



Devices always involve interfaces:

1. metal-semiconductor

- *Schottky Junction*
- *Ohmic Contact*

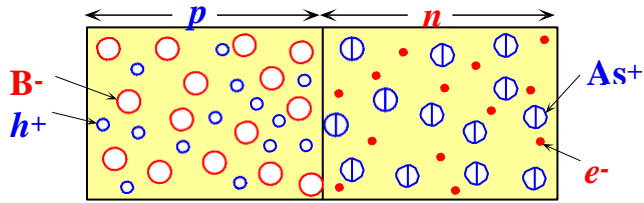
2. semiconductor-semiconductor

- *pn Junction (1)*
- *pn Junction (2)*
- ***Tutorial***

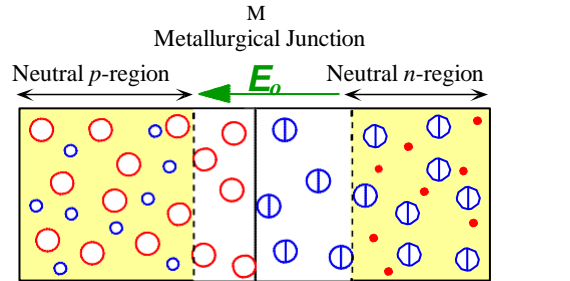
Metal-Oxide-Semiconductor Field-Effect Transistor

(MOSFET, 金属氧化物半导体场效应晶体管)

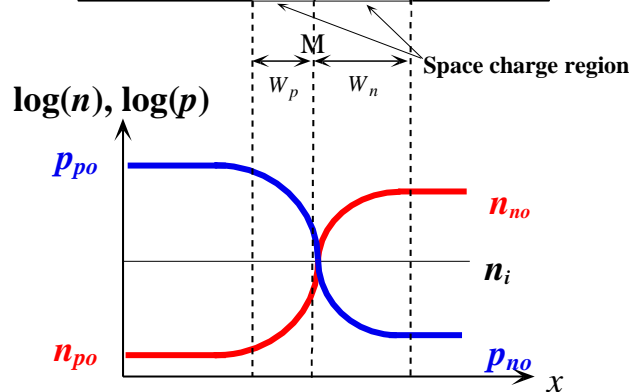
pn Junction: Space Charge Layer



Each electron moves over the interface will combine with one hole.



The total number of negative charge on p side equals to that of positive charge on n side to remain charge neutrality

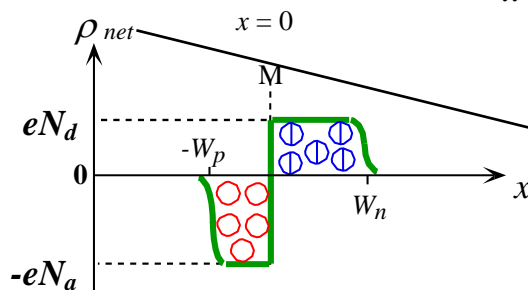


p_{p0} : majority carrier concentration

n_{p0} : minority carrier concentration

$$N_a W_p = N_d W_n$$

N : doping concentration, W : space charge layer width



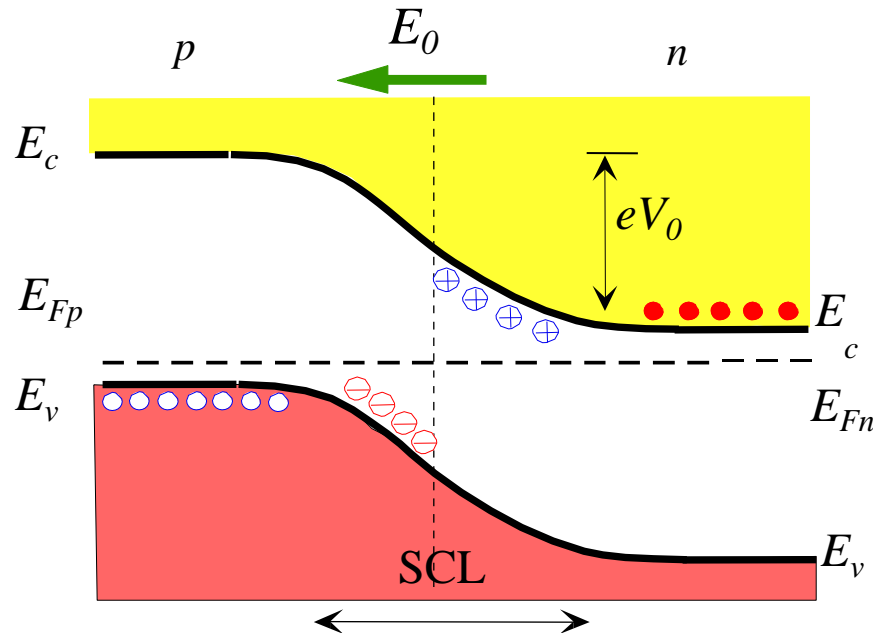
ρ_{net} : net space charge density (净空间电荷密度)

pn Junction: Space Charge Layer

Probability of electrons occupying energy E is determined by **Fermi-Dirac statistics**, which is reduced to **Boltzmann statistics** when $E - E_F \gg k_B T$

$$f(E) = A \exp[-(E - E_F)/k_B T]$$

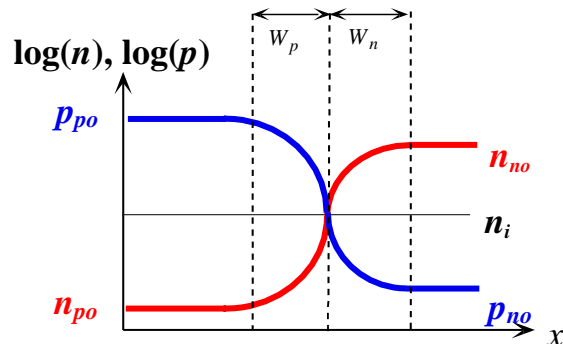
$$\frac{n_2}{n_1} = \exp\left[-\frac{(E_2 - E_1)}{kT}\right]$$



$$\frac{n_{po}}{n_{no}} = \exp\left(-\frac{eV_o}{k_B T}\right)$$

$$\frac{p_{no}}{p_{po}} = \exp\left(-\frac{eV_o}{k_B T}\right)$$

$$p_{po} = N_a \quad p_{no} = \frac{n_i^2}{n_{no}} = \frac{n_i^2}{N_d}$$



$$V_o = \frac{k_B T}{e} \ln\left(\frac{N_a N_d}{n_i^2}\right)$$

$$W_o = \sqrt{\frac{2\epsilon_o \epsilon_r (N_a + N_d) V_o}{e N_a N_d}}$$

Diffusion

Electron diffusion current density

$$\begin{aligned} J_{D,e} &\propto -\frac{dn}{dx} \\ &= eD_e \frac{dn}{dx} \end{aligned}$$

