What can we get from magnets?

Microelectronics

Healthcare

EVs

Electrical Vehicle based on rare-earth motors.
Courtesy of XIAOMI, Inc.

Rare-earth magnet based DC motor. Courtesy of Maxon Precision Motors, Inc.

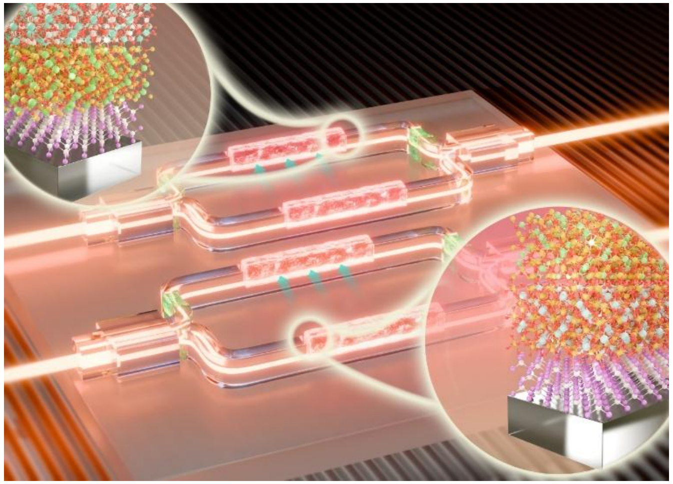
Entertainments

Neodymium magnet based speakers.
Courtesy of Eminence Speaker, LLC.

Maglev

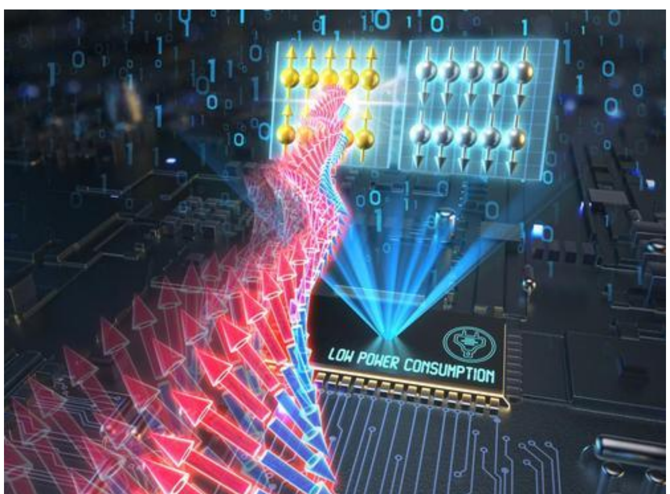
What can we get from magnets?

Communication



On-chip Magneto-Optical Isolator
https://doi.org/10.3390/nano14050400

Low-power-consumption chips



Magnetization switching by magnons
https://doi.org/10.1126/science.aav8076

Chapter 7 Magnetic properties

1. **Magnetization**
2. **Classification of magnetic materials**
3. **Ferromagnetism origin and exchange coupling**
4. **Saturation magnetization and Curie temperature**
5. Magnetic domains
6. Hysteresis
7. Simulation of hysteresis curve

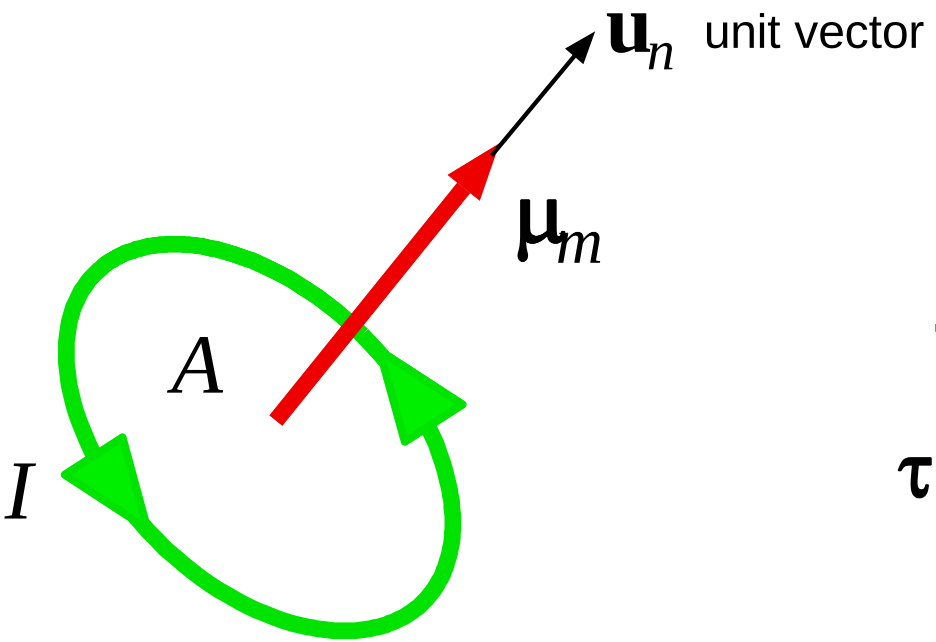
   Tutorial

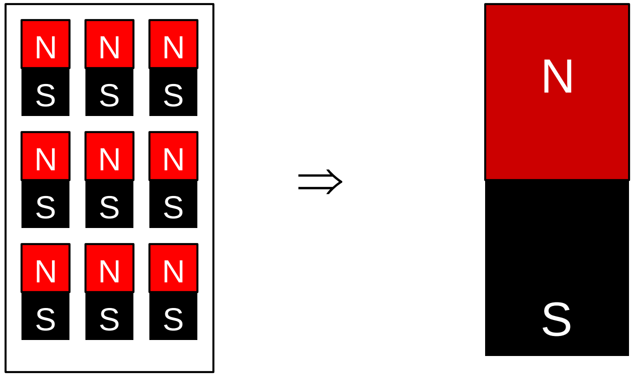
   Superconductivity

Magnetization of matter

Definition of a **magnetic dipole**

磁偶极子

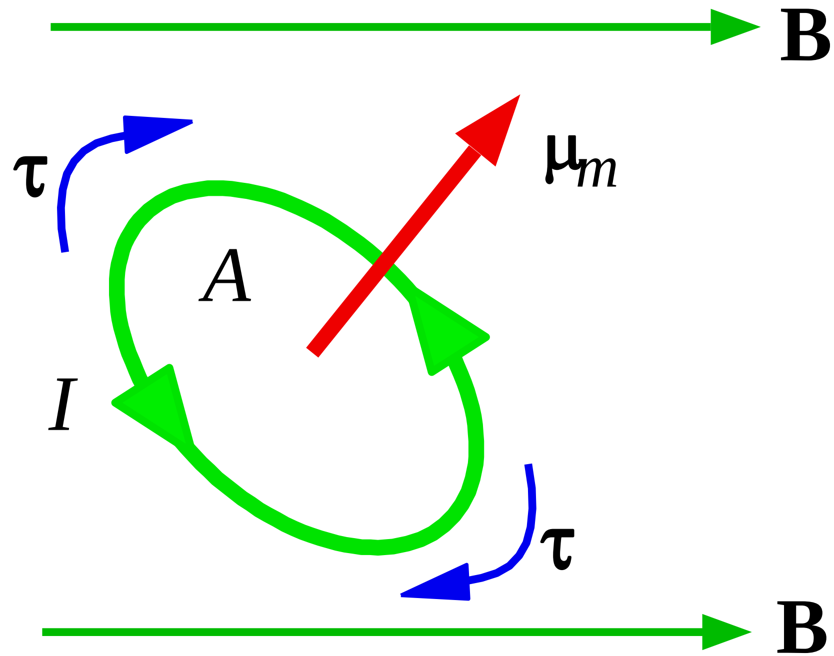




Magnetic moment （磁矩） μ_m:

