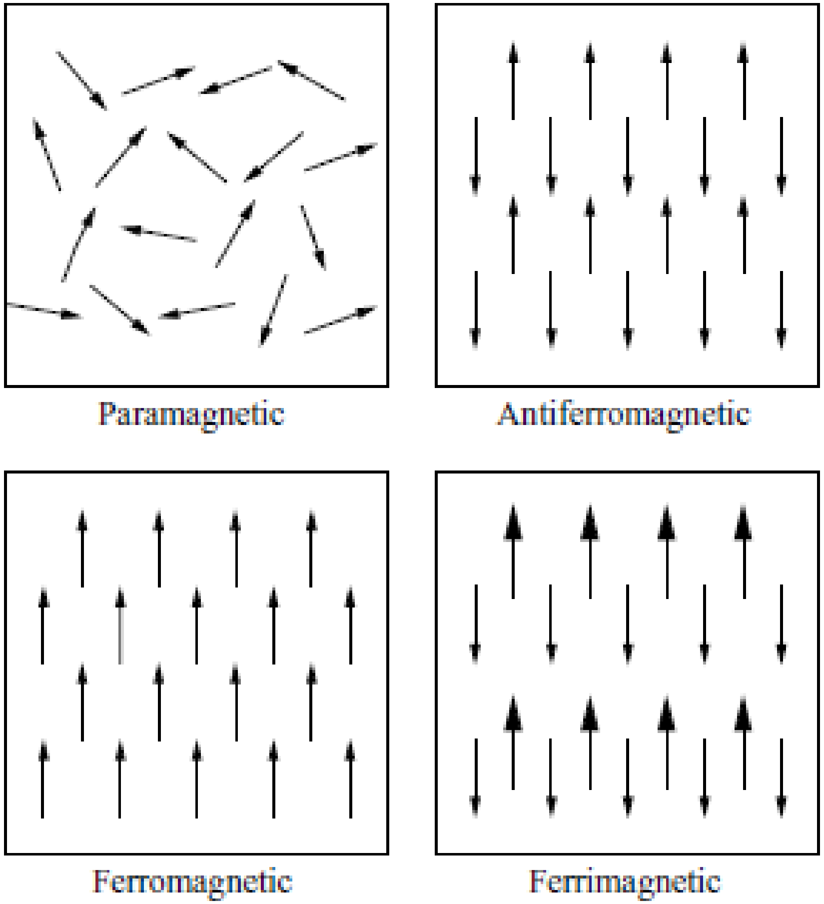
Chapter 7 Magnetic properties

1. Magnetization
2. Classification of magnetic materials
3. Ferromagnetism origin and exchange coupling
4. Saturation magnetization and Curie temperature
5. Magnetic domains
6. **Hysteresis**
7. **Simulation of hysteresis curve**

Tutorial

Superconductivity

Ordering of magnetic dipoles of different magnetic materials



	Diamagnetic	Paramagnetic	Ferromagnetic	Antiferromagnetic	Ferrimagnetic
χ_m	Negative and small	Positive and small	Positive and very large	Positive and small	Positive and very large

Assignment 7.1

Question 1:

Consider a long solenoid with a core that is an iron alloy. Suppose that the diameter of the solenoid is 2 cm and the length of the solenoid is 20 cm. The number of turns on the solenoid is 200. The current is increased until the core is magnetized to saturation at about $I = 2$ A and the saturated magnetic field B_{sat} ~~is 1.6 T.~~

a. What is the magnetic field intensity, B_0 at the center of the solenoid and the applied magnetic field, H?

b. What is the saturation magnetization M_{sat} of this iron alloy?

c. If we were to have the same magnetic field of 1.6 T inside the solenoid *without* the iron-alloy core, how much current would we need? Is there a practical way of doing this?

Q1. (a)

$$B_0 = \mu_0 n I = \frac{\mu_0 N I}{L} = \frac{(4\pi \times 10^{-7})(200)(2)}{0.2} = 2.51 \times 10^{-3}\ \text{T}$$

$$B_0 = \mu_0 H \Rightarrow H = \frac{2.51 \times 10^{-3}}{4\pi \times 10^{-7}} = 2000\ \text{A/m}$$

(b) $$B = \mu_0 H + \mu_0 M \Rightarrow M_{sat} = \frac{B_{sat} - \mu_0 H_{sat}}{\mu_0}$$

$$= \frac{1.6 - 2.51 \times 10^{-3}}{4\pi \times 10^{-7}} = 1.27 \times 10^{6}\ \text{A/m}$$

(c) $$B_0 = \mu_0 n I = \frac{\mu_0 N I}{L}$$

$$I = \frac{B_0 L}{\mu_0 N} = \frac{1.6 \times 0.2}{4\pi \times 10^{-7} \times 200} = 1273\ \text{A}$$

Not practical! Can be achieved using a superconducting solenoid.

