Outline

- **Chapter 1.1** Periodic Array of Atoms (原子的周期性排列)
- **Chapter 1.2** Symmetry of Crystals (晶体的对称性)
- **Chapter 1.3** Typical Crystal Structures (典型晶体结构)
- **Chapter 1.4** Reciprocal Lattice (倒易点阵)

Objectives

- To understand the concepts of **reciprocal lattice**;
- To understand the connection between a reciprocal lattice and its direct lattice;
- To learn the typical reciprocal lattices.

Reciprocal Lattice (倒格子)

- **Reciprocal Lattice (倒格子)**

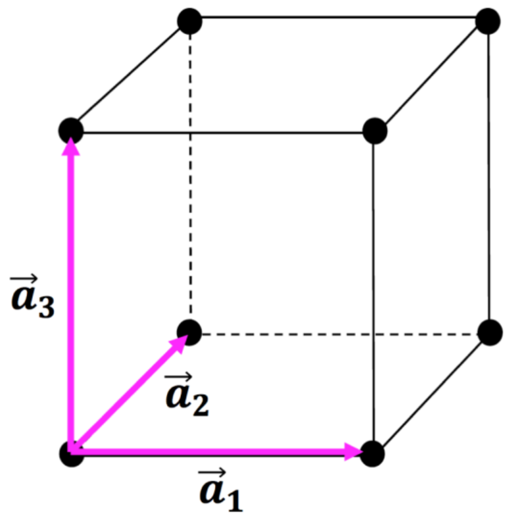
❖ Assuming that we define **two sets of lattices**:

Lattice 1:

Basis Vectors (基矢): $\vec{\boldsymbol{a}}_1$, $\vec{\boldsymbol{a}}_2$, and $\vec{\boldsymbol{a}}_3$

Lattice Vectors (格矢): $\vec{\boldsymbol{R}} = \boldsymbol{n}_1\vec{\boldsymbol{a}}_1 + \boldsymbol{n}_2\vec{\boldsymbol{a}}_2 + \boldsymbol{n}_3\vec{\boldsymbol{a}}_3$

Cell Volume (晶胞体积): $\boldsymbol{\Omega} = \vec{\boldsymbol{a}}_1 \cdot (\vec{\boldsymbol{a}}_2 \times \vec{\boldsymbol{a}}_3)$



➢ Reciprocal Lattice (倒格子)

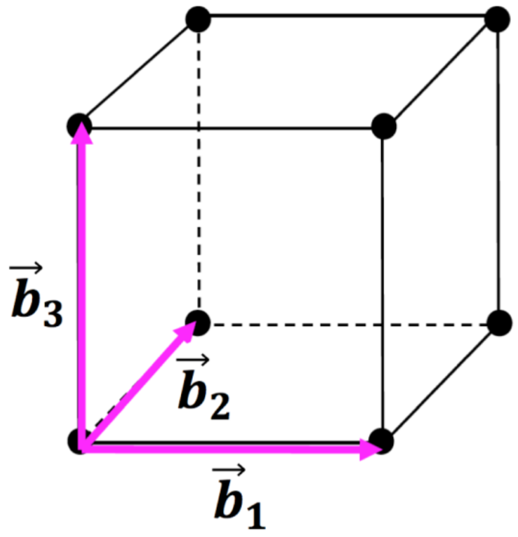
❖ Assuming that we define **two sets of lattices**:

Lattice 2:

Basis Vectors (基矢): $\vec{b}_1$, $\vec{b}_2$, and $\vec{b}_3$

Lattice Vectors (格矢): $\vec{G} = h\vec{b}_1 + k\vec{b}_2 + l\vec{b}_3$

Cell Volume (晶胞体积): $\Omega^* = \vec{b}_1 \cdot \left(\vec{b}_2 \times \vec{b}_3\right)$

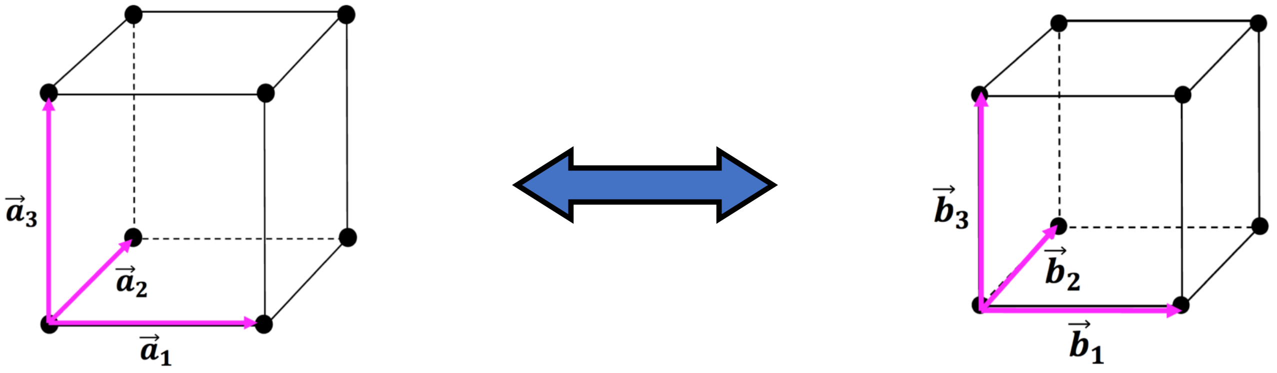


Chapter 1.4: Reciprocal Lattice (倒易点阵)

➢ Reciprocal Lattice (倒格子)

❖ The two sets of lattices defined above are **reciprocal to each other (互为正倒格子)** if the following is satisfied:

$$\vec{a}_i \cdot \vec{b}_j = 2\pi\delta_{ij} = \begin{cases} 2\pi, & i = j \\ 0, & i \neq j \end{cases} \qquad i, j = 1, 2, 3$$



Chapter 1.4: Reciprocal Lattice (倒易点阵)

➢ Reciprocal Lattice (倒格子)

❖ If the lattice defined by $\vec{R}$ is called **direct lattice (正格子)**, the one defined by $\vec{G}$ is called **reciprocal lattice (倒格子，或“倒易点阵”)**, or *vice versa*.

❖ The space in which the **direct lattice** is defined is called **real space (实空间)**.

The space in which the **reciprocal lattice** is defined is called **reciprocal space (倒空间)**.

❖ The lattice vector $\vec{G} = h\vec{b}_1 + k\vec{b}_2 + l\vec{b}_3$ defining the reciprocal lattice is called **reciprocal lattice vector (倒格矢)**.

Chapter 1.4: Reciprocal Lattice (倒易点阵)

Reciprocal Lattice (倒格子)

- Alternatively, the **basis vectors of reciprocal lattice** can be defined as:

$$\vec{b}_1 = 2\pi \frac{\vec{a}_2 \times \vec{a}_3}{\vec{a}_1 \cdot (\vec{a}_2 \times \vec{a}_3)} = 2\pi \frac{\vec{a}_2 \times \vec{a}_3}{\Omega}$$

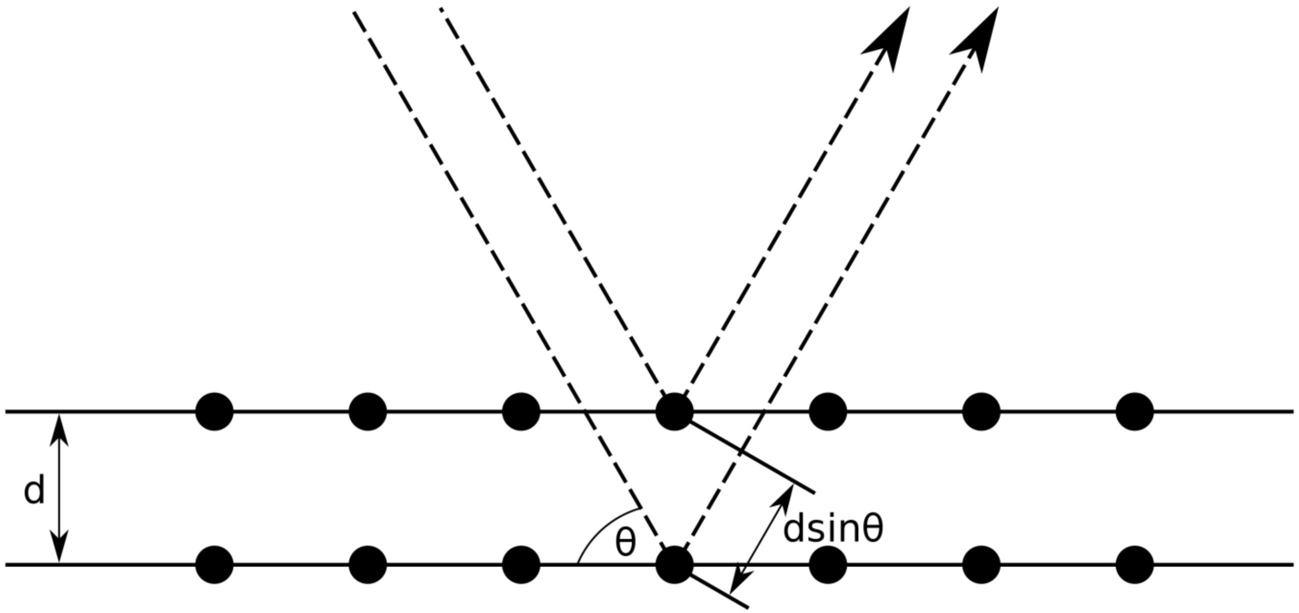
$$\vec{b}_2 = 2\pi \frac{\vec{a}_3 \times \vec{a}_1}{\vec{a}_1 \cdot (\vec{a}_2 \times \vec{a}_3)} = 2\pi \frac{\vec{a}_3 \times \vec{a}_1}{\Omega}$$

$$\vec{b}_3 = 2\pi \frac{\vec{a}_1 \times \vec{a}_2}{\vec{a}_1 \cdot (\vec{a}_2 \times \vec{a}_3)} = 2\pi \frac{\vec{a}_1 \times \vec{a}_2}{\Omega}$$