$$\vec{\mu}_m = IA\vec{u}_n$$

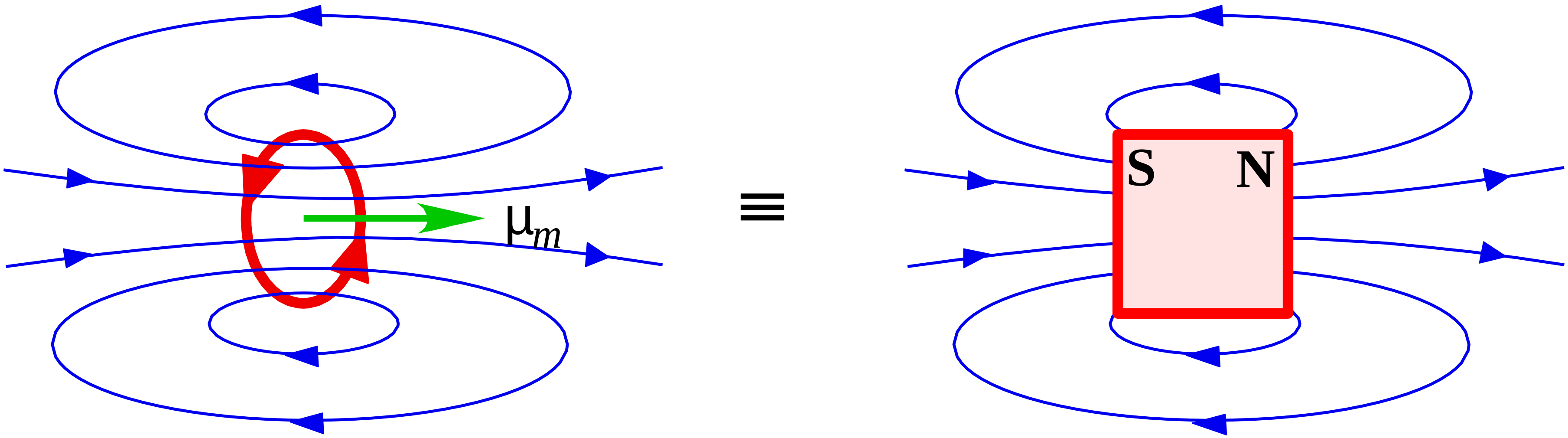
magnetic strength and orientation of a magnetic dipole

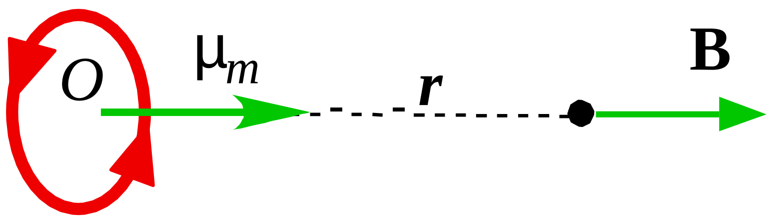


A magnetic dipole moment in an external field experiences a torque (扭矩) to align with the field

Magnetic field B

a magnetic moment gives rise to a magnetic field **B** around it



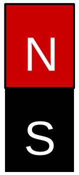


A magnetic dipole moment puts out a magnetic field just like bar magnet. The field **B** depends on $\boldsymbol{\mu}_m$.

Magnetic field **B**:

$$\vec{B} \propto \vec{\mu}_m / r^3$$

Atomic magnetic moment（原子磁矩）



$$I = charge\ flowing\ per\ unit\ time = -\frac{e}{period} = -\frac{e\omega}{2\pi}$$

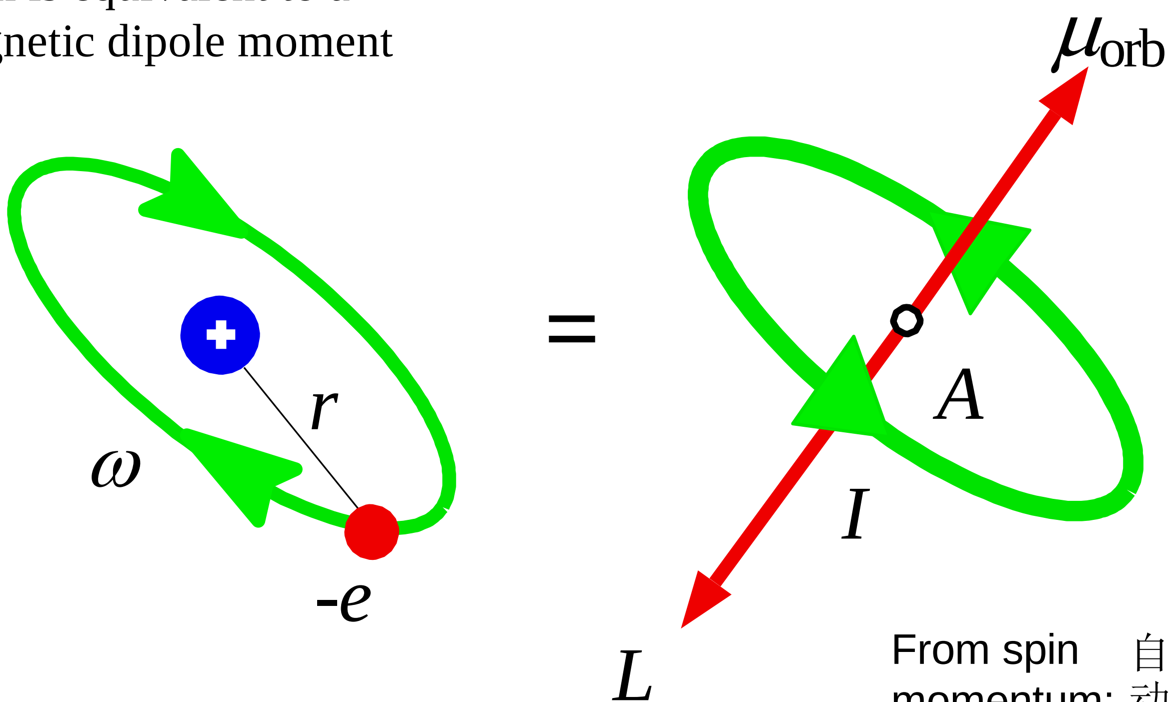
ω is the angular frequency of the electron = $2\pi f$

