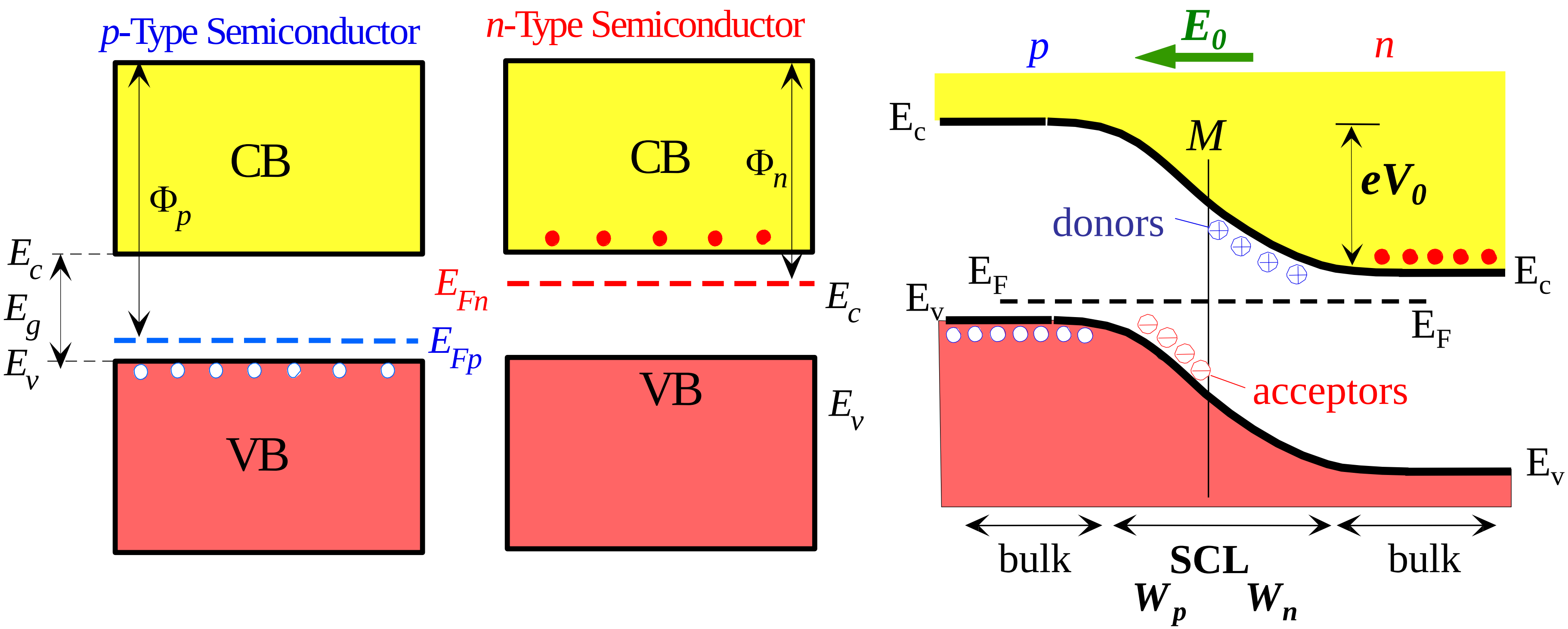
Devices always involve interfaces:

1. *metal-semiconductor*
   - *Schottky Junction*
   - *Ohmic Contact*

2. *semiconductor-semiconductor*
   - *pn Junction (1)*
   - ***pn Junction (2)***
   - *Tutorial + metal oxide field-effect transistor (MOSFET)*

Band Diagram of pn Junction: No Bias (Open Circuit)



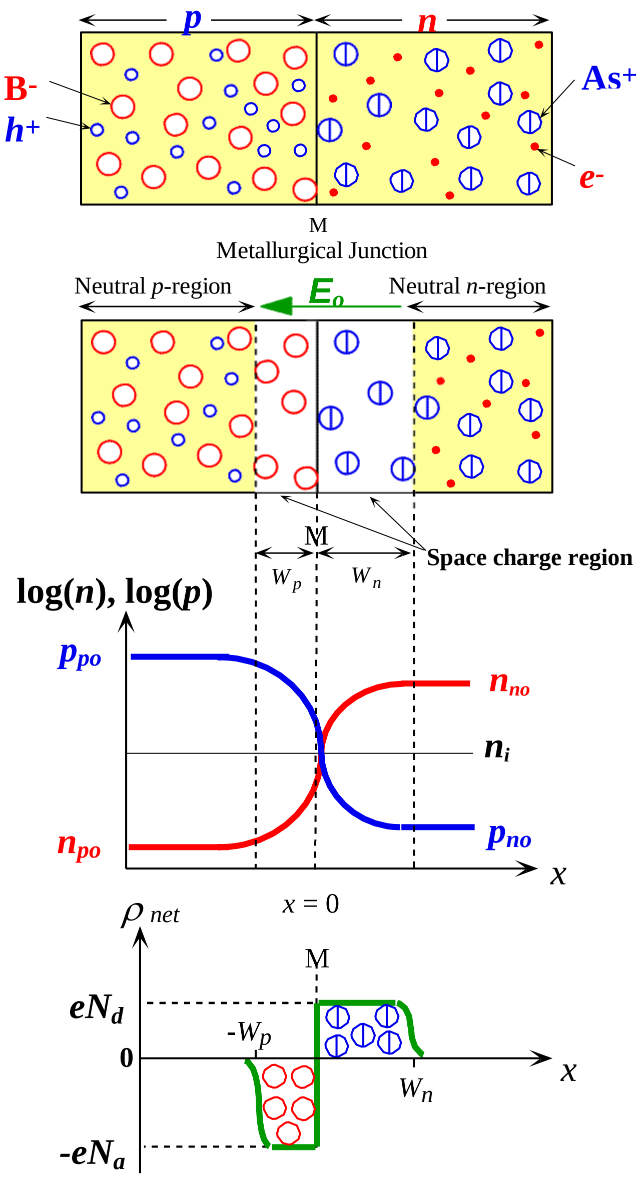
Space charge layer width $W = W_p + W_n$

The region around the M contains the **space charge layer (SCL)**. On the *n*-side of M, SCL has the exposed positively charged donors whereas on the *p*-side it has the exposed negatively charged accepters.

SCL: space charge layer（空间电荷层）/depletion region/depletion layer (耗尽层)

M: metallurgical junction (冶金结), Bulk (块体)

pn Junction: Space Charge Layer



Each electron moves over the interface will combine with one hole.

