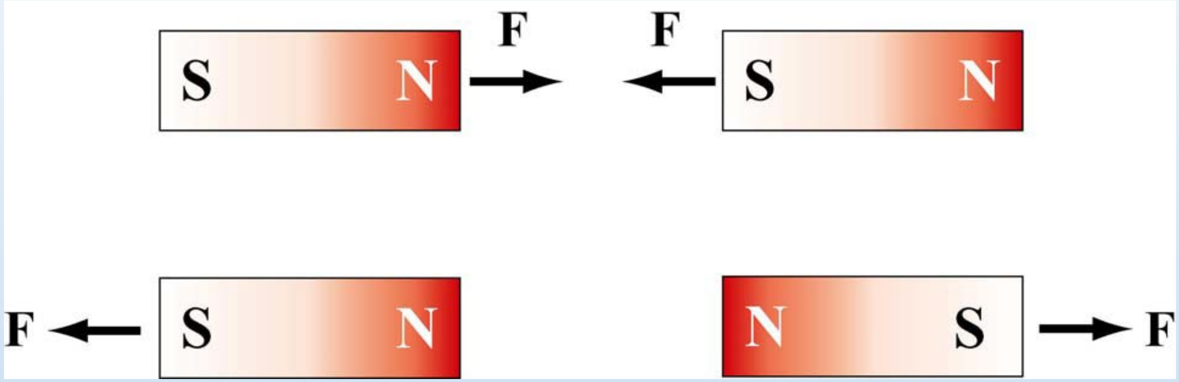
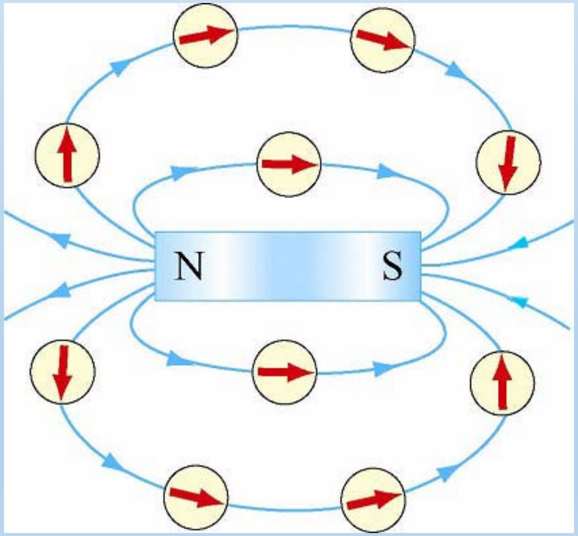
Lecture 2: Magnetostatics and Faraday's Law

Outline

- Magnetic Fields
- Lorentz Force
- Magnetic Dipoles
- Force and Torque on Magnetic Dipoles
- Biot-Savart Law
- Ampere's Law
- Faraday's Law

Magnetism –Bar Magnet

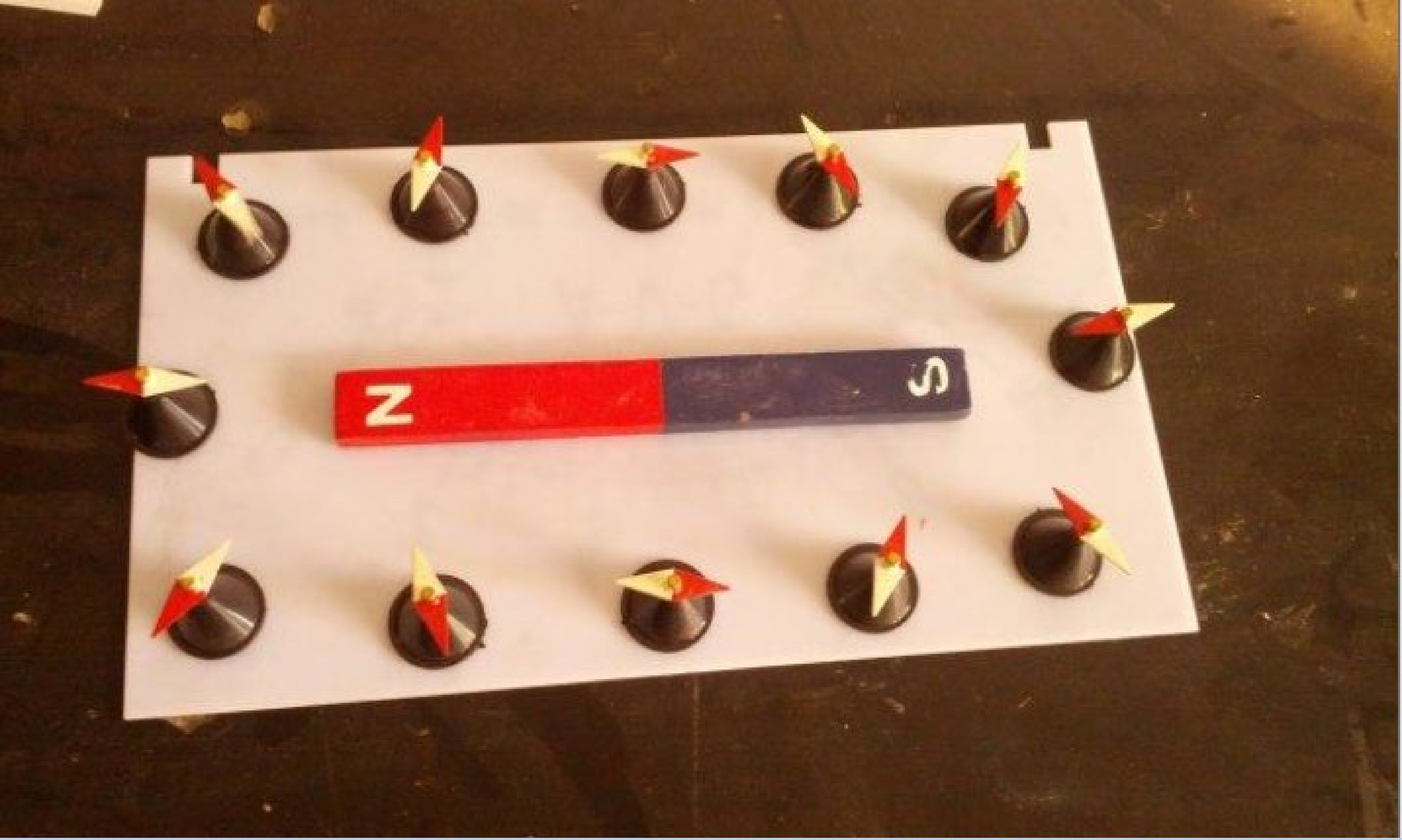




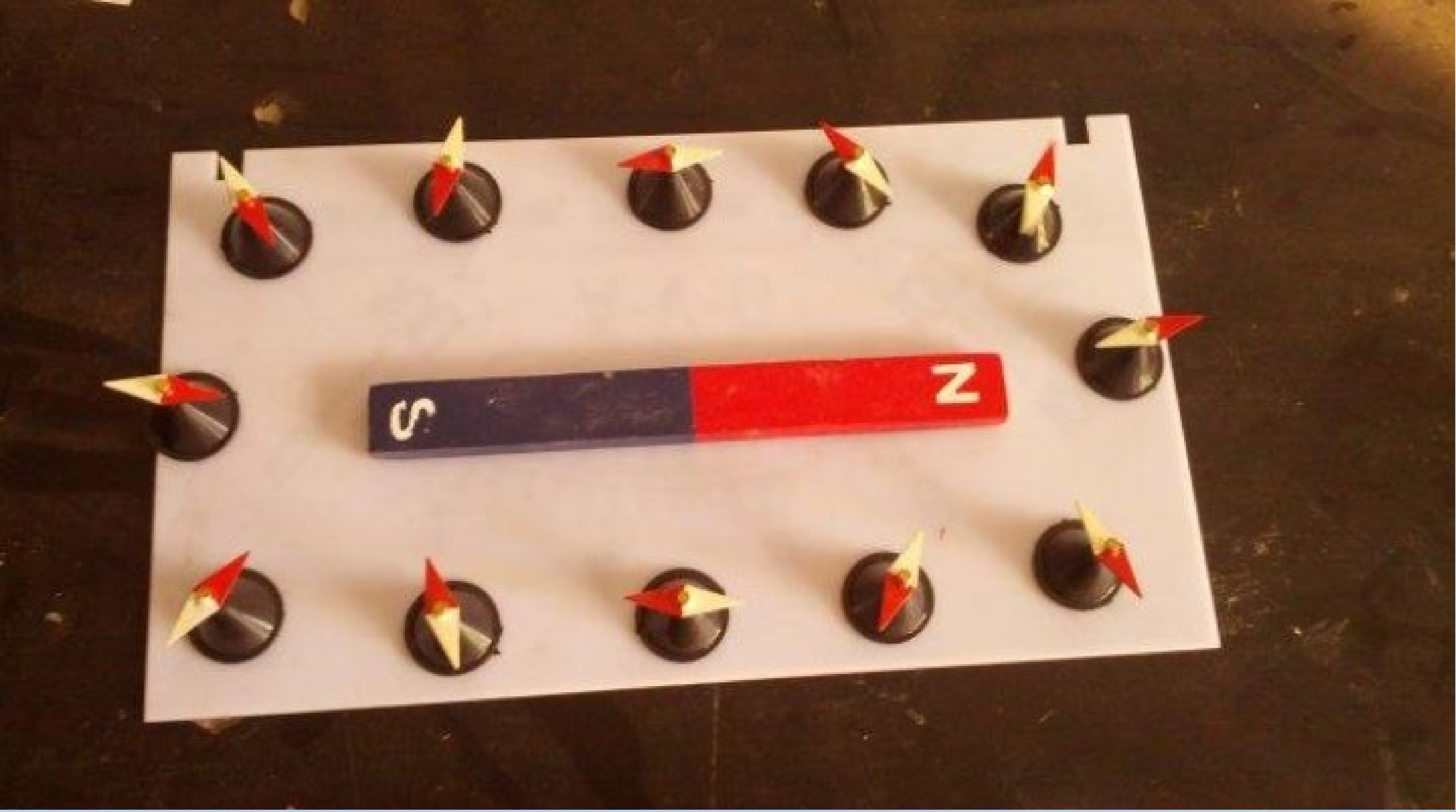
Magnetic Field of Bar Magnet:

