Lecture 6: Reflection and Transmission

Outline

- Review of Maxwell equations
- Reflection and Transmission of TE waves
- Reflection and Transmission of TM waves
- Phase Matching
- Total Reflection and Critical angle
- Total Transmission and Brewster Angle
- Reflection and Transmission by a Layered Medium

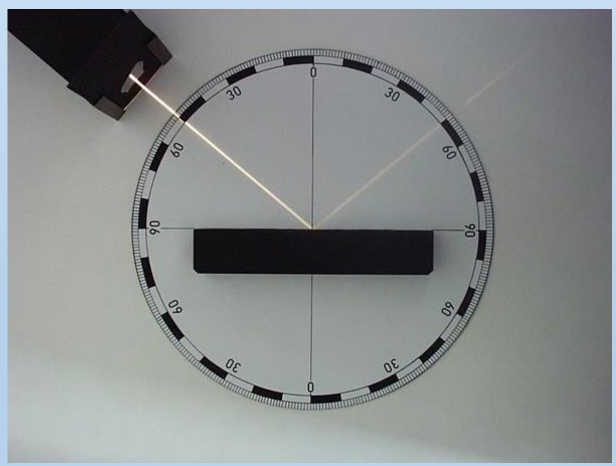
EM wave reflection/transmission

Mirror

Metal reflection

Highway mirage

The reflection of Mount Hood

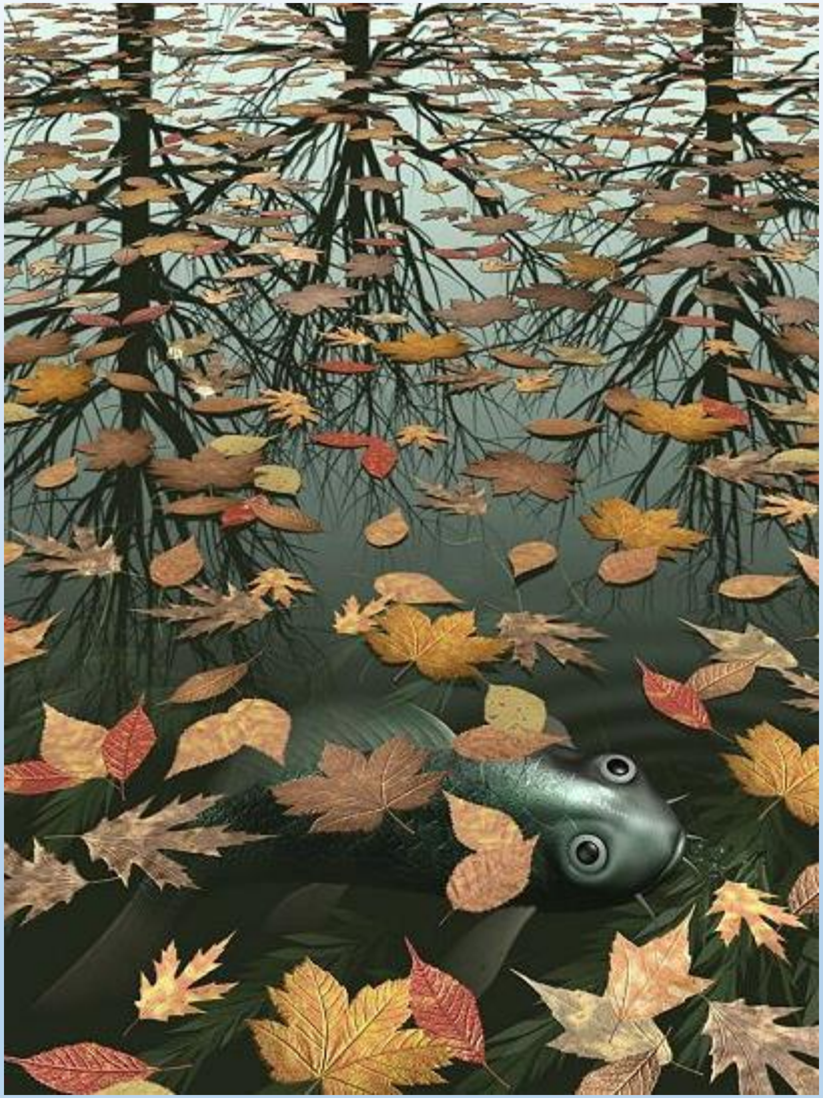


the law of reflection

A pencil in water looks bent

M. C. Escher

Three Worlds



Time-Harmonic Form of Maxwell's Equations

$$\nabla \cdot \vec{D} = \rho_{free}$$

$$\nabla \times \vec{E} = -j\omega\vec{B}$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{H} = \vec{J}_{free} + j\omega\vec{D}$$

(Source-free)

$$\overline{J} = \rho = 0$$

$$\nabla \rightarrow -j\overline{k}$$

(Plane wave)

$$\overline{k} \cdot \overline{E} = 0$$

$$\overline{k} \times \overline{E} = \omega\mu\overline{H}$$

$$\overline{k} \cdot \overline{H} = 0$$

$$\overline{k} \times \overline{H} = -\omega\varepsilon\overline{E}$$

Constitutive Relations

$$\vec{D} = \varepsilon\vec{E} \qquad \vec{B} = \mu\vec{H}$$

Plane wave solution:

$$\vec{E}\left(\vec{r}\right) = \vec{E}_0 \exp\left(-j\vec{k}\cdot\vec{r}\right)$$

$$\vec{k} = \hat{x}k_x + \hat{y}k_y + \hat{z}k_z$$

$$k_x^2 + k_y^2 + k_y^2 = \omega^2\mu\varepsilon = k^2$$

$$\vec{k}\cdot\vec{r} = k_x x + k_y y + k_z z$$

General Boundary Conditions

(Electric)

$$\hat{n}\cdot\left(\vec{D}_1-\vec{D}_2\right)=\rho_s$$

$$\hat{n}\times\left(\vec{E}_1-\vec{E}_2\right)=0$$

(Magnetic)

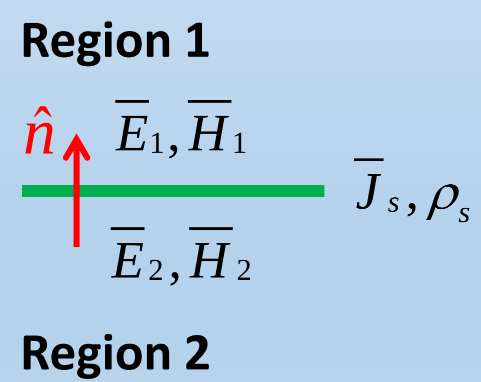
$$\hat{n}\cdot\left(\vec{B}_1-\vec{B}_2\right)=0$$

$$\hat{n}\times\left(\vec{H}_1-\vec{H}_2\right)=\vec{J}_s$$

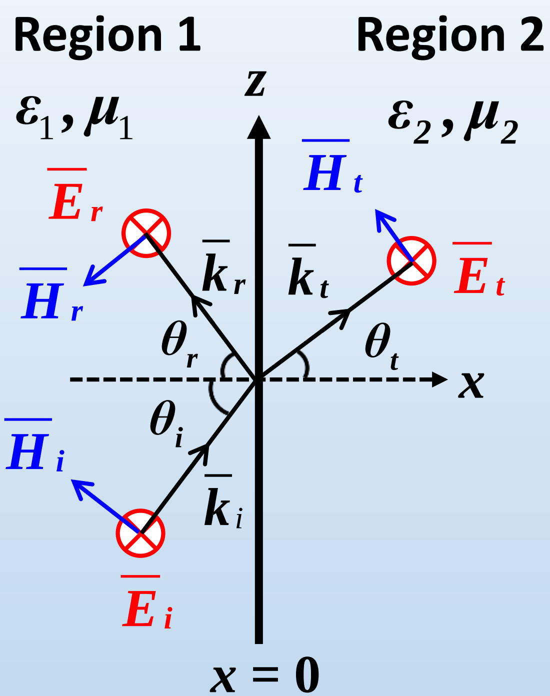
ρ_s **(surface charge density [C/m²])**

$\vec{J}_s$ **(surface current density [A/m])**

$\hat{n}$ **(Points from region 2 to region 1)**



Reflection and Transmission of TE Waves



Incident wave:

$$\boxed{k_{ix}, k_{iz}}$$

$$\overline{E}_i = \hat{y}\exp\left(-j\overline{k}_i \cdot \overline{r}\right)$$

$$\overline{H}_i = \frac{1}{\omega\mu_1}\overline{k}_i \times \overline{E}_i = \frac{1}{\omega\mu_1}(-\hat{x}k_{iz} + \hat{z}k_{ix})\exp\left(-j\overline{k}_i \cdot \overline{r}\right)$$

Reflected wave:

$$\boxed{k_{rx}, k_{rz}, R^{TE}}$$

$$\overline{E}_r = \hat{y}R^{TE}\exp\left(-j\overline{k}_r \cdot \overline{r}\right)$$

$$\overline{H}_r = \frac{1}{\omega\mu_1}\overline{k}_r \times \overline{E}_r = \frac{1}{\omega\mu_1}(-\hat{x}k_{rz} + \hat{z}k_{rx})R^{TE}\exp\left(-j\overline{k}_r \cdot \overline{r}\right)$$

Wave vectors:

$$\overline{k}_i = \hat{x}k_{ix} + \hat{z}k_{iz}$$

$$\overline{k}_r = \hat{x}k_{rx} + \hat{z}k_{rz}$$

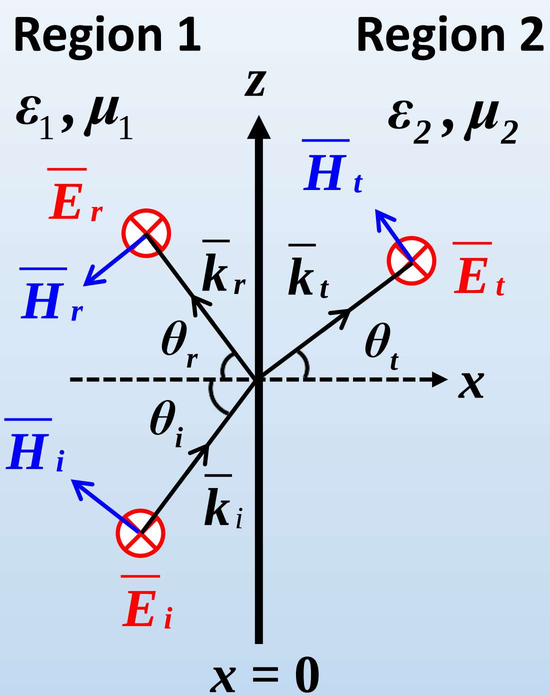
$$\overline{k}_t = \hat{x}k_{tx} + \hat{z}k_{tz}$$

Transmitted wave:

$$\boxed{k_{tx}, k_{tz}, T^{TE}}$$

$$\overline{E}_t = \hat{y}T^{TE}\exp\left(-j\overline{k}_t \cdot \overline{r}\right)$$

$$\overline{H}_t = \frac{1}{\omega\mu_2}\overline{k}_t \times \overline{E}_t = \frac{1}{\omega\mu_2}(-\hat{x}k_{tz} + \hat{z}k_{tx})T^{TE}\exp\left(-j\overline{k}_t \cdot \overline{r}\right)$$



