Problem Solving 7: Wave Guidance

OBJECTIVES:

1. To learn the calculation of guided modes.

2. To learn the calculation of cutoff frequency.

REFERENCE: Chapter 7, Wave Guidance

PROBLEM SOLVING STRATEGIES

A. Guidance by Conducting Parallel Plates

The cutoff spatial frequency k_{cm} is

$$k_{cm} = \frac{m\pi}{d}$$

For each mode, the *phase velocity* is calculated by

$$v_p = \frac{\omega}{k_z}$$

and the *group velocity* is

$$v_g = \frac{d\omega}{dk_z}$$

B. Rectangular Waveguide

The guidance condition is

$$k_x a = m\pi$$
$$k_y b = n\pi$$

The cutoff frequency is

$$k_{cmn} = \sqrt{\left(m\pi/a\right)^2 + \left(n\pi/b\right)^2}$$

C. Dielectric Slab Waveguide

The *guidance condition* for TE mode is

$$\alpha d = k_x d \tan\left(k_x d - \frac{m\pi}{2}\right)$$

Here, m is an even number (m = 0, 2, 4, …) for a symmetric mode and an odd number (m = 1, 3, 5, …) for an anti-symmetric mode.

We find the cutoff frequency for the m-th TE mode is

$$\omega = c_0 k_0 = \frac{m\pi}{\sqrt{n_1^2 - n_2^2}\, d}$$

PROBLEM 1

An AM (535-1605 kHz) radio in an automobile cannot receive any signal when the car is inside a tunnel. Model the tunnel as a rectangular waveguide of dimension 6.55m $\times$4.19m.

(a) Give the range of frequencies for which only the dominant mode, TE10, may propagate.

(b) Explain why AM signals cannot be received.

(c) Can FM (88-108 MHz) signals be received? Above what frequencies?

Solution:

(a)

$$f_{c10} = \frac{\omega_{c10}}{2\pi} = \frac{c}{2\pi}\left(\frac{\pi}{a}\right) = \frac{3\times10^8}{2\pi} \times \frac{\pi}{6.55} = 22.9\left(MHz\right)$$

$$f_{c01} = \frac{\omega_{c01}}{2\pi} = \frac{c}{2\pi}\left(\frac{\pi}{b}\right) = \frac{3\times10^8}{2\pi} \times \frac{\pi}{4.19} = 35.8\left(MHz\right)$$

$$f_{c10} < f < f_{c01}$$

(b) An AM radio operates in the range of 500 to 1600 (KHz) is below the cutoff frequency of the fundamental mode TE10. Therefore, AM signals cannot be received in the tunnel.

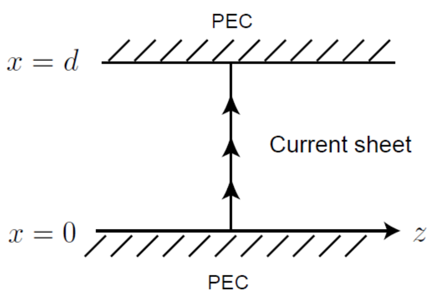
(c) FM singals operate in the range of 88 to 108 (MHz) can be received in the tunnel.

PROBLEM 2

Consider the excitation of a parallel-plate waveguide by a current sheet with

$$\vec{J}_s = \hat{x} J_s \cos\frac{3\pi x}{d}$$

The plates are at $x = 0$ and $x = d$ and the propagation direction is $+z$. Find the amplitudes of the excited modes. (Hint: Only TM modes are excited in this case.)



Solution:

In the region of z > 0

$$H_y = \sum_{m=1}^{\infty} H_m^{(1)} \cos\left(\frac{m\pi x}{d}\right) \exp\left(-jk_z z\right)$$

$$E_x = \sum_{m=1}^{\infty} \frac{k_z}{\omega\varepsilon} H_m^{(1)} \cos\left(\frac{m\pi x}{d}\right) \exp\left(-jk_z z\right)$$

In the region of z < 0

$$H_y = \sum_{m=1}^{\infty} H_m^{(2)} \cos\left(\frac{m\pi x}{d}\right) \exp\left(jk_z z\right)$$

$$E_x = \sum_{m=1}^{\infty} \frac{-k_z}{\omega\varepsilon} H_m^{(2)} \cos\left(\frac{m\pi x}{d}\right) \exp\left(jk_z z\right)$$

At the boundary z = 0

$$\sum_{m=1}^{\infty} \left(H_m^{(2)} - H_m^{(1)}\right) \cos\left(\frac{m\pi x}{d}\right) = J_s \cos\frac{3\pi x}{d}$$

$$\sum_{m=1}^{\infty} \frac{k_z}{\omega\varepsilon} H_m^{(1)} \cos\left(\frac{m\pi x}{d}\right) = \sum_{m=1}^{\infty} \frac{-k_z}{\omega\varepsilon} H_m^{(2)} \cos\left(\frac{m\pi x}{d}\right)$$

So m = 3 and $H_m^{(2)} = -H_m^{(1)} = J_s/2$

In the region of z > 0

$$H_y = -\frac{J_s}{2} \cos\left(\frac{3\pi x}{d}\right) \exp\left(-jk_z z\right)$$

$$E_x = -\frac{k_z J_s}{2\omega\varepsilon} \cos\left(\frac{3\pi x}{d}\right) \exp\left(-jk_z z\right)$$

$$E_z = -\frac{jJ_s}{2\omega\varepsilon} \left(\frac{3\pi}{d}\right) \sin\left(\frac{3\pi x}{d}\right) \exp\left(-jk_z z\right)$$

In the region of z < 0

$$H_y = \frac{J_s}{2} \cos\left(\frac{3\pi x}{d}\right) \exp\left(jk_z z\right)$$

$$E_x = -\frac{k_z J_s}{2\omega\varepsilon} \cos\left(\frac{3\pi x}{d}\right) \exp\left(jk_z z\right)$$

$$E_z = \frac{jJ_s}{2\omega\varepsilon} \left(\frac{3\pi}{d}\right) \sin\left(\frac{3\pi x}{d}\right) \exp\left(jk_z z\right)$$