Fundamentals of Information Science: Homework 6

April 1, 2025

Problem 1.

For any linear block code over $\mathbb{F}_2$ with minimum Hamming distance at least 2t+1 between codewords, show that:

$$2^{n-k} \geq 1 + \binom{n}{1} + \binom{n}{2} + ... + \binom{n}{t}.$$

Hint: How many errors can such a code always correct?

For each (n,k,d) combination below, state whether a linear block code with those parameters exists or not. Please provide a brief explanation for each case: if such a code exists, give an example; if not, you may rely on a suitable necessary condition.

(a) (31,26,3): Yes / No

(b) (32,27,3): Yes / No

(c) (43,42,2): Yes / No

(d) (27,18,3): Yes / No

(e) (11,5,5): Yes / No

Problem 2.

Generate random regular $(3, 5)$ LDPC codes with blocklength $n = 1000$. Evaluate the bit error probability curves for communication over the BEC(ϵ), i.e., binary erasure channel with erasure probability ϵ.

I expect to receive

1. A print-out of the program you used.

2. A plot of error probability curves (the probability with some erasures cannot be correctly recovered) versus ϵ with ϵ chosen from $[0.1, 0.2, ..., 0.6]$. (for each value of ϵ, assume that the transmitted codeword is 0000..0, and decode the received codewords with random erasures, repeating for 100 times. The maximum number of iterations for message passing chooses 20.)

3. Description of the following features of your simulation: How much CPU time did the simulation take to construct and process the parity-check matrix? How much CPU time did the simulation take for each value of ϵ?