Boundary conditions

$$\hat{n}\times\left(\overline{E}_1-\overline{E}_2\right)=0$$

$$\hat{n}\times\left(\overline{H}_1-\overline{H}_2\right)=0$$

Tangential components of E and H are continuous

Continuity of E_y at x = 0:

$$e^{-jk_{iz}z}+R^{TE}e^{-jk_{rz}z}=T^{TE}e^{-jk_{tz}z}$$

for All z

$$\boxed{k_{iz}=k_{rz}=k_{tz}}$$

→ **Phase matching condition**

$$k_{rx}=-k_{ix},k_{tx}=\sqrt{k_t^2-k_{iz}^2}$$

Dispersion relations

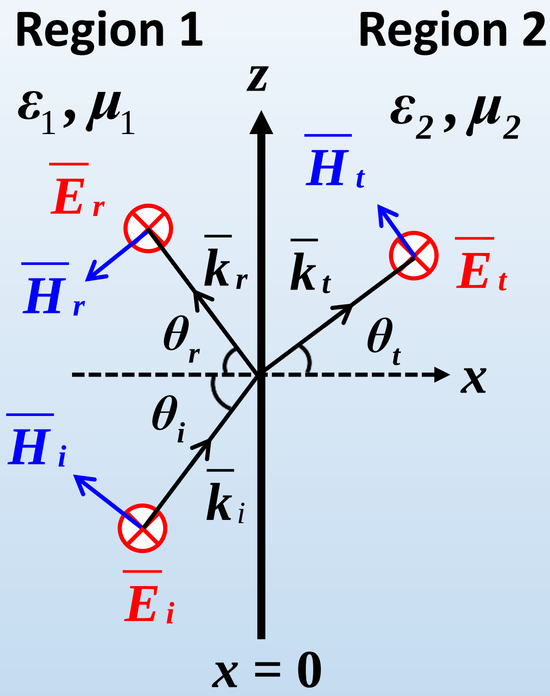
$$k_{ix}^2+k_{iz}^2=\omega^2\mu_1\varepsilon_1=n_1^2k_0^2$$

$$k_{rx}^2+k_{rz}^2=\omega^2\mu_1\varepsilon_1=n_1^2k_0^2$$

$$k_{tx}^2+k_{tz}^2=\omega^2\mu_2\varepsilon_2=n_2^2k_0^2$$

Continuity of H_z at x = 0:

$$\frac{k_{ix}}{\mu_1}\left(1-R^{TE}\right)=\frac{k_{tx}}{\mu_2}T^{TE}$$



Boundary conditions at x = 0 give:

$$1 + R^{TE} = T^{TE}$$

$$\frac{k_{ix}}{\mu_1}\left(1 - R^{TE}\right) = \frac{k_{tx}}{\mu_2} T^{TE}$$

$$\boxed{R^{TE} = \frac{\mu_2 k_{ix} - \mu_1 k_{tx}}{\mu_2 k_{ix} + \mu_1 k_{tx}} = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}}$$

(Reflection coefficient)

$$\boxed{T^{TE} = \frac{2\mu_2 k_{ix}}{\mu_2 k_{ix} + \mu_1 k_{tx}} = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}}$$

(Transmission coefficient)

$$\eta_1 = \sqrt{\mu_1/\varepsilon_1} \qquad \eta_2 = \sqrt{\mu_2/\varepsilon_2}$$

(Characteristic Impedance)

Dispersion relations

$$k_{ix}^2 + k_{iz}^2 = \omega^2 \mu_1 \varepsilon_1 = n_1^2 k_0^2$$

$$k_{rx}^2 + k_{rz}^2 = \omega^2 \mu_1 \varepsilon_1 = n_1^2 k_0^2$$

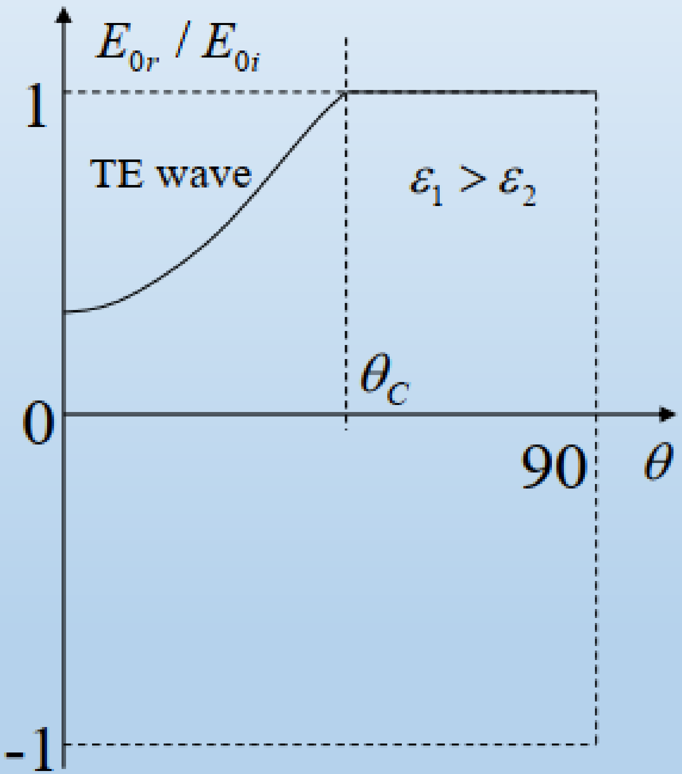
$$k_{tx}^2 + k_{tz}^2 = \omega^2 \mu_2 \varepsilon_2 = n_2^2 k_0^2$$

Reflection at medium interface (TE)

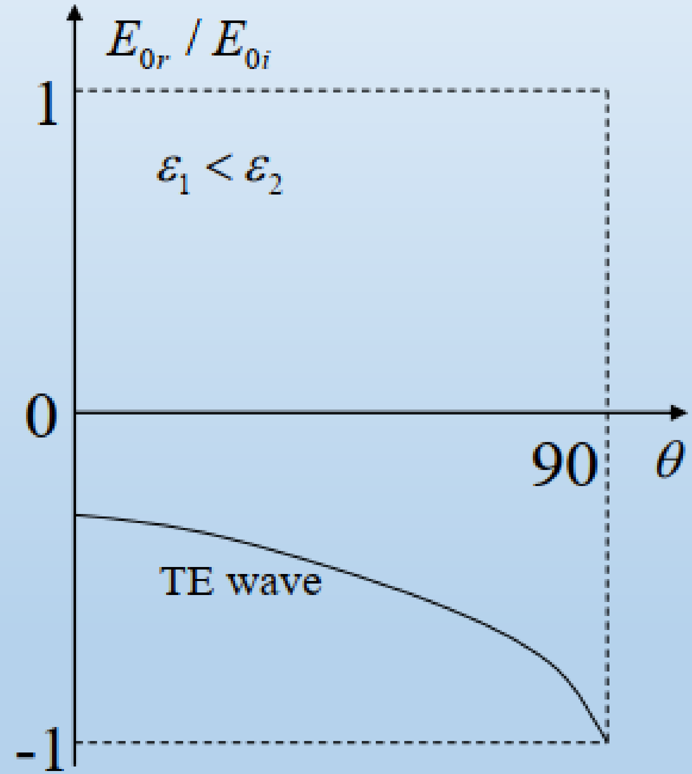
$$\boxed{R^{TE}=\frac{\mu_2 k_{ix}-\mu_1 k_{tx}}{\mu_2 k_{ix}+\mu_1 k_{tx}}=\frac{\eta_2\cos\theta_i-\eta_1\cos\theta_t}{\eta_2\cos\theta_i+\eta_1\cos\theta_t}}$$

$$\eta_1=\sqrt{\mu_1/\varepsilon_1}$$

$$\eta_2=\sqrt{\mu_2/\varepsilon_2}$$

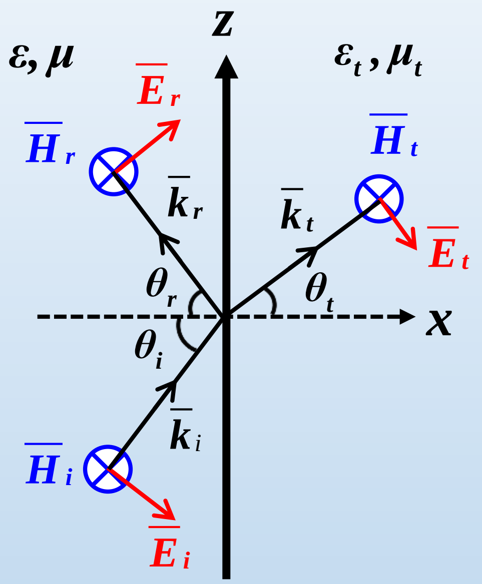


Internal reflection



External reflection

Reflection and Transmission of TM Waves



Wave vectors:

$$\overline{k}_i = \hat{x}k_{ix} + \hat{z}k_{iz}$$

$$\overline{k}_r = \hat{x}k_{rx} + \hat{z}k_{rz}$$

$$\overline{k}_t = \hat{x}k_{tx} + \hat{z}k_{tz}$$

Incident wave:

$$\boxed{k_{ix}, k_{iz}}$$

$$\overline{H}_i = \hat{y}\exp\left(-j\overline{k}_i \cdot \overline{r}\right)$$

$$\overline{E}_i = -\frac{1}{\omega\varepsilon}\overline{k}_i \times \overline{H}_i = -\frac{1}{\omega\varepsilon}\left(-\hat{x}k_{iz} + \hat{z}k_{ix}\right)\exp\left(-j\overline{k}_i \cdot \overline{r}\right)$$

$$\overline{S}_i = \overline{E}_i \times \overline{H}_i^* = \overline{k}_i \frac{1}{\omega\varepsilon}\left|\overline{H}_i\right|^2$$

Reflected wave:

$$\boxed{k_{rx}, k_{rz}, R^{TM}}$$

$$\overline{H}_r = \hat{y}R^{TM}\exp\left(-j\overline{k}_r \cdot \overline{r}\right)$$

$$\overline{E}_r = -\frac{1}{\omega\varepsilon}\overline{k}_r \times \overline{H}_r = -\frac{1}{\omega\varepsilon}\left(-\hat{x}k_{rz} + \hat{z}k_{rx}\right)R^{TM}\exp\left(-j\overline{k}_r \cdot \overline{r}\right)$$

$$\overline{S}_r = \overline{E}_r \times \overline{H}_r^* = \overline{k}_r \frac{1}{\omega\varepsilon}\left|\overline{H}_r\right|^2$$