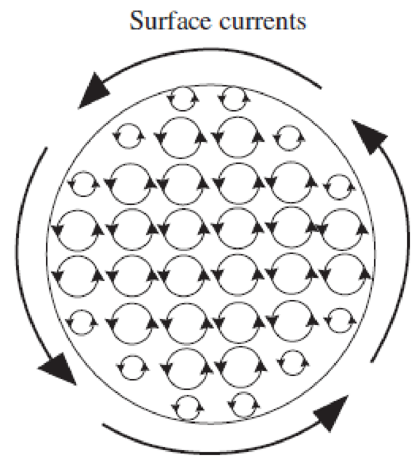
Magnetizing field **H**

Total magnetic moment = M × Volume = MAℓ

Total magnetic moment = (Total current) × (Cross-sectional area) = I_mℓA

M = I_m, net surface current

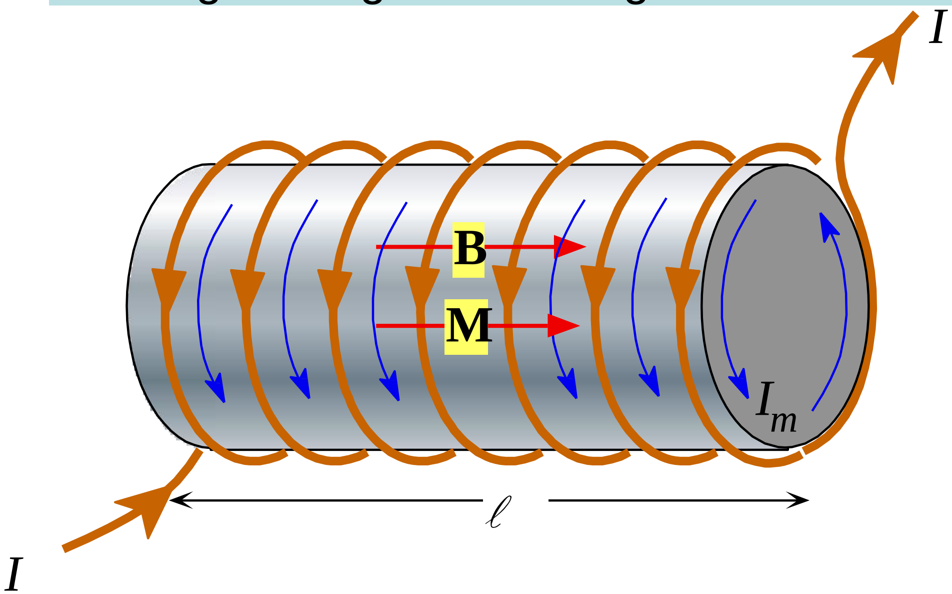
I_m magnetization current on the surface per unit length



The total magnetic field:

$$\vec{B} = \vec{B}_0 + \mu_0 \vec{M} = \mu_0 \vec{H} + \mu_0 \vec{M} = \mu_o (I' + I_m)$$

H: magnetizing field or magnetic field intensity（磁化磁场）, unit: A/m



$$\vec{H} = \frac{1}{\mu_0} \vec{B}_0$$

H is related to the external conduction currents (through Ampere's law). **H = nI** = total conduction current per unit length

The field **B** in the material inside the solenoid is due to the **conduction current I'** through the wires and the **magnetization current I_m** on the surface of the magnetized medium

Question 2:
Sometimes magnetic susceptibilities are reported as molar or mass susceptibilities. **Mass susceptibility** (in $m^3\ kg^{-1}$) is χ_m/ρ where ρ is the density. **Molar susceptibility** (in $m^3\ mol^{-1}$) is $\chi_m(M_{at}/\rho)$ where M_{at} is the atomic mass. Terbium (Tb) has a magnetic molar susceptibility of 2 $cm^3\ mol^{-1}$. Tb has a density of 8.2 g cm^{-3} and an atomic mass of 158.93 g mol^{-1}. What is its susceptibility, mass susceptibility and relative permeability? What is the magnetization in the sample in an applied magnetic field of 2 T?

Q2. Molar susceptibility : $\frac{\chi_m M_{at}}{\rho} = 2 cm^3/mol$

$\Rightarrow \chi_m = 2\left(\frac{\rho}{M_{at}}\right) = 2\left(\frac{8.2}{158.93}\right) = 0.1032$ susceptibility

$\frac{\chi_m}{\rho} = \frac{0.1032}{8.2} = 1.26 \times 10^{-5}\ m^3/kg$ mass susceptibility

$\mu_r = 1 + \chi_m = 1 + 0.1032 = 1.1032$ relative permeability

$M = \chi_m H = \chi_m\left(\frac{B_o}{\mu_o}\right) = \frac{0.1032 \times 2}{4\pi \times 10^{-7}} = 1.64 \times 10^5\ A/m$

Question 3:

Consider bismuth with χ_m =-17×10^{-5} and aluminum with χ_m = 2×10^{-5}. Suppose that we subject each sample to an applied magnetic field B_o of 1 T applied in the +x direction. What is the magnetization **M** and the equivalent magnetic field $\mu_o M$ in each sample? Which is paramagnetic and which is diamagnetic?

Q3. $M = \chi_m H = \chi_m \left(\frac{B_o}{\mu_o}\right)$

Bismuth: $\chi_m = -17 \times 10^{-5}$, $M = (-17 \times 10^{-5})\left(\frac{1}{4\pi \times 10^{-7}}\right) = -135.28\,A/m$

χ_m negative and small $\Rightarrow$ diamagnetic. $\mu_o M = \chi_m B_o = -17 \times 10^{-5}\,T$.