- A magnet has two poles, North (N) and South (S)
- Like poles repel, opposite poles attract
- Magnetic field lines leave from N, end at S

- Magnetic field lines leave from N, end at S

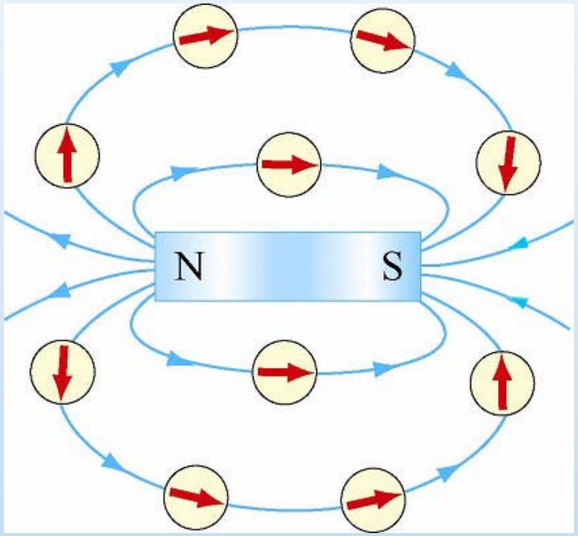


- **Magnetic field lines leave from N, end at S**



- **Magnetic field lines leave from N, end at S**

Bar Magnets Are Dipoles!



Magnetic Field of Bar Magnet:

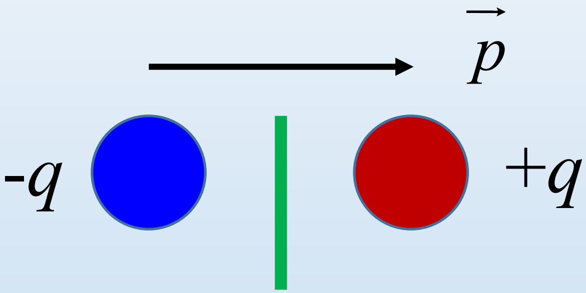
- Create Dipole Field
- Rotate to orient with Field

Is there magnetic "mass" or magnetic "charge?"

NO! Magnetic monopoles do not exist in isolation

Magnetic Monopoles?

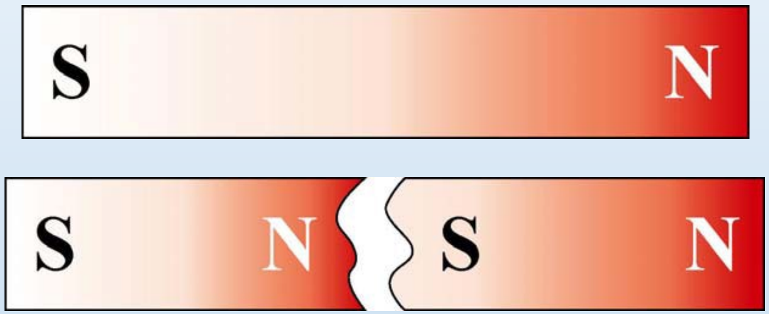
Electric dipole



When cut

2 monopoles (charges)

Magnetic dipole



When cut: 2 dipoles

Magnetic monopoles do not exist in isolation

Another Maxwell's Equation! (2 of 4)

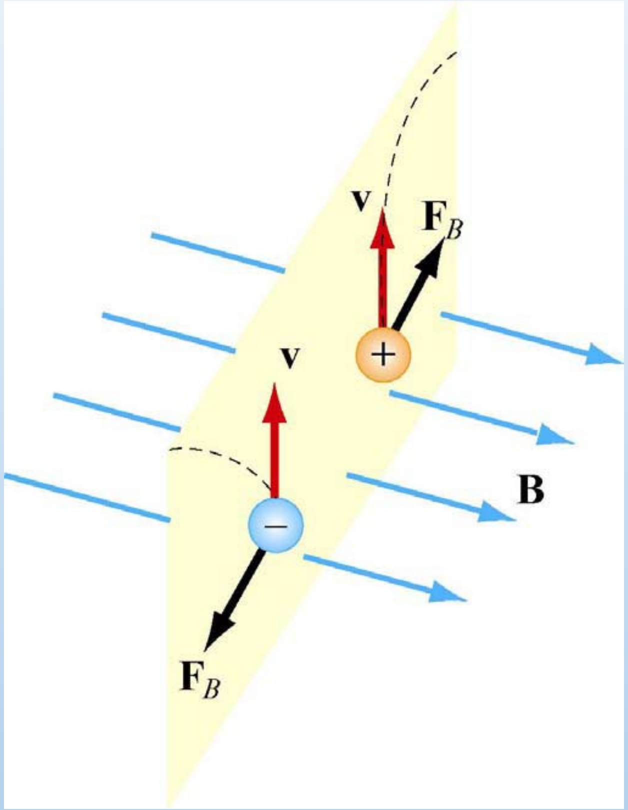
$$\oiint_S \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\varepsilon_0}$$

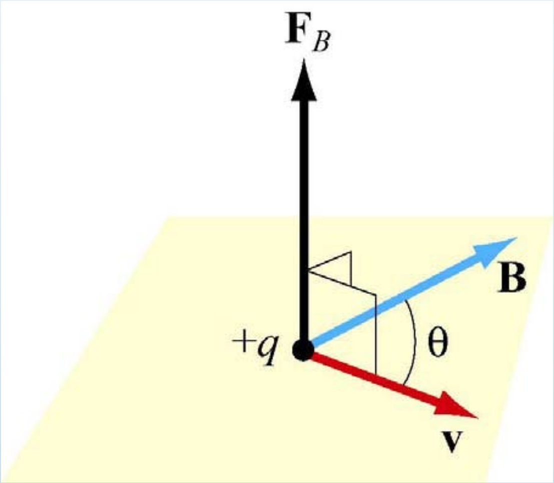
Gauss's Law

$$\oiint_S \vec{B} \cdot d\vec{A} = 0$$

Magnetic Gauss's Law

Moving Charges Feel Magnetic Force





$$\boxed{\vec{F}_B = q\vec{v} \times \vec{B}}$$

Magnetic force perpendicular both to:
Velocity v of charge and magnetic field B

Putting it Together: Lorentz Force

Charges Feel…

$$\boxed{\vec{F}_E = q\vec{E}}$$

Electric fields

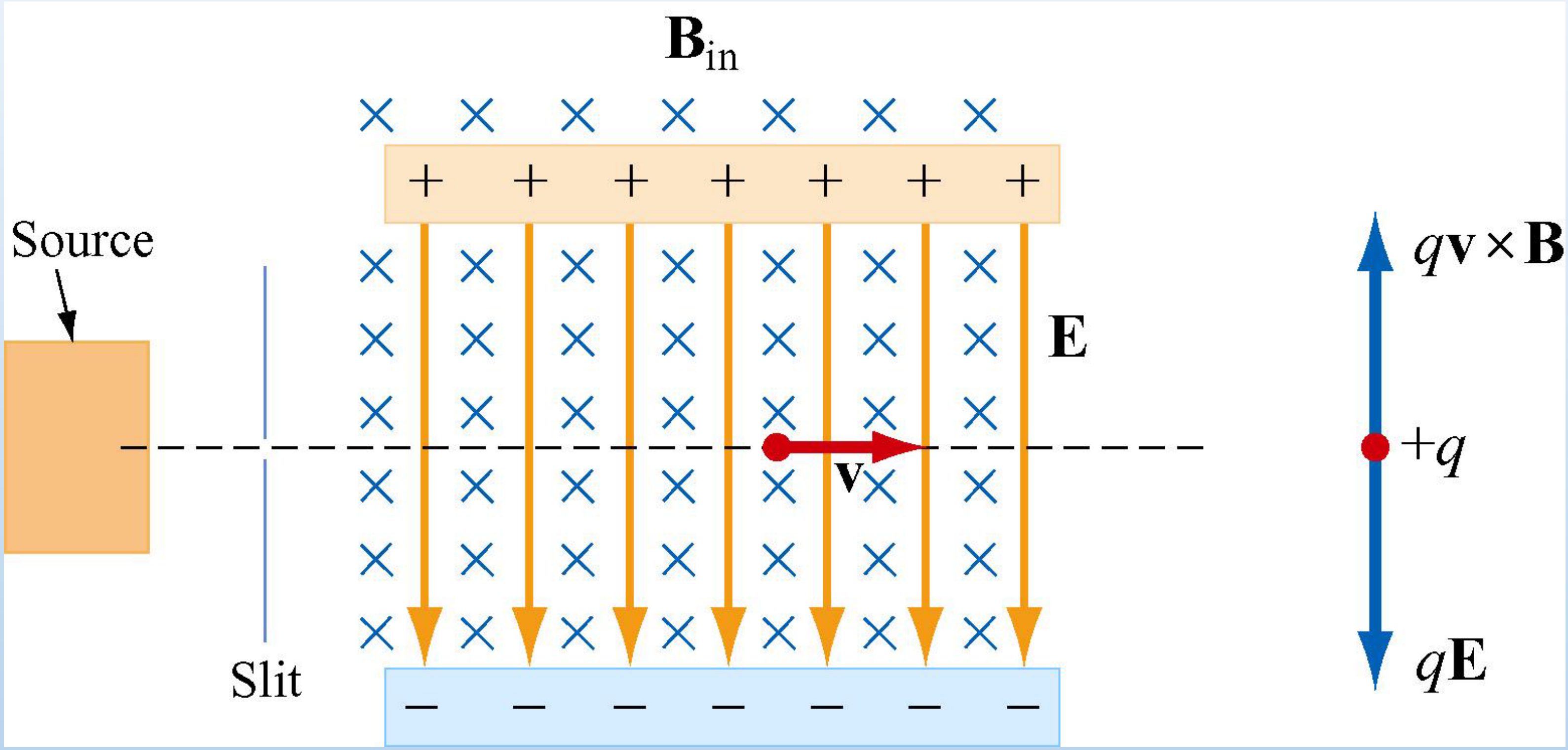
$$\boxed{\vec{F}_B = q\vec{v}\times\vec{B}}$$