Chapter 1.4: Reciprocal Lattice (倒易点阵)

➢ **Reciprocal Lattice and the Bragg Law (倒格子与布拉格定律)**

❖ The **Bragg law** is the condition for **Bragg diffraction** (布拉格衍射):



Bragg Diffraction

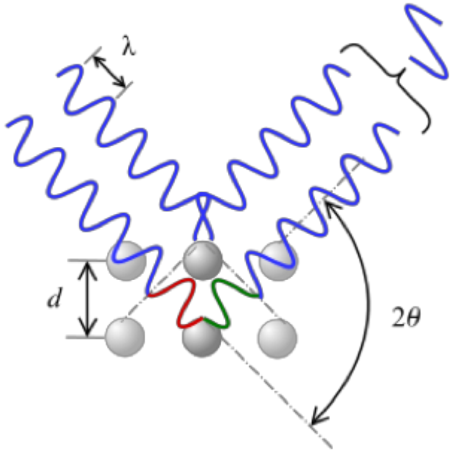
Chapter 1.4: Reciprocal Lattice (倒易点阵)

➢ Reciprocal Lattice and the Bragg Law (倒格子与布拉格定律)

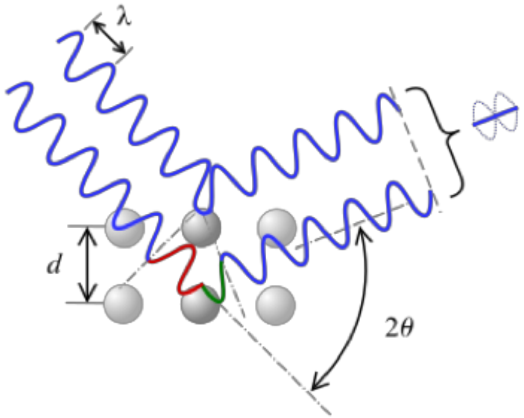
❖ The **Bragg law** is the condition for **Bragg diffraction** (布拉格衍射):

$$\boxed{2d \sin\theta = n\lambda}$$

(n is an integer and $\lambda \leq 2d$)



Constructive Interference
(相长干涉)

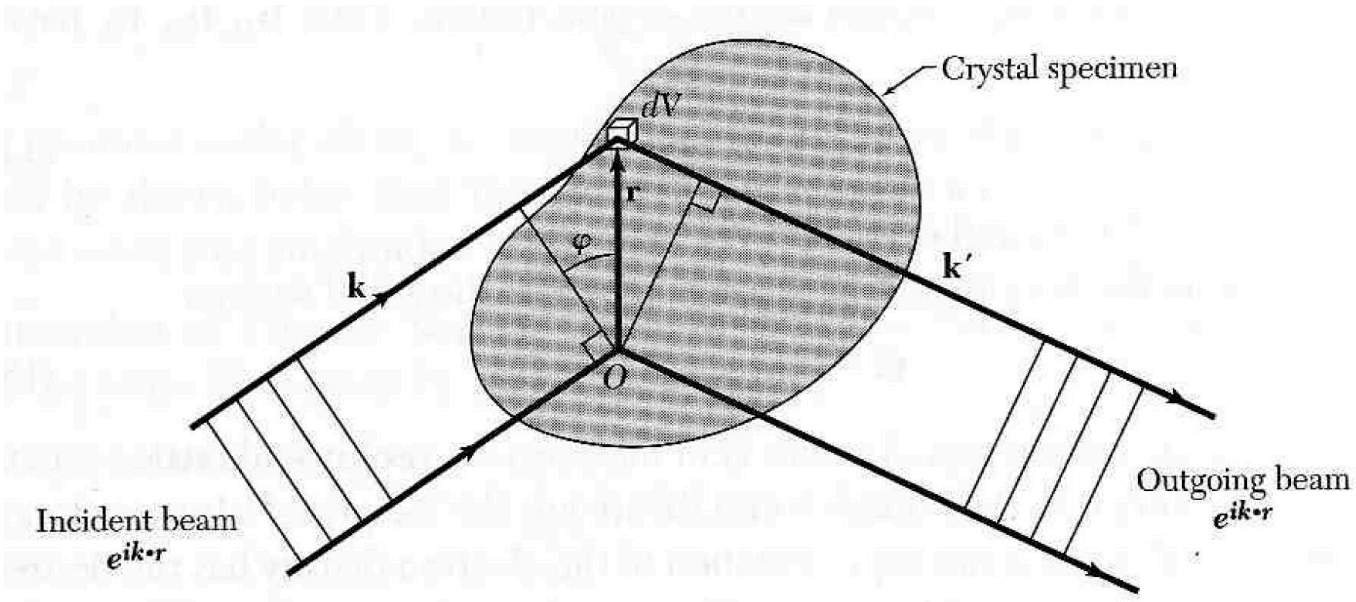


Destructive Interference
(相消干涉)

Chapter 1.4: Reciprocal Lattice (倒易点阵)

- **Reciprocal Lattice and the Bragg Law (倒格子与布拉格定律)**

  - **Laue condition** for the diffraction **(劳厄衍射条件)**:



$\vec{k}$: wave vector of the incident beam.

$\vec{k'}$: wave vector of the outgoing beam.

Chapter 1.4: Reciprocal Lattice (倒易点阵)

➢ Reciprocal Lattice and the Bragg Law (倒格子与布拉格定律)

❖ **Laue condition** for the diffraction **(劳厄衍射条件)**:

The possible reflection is determined by a set of reciprocal lattice vectors $\vec{\boldsymbol{G}}$ that satisfy:

$$\vec{\boldsymbol{k}}' - \vec{\boldsymbol{k}} = \vec{\boldsymbol{G}} \xrightarrow{k' = k} \boxed{2\vec{\boldsymbol{k}} \cdot \vec{\boldsymbol{G}} = \boldsymbol{G}^2}$$

Chapter 1.4: Reciprocal Lattice (倒易点阵)

➢ **Relationships between Reciprocal and Direct Lattices (倒格子与正格子的关系)**

1) **The basis vectors are orthogonal:**

$$\vec{a}_i \cdot \vec{b}_j = 2\pi\delta_{ij}$$

$$\delta_{ij} = \begin{cases} 2\pi, & i = j \\ 0, & i \neq j \end{cases} \qquad i, j = 1, 2, 3$$

Chapter 1.4: Reciprocal Lattice (倒易点阵)

➢ **Relationships between Reciprocal and Direct Lattices (倒格子与正格子的关系)**

2) **The dot product of the lattice vectors is an integral multiple of 2π:**

$$\vec{\boldsymbol{R}} \cdot \vec{\boldsymbol{G}} = \boldsymbol{2\pi m} \qquad (\boldsymbol{m} \text{ is an integer})$$

Chapter 1.4: Reciprocal Lattice (倒易点阵)

➢ **Relationships between Reciprocal and Direct Lattices (倒格子与正格子的关系)**

3) **The volumes of the primitive cells satisfy:**

$$\boldsymbol{\Omega}^* = \vec{\boldsymbol{b}}_1 \cdot \left(\vec{\boldsymbol{b}}_2 \times \vec{\boldsymbol{b}}_3\right) = \frac{(2\boldsymbol{\pi})^3}{\boldsymbol{\Omega}}$$

➢ Relationships between Reciprocal and Direct Lattices (倒格子与正格子的关系)

4) **The reciprocal lattice vector $\vec{G} = h\vec{b}_1 + k\vec{b}_2 + l\vec{b}_3$ is perpendicular to the crystal planes (hkl) of the direct lattice, and the interplanar spacing (晶面间距) d_{hkl} can be written as:**

$$d_{hkl} = \frac{2\pi}{|\vec{G}|}$$

Conclusion: A lattice vector in the reciprocal lattice corresponds to a family of crystal planes in the direct lattice:

1) The direction of the vector is parallel to the normal direction of the planes;
2) The magnitude of the vector is 2π times of the reciprocal of interplanar spacing.