Aluminum: $\chi_m = 2 \times 10^{-5}$

$M = \chi_m H = \chi_m \left(\frac{B_o}{\mu_o}\right) = (2 \times 10^{-5})\left(\frac{1}{4\pi \times 10^{-7}}\right) = 15.92\,A/m$

$\mu_o M = \chi_m B_o = 2 \times 10^{-5}\,T.$

Assignment 7.2

Question 1:

Consider dysprosium (Dy), which is a rare earth metal with a density of 8.54 g cm^{-3} and atomic mass of 162.50 g mol^{-1}. If the saturation magnetization of Dy near absolute zero of temperature is 2.4×10^{6} Am^{-1}, what is the magnetic moment per atom in Bohr-magneton μ_B? What is the exchange energy E_{ex} in eV per atom in Dy if the Curie temperature is 85 K?

Q1. $M_{sat} = n_{at} \vec{\mu}_{at}$

$$n_{at} = \frac{\rho N_A}{M_{at}} = \frac{(8.54 \times 10^{6})(6.022 \times 10^{23})}{(162.50)} = 3.16 \times 10^{28} \text{ atoms/m}^3$$

$$\vec{\mu}_{at} = \frac{M_{sat}}{n_{at}} = \frac{2.4 \times 10^{6}}{3.16 \times 10^{28}} \times \frac{\mu_B}{9.27 \times 10^{-24} \text{ Am}^2} = 8.18 \mu_B$$

$$E_{ex} = kT_c = (1.38 \times 10^{-23}) \cdot 85 = 1.173 \times 10^{-21} \text{ J}.$$

Question 2:

The energy of a domain wall depends on two main factors: the exchange energy E_{ex} (J/atom) and magnetocrystalline energy K (J m^{-3}). If a is the interatomic distance, $\delta'$is the wall thickness, then it can be shown that the potential energy per unit area of the wall is

$$U_{wall} = \frac{\pi^2 E_{ex}}{2a\delta} + K\delta \qquad \textit{Potential energy of a domain wall}$$

Show that the minimum energy occurs when the wall has the thickness

$$\delta' = \left(\frac{\pi^2 E_{ex}}{2aK}\right)^{1/2} \qquad \textit{domain wall thickness}$$

and show that when $\delta = \delta'$, the exchange and anisotropy energy contributions are *equal*.

Q2. $U_{wall} = \frac{\pi^2 E_{ex}}{2a\delta} + k\delta$

$\therefore \frac{dU_{wall}}{d\delta} = -\frac{\pi^2 E_{ex}}{2a\delta^2} + k = 0 \qquad \therefore \delta' = \left(\frac{\pi^2 E_{ex}}{2ak}\right)^{1/2}$

$U_{wall} = \frac{\pi^2 E_{ex}}{2a\delta} + k\delta = \left(\frac{\pi^2 E_{ex}}{2a}\right)\left(\frac{2ak}{\pi^2 E_{ex}}\right)^{1/2} + k\left(\frac{\pi^2 E_{ex}}{2ak}\right)^{1/2}$

$= \left(\frac{\pi^2 E_{ex} k}{2a}\right)^{1/2} + \left(\frac{\pi^2 E_{ex} k}{2a}\right)^{1/2}$

$\therefore$ when $\delta = \delta'$, $\frac{\pi^2 E_{ex}}{2a\delta} = k\delta$

Question 3:

Estimate the potential energy and wall thickness of a domain wall for Ni. The properties of Ni are given in Table 8.4 from lecture notes.

Q3. For Ni: $a = 0.3\,nm$, $E_{ex} = 50\,meV$, $k = 5\,mJ\,cm^{-3}$

$$\delta' = \left(\frac{\pi^2 E_{ex}}{2ak}\right)^{1/2} = \left[\frac{\pi^2 (50\times10^{-3})(1.6\times10^{-19})}{2(0.3\times10^{-9})(5\times10^{-3}\times10^{6})}\right]^{1/2}$$