Magnetic fields

$$\boxed{\vec{F} = q\left(\vec{E}+\vec{v}\times\vec{B}\right)}$$

This is the final word on the force on a charge (Lorentz Force)

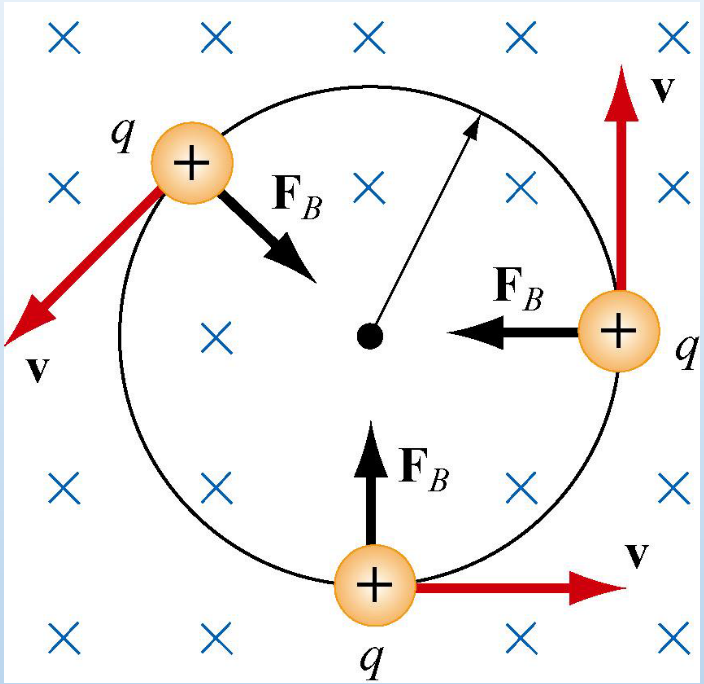
Application: Velocity Selector



Particle moves in a straight line when

$$\vec{F} = q\left(\vec{E} + \vec{v}\times\vec{B}\right) = 0 \quad \Rightarrow \quad v = \frac{E}{B}$$

Application: Cyclotron Motion



(1) r : radius of the circle

$$qvB = \frac{mv^2}{r} \Rightarrow r = \frac{mv}{qB}$$

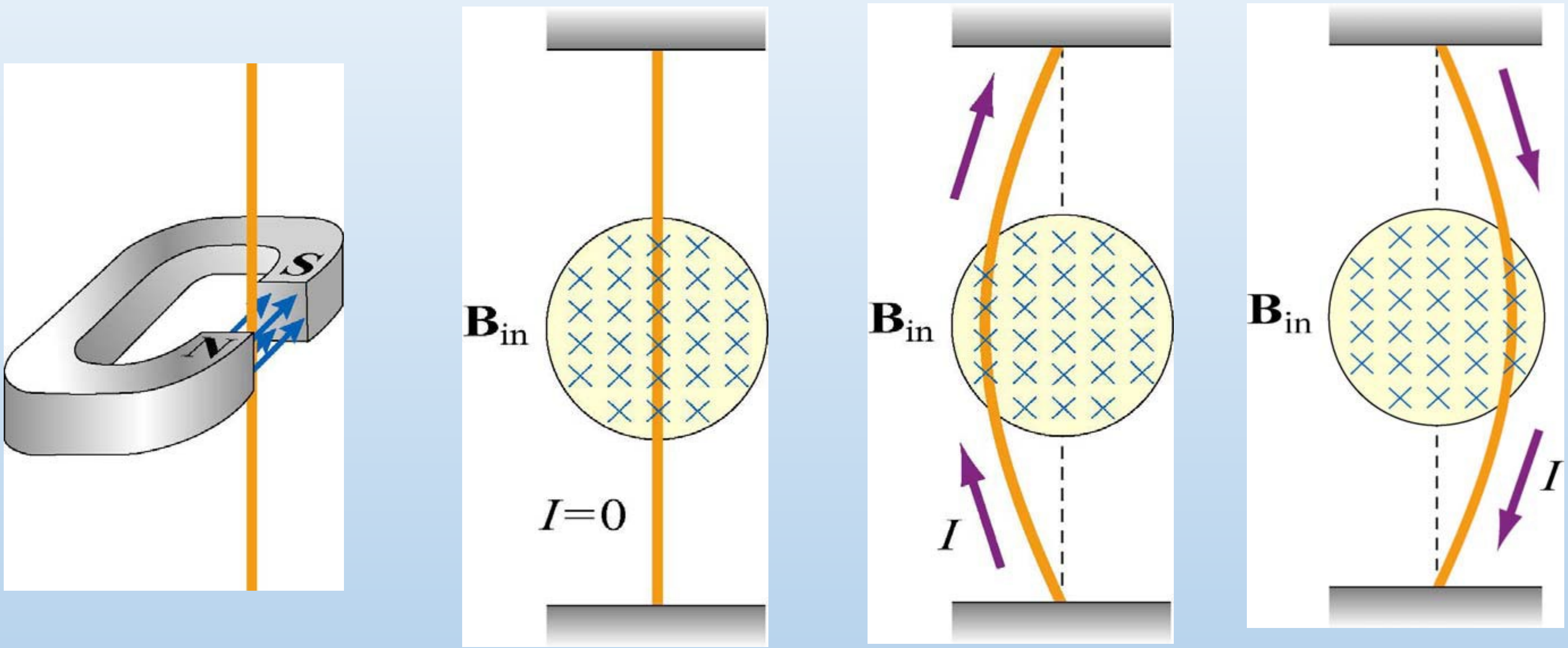
(2) T : period of the motion

$$T = \frac{2\pi r}{v} = \frac{2\pi m}{qB}$$

(3) ω : cyclotron frequency

$$\omega = 2\pi f = \frac{v}{r} = \frac{qB}{m}$$

Magnetic Force on Current-Carrying Wire



Current is the collective <u>moving charges</u>, and <u>moving charges</u> feel a force in a magnetic field

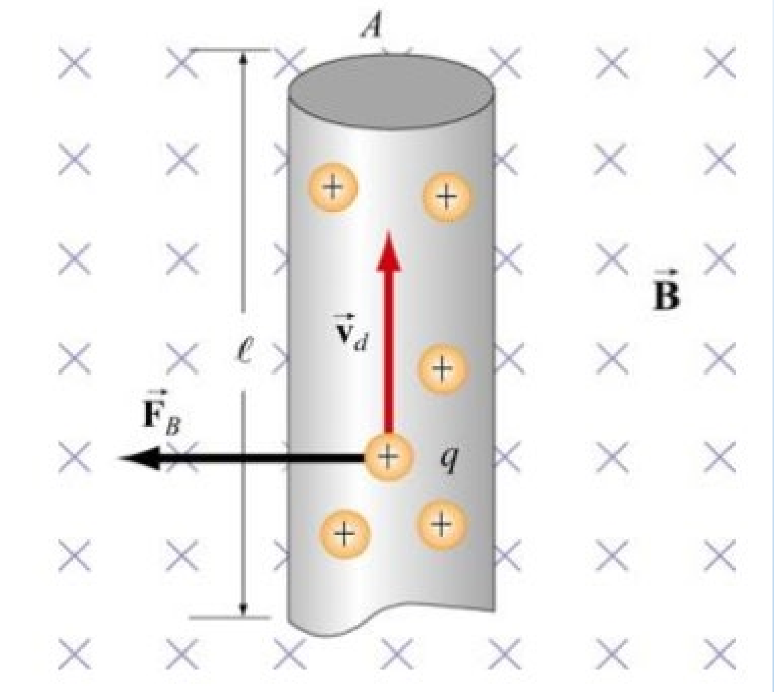
Magnetic Force on Current-Carrying Wire

$$\vec{F}_B = q\vec{v} \times \vec{B}$$

$$= (\text{charge})\frac{\text{m}}{\text{s}} \times \vec{B}$$

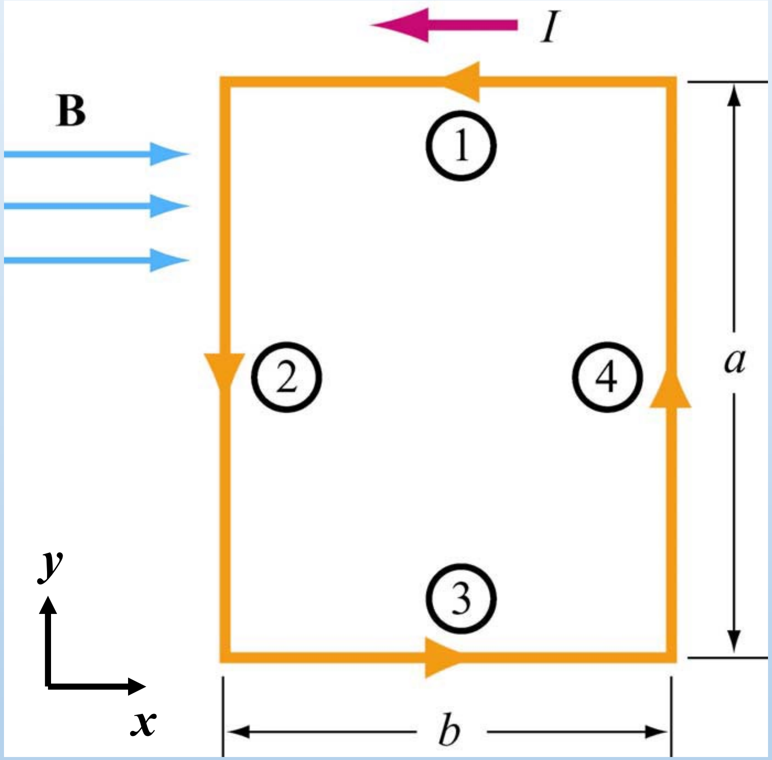
$$= \frac{\text{charge}}{\text{s}}\text{m} \times \vec{B}$$

$$\boxed{\vec{F}_B = I\left(\vec{L} \times \vec{B}\right)}$$



Rectangular Current Loop

Place rectangular current loop in uniform B field



$$\vec{F}_1 = \vec{F}_3 = 0 \qquad \left(I\vec{L} \parallel \vec{B}\right)$$

$$\vec{F}_2 = I\left(-a\hat{y}\right)\times\left(B\hat{x}\right) = IaB\hat{z}$$

$$\vec{F}_4 = I\left(a\hat{y}\right)\times\left(B\hat{x}\right) = -IaB\hat{z}$$

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 = 0$$

No net force on the loop!!

