Chapter 3

模拟信号的发送和接收

主讲教师：朱维红

Analog Signal Transmission and Reception

- 3.1. 调制简介
- 3.2. 振幅调制 (AM)
- 3.3. 角度调制
- 3.4. 无线电广播和电视广播(自学)
- 3.5. 移动无线系统(自学)

3.1 INTRODUCTION TO MODULATION

☺ 载波调制的基本概念

待发送的消息信号用 $m(t)$（调制信号）表示，它是一个带宽为 W 的低通功率型信号，其功率用 P_m 表示.

$$P_m = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}|m(t)|^2 dt$$

载波调制: 消息信号记录在

$$c(t) = A_c cos(2\pi f_c t + \Phi_c)$$

的载波信号上。式中:A_c 是载波幅度，f_c 是载波频率，Φ_c 是载波相位。

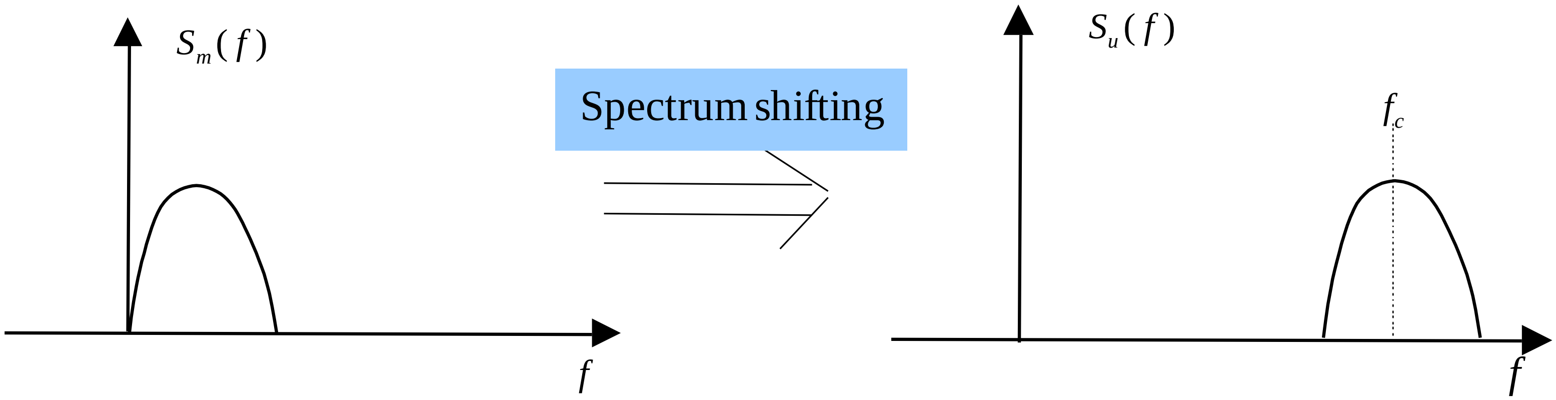
分类：振幅调制（AM）、频率调制（FM）和相位调制（PM）

3.1 INTRODUCTION TO MODULATION

☺ 调制的目标

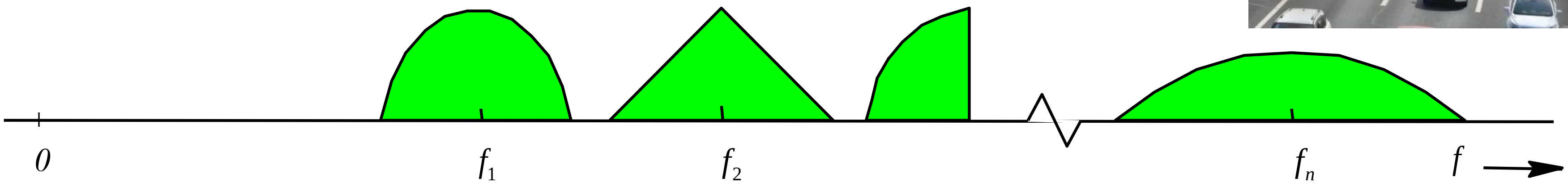
1. 通过频谱搬移,使信号适应信道。

调制将低通信号（基带信号）变成了带通信号。

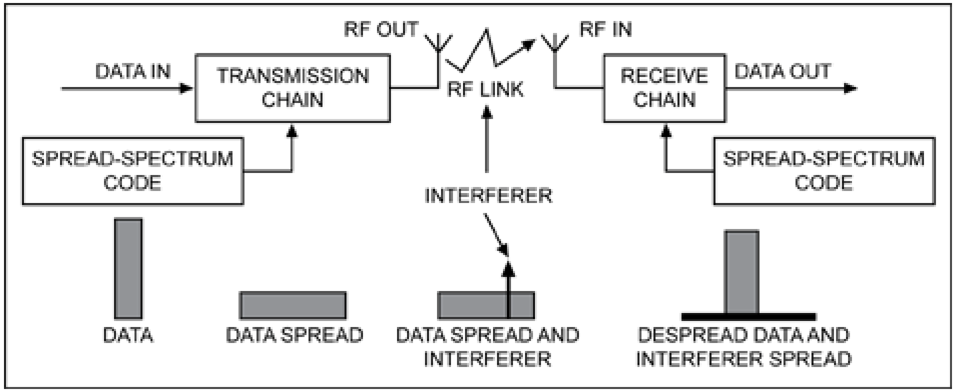


3.1 INTRODUCTION TO MODULATION

2. FDM，频分复用.



3. 扩展带宽抗噪



主观题 10分

设置

上网搜一下，调制还有哪些作用?

作答

3.2 Amplitude Modulation (AM, 幅度调制)

- 振幅调制中,带宽为W的消息信号$m(t)$记录在载波$c(t)=A_c\cos(2\pi f_c t+\phi_c)$的振幅上.

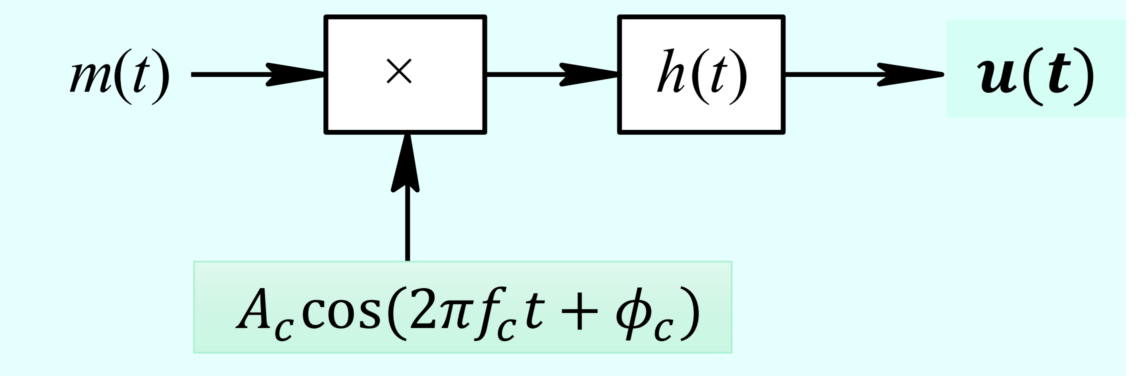


Fig. 3.2.1 Block diagram of AM

3.2.1. Double-Sideband suppressed Carrier AM (抑止载波的双边带调幅, DSB-SC AM)

- 实现

$$u(t) = m(t)c(t) = A_c m(t)\cos(2\pi f_c t + \phi_c)$$

- 带宽要求

$$U(f) = F\left[m(t)\right] * F\left[A_c \cos(2\pi f_c t + \phi_c\right]$$

$$= M(f) * \frac{A_c}{2}\left[e^{j\phi_c}\delta(f - f_c) + e^{-j\phi_c}\delta(f + f_c)\right]$$

$$= \frac{A_c}{2}\left[M(f - f_c)e^{j\phi_c} + M(f + f_c)e^{-j\phi_c}\right]$$

3.2.1. Double-Sideband suppressed Carrier AM

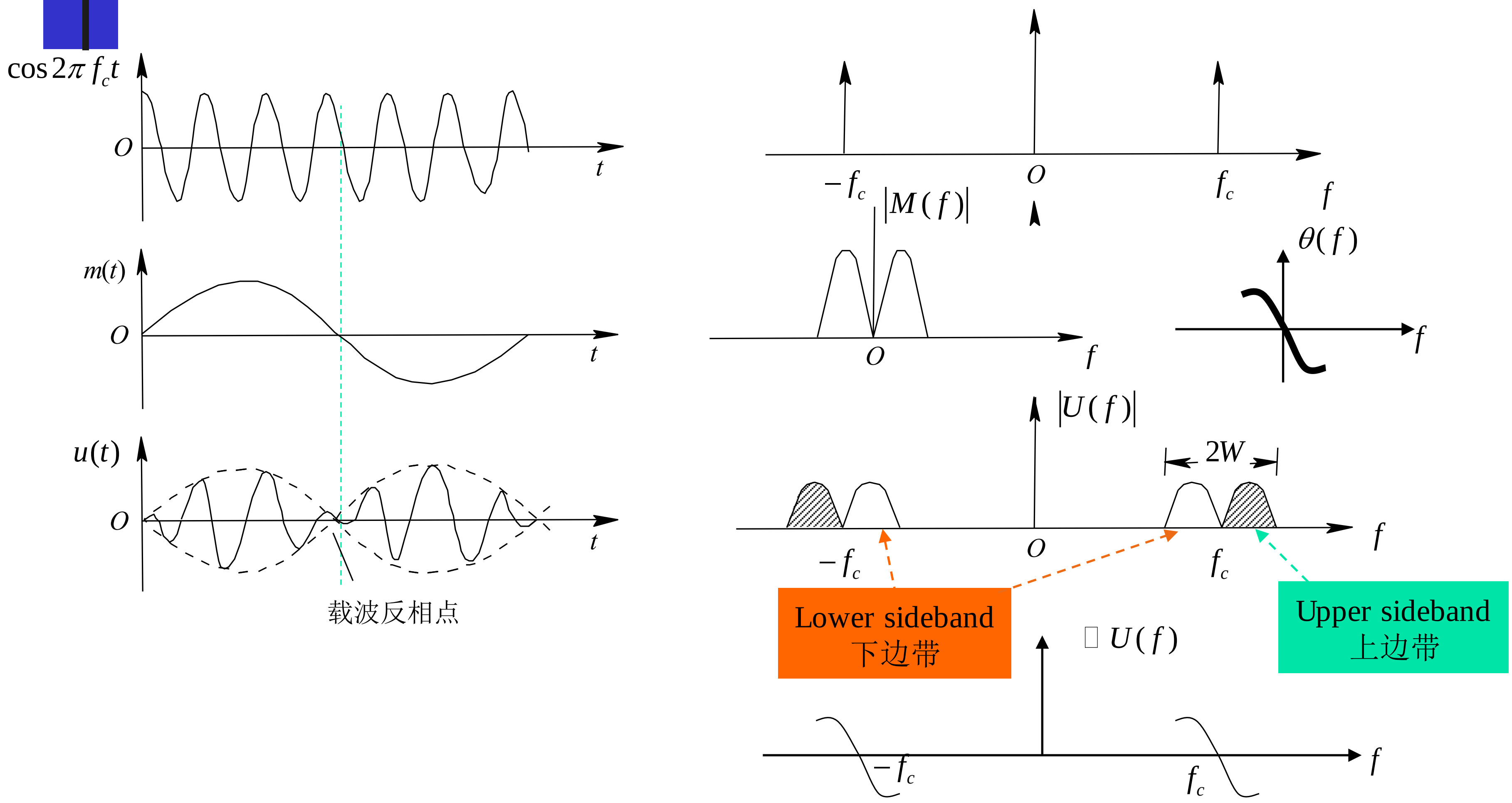
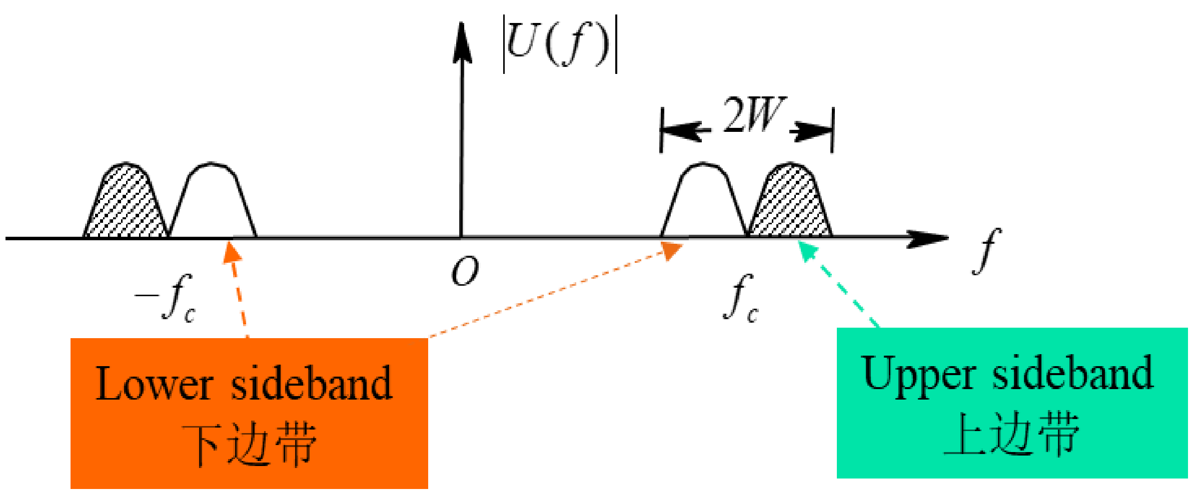


Fig. 3.2.2 DSB-SC AM

3.2.1. Double-Sideband suppressed Carrier AM

Note1. **AM信号的带宽为** $2W$.→ **从而传输该已调信号** $u(t)$**的信道带宽至少为** $B_c=2W$.

