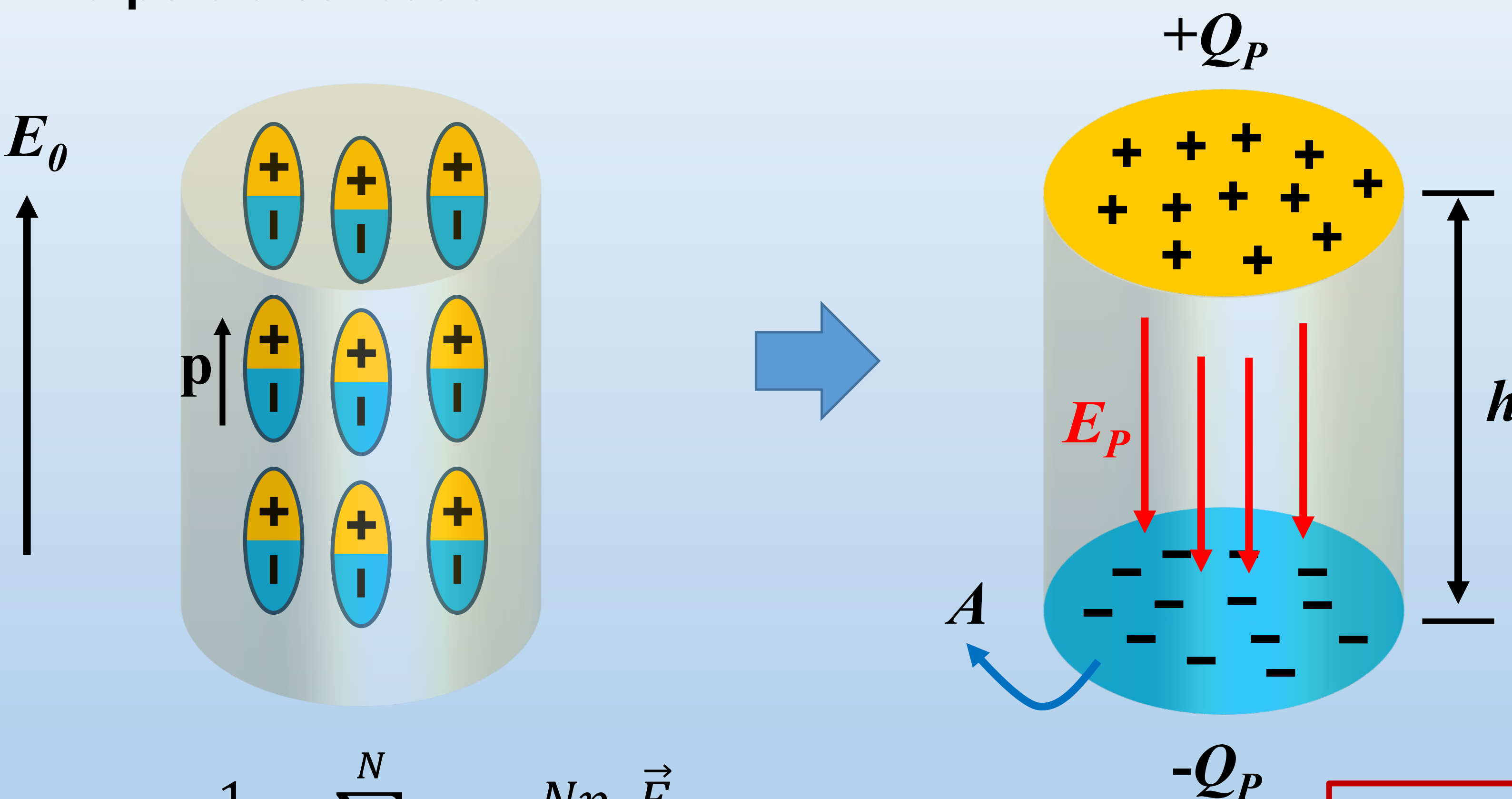
CH_4



Polarization

A cylinder with uniform dipole distribution

Equivalent charge distribution



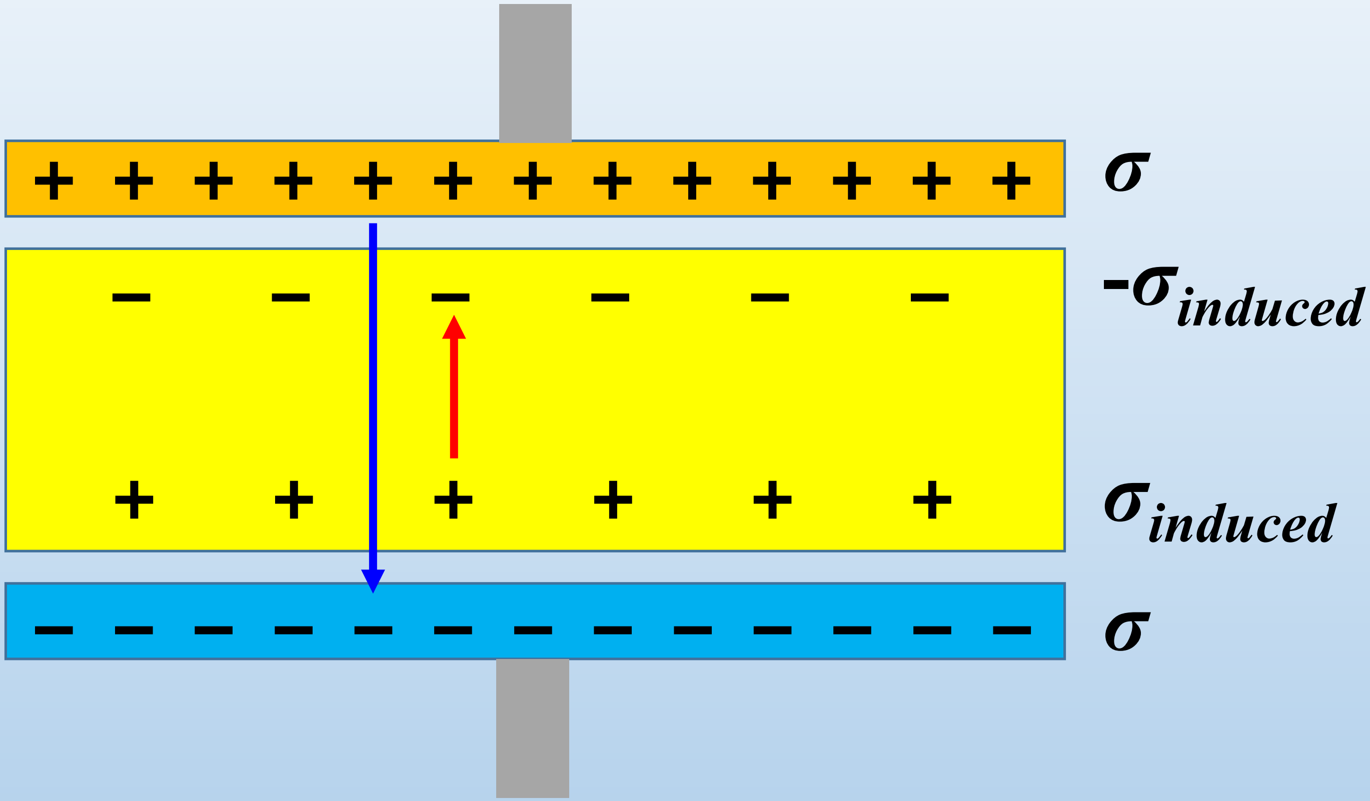
$$\vec{P} = \frac{1}{volume}\sum_{i=1}^{N}\vec{p}_i = \frac{Np}{Ah}\frac{\vec{E}_0}{|E_0|}$$

(polarization density)

$$Q_P = \frac{Np}{h}$$

$$\boxed{\vec{E}_P = -\frac{\vec{P}}{\varepsilon_0}}$$

Dielectric in Capacitor

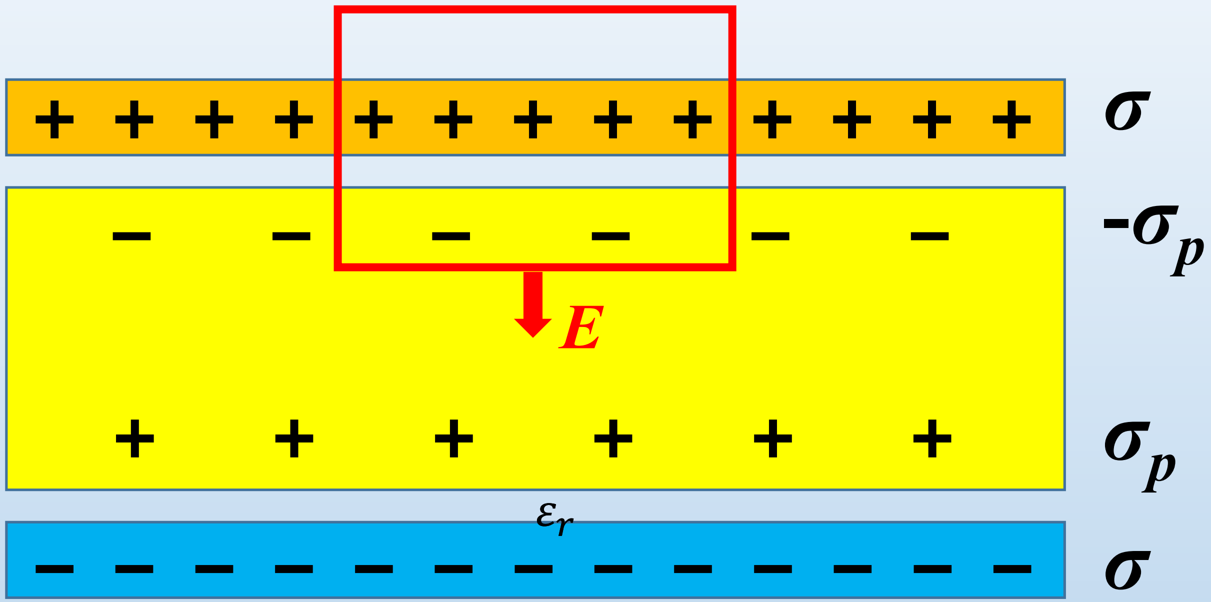


Potential difference decreases because dielectric polarization decreases Electric Field!

$$\vec{P} = \varepsilon_0 \chi_e \vec{E} \qquad \varepsilon_r = 1 + \chi_e$$

(Dielectric Constant)