Torque on Rectangular Loop

Recall: $\vec{\tau} = \vec{r} \times \vec{F}$

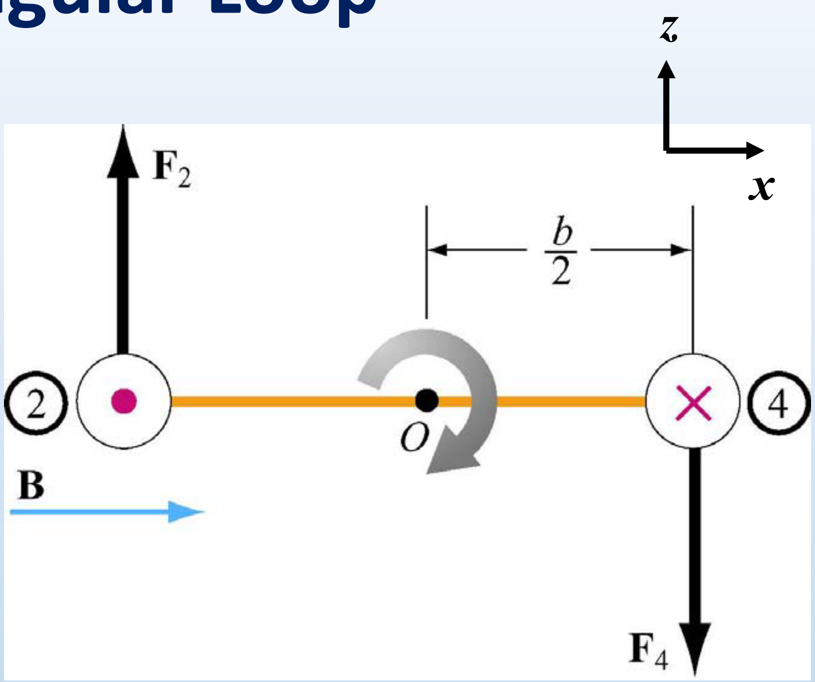
$$\vec{\tau} = \left(-\frac{b}{2}\hat{x}\right) \times \vec{F}_2 + \left(\frac{b}{2}\hat{x}\right) \times \vec{F}_4$$

$$= \left(-\frac{b}{2}\hat{x}\right) \times \left(IaB\hat{z}\right) + \left(\frac{b}{2}\hat{x}\right) \times \left(-IaB\hat{z}\right)$$

$$= \frac{IabB}{2}\hat{y} + \frac{IabB}{2}\hat{y} = IabB\hat{y}$$

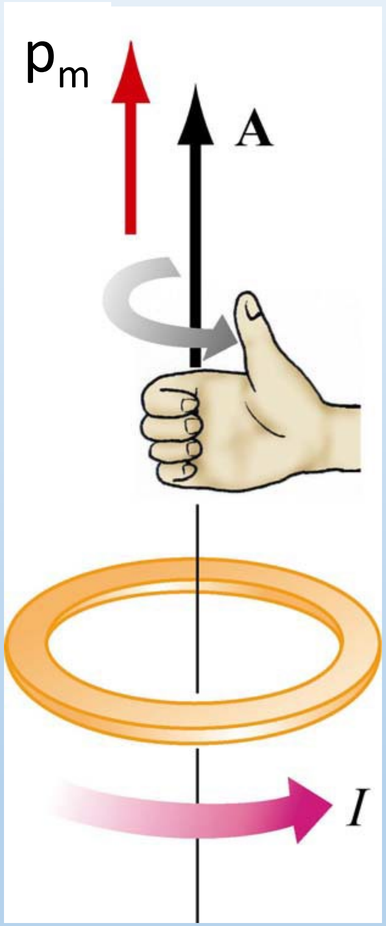
$$\boxed{\vec{\tau} = I\vec{A} \times \vec{B}}$$

$$\vec{A} = A\hat{n}$$



Torque Direction:
Thumb in torque direction, fingers rotate with object

Magnetic Dipole Moment



Define Magnetic Dipole Moment

$$\boxed{\vec{p}_m = IA\hat{n} = I\vec{A}}$$

Then:

$$\boxed{\vec{\tau} = \vec{p}_m \times \vec{B}}$$

Analogous to $\vec{\tau} = \vec{p}_e \times \vec{E}$

τ tends to align p_m with B

Dipoles in Non-Uniform Fields: force

To determine force, we need to know energy

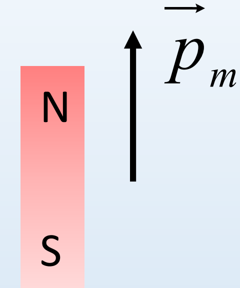
$$U_{Dipole} = -\vec{p}_m \cdot \vec{B}$$

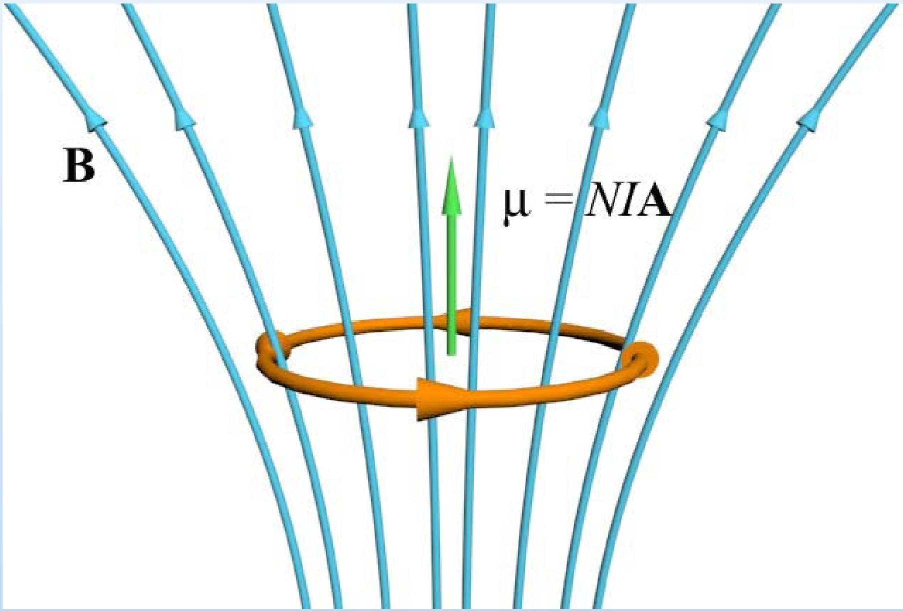
Force tells how the energy changes with position

$$\vec{F}_{Dipole} = -\nabla U_{Dipole} = \nabla\left(\vec{p}_m \cdot \vec{B}\right)$$

Dipoles only feel force in non-uniform field

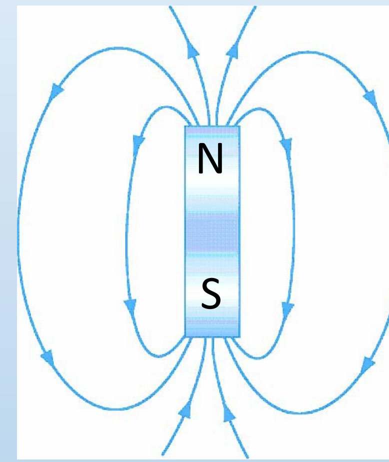
Force on Magnetic Dipole





$$\frac{\partial \vec{B}}{\partial z}$$ **negative**

Force down

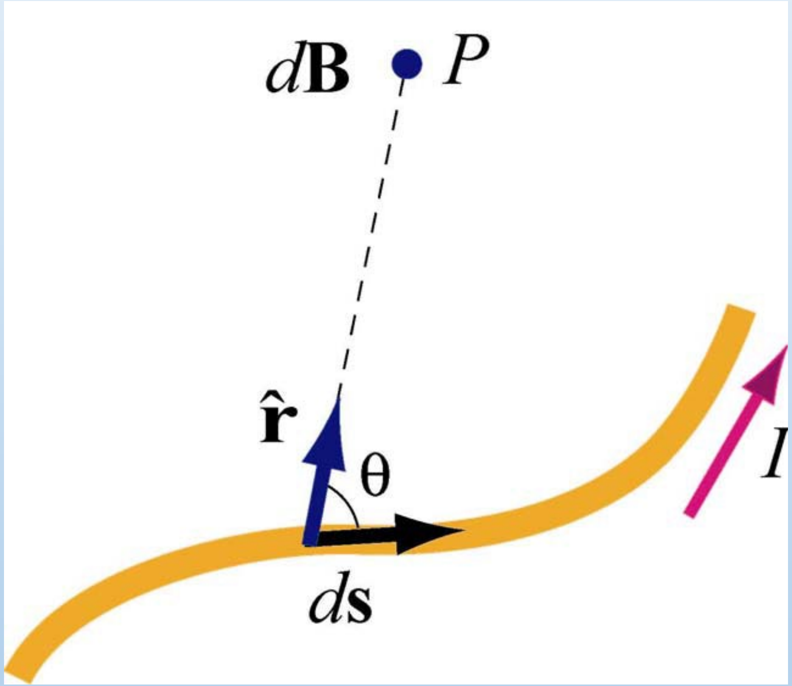


$$\vec{F}_{Dipole} = \nabla\left(\vec{p}_m \cdot \vec{B}\right) = p_m \frac{\partial \vec{B}}{\partial z}$$

Bar magnet below dipole, with N pole on top
It is aligned with the dipole pictured, they attract!