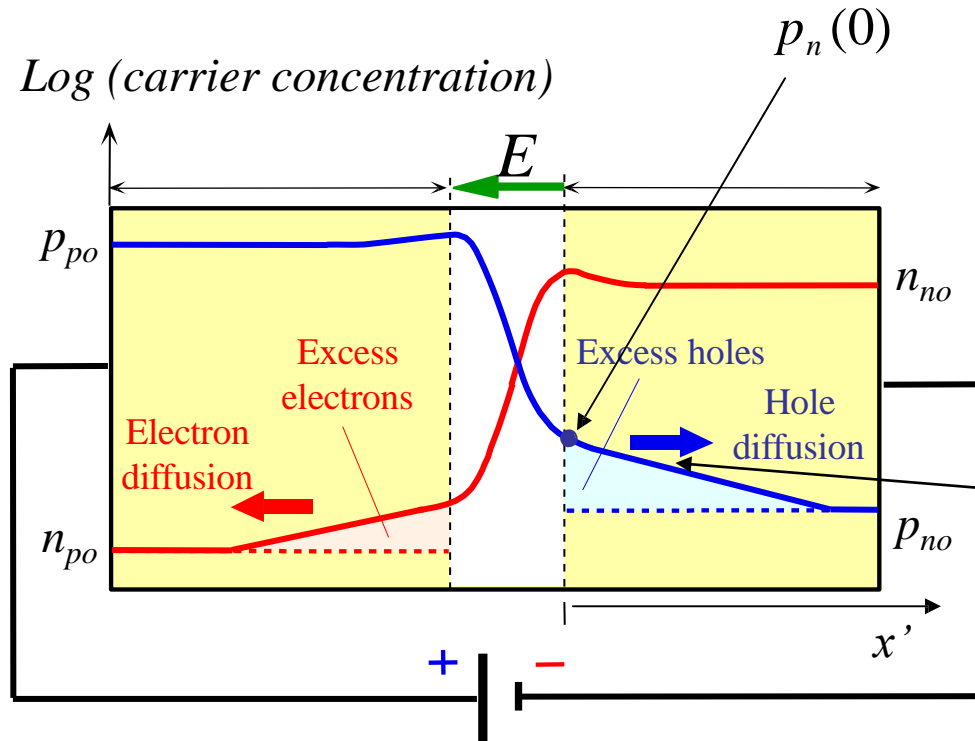
Hole diffusion current density

$$\begin{aligned} J_{D,h} &\propto -\frac{dp}{dx} \\ &= -eD_h \frac{dp}{dx} \end{aligned}$$

Diffusion coefficient, D_e or D_h , is a measure of the ease of carrier diffusion motion in a medium. Mobility, μ_n or μ_p , is a measure of the ease of carrier drift motion in a medium. The two quantities are related by the **Einstein Relation**.

$$\frac{D_e}{\mu_e} = \frac{kT}{e} \quad \text{and} \quad \frac{D_h}{\mu_h} = \frac{kT}{e}$$

Current Across a **Forward Biased** *pn* Junction: **Diffusion**



Law of the junction

$$p_n(0) = p_{no} \exp\left[\frac{eV}{k_B T}\right]$$

$$J_{D,hole} = -eD_h \frac{dp_n(x')}{dx'}$$

$$\Delta p_n(x') = \Delta p_n(0) \exp\left(-\frac{x'}{L_h}\right)$$

$$\Delta p_n(x') = p_n(x') - p_{no}$$

$$J_{D,hole} = \left(\frac{eD_h}{L_h}\right) \Delta p_n(0) \exp\left(-\frac{x'}{L_h}\right)$$

At $x'=0$: $J_{D,hole} = \left(\frac{eD_h}{L_h}\right) \Delta p_n(0)$

$$J_{D,hole} = \left(\frac{eD_h p_{no}}{L_h}\right) \left[\exp\left(\frac{eV}{k_B T}\right) - 1\right]$$

$$p_{no} = \frac{n_i^2}{n_{no}} = \frac{n_i^2}{N_d}$$

Minority Carrier **Diffusion Length**

$$L_h = \sqrt{D_h \tau_h}$$

τ_h is the mean hole **recombination lifetime** (minority carrier lifetime) in the n-region

Total Diffusion Current: Electron and Hole

$$J = J_{\text{elec}} + J_{\text{hole}}$$

$$J_{D,\text{elec}} = \left(\frac{eD_e n_i^2}{L_e N_a} \right) \left[\exp\left(\frac{eV}{k_B T}\right) - 1 \right]$$

$$J_{D,\text{hole}} = \left(\frac{eD_h n_i^2}{L_h N_d} \right) \left[\exp\left(\frac{eV}{k_B T}\right) - 1 \right]$$

$$J = \left(\frac{eD_e}{L_e N_a} + \frac{eD_h}{L_h N_d} \right) n_i^2 \left[\exp\left(\frac{eV}{k_B T}\right) - 1 \right]$$

$$J = J_{so} \left[\exp\left(\frac{eV}{kT}\right) - 1 \right]$$

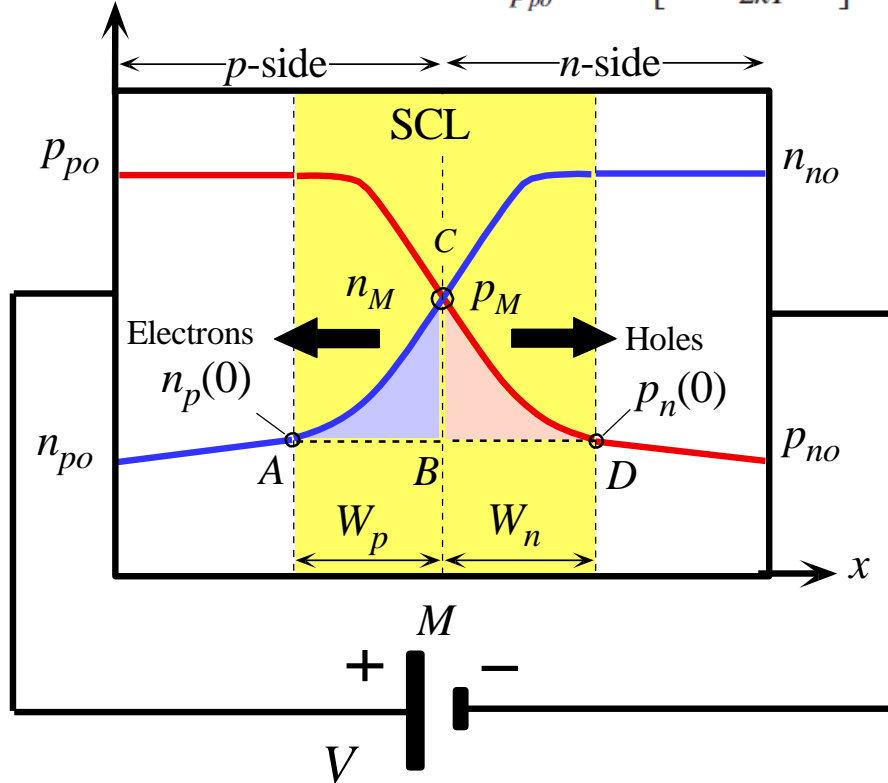
**Ideal diode
(Shockley) equation**

$$J_{so} = \left[\left(\frac{eD_h}{L_h N_d} \right) + \left(\frac{eD_e}{L_e N_a} \right) \right] n_i^2$$

reverse saturation
current density

Current Across a Forward Biased pn Junction: Recombination

Log (carrier concentration) $\frac{p_M}{p_{po}} = \exp\left[-\frac{e(V_o - V)}{2kT}\right] \Rightarrow p_M = n_i \exp\left(\frac{eV}{2kT}\right)$



$$J_{recom} \approx \frac{e \frac{1}{2} W_p n_M}{\tau_e} + \frac{e \frac{1}{2} W_n p_M}{\tau_h}$$

mean electron recombination time
in W_p is τ_e .