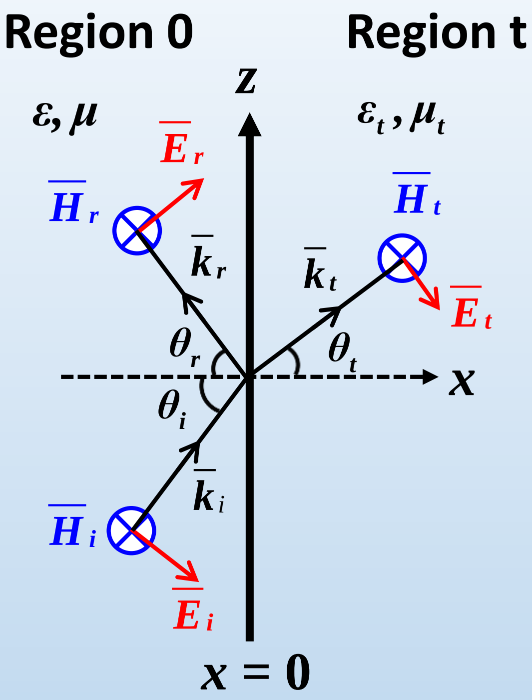
Transmitted wave:

$$\boxed{k_{tx}, k_{tz}, T^{TM}}$$

$$\overline{H}_t = \hat{y}T^{TM}\exp\left(-j\overline{k}_t \cdot \overline{r}\right)$$

$$\overline{E}_t = -\frac{1}{\omega\varepsilon_t}\overline{k}_t \times \overline{H}_t = -\frac{1}{\omega\varepsilon_t}\left(-\hat{x}k_{tz} + \hat{z}k_{tx}\right)T^{TM}\exp\left(-j\overline{k}_t \cdot \overline{r}\right)$$

$$\overline{S}_t = \overline{E}_t \times \overline{H}_t^* = \overline{k}_t \frac{1}{\omega\varepsilon_t}\left|\overline{H}_t\right|^2$$



Boundary conditions

$$\hat{n}\times\left(\overline{E}_1-\overline{E}_2\right)=0$$

$$\hat{n}\times\left(\overline{H}_1-\overline{H}_2\right)=0$$

Tangential components of E and H are continuous

Continuity of H_y at x = 0:

$$e^{-jk_{iz}z}+R^{TM}e^{-jk_{rz}z}=T^{TM}e^{-jk_{tz}z} \qquad \textbf{(1)}$$

for All z

$$\boxed{k_{iz}=k_{rz}=k_{tz}}$$

→ **Phase matching condition**

$$k_{rx}=-k_{ix},\, k_{tx}=\sqrt{k_t^2-k_{iz}^2}$$

Dispersion relations

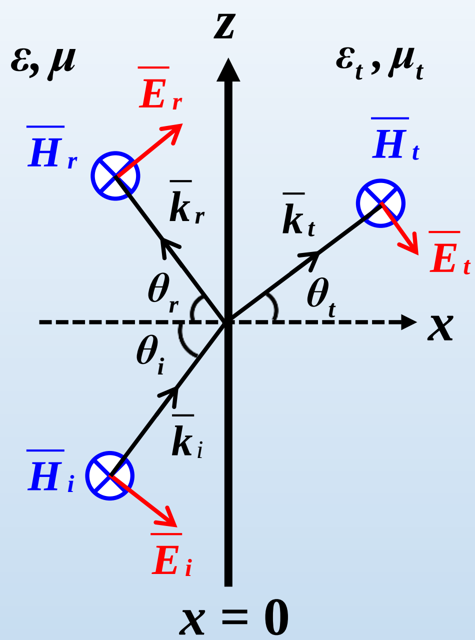
$$k_{ix}^2+k_{iz}^2=\omega^2\mu\varepsilon=k^2$$

$$k_{rx}^2+k_{rz}^2=\omega^2\mu\varepsilon=k^2$$

$$k_{tx}^2+k_{tz}^2=\omega^2\mu_t\varepsilon_t=k_t^2$$

Continuity of E_z at x = 0:

$$\frac{k_{ix}}{\varepsilon}\left(1-R^{TM}\right)=\frac{k_{tx}}{\varepsilon_t}T^{TM} \qquad \textbf{(2)}$$



Boundary conditions at x = 0 give:

$$1 + R^{TM} = T^{TM}$$

$$\frac{k_{ix}}{\varepsilon}\left(1 - R^{TM}\right) = \frac{k_{tx}}{\varepsilon_t} T^{TM}$$

$$R^{TM} = R_{0t}^{TM} = \frac{\varepsilon_t k_{ix} - \varepsilon k_{tx}}{\varepsilon_t k_{ix} + \varepsilon k_{tx}}$$ → **Reflection coefficient**

$$T^{TM} = T_{0t}^{TM} = \frac{2\varepsilon_t k_{ix}}{\varepsilon_t k_{ix} + \varepsilon k_{tx}}$$ → **Transmission coefficient**

Wave vectors:

$$\overline{k}_i = \hat{x}k_{ix} + \hat{z}k_{iz}$$

$$\overline{k}_r = \hat{x}k_{rx} + \hat{z}k_{rz}$$

$$\overline{k}_t = \hat{x}k_{tx} + \hat{z}k_{tz}$$

Time-averaged Poynting power vectors:

$$\langle \overline{S}_i \rangle = \frac{1}{2}\mathrm{Re}\left\{\frac{\overline{k}_i}{\omega\varepsilon}\right\} \qquad \langle \overline{S}_r \rangle = \frac{1}{2}\mathrm{Re}\left\{\frac{\overline{k}_r}{\omega\varepsilon}\left|R^{TM}\right|^2\right\}$$

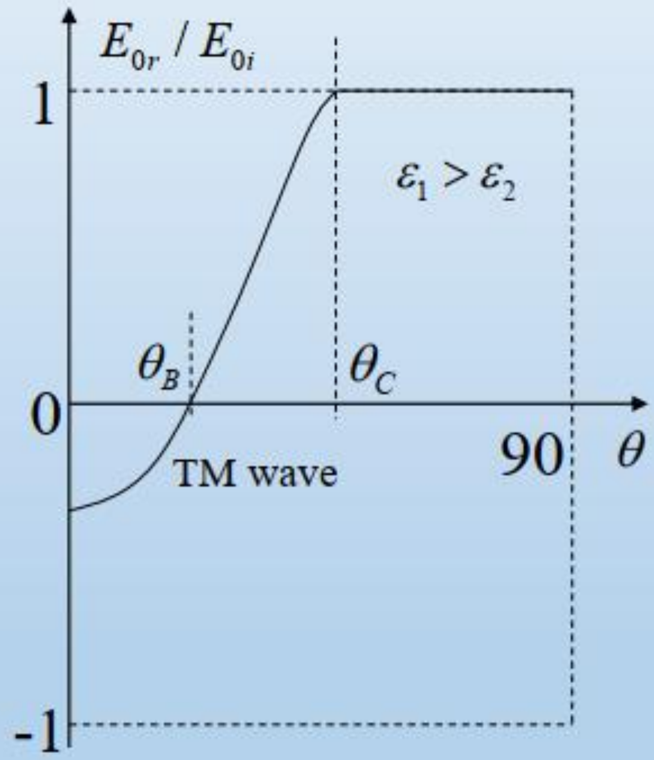
$$\langle \overline{S}_t \rangle = \frac{1}{2}\mathrm{Re}\left\{\frac{\overline{k}_t}{\omega\varepsilon_t}|T^{TM}|^2 e^{-j(k_{tx}-k_{tx}^*)x}\right\}$$

Reflection at medium interface (TM)

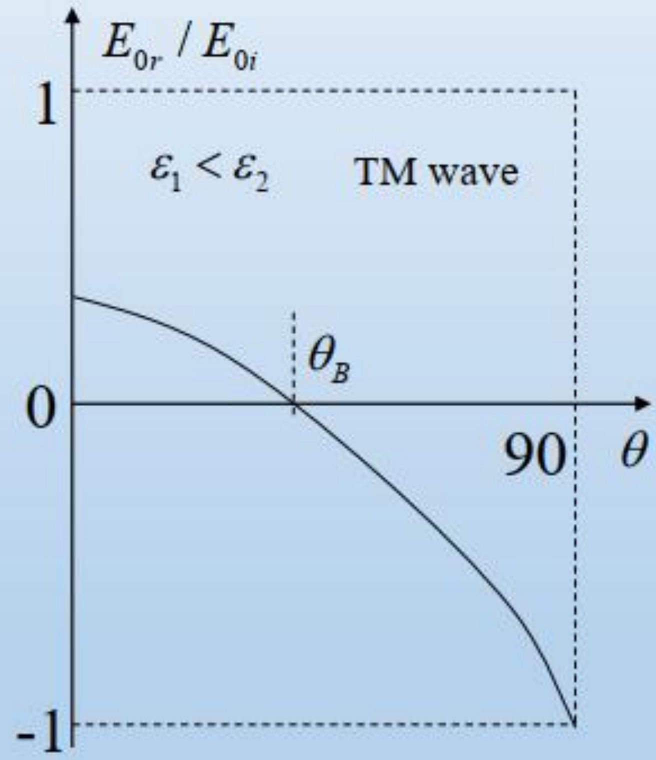
$$\boxed{R^{TM}=\frac{\varepsilon_2 k_{ix}-\varepsilon_1 k_{tx}}{\varepsilon_2 k_{ix}+\varepsilon_1 k_{tx}}=\frac{\eta_1\cos\theta_i-\eta_2\cos\theta_t}{\eta_1\cos\theta_i+\eta_2\cos\theta_t}}$$

$$\eta_1=\sqrt{\mu_1/\varepsilon_1}$$

$$\eta_2=\sqrt{\mu_2/\varepsilon_2}$$



Internal reflection



External reflection

TM case is the dual of TE

$$\overline{k}\cdot\overline{E}=0$$

$$\overline{k}\times\overline{E}=\omega\mu\overline{H}$$

$$\overline{k}\cdot\overline{H}=0$$

$$\overline{k}\times\overline{H}=-\omega\varepsilon\overline{E}$$

$$\overline{E}\rightarrow\overline{H}$$

$$\overline{H}\rightarrow-\overline{E}$$

$$\mu\rightarrow\varepsilon$$

$$\varepsilon\rightarrow\mu$$

$$\overline{k}\cdot\overline{H}=0$$

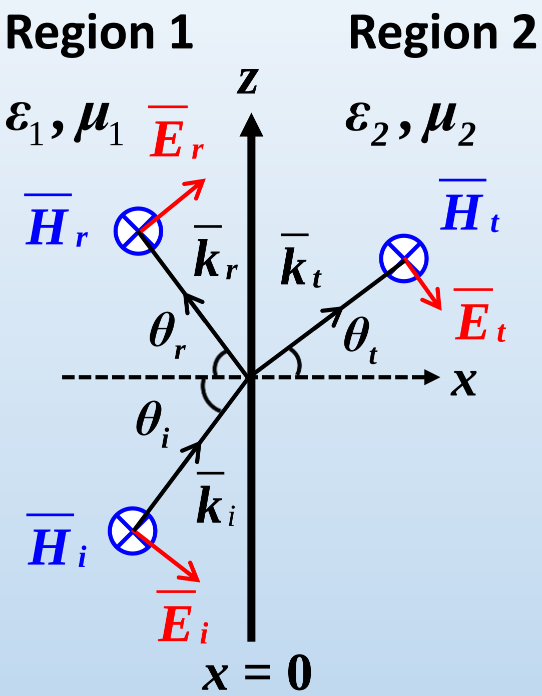
$$\overline{k}\times\overline{H}=-\omega\varepsilon\overline{E}$$

$$\overline{k}\cdot\overline{E}=0$$

$$\overline{k}\times\overline{E}=\omega\mu\overline{H}$$

The TM solution can be recovered from the TE solution. So, consider only the TE solution in detail.