Gauss's Law for Dielectrics

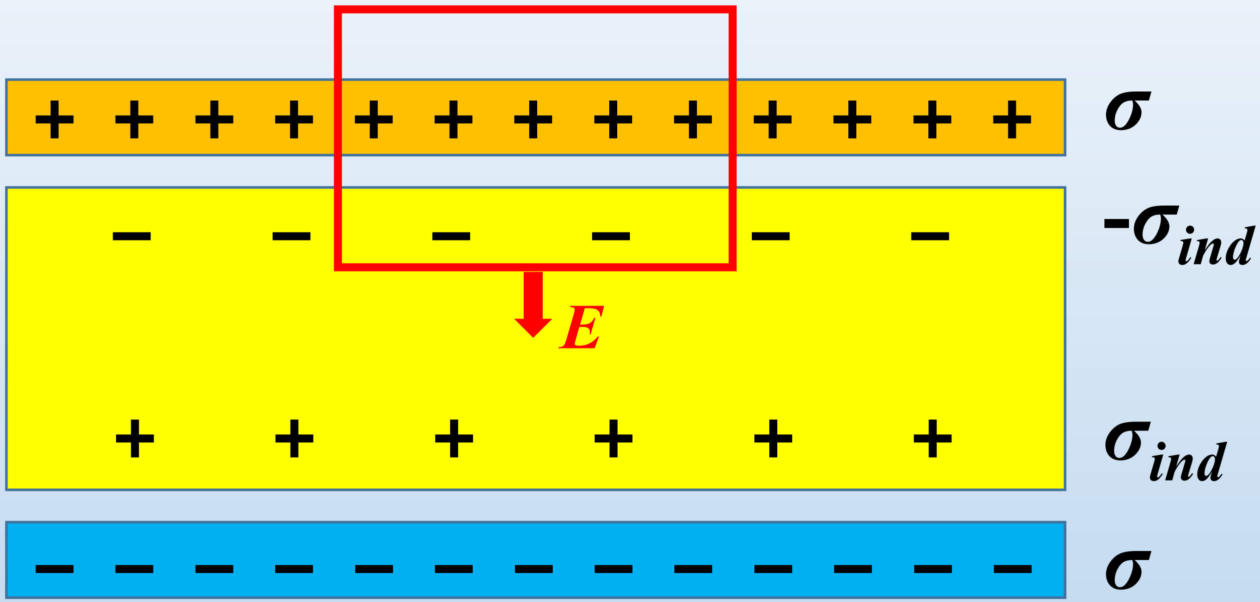


$$\oiint_S \vec{E}\cdot d\vec{S} = EA = \frac{Q_{inside}}{\varepsilon_0} = \frac{(\sigma-\sigma_p)A}{\varepsilon_0} \Rightarrow E = \frac{(\sigma-\sigma_p)}{\varepsilon_0}$$

$$E = \frac{(\sigma-\sigma_p)}{\varepsilon_0} = \frac{E_0}{\varepsilon_r} = \frac{\sigma}{\varepsilon_r\varepsilon_0} \Rightarrow \vec{P} = \varepsilon_0\chi_e\vec{E} \qquad \varepsilon_r = 1+\chi_e$$

(Dielectric Constant)

Gauss's Law for Dielectrics



$$\oiint_S \left(\vec{E} + \frac{\vec{P}}{\varepsilon_0} \right) \cdot d\vec{S} = \frac{Q_{free}}{\varepsilon_0} \quad \Rightarrow \quad \boxed{\oiint_S \vec{D} \cdot d\vec{S} = Q_{free}}$$

(Gauss's Law)

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$$ **(Electric displacement [C/m^2])**

Displacement Fields

$$\oint\!\!\oint_S \vec{D}\cdot d\vec{S} = \oint\!\!\oint_S \left(\varepsilon_0 \vec{E} + \vec{P}\right)\cdot d\vec{S} = \iiint_V \rho_{free} dV$$

The electric displacement field is defined as

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$$

Displacement field D accounts for the effects of unbound ("free") charges within materials.

$$\oint\!\!\oint_S \varepsilon_0 \vec{E}\cdot d\vec{S} = \iiint_V \rho_{total} dV$$

Electric field E accounts for the effects of total charges (both "bound" and "free") within materials.

Integral Form

$$\oiint_S \vec{D}\cdot d\vec{S} = \oiint_S \left(\varepsilon_0 \vec{E} + \vec{P}\right)\cdot d\vec{S} = \iiint_V \rho_{free} dV$$

Divergence theorem

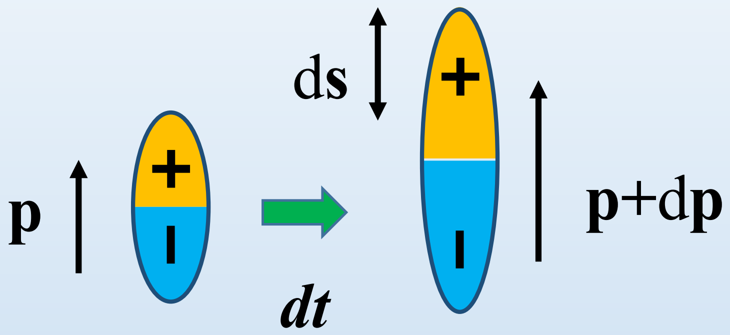
$$\oiint_S \vec{F}\cdot d\vec{S} = \iiint_V \left(\nabla\cdot\vec{F}\right) dV$$

$$\boxed{\nabla\cdot\vec{D} = \rho_{free}}$$ **(Gauss's Law)**

$$\nabla\cdot\varepsilon_0\vec{E} = \rho_{total}$$

$$\nabla\cdot\vec{P} = -\rho_{bound}$$

Polarization in changing fields



$$\vec{J}_{bound} = \rho\vec{v} = Nq\frac{d\vec{s}}{dt} = N\frac{d\vec{p}}{dt} \Rightarrow \boxed{\vec{J}_{bound} = \frac{d\vec{P}}{dt}}$$

$$\nabla \times \vec{B} = \mu_0\varepsilon_0\frac{\partial \vec{E}}{\partial t} + \mu_0\frac{d\vec{P}}{dt} + \mu_0\vec{J}_{free}$$ **(Ampere's Law)**

Bound-charge current density

Free-charge current density

Ampere’s Law for Dielectrics

Vacuum displacement current density

$$\nabla \times \vec{B} = \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} + \mu_0 \frac{d\vec{P}}{dt} + \mu_0 \vec{J}_{free}$$

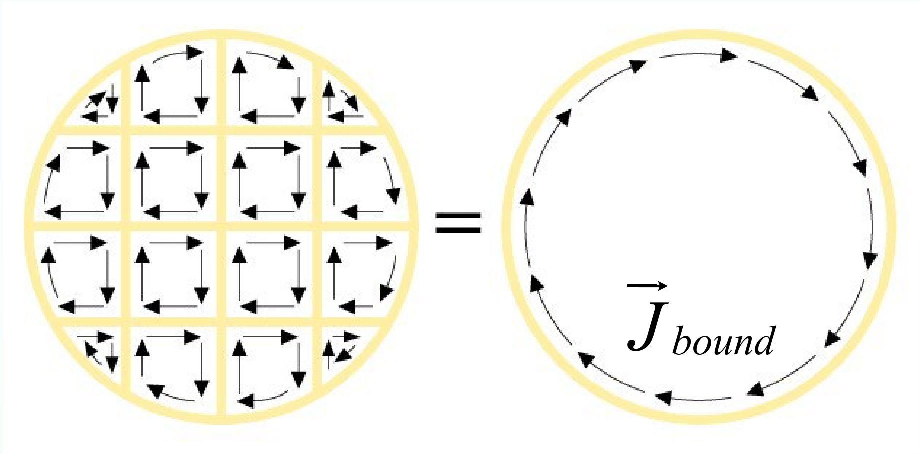
Bound-charge current density

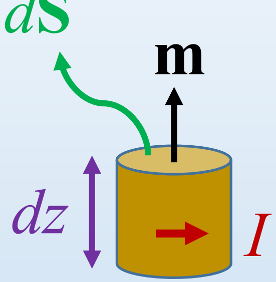
Free-charge current density

$$\boxed{\nabla \times \vec{B} = \mu_0 \frac{\partial \vec{D}}{\partial t} + \mu_0 \vec{J}_{free}}$$

(Ampere’s Law)

Magnetization and Bound Currents





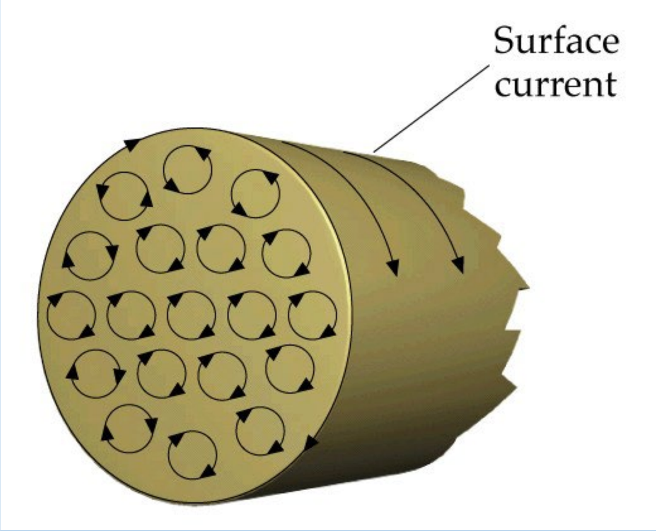
$$\mathbf{m} = I\, d\mathbf{S}$$

$$I = M\, dz$$

$$\vec{M} = \frac{1}{volume}\sum_{i=1}^{N}\vec{m}_i$$

(Magnetization)