The Biot-Savart Law

Current element of length ds carrying current I produces a magnetic field:



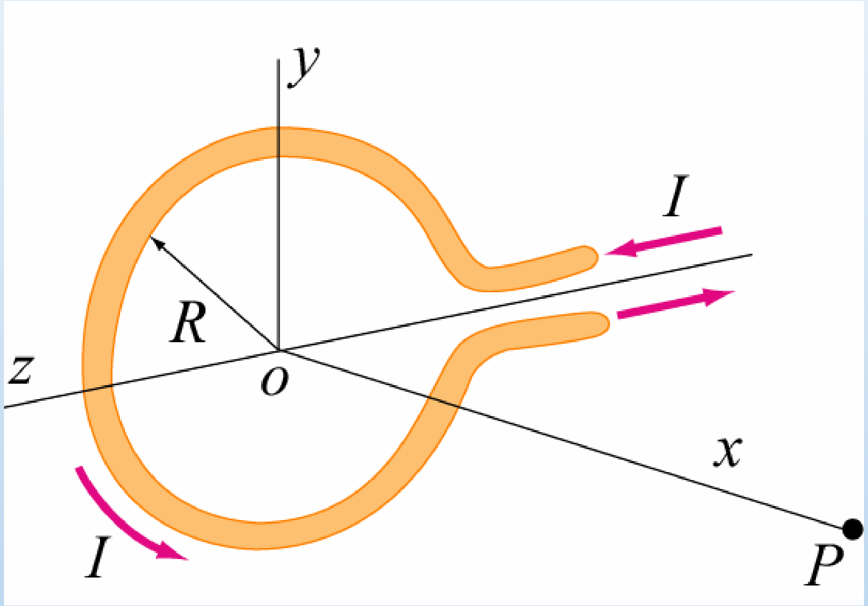
$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{s} \times \hat{r}}{r^2}$$

$$\mu_0 = 4\pi \times 10^{-7}\, \mathrm{T \cdot m/A}$$

$$\vec{B} = \int_{wire} d\vec{B} = \frac{\mu_0 I}{4\pi} \int_{wire} \frac{d\vec{s} \times \hat{r}}{r^2}$$

similar to the Coulomb's law $d\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{dq}{r^2} \hat{r}$

Example : Coil of Radius R



In the circular part of the coil…

$$d\vec{s} \perp \hat{r} \quad \Rightarrow \quad \left| d\vec{s} \times \hat{r} \right| = ds$$

Biot-Savart:

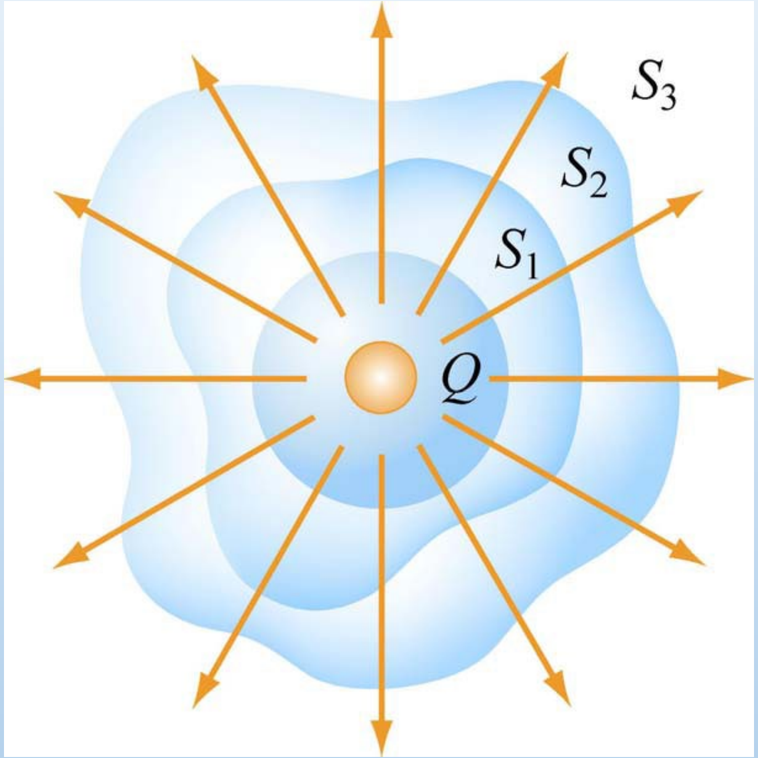
$$dB = \frac{\mu_0 I}{4\pi} \frac{\left| d\vec{s} \times \hat{r} \right|}{r^2} = \frac{\mu_0 I}{4\pi} \frac{ds}{r^2}$$

$$= \frac{\mu_0 I}{4\pi} \frac{R d\theta}{R^2} = \frac{\mu_0 I}{4\pi} \frac{d\theta}{R}$$

At point O: $\vec{B} = \hat{x} \int dB = \hat{x} \int_0^{2\pi} \frac{\mu_0 I}{4\pi} \frac{d\theta}{R} = \hat{x} \frac{\mu_0 I}{2R}$

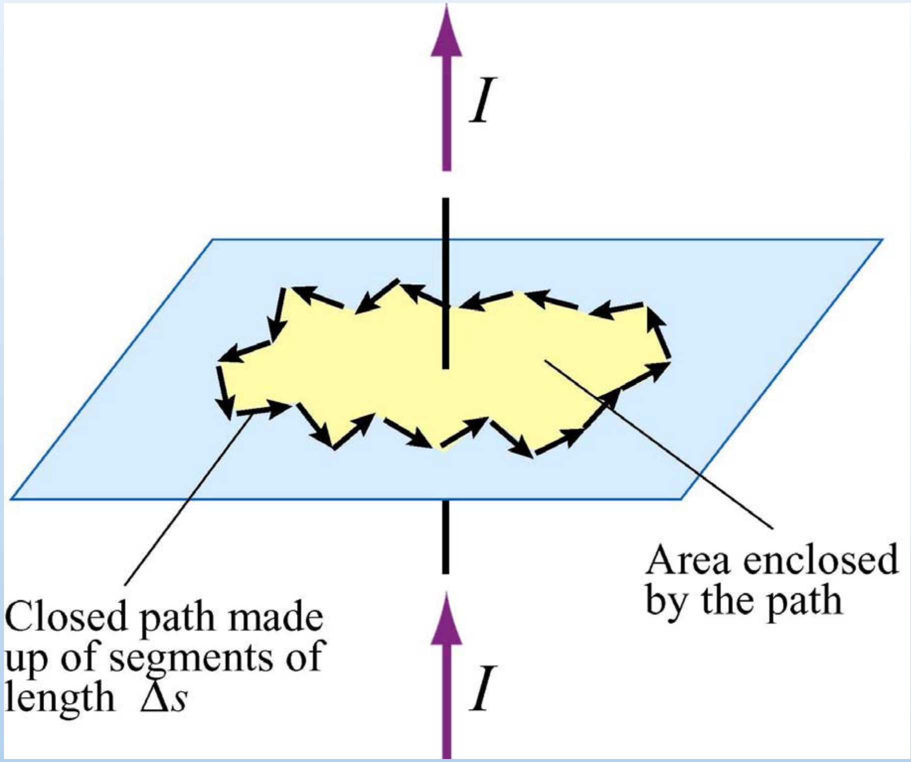
Ampere's Law: The Idea

Gauss's Law



$$\Phi_E = \oiint_S \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\varepsilon_0}$$

Ampere's Law



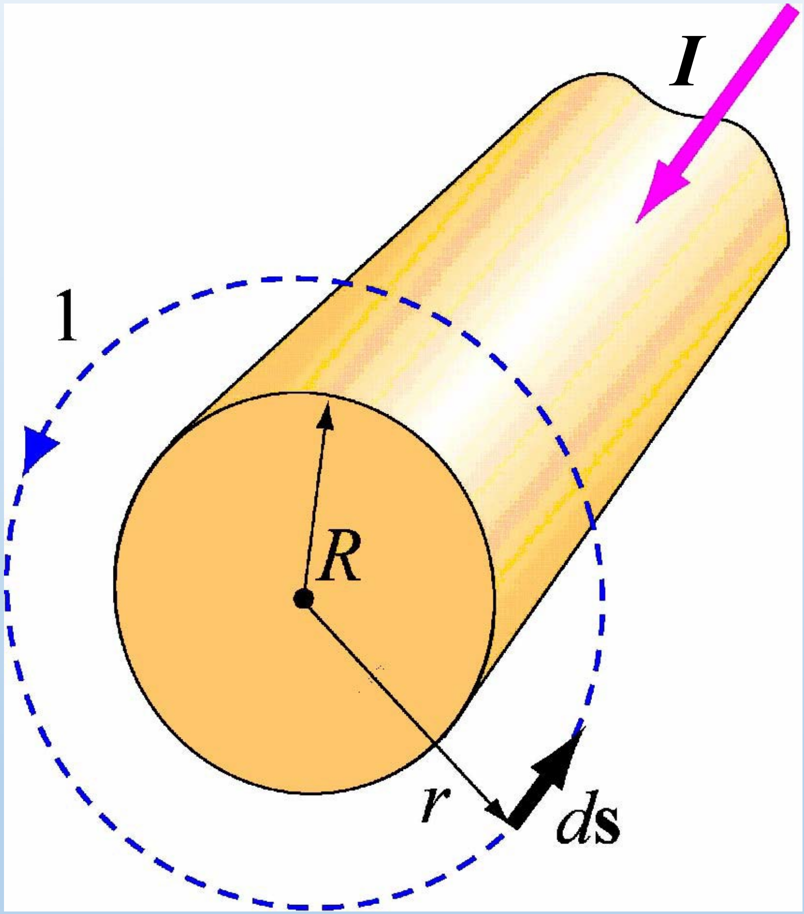
$$\boxed{\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{enc} = \mu_0 \oiint_S \vec{J} \cdot d\vec{A}}$$