$$\Rightarrow \quad \mu_{orb} = I\left(\pi r^2\right) = -\frac{e\omega r^2}{2}$$

An orbitting electron in an atom is equivalent to a magnetic dipole moment

From angular momentum: 轨道角动量

$$L = m_e v r = m_e \omega r^2$$



Orbital magnetic moment
电子轨道磁矩

$$\mu_{orb} = -\frac{e}{2m_e}L$$

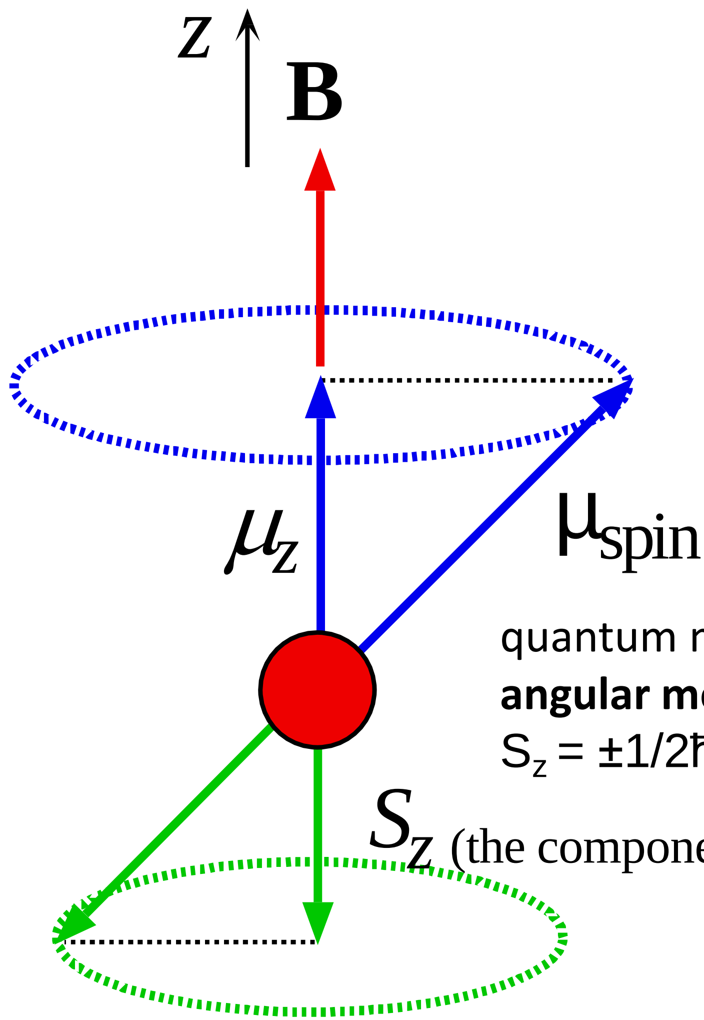
Spin magnetic moment
电子自旋磁矩

From spin momentum: 自旋角动量

$$\mu_{spin} = -\frac{e}{m_e}S$$

μ_{atom} only **unfilled subshells**（电子亚层）contribute to the magnetic moment of an atom

Bohr - magneton（玻尔磁子）



(the component of **S** along z)

For a s-electron, L=0 and $S_z = -1/2\hbar$ ($\mu_{atom} = \mu_{spin}$)

$$\mu_z = -\frac{e}{m_e} S_z = -\frac{e}{m_e}(m_s\hbar) = \frac{e\hbar}{2m_e}$$

$$= bohr - magneton = \boldsymbol{\mu}_{\mathbf{B}} = \beta$$

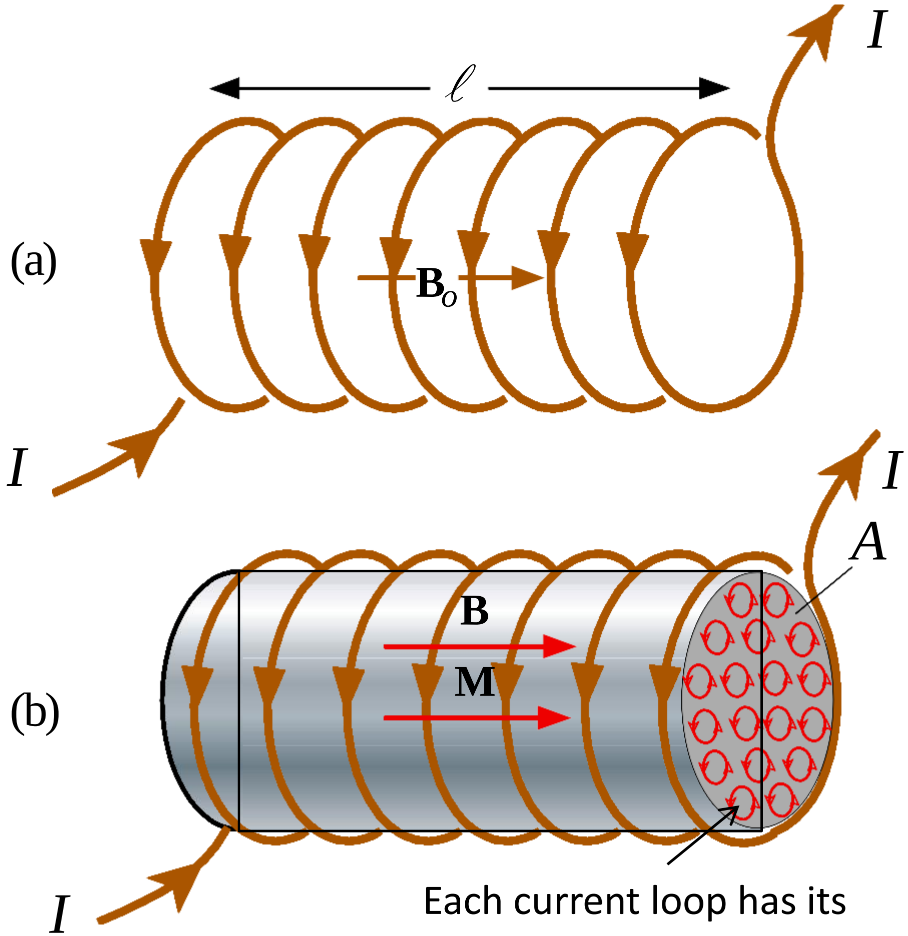
$\beta = \mu_B = 9.27\mathrm{x}10^{-24}\ \mathrm{Am}^2$.

quantum mechanics requires the **spin angular momentum** to be space **quantized**:
$S_z = \pm 1/2\hbar$, where spin magnetic quantum number $m_s = \pm 1/2$

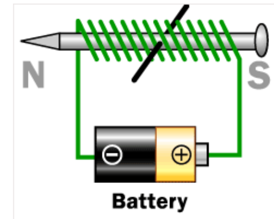
The spin magnetic moment precesses（进动）about an external magnetic field along *z* and has an average value of μ_Z along *z*.

The spin of a single electron has a magnetic moment of one Bohr magneton along the field.

Magnetization vector M（磁化矢量）



Each current loop has its current plane normal to $\mathbf{B_0}$



Magnetic field $\mathbf{B_0}$ in free space:

$$B_0 = \mu_0 n I = \mu_0 I'$$

where n = N/l number of turns per unit length

I' = nI, current per unit length

μ_0: absolute permeability of free space

真空磁导率 $\mu_0 = 4\pi \times 10^{-7}$ H/m

Magnetization **M**: (magnetic dipole moment per unit volume)

$$\vec{M} = \frac{1}{\Delta V}\sum_{i=1}^{N}\vec{\mu}_{mi} = n_{at}\vec{\mu}_{av}$$

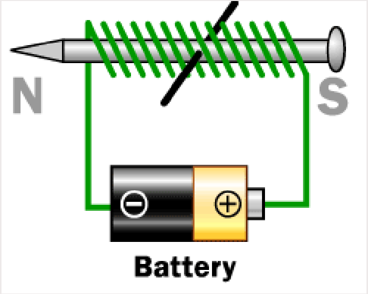
n_{at}: the number of atoms per unit volume

μ_{av}: average magnetic moment per atom

(a) A long solenoid, with free space as medium inside, the magnetic field is $\mathbf{B}_o$