- **Relationships between Reciprocal and Direct Lattices (倒格子与正格子的关系)**

5) **The direct lattice and the reciprocal lattice are reciprocal to each other, i.e., the reciprocal lattice of a reciprocal lattice is its direct lattice.**

$$\vec{b}_1 = 2\pi\frac{\vec{a}_2\times\vec{a}_3}{\Omega} \qquad \vec{a}_1 = 2\pi\frac{\vec{b}_2\times\vec{b}_3}{\Omega^*}$$

$$\vec{b}_2 = 2\pi\frac{\vec{a}_3\times\vec{a}_1}{\Omega} \quad \Rightarrow \quad \vec{a}_2 = 2\pi\frac{\vec{b}_3\times\vec{b}_1}{\Omega^*}$$

$$\vec{b}_3 = 2\pi\frac{\vec{a}_1\times\vec{a}_2}{\Omega} \qquad \vec{a}_3 = 2\pi\frac{\vec{b}_1\times\vec{b}_2}{\Omega^*}$$

➢ Relationships between Reciprocal and Direct Lattices (倒格子与正格子的关系)

6) **The direct lattice and reciprocal lattice of the same crystal have the same point group symmetry.**

Proof

Assuming $\boldsymbol{\alpha}$ is a symmetry operation of a point group, for a direction lattice vector $\vec{\boldsymbol{R}}$, it is obvious that $\boldsymbol{\alpha}\vec{\boldsymbol{R}}$ and $\boldsymbol{\alpha}^{-1}\vec{\boldsymbol{R}}$ are both direct lattice vectors. Thus:

$$\vec{G}\cdot\vec{R} = 2\pi m \longrightarrow \vec{G}\cdot\alpha\vec{R} = 2\pi m \qquad \vec{G}\cdot\alpha^{-1}\vec{R} = 2\pi m$$

Since $\boldsymbol{\alpha}$ is orthogonal transformation, we obtain:

$$\alpha\vec{G}\cdot\vec{R} = \alpha\vec{G}\cdot\alpha\alpha^{-1}\vec{R} = \vec{G}\cdot\alpha^{-1}\vec{R} = 2\pi m$$

$$\alpha^{-1}\vec{G}\cdot\vec{R} = \alpha^{-1}\vec{G}\cdot\alpha^{-1}\alpha\vec{R} = \vec{G}\cdot\alpha\vec{R} = 2\pi m$$

Therefore, for any symmetry operation $\boldsymbol{\alpha}$ of a point group, $\boldsymbol{\alpha}\vec{\boldsymbol{G}}$ and $\boldsymbol{\alpha}^{-1}\vec{\boldsymbol{G}}$ are also reciprocal lattice vectors.

Chapter 1.4: Reciprocal Lattice (倒易点阵)

➢ **The Physical Meaning of Reciprocal Lattice (倒格子的物理意义)**

❖ The reciprocal lattice is the **Fourier Transform** of the direct lattice.

❖ The direct lattice is the **inverse Fourier Transform** of the reciprocal lattice.

$$F(\vec{r}) = \sum_{\vec{G}_{hkl}} A(\vec{G}_{hkl}) e^{i\vec{G}_{hkl}\cdot\vec{r}}$$

$$A(\vec{G}_{hkl}) = \frac{1}{\Omega}\int F(\vec{r}) e^{-i\vec{G}_{hkl}\cdot\vec{r}} d\vec{r}$$

Chapter 1.4: Reciprocal Lattice (倒易点阵)

➢ The Physical Meaning of Reciprocal Lattice (倒格子的物理意义)

❖ Each crystal structure corresponds to **2 sets of lattices**, i.e., the **direct lattice** (Bravais lattice) and the **reciprocal lattice**.

- The **direct lattice** shows the periodic array of atoms in real space;
- The **reciprocal lattice** shows the periodicity of physical properties in reciprocal space.

Examples of Reciprocal Lattice (倒格子实例)

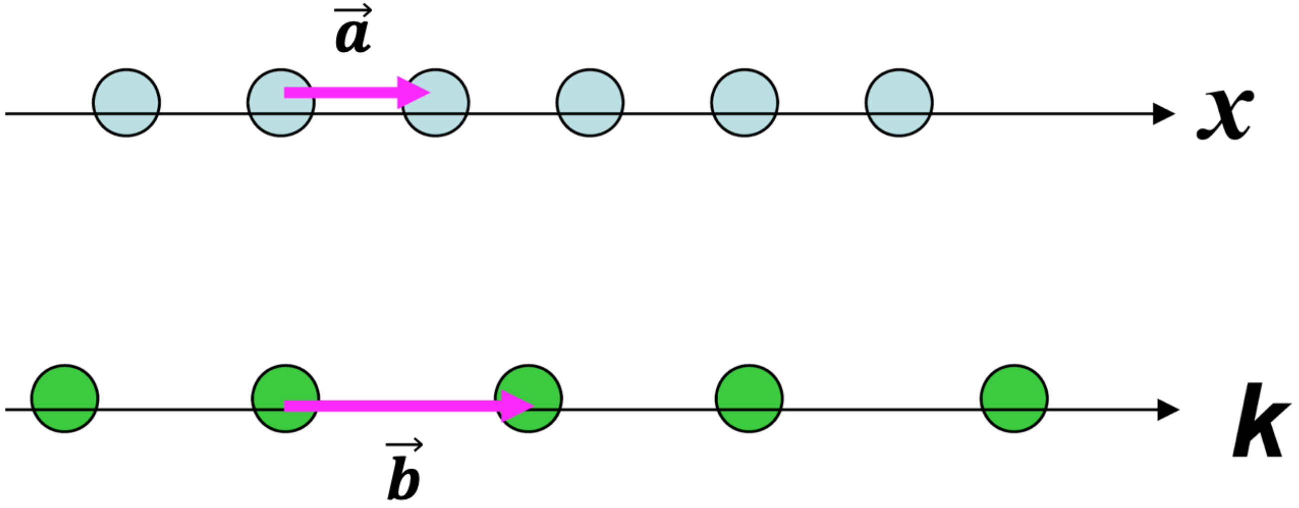
Chapter 1.4: Reciprocal Lattice (倒易点阵)

Examples of Reciprocal Lattice

- 1D

Direct Lattice

Reciprocal Lattice

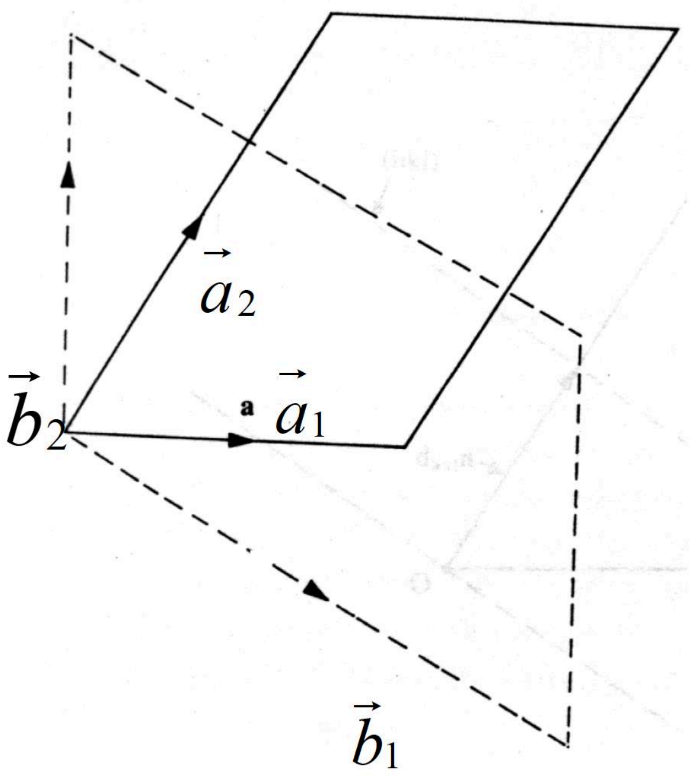


$$\boldsymbol{b} = \boldsymbol{2\pi/a} \qquad \vec{b}_1 = 2\pi \frac{\vec{a}_2 \times \vec{a}_3}{\vec{a}_1 \cdot (\vec{a}_2 \times \vec{a}_3)}$$

Chapter 1.4: Reciprocal Lattice (倒易点阵)

➢ Examples of Reciprocal Lattice

❖ 2D



$$\vec{b}_1 \perp \vec{a}_2$$

$$\vec{b}_2 \perp \vec{a}_1$$

$$\vec{b}_1 \cdot \vec{a}_1 = \vec{b}_2 \cdot \vec{a}_2 = 2\pi$$

$$\vec{b}_1 \cdot \vec{a}_2 = \vec{b}_2 \cdot \vec{a}_1 = 0$$

$$\vec{b}_1 = 2\pi \frac{\vec{a}_2 \times \vec{a}_3}{\vec{a}_1 \cdot (\vec{a}_2 \times \vec{a}_3)}$$

$$\vec{b}_2 = 2\pi \frac{\vec{a}_3 \times \vec{a}_1}{\vec{a}_1 \cdot (\vec{a}_2 \times \vec{a}_3)}$$

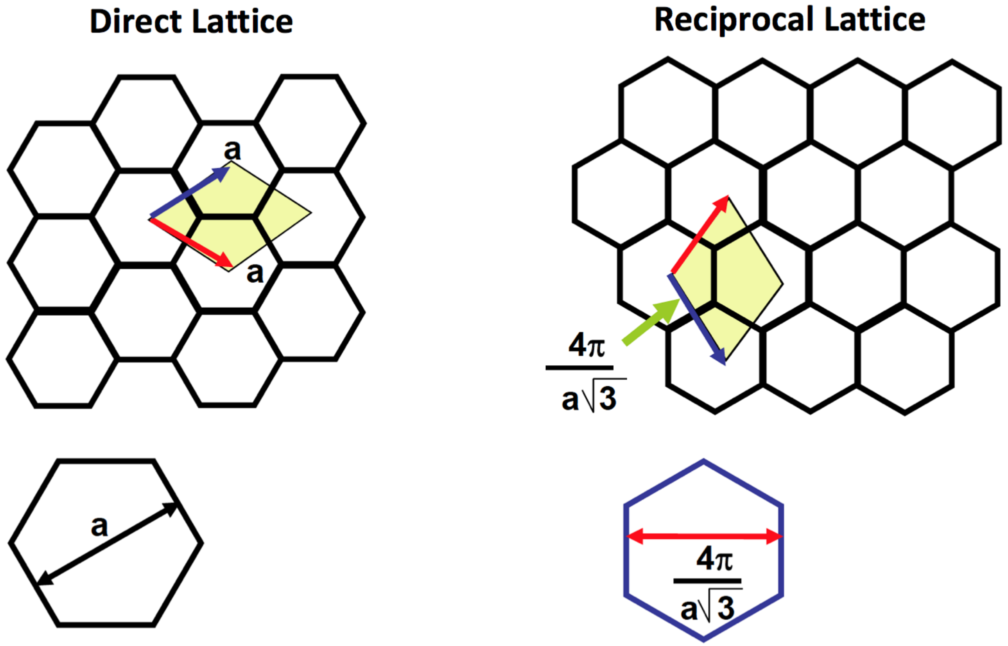
Here, "$\boldsymbol{\vec{a}_3}$" is the normal vector perpendicular to the 2D plane.