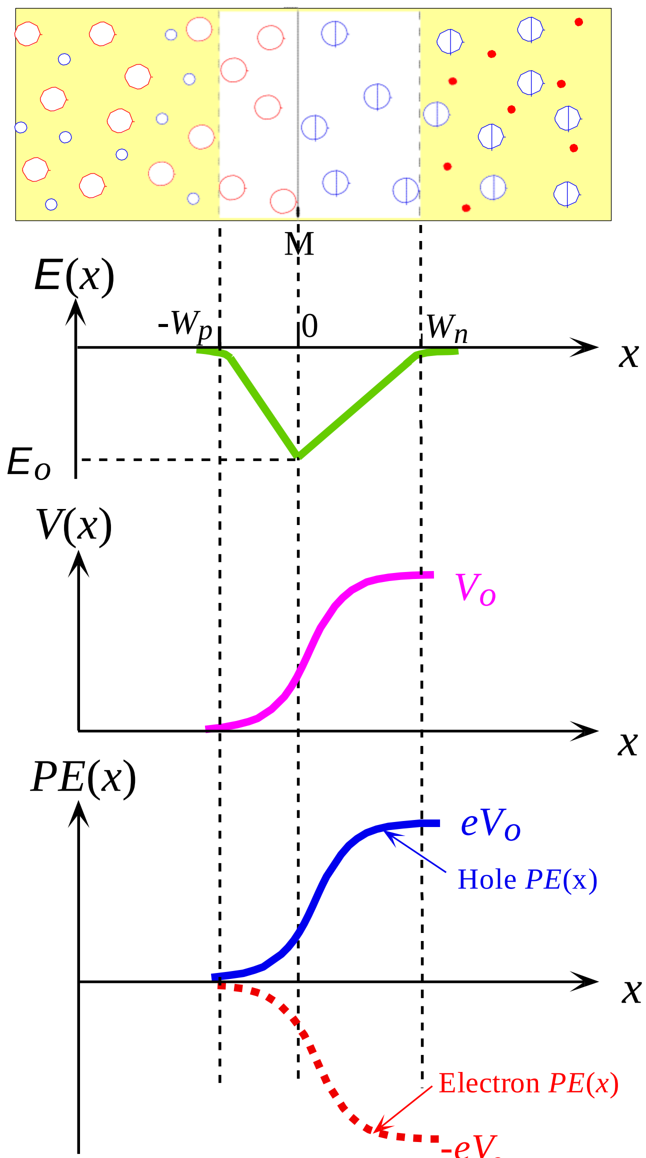
The total number of negative charge on *p* side equals to that of positive charge on *n* side to remain charge neutrality

p_{p0}: majority carrier concentration on *p* side
n_{p0}: minority carrier concentration on *p* side
p_{n0}: minority carrier concentration on *n* side
n_{n0}: majority carrier concentration on *n* side

$$N_a W_p = N_d W_n$$

ρ_{net}: net space charge density
N: doping concentration
W: space charge layer width

pn Junction: Space Charge Layer



Gauss's law in point form:

ε_o and ε_r: absolute and relative permittivity （介电常数）of the semiconductor material

$$\frac{dE}{dx} = \frac{\rho_{net}(x)}{\varepsilon_r \varepsilon_o} \Rightarrow E(x) = \frac{\rho_{net}}{\varepsilon_r \varepsilon_o} x + C_1$$

$$\boxed{E_0 = -\frac{eN_d W_n}{\varepsilon_r \varepsilon_o} = -\frac{eN_a W_p}{\varepsilon_r \varepsilon_o}}$$

The negative field means −x direction

$$E(x) = -\frac{dV}{dx} \Rightarrow V(x) = -\int_{-W_p}^{x} E(x)dx$$

By putting x = W_n ➡

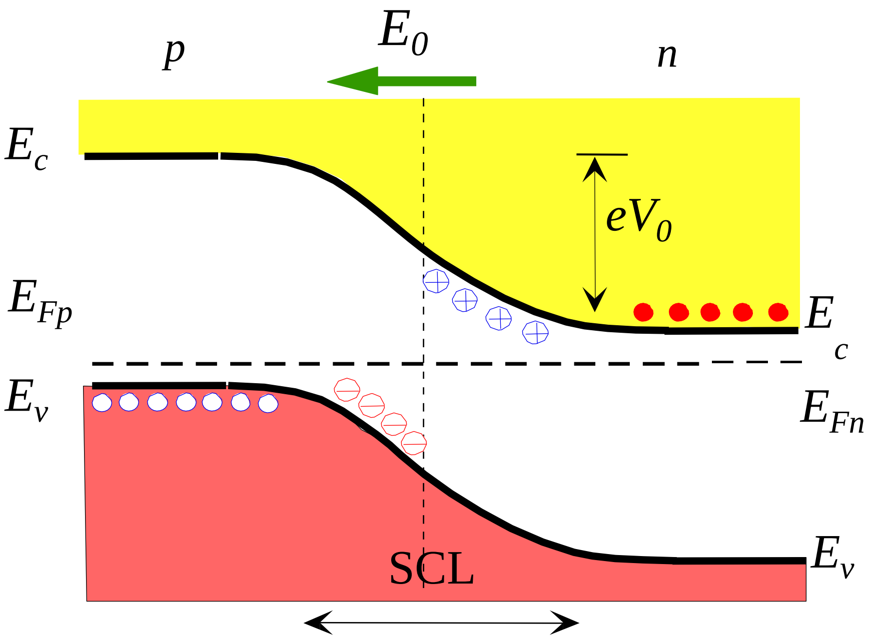
$$\boldsymbol{V}_0 = \frac{1}{2} E_0 W_0 = \frac{eN_a N_d \boldsymbol{W}_0^{\,2}}{2\varepsilon_r \varepsilon_0 (N_a + N_d)}$$