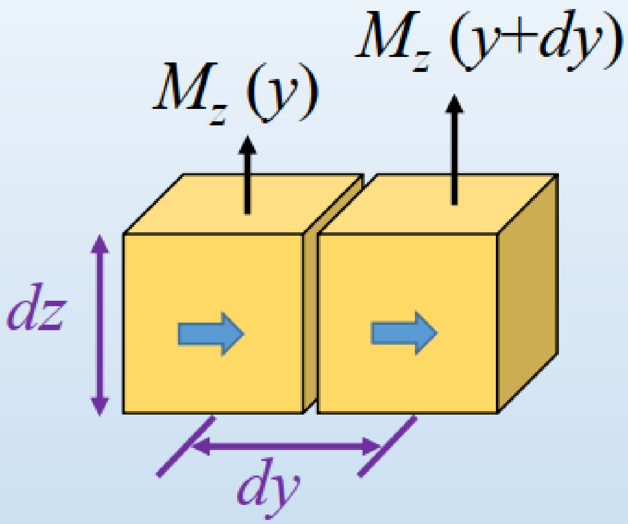
$$J^{s}_{bound} = \frac{I}{dz} = M$$

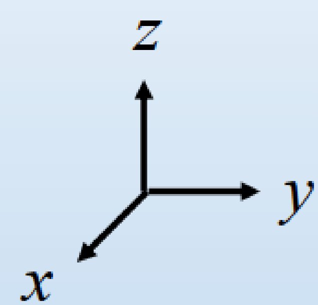


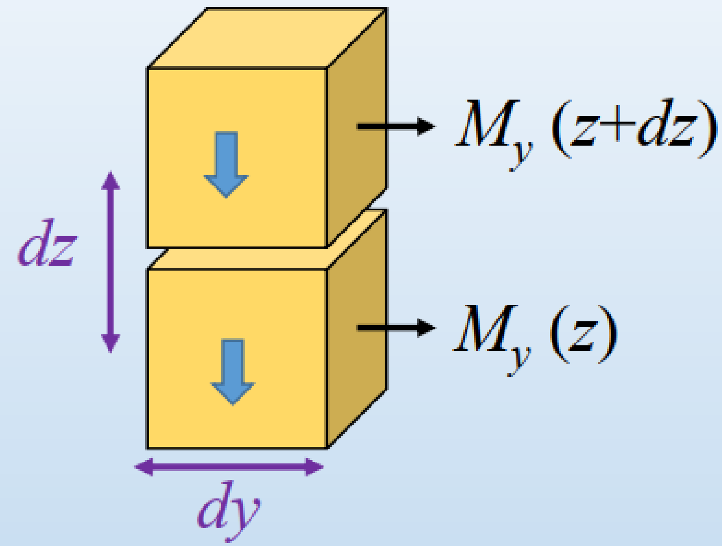
For nonuniform magnetization:

$$\boxed{\vec{J}_{bound} = \nabla \times \vec{M}}$$ **(bound currents)**

Nonuniform magnetization: $I_{\mathrm{m}} = \int nId\mathbf{a}d\mathbf{l} = \oint_L \mathbf{M}\cdot d\mathbf{l}$







A net current in the x direction

$$I_x = \left[M_z\left(y+dy\right) - M_z\left(y\right)\right]dz$$

$$= \frac{\partial M_z}{\partial y}dydz$$

A net current in the x direction

$$I_x = -\left[M_y\left(z+dz\right) - M_y\left(z\right)\right]dy$$

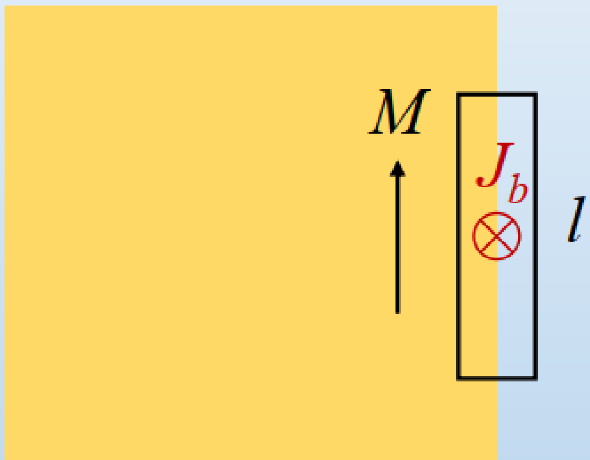
$$= -\frac{\partial M_y}{\partial z}dydz$$

$$\boxed{\left(J_{bound}\right)_x = \frac{I_x}{dydz} = \frac{\partial M_z}{\partial y} - \frac{\partial M_y}{\partial z}}$$

$$\boxed{\vec{J}_{bound} = \nabla \times \vec{M}}$$

(bound current)

*__Example:__ show that $J^s_{bound} = M$ follows from $\vec{J}_{bound} = \nabla \times \vec{M}$



$$\iint_S \vec{J}_{bound} \cdot d\vec{S} = \iint_S \left(\nabla \times \vec{M} \right) \cdot d\vec{S}$$

$$= \oint_C \vec{M} \cdot d\vec{l}$$

$$\Rightarrow \quad J^s_{bound} l = Ml$$

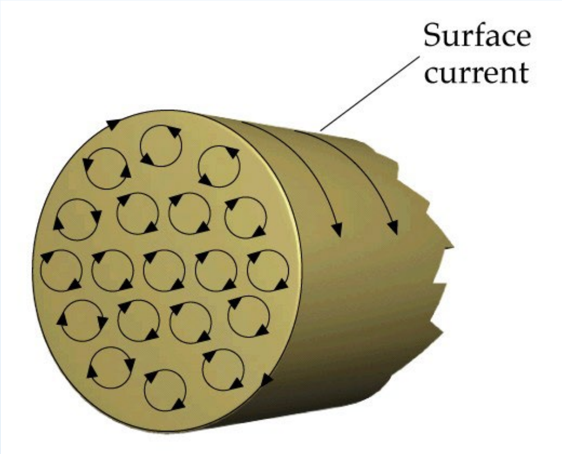
$$\Rightarrow \quad \boxed{J^s_{bound} = M}$$

The field due to magnetization of the medium is just the field produced by the bound currents.

$$\boxed{\vec{J}_{bound} = \nabla \times \vec{M}}$$

Ampere's Law for Magnets

$$\nabla \times \vec{B} = \mu_0 \frac{\partial \vec{D}}{\partial t} + \mu_0 \left(\vec{J}_{free} + \vec{J}_{bound} \right)$$



$$\nabla \times \vec{B} = \mu_0 \frac{\partial \vec{D}}{\partial t} + \mu_0 \left(\vec{J}_{free} + \nabla \times \vec{M} \right)$$

$$\nabla \times \left(\frac{\vec{B}}{\mu_0} - \vec{M} \right) = \frac{\partial \vec{D}}{\partial t} + \vec{J}_{free} \Rightarrow \boxed{\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} + \vec{J}_{free}}$$

(Ampere's Law)

$$\boxed{\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}}$$

(Magnetic field strength [A/m])

Maxwell's Equations in Matter

$$\nabla \cdot \vec{D} = \rho_{free}$$ **(Gauss's Law)**

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$ **(Faraday's Law)**

$$\nabla \cdot \vec{B} = 0$$ **(Magnetic Gauss's Law)**

$$\nabla \times \vec{H} = \vec{J}_{free} + \frac{\partial \vec{D}}{\partial t}$$ **(Ampere's Law)**

$$\nabla \cdot \vec{J} = -\partial \rho / \partial t$$ **(The continuity equation)**

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P} = \varepsilon_0 \left(1 + \chi_e\right) \vec{E} = \varepsilon \vec{E}$$ **ε : permittivity**

$$\vec{B} = \mu_0 \left(\vec{H} + \vec{M}\right) = \mu_0 \left(1 + \chi_m\right) \vec{H} = \mu \vec{H}$$ **μ : permeability**

Integral Form

$$\oiint_S \vec{D}\cdot d\vec{S} = \iiint_V \rho_{free} dV$$ (Gauss's Law)

$$\oint_C \vec{E}\cdot d\vec{l} = -\iint_S \frac{\partial \vec{B}}{\partial t}\cdot d\vec{S}$$ (Faraday's Law)

$$\oiint_S \vec{B}\cdot d\vec{S} = 0$$ (Magnetic Gauss's Law)

$$\oint_C \vec{H}\cdot d\vec{l} = \iint_S \left(\vec{J}_{free} + \frac{\partial \vec{D}}{\partial t} \right)\cdot d\vec{S}$$ (Ampere's Law)

$$\nabla \cdot \vec{J} = -\partial \rho / \partial t$$ (The continuity equation)

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P} = \varepsilon_0 \left(1+\chi_e\right)\vec{E} = \varepsilon \vec{E}$$ ε : permittivity

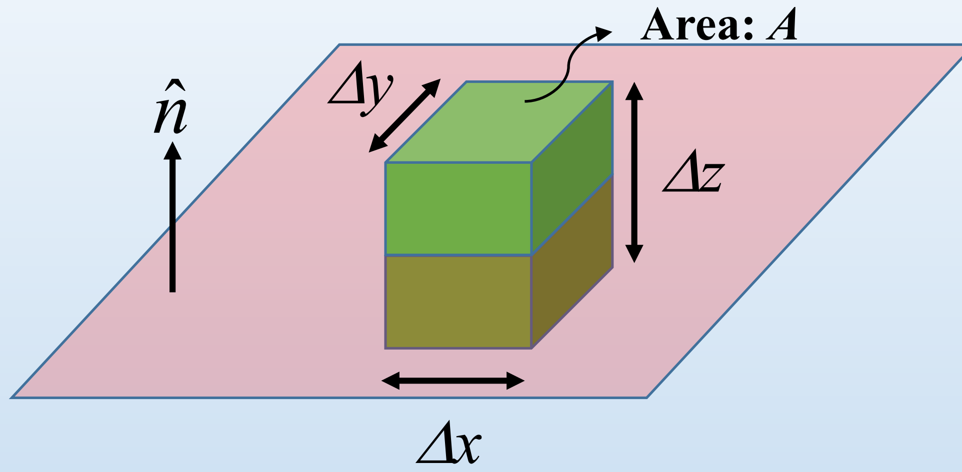
$$\vec{B} = \mu_0 \left(\vec{H} + \vec{M}\right) = \mu_0 \left(1+\chi_m\right)\vec{H} = \mu \vec{H}$$ μ : permeability