(b) Material inserted into the solenoid develops a magnetization **M**（ 磁化强度 ）

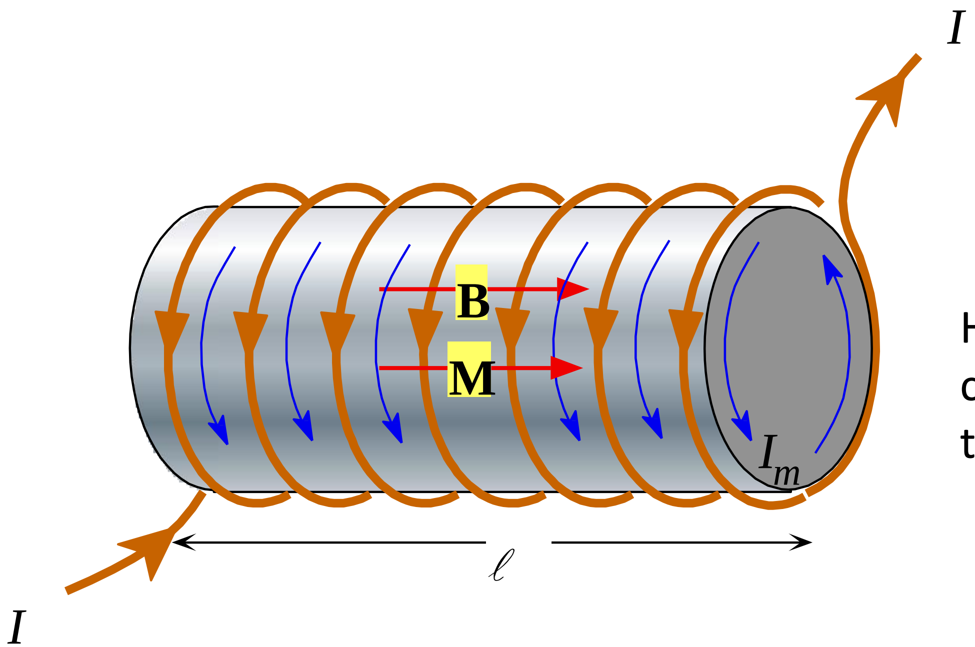
Magnetizing field **H**



The total magnetic field ($M = I_m$, net surface current):

$$\vec{B} = \vec{B}_0 + \mu_0 \vec{M} = \mu_0 \vec{H} + \mu_0 \vec{M}$$

H: magnetizing field or magnetic field intensity（磁化磁场）, unit: A/m



$$\vec{H} = \frac{1}{\mu_0} \vec{B}_0$$

H is related to the external conduction currents (through Ampere's law). **H = nI** = total conduction current per unit length

The field **B** in the material inside the solenoid is due to the conduction current *I* through the wires and the magnetization current I_m on the surface of the magnetized medium

Magnetic permeability and magnetic susceptibility

Magnetic permeability μ is defined as:

磁导率

$$\mu = \frac{B}{H}$$

Magnetic susceptibility χ_m is defined as:

磁化系数

$$\vec{M} = \chi_m \vec{H}$$

From

$$B = \mu_0 H + \mu_0 M = \mu_0 H + \mu_0 \chi_m H = \mu_0 (1 + \chi_m) H = \mu H$$

$$\Rightarrow \quad \mu = \mu_0 (1 + \chi_m)$$

or

$$\mu_r = 1 + \chi_m$$

Where $\mu_r = \mu/\mu_0$, the relative permeability

相对磁导率

Summary of important magnetic quantities

Magnetic Quantity	Symbol	Definition	Units	Comment
Magnetic field; magnetic induction	$\mathbf{B}$	$\mathbf{F} = q\mathbf{v} \times \mathbf{B}$	T = tesla = webers m^{-2}	Produced by moving charges or currents, acts on moving charges or currents.
Magnetic flux	Φ	$\Delta\Phi = B_{\text{normal}}\,\Delta A$	Wb = weber	$\Delta\Phi$ is flux through ΔA and B_{normal} is normal to ΔA. Total flux through any closed surface is zero.
Magnetic dipole moment	$\boldsymbol{\mu}_m$	$\mu_m = IA$	A m^2	Experiences a torque in $\mathbf{B}$ and a net force in a nonuniform $\mathbf{B}$.
Bohr magneton	β	$\beta = e\hbar/2m_e$	A m^2 or J T^{-1}	Magnetic moment due to the spin of the electron. $\beta = 9.27 \times 10^{-24}$ A m^2
Magnetization vector	$\mathbf{M}$	Magnetic moment per unit volume	A m^{-1}	Net magnetic moment in a material per unit volume.
Magnetizing field; magnetic field intensity	$\mathbf{H}$	$\mathbf{H} = \mathbf{B}/\mu_o - \mathbf{M}$	A m^{-1}	$\mathbf{H}$ is due to external conduction currents only and is the cause of $\mathbf{B}$ in a material.
Magnetic susceptibility	χ_m	$\mathbf{M} = \chi_m \mathbf{H}$	None	Relates the magnetization of a material to the magnetizing field $\mathbf{H}$.
Absolute permeability	μ_o	$c = [\varepsilon_o \mu_o]^{-1/2}$	H m^{-1} = Wb m^{-1} A^{-1}	A fundamental constant in magnetism. In free space, $\mu_o = B/H$.
Relative permeability	μ_r	$\mu_r = B/\mu_o H$	None	
Magnetic permeability	μ	$\mu = \mu_o \mu_r$	H m^{-1}	Not to be confused with magnetic moment.

Classification of magnetic materials

In general, magnetic materials are classified into five distinct groups:

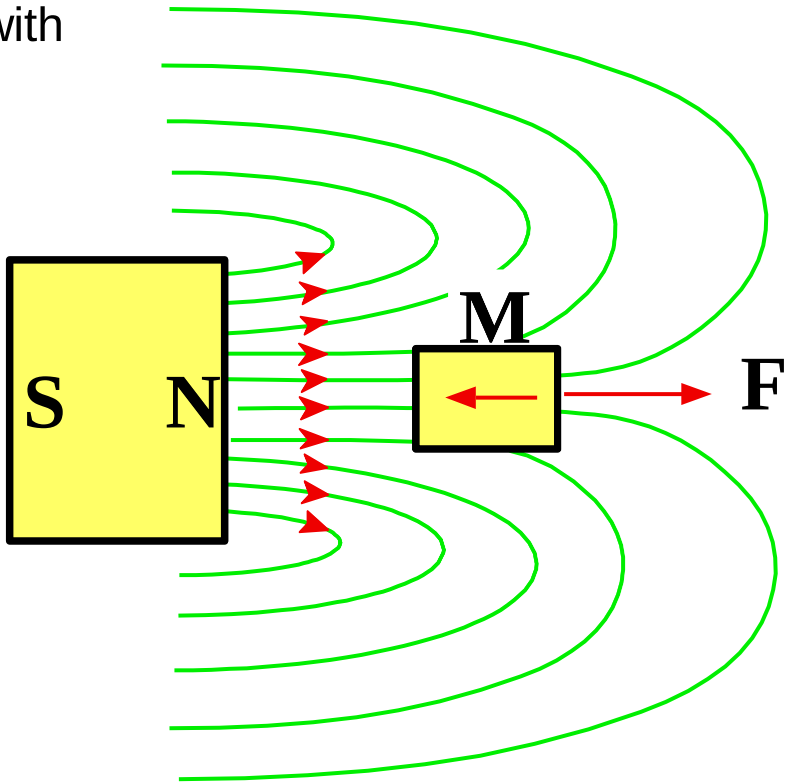
- **Diamagnetic**（抗磁性）
- **Paramagnetic**（ 顺磁性）
- **Ferromagnetic**（ 铁磁性 ）
- **Antiferromagnetic**（ 反铁磁性 ）
- **Ferrimagnetic**（亚铁磁性）