V_0: the built-in potential（内置电位）
$W_0 = W_p + W_n$: the total width of the space charge layer under a zero applied voltage

pn Junction: Built-in Potential

Probability of electrons occupying energy E is determined by **Fermi-Dirac statistics**, which is reduced to **Boltzmann statistics** when $E-E_F >> k_BT$, it demands the concentrations n_1 and n_2 of potential energies E_1 and E_2 are related by:

$$\frac{n_2}{n_1} = \exp\left[-\frac{(E_2 - E_1)}{kT}\right]$$

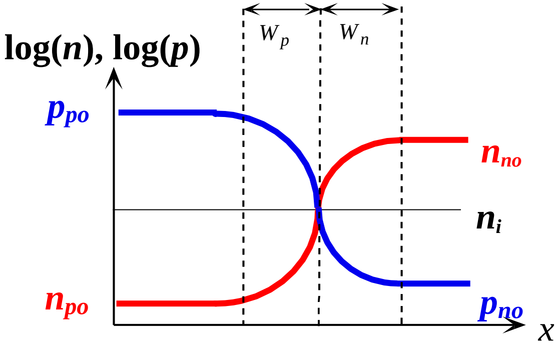


$$\frac{n_{po}}{n_{no}} = \exp\left(-\frac{eV_o}{k_BT}\right) \qquad \frac{p_{no}}{p_{po}} = \exp\left(-\frac{eV_o}{k_BT}\right)$$

$$p_{po} = N_a \qquad p_{no} = \frac{n_i^2}{n_{no}} = \frac{n_i^2}{N_d}$$

$$V_o = \frac{k_BT}{e}\ln\left(\frac{N_aN_d}{n_i^2}\right)$$

$$W_o = \sqrt{\frac{2\varepsilon_o\varepsilon_r(N_a+N_d)V_o}{eN_aN_d}}$$



Diffusion

Electron diffusion current density

$$J_{D,e} \propto -\frac{dn}{dx}$$
$$= eDe\frac{dn}{dx}$$

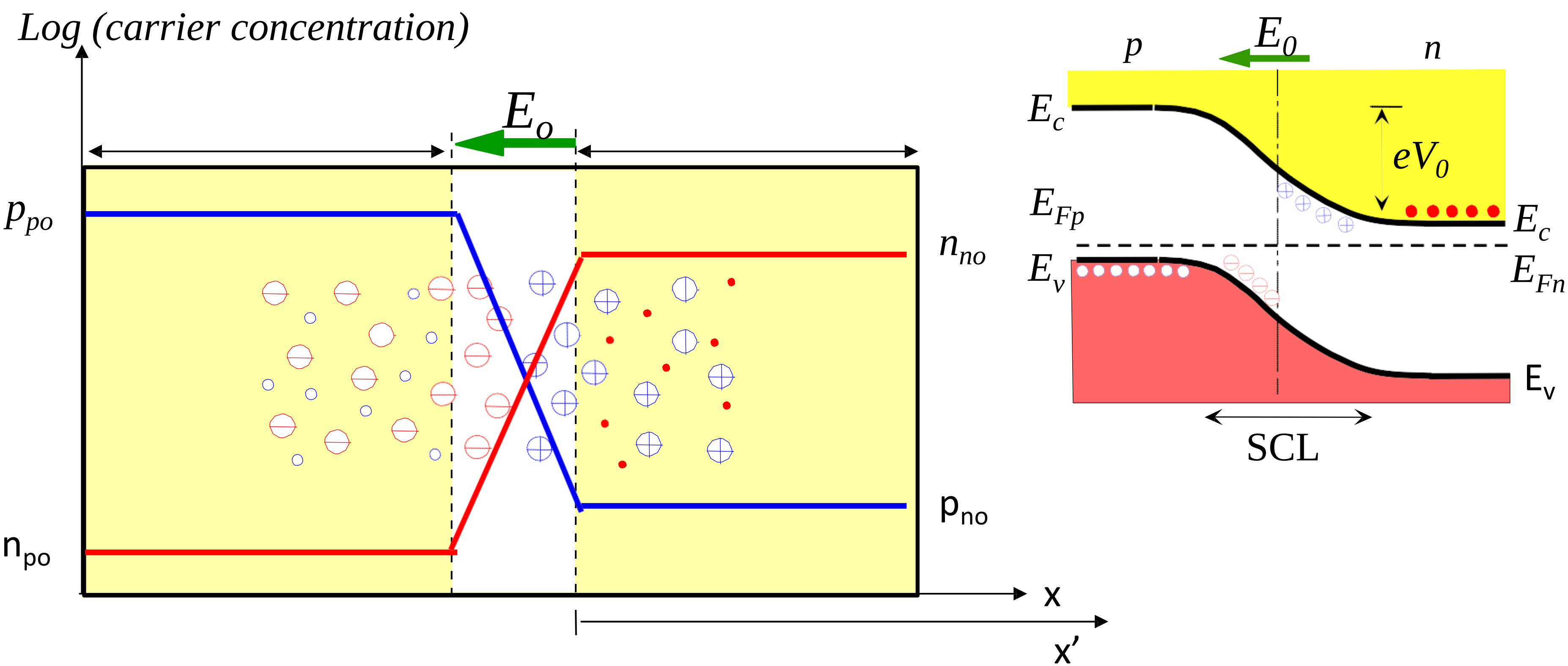
Hole diffusion current density

$$J_{D,h} \propto -\frac{dp}{dx}$$
$$= -eDh\frac{dp}{dx}$$

Diffusion coefficient（扩散系数）, $\mathbf{D_e}$ or $\mathbf{D_h}$, is a measure of the ease of carrier ***diffusion*** motion in a medium. Mobility, $\boldsymbol{\mu_e}$ or $\boldsymbol{\mu_h}$, is a measure of the ease of carrier ***drift*** motion in a medium. The two quantities are related by the **Einstein Relation.**

$$\boxed{\frac{D_e}{\mu_e} = \frac{kT}{e} \quad \text{and} \quad \frac{D_h}{\mu_h} = \frac{kT}{e}}$$