$$= 1.62\times10^{-7}\,m = 162\,nm$$

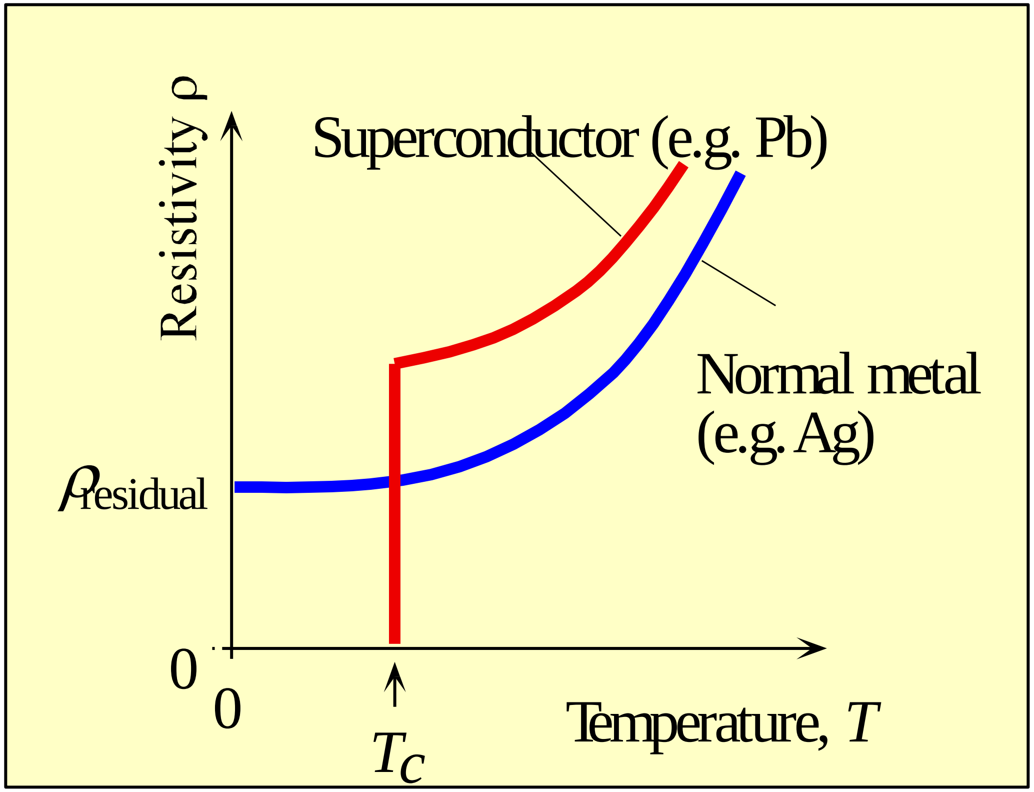
$$U_{wall} = \frac{\pi^2 E_{ex}}{2a\delta} + k\delta = \frac{\pi^2 (50\times10^{-3})(1.6\times10^{-19})}{2(0.3\times10^{-9})(1.62\times10^{-7})} + (5\times10^{-3}\times10^{6})\cdot(1.62\times10^{-7})$$

$$= 1.62\times10^{-3}\,J/m^2$$

Superconductivity

Zero resistance and the **Meissner effect** (迈斯纳效应)

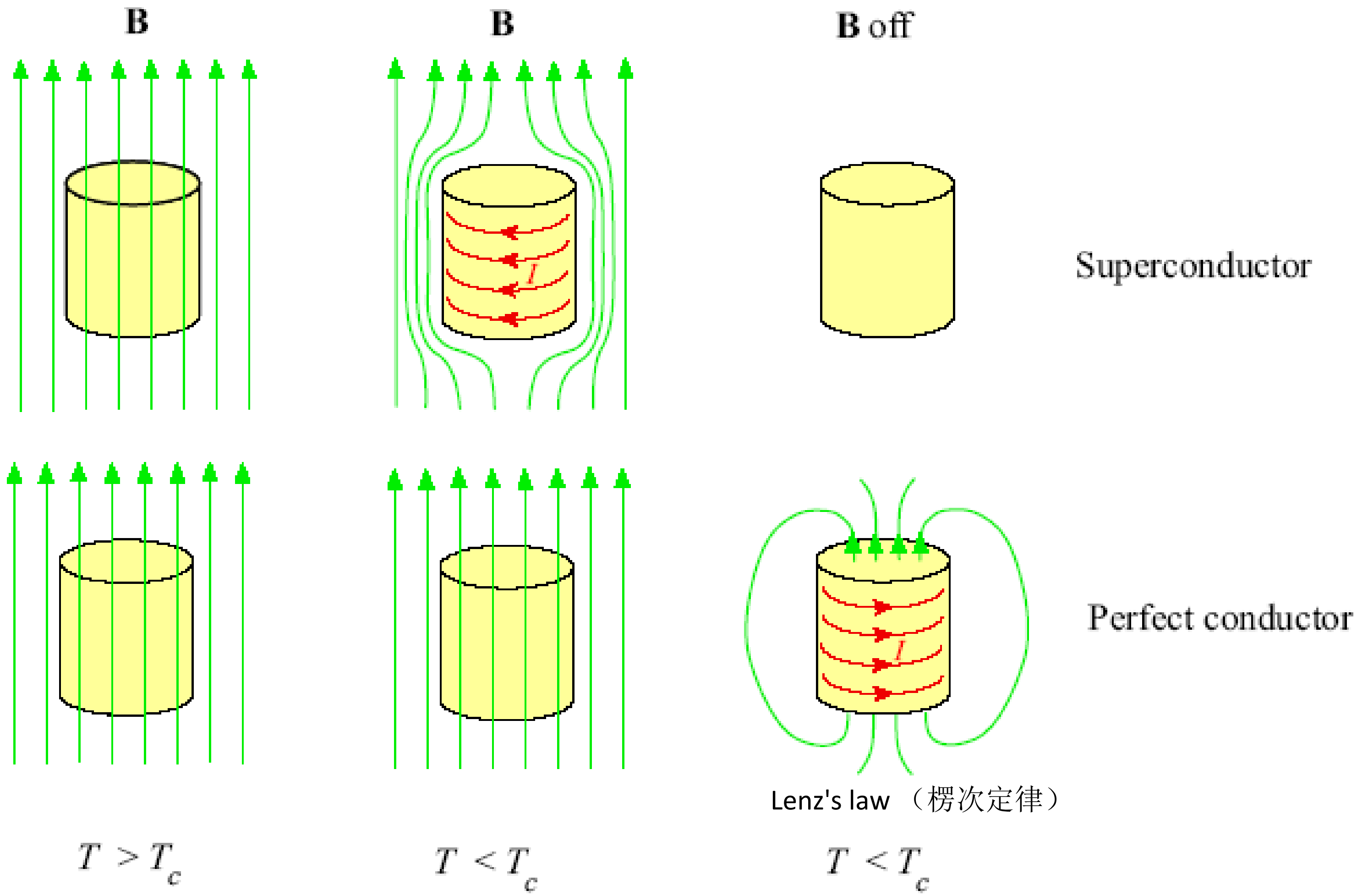
The first superconductor - mercury with T_c = 4.2K was discovered in 1911.