Chapter 1.4: Reciprocal Lattice (倒易点阵)

➢ Examples of Reciprocal Lattice

❖ 2D

Example: **Graphene**



$$\vec{b}_1 = 2\pi \frac{\vec{a}_2 \times \vec{a}_3}{\vec{a}_1 \cdot (\vec{a}_2 \times \vec{a}_3)}$$

$$\vec{b}_2 = 2\pi \frac{\vec{a}_3 \times \vec{a}_1}{\vec{a}_1 \cdot (\vec{a}_2 \times \vec{a}_3)}$$

Here, "$\vec{\boldsymbol{a}}_3$" is the normal vector perpendicular to the 2D plane.

Examples of Reciprocal Lattice

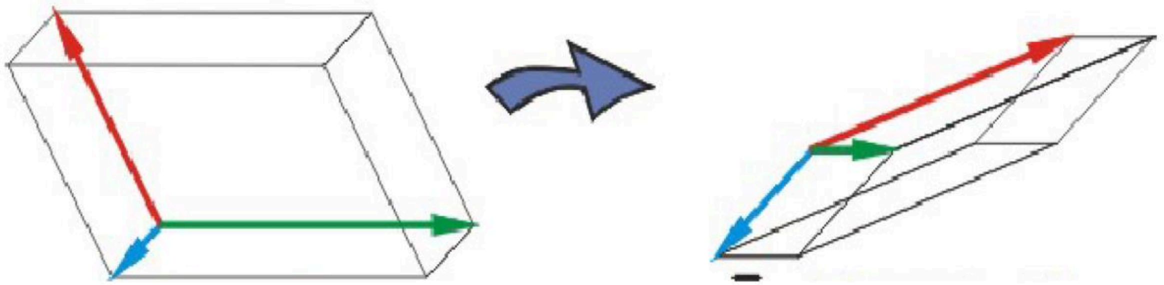
3D- Simple Lattice (简单晶格)

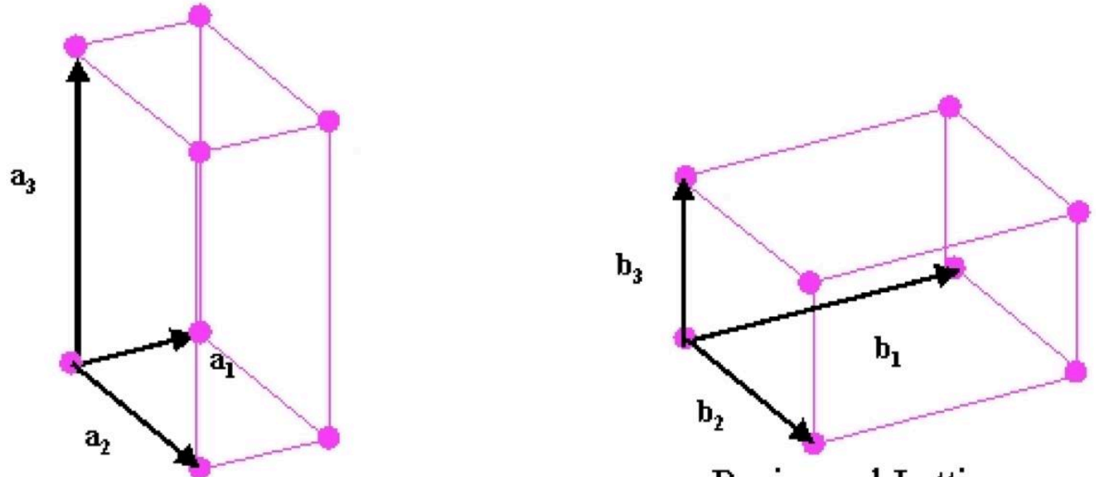
$$\vec{b}_1 = 2\pi \frac{\vec{a}_2 \times \vec{a}_3}{\vec{a}_1 \cdot (\vec{a}_2 \times \vec{a}_3)}$$

$$\vec{b}_2 = 2\pi \frac{\vec{a}_3 \times \vec{a}_1}{\vec{a}_1 \cdot (\vec{a}_2 \times \vec{a}_3)}$$

$$\vec{b}_3 = 2\pi \frac{\vec{a}_1 \times \vec{a}_2}{\vec{a}_1 \cdot (\vec{a}_2 \times \vec{a}_3)}$$

Direct Lattice　　Reciprocal Lattice





Simple Orthorhombic Bravais Lattice with $a_3 > a_2 > a_1$

Reciprocal Lattice Note: $b_1 > b_2 > b_3$

Chapter 1.4: Reciprocal Lattice (倒易点阵)

➢ Examples of Reciprocal Lattice

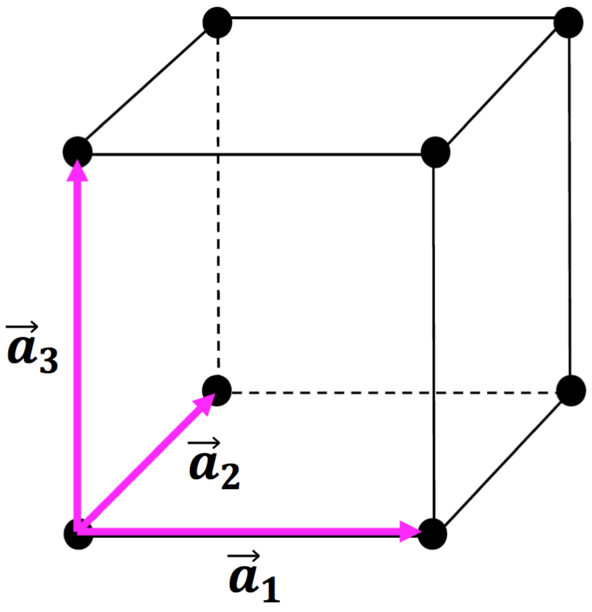
❖ 3D- *sc* Lattice (简单立方晶格)

$$\vec{b}_1 = 2\pi \frac{\vec{a}_2 \times \vec{a}_3}{\vec{a}_1 \cdot (\vec{a}_2 \times \vec{a}_3)}$$

$$\vec{b}_2 = 2\pi \frac{\vec{a}_3 \times \vec{a}_1}{\vec{a}_1 \cdot (\vec{a}_2 \times \vec{a}_3)}$$

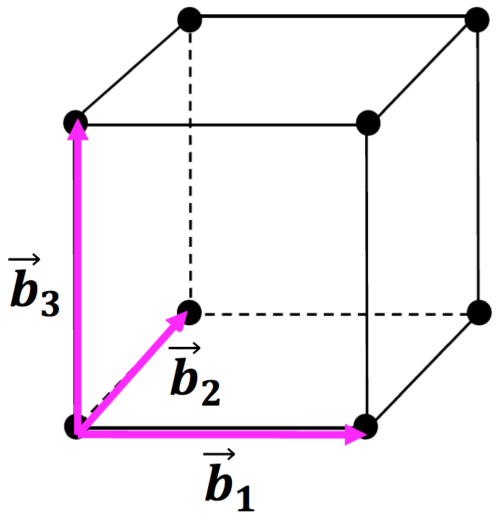
$$\vec{b}_3 = 2\pi \frac{\vec{a}_1 \times \vec{a}_2}{\vec{a}_1 \cdot (\vec{a}_2 \times \vec{a}_3)}$$

Direct Lattice



$$|\vec{a}_i| = a$$

Reciprocal Lattice



$$|\vec{b}_i| = 2\pi/a$$

Chapter 1.4: Reciprocal Lattice (倒易点阵)

➢ **Examples of Reciprocal Lattice**

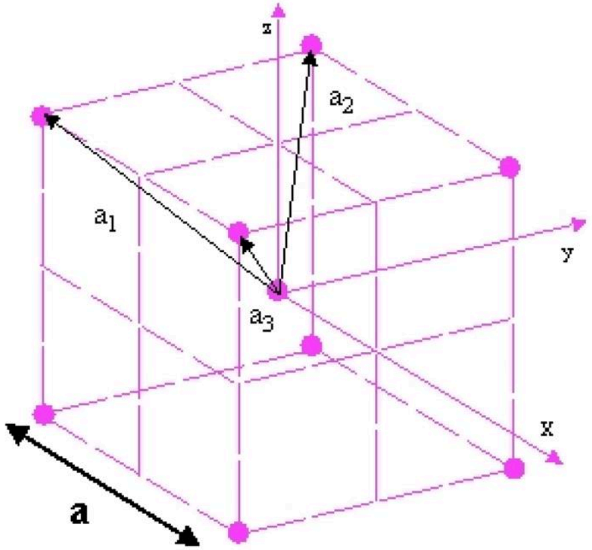
❖ **3D- *bcc* Lattice (体心立方晶格)**

$$\vec{b}_1 = 2\pi \frac{\vec{a}_2 \times \vec{a}_3}{\vec{a}_1 \cdot (\vec{a}_2 \times \vec{a}_3)}$$

$$\vec{b}_2 = 2\pi \frac{\vec{a}_3 \times \vec{a}_1}{\vec{a}_1 \cdot (\vec{a}_2 \times \vec{a}_3)}$$

$$\vec{b}_3 = 2\pi \frac{\vec{a}_1 \times \vec{a}_2}{\vec{a}_1 \cdot (\vec{a}_2 \times \vec{a}_3)}$$