Applying Ampere's Law

1. Identify regions in which to calculate B field Get B direction by right hand rule
2. Choose Amperian Loops S: Symmetry B is 0 or constant on the loop!
3. Calculate $\oint \vec{B} \cdot d\vec{s}$
4. Calculate current enclosed by loop S
5. Apply Ampere's Law to solve for B

$$\boxed{\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{enc}}$$

Example: Wire of Radius R



Region 1: Outside wire (r ≥ R)

Cylindrical symmetry

=>

Amperian Circle

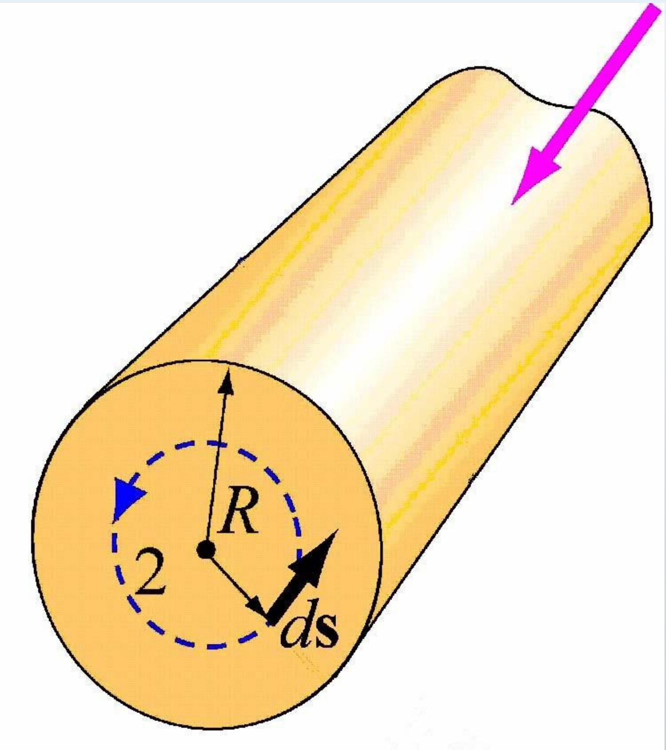
B-field counterclockwise

$$\oint \vec{B} \cdot d\vec{s} = B\oint ds = 2\pi r B$$

$$= \mu_0 I_{enc} = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r}$$ **counterclockwise**

Example: Wire of Radius R



Region 2: Inside wire (r < R)

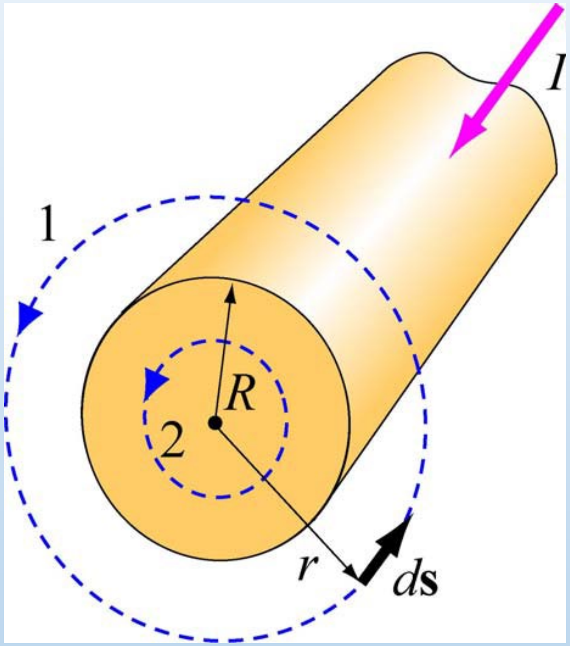
$$\oint \vec{B} \cdot d\vec{s} = B \oint ds = 2\pi r B$$

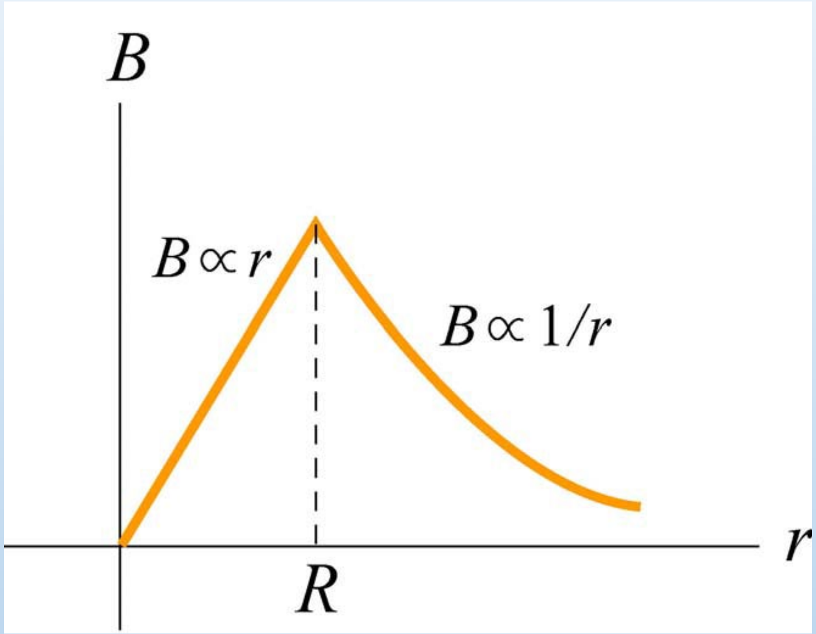
$$= \mu_0 I_{enc} = \mu_0 I \frac{\pi r^2}{\pi R^2}$$

$$B = \frac{\mu_0 I r}{2\pi R^2}$$ **counterclockwise**

Could also say $J = \dfrac{I}{A} = \dfrac{I}{\pi R^2}; \quad I_{enc} = J A_{enc} = \dfrac{I}{\pi R^2}\left(\pi r^2\right)$

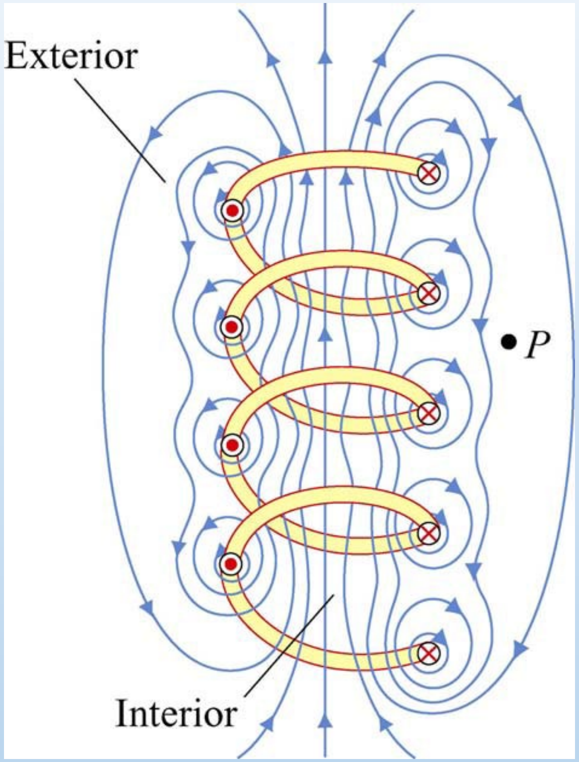
Example: Wire of Radius R



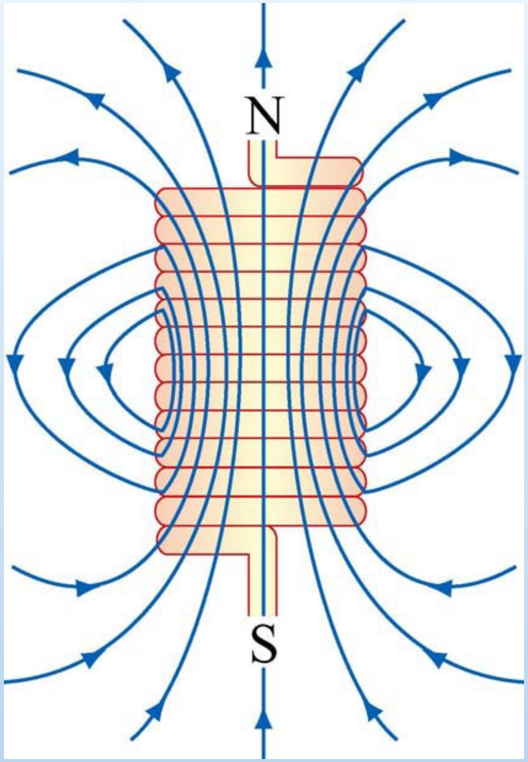


$$B_{in} = \frac{\mu_0 I r}{2\pi R^2} \qquad B_{out} = \frac{\mu_0 I}{2\pi r}$$

Magnetic Field of Solenoid



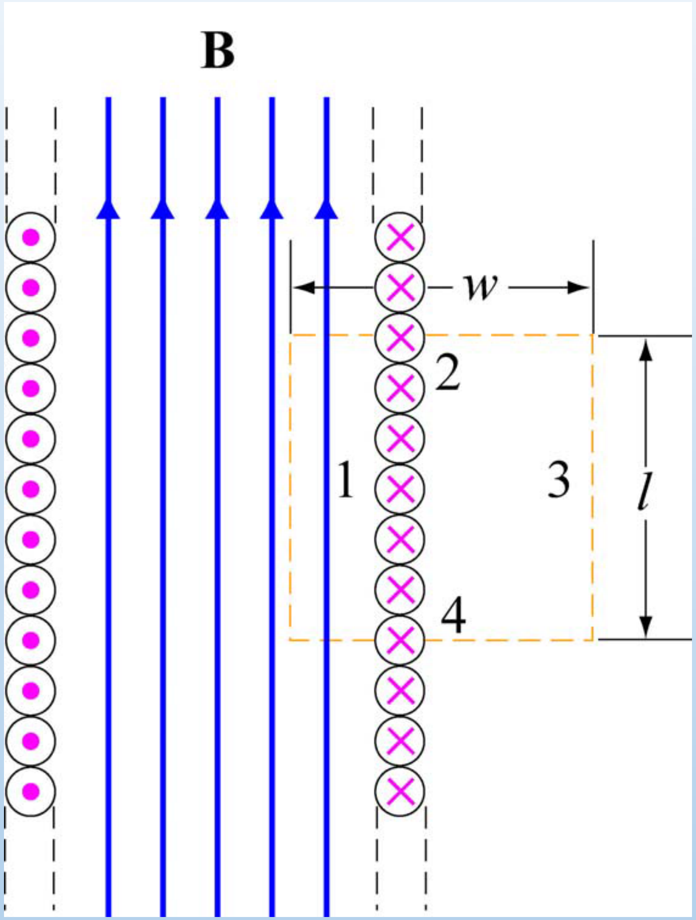
loosely wound



tightly wound

For ideal solenoid, B is uniform inside & zero outside

Magnetic Field of Ideal Solenoid



Using Ampere's law: Think!