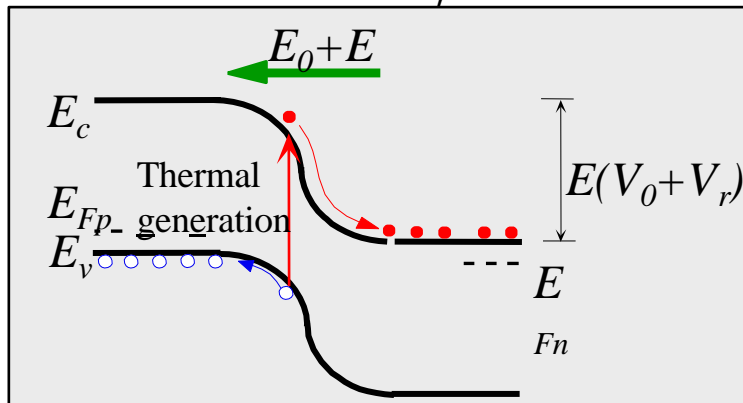
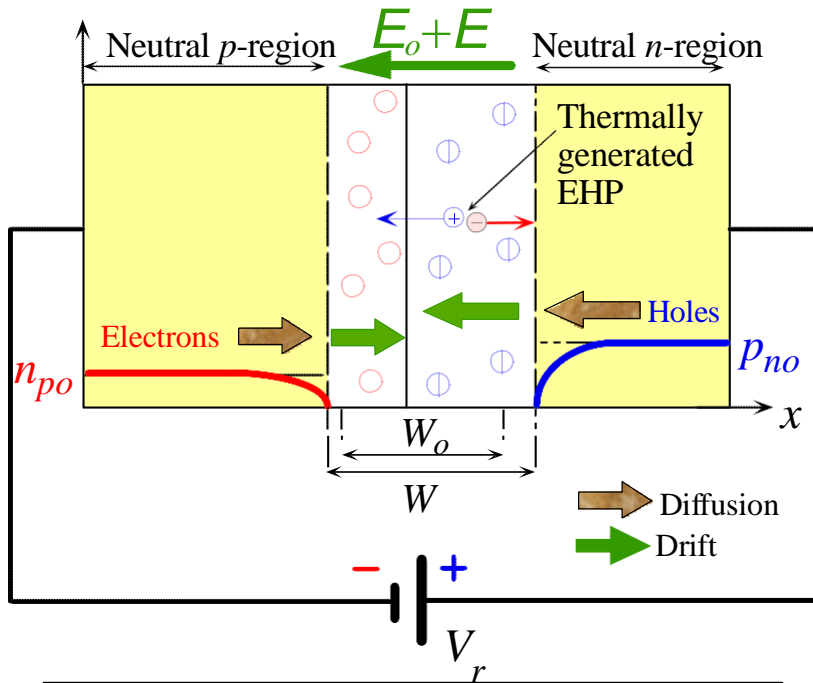
$$J_{recom} = J_{ro} \left[\exp\left(\frac{eV}{2k_B T}\right) - 1 \right]$$

where $J_{ro} = \frac{en_i}{2} \left(\frac{W_p}{\tau_e} + \frac{W_n}{\tau_h} \right)$

Forward biased pn junction: the injection of carriers and their recombination in the SCL

Current Across a pn Junction: **Reverse Bias**

Minority Carrier



(a) Minority carrier extracted and swept by the field across the SCL

Essentially **Shockley equation**:

$$J = \left(\frac{eD_e}{L_e N_a} + \frac{eD_h}{L_h N_d} \right) n_i^2 \left[\exp\left(\frac{eV}{k_B T}\right) - 1 \right]$$

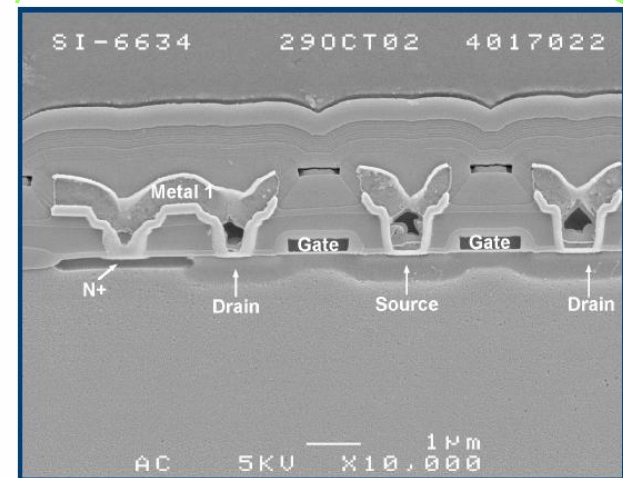
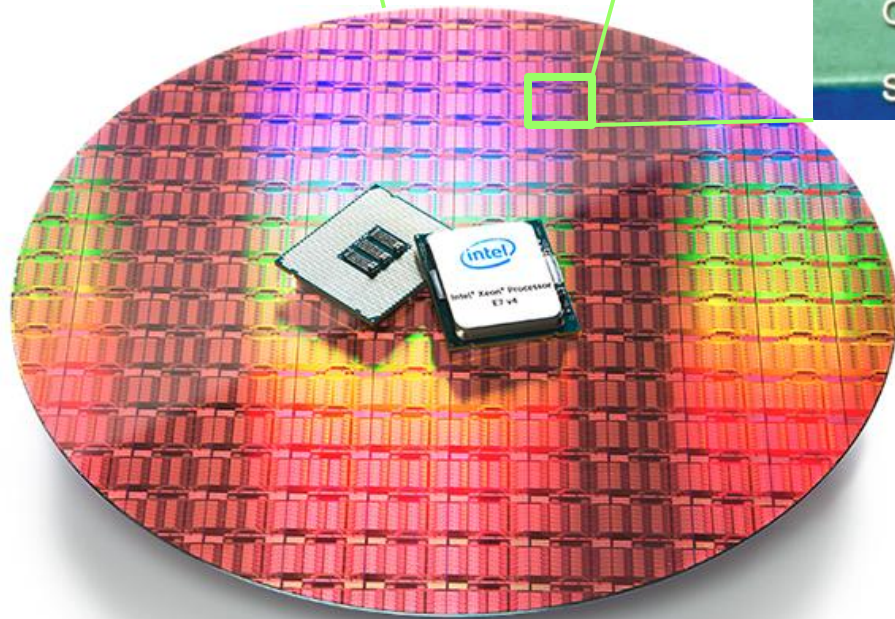
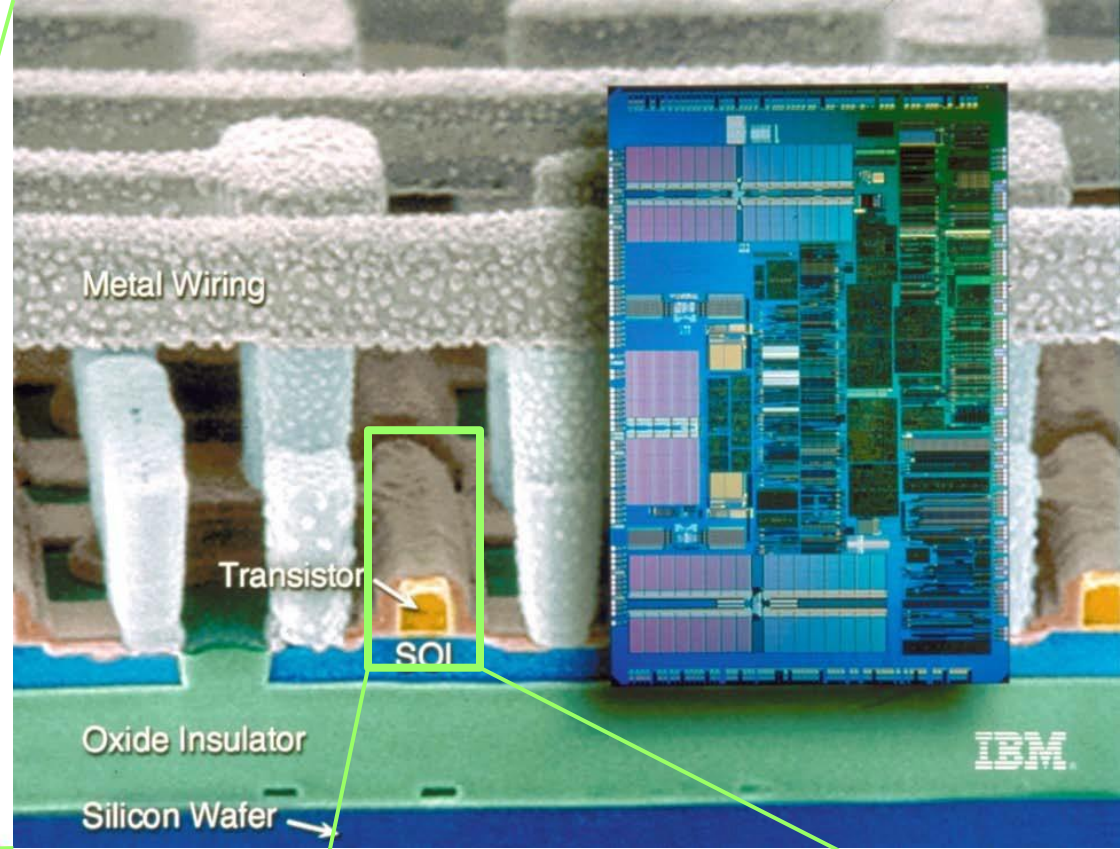
$$\approx - \left(\frac{eD_e}{L_e N_a} + \frac{eD_h}{L_h N_d} \right) n_i^2$$

reverse saturation current density, $-J_0$
independent of voltage ($V_r > kT/e$)