Reflection and Transmission of TM Waves



$$\boxed{R^{TM} = \frac{\varepsilon_2 k_{ix} - \varepsilon_1 k_{tx}}{\varepsilon_2 k_{ix} + \varepsilon_1 k_{tx}} = \frac{\eta_1 \cos\theta_i - \eta_2 \cos\theta_t}{\eta_1 \cos\theta_i + \eta_2 \cos\theta_t}}$$

(Reflection coefficient)

$$\boxed{T^{TM} = \frac{2\varepsilon_2 k_{ix}}{\varepsilon_2 k_{ix} + \varepsilon_1 k_{tx}} = \frac{2\eta_1 \cos\theta_i}{\eta_1 \cos\theta_i + \eta_2 \cos\theta_t}}$$

(Transmission coefficient)

Wave vectors:

$$\overline{k}_i = \hat{x}k_{ix} + \hat{z}k_{iz}$$

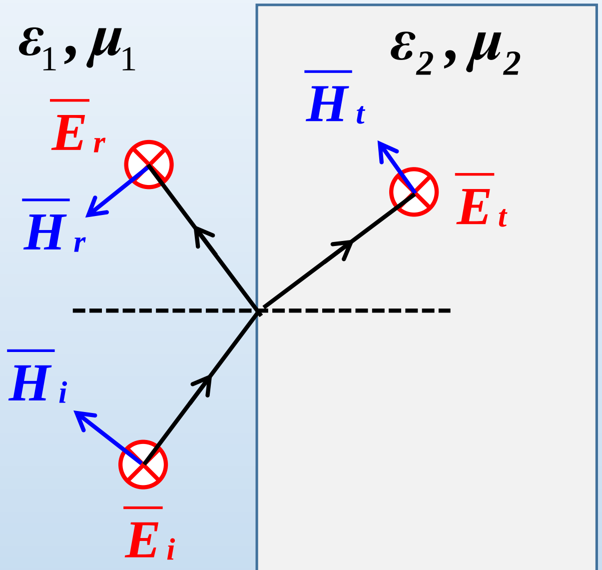
$$\overline{k}_r = \hat{x}k_{rx} + \hat{z}k_{rz}$$

$$\overline{k}_t = \hat{x}k_{tx} + \hat{z}k_{tz}$$

$$\eta_1 = \sqrt{\mu_1 / \varepsilon_1} \qquad \eta_2 = \sqrt{\mu_2 / \varepsilon_2}$$

(Characteristic Impedance)

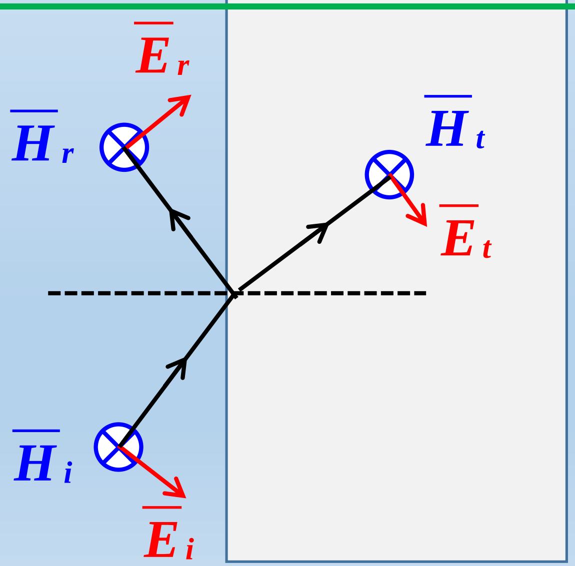
Fresnel Equations - Summary



TE-polarization

$$R^{TE} = \frac{\mu_2 k_{ix} - \mu_1 k_{tx}}{\mu_2 k_{ix} + \mu_1 k_{tx}} = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$$

$$T^{TE} = \frac{2\mu_2 k_{ix}}{\mu_2 k_{ix} + \mu_1 k_{tx}} = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$$



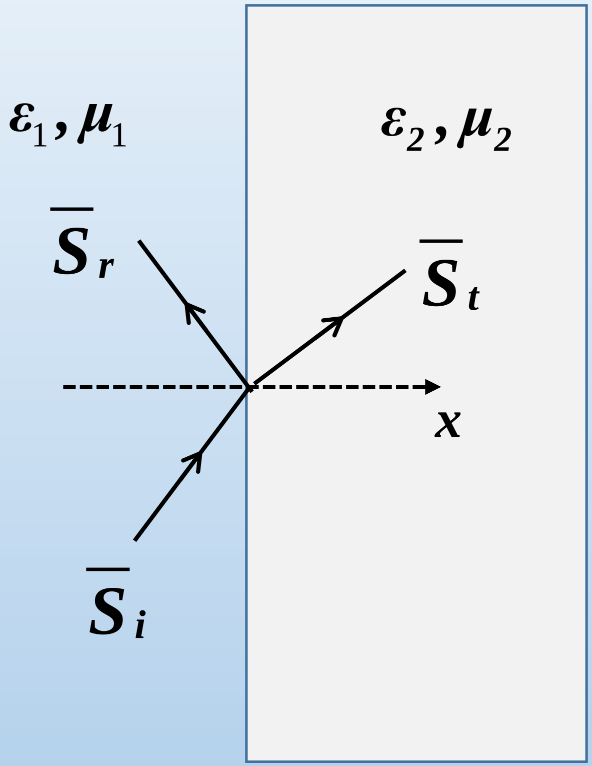
TM-polarization

$$R^{TM} = \frac{\varepsilon_2 k_{ix} - \varepsilon_1 k_{tx}}{\varepsilon_2 k_{ix} + \varepsilon_1 k_{tx}} = \frac{\eta_1 \cos\theta_i - \eta_2 \cos\theta_t}{\eta_1 \cos\theta_i + \eta_2 \cos\theta_t}$$

$$T^{TM} = \frac{2\varepsilon_2 k_{ix}}{\varepsilon_2 k_{ix} + \varepsilon_1 k_{tx}} = \frac{2\eta_1 \cos\theta_i}{\eta_1 \cos\theta_i + \eta_2 \cos\theta_t}$$

Energy Transport

$$\vec{S}_r(t) = \vec{E}(t) \times \vec{H}(t)$$



TE-polarization

$$r = \frac{-\hat{x} \cdot \langle \vec{S}_r \rangle}{\hat{x} \cdot \langle \vec{S}_i \rangle} = \left| R^{TE} \right|^2$$

$$t = \frac{\hat{x} \cdot \langle \vec{S}_t \rangle}{\hat{x} \cdot \langle \vec{S}_i \rangle} = \frac{\eta_1 \cos\theta_t}{\eta_2 \cos\theta_i} \left| T^{TE} \right|^2$$

$$\boxed{\begin{aligned} R^{TE} &= E_r / E_i \\ T^{TE} &= E_t / E_i \end{aligned}}$$

TM-polarization

$$r = \frac{-\hat{x} \cdot \langle \vec{S}_r \rangle}{\hat{x} \cdot \langle \vec{S}_i \rangle} = \left| R^{TM} \right|^2$$

$$t = \frac{\hat{x} \cdot \langle \vec{S}_t \rangle}{\hat{x} \cdot \langle \vec{S}_i \rangle} = \frac{\eta_2 \cos\theta_t}{\eta_1 \cos\theta_i} \left| T^{TM} \right|^2$$

$$\boxed{\begin{aligned} R^{TM} &= H_r / H_i \\ T^{TM} &= H_t / H_i \end{aligned}}$$

***r*: reflectivity**
***t*: transmission**

Energy Conservation

TE-polarization

$$r+t=\left|\frac{\eta_2\cos\theta_i-\eta_1\cos\theta_t}{\eta_2\cos\theta_i+\eta_1\cos\theta_t}\right|^2+\frac{\eta_1\cos\theta_t}{\eta_2\cos\theta_i}\left|\frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i+\eta_1\cos\theta_t}\right|^2=1$$

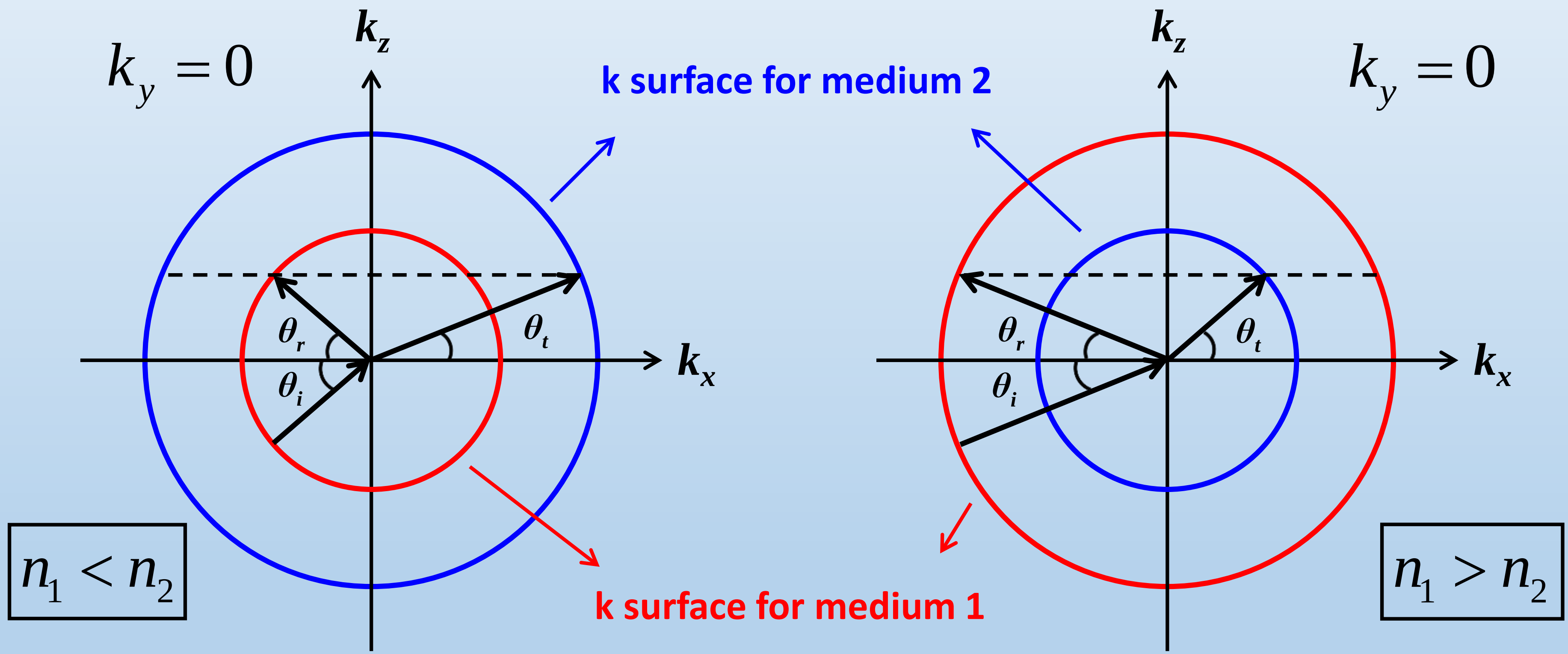
TM-polarization

$$r+t=\left|\frac{\eta_1\cos\theta_i-\eta_2\cos\theta_t}{\eta_1\cos\theta_i+\eta_2\cos\theta_t}\right|^2+\frac{\eta_2\cos\theta_t}{\eta_1\cos\theta_i}\left|\frac{2\eta_1\cos\theta_i}{\eta_1\cos\theta_i+\eta_2\cos\theta_t}\right|^2=1$$