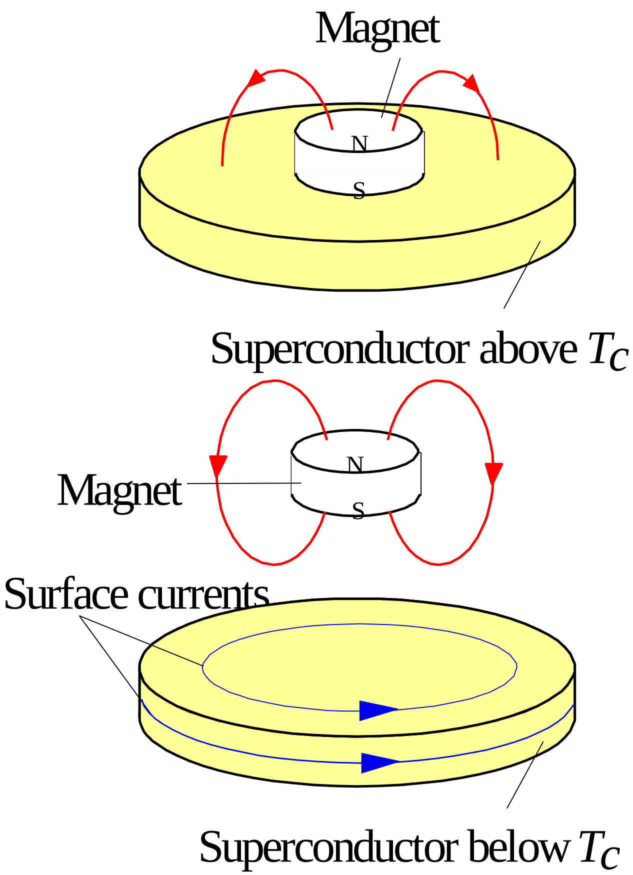
A superconductor such as lead evinces a transition to zero resisitivity at a critical temperature T_C (7.2 K for Pb) whereas a normal conductor such as silver does not

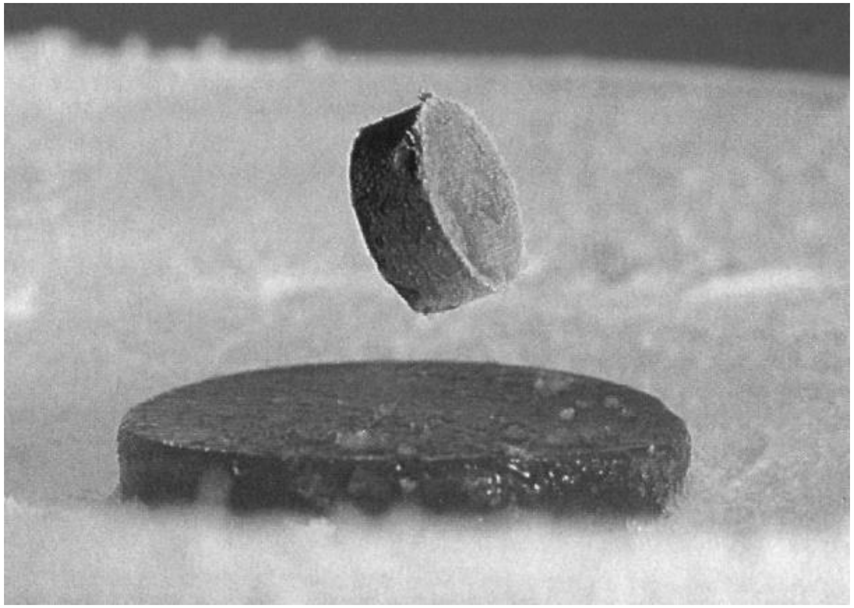
Another important property of superconductors is the **ideal diamagnetism** – **Meissner effect**.

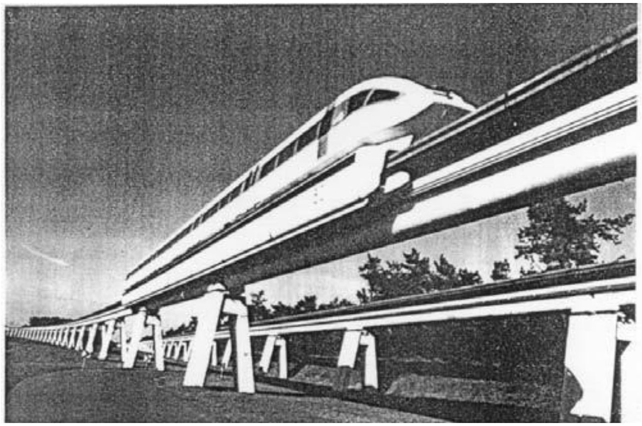
$\mu_0 M = -\mu_0 H$, so that $B = \mu_0 H + \mu_0 M = 0$



The Meissner effect. A superconductor cooled below its critical temperature expels all magnetic field lines from the bulk by setting up a surface current. A perfect conductor ($\sigma = \infty$) shows no Meissner effect.