Electric Boundary Conditions

$$\oiint_S \vec{D} \cdot d\vec{S} = \iiint_V \rho_{free} dV$$

$$\left(D_{1\perp} - D_{2\perp}\right) A = \rho_s A$$

$$\boxed{\hat{n} \cdot \left(\vec{D}_1 - \vec{D}_2\right) = \rho_s}$$

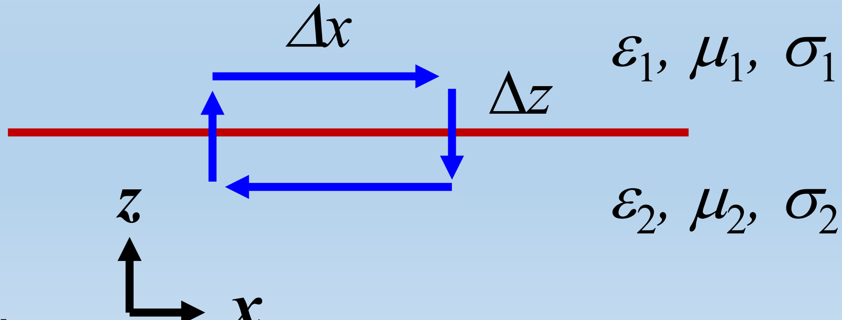


ρ_s **(surface charge density [C/m²])**

$$\iint_S \left(\nabla \times \vec{E}\right) \cdot d\vec{S} = \oint_C \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \iint_S \vec{B} \cdot d\vec{S}$$

$$\vec{E}_1 \cdot \hat{x}\Delta x - \vec{E}_2 \cdot \hat{x}\Delta x = 0$$

$$\boxed{\hat{n} \times \left(\vec{E}_1 - \vec{E}_2\right) = 0}$$



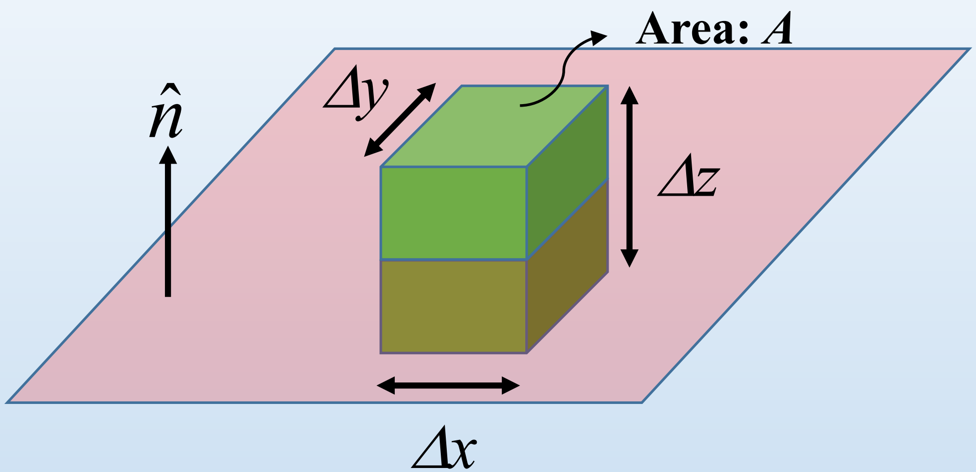
$\hat{n}$: **Points from region 2 to region 1**

Magnetic Boundary Conditions

$$\oiint_S \vec{B}\cdot d\vec{S} = 0$$

$$\left(B_{1\perp} - B_{2\perp}\right)A = 0$$

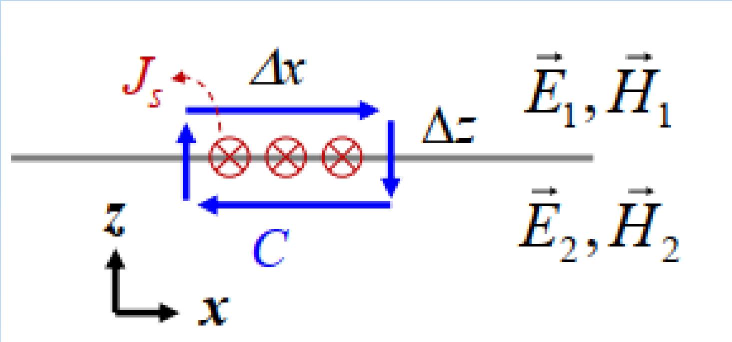
$$\boxed{\hat{n}\cdot\left(\vec{B}_1 - \vec{B}_2\right) = 0}$$



$$\iint_S \left(\nabla\times\vec{H}\right)\cdot d\vec{S} = \oint_C \vec{H}\cdot d\vec{l} = \iint_S \left(\vec{J}_{free} + \frac{\partial\vec{D}}{\partial t}\right)\cdot d\vec{S}$$

$$\vec{H}_1\cdot\hat{x}\Delta x - \vec{H}_2\cdot\hat{x}\Delta x = \vec{J}_{free}\cdot\left(\hat{y}\right)\Delta x\Delta z$$

$$\boxed{\hat{n}\times\left(\vec{H}_1 - \vec{H}_2\right) = \vec{J}_s}$$



$\hat{n}$: **Points from region 2 to region 1**

General Boundary Conditions

(Electric)

$$\hat{n} \cdot \left(\overrightarrow{D}_1 - \overrightarrow{D}_2 \right) = \rho_s$$

$$\hat{n} \times \left(\overrightarrow{E}_1 - \overrightarrow{E}_2 \right) = 0$$

(Magnetic)

$$\hat{n} \cdot \left(\overrightarrow{B}_1 - \overrightarrow{B}_2 \right) = 0$$

$$\hat{n} \times \left(\overrightarrow{H}_1 - \overrightarrow{H}_2 \right) = \overrightarrow{J}_s$$

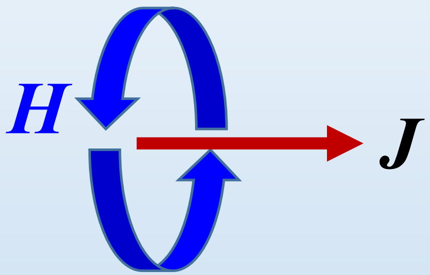
ρ_s **(surface charge density [C/m^2])**

$\overrightarrow{J}_s$ **(surface current density [A/m])**

$\hat{n}$ **(Points from region 2 to region 1)**

Perfect Electric Conductors

Electric Fields:



If $\sigma \to \infty$ and $\vec{E} \neq 0$

Then $\vec{J} = \sigma \vec{E} \to \infty$

Then $\vec{H} \to \infty$

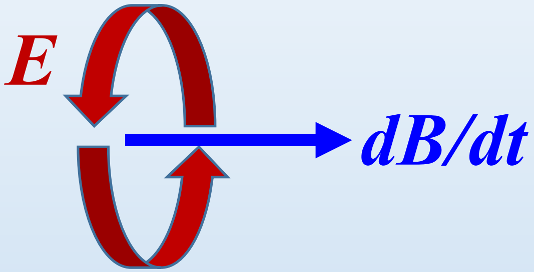
Then $u_B = \dfrac{\mu H^2}{2} \to \infty$ **Impossible!**

Therefore: (1) E = 0 inside perfect electric conductors
(2) ρ = 0 inside perfect electric conductors

$$\oiint_S \varepsilon_0 \vec{E} \cdot d\vec{S} = \iiint_V \rho_{total} dV$$

Perfect Electric Conductors

Magnetic Fields:



$\vec{E} = 0$ **and** $\nabla \times \vec{E} = -jw\vec{B}$

Then $\mathbf{B} = 0$

Therefore: H = 0 inside perfect electric conductors

Boundary Conditions in PEC

Inside PEC:

$$\vec{D}_2 = \vec{E}_2 = \vec{B}_2 = \vec{H}_2 = 0$$

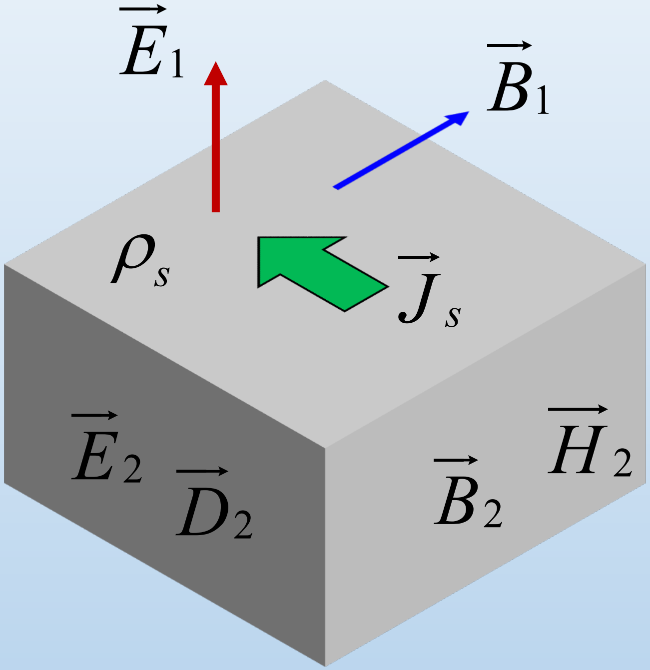
At the boundary:

$$\hat{n} \cdot \vec{D}_1 = \rho_s$$

$$\hat{n} \cdot \vec{B}_1 = 0$$

$$\hat{n} \times \vec{H}_1 = \vec{J}_s$$

$$\hat{n} \times \vec{E}_1 = 0$$



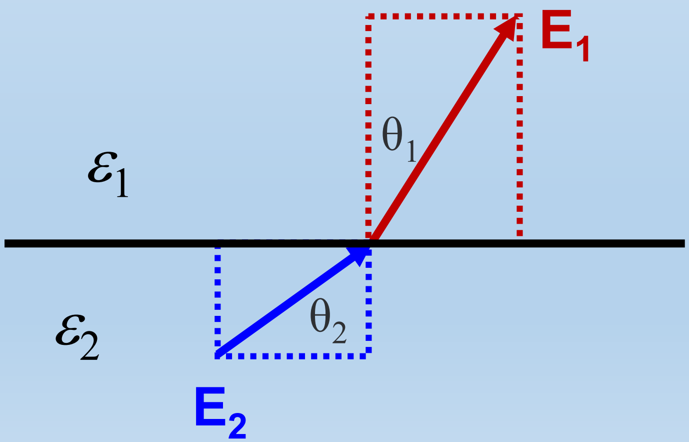
B is parallel to perfect conductors
E is perpendicular to perfect conductors

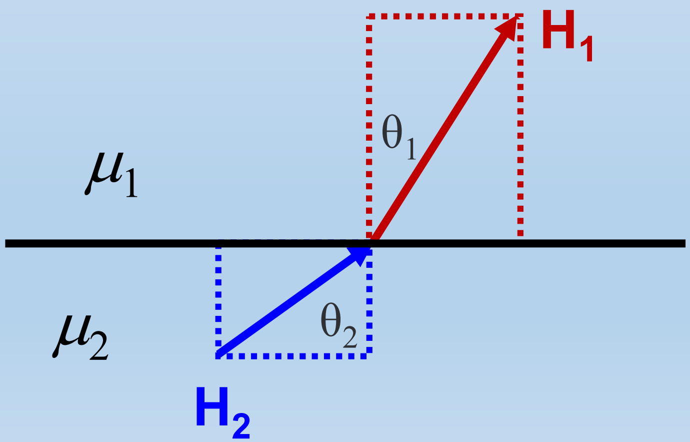
Boundary Conditions in Ideal Dielectrics