- *Note2.* **上边带(USB):** $f > f_c$;**下边带(LSB):**$f < f_c$
- *Note3*. $m(t)$ **不含直流分量 (meaning?). 因此** $u(t)$ **不含载波分量**



抑止载波的双边带调幅, DSB-SC AM

Exapmle 3.2.1 假设调制信号 ***m(t)*** 是具有如下形式的正弦波,

$$m(t) = a\cos 2\pi f_m t \qquad f_m \ll f_c$$

试确定**DSB-SC AM**信号及其上下边带.

3.2.1. Double-Sideband suppressed Carrier AM

Solution **DSB-SC AM 信号在时域的表达式为:**

$$u(t)=m(t)c(t)=a\cos 2\pi f_m t\cdot A_c\cos\left(2\pi f_c t+\phi_c\right)$$

$$=\underbrace{\frac{A_c a}{2}\cos\left[2\pi\left(f_c-f_m\right)t+\phi_c\right]}_{\text{下边带}}+\underbrace{\frac{A_c a}{2}\cos\left[2\pi\left(f_c+f_m\right)t+\phi_c\right]}_{\text{上边带}}$$

$$U\left(f\right)=\frac{A_c a}{4}\left[e^{j\phi_c}\delta\left(f-f_c+f_m\right)+e^{-j\phi_c}\delta\left(f+f_c-f_m\right)\right]$$

$$+\frac{A_c a}{4}\left[e^{j\phi_c}\delta\left(f-f_c-f_m\right)+e^{-j\phi_c}\delta\left(f+f_c+f_m\right)\right]$$

3.2.1. DSB-SC AM

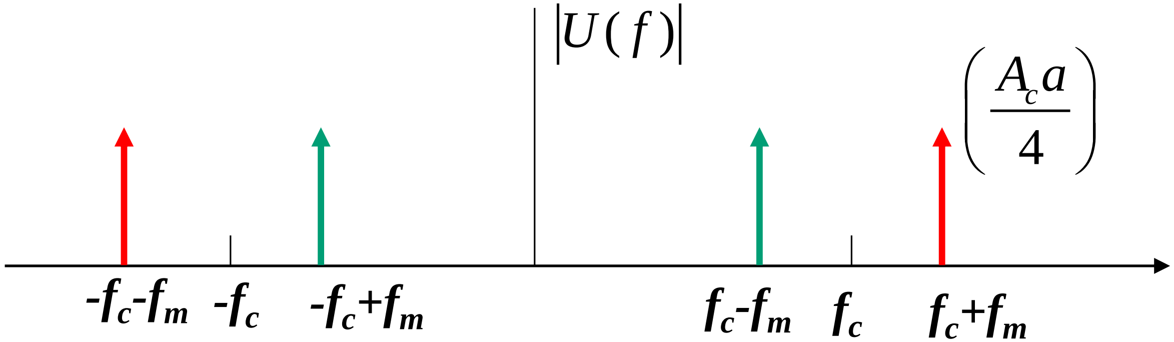
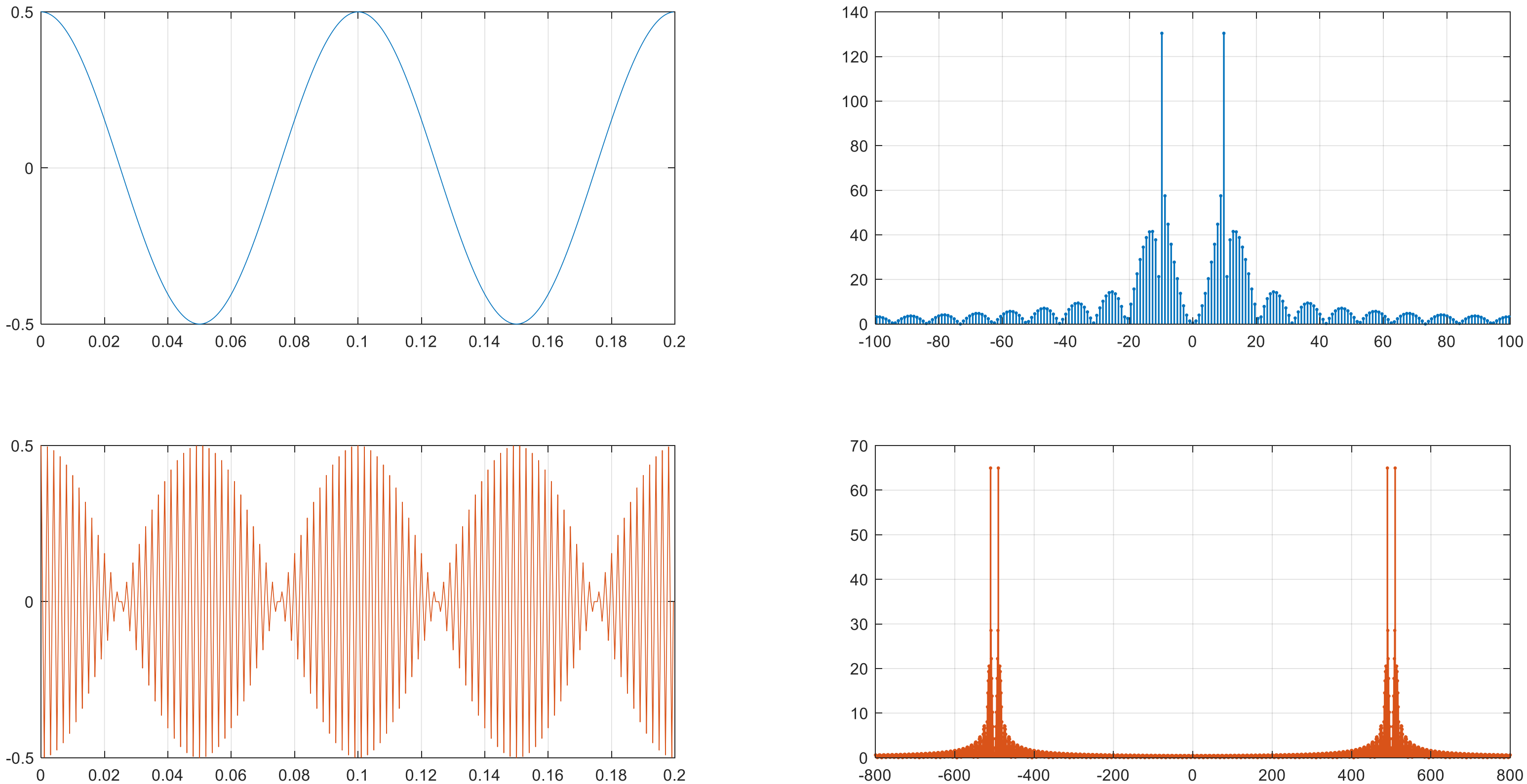


Figure example 3.2.1 Spectrum of the DSB-SC AM signals

注：绿色为下边带，红色为上边带

3.2.1. DSB-SC AM

Matlab 运行结果



3.2.1. 抑止载波的双边带调幅, DSB-SC AM

- *DSB-SC AM*信号的功率谱和功率: 设信号初始相位为零(?)

$$R_u(\tau)=\lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}u(t)u(t-\tau)dt$$

$$=\lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}A_c^2 m(t)m(t-\tau)\cos(2\pi f_c t)\cos(2\pi f_c(t-\tau))dt$$

$$=\frac{A_c^2}{2}\lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}m(t)m(t-\tau)\left[\cos(4\pi f_c t-2\pi f_c\tau)+\cos(2\pi f_c\tau)\right]dt$$

3.2.1. 抑止载波的双边带调幅, DSB-SC AM

DSB-SC AM信号的功率谱和功率（续）：

$$\boldsymbol{R_u}(\boldsymbol{\tau}) = \frac{A_c^2}{2} R_m(\tau)\cos(2\pi f_c\tau) + \int_{-\infty}^{\infty} m(t)m(t-\tau)\cos(4\pi f_c t - 2\pi f_c\tau)dt$$

$$= \frac{A_c^2}{2} R_m(\tau)\cos(2\pi f_c\tau)$$

$$\int_{-\infty}^{\infty} x(t)y^*(t)dt = \int_{-\infty}^{\infty} X(f)Y^*(f)df$$

$$+ \int_{-\infty}^{\infty} \mathcal{F}[m(t-\tau)]\{\mathcal{F}[m(t)\cos(4\pi f_c t - 2\pi f_c\tau)]\}^* df$$

$$= \frac{A_c^2}{2} R_m(\tau)\cos(2\pi f_c\tau)$$

$$+ \int_{-\infty}^{\infty} e^{-j2\pi f\tau} M(f)\left[\frac{M(f-2f_c)e^{-j2\pi f\tau}}{2} + \frac{M(f+2f_c)e^{j2\pi f\tau}}{2}\right]^* df$$

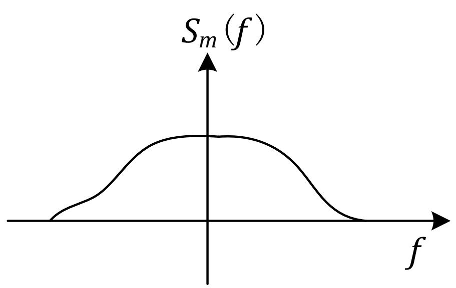
$$= \frac{A_c^2}{2} R_m(\tau)\cos(2\pi f_c\tau)$$

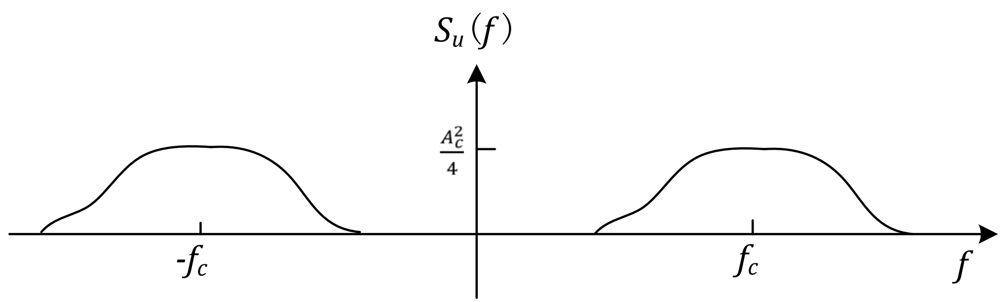
3.2.1. Double-Sideband suppressed Carrier AM

$$S_u(f) = F[R_u(\tau)] = F\left[\frac{A_c^2}{2} R_m(\tau)\cos 2\pi f_c \tau\right]$$

$$= \frac{A_c^2}{4}[S_m(f - f_c) + S_m(f + f_c)]$$

$$P_u = \frac{A_c^2}{2} R_m(0) = \frac{A_c^2}{2} P_m$$





DSB

- **Example 3.2.2** 确定例3.2.1中已调信号的功率及每个边带的功率.
- **Solution**

$$S_u(f) = \frac{A_c^2 a^2}{16}[\delta(f - f_m - f_c) + \delta(f + f_m - f_c) + \delta(f - f_m + f_c) + \delta(f + f_m + f_c)]$$

$$P_u = \frac{A_c^2 a^2}{4}, P_{uu} = P_{ul} = \frac{A_c^2 a^2}{8}$$

3.2.1. Double-Sideband suppressed Carrier AM

DSB-SC AM 信号的解调.