Diamagnetic（抗磁性）

Diamagnetism: many covalent and ionic crystals are diamagnetic (constituent atoms have **no unfilled subshells**, and no permanent magnetic moment in the absence of an applied field). The magnetic susceptibility is ***negative*** and *small*. For example, the silicon crystal is diamagnetic with $\chi_m = -5.2 \times 10^{-6}$

Superconductors are ideal diamagnets with χ_m = -1 below its critical temperature

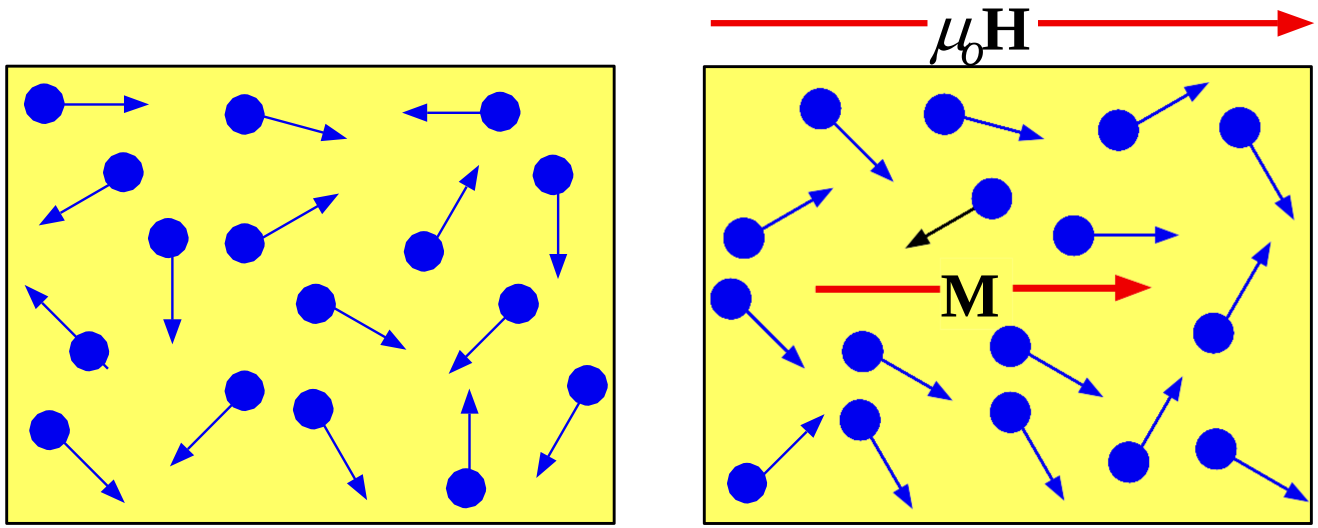
The negative susceptibility can be interpreted as the diamagnetic substance trying to expel the applied field from the material



The magnetization **M** of the substance is in the *opposite* direction to **B** and experiences a force towards smaller fields. This repels the diamagnetic material away from a permanent magnet.

Paramagnetic（顺磁性）

Paramagnetism: a result of the competition of thermal fluctuation and alignment under the application of a magnetizing field (applied field), *small* and *positive* magnetic susceptibility χ_m

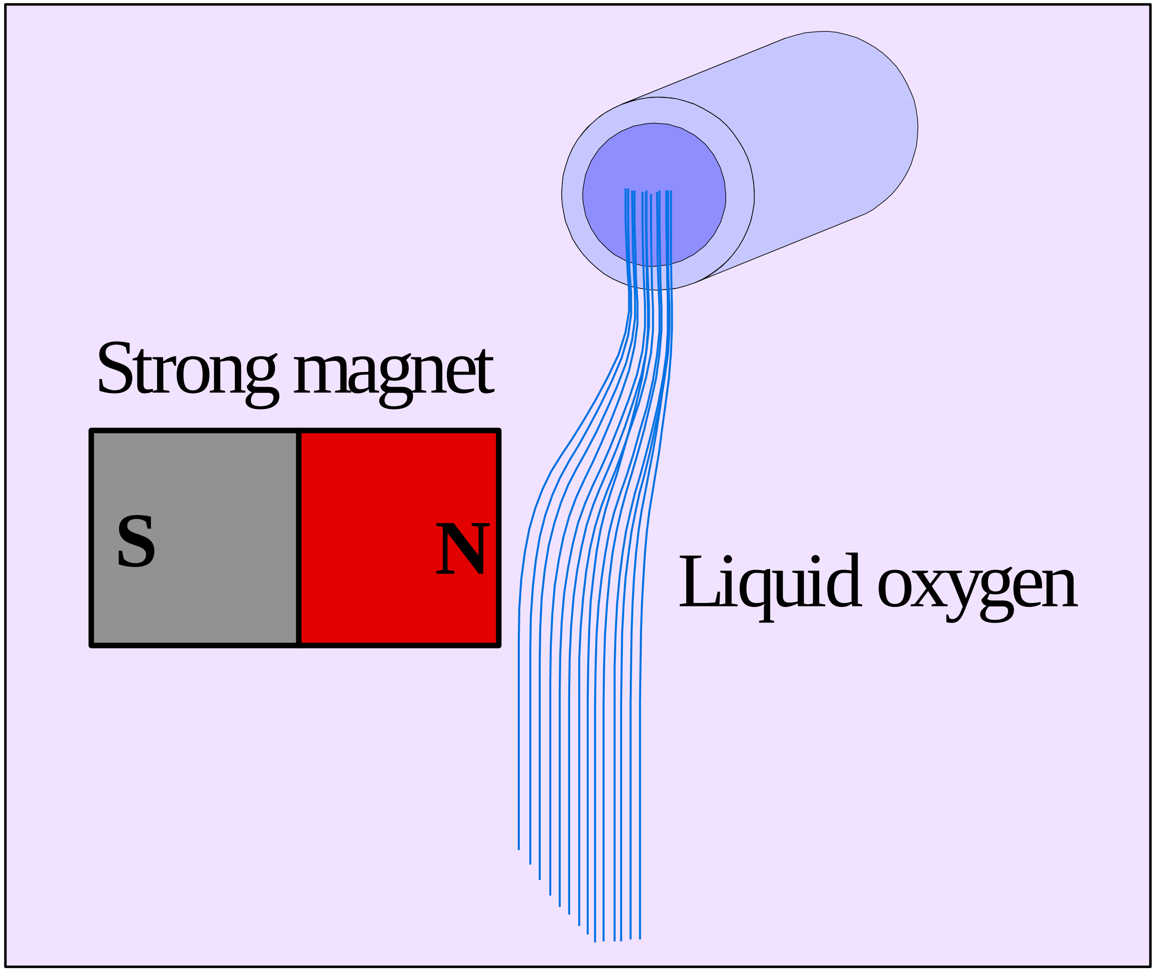


$\mu_{av} = 0$ and $\mathbf{M} = 0$ $\mu_{av} \neq 0$ and $\mathbf{M} = \chi_m \mathbf{H}$

(a) (b)

(a) In a paramagnetic material each individual atom possesses a permanent magnetic moment but due to thermal agitation there is no average moment per atom and **M** = 0. (b) In the presence of an applied field, individual magnetic moments take alignments along the applied field and **M** is finite and along **B**.