Carrier Concentration Profiles Across a *pn* Junction: **No Bias**

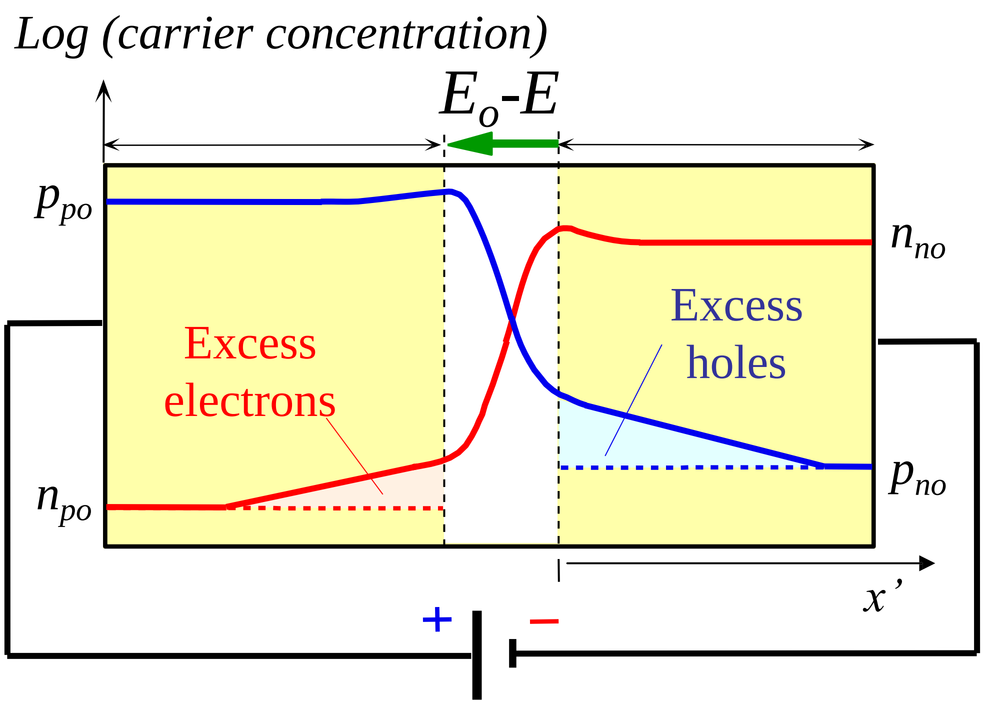


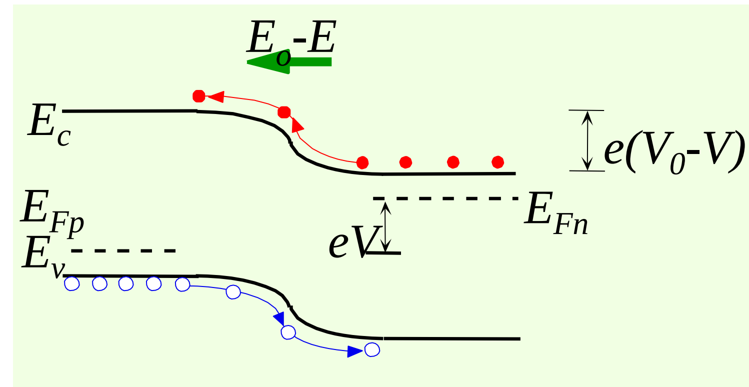
$$J_e = en\mu_e E_x + eD_e \frac{dn}{dx}$$

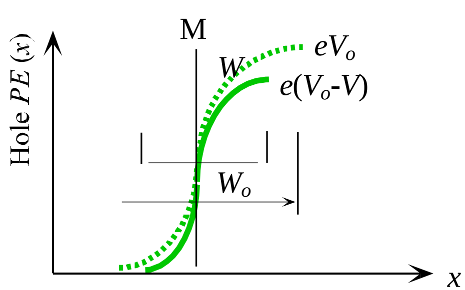
$$J_h = ep\mu_h E_x - eD_h \frac{dp}{dx}$$

When there is no electric field applied to a *pn* junction, there is no current. The diffusion current and drift current balance each other within the space charge layer.

Carrier Concentration Profiles Across a *pn* Junction:
Forward Bias







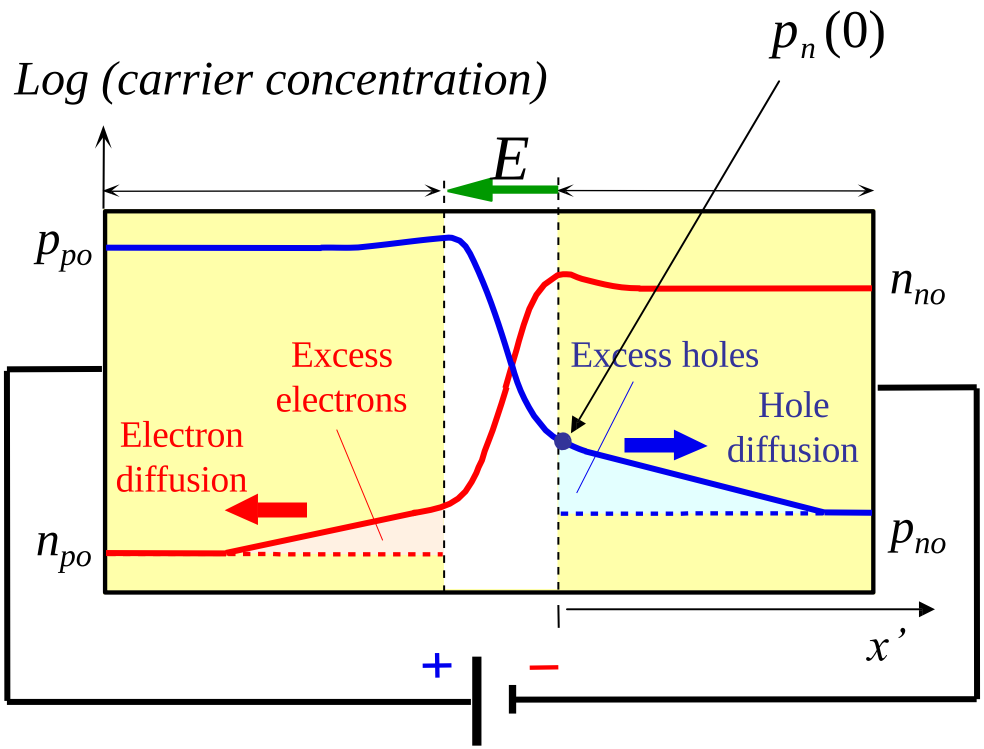
(1) Voltage drops mainly across the SCL.

(2) The potential barrier against diffusion is reduced to $(V_o$-$V)$.

(3) The probability that a carrier will surmount the barrier becomes proportional to $\exp[-\frac{(V_o - V)}{k_B T}]$

More carriers can diffuse to the opposite sides of the junction. This is called the **injection of excess minority carriers**（少数载流子注入）.