$$\begin{cases} \vec{B} \perp d\vec{s} & \text{along sides 2 and 4} \\ \vec{B} = 0 & \text{along side 3} \end{cases}$$

$$\oint \vec{B} \cdot d\vec{s} = \int_1 \vec{B} \cdot d\vec{s} + \int_2 \vec{B} \cdot d\vec{s} + \int_3 \vec{B} \cdot d\vec{s} + \int_4 \vec{B} \cdot d\vec{s}$$

$$= Bl + 0 + 0 + 0$$

$I_{enc} = nlI$ $\quad$ n: turn density

$$\oint \vec{B} \cdot d\vec{s} = Bl = \mu_0 nlI$$

$n = N/L$: turns/unit length

$$\boxed{B = \mu_0 nI}$$

Maxwell’s Equations (So Far)

Gauss's Law: $$\oiint_S \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\varepsilon_0}$$

Electric charges make diverging Electric Fields

Magnetic Gauss's Law: $$\oiint_S \vec{B} \cdot d\vec{A} = 0$$

No Magnetic Monopoles! (No diverging B Fields)

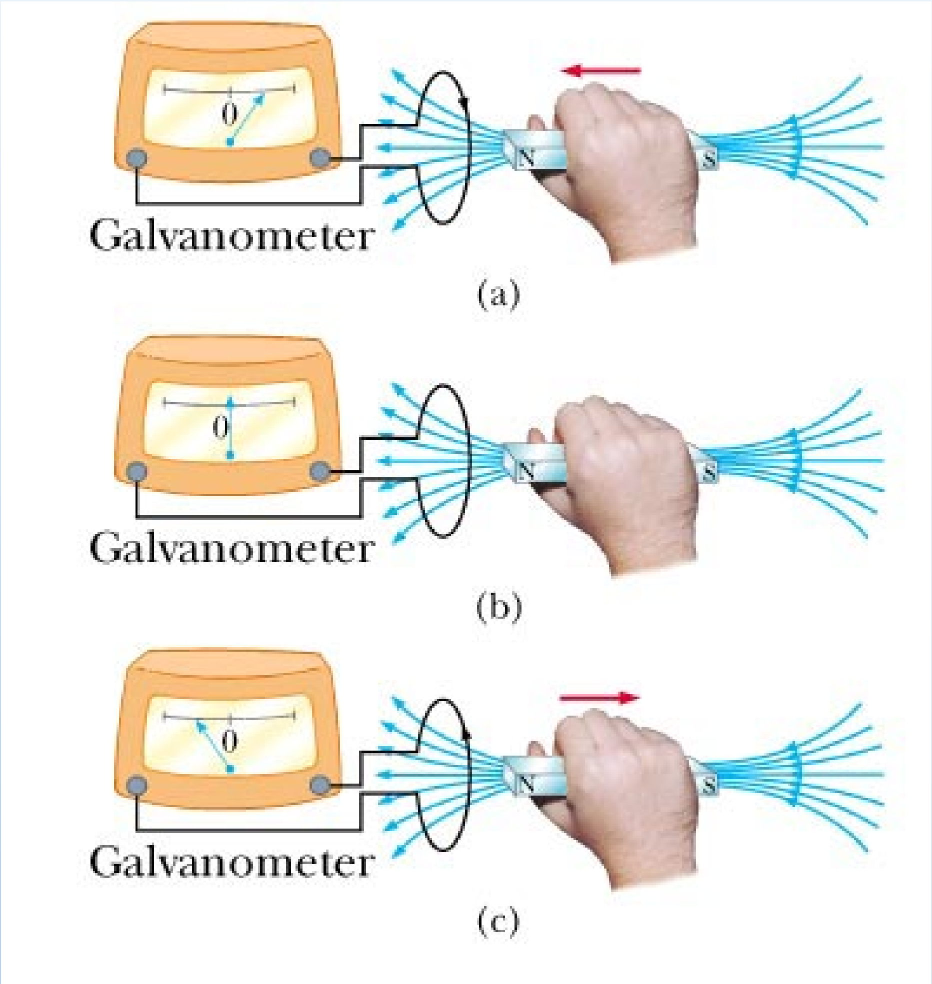
Ampere’s Law: $$\oint_C \vec{B} \cdot d\vec{s} = \mu_0 I_{enc}$$

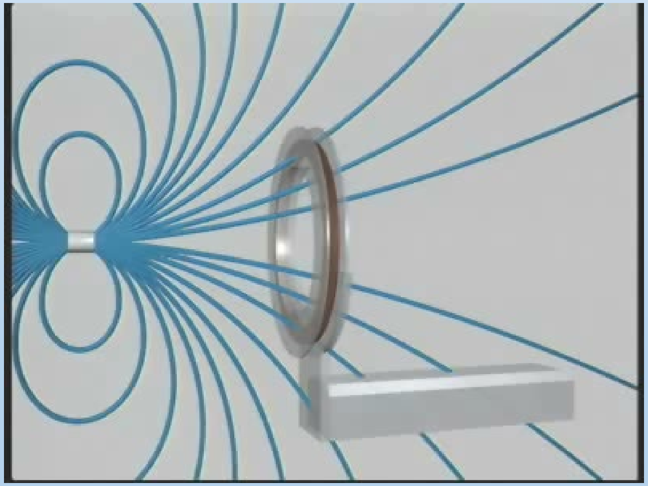
Currents make curling Magnetic Fields

This Time: Faraday’s Law

-Fourth (Final) Maxwell’s Equation

Electromagnetic Induction

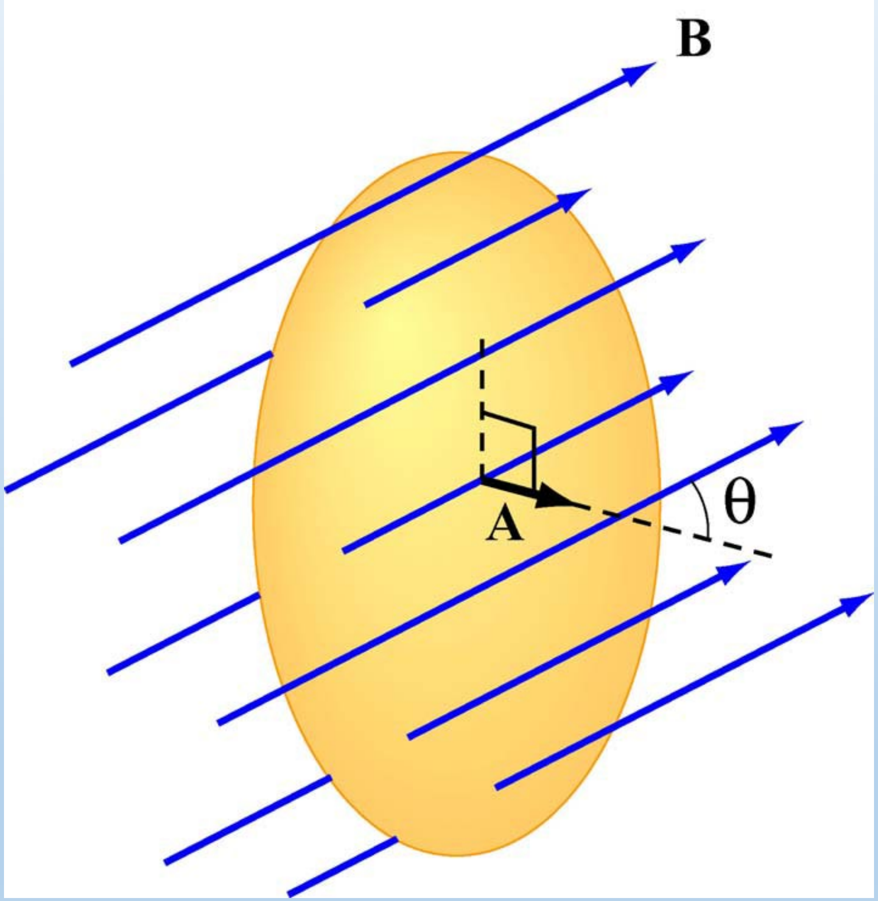




https://ocw.mit.edu/ans7870/8/8.02T/f04/visualizations/faraday/15-inductance/15-1_wmv320.html

Magnetic Flux Thru Wire Loop

Analogous to Electric Flux (Gauss' Law)