Phase Matching

Phase matching condition: $\boxed{k_{iz} = k_{rz} = k_{tz}}$

k surface: $k_x^2 + k_y^2 + k_z^2 = \omega^2 \mu \varepsilon = n^2 k_0^2$



Snell's law: $\begin{cases} k_{iz} = k_{rz} \\ k_{ix} = -k_{rx} \end{cases} \Rightarrow \theta_i = \theta_r$

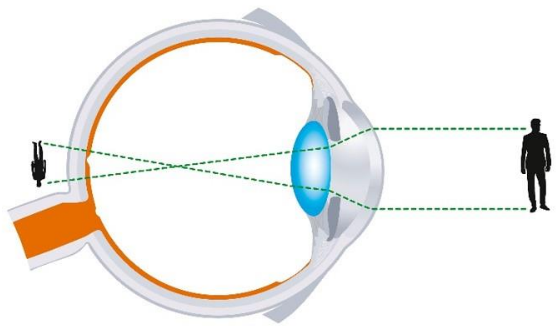
$$\frac{\sin\theta_i}{\sin\theta_t} = \frac{k_{iz}/k_i}{k_{tz}/k_t} = \frac{k_t}{k_i} = \frac{n_2}{n_1}$$

History of Snell's Law

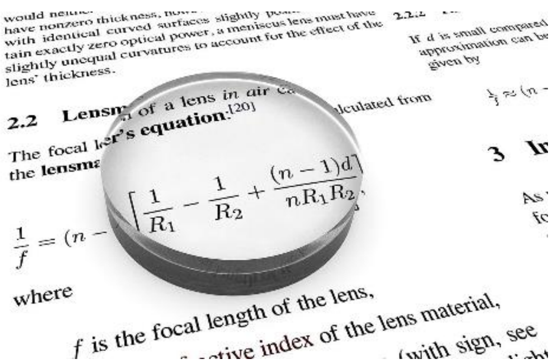
- **Snell's Law describing refraction was first recorded by Ptolemy in 140 A.D**
- **First described by relationship by Snell in 1621**
- **First explained in 1662 by Fermat's principle of least time.**

EXAMPLES:

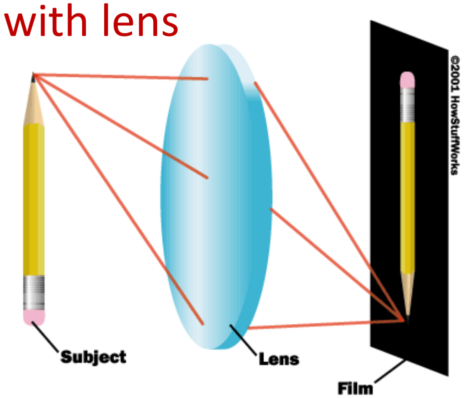
Eyes

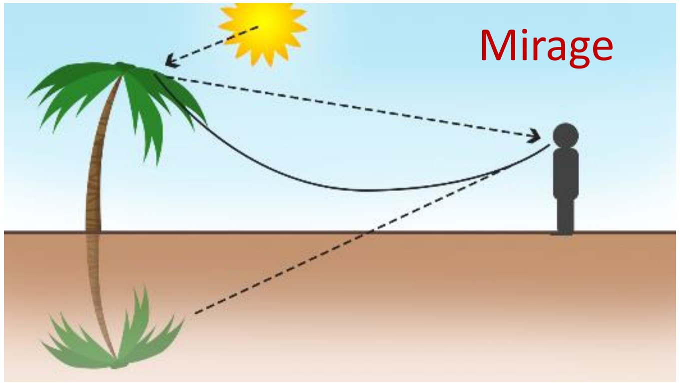


Magnifying lens



Imaging with lens

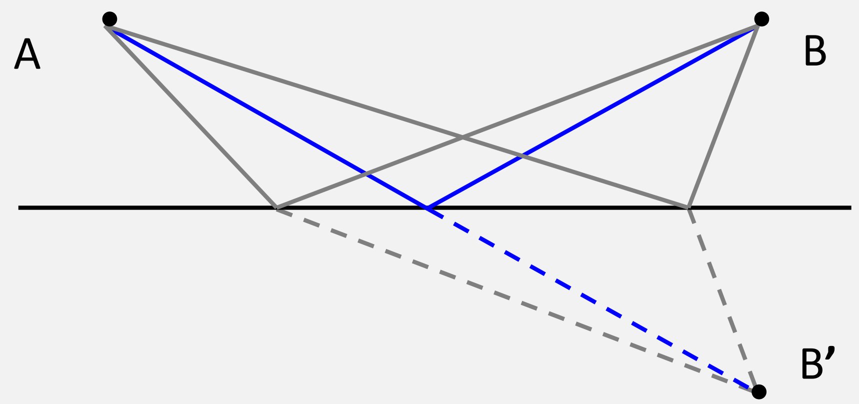




Fermat's Principle of Least Time

Fermat's principle of minimum time argues that light will travel from one point to another along a path that requires the minimum time.

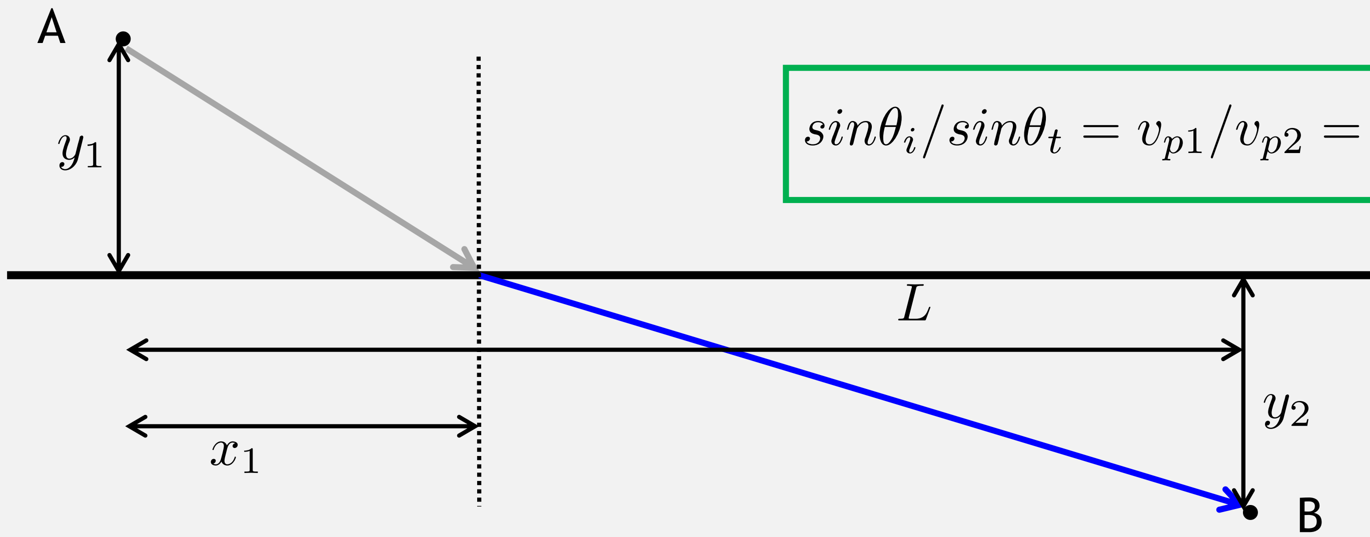
Applied to Reflection



Since it is straight, the blue path is the shortest path from A to B'. So, the blue path is also the shortest reflecting path to B since it images the path to B'. For the blue path, the incidence and reflection angles equal.

Fermat's Principle of Least Time

Applied to Refraction



$$sin\theta_i/sin\theta_t = v_{p1}/v_{p2} = n_2/n_1$$

The time t require for light to travel from A to B is given by

$$t = \frac{\sqrt{x_1^2 + y_1^2}}{v_1} + \frac{\sqrt{((L - x_1)^2 + y_2^2)}}{v_2}$$

From $dt/dx_1 = 0$**, it follows that**

$$\frac{x_1}{v_1\sqrt{(x_1^2 + y_1^2)}} = \frac{x_2}{v_2\sqrt{((L - x_1)^2 + y_2^2)}} \Rightarrow sin\theta_i/sin\theta_t = v_{p1}/v_{p2} = n_2/n_1$$

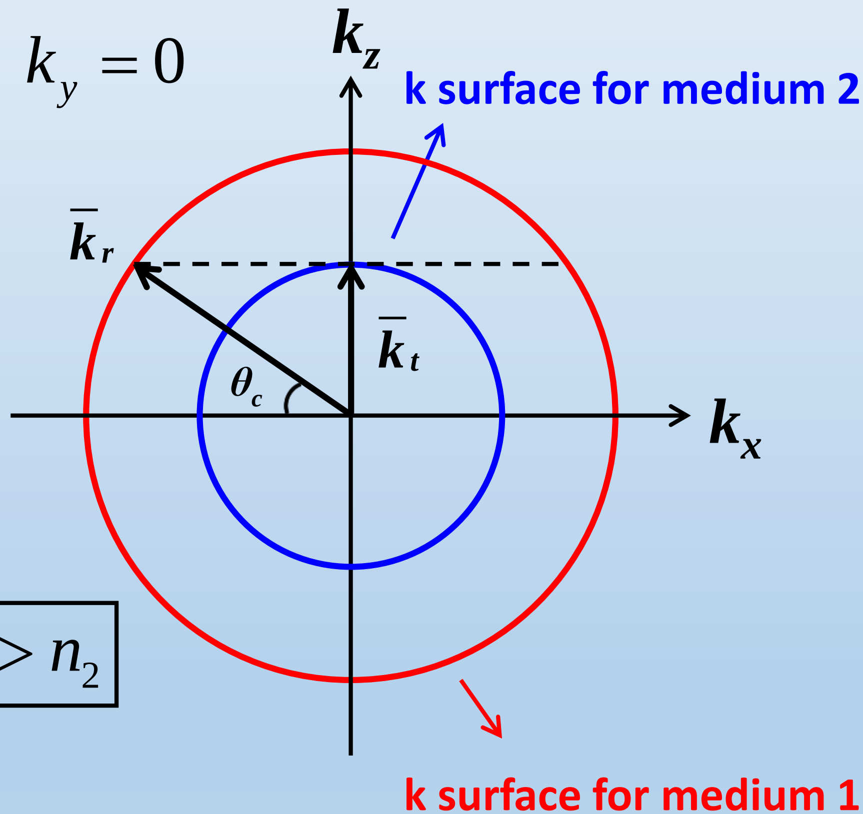
Total Reflection and Critical angle

Phase matching condition: $\boxed{k_{iz} = k_{rz} = k_{tz}}$

$$n_1 > n_2, k_{iz} > k_t \ (\theta_i > \theta_c)$$

k surface: $k_x^2 + k_y^2 + k_z^2 = \omega^2 \mu\varepsilon = n^2 k_0^2$

$$k_y = 0$$



$$\boxed{n_1 > n_2}$$

$$k_{tx} = \sqrt{k_t^2 - k_{iz}^2} = -jk''_{tx}$$

(purely imaginary)

$$\langle \bar{S}_t \rangle = \hat{z} \frac{k_z}{2\omega\varepsilon_t} |T^{TM}|^2 e^{-2k''_{tx}x} \quad \textbf{(TM waves)}$$

$$\langle \bar{S}_t \rangle = \hat{z} \frac{k_z}{2\omega\mu_t} |T^{TE}|^2 e^{-2k''_{tx}x} \quad \textbf{(TE waves)}$$

$$\langle \bar{S}_t \rangle = \frac{1}{2} \mathrm{Re} \left\{ \frac{\bar{k}_t}{\omega\varepsilon_t} |T^{TM}|^2 e^{-j(k_{tx} - k_{tx}^*)x} \right\}$$