Current Across a **Forward Biased** *pn* Junction: **Diffusion**



$$\frac{n_2}{n_1} = \exp\left[-\frac{(E_2 - E_1)}{kT}\right]$$

$$p_n(0) = p_n(x'=0)$$

$$= p_{po}\exp\left[-\frac{e(V_o - V)}{k_B T}\right]$$

$$\frac{p_{no}}{p_{po}} = \exp\left(-\frac{eV_o}{k_B T}\right)$$

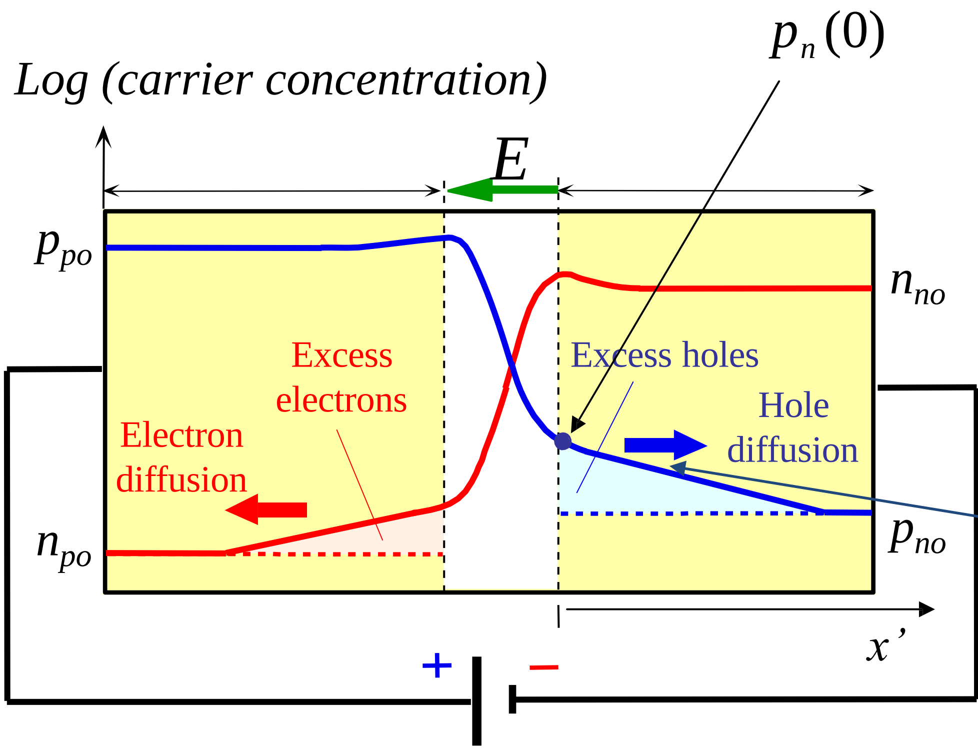
$$\boxed{p_n(0) = p_{no}\exp\left[\frac{eV}{k_B T}\right]}$$

The **diffusion of minority carriers** contributes to the current density of a forward biased *pn* junction.

Law of the junction

relates the injected minority carrier concentration just outside SCL to the applied voltage.

Current Across a **Forward Biased** *pn* Junction: **Diffusion**



Law of the junction

$$p_n(0) = p_{no} \exp[\frac{eV}{k_B T}]$$

$$J_{D,hole} = -eD_h \frac{dp_n(x')}{dx'}$$

hole concentration $p_n(x')$ fall **exponentially** toward the thermal equilibrium value p_{no}

$$\Delta p_n(x') = \Delta p_n(0) \exp\left(-\frac{x'}{L_h}\right)$$

Where: $\Delta p_n(x') = p_n(x') - p_{no}$

Minority Carrier **Diffusion Length**

$$\boxed{L_h = \sqrt{D_h \tau_h}}$$

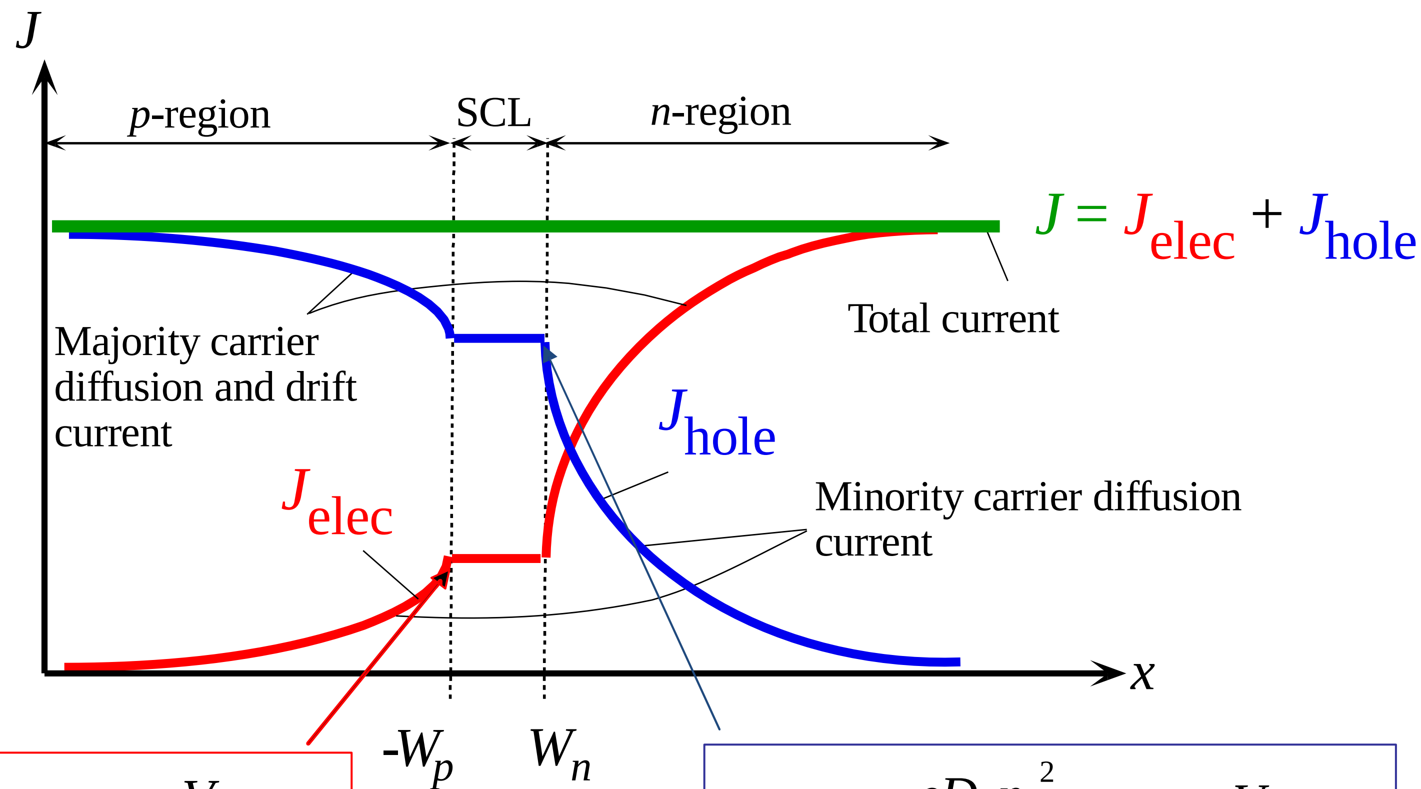
τ_h is the mean hole **recombination lifetime** (minority carrier lifetime) in the n-region

At x'=0:

$$\boxed{J_{D,hole} = (\frac{eD_h p_{no}}{L_h})[\exp(\frac{eV}{k_B T}) - 1]}$$

$$p_{no} = \frac{n_i^2}{n_{no}} = \frac{n_i^2}{N_d}$$

Total **Diffusion Current**: Electron and Hole



$$J_{D,elec} = \left(\frac{eD_e n_i^2}{L_e N_a}\right)\left[\exp\left(\frac{eV}{k_B T}\right) - 1\right]$$

$$J_{D,hole} = \left(\frac{eD_h n_i^2}{L_h N_d}\right)\left[\exp\left(\frac{eV}{k_B T}\right) - 1\right]$$

The total current anywhere in the device is constant.

