Fundamentals of Information Science: Homework 10

May 14, 2025

Problem 1. Ciphertext expansion vs. security

Let $\mathcal{E} = (E, D)$ be an encryption scheme messages and ciphertexts are bit strings.

- (a) Suppose that for all keys and all messages m, the encryption of m is the exact same length as m. Show that (E, D) cannot be semantically secure under a chosen plaintext attack.
- (b) Suppose that for all keys and all messages m, the encryption of m is exactly ℓ bits longer than the length of m. Show an attacker that can win the CPA security game using $\approx 2^{\ell}/2$ queries and advantage $\approx 1/2$. You may assume the message space contains more than $\approx 2^{\ell}/2$ messages.

Problem 2. Understand public-key encryption

Given two random primers (p, q) = (31, 43), you are asked to construct an RSA encryption based on the two primes (p, q) (although the primes are two small to guarantee security).

- (a) Construct a pair of public key and secret key.
- (b) Demonstrate the process of encrypting a message m = 100 with a random number x = 13 using the generated key pair, and then decrypt the resulting ciphertext.