Paramagnetic



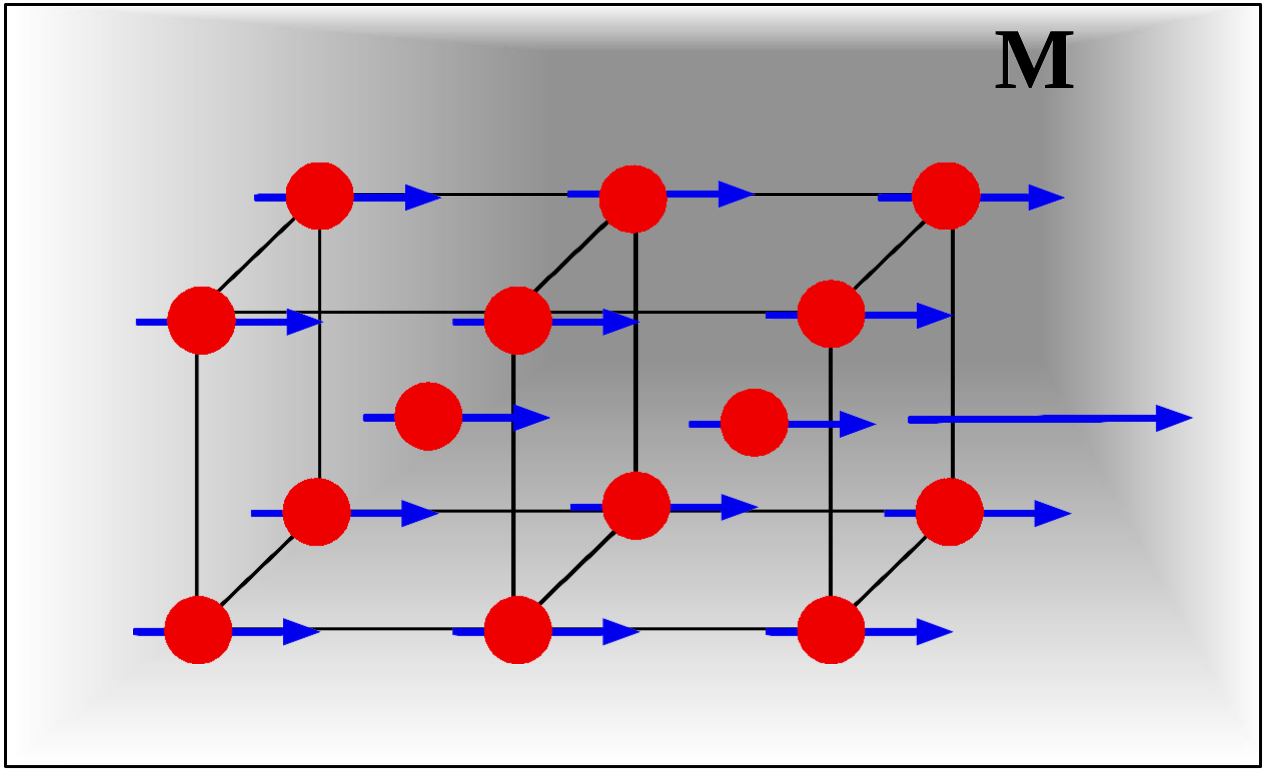
An example of paramagnetic substance is oxygen gas, with $\chi_m = 2.1 \times 10^{-6}$ at atmospheric pressure and room temperature.

Magnetization M decreases with increasing temperature because at higher temperatures there are more molecular collisions, which destroy the alignments of molecular magnetic moments with the applied field.

A paramagnetic material placed in a non-uniform magnetic field experiences a force towards greater fields. This attracts the paramagnetic material (e.g. liquid oxygen) towards a permanent magnet.

Ferromagnetic（铁磁性）

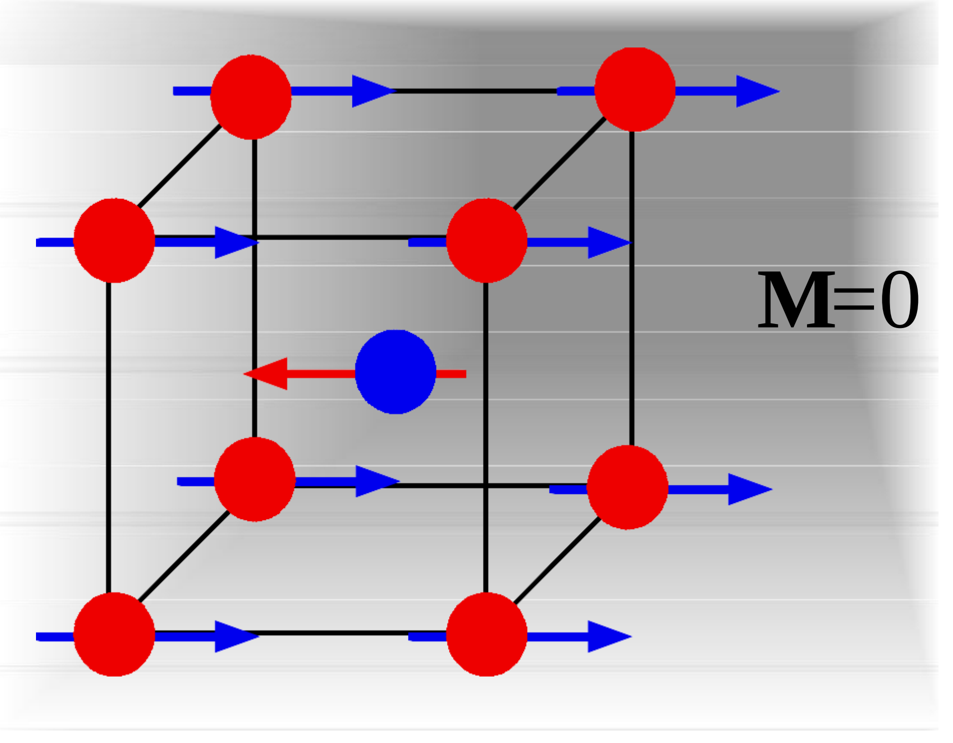
Ferromagnetism: parallel orientation of permanent dipoles due to strong exchange coupling. The ordering temperature is called the Curie temperature. There is a *strong* magnetization even when no applied magnetic field is present. Above the Curie temperature (T_c), the substance is paramagnetic.



In a magnetized region of a ferromagnetic material, all the magnetic moments are spontaneously aligned in the same direction（自发磁化）. There is a strong magnetization vector **M** even in the absence of an applied field. The magnetic susceptibility χ_m is *positive* and *very large* (even infinite) and, depends on the applied field intensity (highly nonlinear relationship between M and $\mu_0 H$).

Antiferromagnetic（反铁磁性）

Antiferromgnetism: below the Neel temperature T_N, magnetic moments of alternating atoms in the crystals align ***opposite*** direction due to the negative sign of magnetic coupling. There is no net magnetization. Above T_N, antiferromagnetic material becomes paramagnetic.



In this antiferromagnetic BCC crystal (Cr), the magnetic moment of the center atom is cancelled by the magnetic moments of the corner atoms (an eighth of the corner atom belongs to the unit cell).

Ferrimagnetic（亚铁磁性）

Ferrimagnetism: there is a net magnetization (below its Curie temperature), due to the difference in magnetic moments A and B.

