Fundamentals of Information Science: Homework 2

February 26, 2025

Problem 1.
You are asked to design a Turing machine that accepts the following languages: some number of 1's followed by the same number of 0's.

$$\#01\#, \#0011\#, \#000111\#, \#00001111\#, ...$$

Explain your design.

Problem 2.
Coin flips. A fair coin is flipped until the first head occurs. Let X denote the number of flips required. Find the entropy $H(X)$ in bits. The following expressions may be useful:

$$\sum_{n=0}^{\infty} r^n = \frac{1}{1-r}, \quad \sum_{n=0}^{\infty} nr^n = \frac{r}{(1-r)^2}.$$

Problem 3.
A die comes up 6 twice as often as it comes up 1. What is the maximum entropy $(p_1, p_2, ..., p_6)$?