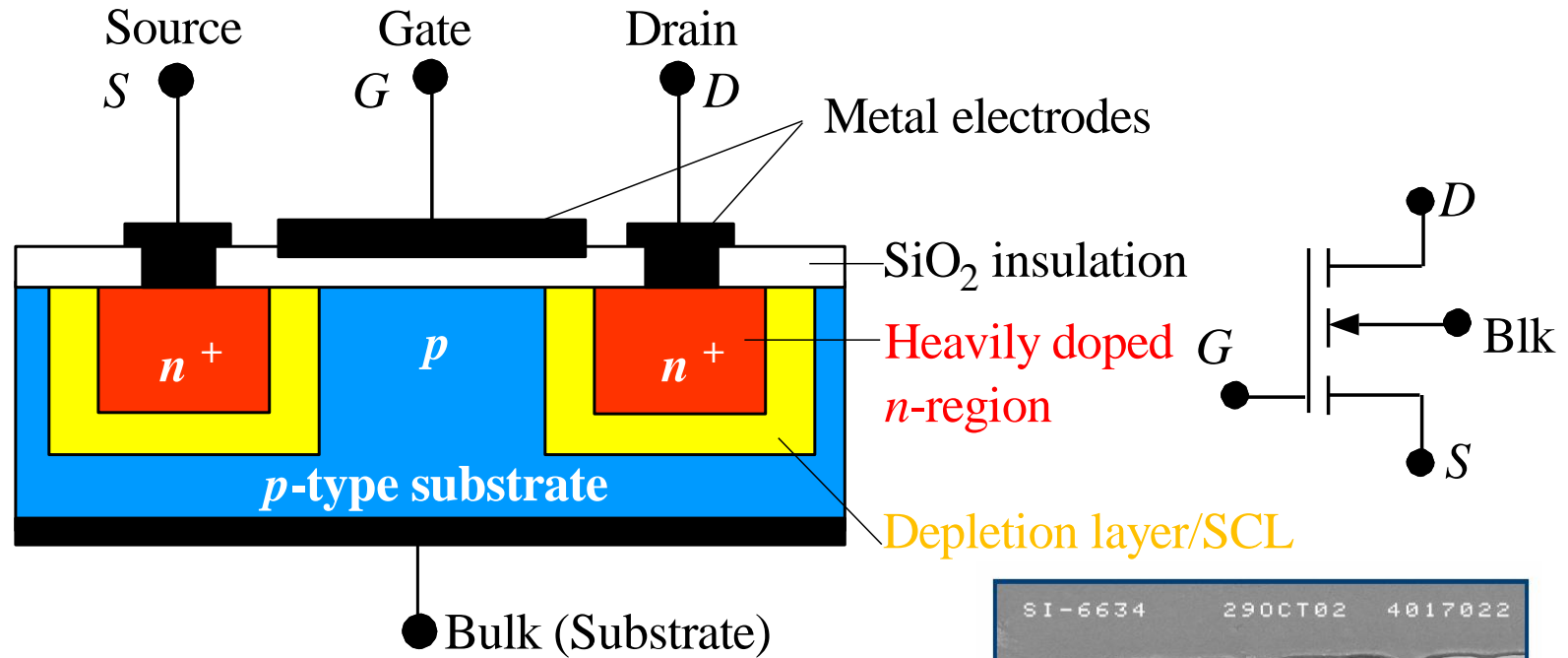
(b) Electron-hole pair generated within the SCL.

$$J_{gen} = \frac{eWn_i}{\tau_g}$$

τ_g is the mean time to generate an EHP

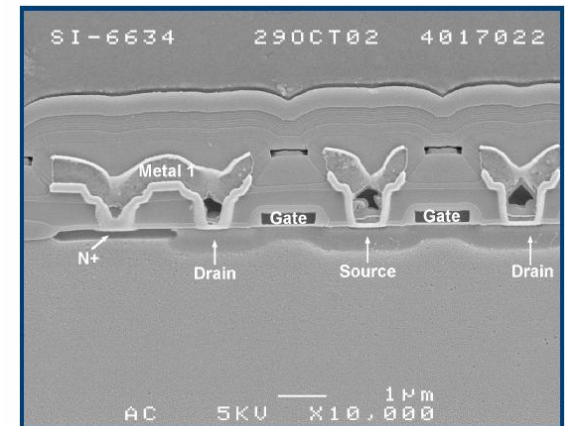


Metal-Oxide-Semiconductor Field-Effect Transistor (MOSFET)



SEM cross section of a MOS Transistor

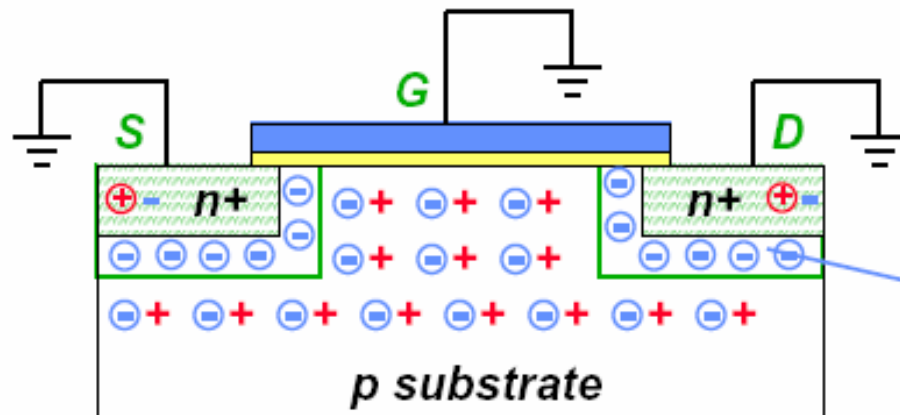
[SOURCE: Courtesy of Don Scansen, Semiconductor Insights, Kanata, Ontario, Canada]



