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[1.22]

解：$\oint_c A\cdot dl = \oint_c xdx + xy^2dy$

令 $x=\rho\cos\theta \quad y=\rho\sin\theta \qquad C:\rho=a$

得 $\oint_c xdx+xy^2dy = \int_0^{2\pi}(-\rho^2\underbrace{\cos\theta\sin\theta}_{\frac{1}{2}\sin2\theta}+\rho^4\underbrace{\cos^2\theta\sin^2\theta}_{(\cos^2\theta-\cos^4\theta)})d\theta$

$= \frac{\pi a^4}{4}$

$\int_S \nabla\times A\, dS = \int_S y^2 dS = \int_0^a\int_0^{2\pi}\rho^2\sin^2\theta\cdot\rho\, d\theta d\rho = \frac{\pi a^4}{4}$

$\because \int_S\nabla\times A\, dS = \oint_c A\cdot dl$

$\therefore$ 得证

[1.23]

证明：(1) $\nabla\cdot r = \frac{\partial r_x}{\partial x}+\frac{\partial r_y}{\partial y}+\frac{\partial r_z}{\partial z} = 3$

(2) $\nabla\times r = \begin{vmatrix} e_x & e_y & e_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ r_x & r_y & r_z \end{vmatrix} = 0$

(3) $\nabla(k\cdot r) = k\nabla r = k\cdot(e_x+e_y+e_z) = \vec{k}$

[2.1]

解：$q=\int_S \rho_S dS = \int_0^{\pi}\rho_{S0}\cos\theta\cdot 2\pi a^2\sin\theta\, d\theta$

$= \int_0^{\pi}\rho_{S0}\cdot\sin2\theta\,\pi a^2 d\theta$

$= 0$

[2.2]

解：$q=\int_V \rho\, dV = \int_0^L\int_0^{2\pi}\int_0^a \rho_0\frac{r}{a}\, r\,dr\,d\theta\,dz$

$= \frac{2\pi\rho_0 a^2 L}{3}$

[2.12]

解: $E = \frac{1}{4\pi\varepsilon_0}\int_S \rho_s \frac{r-r'}{|r-r'|^3} dS$

$= \frac{1}{4\pi\varepsilon_0}\int_S \rho_s \frac{z_0 e_z - r' e_r}{|z_0 - r'|^3} dS$

$= \frac{\rho_s}{4\pi\varepsilon_0}\int_0^r\int_0^{2\pi} \frac{e_z z_0 - r' e_r}{(\sqrt{(z_0^2+r'^2)})^3} r' d\theta dr$

$= \frac{\rho_s}{2\varepsilon_0}\int_0^r \frac{e_z \cdot z_0 - r' e_r}{(z_0^2+r'^2)^{\frac{3}{2}}} r' dr$

$= \frac{-\rho_s z_0}{2\varepsilon_0} \cdot \frac{1}{(z_0^2+r'^2)^{\frac{1}{2}}}\Big|_0^r$

$r' \to \infty$ 时 $E = \frac{\rho_s}{2\varepsilon_0}$

$r' \to \sqrt{3} z_0$ 时 $E = \frac{\rho_s}{4\varepsilon_0} = \frac{1}{2} \cdot \frac{\rho_s}{2\varepsilon_0}$ 　　得证

[2.14]

(1) 解: $\mathrm{div} E = 2yz + (-2x)$.

在 (2, 3, -1) 处 得

$\mathrm{div} E = -6 - 4 = -10$

(2) 解: $\mathrm{div} E = 4z^2 \sin\phi^2 + 2z^2\cos 2\phi + 2\rho^2 \sin^2\phi$

在 $\rho = 2$, $\phi = 110°$ $z = -1$ 时

$\mathrm{div} E = 9.06$

(3) 解: $\mathrm{div} E = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 E_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta E_\theta) + \frac{1}{r\sin\theta}\frac{\partial E_\phi}{\partial\phi}$

$= 6\sin\theta\cos\phi + \frac{\cos 2\theta\cos\phi}{\sin\theta} - \frac{\cos\phi}{\sin\theta}$

当 $r = 1.5$ $\theta = 30°$ $\phi = 50°$ 时

$\mathrm{div} E = 1.29$.