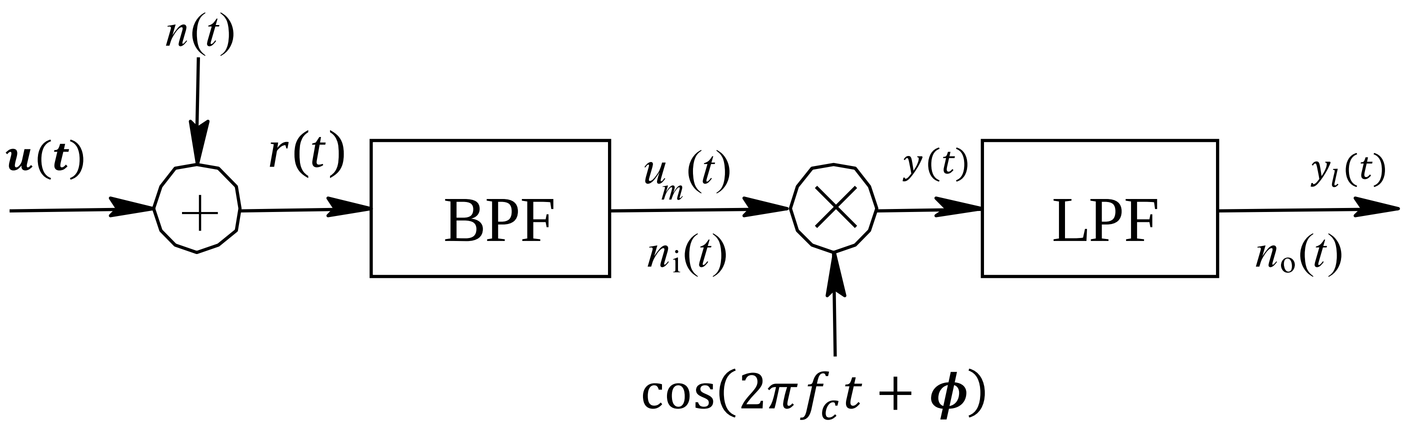


Fig3.2.3. Block diagram of the demodulator

Coherent demodulating **or** *synchronous demodulating*

(相干解调或同步解调）

图中，$\boldsymbol{n(t)}$是均值为零的加性高斯白噪声。

BPF的带宽为 2WHz，允许已调信号$\boldsymbol{u(t)}$无失真通过。LPF带宽为W Hz。

3.2.1. Double-Sideband suppressed Carrier AM

- **Assuming *n(t)=0*, 忽略噪声， then**

$$r(t) = u_m(t) = A_c m(t)\cos(2\pi f_c t + \phi_c)$$

$$y(t) = u_m(t)\cos(2\pi f_c t + \phi)$$

$$= A_c m(t)\cos(2\pi f_c t + \phi)\cos(2\pi f_c t + \phi_c)$$

$$= \frac{A_c}{2} m(t)\cos(\phi_c - \phi) + \frac{A_c}{2} m(t)\cos(4\pi f_c t + \phi + \phi_c)$$

$$y_l(t) = \frac{A_c}{2} m(t)\cos(\phi_c - \phi)$$

$\phi_c - \phi = 0$ **is desired**

- $\phi = \phi_c$时，称为相干解调或同步解调。

3.2.1. Double-Sideband suppressed Carrier AM

- 载波获取的两种方法

1) Inserting pilot tone(插入导频，图3.24发送端，图3.25接收端）

2) Using phase locked loop(PLL,锁相环）(learning after)

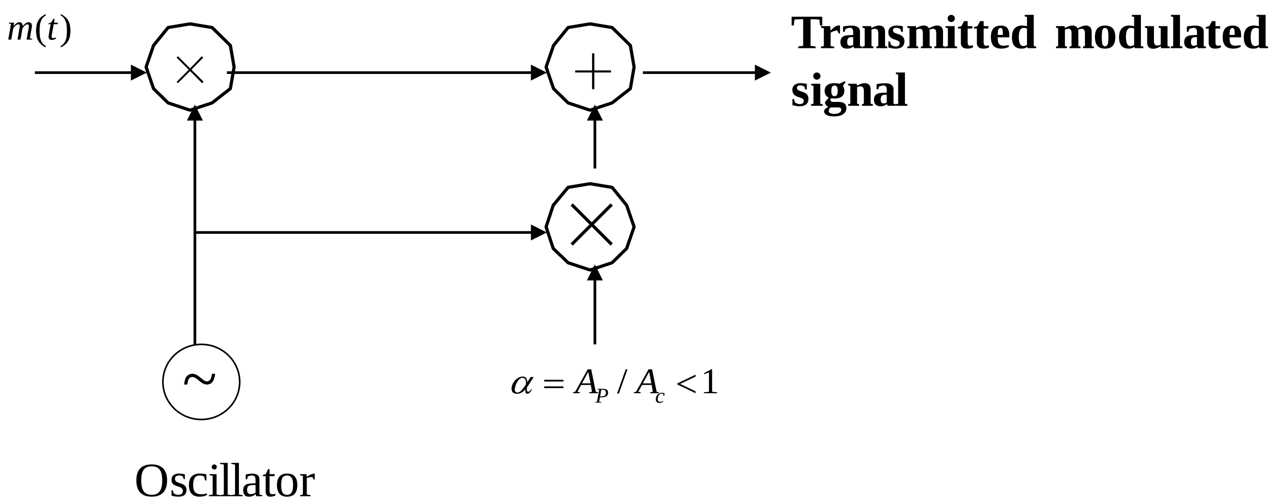


Fig 3.2.4 Addition of a pilot tone to a DSB AM signal

3.2.1. Double-Sideband suppressed Carrier AM

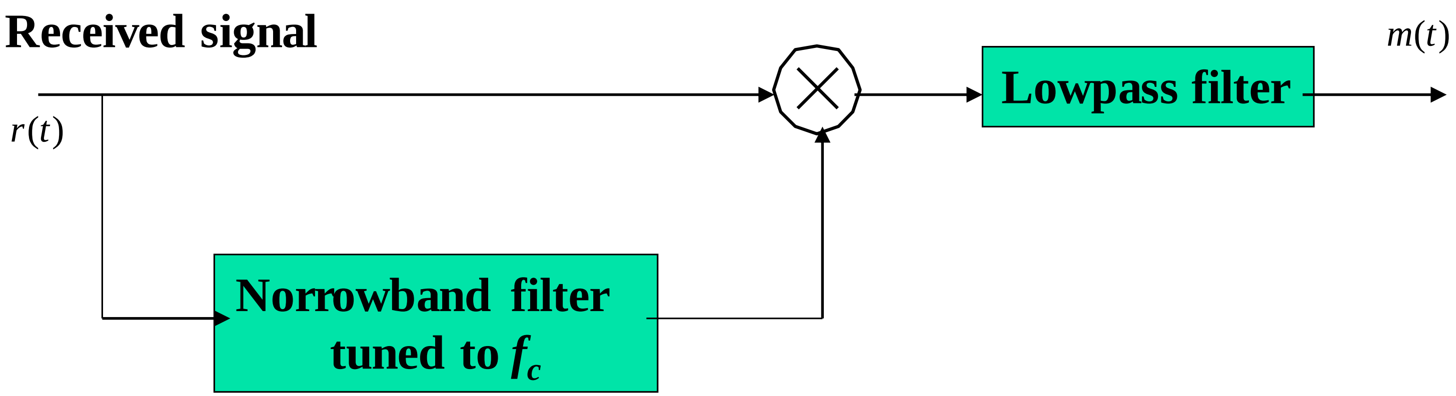


Fig3.2.5. Use of a pilot tone to demodualte a DSB AM signal

单选题 1分

设置

对消息信号$m(t) = a\cos 20\pi t$进行抑制载波双边带调制，已知载波$c(t) = A_c cos200\pi t$，则已调信号的上边带频率为

A 90Hz

B 110Hz

C 180Hz

D 220Hz

提交

对消息信号 $m(t) = \cos 20\pi t$ 进行抑制载波双边带调制，已知载波 $c(t) = 10cos200\pi t$ ，则已调信号的功率是（ ）瓦特

A 25

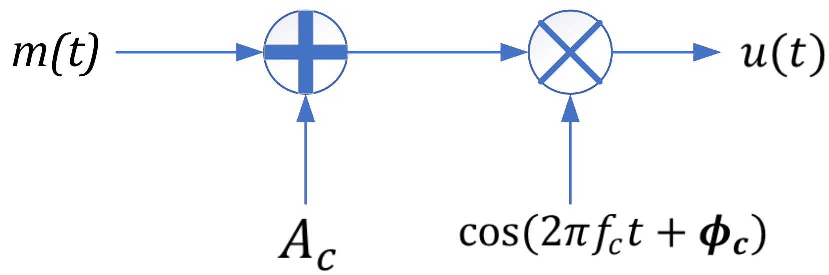
B 50

C 75

D 100

提交

3.2.2 Conventional Amplitude Modulation(常规调幅)



常规调幅实现框图

$$u(t) = \left(A_c + m(t)\right)\cos(2\pi f_c t + \boldsymbol{\phi_c}),$$

要求

$$A_c \geq |m(t)|_{\max}$$

定义：

$$\alpha = \frac{|m(t)|_{\max}}{A_c}$$

为调制指数，则 $0 < \alpha \leq 1$。

3.2.2 Conventional Amplitude Modulation(常规调幅)

- 定义

归一化
消息信号

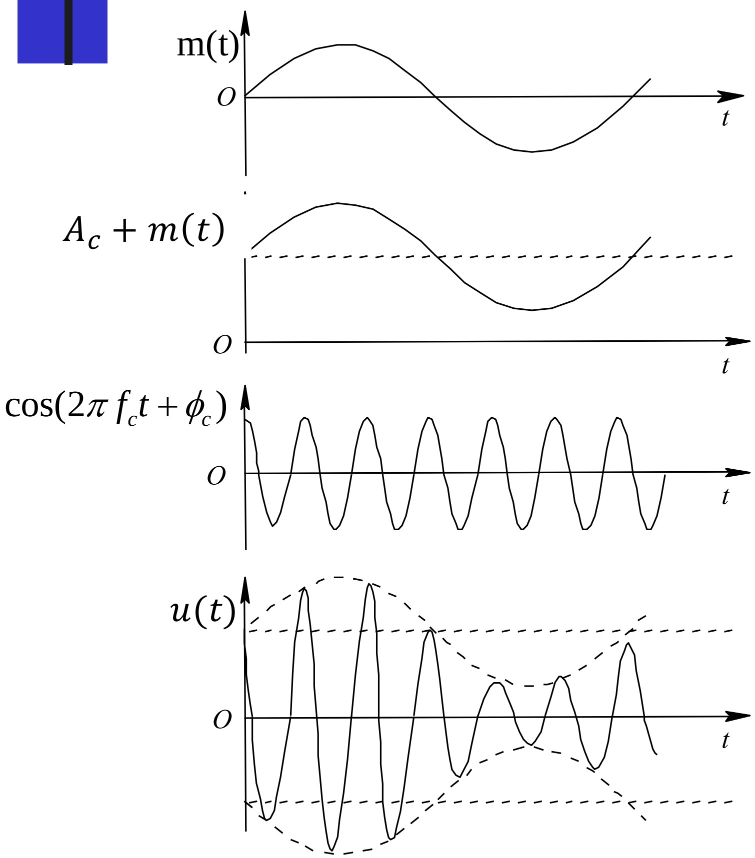
$$m_n(t) = \frac{m(t)}{|m(t)|_{\max}}$$

则

$$u(t) = A_c[1 + \alpha m_n(t)]\cos(2\pi f_c t + \boldsymbol{\phi}_c)$$

$$U(f) = \frac{A_c}{2}[e^{j\varphi_c} a M_n(f - f_c) + e^{j\varphi_c}\delta(f - f_c) + e^{-j\varphi_c} a M_n(f + f_c) + e^{-j\varphi_c}\delta(f + f_c)]$$