The basic structure of MOSFET and its circuit symbol

Operation of Field Effect Transistor

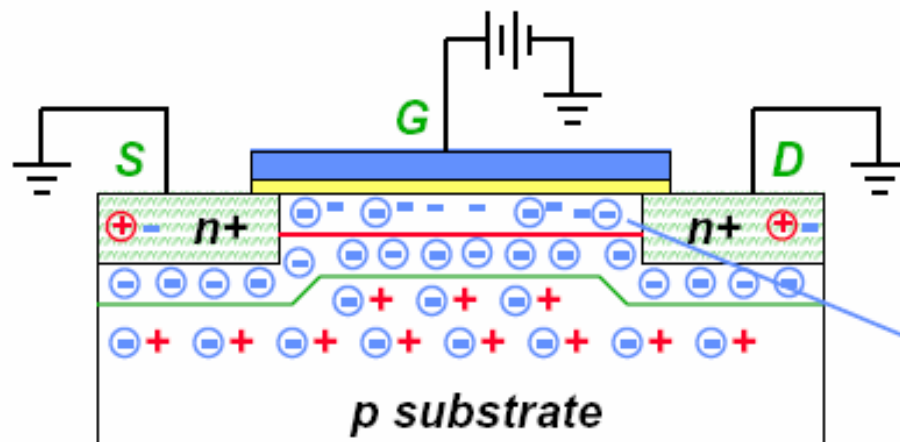
FET Under Gate Bias



- + mobile holes
- mobile electrons
- ⊖ immobile acceptors
- ⊕ immobile donors

Depletion layer

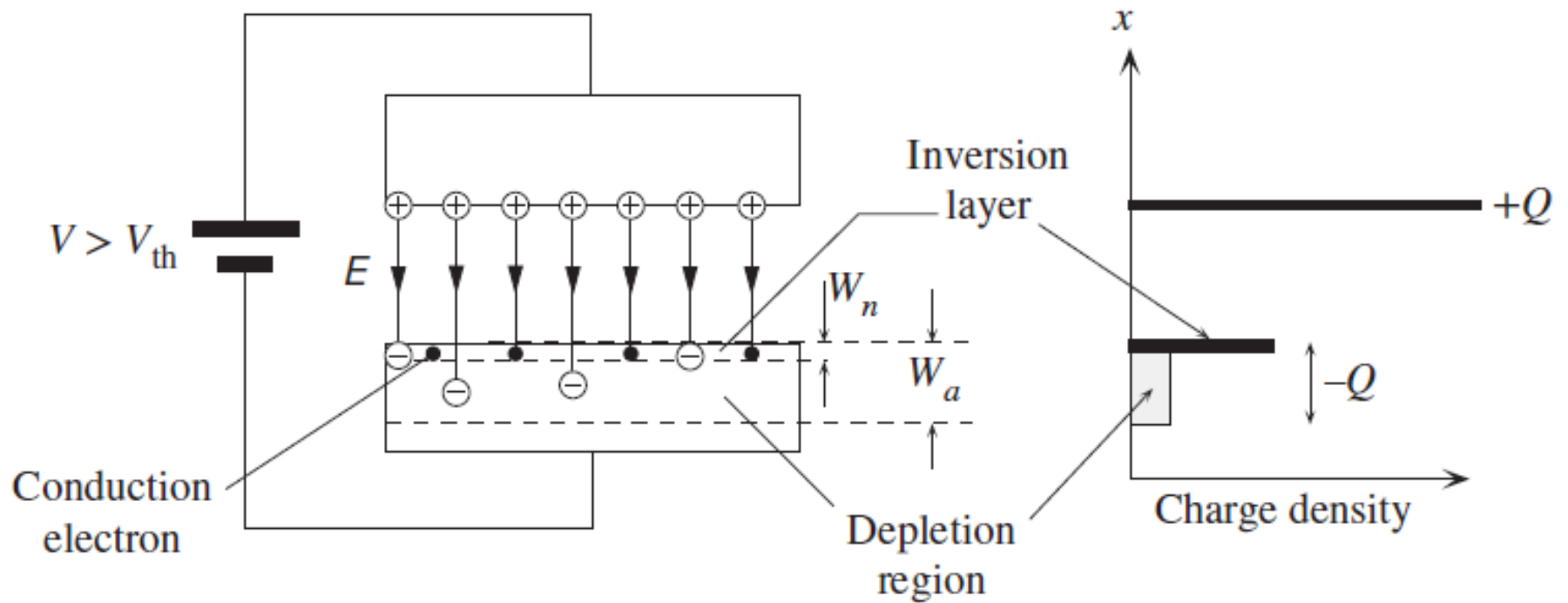
$V_{GS} < V_{TH}$
Cutoff region



$V_{GS} \geq V_{TH}$

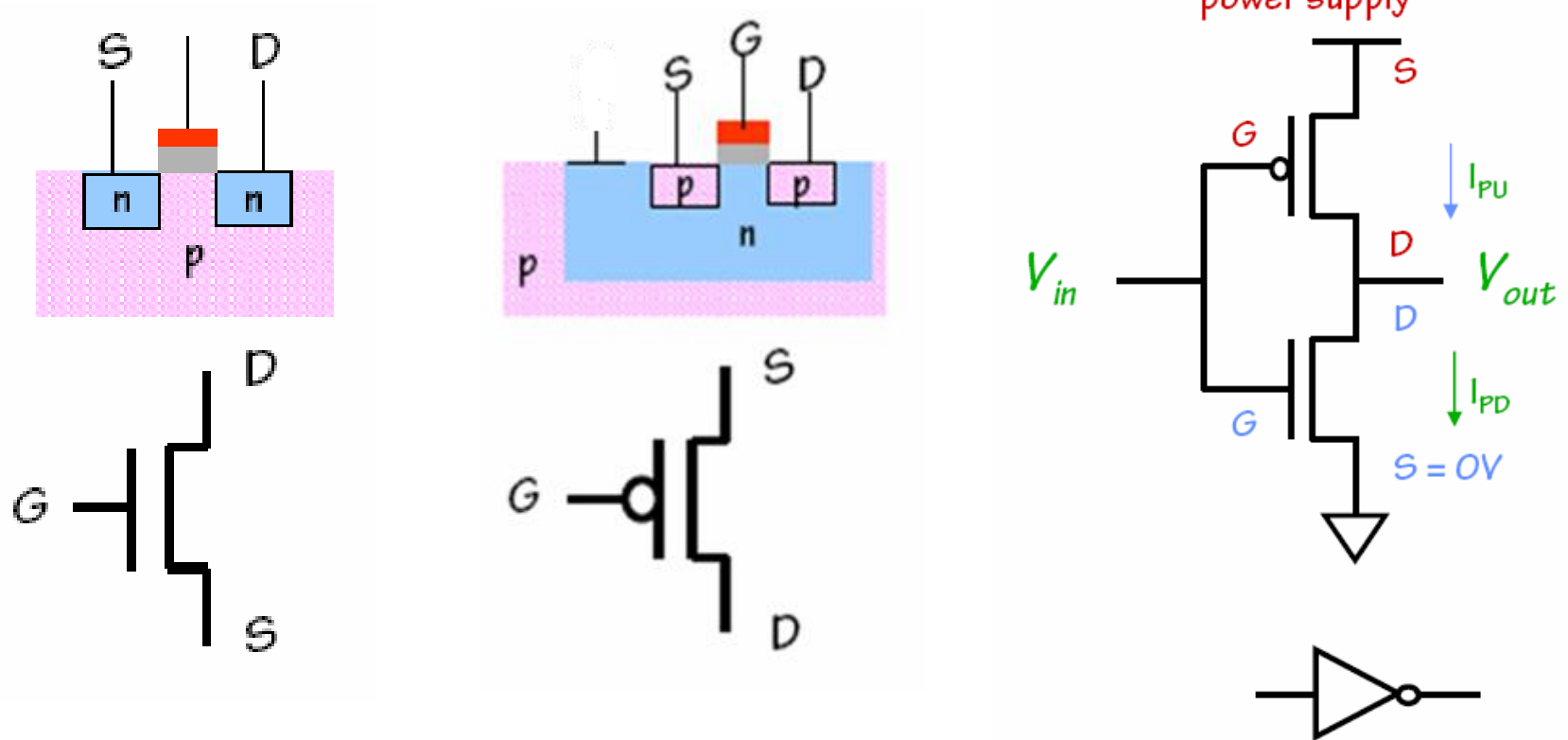
Inversion layer forms

Field Effect and Inversion



CMOS: The Heart of Modern Computer

By embedding p-type source and drain in a n-type substrate, we can fabricate a complement to the n-FET



The use of both n-FET and p-FET is a key to **CMOS (complementary Metal Oxide Semiconductor)** logic function. 只有需要切换启动与关闭时才需消耗能量