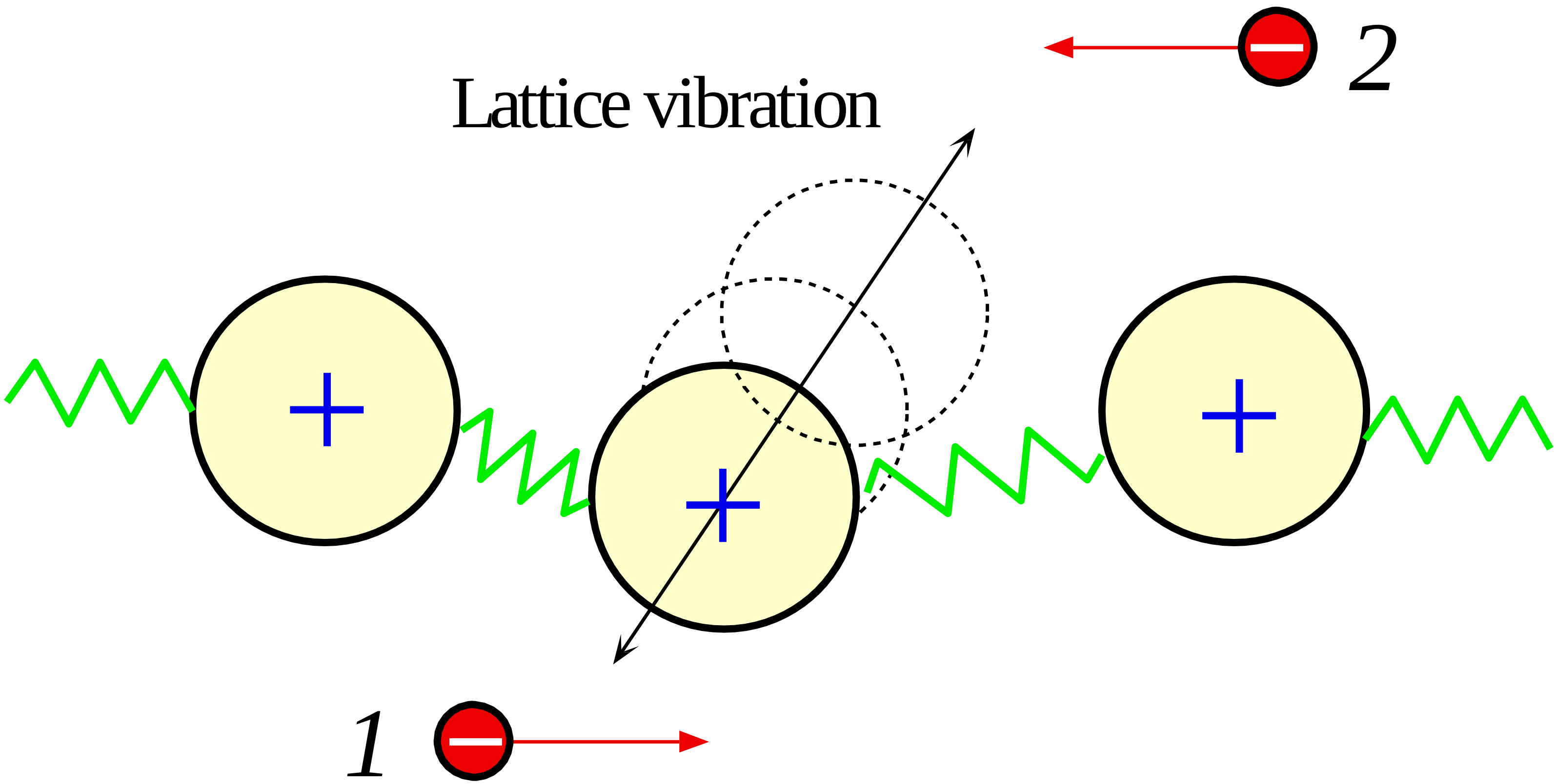




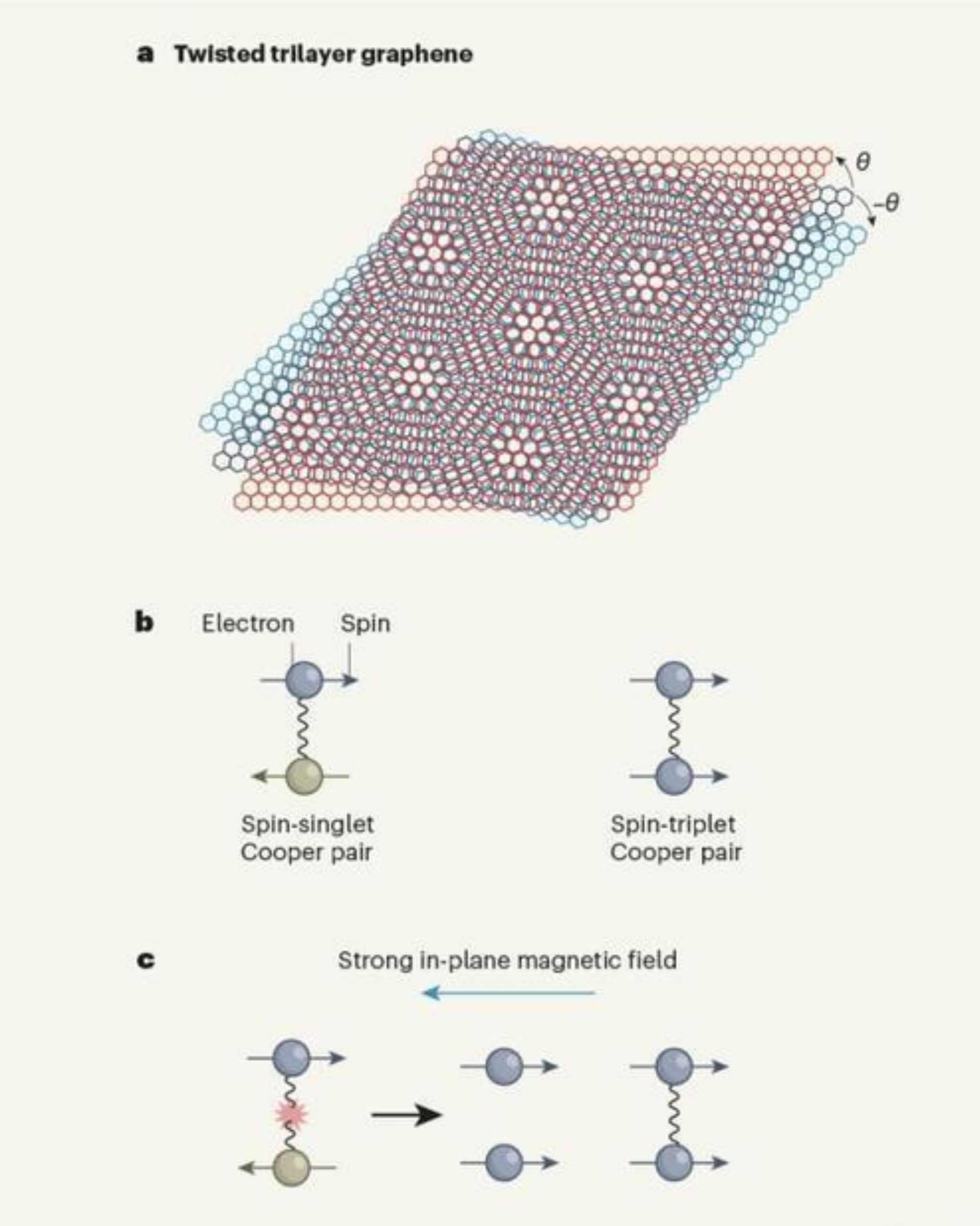
Left: A magnet over a superconductor becomes levitated. The superconductor is a perfect diamagnet which means that there can be no magnetic field inside the superconductor. Right: Photograph of a magnet levitating above a superconductor immersed in liquid nitrogen (77K). This is the Meissner effect.