3.2.2 Conventional Amplitude Modulation



$$U(f) = \frac{A_c}{2}[e^{j\varphi_c} a M_n(f - f_c) + e^{j\varphi_c}\delta(f - f_c)$$
$$+e^{-j\varphi_c} a M_n(f + f_c) + e^{-j\varphi_c}\delta(f + f_c)]$$

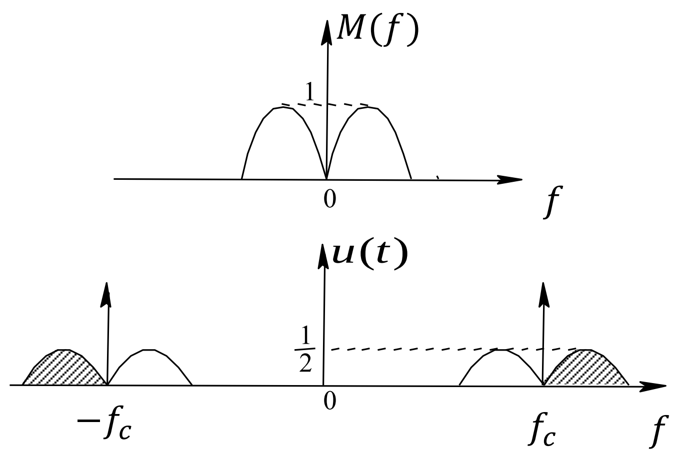


Fig. 3.2.7 Block diagram of AM

3.2.2 Conventional Amplitude Modulation

- AM 信号的带宽= $2W$.
- AM 信号的功率

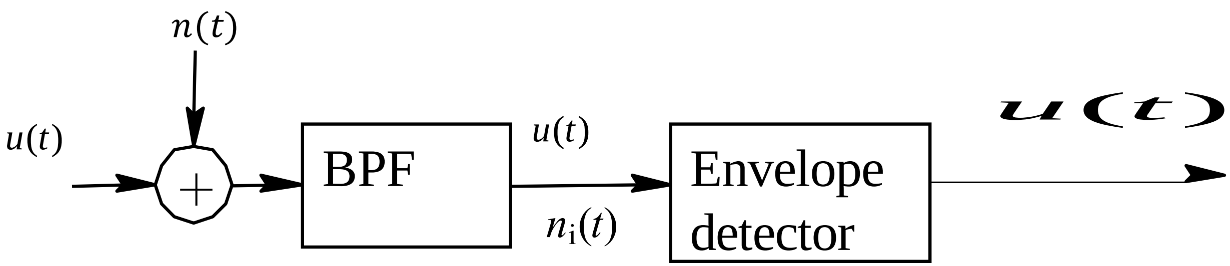
$$P_u = \frac{A_c^2}{2} + P_m = \frac{A_c^2}{2} + \frac{A_c^2}{2}\alpha^2 P_{m_n}$$

常规AM信号的**功率效率**远小于DSB-SC AM,但其解调容易.

3.2.2 Conventional Amplitude Modulation

▪ *DSB AM 信号的解调*

Envelope detector(包络检波)



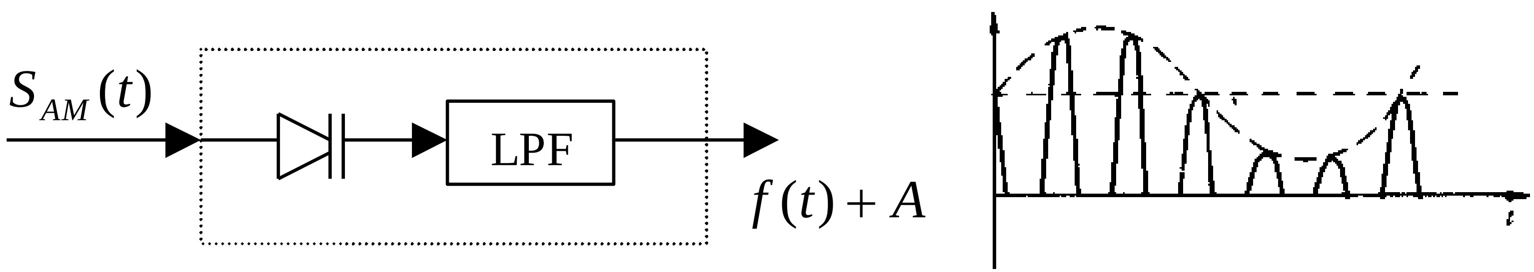


Fig. 3.2.8 Envelope detector

AM

例（problem3.7））已知AM信号如下

$$u(t) = [20 + 2cos30\pi t + 10cos60\pi t]cos(2 \times 10^5 \pi t)$$

求：（1）调制指数；

（2）每个频率分量的功率。

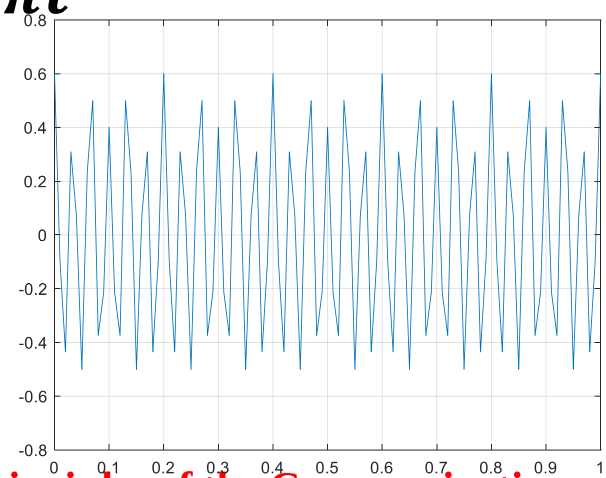
解：

$$u(t) = [20 + 2cos30\pi t + 10cos60\pi t]cos(2 \times 10^5 \pi t)$$

$$= 20[1 + 0.1\cos30\pi t + 0.5cos60\pi t]\ cos(2 \times 10^5 \pi t)$$

故: $\alpha m_n(t) = 0.1\cos30\pi t + 0.5cos60\pi t$

调制指数α=0.6



（2）

$$u(t)=[20+2cos30\pi t+10cos60\pi t]cos\left(2\times 10^{5}\pi t\right)$$

$$=20cos\left(2\times 10^{5}\pi t\right)+cos\left((2\times 10^{5}-30)\pi t\right)+cos\left((2\times 10^{5}+30)\pi t\right)+5\,cos\left((2\times 10^{5}-60)\pi t\right)+5cos\left((2\times 10^{5}+60)\pi t\right)$$

从而5个频率分量的功率分别为：200、0.5、0.5、12.5、12.5。

主观题 10分

设置

请写出AM信号的时域表达式，并基于你写出的表达式画出该信号的功率谱，计算出功率。假设消息信号用$m(t)$表示，它的功率谱为$S_m(f)$。

正常使用主观题需2.0以上版本雨课堂

作答

解： $u(t) = \left(A_c + m(t)\right)cos2\pi f_c t$
$= A_c cos2\pi f_c t + m(t) cos2\pi f_c t$

故

$$S_u(f) = \frac{A_c^2}{4}[\delta(f+f_c)+\delta(f-f_c)] + \frac{1}{4}[S_m(f+f_c)+S_m(f-f_c)]$$

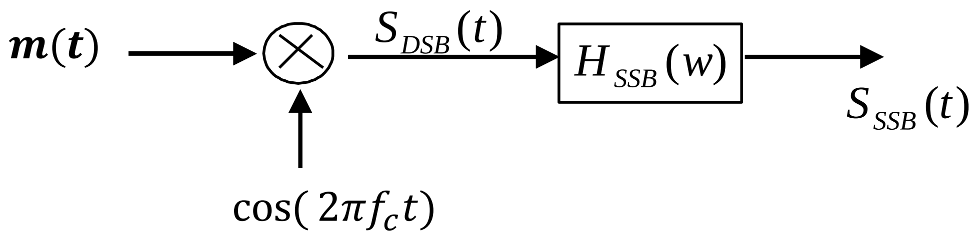
功率为

$$P_u = \frac{A_c^2}{2} + P_m$$

3.2.3 Single-Sideband AM (单边带调幅)

- 单边带 AM 信号的产生.

1. 滤波法



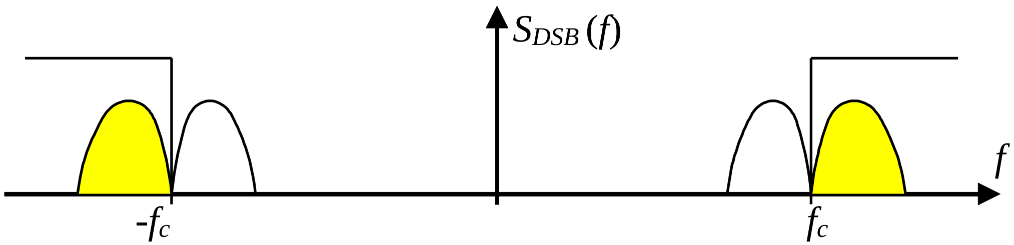
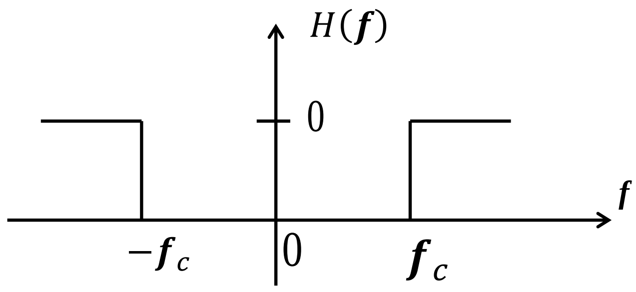
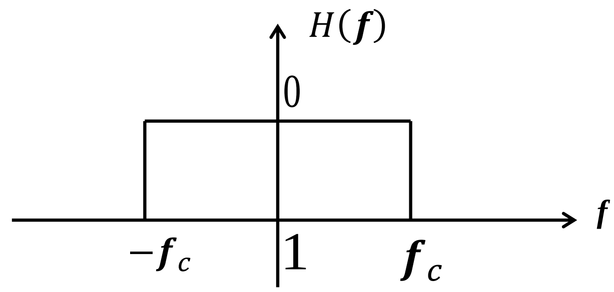


Fig. 3.2.9 Block diagram of SSB-AM

3.2.3 SSB AM



Filter for USB signal



Filter for LSB signal

对 USB 信号, $H_U(f)$ 的频率响应为

$$H_U(f) = u(f - f_c) + u(-f - f_c)$$

对 LSB 信号, $H_L(f)$的频率响应为

$$H_L(f) = \frac{1}{2}[\mathrm{sgn}(f + f_c) - \mathrm{sgn}(f - f_c)]$$
$$= u(f + f_c) - u(f - f_c)$$

3.2.3 Single-Sideband AM

以 USB信号为例，推导单边带信号的时域表达式

$$M_{USB}(f) = A_c M(f-f_c)u(f-f_c) + A_c M(f+f_c)u(-f-f_c)$$
$$= A_c M(f)u(f)\big|_{f=f-f_c} + A_c M(f)u(-f)\big|_{f=f+f_c}$$

$$s_{USB}(t)$$
$$= A_c m(t) * F^{-1}[u(f)]e^{j2\pi f_c t} + A_c m(t) * F^{-1}[u(-f)]e^{-j2\pi f_c t}$$
$$= A_c m(t) * \left[\frac{1}{2}\delta(t) + \frac{j}{2\pi t}\right]e^{j2\pi f_c t} + A_c m(t)$$
$$* \left[\frac{1}{2}\delta(t) - \frac{j}{2\pi t}\right]e^{-j2\pi f_c t}$$
$$= \frac{1}{2}A_c[m(t) + j\hat{m}(t)]e^{j2\pi f_c t} + \frac{1}{2}A_c[m(t) - j\hat{m}(t)]e^{-j2\pi f_c t}$$
$$= A_c m(t)\cos 2\pi f_c t - A_c\hat{m}(t)\sin 2\pi f_c t$$