Innovation-Enabled Technology Pipeline is Full

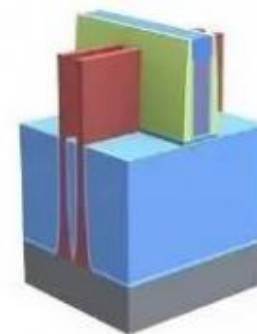
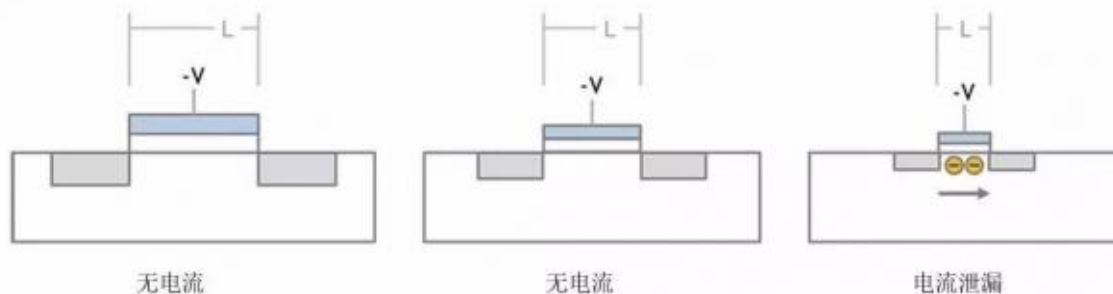


Our limit to visibility goes out ~10 years

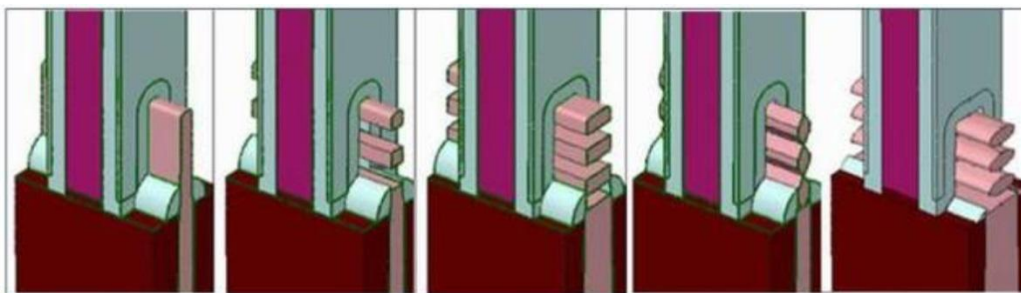
INVESTOR MEETING 2010



FinFET(鳍式场效应晶体管)



变种FinFET：环绕栅极



Ref: S.C. Song, Qualcomm; V. Moroz, Synopsys

Traditional FinFET	Round/Square Wire	Horizontal Nanoslab/sheet/multi-bridge channel	Hexagonal Nanowire	Nano-Ring
--------------------	-------------------	--	--------------------	-----------

发展趋势：
形状、材料、界面

Assignment 6.3

Question 1:

Sketch the energy diagrams of a pn junction, indicating the Fermi energy (E_F), the bottom of the conduction band (E_C), the top of the valence band (E_V), built-in potential (V_o), and the direction of the internal field.

Question 2:

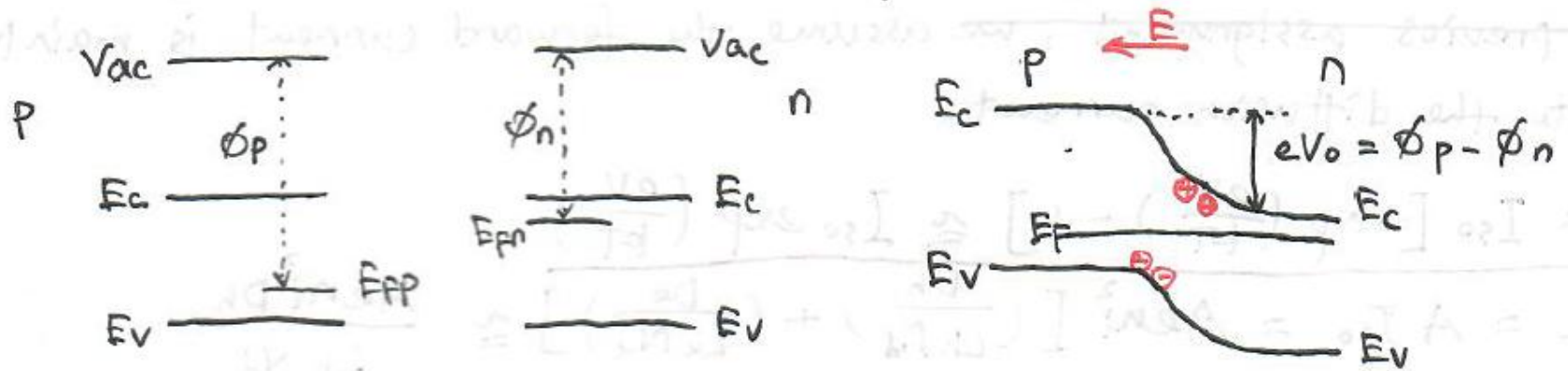
In the lecture, we used Boltzmann statistics to derive the built-in potential, V_o , of a pn junction. The energy band treatment allows a simple way to calculate V_o . When the junction is formed, E_{Fp} and E_{Fn} must shift and line up. The shift in E_{Fp} and E_{Fn} to line up is clearly $\Phi_p - \Phi_n$, the work function difference.

Using the energy band diagrams and semiconductor equations, derive an expression for the built-in potential V_o in terms of N_d , N_a , and n_i .

Question 3:

Consider a p^+n junction, which has a heavily doped p-side relative to the n-side, that is, $N_a \gg N_d$. What is your comment on the depletion width on the n-side and the p-side? What is the total depletion width (W_o) for the p^+n junction Si diode that has been doped with 10^{18} acceptor atoms cm^{-3} on the p-side and 10^{16} donor atoms cm^{-3} on the n-side?

\therefore The shift in E_{FP} and E_{Fn} to line up is $\phi_p - \phi_n$.



$$\therefore eV_0 = \phi_p - \phi_n = (E_c - E_{FP}) - (E_c - E_{Fn})$$

$$\therefore n = N_c \exp\left(-\frac{E_c - E_F}{kT}\right) \Rightarrow n_{no} = N_c \exp\left(-\frac{E_c - E_{Fn}}{kT}\right)$$

$$\Rightarrow n_{po} = N_c \exp\left(-\frac{E_c - E_{FP}}{kT}\right)$$

$$\Rightarrow \ln\left(\frac{n_{no}}{n_{po}}\right) = \frac{1}{kT} \cdot [(E_c - E_{FP}) - (E_c - E_{Fn})] = \frac{eV_0}{kT}$$

$$\therefore n_{po} = n_i^2 / N_a, \quad n_{no} = N_d$$

$$\therefore V_0 = \frac{kT}{e} \ln\left(\frac{N_a N_d}{n_i^2}\right)$$

THE p^+n JUNCTION Consider a p^+n junction, which has a heavily doped p -side relative to the n -side, that is, $N_a \gg N_d$. Since the amount of charge Q on both sides of the metallurgical junction must be the same (so that the junction is overall neutral)

$$Q = eN_aW_p = eN_dW_n$$

it is clear that the depletion region essentially extends into the n -side. According to Equation 6.7, when $N_d \ll N_a$, the width is

$$W_o = \left[\frac{2eV_o}{eN_d} \right]^{1/2}$$

What is the depletion width for a pn junction Si diode that has been doped with 10^{18} acceptor atoms cm^{-3} on the p -side and 10^{16} donor atoms cm^{-3} on the n -side?