Superconductivity origin

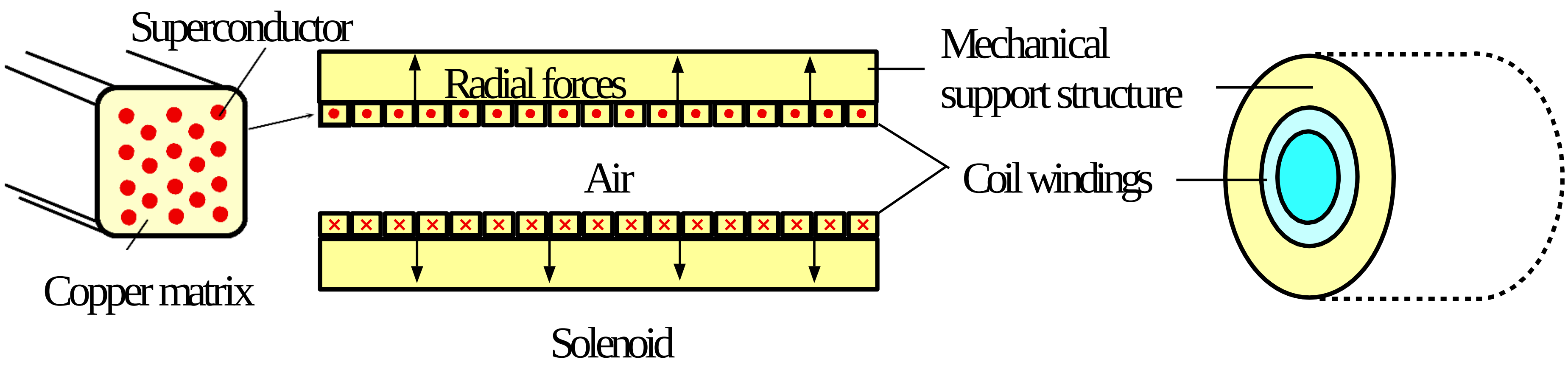
BCS theory (1957): Cooper pairs are formed through electron-phonon interactions. A energy gap appears because of condensation of the **Cooper pairs** to the lowest energy level.



An indirect attraction between two oppositely travelling electrons via a lattice distorsion and vibration.