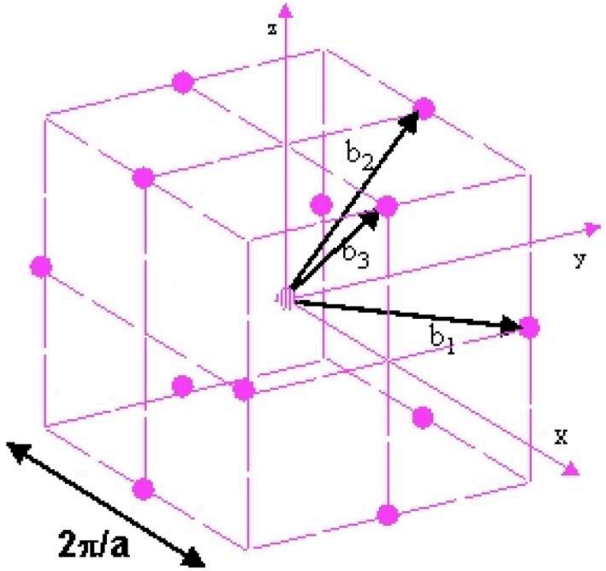
Direct Lattice



Primitive vectors and the conventional cell of bcc lattice

bcc

Reciprocal Lattice



Reciprocal lattice is Face Centered Cubic

fcc

➢ Examples of Reciprocal Lattice

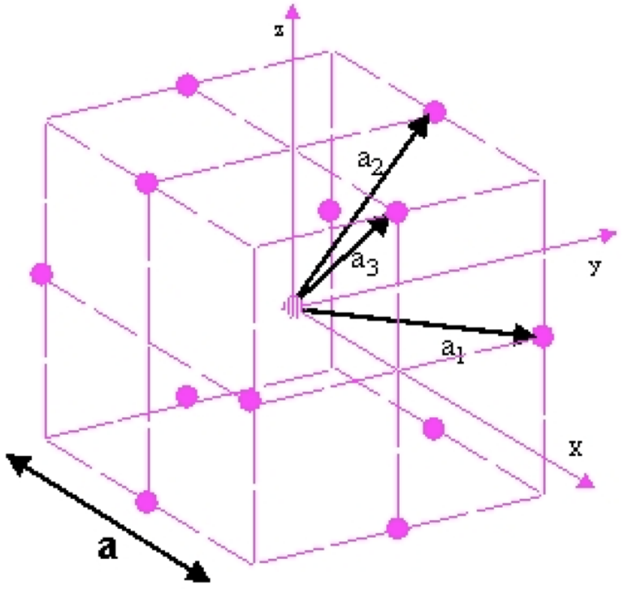
❖ 3D- *fcc* Lattice (面心立方晶格)

$$\vec{b}_1 = 2\pi \frac{\vec{a}_2 \times \vec{a}_3}{\vec{a}_1 \cdot (\vec{a}_2 \times \vec{a}_3)}$$

$$\vec{b}_2 = 2\pi \frac{\vec{a}_3 \times \vec{a}_1}{\vec{a}_1 \cdot (\vec{a}_2 \times \vec{a}_3)}$$

$$\vec{b}_3 = 2\pi \frac{\vec{a}_1 \times \vec{a}_2}{\vec{a}_1 \cdot (\vec{a}_2 \times \vec{a}_3)}$$

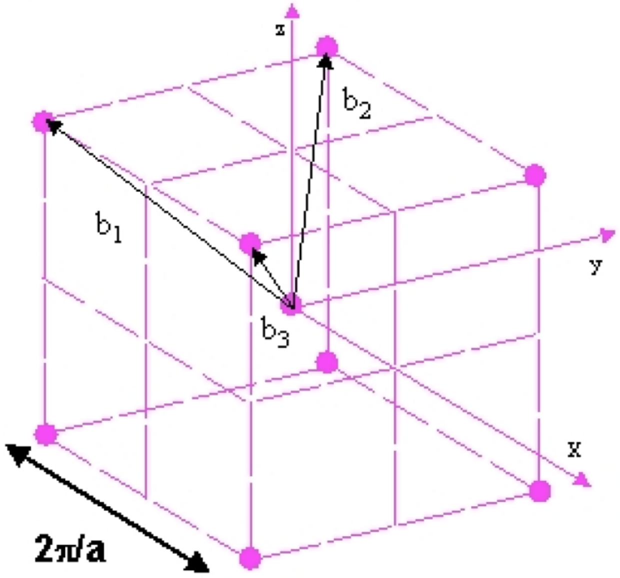
Direct Lattice



Primitive vectors and the conventional cell of fcc lattice

fcc

Reciprocal Lattice



Reciprocal lattice is Body Centered Cubic

bcc

➢ **Examples of Reciprocal Lattice**

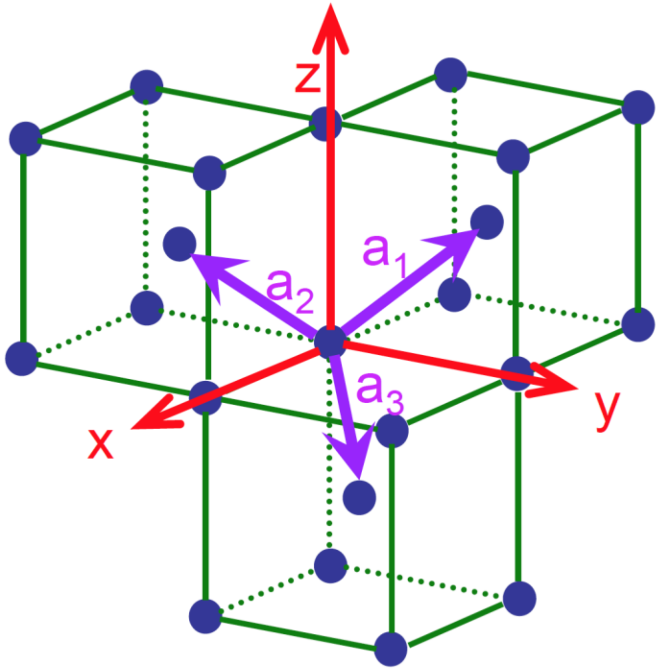
bcc **lattice**

$$\vec{a}_1 = \frac{a}{2}(-\vec{x} + \vec{y} + \vec{z})$$

$$\vec{a}_2 = \frac{a}{2}(\vec{x} - \vec{y} + \vec{z})$$

$$\vec{a}_3 = \frac{a}{2}(\vec{x} + \vec{y} - \vec{z})$$

$$\Omega = a^3/2$$



fcc **lattice**

$$\vec{b}_1 = \frac{2\pi}{a}(\vec{y} + \vec{z})$$

$$\vec{b}_2 = \frac{2\pi}{a}(\vec{z} + \vec{x})$$

$$\vec{b}_3 = \frac{2\pi}{a}(\vec{x} + \vec{y})$$

$$\Omega^* = 16(\pi/a)^3$$

- **Examples of Reciprocal Lattice**

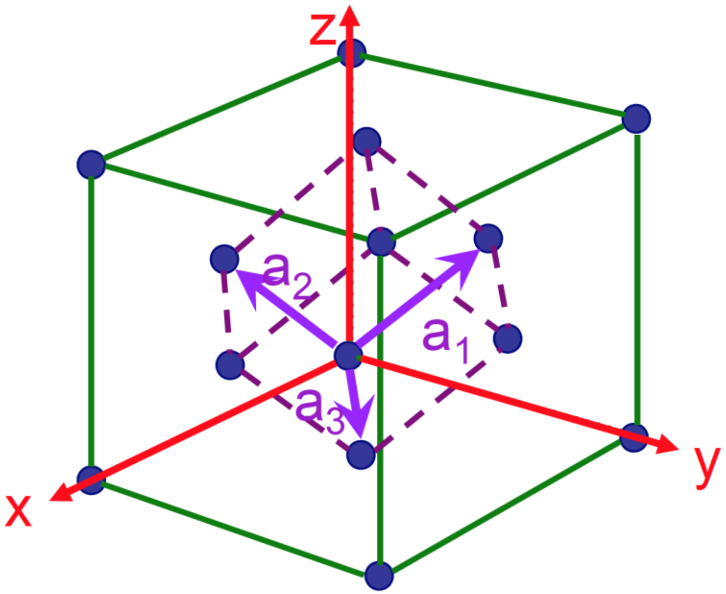
fcc **lattice**

$$\vec{a}_1 = \frac{a}{2}(\vec{y} + \vec{z})$$

$$\vec{a}_2 = \frac{a}{2}(\vec{z} + \vec{x})$$

$$\vec{a}_3 = \frac{a}{2}(\vec{x} + \vec{y})$$

$$\Omega = a^3/4$$



bcc **lattice**

$$\vec{b}_1 = \frac{2\pi}{a}(-\vec{x} + \vec{y} + \vec{z})$$

$$\vec{b}_2 = \frac{2\pi}{a}(\vec{x} - \vec{y} + \vec{z})$$

$$\vec{b}_3 = \frac{2\pi}{a}(\vec{x} + \vec{y} - \vec{z})$$

$$\Omega^* = 32(\pi/a)^3$$

Brillouin Zones (布里渊区)