SOLUTION

To apply the above equation for W_o , we need the built-in potential, which is

$$V_o = \left(\frac{kT}{e} \right) \ln \left(\frac{N_d N_a}{n_i^2} \right) = (0.0259 \text{ V}) \ln \left[\frac{(10^{16})(10^{18})}{(1.0 \times 10^{10})^2} \right] = 0.835 \text{ V}$$

Then with $N_d = 10^{16} \text{ cm}^{-3}$, that is, 10^{22} m^{-3} , $V_o = 0.835 \text{ V}$, and $\epsilon_r = 11.9$ in the equation for W_o

$$\begin{aligned} W_o &= \left[\frac{2\epsilon V_o}{eN_d} \right]^{1/2} = \left[\frac{2(11.9)(8.85 \times 10^{-12})(0.835)}{(1.6 \times 10^{-19})(10^{22})} \right]^{1/2} \\ &= 3.32 \times 10^{-7} \text{ m} \quad \text{or} \quad 0.33 \text{ } \mu\text{m} \end{aligned}$$

Nearly all of this region (99 percent of it) is on the n -side.

Assignment 6.3 (2)

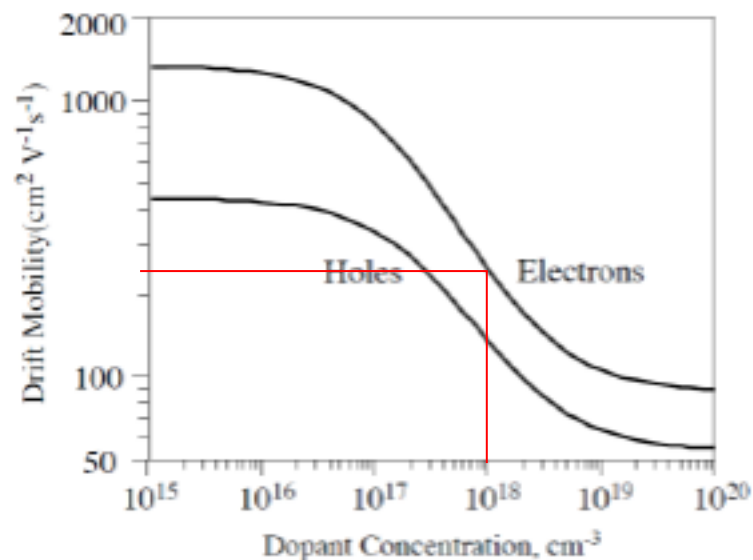


Figure 1. The variation of the drift mobility with dopant concentration in Si for electrons and holes at 300 K

Question 1:

Consider a Si ($n_i = 1.45 \times 10^{10} \text{ cm}^{-3}$, ϵ_r is 11.9) *pn* junction diode, with an acceptor concentration N_a of 10^{18} cm^{-3} on the p-side and donor concentration N_d of 10^{15} cm^{-3} on the n-side. The drift mobility refers to Figure 1. The diode is forward biased and has a voltage of 0.6 V across it. The diode cross-sectional area is 1 mm^2 . The minority carrier recombination time, τ , depends on the dopant concentration, N_{dopant} (cm^{-3}), through the following approximate relation

$$\tau = \frac{5 \times 10^{-7}}{(1 + 2 \times 10^{-17} N_{\text{dopant}})}$$

Calculate the diffusion current and the recombination current. What is your conclusion on the contributions to the total diode current?

Q1. This is a p^+n diode: $N_d = 10^{15} \text{ cm}^{-3}$. Hole^v lifetime τ_h : recombination

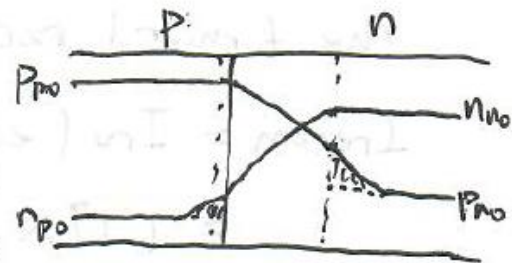
$$\tau_h = \frac{5 \times 10^{-7}}{(1 + 2 \times 10^{-17} N_{\text{dopant}})} = \frac{5 \times 10^{-7}}{1 + 2 \times 10^{-17} \times 10^{15} \text{ cm}^{-3}} = 490.2 \text{ ns}$$

similarly: $\tau_e = 23.81 \text{ ns}$

1) Diffusion diode current:

From Figure 1, $N_a = 10^{18} \text{ cm}^{-3} \Rightarrow \mu_e \approx 250 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}$

$N_d = 10^{15} \text{ cm}^{-3} \Rightarrow \mu_h \approx 450 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}$



Einstein Relation: $D_e = \frac{kT}{e} \cdot \mu_e = (0.02586 \text{ V})(250 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}) = 6.465 \text{ cm}^2 \text{ s}^{-1}$

$D_h = \frac{kT}{e} \cdot \mu_h = (0.02586 \text{ V})(450 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}) = 11.64 \text{ cm}^2 \text{ s}^{-1}$

Thus diffusion length: $L_e = \sqrt{D_e \tau_e} = \sqrt{(6.465 \text{ cm}^2 \text{ s}^{-1})(23.81 \times 10^{-9} \text{ s})}$
 $= 3.923 \times 10^{-4} \text{ cm}$

$L_h = \sqrt{D_h \tau_h} = \sqrt{(11.64 \text{ cm}^2 \text{ s}^{-1})(490.2 \times 10^{-9} \text{ s})}$
 $= 2.389 \times 10^{-3} \text{ cm}$

Diffusion current: $I_{diff} = A \cdot J_{diff} = A \cdot J_{so} [\exp(eV/kT) - 1]$
 $\approx A \cdot J_{so} \exp(eV/kT) \quad \because V \gg kT/e = 0.02586 \text{ V}$

where $I_{so} = A \cdot J_{so} = A e n_i^2 [D_h/(L_h N_d) + D_e/(L_e N_a)] \approx A e n_i^2 D_h/(L_h N_d)$

$\because N_a \gg N_d$. In other words, the current is mainly due to the diffusion of the holes in n-region.

$$\therefore I_{so} = \frac{(1 \times 10^{-2} \text{ cm}^2) (1.602 \times 10^{-19} \text{ C}) (\overset{1.0}{1.45 \times 10^{10} \text{ cm}^{-3}})^2 (11.64 \text{ cm}^2 \text{ s}^{-1})}{(2.389 \times 10^{-3} \text{ cm}) (1 \times 10^{15} \text{ cm}^{-3})}$$

\therefore Forward current due to diffusion: $= 1.641 \times 10^{-12} \text{ A}$

$$I_{diff} = I_{so} \exp(eV/kT) = (1.641 \times 10^{-12} \text{ A}) \exp(0.6 \text{ V} / 0.02586 \text{ V})$$

$$= 0.0196 \text{ A} \quad \text{or} \quad 19.6 \text{ mA} \checkmark$$

2) Recombination current.