主观题 10分

设置

请推导下边带信号的表达式。

正常使用主观题需2.0以上版本雨课堂

作答

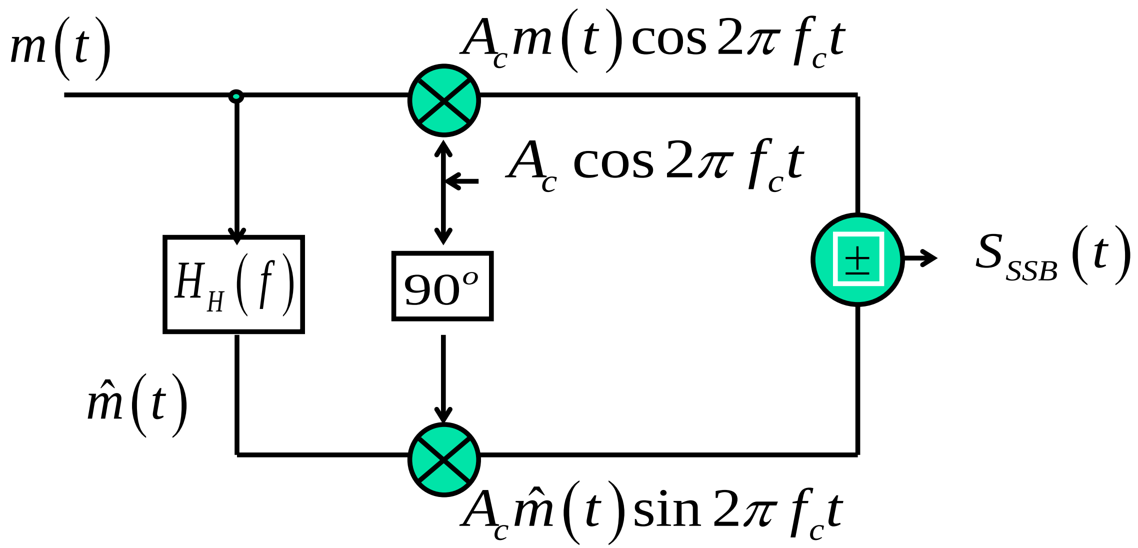
3.2.3 Single-Sideband AM

- **LSB信号的时域表达式为：**

$$s_{LSB}(t) = A_c m(t)\cos 2\pi f_c t + A_c \hat{m}(t)\sin 2\pi f_c t$$

***Note*: 这里DSB AM 信号为 $2A_c m(t)\cos 2\pi f_c t$**

2. 相移法产生 SSB 信号



3.2.3 Single-Sideband AM

- **Example 3.2.4**. 假设调制信号如下:

$$m(t) = \cos 2\pi f_m t, \quad f_m \ll f_c$$

确定两个可能的SSB AM信号.

Solution **The Hilbert transform of *m(t)* is**

$$\hat{m}(t) = \sin 2\pi f_c t$$

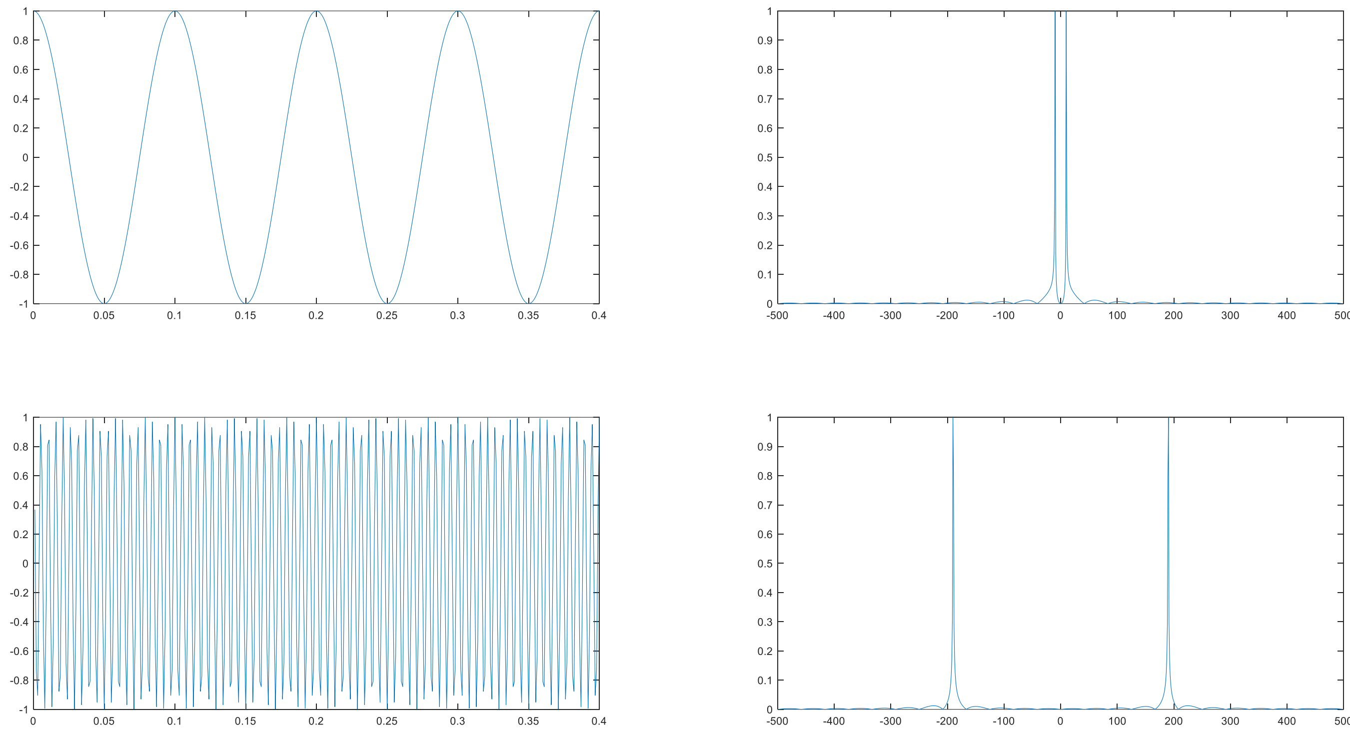
$$u_{SSB}(t) = A_c \cos 2\pi f_m t \cos 2\pi f_c t \mp A_c \sin 2\pi f_m t \sin 2\pi f_c t$$

$$= A_c \cos 2\pi (f_c \pm f_m)$$

Note: **SSB AM 信号的带宽为 *W*.**

SSB AM

$$f_c = 200Hz, m(t) = cos(20\pi t)$$



3.2.3 Single-Sideband AM

- **SSB AM 信号的解调:相干或同步解调.**

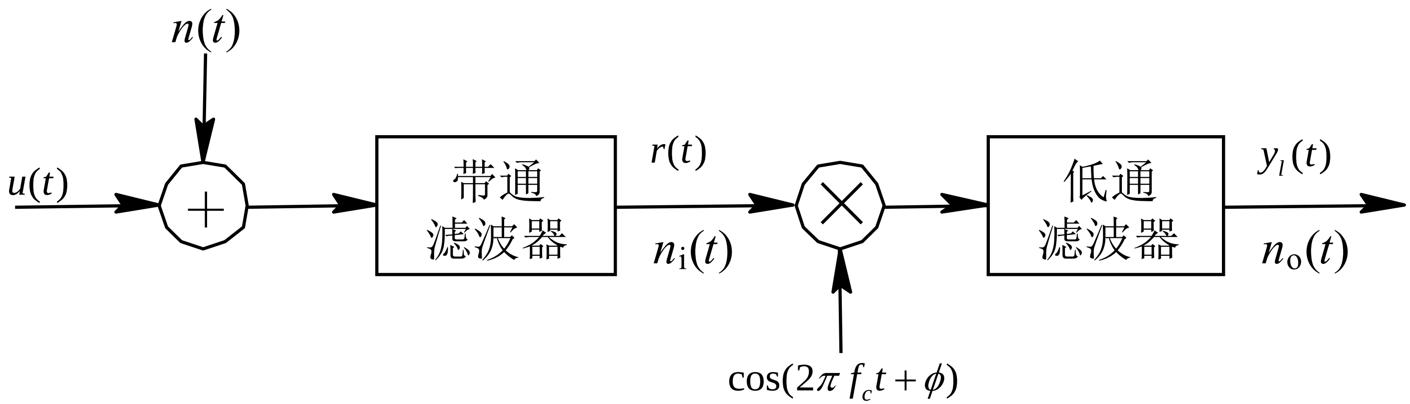


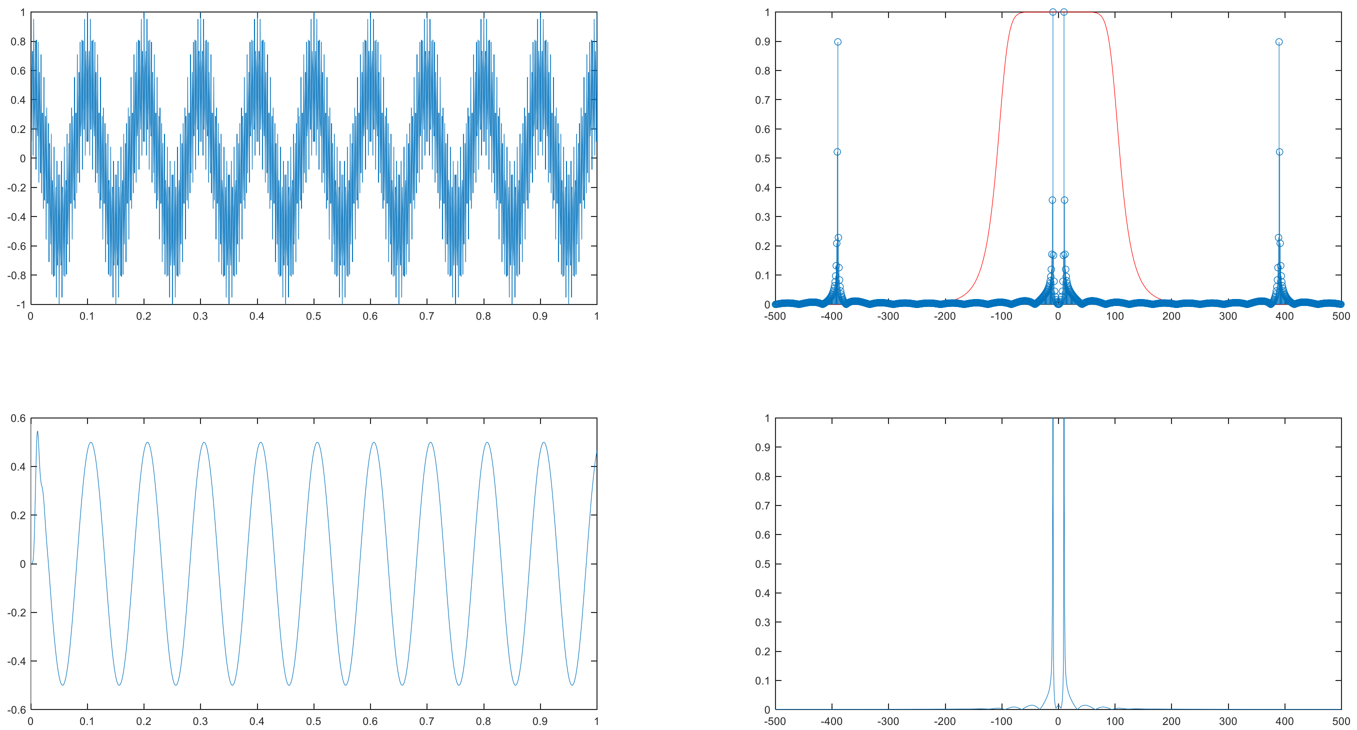
Fig. 3.2.10 Demodulator of SSB-AM

$$r(t)\cos(2\pi f_c t+\boldsymbol{\phi}) = u_{LSSB}(t)\cos(2\pi f_c t+\boldsymbol{\phi})$$
$$= [A_c m(t)\cos 2\pi f_c t + A_c\hat{m}(t)\sin 2\pi f_c t]\cos(2\pi f_c t+\boldsymbol{\phi})$$
$$= \frac{1}{2}A_c m(t)\cos\boldsymbol{\phi} + \frac{1}{2}A_c\hat{m}(t)\sin\boldsymbol{\phi} + \frac{1}{2}A_c m(t)\cos(4\pi f_c t+\boldsymbol{\phi})$$
$$+ \frac{1}{2}A_c\hat{m}(t)\sin(4\pi f_c t+\boldsymbol{\phi})$$

$$y_l(t) = \frac{1}{2}A_c m(t)\cos\boldsymbol{\phi} + \frac{1}{2}A_c\hat{m}(t)\sin\boldsymbol{\phi}$$

Φ=0非常重要，若Φ≠0，会引入正交干扰

SSB AM


1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
1 0.9 0.8 0.7 0.6 0.5 0.4 0.3 0.2 0.1 0
-500 -400 -300 -200 -100 0 100 200 300 400 500
0.6 0.4 0.2 0 -0.2 -0.4 -0.6
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
1 0.9 0.8 0.7 0.6 0.5 0.4 0.3 0.2 0.1 0
-500 -400 -300 -200 -100 0 100 200 300 400 500

3.2.4 Vestigial-sideband AM (残余边带调幅, VSB AM)

SSB对边带滤波器的频率响应严格。如何克服?