No free charge and current

$$\hat{n}\cdot\left(\vec{D}_1-\vec{D}_2\right)=0 \qquad \hat{n}\cdot\left(\vec{B}_1-\vec{B}_2\right)=0$$

$$\hat{n}\times\left(\vec{E}_1-\vec{E}_2\right)=0 \qquad \hat{n}\times\left(\vec{H}_1-\vec{H}_2\right)=0$$



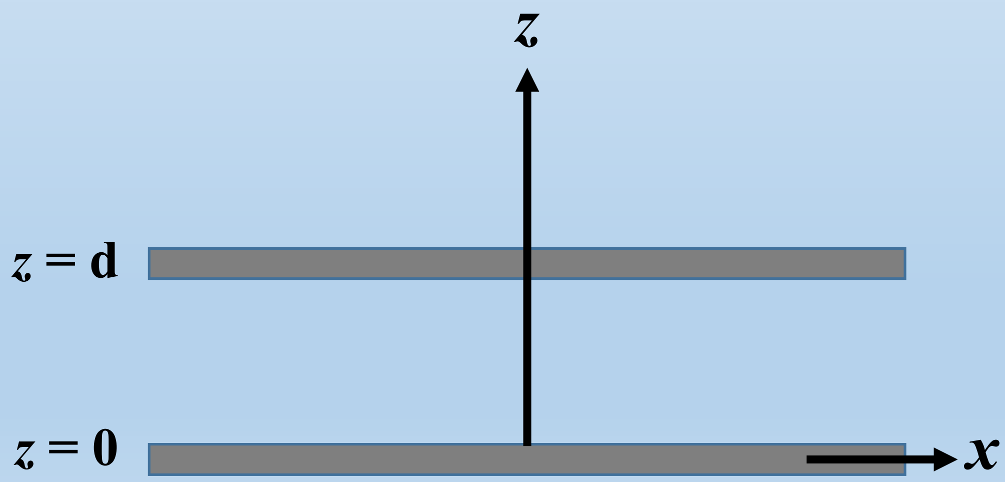


Example

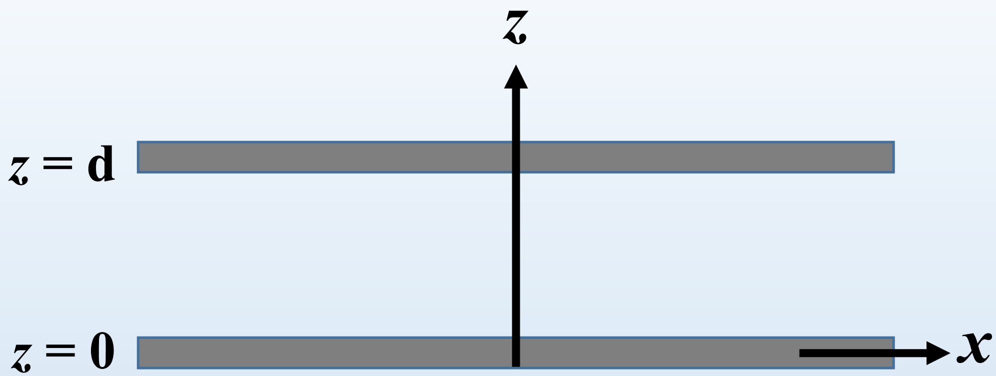
Two infinite perfect electric conductors are placed at z = 0 and z = d, with the electric field between them:

$$\vec{E}(x,y,t) = \hat{y}E_0 \sin\left(\frac{\pi z}{d}\right)\cos\left(k_x x - \omega t\right)\ \text{V/m}$$

(a) What is the magnetic field H?
(b) What is surface current density J_s?
(c) What is the surface charge density ρ_s?



Solution



(a) $$\vec{E}(x,y,t) = \hat{y}E_0 \sin\left(\frac{\pi z}{d}\right)\cos(k_x x - \omega t) \text{ [V/m]}$$

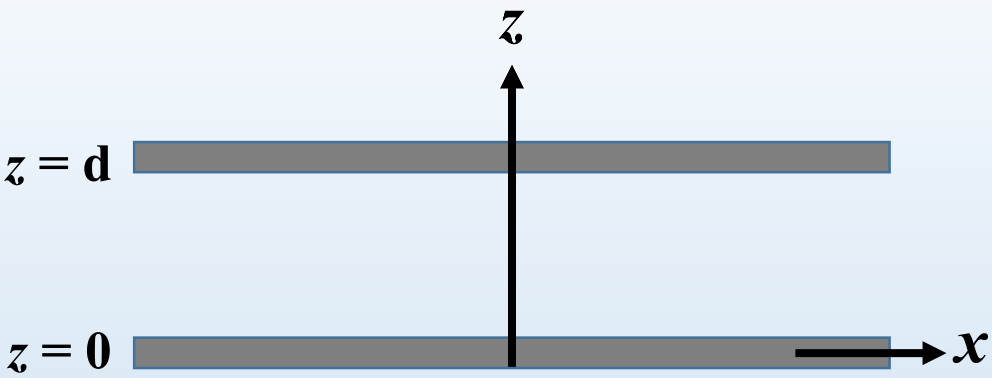
$$\frac{\partial \vec{H}}{\partial t} = -\frac{1}{\mu_0}\nabla\times\vec{E} = -\frac{1}{\mu_0}\left(\hat{z}\frac{\partial}{\partial x} - \hat{x}\frac{\partial}{\partial z}\right)E_y(x,z,t)$$

$$= \hat{z}\frac{E_0 k_x}{\mu_0}\sin\left(\frac{\pi z}{d}\right)\sin(k_x x - \omega t) + \hat{x}\frac{E_0 \pi}{\mu_0 d}\cos\left(\frac{\pi z}{d}\right)\cos(k_x x - \omega t)$$

$$\vec{H} = \int \frac{\partial \vec{H}}{\partial t}dt = \hat{z}\frac{k_x E_0}{\omega\mu_0}\sin\left(\frac{\pi z}{d}\right)\cos(k_x x - \omega t)$$

$$-\hat{x}\frac{\pi E_0}{\omega\mu_0 d}\cos\left(\frac{\pi z}{d}\right)\sin(k_x x - \omega t)$$

Solution



$$\vec{H} = \hat{z}\frac{k_x E_0}{\omega\mu_0}\sin\left(\frac{\pi z}{d}\right)\cos\left(k_x x - \omega t\right) - \hat{x}\frac{\pi E_0}{\omega\mu_0 d}\cos\left(\frac{\pi z}{d}\right)\sin\left(k_x x - \omega t\right)$$

(b, c)

At z = 0:

$$\vec{J}_s = \hat{z}\times\vec{H}\left(z=0\right) = -\hat{y}\frac{\pi E_0}{\omega\mu_0 d}\sin\left(k_x x - \omega t\right)\ \text{[A/m]}$$

$$\rho_s = \hat{z}\cdot\vec{D}\left(z=0\right) = 0$$

At z = d:

$$\vec{J}_s = -\hat{z}\times\vec{H}\left(z=d\right) = -\hat{y}\frac{\pi E_0}{\omega\mu_0 d}\sin\left(k_x x - \omega t\right)\ \text{[A/m]}$$

$$\rho_s = -\hat{z}\cdot\vec{D}(z=d) = 0$$

Scalar and Vector potentials in Static Fields

$$\nabla \cdot \vec{D} = \rho_{free} \qquad \boxed{\nabla \cdot \vec{B} = 0}$$

$$\boxed{\nabla \times \vec{E} = 0} \qquad \nabla \times \vec{H} = \vec{J}_{free}$$

Let $\vec{E} = -\nabla \underline{\varphi}$

(electric potential)

$$\nabla \cdot \left(-\varepsilon \nabla \varphi \right) = \rho_{free}$$

$$\boxed{\nabla^2 \varphi = -\rho_{free} / \varepsilon}$$

Let $\vec{B} = \nabla \times \underline{\vec{A}}$ **and** $\nabla \cdot \vec{A} = 0$

(magnetic vector potential)

$$\nabla \times \left(\nabla \times \vec{A} \right) = \mu \vec{J}_{free}$$

$$\boxed{\nabla^2 \vec{A} = -\mu \vec{J}_{free}}$$

The problem turns into the solution of two Poisson's Equations

Boundary Conditions and Uniqueness Theorems

$$\boxed{\nabla^2 \varphi = -\rho_{free} / \varepsilon} \qquad \boxed{\nabla^2 \vec{A} = -\mu \vec{J}_{free}}$$

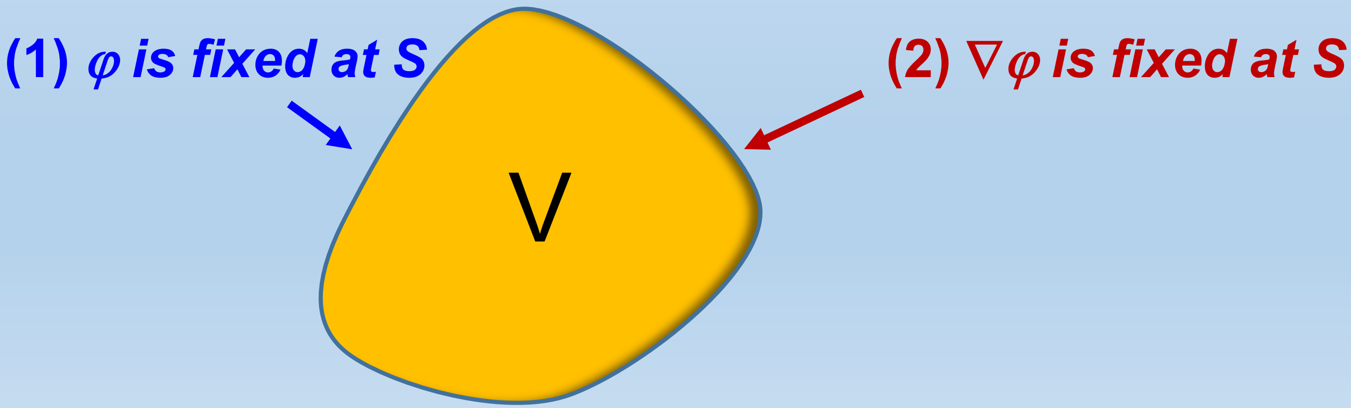
Poisson's Equations

Unique solution exists when the one of the following boundary equations is satisfied.