Built-in potential: $V_0 = (kT/e) \ln(N_a N_d / n_i^2) = (0.02586 \text{ V}) \ln(10^{18} \text{ cm}^{-3} \times 10^{15} \text{ cm}^{-3} / (\overset{1.0}{1.45 \times 10^{10} \text{ cm}^{-3}})^2) = 0.7549 \text{ V}$

depletion region width W is mainly on the n-side:

$$W = \left[\frac{2 \epsilon (N_a + N_d) (V_0 - V)}{e N_a N_d} \right]^{1/2} \approx \left[\frac{2 \epsilon (V_0 - V)}{e N_d} \right]^{1/2} (= W_n)$$

$$= \left[\frac{2 (11.9) (8.854 \times 10^{-12} \text{ F m}^{-1}) (0.7549 \text{ V} - 0.6 \text{ V})}{(1.602 \times 10^{-19} \text{ C}) (10^{21} \text{ m}^{-3})} \right]^{1/2} = 0.4514 \times 10^{-4} \text{ cm}$$

$$I_{\text{recom}} = I_{\text{ro}} \left[\exp\left(\frac{eV}{2kT}\right) - 1 \right] \quad \text{where } I_{\text{ro}} = \frac{en_i}{2} \left(\frac{W_p}{\tau_e} + \frac{W_n}{\tau_h} \right)$$

$$I_{\text{ro}} = A \cdot \frac{en_i}{2} \cdot W_n / \tau_h = \frac{(1 \times 10^{-2} \text{ cm}^2)(1.602 \times 10^{-19} \text{ C})(0.4514 \times 10^{-4} \text{ cm})}{2(490 \times 10^{-9} \text{ s})} \quad (1.45 \times 10^{10} \text{ cm}^{-3})$$

$$= 1.070 \times 10^{-9} \text{ A}$$

\therefore The forward recombination current is:

$$I_{\text{recom}} = I_{\text{ro}} \left(\exp\left(\frac{eV}{2kT}\right) \right) = (1.070 \times 10^{-9} \text{ A}) \exp \left[(0.6 \text{ V}) / 2(0.02586 \text{ V}) \right]$$

$$= 1.17 \times 10^{-4} \text{ A} \quad \text{or} \quad 0.117 \text{ mA} \quad \checkmark$$

Conclusion: the diffusion current dominates the total diode current in this diode. \checkmark

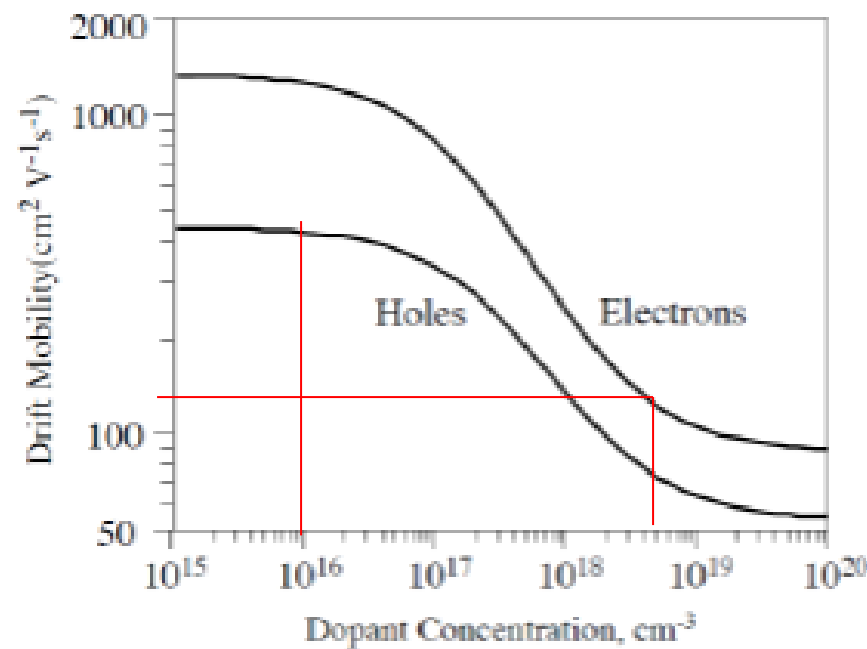


Figure 1. The variation of the drift mobility with dopant concentration in Si for electrons and holes at 300 K

Question 2:

An Si p^+n junction diode has a cross-sectional area of 1 mm^2 , an acceptor concentration of $5 \times 10^{18} \text{ cm}^{-3}$ on the p-side, and a donor concentration of 10^{16} cm^{-3} on the n-side. The recombination lifetime of holes in the n-region is 420 ns, whereas that of electrons in the p-region is 5 ns due to a greater concentration of impurities (recombination centers) on that side. Mean thermal generation lifetime (τ_g) is about $1 \mu\text{s}$.

- Calculate the minority carrier diffusion lengths.
- What is the built-in potential across the junction?
- What is the current when there is a forward bias of 0.6 V across the diode at 27°C ? Assume that the current is by minority carrier diffusion.
- What is the reverse current when the diode is reverse biased by a voltage $V_r = 5 \text{ V}$?

a) From Figure 1. $N_a = 5 \times 10^{18} \text{ cm}^{-3} \Rightarrow \mu_e \approx 150 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}$
 $N_d = 10^{16} \text{ cm}^{-3} \Rightarrow \mu_h \approx 430 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}$

Einstein relation; $D_e = \frac{kT}{e} \cdot \mu_e = (0.02586 \text{ V}) (150 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}) = 3.88 \text{ cm}^2 \text{ s}^{-1}$
 $D_h = \frac{kT}{e} \cdot \mu_h = (0.02586 \text{ V}) (430 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}) = 11.12 \text{ cm}^2 \text{ s}^{-1}$

Diffusion lengths; $L_e = \sqrt{D_e \tau_e} = \sqrt{(3.88 \text{ cm}^2 \text{ s}^{-1}) (5 \times 10^{-9} \text{ s})} = 1.39 \times 10^{-4} \text{ cm}$
 $L_h = \sqrt{D_h \tau_h} = \sqrt{(11.12 \text{ cm}^2 \text{ s}^{-1}) (420 \times 10^{-9} \text{ s})} = 21.6 \times 10^{-4} \text{ cm}$

(b) Built-in potential:

$$V_0 = \left(\frac{kT}{e} \right) \ln \left(\frac{N_d N_a}{n_i^2} \right) = (0.02586 \text{ V}) \ln \left[\frac{10^{16} \times 5 \times 10^{18}}{(1.45 \times 10^{10})^2} \right] = 0.875 \text{ V}$$

(c) From previous assignment, we assume the forward current is mainly due to the diffusion current.

$$I = I_{so} \left[\exp\left(\frac{eV}{kT}\right) - 1 \right] \approx I_{so} \exp\left(\frac{eV}{kT}\right)$$

where $I_{so} = A J_{so} = A e n_i^2 \left[\left(\frac{D_h}{L_h N_d} \right) + \left(\frac{D_e}{L_e N_a} \right) \right] \approx \frac{A e n_i^2 D_h}{L_h N_d}$

as $N_a \gg N_d$, the diffusion current mainly due to holes diffusing in n-region.

$$I_{so} = \frac{(0.01 \text{ cm}^2) (1.602 \times 10^{-19} \text{ C}) (1.45 \times 10^{10} \text{ cm}^{-3})^2 (11.12 \text{ cm}^2 \text{ s}^{-1})}{(21.6 \times 10^{-4} \text{ cm}) (10^{16} \text{ cm}^{-3})}$$

$$= 8.24 \times 10^{-14} \text{ A}$$

$$I \approx I_{so} \exp\left(\frac{eV}{kT}\right) = (8.24 \times 10^{-14} \text{ A}) \exp\left(\frac{0.6 \text{ V}}{0.02586 \text{ V}}\right)$$

$$= 0.99 \times 10^{-3} \text{ A} = 1.0 \text{ mA}$$

(d) Reverse saturation current: $I = I_{so} = 8.24 \times 10^{-14} \text{ A}$

Thermal generation current:

$$I_{gen} = A \cdot J_{gen} = A \cdot \frac{e W n_i}{\tau_g}$$

$$W = \left[\frac{2 \epsilon (V_0 + V_r)}{e N_d} \right]^{1/2} = \left[\frac{2 (11.9) (8.85 \times 10^{-12}) (0.875 + 5)}{(1.6 \times 10^{-19}) (10^{22})} \right]^{1/2}$$

$$= 0.88 \times 10^{-4} \text{ cm}$$

$$\therefore I_{gen} = (0.01 \text{ cm}^2) \frac{(1.602 \times 10^{-19} \text{ C}) (0.88 \times 10^{-4} \text{ cm}) (1.45 \times 10^{10} \text{ cm}^{-3})}{(10^{-6} \text{ s})}$$

$$= 1.41 \times 10^{-9} \text{ A}$$

$I_{gen} \gg I_{so} \Rightarrow$ The reverse current is dominated by thermal generation current.