Critical angle: $\theta_c = \sin^{-1} \dfrac{k_t}{k_i} = \sin^{-1} \dfrac{n_2}{n_1}$

No power transmitted in the x direction into the region t

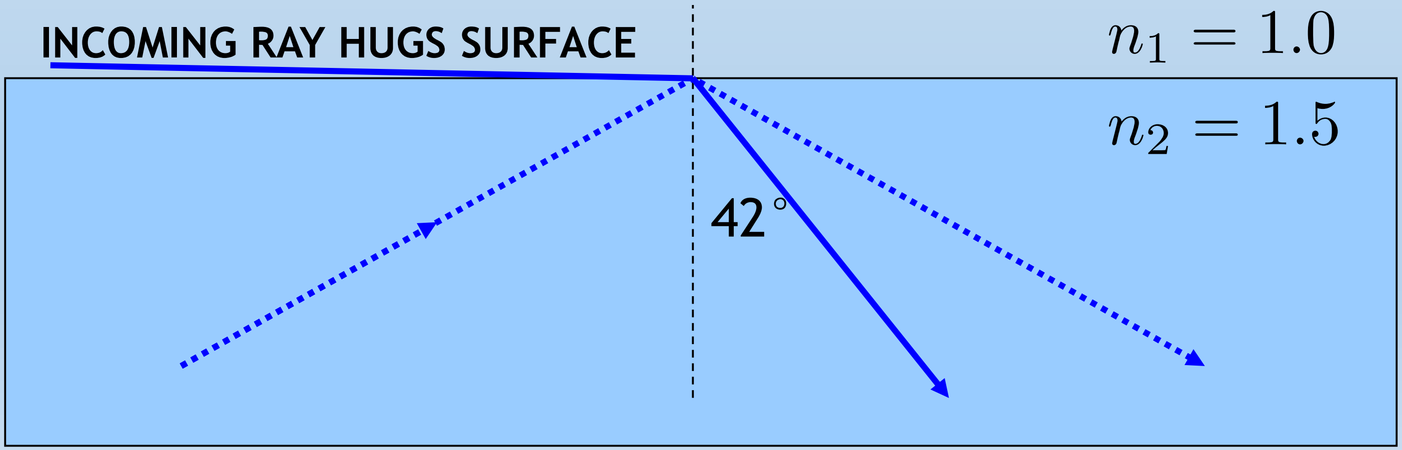
Phenomena of Total Internal Reflection

Beyond the critical angle θ_c , a ray within the higher index medium cannot escape at shallower angles

$$n_2 sin\theta_2 = n_1 sin\theta_1 \quad \theta_c = sin^{-1}(n_1/n_2)$$

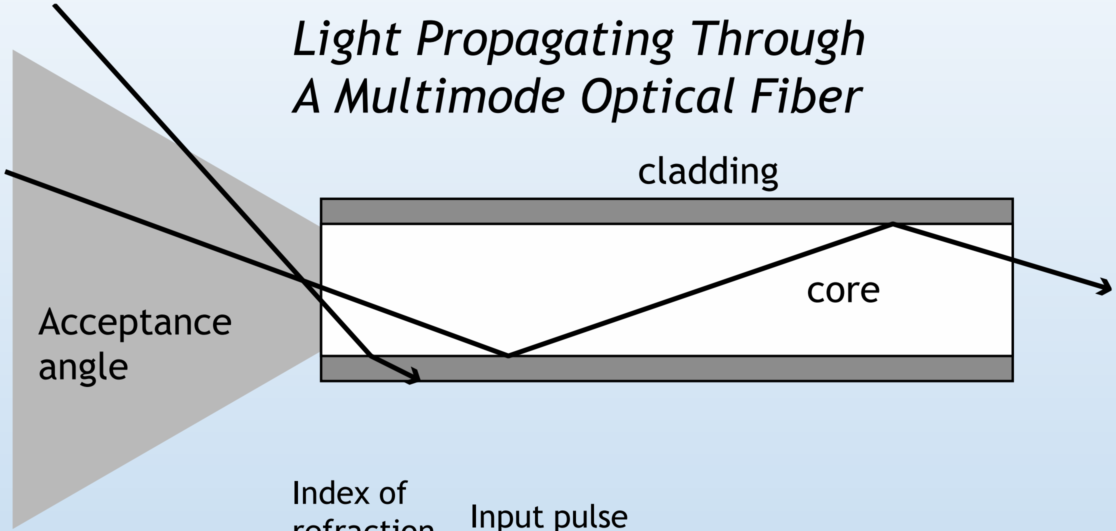
For glass (n=1.5), the critical internal angle is 42°

For water (n=1.33), it is 49°

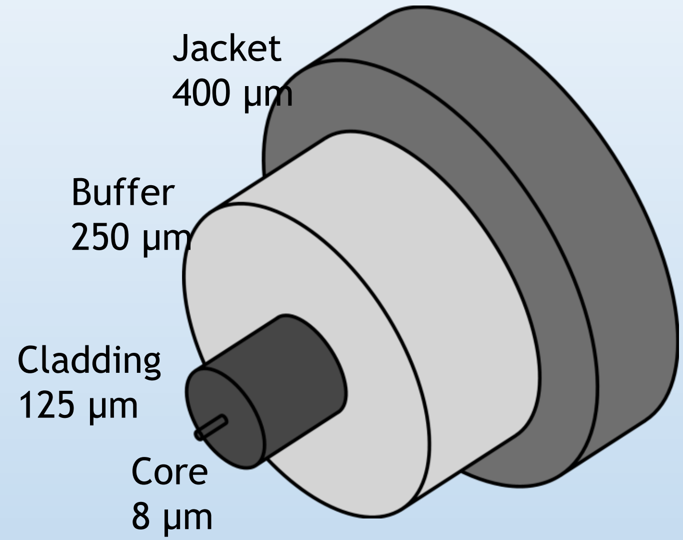


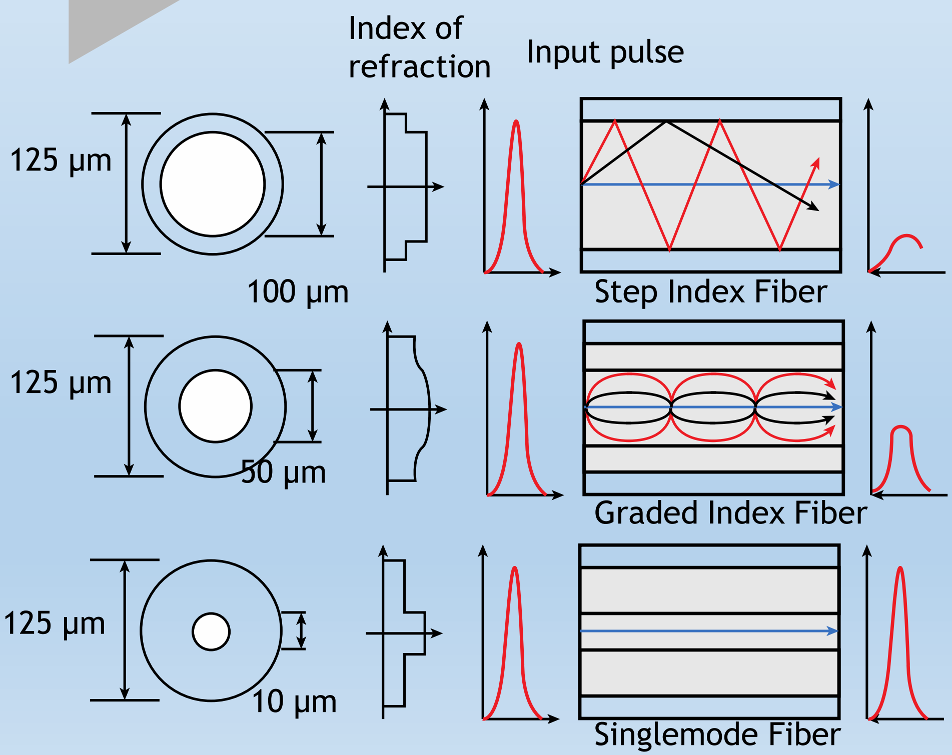
Optical Fibers

Light Propagating Through A Multimode Optical Fiber



Single Mode Fiber Structure





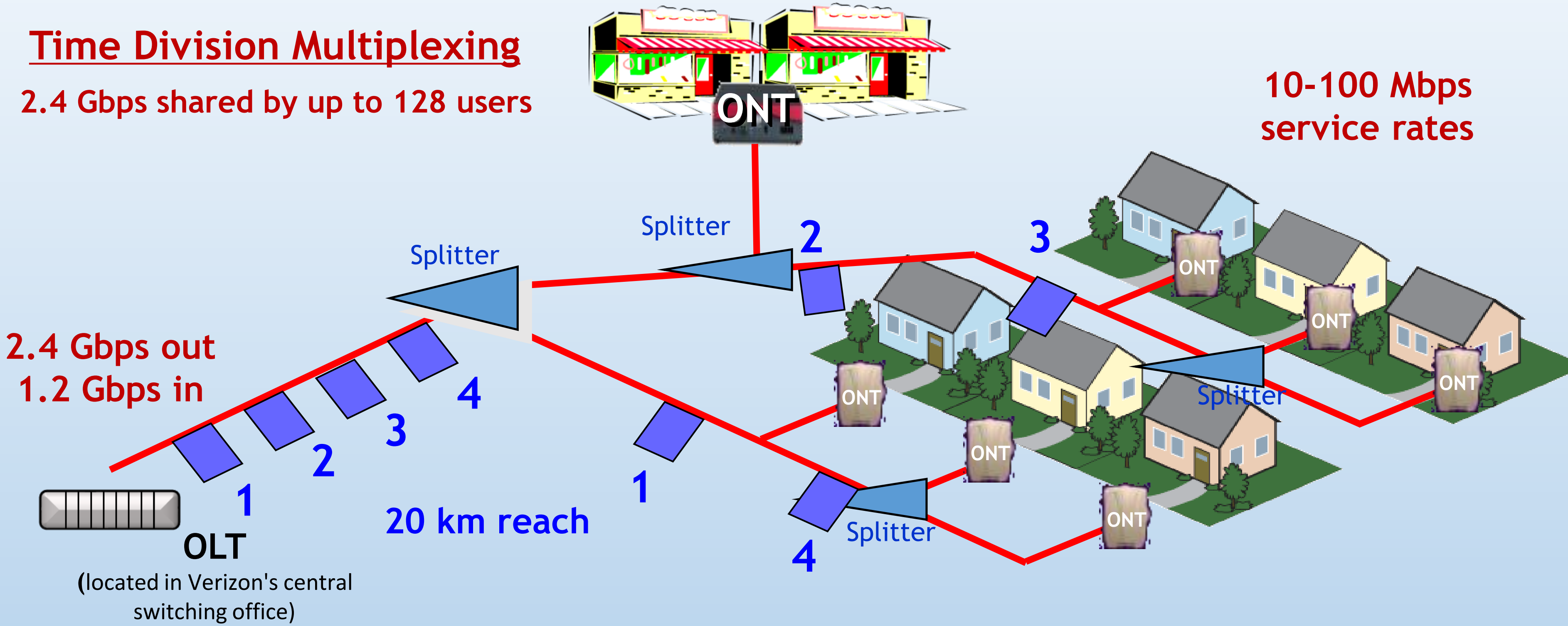
Modal dispersion is a distortion mechanism occurring in multimode fibers and other waveguides, in which the signal is spread in time because the propagation velocity of the optical signal is not the same for all modes