Total Diffusion Current: Electron and Hole

$$J = J_{\text{elec}} + J_{\text{hole}}$$

$$\boxed{J_{D,elec} = \left(\frac{eD_e n_i^{\ 2}}{L_e N_a}\right)\left[\exp\left(\frac{eV}{k_B T}\right) - 1\right]}$$

$$\boxed{J_{D,hole} = \left(\frac{eD_h n_i^{\ 2}}{L_h N_d}\right)\left[\exp\left(\frac{eV}{k_B T}\right) - 1\right]}$$

$$\boxed{J = \left(\frac{eD_e}{L_e N_a} + \frac{eD_h}{L_h N_d}\right) n_i^{\ 2}\left[\exp\left(\frac{eV}{k_B T}\right) - 1\right]}$$

$$J = J_{so}\left[\exp\left(\frac{eV}{kT}\right) - 1\right]$$

Ideal diode equation (Shockley equation)

$$J_{so} = \left[\left(\frac{eD_h}{L_h N_d}\right) + \left(\frac{eD_e}{L_e N_a}\right)\right] n_i^2$$

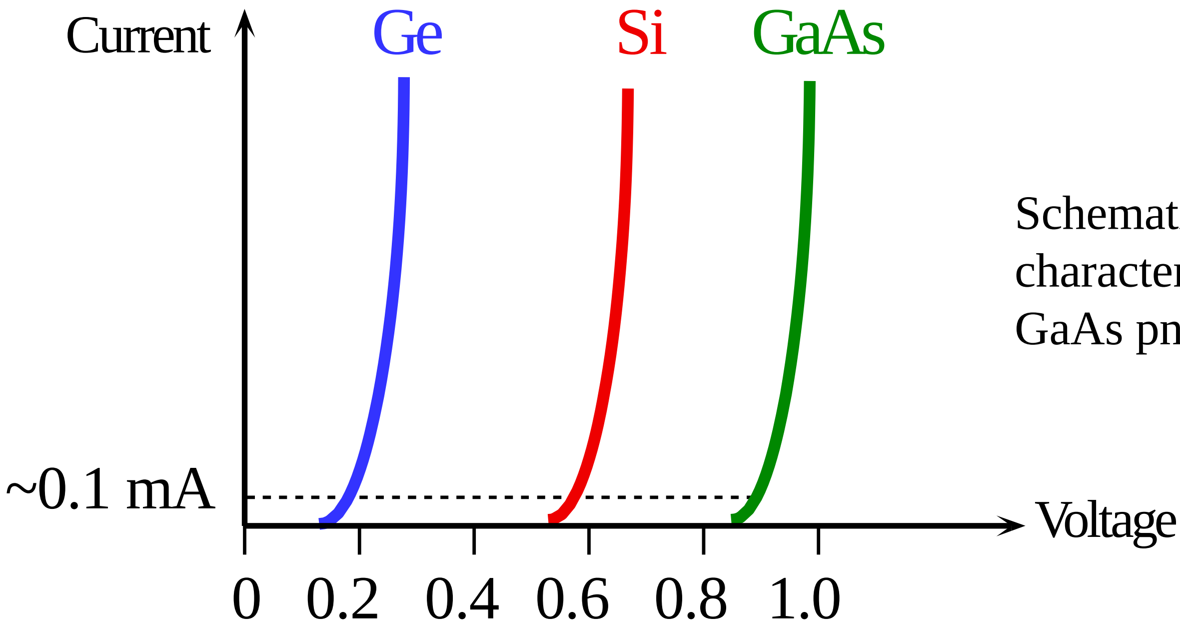
reverse saturation current density

Total Diffusion Current: Electron and Hole

$$J = \left(\frac{eD_e}{L_e N_a} + \frac{eD_h}{L_h N_d}\right) n_i^{\,2}\left[\exp\left(\frac{eV}{k_B T}\right) - 1\right]$$

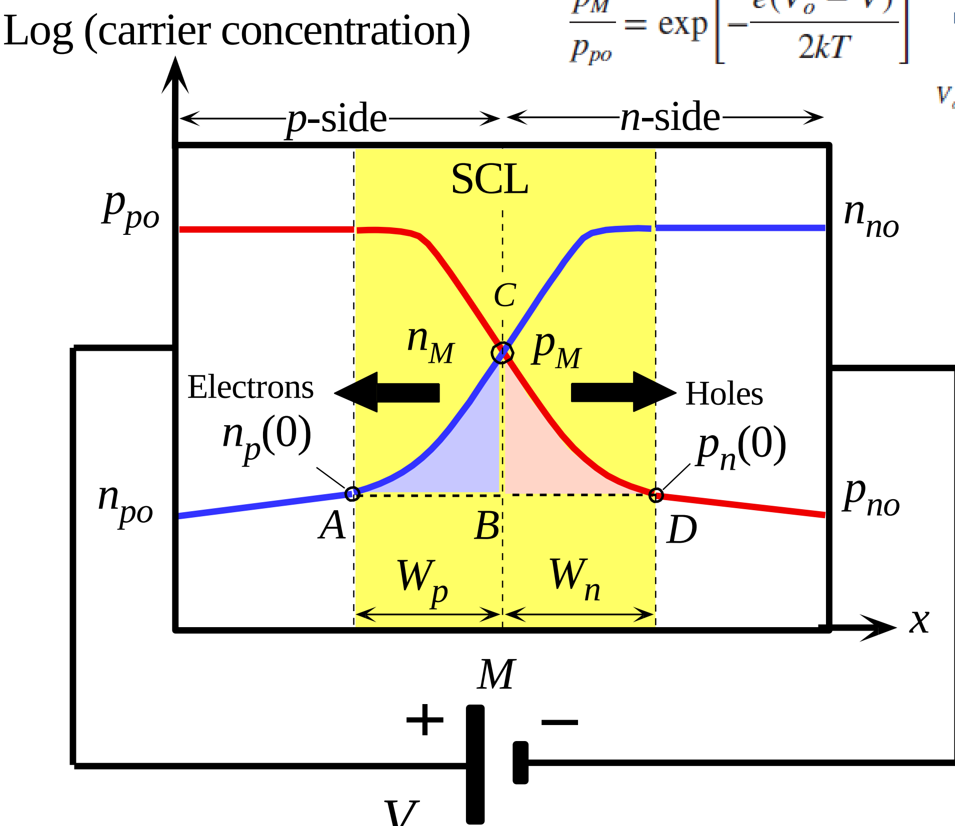
$$n_i^2 = N_c N_v \exp\left(-\frac{E_g}{k_B T}\right)$$

$$\boxed{J \approx \left(\frac{eD_e}{L_e N_a} + \frac{eD_h}{L_h N_d}\right) N_c N_v \exp\left[\frac{e(V - V_g)}{k_B T}\right]}$$