(1) Dirichlet boundary condition
- **φ is well defined at the boundary S**

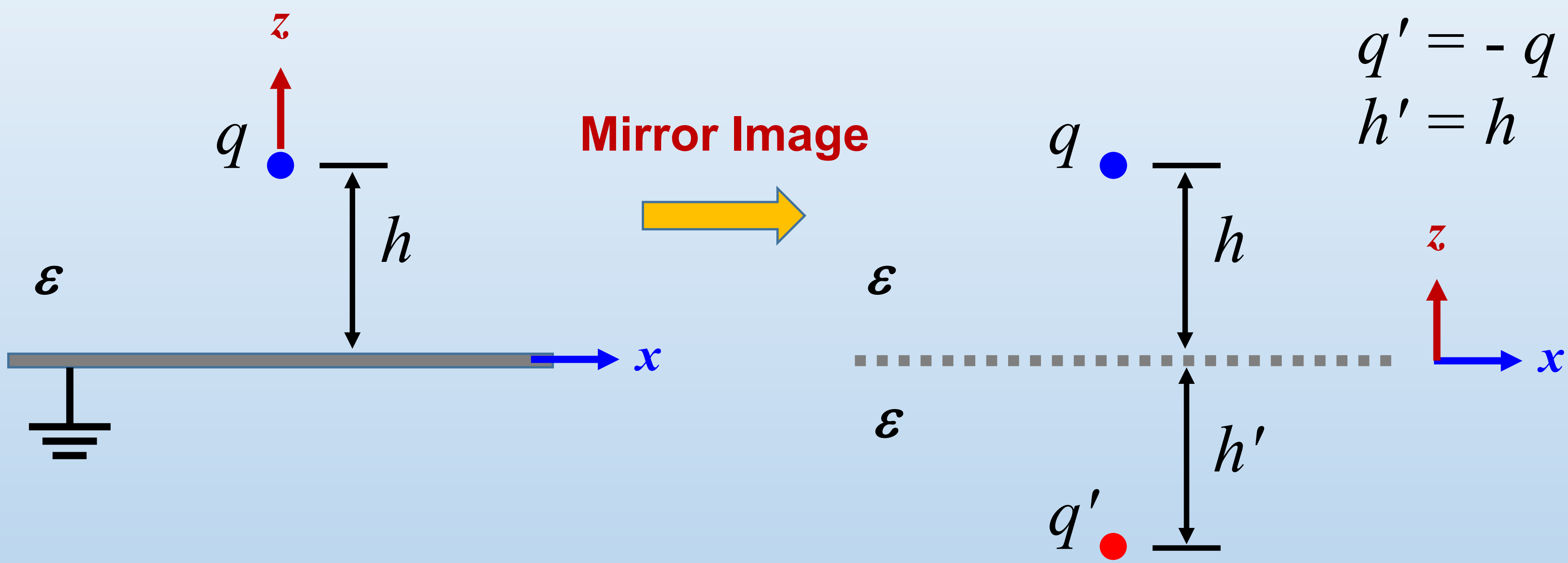
(2) Neumann boundary condition
- **$\nabla\varphi$ is well defined at the boundary S**

(3) Mixed boundary conditions
- **mix of 1 and 2**



The method of images

(a) A point charge is held a distance h above an infinite grounded conducting plane



Solve the Poisson's Equation with two boundary conditions:

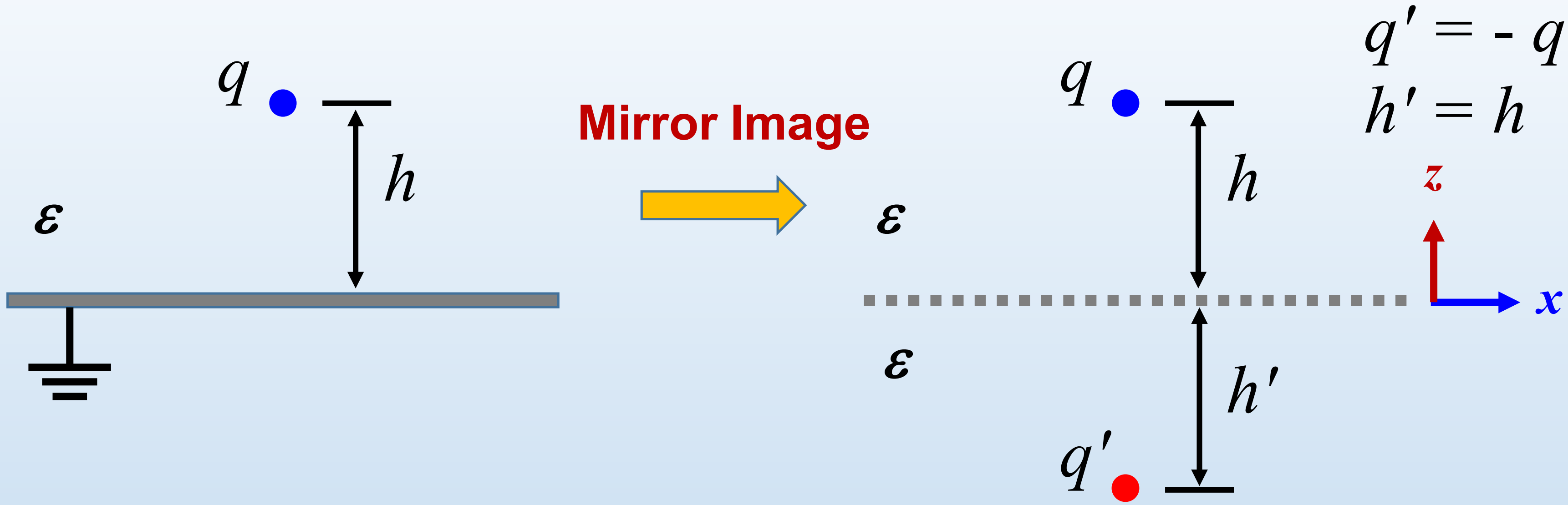
(1) φ = 0 at z = 0

(2) φ -> 0 far from the charge

Uniqueness Theorems

The region of interest

$$\varphi(x,y,z)=\frac{q}{4\pi\varepsilon}\left[\frac{1}{\sqrt{x^2+y^2+(z-h)^2}}-\frac{1}{\sqrt{x^2+y^2+(z+h)^2}}\right],\quad z\geq 0$$



Induced Surface Charge Density at z = 0 :

$$\rho_s = \hat{n}\cdot\left(\vec{D}_1 - \vec{D}_2\right) = -\varepsilon\frac{\partial\varphi}{\partial z}\Big|_{z=0} = -\frac{qh}{2\pi\left(x^2+y^2+h^2\right)^{3/2}}$$

Induced Total Charge at z = 0:

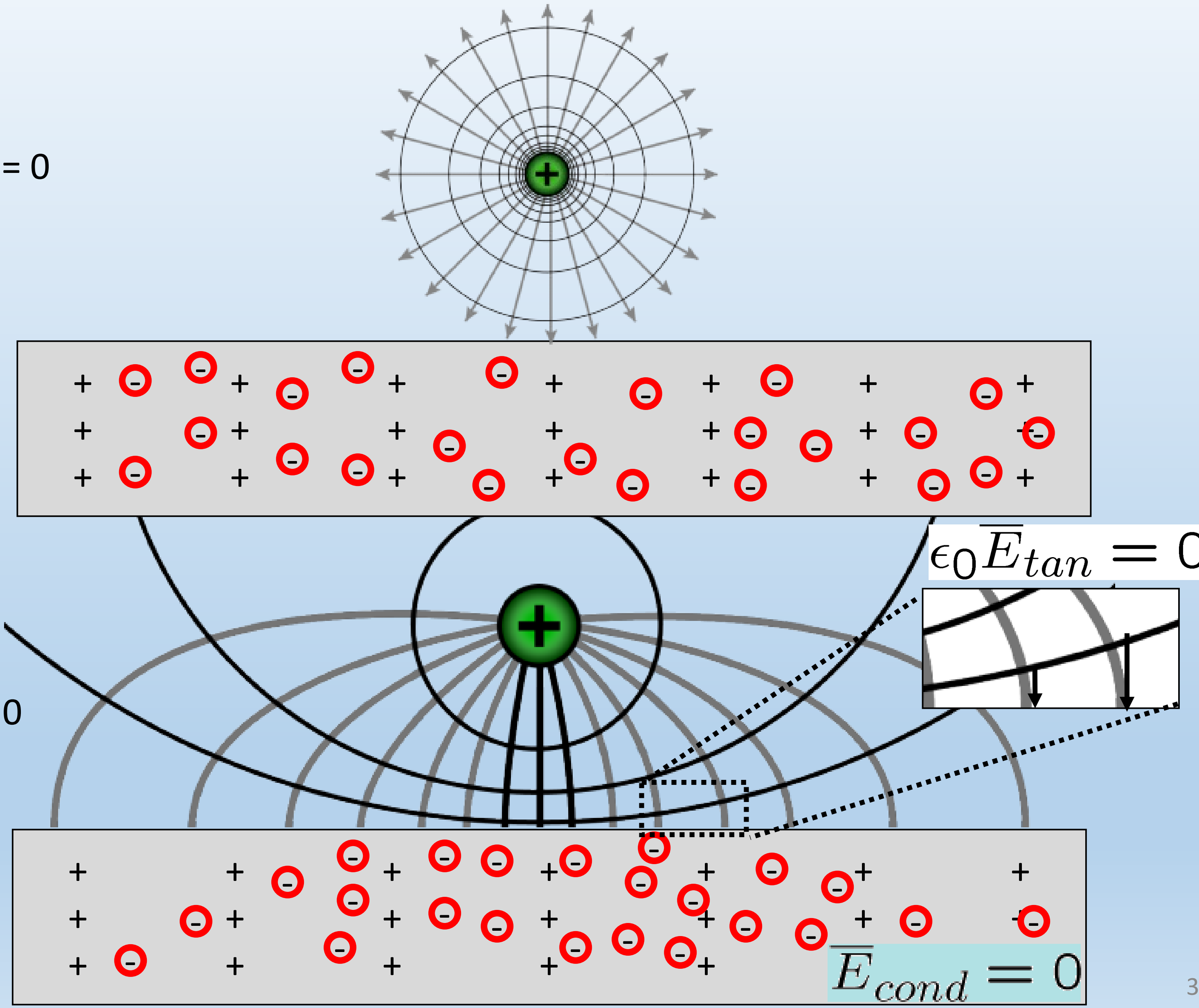
$$q_{in} = \int_S \rho_s dS = -\frac{qh}{2\pi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\frac{dxdy}{\left(x^2+y^2+h^2\right)^{3/2}} = -\frac{qh}{2\pi}\int_0^{2\pi}\int_0^{+\infty}\frac{\rho d\rho d\phi}{\left(\rho^2+h^2\right)^{3/2}} = -q$$

The total charge induced on the plane is −*q*, as expected.