Fiber to the Home

Time Division Multiplexing

2.4 Gbps shared by up to 128 users

10-100 Mbps service rates



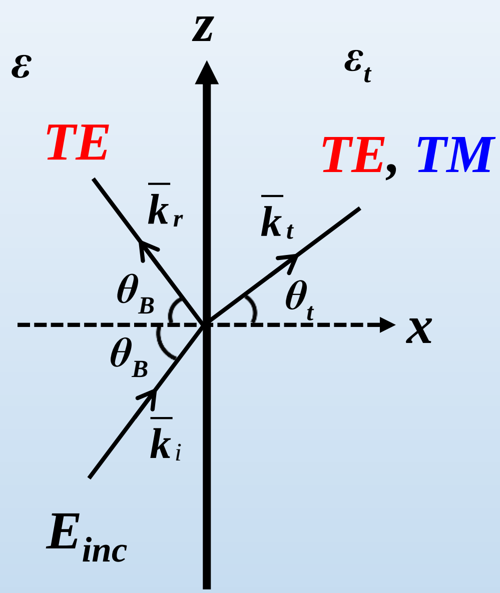
An ONT (Optical Network Terminal) is a media converter that is installed by Verizon either outside or inside your premises, during FiOS installation. The ONT converts fiber-optic light signals to copper/electric signals. Three wavelengths of light are used between the ONT and the OLT (Optical Line Terminal):

- λ = 1310 nm voice/data transmit
- λ = 1490 nm voice/data receive

Each ONT is capable of delivering:
Multiple POTS (plain old telephone service) lines, Internet data, Video

Image by Raj from Chennai, India
http://commons.wikimedia.org/wiki/File:Strings_of_lights.jpg
on Wikimedia Commons

Total Transmission and Brewster Angle



(TM waves)

If $\eta_1 \cos\theta_i = \eta_2 \cos\theta_t$

$$R^{TM} = \frac{\eta_1 \cos\theta_i - \eta_2 \cos\theta_t}{\eta_1 \cos\theta_i + \eta_2 \cos\theta_t} = 0$$

$$T^{TM} = \frac{2\eta_1 \cos\theta_i}{\eta_1 \cos\theta_i + \eta_2 \cos\theta_t} = 1$$

$$\Rightarrow \quad \frac{\cos\theta_i}{\cos\theta_t} = \frac{n_1}{n_2}$$

$$\frac{\sin\theta_i}{\sin\theta_t} = \frac{n_2}{n_1}$$

(Snell Law)

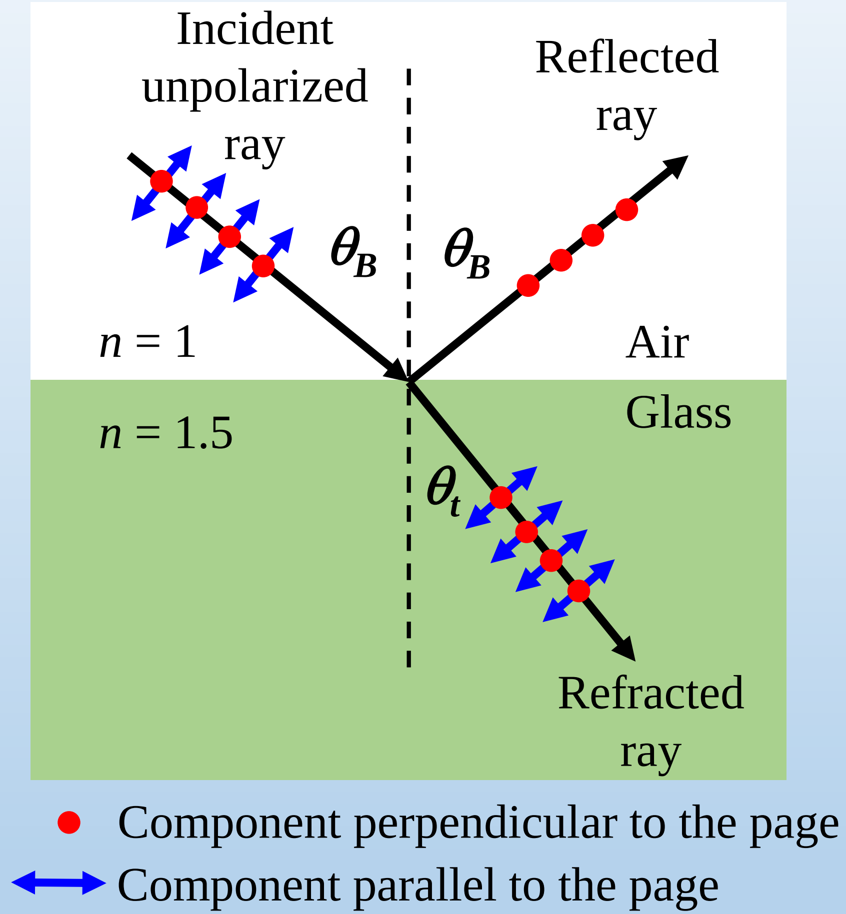
$$\Rightarrow \quad \boxed{\theta_i + \theta_t = \frac{\pi}{2}}$$

Brewster Angle: θ_B

Polarization by reflection

Different polarization of light get reflected and refracted with different amplitudes.

At one particular angle, the parallel polarization is NOT reflected at all! This is the "Brewster angle" θ_B, and $\theta_B + \theta_t = 90^o$.



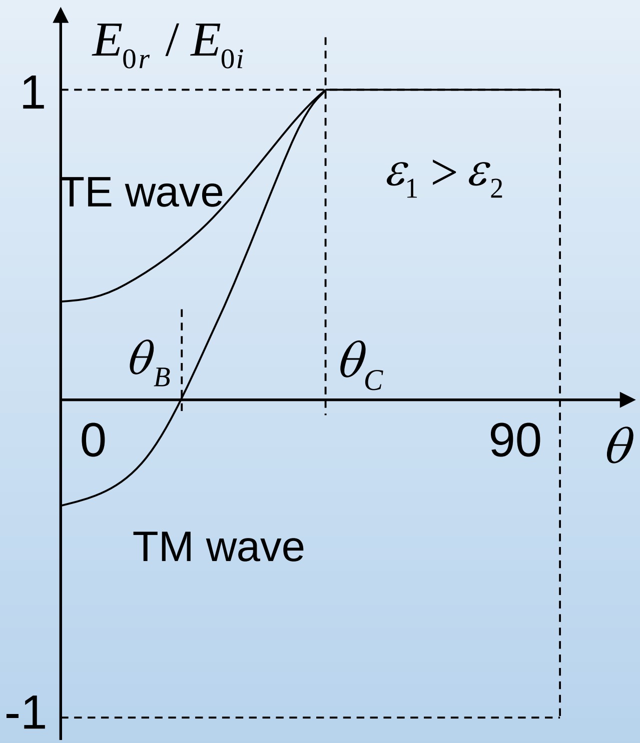
$$n_1 \sin\theta_B = n_2 \sin(90^o - \theta_B) = n_2 \cos\theta_B$$

$$\boxed{\tan\theta_B = \frac{n_2}{n_1}}$$

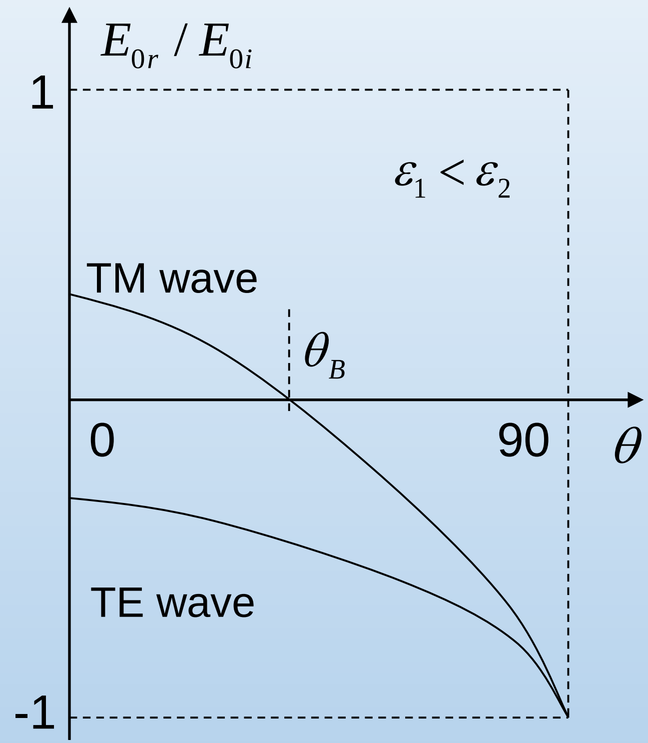
Polarizing Filter Camera

Photographs taken of a window with a camera polarizer filter rotated to two different angles. In the picture at left, the polarizer is aligned with the polarization angle of the window reflection. In the picture at right, the polarizer has been rotated 90° eliminating the heavily polarized reflected sunlight.