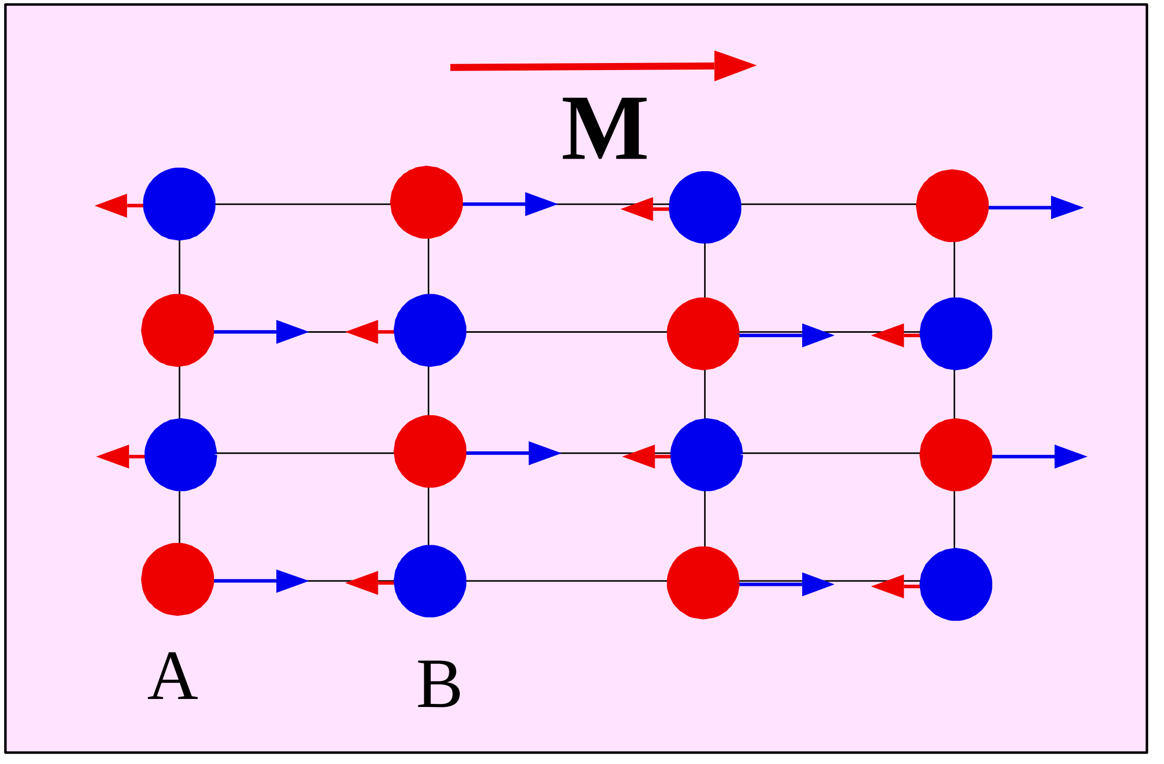
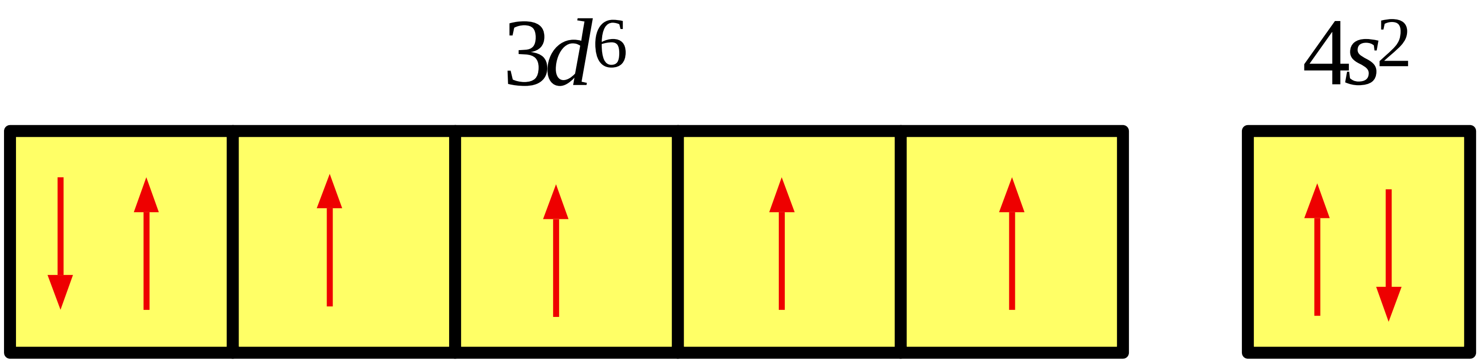


Illustration of magnetic ordering in a ferrimagnetic crystal. All A-atoms have their spins aligned in one direction and all B-atoms have their spins aligned in the opposite direction. As the magnetic moment of an A-atom is greater than that of a B-atom, there is net magnetization, M, in the crystal. Ferrimagnetic materials such as ferrites (e.g., Fe_3O_4) exhibit magnetic behavior similar to ferromagnetism below T_C. They are typically non-conducting, therefore do not suffer from eddy current losses, widely used in high-frequency electronics applications

Ferromagnetism origin and the exchange interaction

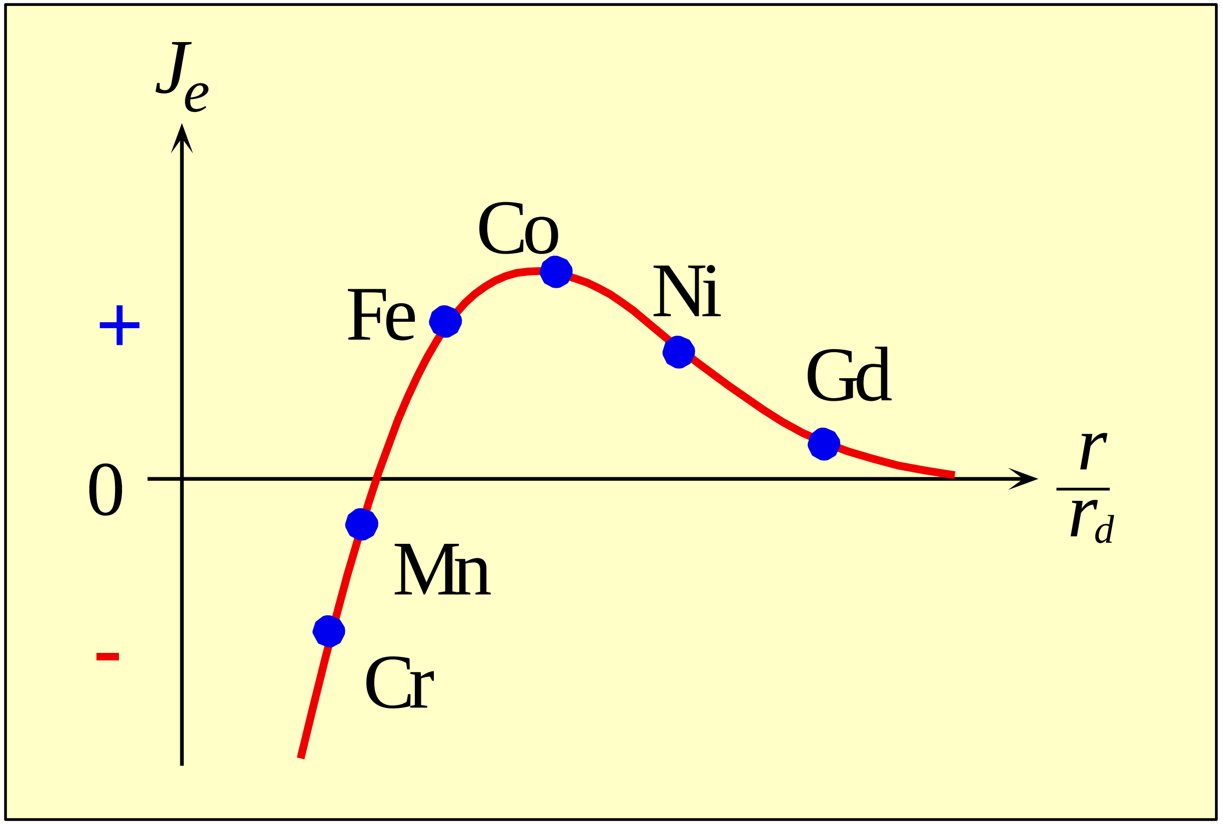
Ferromagnetism: permanent magnetic moment of an atom because of the presence of unpaired electrons, such as 3d-metals and 4f-rare earth elements, only elements with not fully occupied inner shells can possess magnetic moment.