(1) Uniform B

$$\Phi_B = B_\perp A = BA\cos\theta = \vec{B}\cdot\vec{A}$$

(1) Non-Uniform B

$$\boxed{\Phi_B = \iint_S \vec{B}\cdot d\vec{A}}$$

Faraday’s Law of Induction

A changing magnetic flux induces an electromotive force (*emf*)

$$emf = -N\frac{d\Phi_B}{dt} = -N\frac{d}{dt}\left(BA\cos\theta\right)$$

Ways to Induce EMF

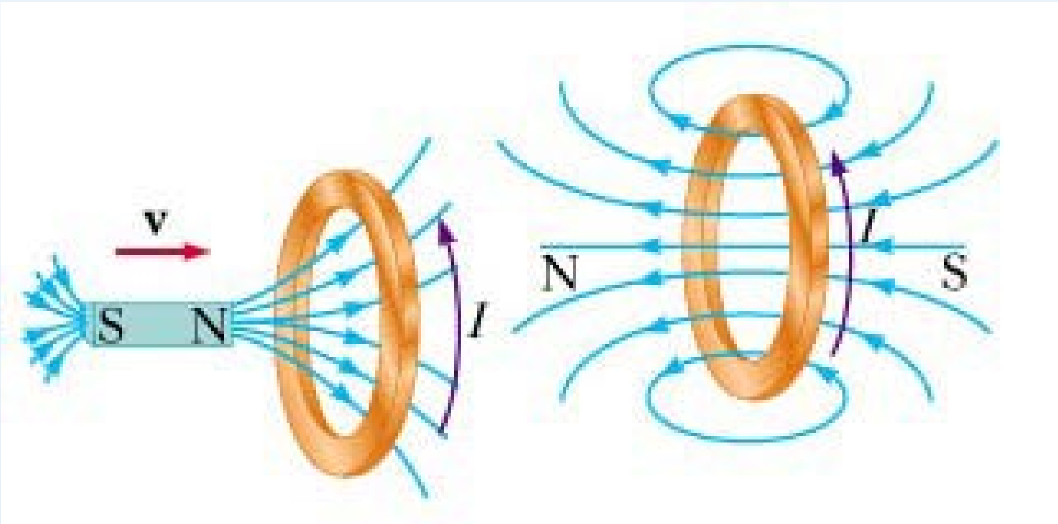
<u>Quantities which can vary with time:</u>

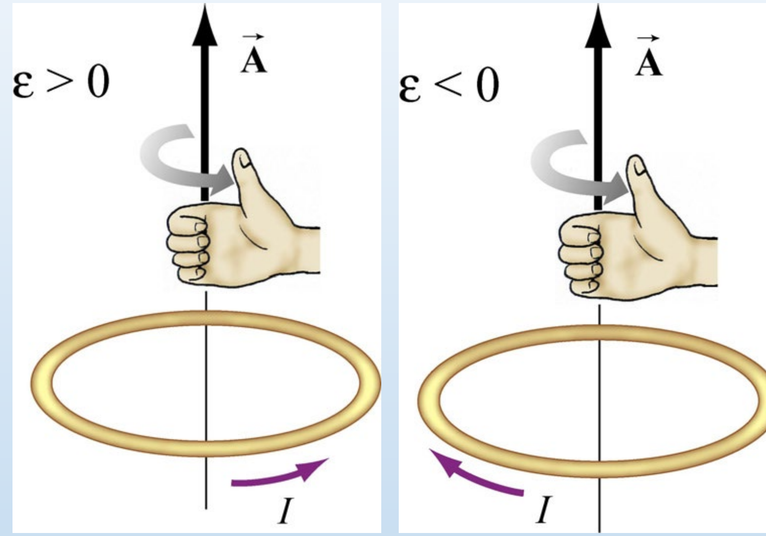
- Magnitude of B, e.g. Falling Magnet
- Area A enclosed by the loop
- Angle θ between B and loop normal

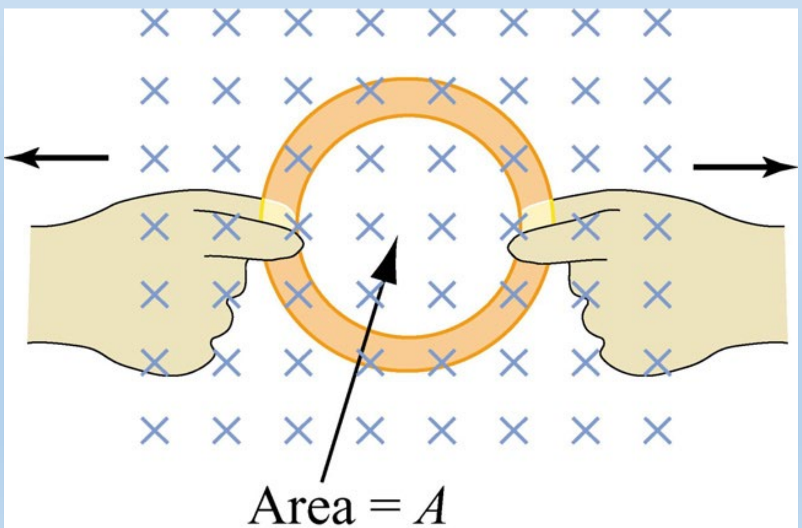
Emf looks like potential. It’s a “driving force”for current

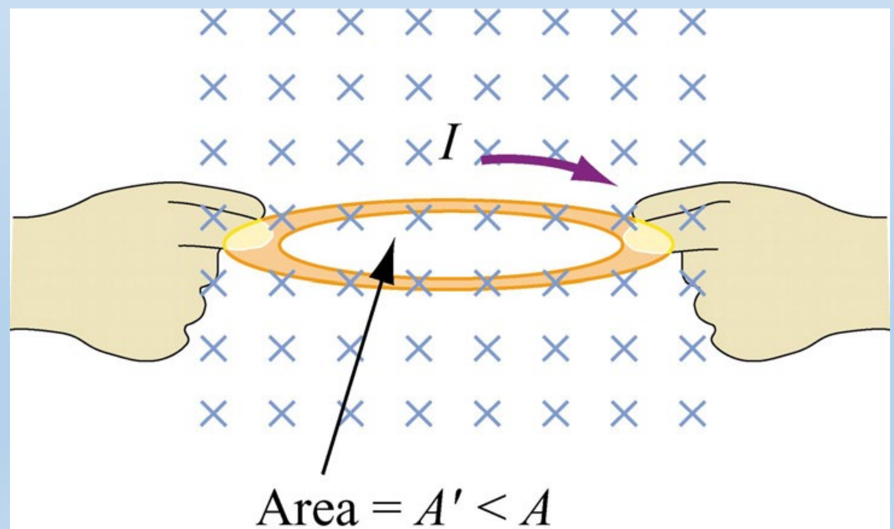
$$emf = \int \vec{E}\cdot d\vec{s}$$

Minus Sign? Lenz's Law







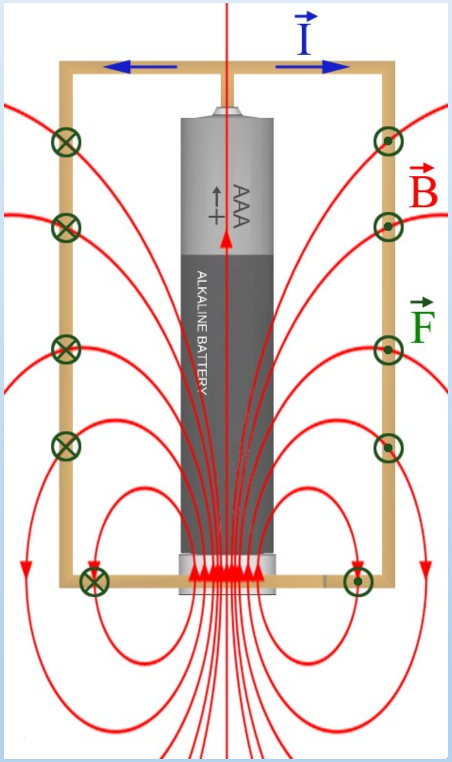


Homopolar motor

Homopolar motor 3D

Homopolar motor 2D

Current, magnetic field lines and Lorentz force on Homopolar motor



https://en.wikipedia.org/wiki/Homopolar_motor

Maxwell's Equations

Creating Electric Fields

$$\oiint_S \vec{E} \cdot d\vec{S} = \frac{Q_{in}}{\varepsilon_0}$$ (Gauss's Law)

$$\oint_C \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$ (Faraday's Law)

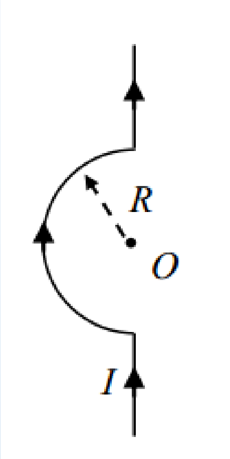
Creating Magnetic Fields

$$\oiint_S \vec{B} \cdot d\vec{S} = 0$$ (Magnetic Gauss's Law)

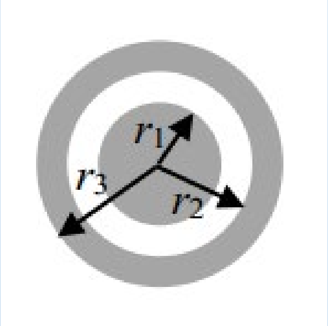
$$\oint_C \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$$ (Ampere's Law)

习题

习题 **1.1**：一条载有电流 I 的无穷长直导线，在一处弯折成半径为 R 的半圆弧，如图所示，**试求半圆弧中心 O 点的磁感应强度 B 。**



习题 **1.2**：电缆由一导体圆柱和一同轴导体圆管构成，使用时，电流 I 从一导体流去，由另一导体流回， I 均匀分布在导线的横截面上，也均匀分布在圆管横截面上。已知圆柱的半径为 $r1$，圆管的内外半径分别为 $r2$和 $r3$，横截面如图所示。**试求离轴线为 r 处的磁感应强度 B 。**



实验作业

通过MATLAB、 COMSOL等软件来仿真课程相关的实例。

第二章静磁场:

无限长通电直导线周围磁场的计算与展示,

半径为R的电流环(磁偶极子)在三维空间的磁场分布