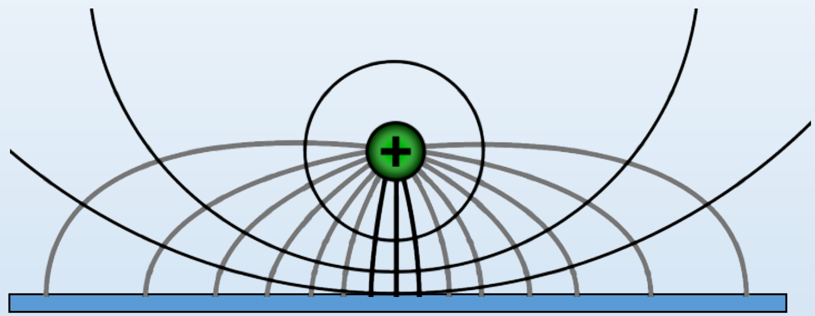
Point Charges Near Perfect Conductors

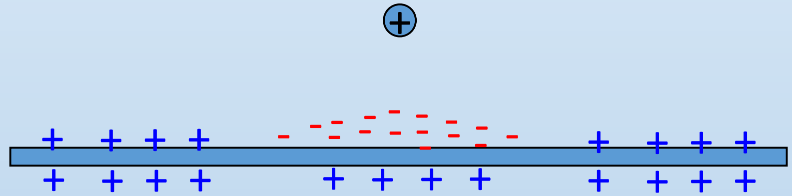
Time t = 0

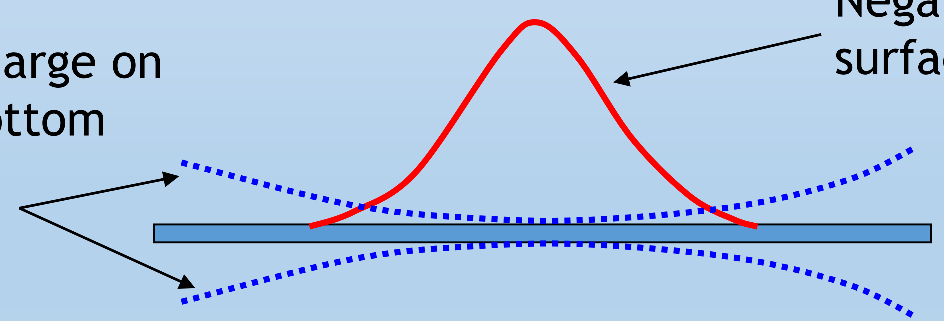
Time t >> 0



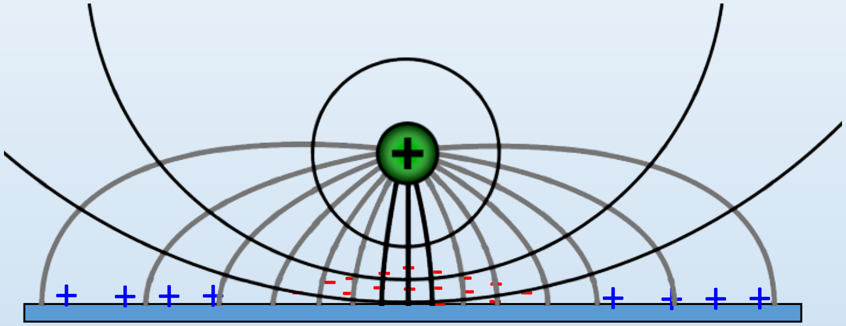
Point Charges Near Perfect Conductors

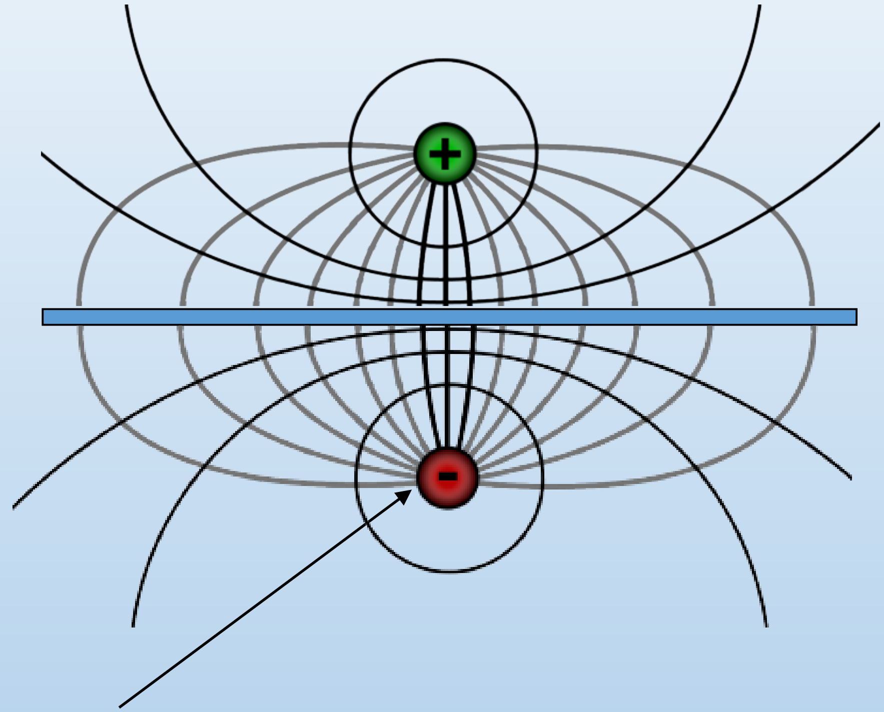




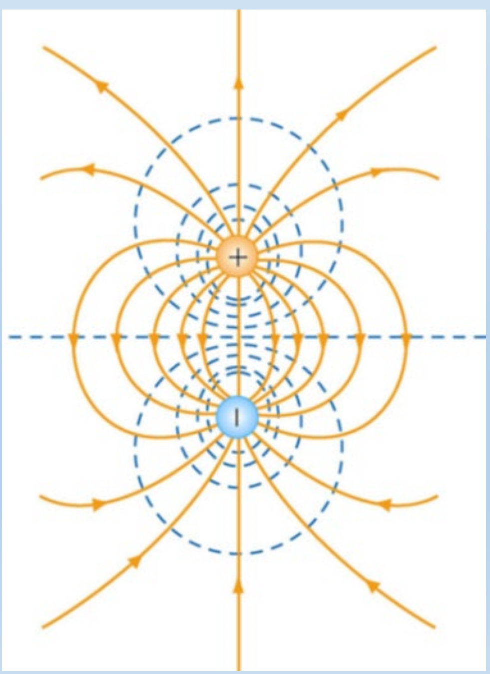


Uniqueness and Equivalent Image Charges





Equivalent Image Charge



Electric dipole

Other image problems near perfect conductors

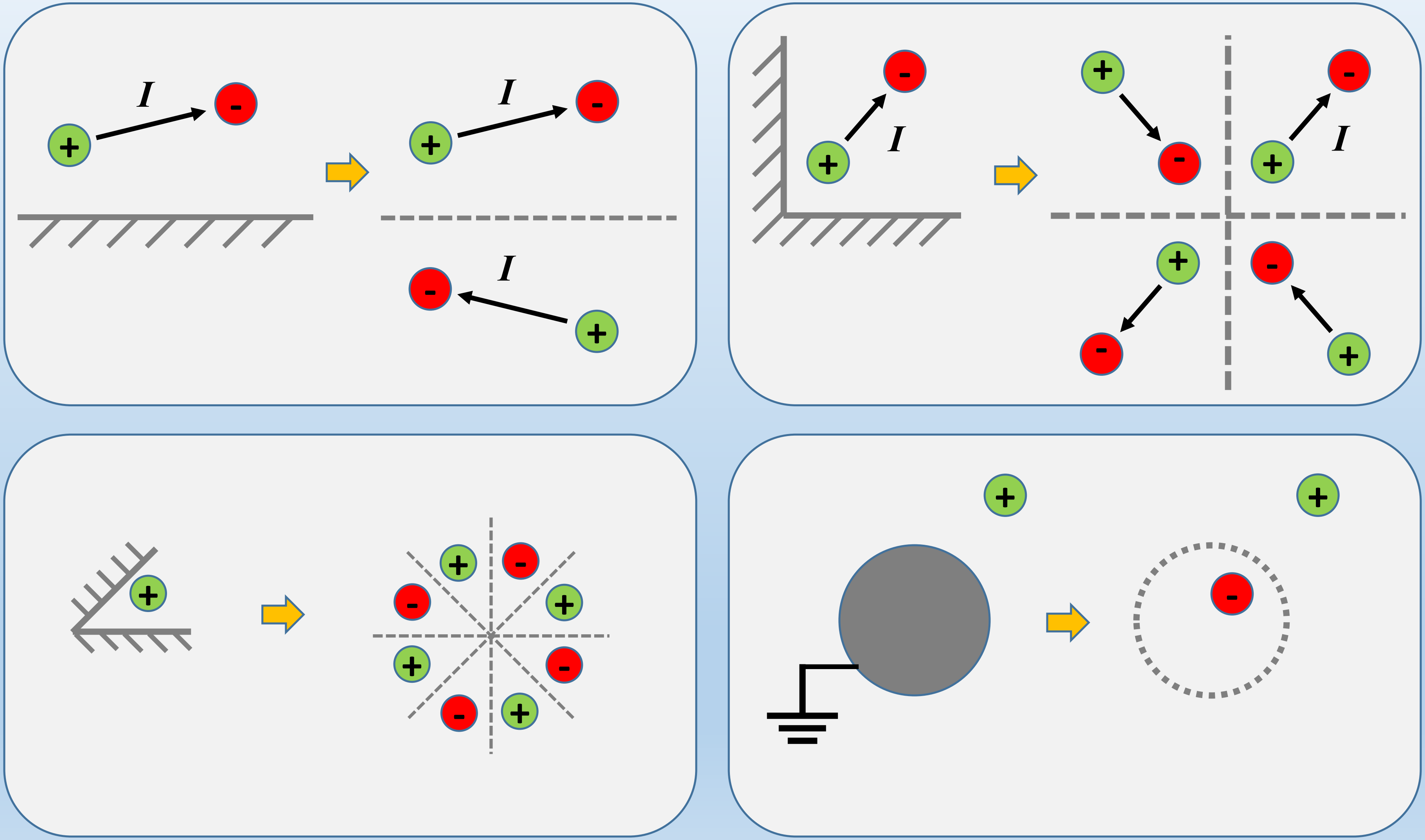
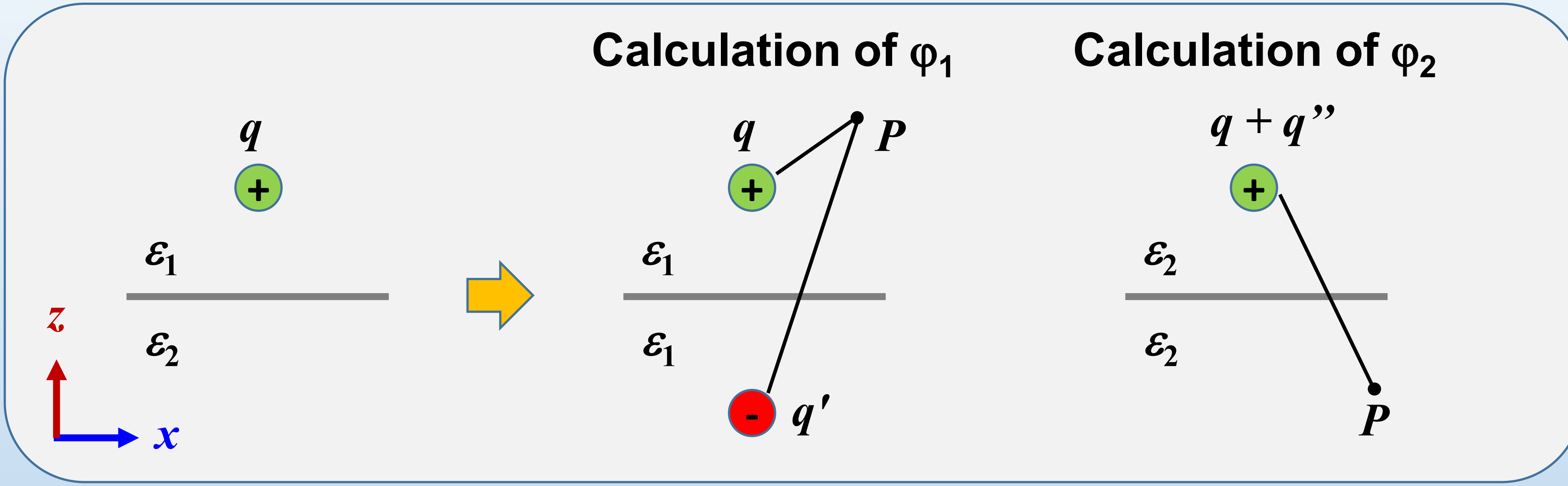


Image problems near dielectrics



$$\varphi_1(x,y,z)=\frac{1}{4\pi\varepsilon_1}\left[\frac{q}{\sqrt{x^2+y^2+(z-h)^2}}+\frac{q'}{\sqrt{x^2+y^2+(z+h)^2}}\right],\quad z\geq 0$$

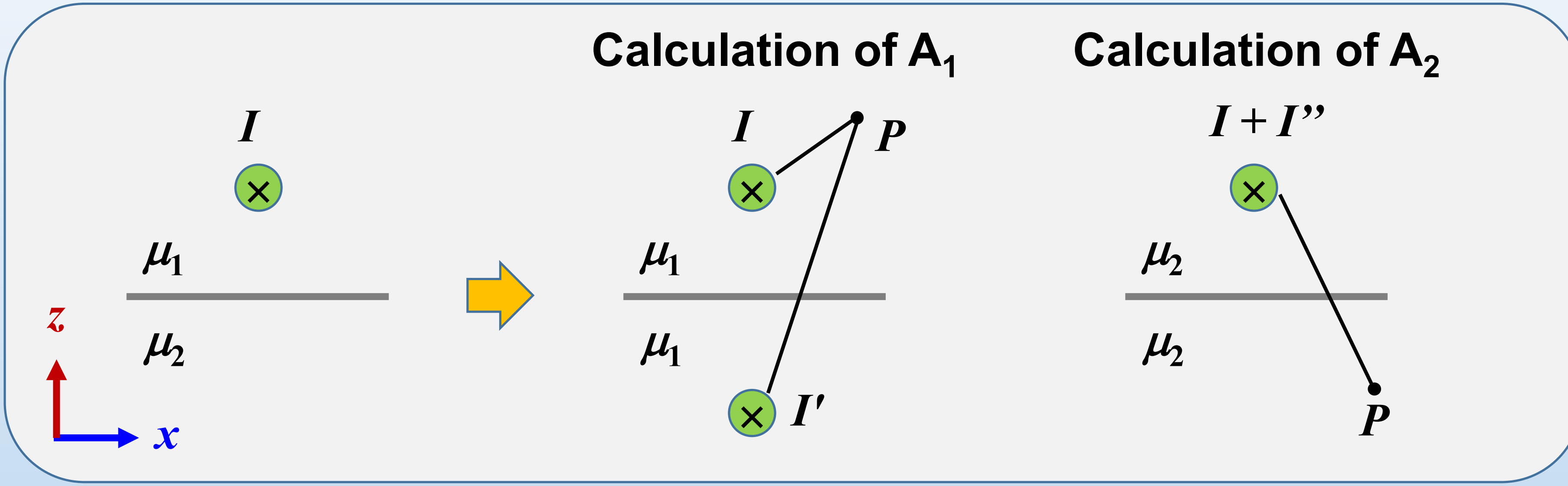
$$\varphi_2(x,y,z)=\frac{1}{4\pi\varepsilon_2}\left[\frac{q+q''}{\sqrt{x^2+y^2+(z-h)^2}}\right],\quad z\leq 0$$

$$q'=\frac{\varepsilon_1-\varepsilon_2}{\varepsilon_1+\varepsilon_2}q$$

$$q''=-\frac{\varepsilon_1-\varepsilon_2}{\varepsilon_1+\varepsilon_2}q$$