放松对边带滤波器的要求，允许上边带中混入少许下边带的频谱，或者下边带中混入少许上边带频谱。

代价：已调信号带宽和功率都稍微增加。

3.2.4 Vestigial-Sideband AM (残余边带调制)

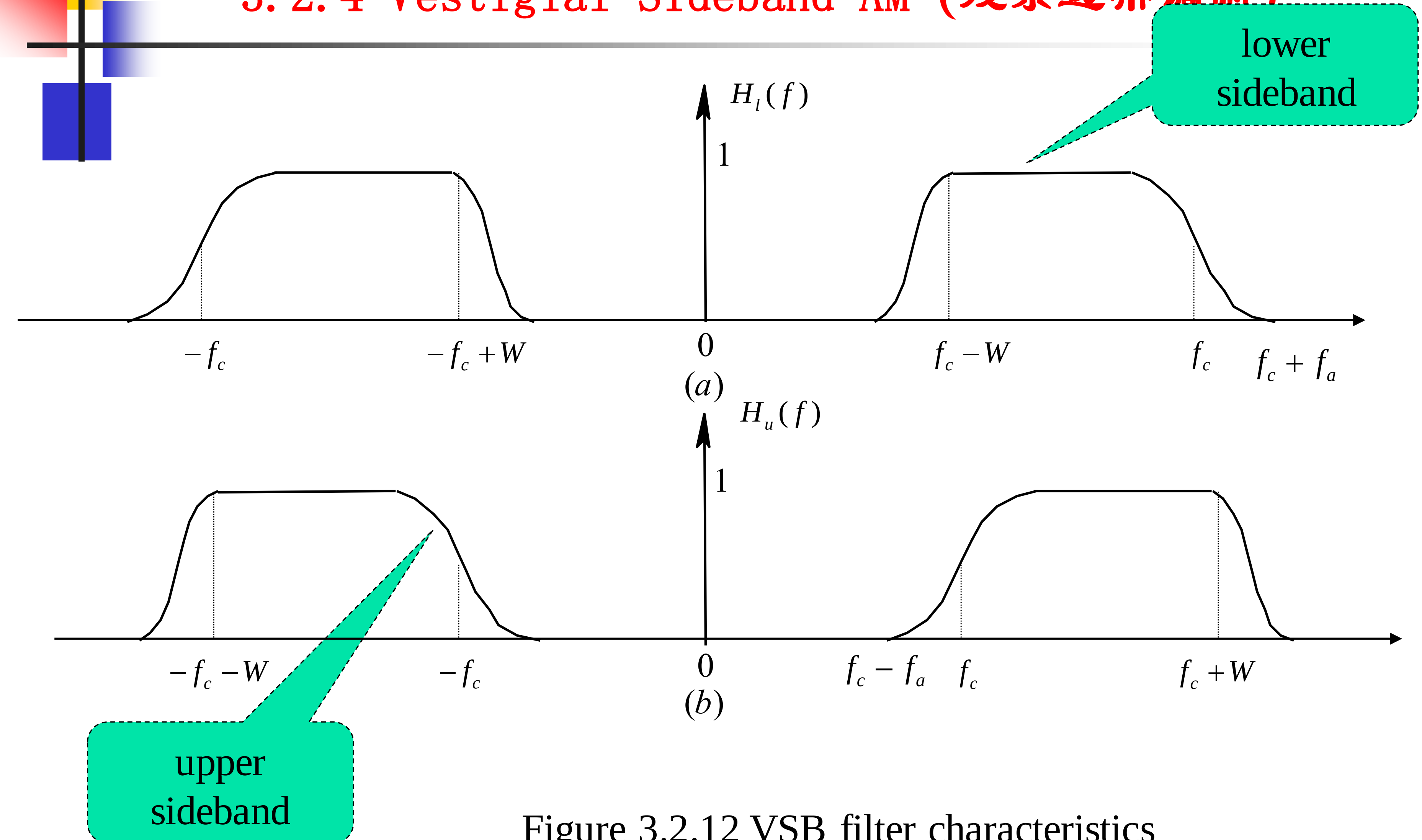
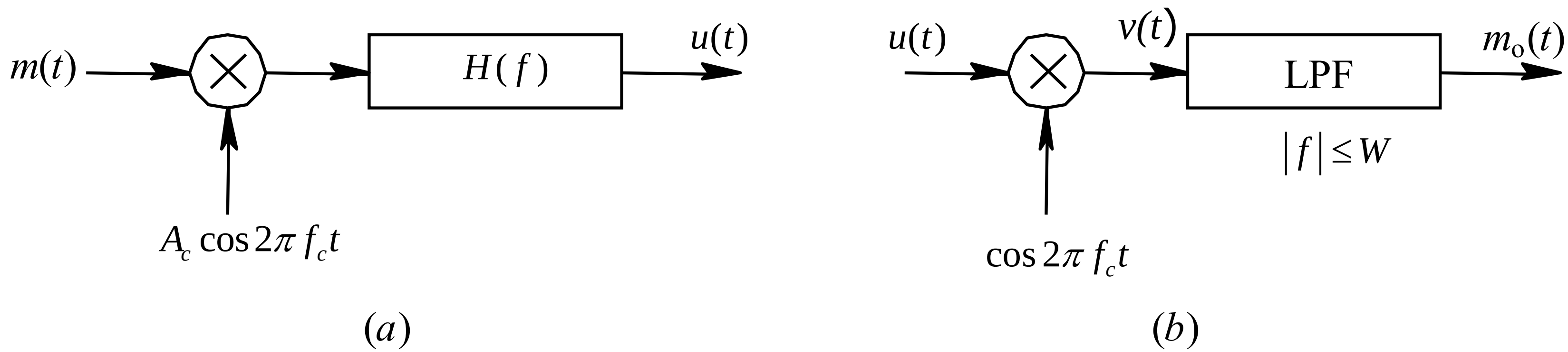


Figure 3.2.12 VSB filter characteristics

The bandwidth of VSB AM signals is a letter large than *W*.

3.2.4 Vestigial-sideband AM (残余边带调幅, VSB AM)



VSB 调制　　　　**VSB 解调**

Fig. 3.2.11 VSB-AM

3.2.4 Vestigial-sideband AM

对$H(f)$的要求

$$u(t)=[A_c m(t)\cos 2\pi f_c t]*h(t)$$

$$U(f)=\frac{A_c}{2}[M(f+f_c)+M(f-f_c)]H(f)$$

$$v(t)=u(t)\cos 2\pi f_c t$$

$$V(f)=\frac{1}{2}[U(f+f_c)+U(f-f_c)]$$

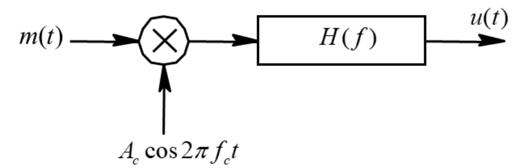
$$=\frac{A_c}{4}[M(f+2f_c)+M(f)]H(f+f_c)$$

$$+\frac{A_c}{4}[M(f)+M(f-2f_c)]H(f-f_c)$$

低通滤波后,得到的$m_o(t)$,必须满足

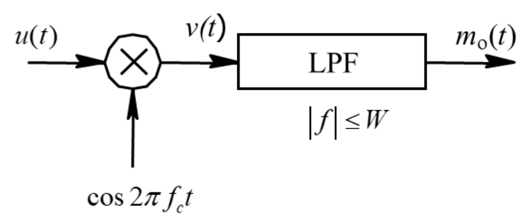
$$m_o(t)=am(t)$$

其中a是常数.



(a)

VSB 调制



(b)

VSB 解调

3.2.4 Vestigial-Sideband AM

Thus,

$$M_o(f)=\frac{A_c}{4}M(f)\left[H(f+f_c)+H(f-f_c)\right]$$

$$\left[H(f+f_c)+H(f-f_c)\right]=\cos\tan t \qquad |f|\le W$$

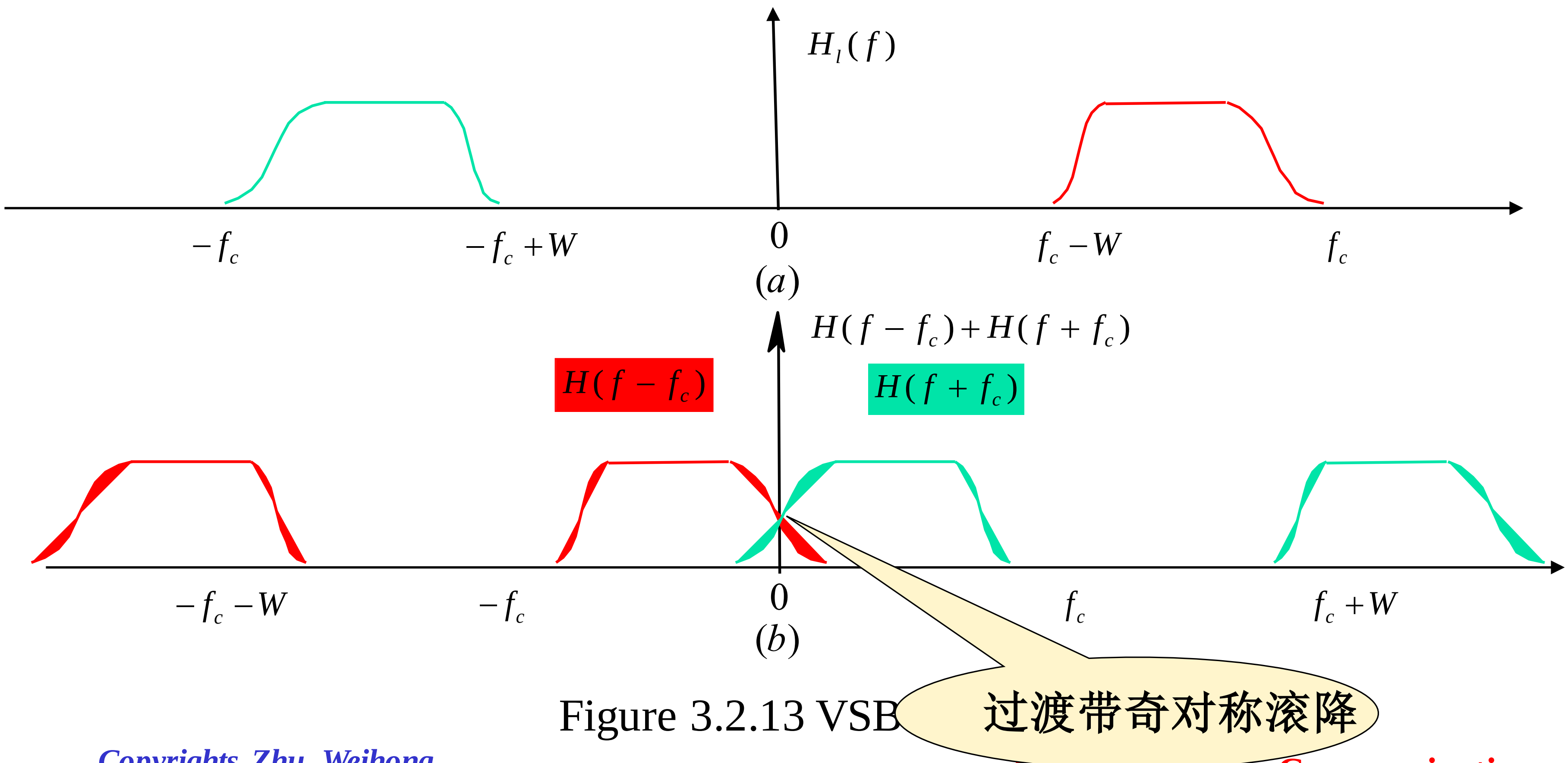


Figure 3.2.13 VSB

3.2.5 AM 信号调制与解调器的实现
（自学，或者高频电路课程学习）

Modulators

1. **Power-Law Modulation (conventional AM)**
2. **Switching Modulation(conventional AM)**
3. **Balanced Modulation (DSB-SC AM)**
4. **Ring Modulator (DSB-SC AM)**