The origin of Hund's rule lies in the fact that when two spins are parallel (same m_s), as a requirements of the Pauli exclusion principle, the electrons must occupy orbitals with different m_l. The parallel arrangement of two spins is one of the results of **exchange interaction** (**Pauli exclusion principle** and minimum of **electrostatic interaction energy**).



The isolated Fe atom has 4 unpaired spins in the 3d subshell and a spin magnetic moment of 4 μ_B (bohr–magneton)

In a solid, the **exchange interaction** between spins may result in ferromagnetism, antiferromagnetism or ferrimagnetism.



The exchange energy:
for two atoms only, depends on the interatomic separation between two interacting atoms and the relative spins of the two outer electrons (labeled as 1 and 2)

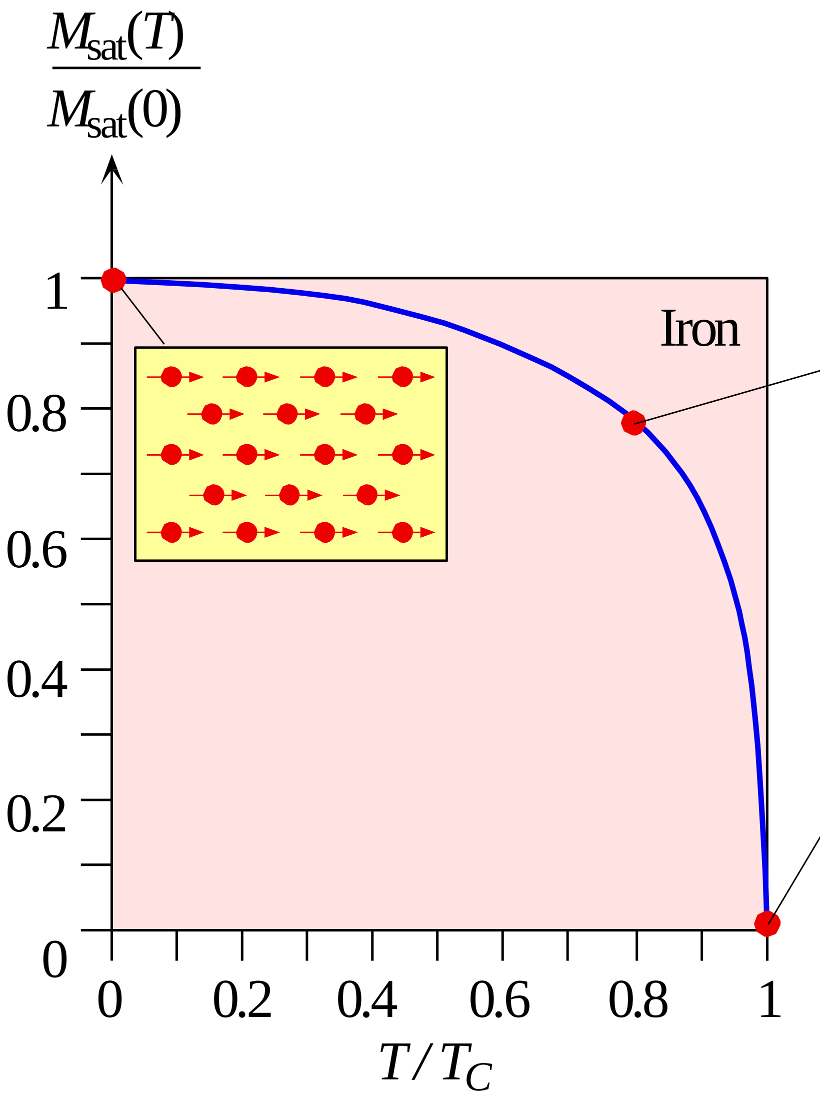
$$E_{ex} = -2J_e \vec{S}_1 \cdot \vec{S}_2$$

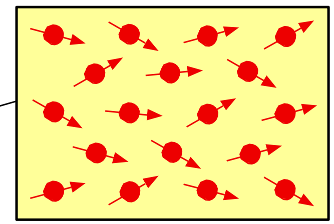
J_e: Exchange integral

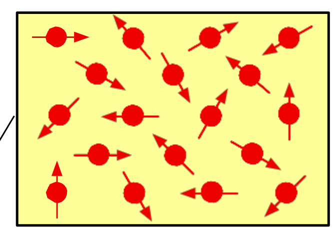
S_1 and S_2: spin moments 1 and 2

The exchange intergral as a function of r/r_d, where r is the interatomic distance and r_d the radius of the d-orbit (or the average d- subshell radius). Cr to Ni are transition metals. For Gd the x-axis is r/r_f where r_f is the radius of the f-orbit.

Saturation magnetization and Curie temperature







Normalized saturated magnetization vs. reduced temperature T/T_C where T_C is the Curie temperature (1043 K).

M_{sat} is a function of temperature, decreases with temperature and becomes zero at the curie temperature T_c.

lattice vibrations become more energetic, which leads to a frequent disruption of the alignments of the spins.

An estimation of the *magnitude* of $\mathbf{E_{ex} = kT_c}$

Example (saturation magnetization in iron): *The maximum magnetization, called saturation magnetization M_{sat}, in iron is about $1.75x10^6$ Am^{-1}. This corresponds to all possible net spins aligning parallel to each other. Calculate the effective number of Bohr magneton per atom, given the density and the relative atomic mass of iron are 7.86 gcm^{-3} and 55.85×10^{-3} kg mol^{-1}, respectively.*

The number of Fe atoms per unit volume:

$$n_{at} = \frac{\rho N_A}{M_{at}} = 8.48x10^{28}\,atoms\,m^{-3}$$

From $M_{sat} = n_{at}(x\beta) = n_{at}(x\mu_B)$:

$$x = \frac{M_{sat}}{n_{at}\beta} = \frac{1.75x10^6}{8.48x10^{28}\cdot 9.27x10^{-24}} \approx 2.2$$

The resulting magnetic field B within the iron (from $B=\mu_0(H+M)$):

$$B = B_{sat} = \mu_0 M_{sat} = 2.2\,Tesla$$

$\mu_0 = 4\pi x10^{-7}\ Hm^{-1}$

完成 **Assignment 7.1**

提交时间：**5月22日**（周四）前

提交方式：电子版（写明姓名、学号），通过本班课代表统一提交