Schematic sketch of the I-V characteristics of Ge, Si and GaAs pn junction

Current Across a Forward Biased pn Junction: **Recombination**



Forward biased *pn* junction: the injection of carriers and their recombination in the SCL

$$\frac{p_M}{p_{po}} = \exp\left[-\frac{e(V_o - V)}{2kT}\right] \Rightarrow p_M = n_i \exp\left(\frac{eV}{2kT}\right)$$

$$V_o = \frac{kT}{e}\ln\left(\frac{N_a N_d}{n_i^2}\right)$$

$$J_{recom} = \frac{eABC}{\tau_e} + \frac{eBCD}{\tau_h}$$

$$J_{recom} \approx \frac{e\frac{1}{2}W_p n_M}{\tau_e} + \frac{e\frac{1}{2}W_n p_M}{\tau_h}$$

mean electron recombination time in $\mathbf{W_p}$ is $\boldsymbol{\tau_e}$.

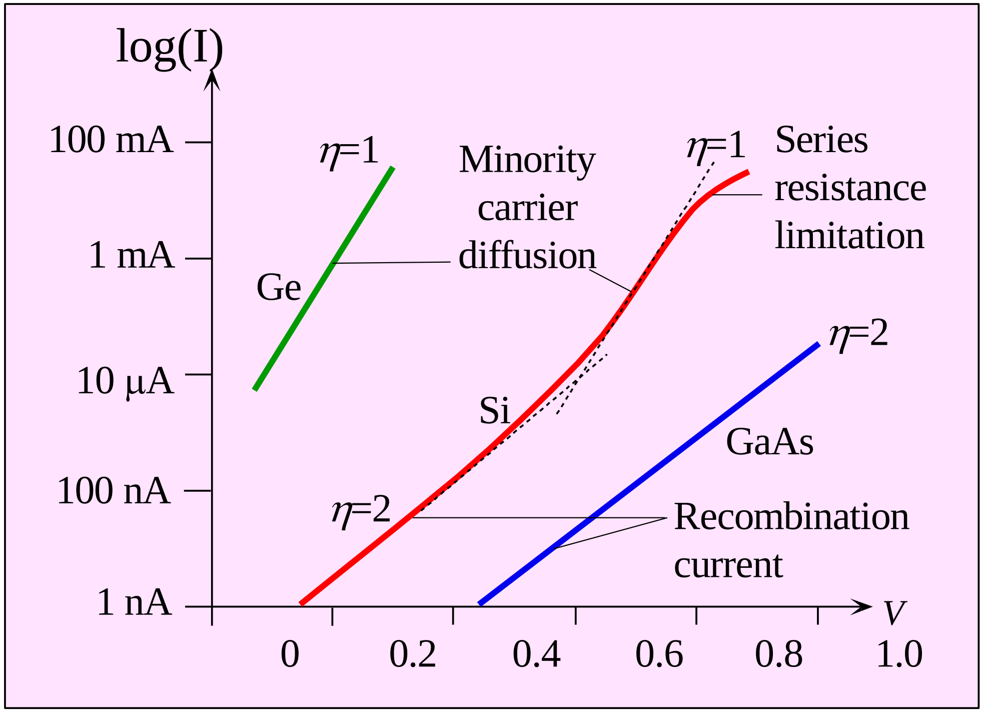
$$J_{recom} = J_{ro}[\exp(\frac{eV}{2k_BT}) - 1]$$

where $J_{ro} = \frac{en_i}{2}(\frac{W_p}{\tau_e} + \frac{W_n}{\tau_h})$

Total Current of a pn Junction under Forward bias

$$J = J_{so}[\exp(\frac{eV}{k_B T}) - 1] + J_{ro}[\exp(\frac{eV}{2k_B T}) - 1]$$

$$\boxed{J \approx J_{so}\exp(\frac{eV}{k_B T}) + J_{ro}\exp(\frac{eV}{2k_B T})}$$