Chapter 1.4: Reciprocal Lattice (倒易点阵)

➢ Brillouin Zones (布里渊区)

❖ In general, **Brillouin zones** are defined in the reciprocal space as follows:

1) Choose one reciprocal lattice point as the starting point;

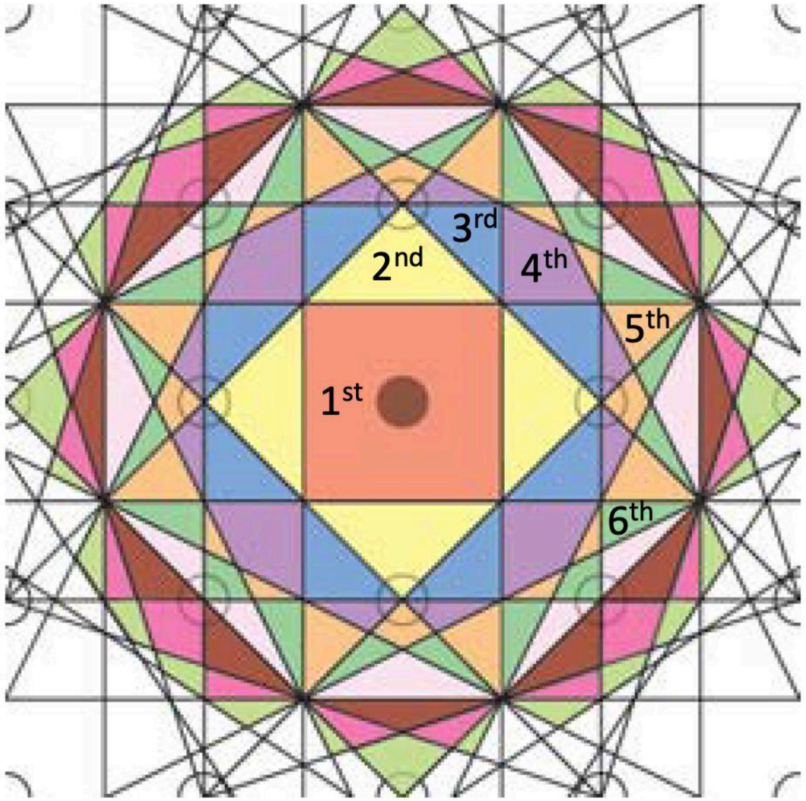
2) Draw perpendicular bisecting planes (垂直平分面) to all the reciprocal lattice vectors from the starting point;

3) The reciprocal space is then divided into different **polyhedral (多面体) zones enclosing the starting point**;

4) These polyhedral zones are called **Brillouin zones**.

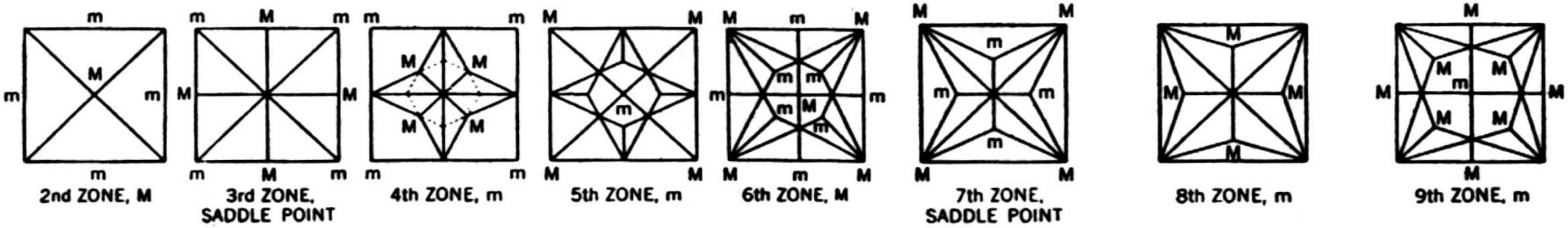
➢ Brillouin Zones (布里渊区)

❖ The Brillouin zones of a 2D square lattice.



➢ **Brillouin Zones (布里渊区)**

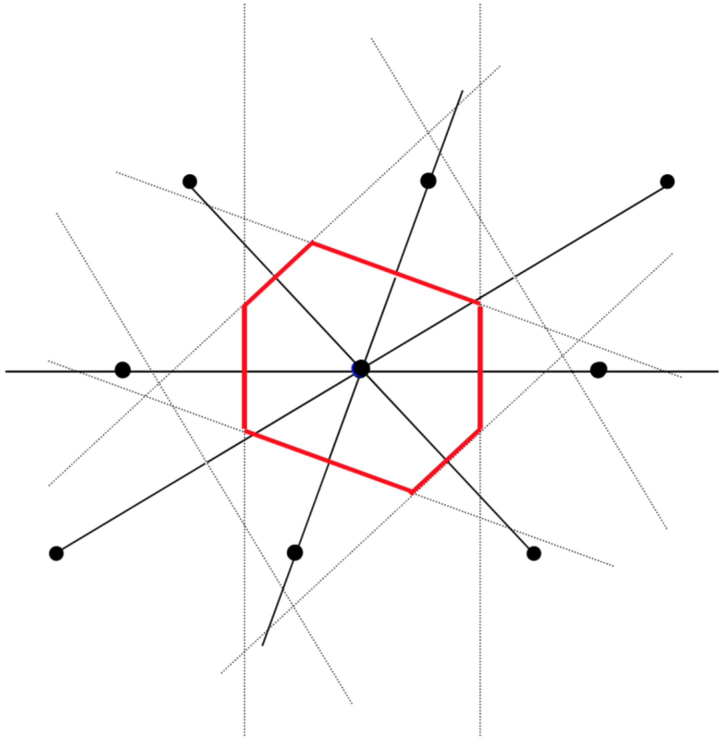
❖ The Brillouin zones of a 2D square lattice.



Reduction to the 1st zone for the second through ninth zones of the 2D square lattice.

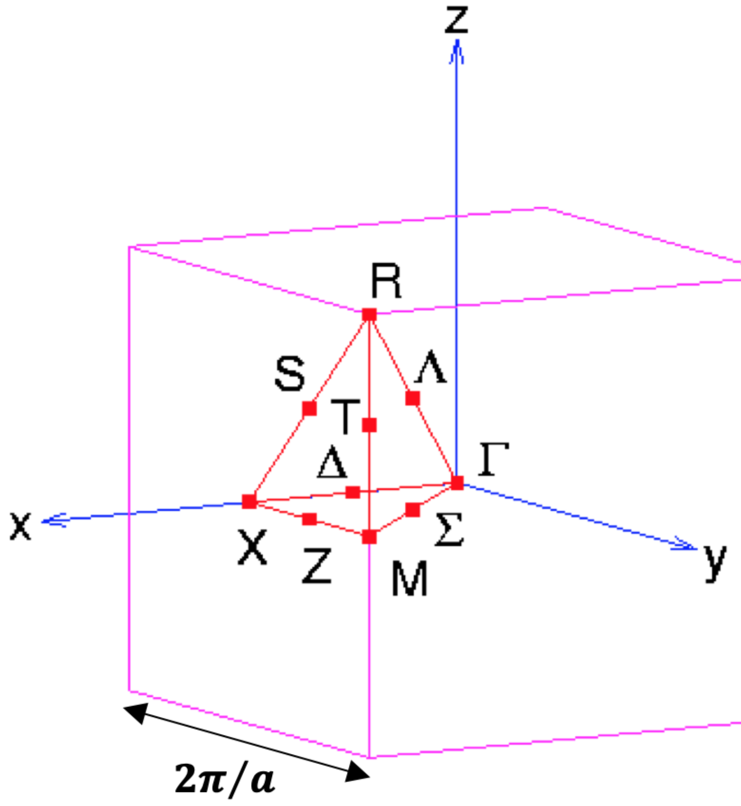
➢ Brillouin Zones (布里渊区)

❖ The **1^{st} Brillouin zone** (**第一布里渊区**, 又称**简约布里渊区**) is defined as the **Wigner-Seitz primitive cell** of the reciprocal lattice.



➢ Brillouin Zones (布里渊区)

❖ The 1st Brillouin zone of a *sc* **lattice** and the **points of high symmetry (高对称点)**.