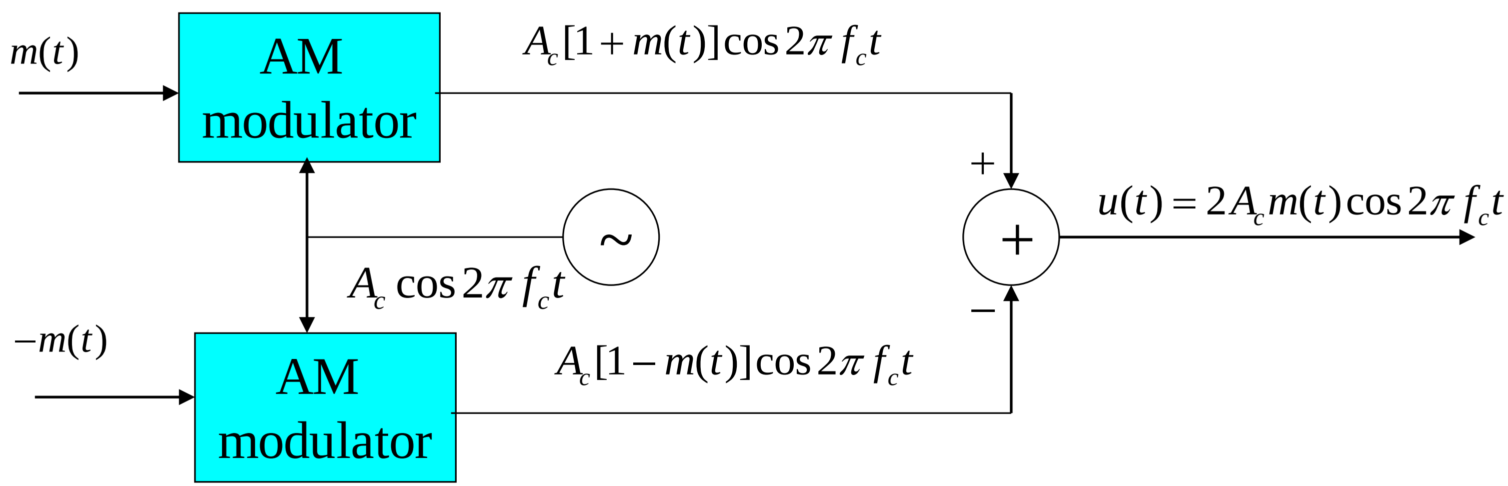


Figure 3.2.14 Balanced Modulator

3.2.5 Implementation of AM Modulators and Demodulator

- **Demodulator**

1. **Envelope Detector**
2. **Demodulation of DSB-SC AM signals (pp93, Fig3.21)**
3. **Demodulation of SSB Signals(pp93, Fig3.22)**
4. **Demodulation of VSB signals**

3.2.6 信号多路复用signal Multiplexing

- Multiplexing (复用). 将许多独立的消息信号合并成一个组合信号,以便在一条公共信道上传输的过程称为多路复用.

TDM (时分复用). **Time-division multiplexing**

FDM(频分复用). **Frequency-division multiplexing.**

3.2.6 signal Multiplexing

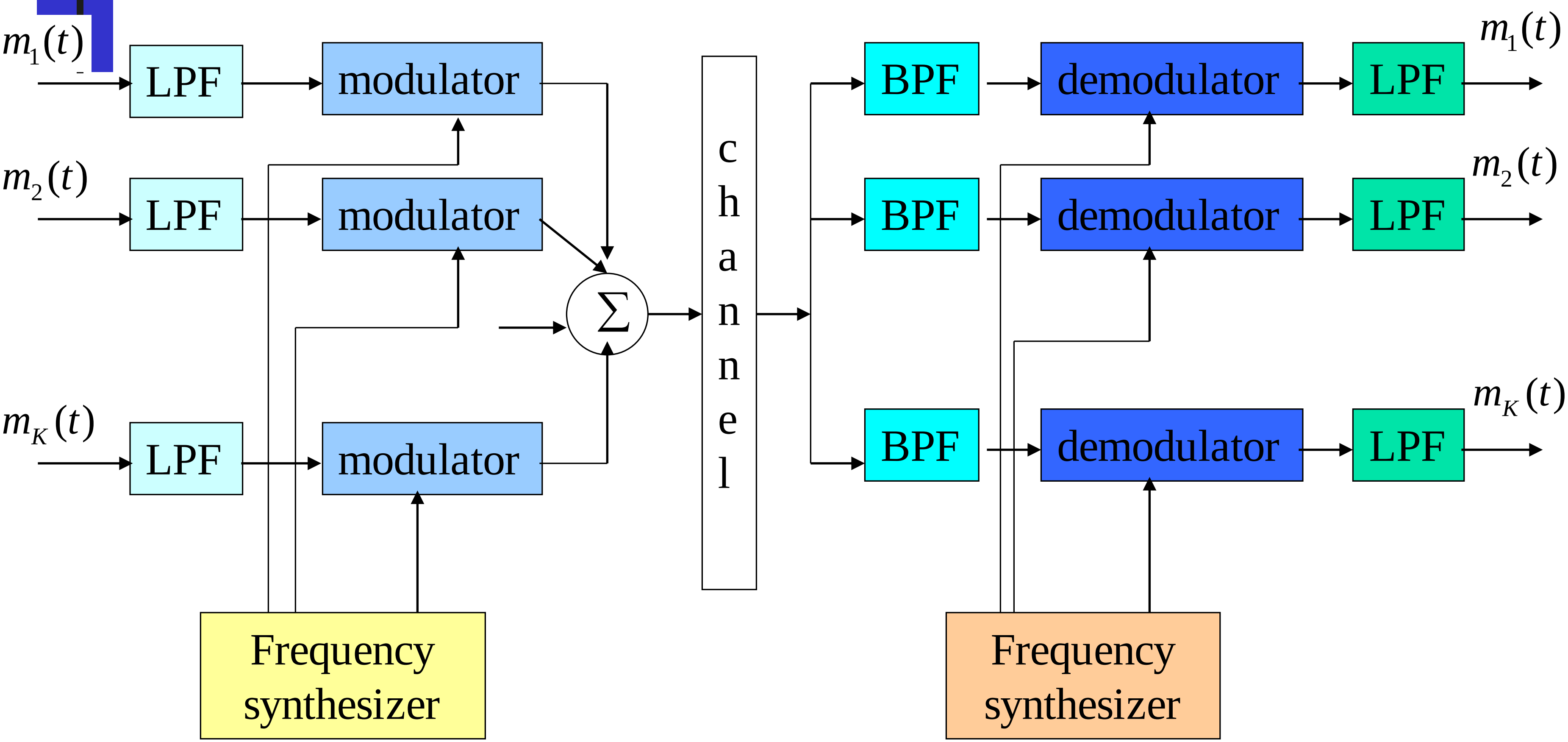
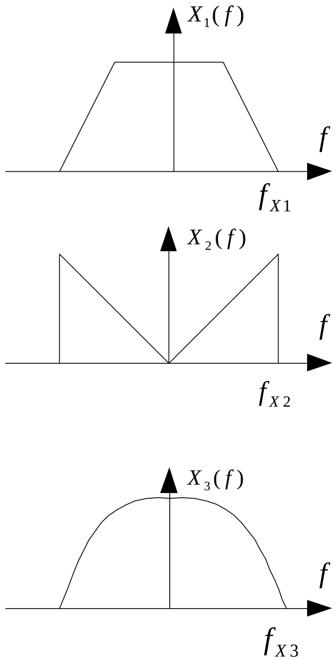
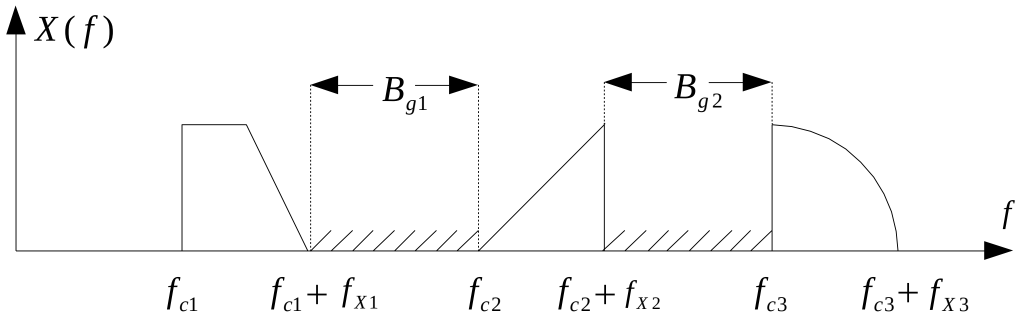


Figure 3.2.15 FDM

3.2.6 signal Multiplexing





B is the guard band

Fig. 3.2.16 Spectrum of FDM signal

FDM 信号的带宽: $W_{FDM}=nW+(n-1)B$

填空题 1分

设置

我们希望用SSB和FDM来传输12路语音信号，已知每一路语音信号的频带范围为（0,3.4k）Hz。现要求每路信号之间的保护间隔为0.6kHz,问合成的FDM信号带宽是 [填空1] Hz?