Boundary Conditions at z = 0: $\varepsilon_1\frac{\partial\varphi_1}{\partial z}=\varepsilon_2\frac{\partial\varphi_2}{\partial z}$ $\quad\varphi_1=\varphi_2$

Image problems near magnets



$$A_{1y}(x,y,z) = \frac{\mu_1 I}{2\pi} \ln \frac{1}{\sqrt{x^2 + (z-h)^2}} + \frac{\mu_1 I'}{2\pi} \ln \frac{1}{\sqrt{x^2 + (z+h)^2}}, \quad z \geq 0$$

$$A_{2y}(x,y,z) = \frac{\mu_2 (I + I'')}{2\pi} \ln \frac{1}{\sqrt{x^2 + (z-h)^2}}, \quad z \leq 0$$

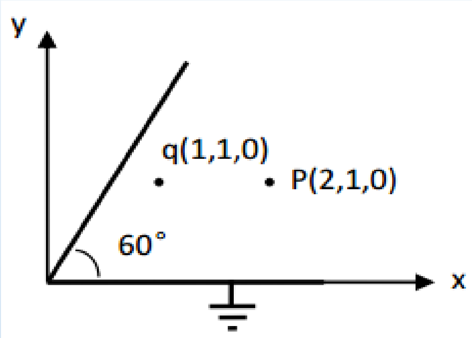
$$I' = \frac{\mu_2 - \mu_1}{\mu_2 + \mu_1} I$$

$$I'' = -\frac{\mu_2 - \mu_1}{\mu_2 + \mu_1} I$$

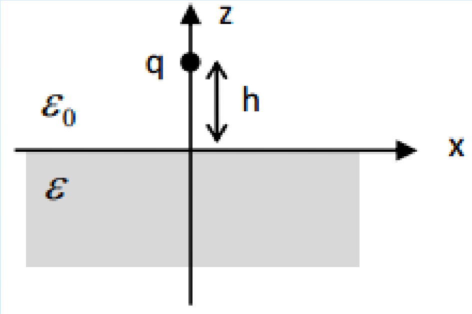
Boundary Conditions at z = 0: $\frac{1}{\mu_1}\frac{\partial A_1}{\partial z} = \frac{1}{\mu_2}\frac{\partial A_2}{\partial z}$ $\quad A_1 = A_2$

习题

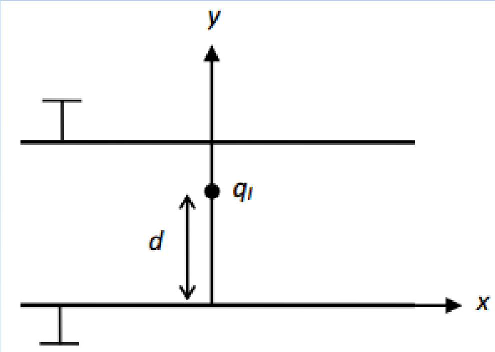
习题4.1 一个点电荷 q 放在 $60°$ 的接地导体角域内的点（1,1,0）处，如图所示。试求：（1）所有镜像电荷的位置和大小；
（2）点 P（2,1,0）处的电位。



习题 4.2：如图所示，在 $z < 0$ 的下半空间是介电常数 ε 的电介质，上半空间为空气，距离介质平面 h 处有一点电荷 q。
试求：（1）$z > 0$ 和 $z < 0$ 的两个半空间内的电位分布；
（2）电介质表面上的极化电荷密度，
并证明表面上的极化电荷总量等于镜像电荷 q'。



习题 4.3：两块无限大接地导体板，两板之间有一与 z 轴平行的线电荷 q_l，其位置为（0，d），求板间的电位分布。



实验作业

通过MATLAB、 COMSOL等软件来仿真课程相关的实例。

第四章介质与边界:

点电荷与金属感应，产生的电场电势和感应电荷;

不同介质板在电容中，对电场电势和电荷的作用。