Γ: $(0,0,0)$

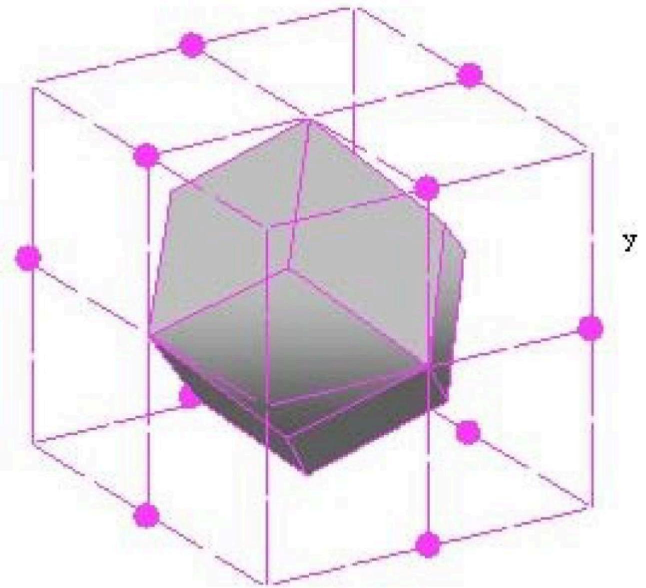
X: $\frac{\pi}{a}(1,0,0)$

R: $\frac{\pi}{a}(1,1,1)$

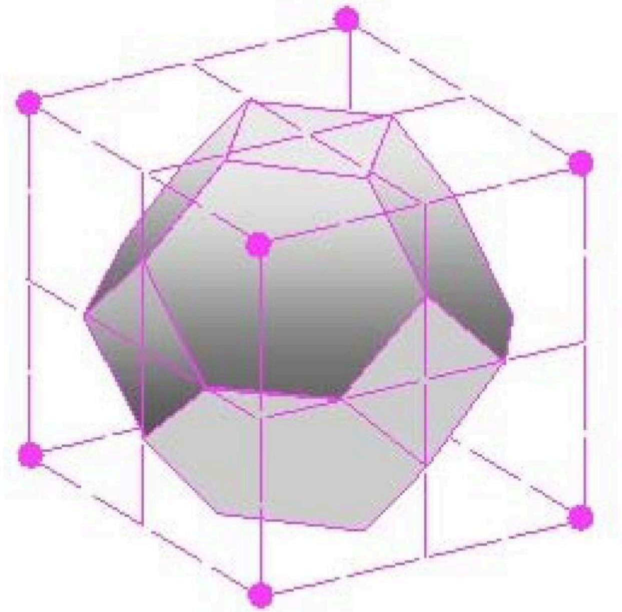
M: $\frac{\pi}{a}(1,1,0)$

➢ Brillouin Zones (布里渊区)

❖ The WS cell and 1st Brillouin zone of a *fcc* **lattice** (*bcc* **reciprocal lattice**).



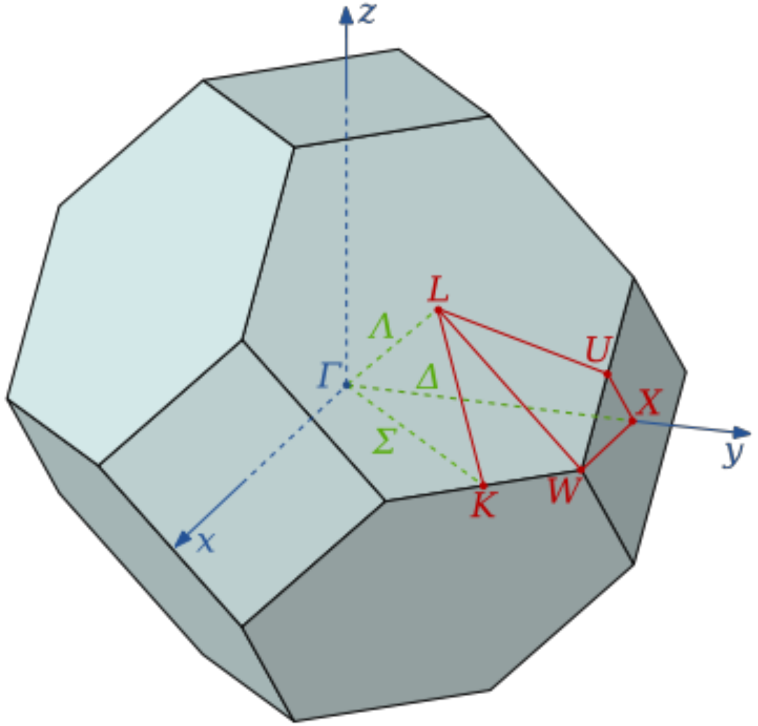
Wigner-Seitz Cell for
Face Centered Cubic Lattice



Brillouin Zone =
Wigner-Seitz Cell for
Reciprocal Lattice

➢ Brillouin Zones (布里渊区)

❖ The 1st Brillouin zone of a fcc **lattice** (bcc **reciprocal lattice**) and the points of high symmetry.



$\boldsymbol{\Gamma}$: $(\mathbf{0}, \mathbf{0}, \mathbf{0})$

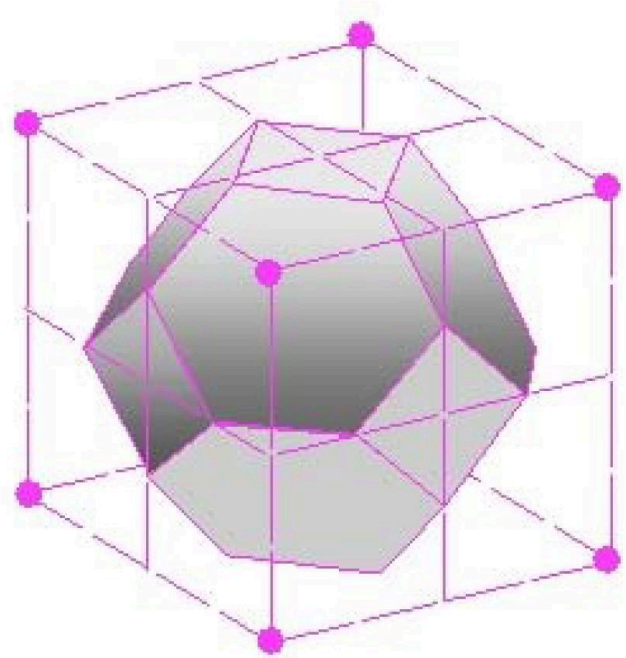
$\mathbf{X}$: $\dfrac{\mathbf{2}\boldsymbol{\pi}}{\boldsymbol{a}}(\mathbf{1}, \mathbf{0}, \mathbf{0})$

$\mathbf{L}$: $\dfrac{\mathbf{2}\boldsymbol{\pi}}{\boldsymbol{a}}\left(\dfrac{\mathbf{1}}{\mathbf{2}}, \dfrac{\mathbf{1}}{\mathbf{2}}, \dfrac{\mathbf{1}}{\mathbf{2}}\right)$

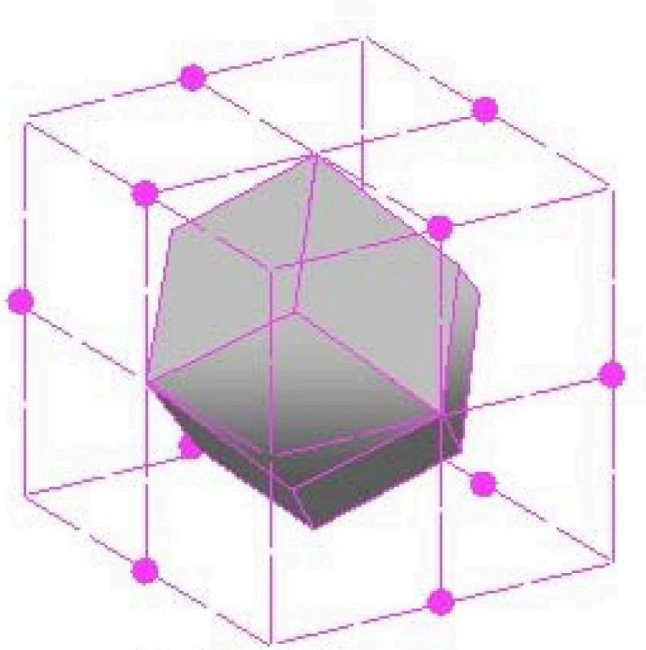
$\mathbf{K}$: $\dfrac{\mathbf{2}\boldsymbol{\pi}}{\boldsymbol{a}}\left(\dfrac{\mathbf{3}}{\mathbf{4}}, \dfrac{\mathbf{3}}{\mathbf{4}}, \mathbf{0}\right)$

➢ Brillouin Zones (布里渊区)

❖ The WS cell and 1st Brillouin zone of a *bcc* **lattice** (*fcc* **reciprocal lattice**).