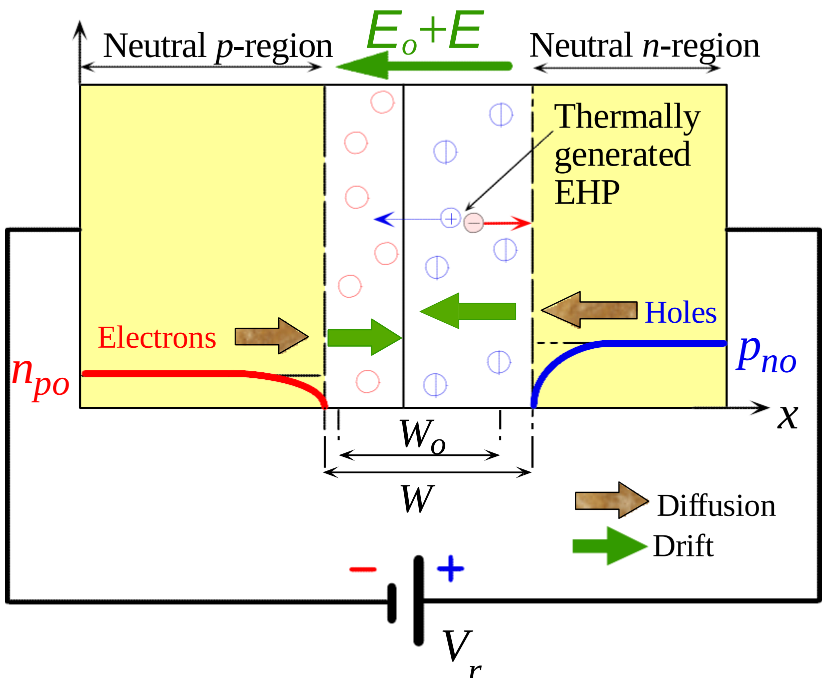


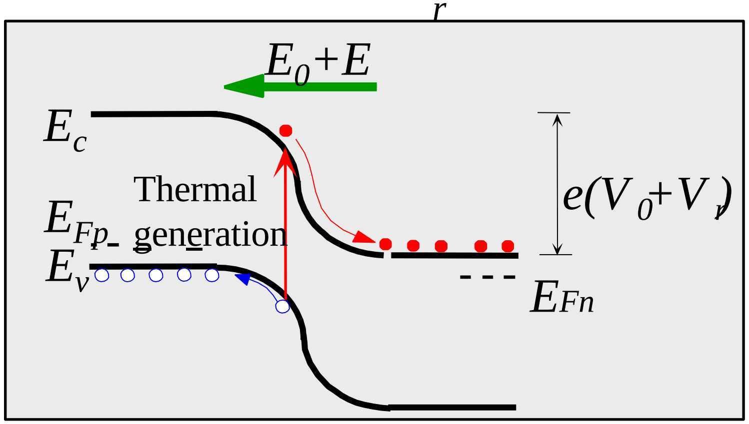
$$I = I_o[\exp(eV/\eta kT) - 1]$$

Schematic sketch of typical I-V characteristics of Ge, Si and GaAs *pn* junctions as log(I) vs. V. the slope indicates $e/(\eta k_B T)$ η: ***ideality factor***

Current Across a pn Junction: **Reverse Bias**

Minority Carrier





(a) Minority carrier extracted and swept by the field across the SCL

Essentially **Shockley equation**:

$$J = \left(\frac{eD_e}{L_eN_a} + \frac{eD_h}{L_hN_d}\right)n_i^{\,2}\left[\exp\left(\frac{eV}{k_BT}\right) - 1\right]$$

$$\approx -\left(\frac{eD_e}{L_eN_a} + \frac{eD_h}{L_hN_d}\right)n_i^{\,2}$$

reverse saturation current density, $-J_0$
independent of voltage V_r

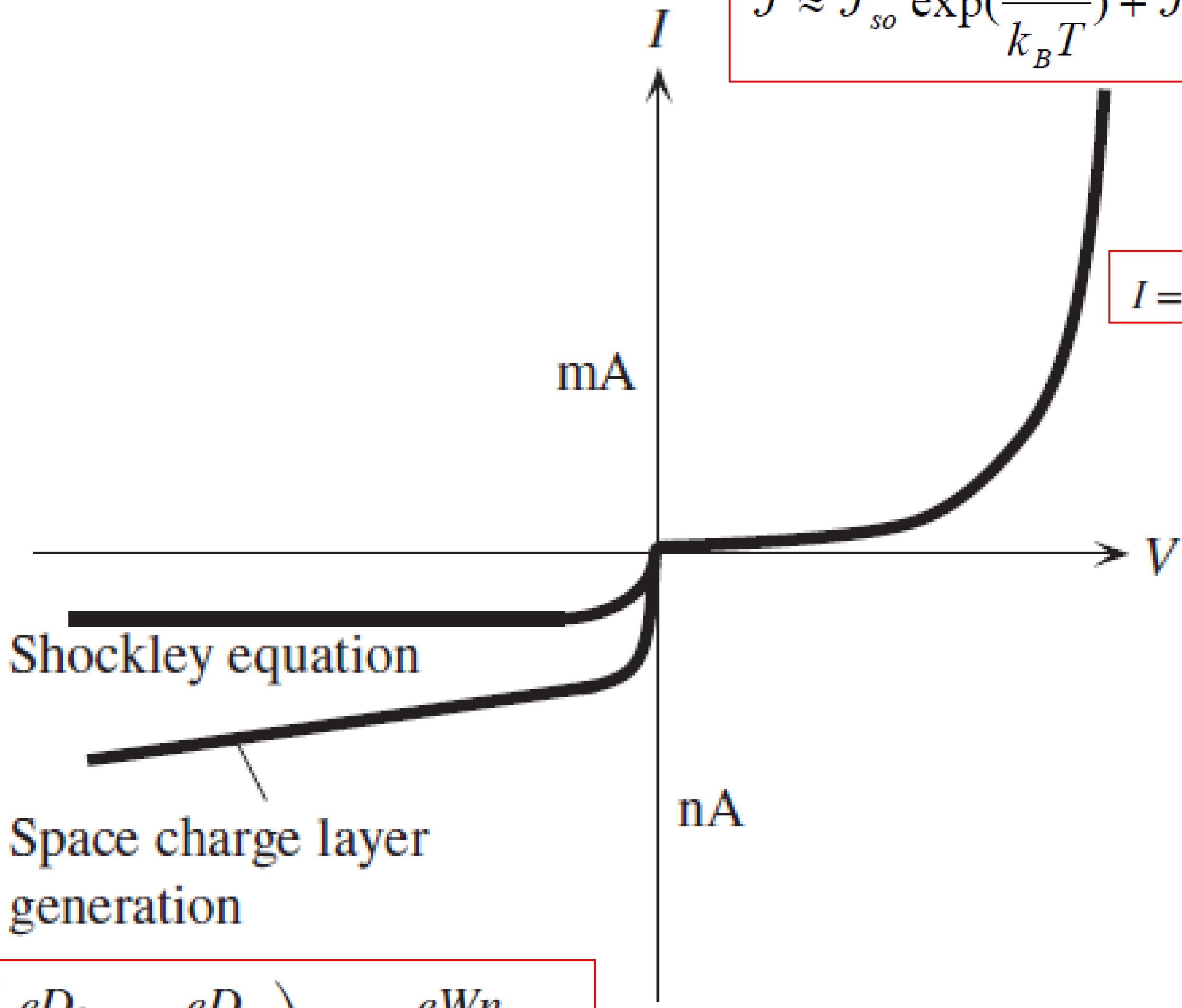
(b) Electron-hole pair (EHP) thermally generated within the SCL.

$$J_{gen} = \frac{eWn_i}{\tau_g}$$

τ_g is the mean time to generate an EHP

I-V Response of a *pn* Junction

$$J \approx J_{so} \exp(\frac{eV}{k_B T}) + J_{ro} \exp(\frac{eV}{2k_B T})$$



$$I = I_o[\exp(eV/\eta kT) - 1]$$

$$J_{\text{rev}} = \left(\frac{eD_h}{L_h N_d} + \frac{eD_e}{L_e N_a}\right) n_i^2 + \frac{eWn_i}{\tau_g}$$

完成并提交 **Assignment 6.3 (1)**
Assignment 6.3 (2)无须提交，在5月12日习题课前尽量完成

提交时间：**5月12日**（周一）中午前提交**Assignment 6.3 (1)**

提交方式：电子版（写明姓名、学号），通过本班课代表统一提交