Reflection at Medium interface-Summary

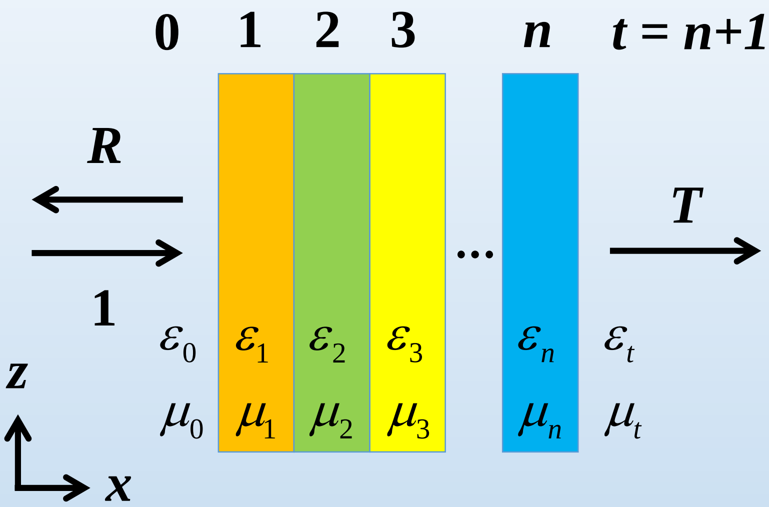


Internal reflection



External reflection

Reflection and Transmission by a Layered Medium



The total field in region *l*:

$$\bar{k}_l = \hat{x}k_{lx} + \hat{z}k_z$$

$$\bar{E}_l = \frac{1}{j\omega\varepsilon_l}\nabla\times\bar{H}_l$$

$$\bar{H}_l = \hat{y}\left(\underbrace{A_l e^{-jk_{lx}x}}_{\text{forward}} + \underbrace{B_l e^{jk_{lx}x}}_{\text{backward}}\right)e^{-jk_z z}$$

In region 0:

$$\begin{cases} A_0 = 1 \\ B_0 = R \end{cases}$$

In region *t*

$$\begin{cases} A_t = T \\ B_t = 0 \end{cases}$$

Boundary conditions:

$$\hat{n}\times\left(\bar{E}_l - \bar{E}_{l+1}\right) = 0$$

$$\hat{n}\times\left(\bar{H}_l - \bar{H}_{l+1}\right) = 0$$

Unknown parameters (2n+2):

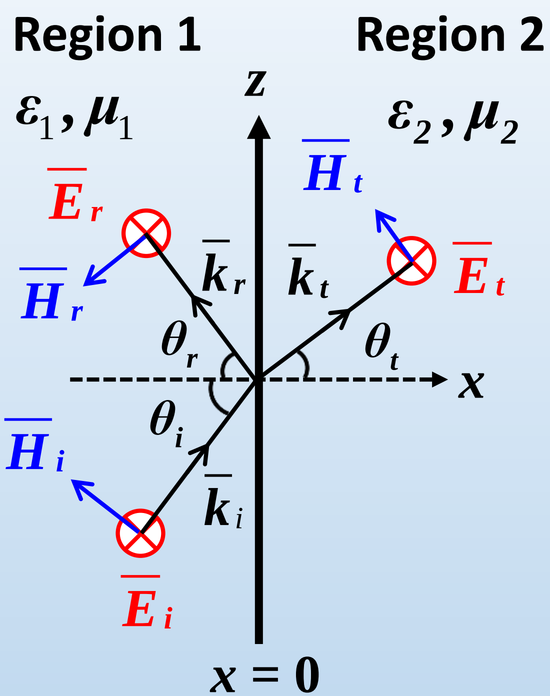
$$\begin{cases} A_l,\ l = 1,2,\cdots,n+1 \\ B_l,\ l = 0,1,2,\cdots,n \end{cases}$$

(2n+2) unknowns & (2n+2) equations

Solve all A_l, B_l

Appendix

Reflection and Transmission of TE Waves



Wave vectors:

$$\overline{k}_i = \hat{x}k_{ix} + \hat{z}k_{iz}$$

$$\overline{k}_r = \hat{x}k_{rx} + \hat{z}k_{rz}$$

$$\overline{k}_t = \hat{x}k_{tx} + \hat{z}k_{tz}$$

Incident wave:

$$\boxed{k_{ix}, k_{iz}}$$

$$\overline{E}_i = \hat{y}\exp\left(-j\overline{k}_i \cdot \overline{r}\right)$$

$$\overline{H}_i = \frac{1}{\omega\mu_1}\overline{k}_i \times \overline{E}_i = \frac{1}{\omega\mu_1}\left(-\hat{x}k_{iz} + \hat{z}k_{ix}\right)\exp\left(-j\overline{k}_i \cdot \overline{r}\right)$$

$$\overline{S}_i = \overline{E}_i \times \overline{H}_i^* = \overline{k}_i \frac{1}{\omega\mu_1}\left|\overline{E}_i\right|^2$$

Reflected wave:

$$\boxed{k_{rx}, k_{rz}, R^{TE}}$$

$$\overline{E}_r = \hat{y}R^{TE}\exp\left(-j\overline{k}_r \cdot \overline{r}\right)$$

$$\overline{H}_r = \frac{1}{\omega\mu_1}\overline{k}_r \times \overline{E}_r = \frac{1}{\omega\mu_1}\left(-\hat{x}k_{rz} + \hat{z}k_{rx}\right)R^{TE}\exp\left(-j\overline{k}_r \cdot \overline{r}\right)$$

$$\overline{S}_r = \overline{E}_r \times \overline{H}_r^* = \overline{k}_r \frac{1}{\omega\mu_1}\left|\overline{E}_r\right|^2$$

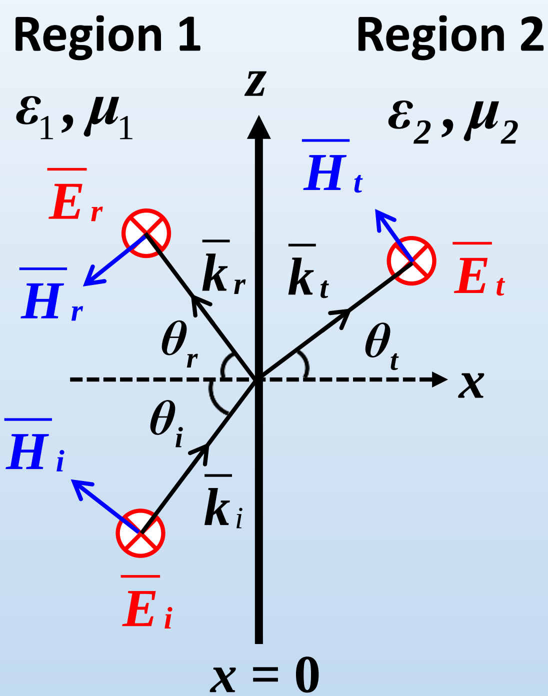
Transmitted wave:

$$\boxed{k_{tx}, k_{tz}, T^{TE}}$$

$$\overline{E}_t = \hat{y}T^{TE}\exp\left(-j\overline{k}_t \cdot \overline{r}\right)$$

$$\overline{H}_t = \frac{1}{\omega\mu_2}\overline{k}_t \times \overline{E}_t = \frac{1}{\omega\mu_2}\left(-\hat{x}k_{tz} + \hat{z}k_{tx}\right)T^{TE}\exp\left(-j\overline{k}_t \cdot \overline{r}\right)$$

$$\overline{S}_t = \overline{E}_t \times \overline{H}_t^* = \overline{k}_t^* \frac{1}{\omega\mu_2}\left|\overline{E}_t\right|^2$$



Boundary conditions

$$\hat{n}\times\left(\overline{E}_1-\overline{E}_2\right)=0$$

$$\hat{n}\times\left(\overline{H}_1-\overline{H}_2\right)=0$$

➡ **Tangential components of E and H are continuous**

Continuity of E_y at x = 0:

$$e^{-jk_{iz}z}+R^{TM}e^{-jk_{rz}z}=T^{TM}e^{-jk_{tz}z}$$

for All z

$$\boxed{k_{iz}=k_{rz}=k_{tz}}$$

→ **Phase matching condition**

$$k_{rx}=-k_{ix},\, k_{tx}=\sqrt{k_t^2-k_{iz}^2}$$

Dispersion relations

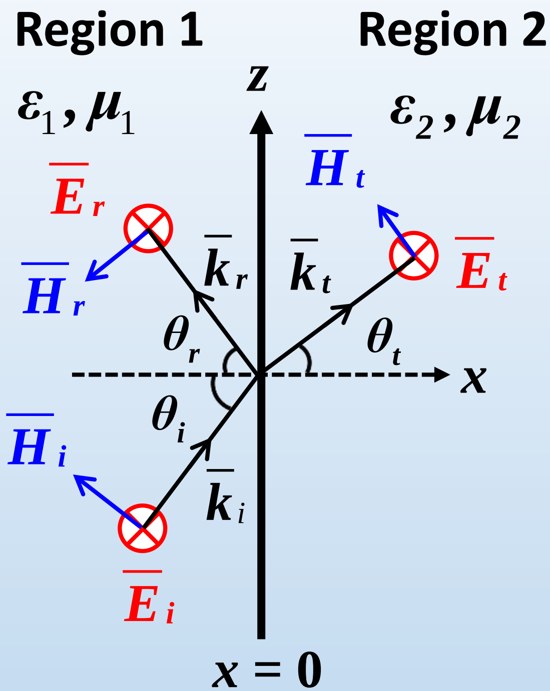
$$k_{ix}^2+k_{iz}^2=\omega^2\mu_1\varepsilon_1=n_1^2k_0^2$$

$$k_{rx}^2+k_{rz}^2=\omega^2\mu_1\varepsilon_1=n_1^2k_0^2$$

$$k_{tx}^2+k_{tz}^2=\omega^2\mu_2\varepsilon_2=n_2^2k_0^2$$

Continuity of H_z at x = 0:

$$\frac{k_{ix}}{\mu_1}\left(1-R^{TE}\right)=\frac{k_{tx}}{\mu_2}T^{TE}$$



Boundary conditions at x = 0 give:

$$1 + R^{TE} = T^{TE}$$

$$\frac{k_{ix}}{\mu_1}\left(1 - R^{TE}\right) = \frac{k_{tx}}{\mu_2} T^{TE}$$

$$R^{TE} = R_{0t}^{TE} = \frac{\mu_2 k_{ix} - \mu_1 k_{tx}}{\mu_2 k_{ix} + \mu_1 k_{tx}}$$ → **Reflection coefficient**

$$T^{TE} = T_{0t}^{TE} = \frac{\mu_2 k_{ix}}{\mu_2 k_{ix} + \mu_1 k_{tx}}$$ → **Transmission coefficient**

Wave vectors:

$$\overline{k}_i = \hat{x}k_{ix} + \hat{z}k_{iz}$$

$$\overline{k}_r = \hat{x}k_{rx} + \hat{z}k_{rz}$$

$$\overline{k}_t = \hat{x}k_{tx} + \hat{z}k_{tz}$$

Time-averaged Poynting power vectors:

$$\langle \overline{S}_i \rangle = \frac{1}{2}\mathrm{Re}\left\{\frac{\overline{k}_i}{\omega\mu_1}\right\} \qquad \langle \overline{S}_r \rangle = \frac{1}{2}\mathrm{Re}\left\{\frac{\overline{k}_r}{\omega\mu_1}\left|R^{TE}\right|^2\right\}$$

$$\langle \overline{S}_t \rangle = \frac{1}{2}\mathrm{Re}\left\{\frac{\overline{k}_t^*}{\omega\mu_2^*}\left|T^{TE}\right|^2 e^{-j\left(k_{tx} - k_{tx}^*\right)x}\right\}$$

实验作业

通过MATLAB、 COMSOL等软件来仿真如下的实例。

第六章 反射与透射：

仿真一维单频率TE波正入射到介质，其反射与透射的情况；

仿真TM波情况，并尝试改变入射角度以观察临界角和布鲁斯特角；

仿真正入射到多层介质的反射透射特性（选做）。