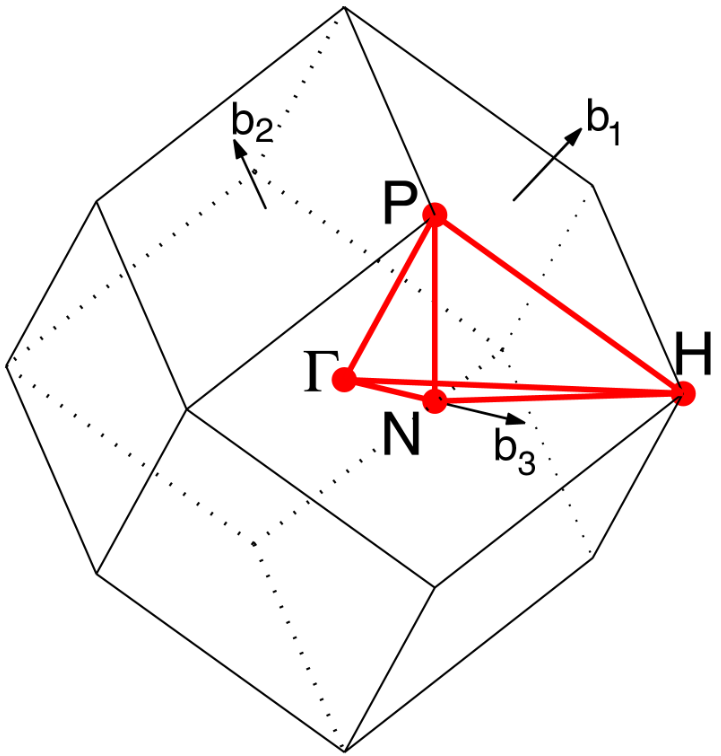
Wigner-Seitz Cell for
Body Centered Cubic Lattice



Brillouin Zone =
Wigner-Seitz Cell for
Reciprocal Lattice

- **Brillouin Zones (布里渊区)**

  - The 1st Brillouin zone of a bcc **lattice** (fcc **reciprocal lattice**) and the points of high symmetry.



$$\boldsymbol{\Gamma}: \ (\mathbf{0}, \mathbf{0}, \mathbf{0})$$

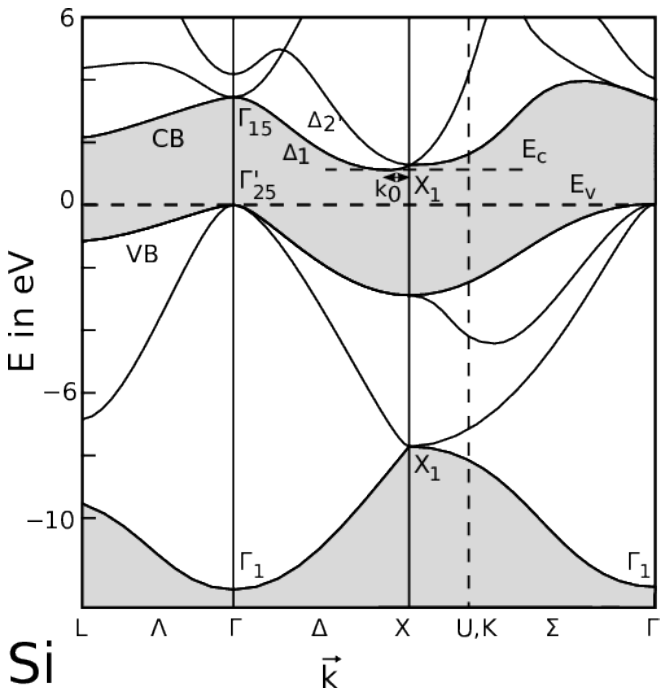
$$\boldsymbol{H}: \ \frac{2\pi}{a}(\mathbf{1}, \mathbf{0}, \mathbf{0})$$

$$\mathbf{P}: \ \frac{2\pi}{a}\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$$

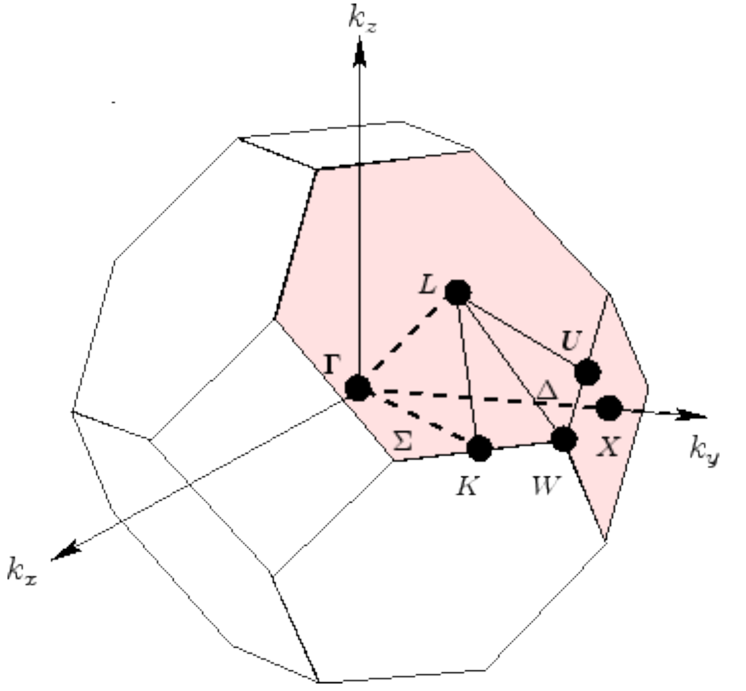
$$\mathbf{N}: \ \frac{2\pi}{a}\left(\frac{1}{2}, \frac{1}{2}, 0\right)$$

➢ Brillouin Zones (布里渊区)

❖ Band dispersion and the points of high symmetry.



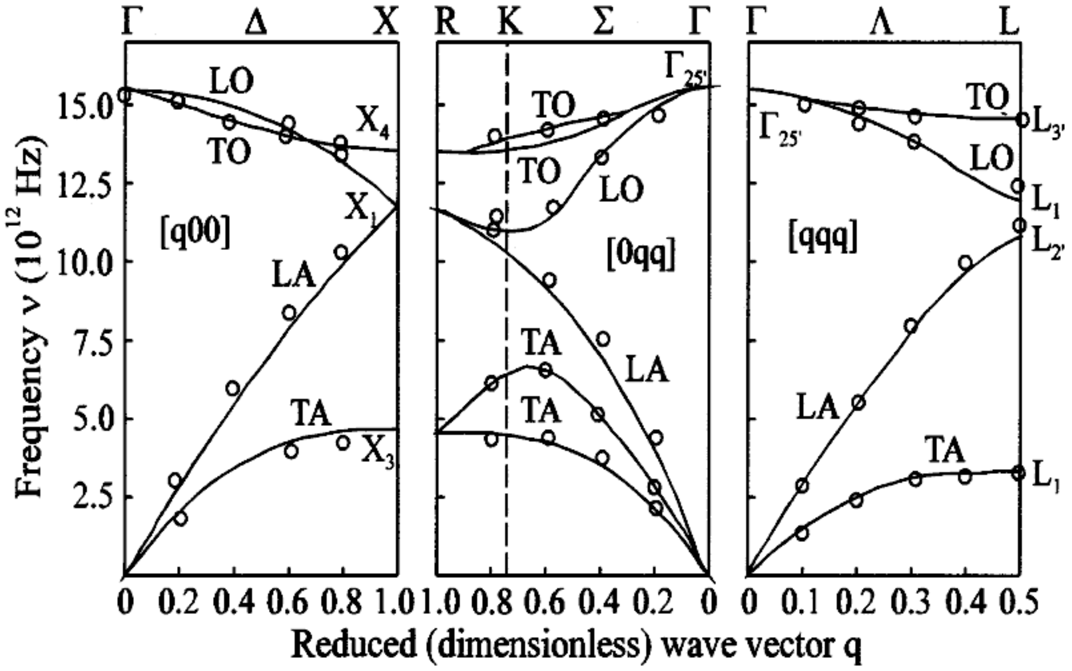
Electron band dispersion of Si: $E_n(k)$, with k the lattice vector in reciprocal space and n the band index.



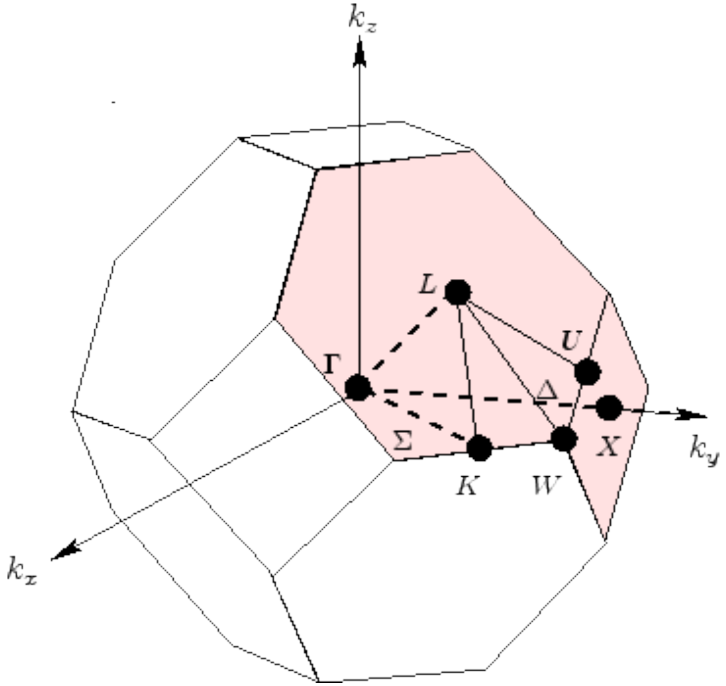
bcc reciprocal lattice

➢ Brillouin Zones (布里渊区)

❖ Band dispersion and the points of high symmetry.



Phonon band dispersion of Si: with q the lattice vector in reciprocal space.



bcc reciprocal lattice

Summary (总结)

- **Summary (总结)**

  - **Reciprocal lattices.**
  - **Connection between reciprocal and direct lattices: 6**
  - **Typical reciprocal lattices:**

sc lattice	⟺	*sc* lattice
fcc lattice	⟺	*bcc* lattice
bcc lattice	⟺	*fcc* lattice

  - **Brillouin zones.**

Chapter 1.4: 课后作业

试证明:

倒格子矢量 $G = h_1 b_1 + h_2 b_2 + h_3 b_3$ 垂直于密勒指数为$(h_1 h_2 h_3)$的晶面系。

提交时间：**3月3日之前**

提交方式：**手写**（**写明姓名学号**）**后拍照**，**通**过本班课代表统一提交电子版