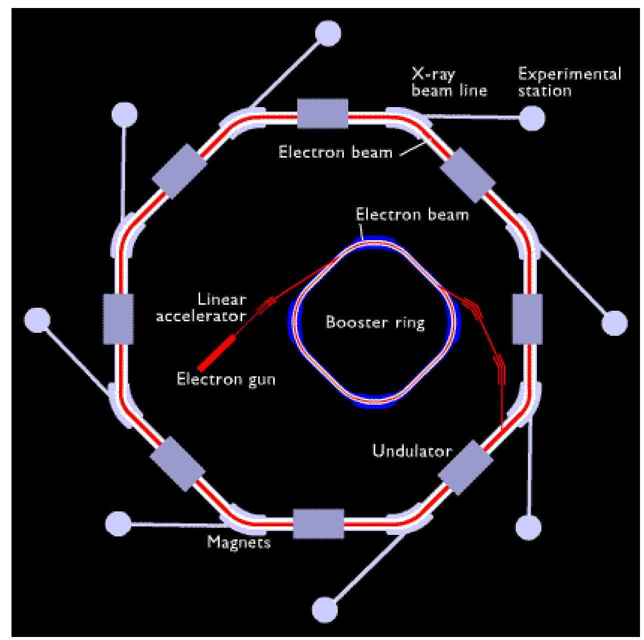
In 2018, MIT scientists Yuan Cao（曹原） discovered that when two sheets of graphene are stacked together at a slightly offset "magic" angle, the new "twisted" graphene structure can become either an insulator or a superconductor.



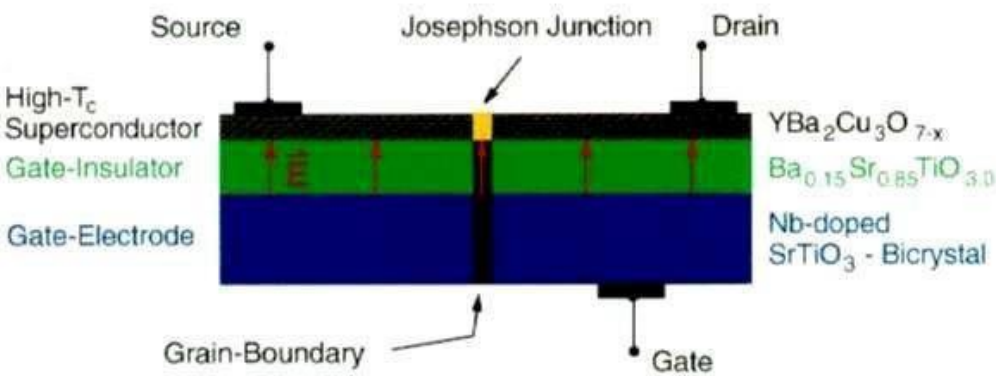
A solenoid carrying a current experiences radial forces pushing the coil apart and axis froces compressing the coil.

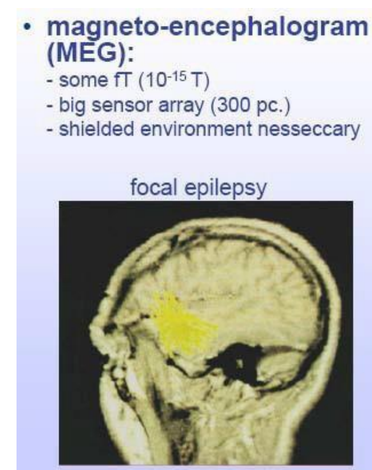
Superconducting electromagnets used on MRI. Provides a magnetic field 0.5–1.5 T.

SOURCE: Courtesy of IGC Magnet Business group.

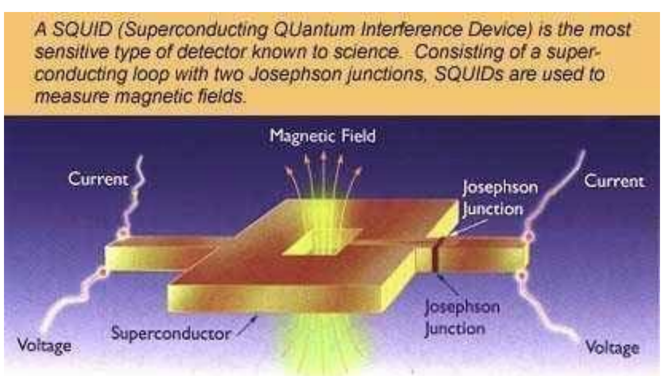


核磁共振





脑电图 / 脑机信号



超导量子干涉仪

暑期学校（国际学分课程）

先进材料与器件（SQ0000218H）

授课时间：7月8日 - 7月11日 （16学时，1学分）
授课形式：线上

数字时代LED器件、性能和应用（ SQ012147EH ）

授课时间：7月14日 - 7月17日 （16学时，1学分）
授课形式：线上

课程由新加坡国立大学（NUS）电子与计算机工程学院教授、新加坡科技局材料研究与工程研究院（A*STAR IMRE）院长，蔡树仁（Chua Soo-Jin）教授主讲。

《先进材料与器件》：光电化合物半导体材料主要用于光的激发、调制和探测，如InGaAsP用于光通讯、GaAs用于安保探测、InGaN和AlInGaP用于显示和照明等。先进化合物半导体被广泛应用于半导体激光器、发光二极管（LED）、高电子迁移率晶体管（HEMT）、异质结双极晶体管（HBT）等。

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