正常使用填空题需3.0以上版本雨课堂

作答

3.2.6 signal Multiplexing

思考：这种实现FDM的方式适合大规模频分复用吗？还有没有其他的实现方法？

习题3.42是其中一种。

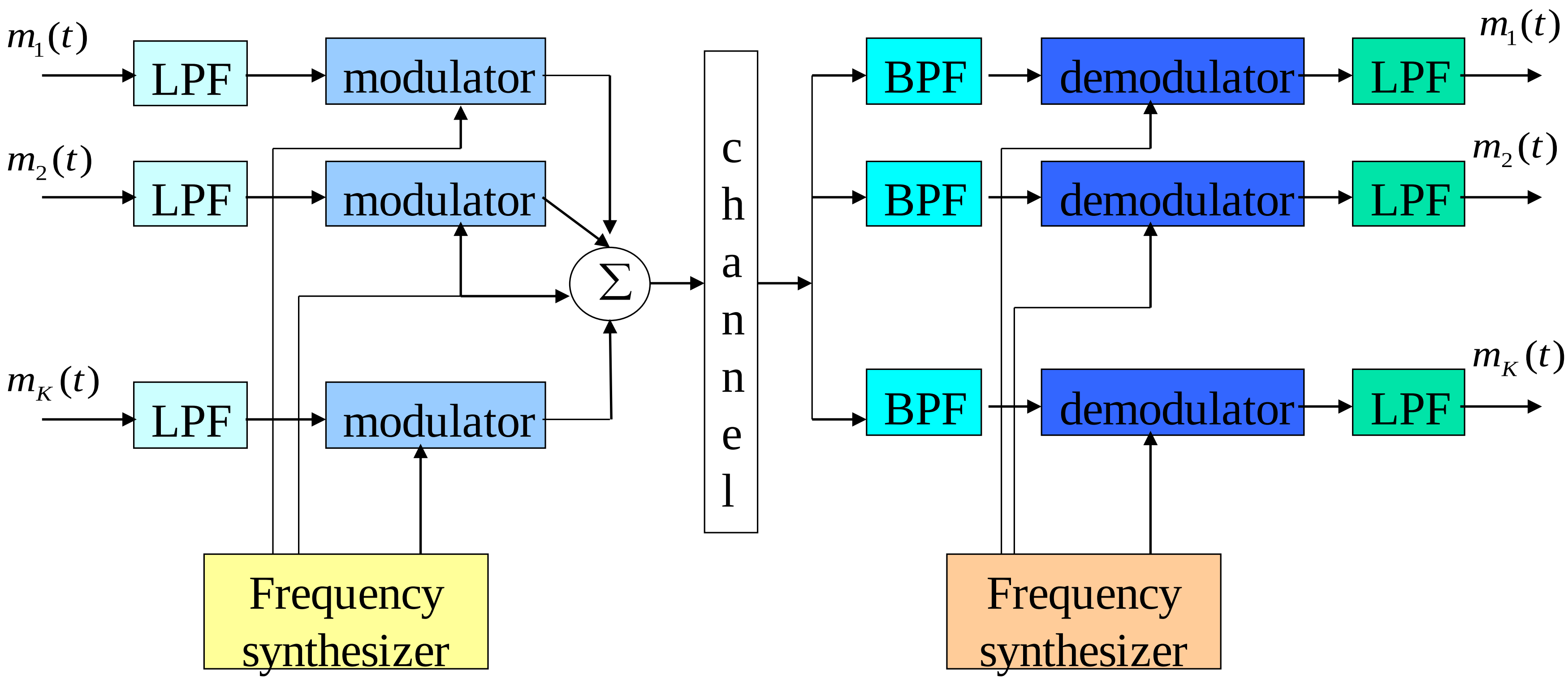


Figure 3.2.15 FDM

3.2.6 signal Multiplexing

- 载波正交多路复用.

利用两个正交载波$A_c cos2\pi f_c t$和$A_c sin2\pi f_c t$，在同一个载频上传送两路消息信号.

$$u(t) = A_c m_1(t)\cos 2\pi f_c t + A_c m_2(t)\sin 2\pi f_c t$$

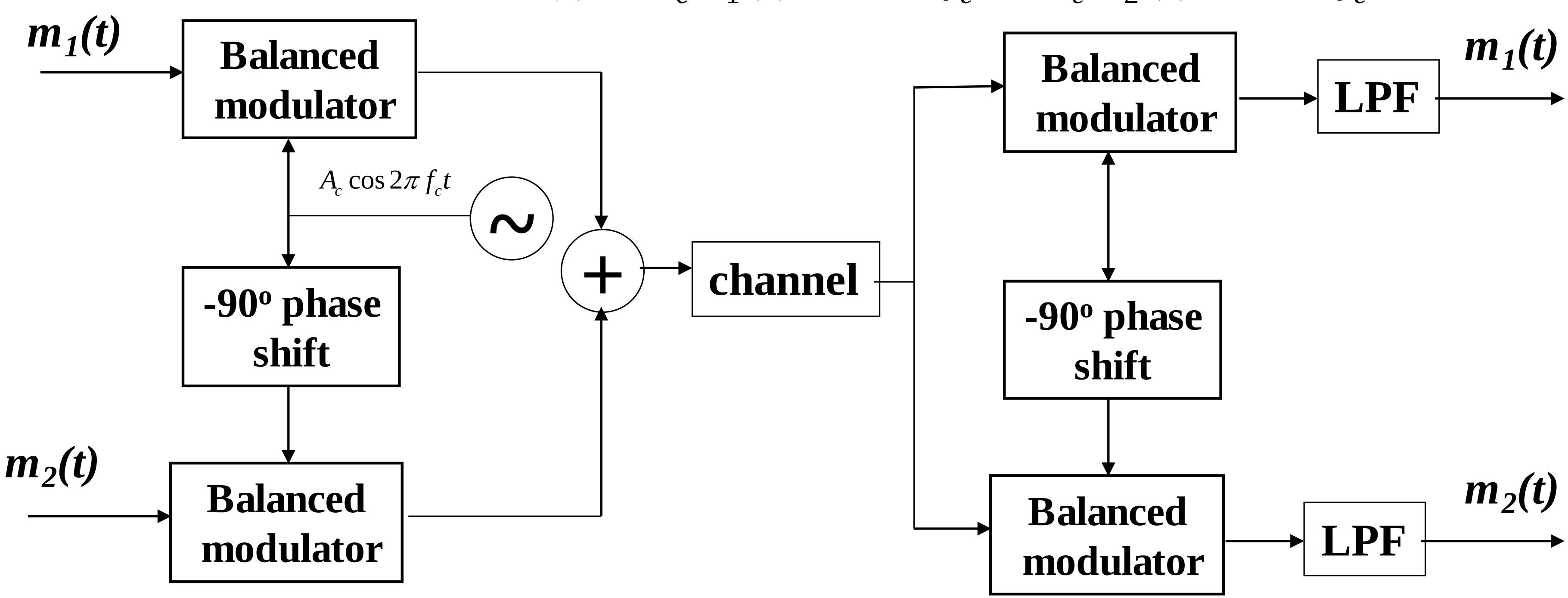


Figure 3.2.17 Quadrature-Carrier Multiplexing

Summarizing

- **Introduction to modulation**
- **Amplitude modulation (DSB-SC AM, Conventional AM, SSB AM, VSB AM)**

问题1: AM 调制一般称为线性调制，它们是线性变换吗?

答：不是。

问题2：请按信号带宽从大到小的顺序排列AM信号。

答： DSB-SC AM =Conventional AM> VSB AM > SSB AM.

3.3 角度调制 (Angle Modulation)

目的：用带宽来换取较高的抗噪声性能。

- **3.3.1 FM信号和PM信号的表示形式**

角度已调信号通常表示如下

$$u(t)=A_c\cos(\theta(t)).$$

其中$\theta(t)$ 是信号的相位，其瞬时频率 $f_i(t)$ 由下式给出

$$f_i(t)=\frac{1}{2\pi}\frac{d}{dt}\theta(t)$$

已调信号 $u(t)$ 是带通信号，它可以表示为

$$u(t)=A_c\cos(2\pi f_c t+\Phi(t))$$

故

$$f_i(t)=f_c+\frac{1}{2\pi}\frac{d}{dt}\boldsymbol{\phi}(t)$$

3.3.1 FM信号和PM信号的表示形式

若 *m(t)* 是消息信号，在 相位调制（PM）系统中

$$\Phi(t)=k_p m(t);$$

在频率调制（FM）系统中，瞬时频率偏移

$$\Delta f = f_i(t) - f_c = k_f m(t) = \frac{1}{2\pi}\frac{d}{dt}\phi(t)$$

So

$$\phi(t) = \begin{cases} k_p m(t), & PM \\ 2\pi k_f \int_{-\infty}^{t} m(\tau)d\tau, & FM \end{cases}$$

K_p 和 K_f 分别是相位偏移常数和频率偏移常数

3.3 Angle Modulation

$$u(t) = A_c cos\left(2\pi f_c t + \Phi(t)\right)$$

$$= \begin{cases} A_c cos(2\pi f_c t + k_p m(t)), & PM \\ A_c cos\left(2\pi f_c t + 2\pi k_f \int_{-\infty}^{t} m(\tau) d\tau, \right), & FM \end{cases}$$

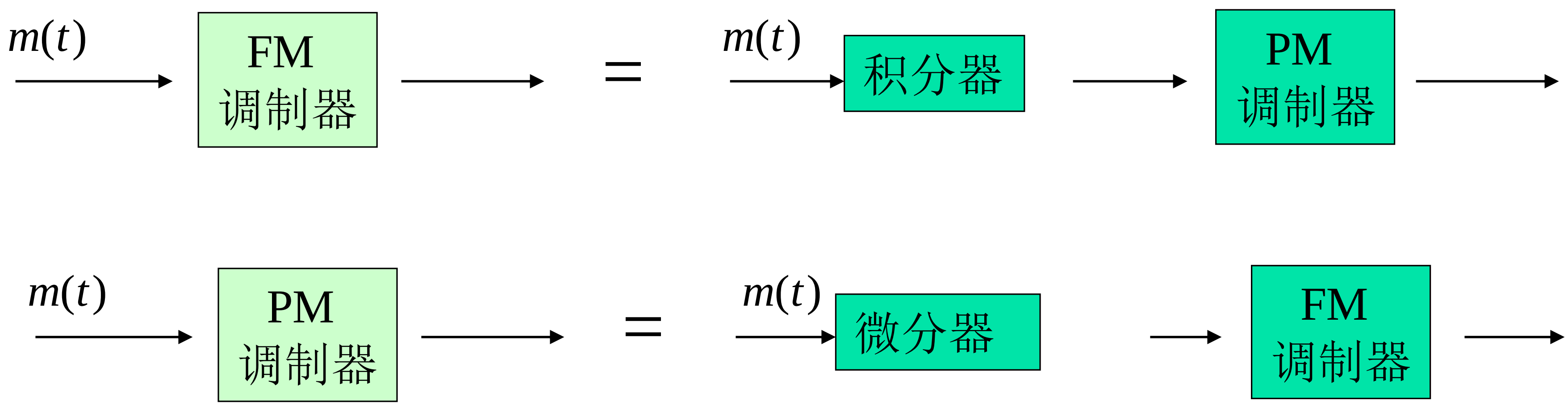


Figure 3.3.1 FM和PM调制器的比较

3.3.1 FM信号和PM信号的表示形式

1. 定义PM系统的最大相位偏移为

$$\Delta\phi_{\max} = k_p \max[|m(t)|]$$

2. 定义FM系统的最大频率偏移为

$$\Delta f_{\max} = k_f \max[|m(t)|]$$

3. 定义信号 ***m(t)*** 的调制指数为

$$\beta_p = k_p \max[|m(t)|] = \Delta\phi_{\max}$$

$$\beta_f = \frac{k_f \max[|m(t)|]}{W} = \frac{\Delta f_{\max}}{W}$$

***W*----信号 *m(t)*的带宽.**

3.3.1 FM信号和PM信号的表示形式

- 例3.3.1 消息信号：

$m(t)=a\cos(2\pi f_m t)$

试确定其PM和FM已调信号的形式。

解：

在PM调制中：$\Phi(t)=k_p m(t)=k_p\, a\cos(2\pi f_m t)$

所以PM已调信号

$u(t)=A_c\cos(2\pi f_c t+\beta_p\cos(2\pi f_m t))$，其中$\beta_p=k_p\, a$

3.3.1 FM信号和PM信号的表示形式

在FM调制中：

$$\phi(t) = 2\pi k_f \int_{-\infty}^{t} m(\tau)\,d\tau$$

$$= 2\pi k_f \int_{-\infty}^{t} a\cos(2\pi f_m \tau)d\tau$$

$$= \frac{ak_f}{f_m}\sin(2\pi f_m t)$$

所以FM已调信号为：

$$u(t) = A_c cos(2\pi f_c t + \beta_f sin(2\pi f_m t))$$

其中

$$\beta_f = ak_f / f_m$$

3.3.1 FM信号和PM信号的表示形式

窄带角度调制.

如果角度调制系统中，偏移常数 k_p 和 k_f 以及消息信号 $m(t)$ 满足条件：对所有的t，都有 $\Phi(t)$ 远小于 1，则

$$u(t) = A_c \cos\phi(t)\cos 2\pi f_c t - A_c \sin\phi(t)\sin 2\pi f_c t$$

$$\approx A_c \cos 2\pi f_c t - A_c\phi(t)\sin 2\pi f_c t$$

$$u_{AM}(t) = Ac\big(1 + m(t)\big)cos2\pi f_c t$$

窄带角度调制≈常规AM 信号 $B_N=2W$

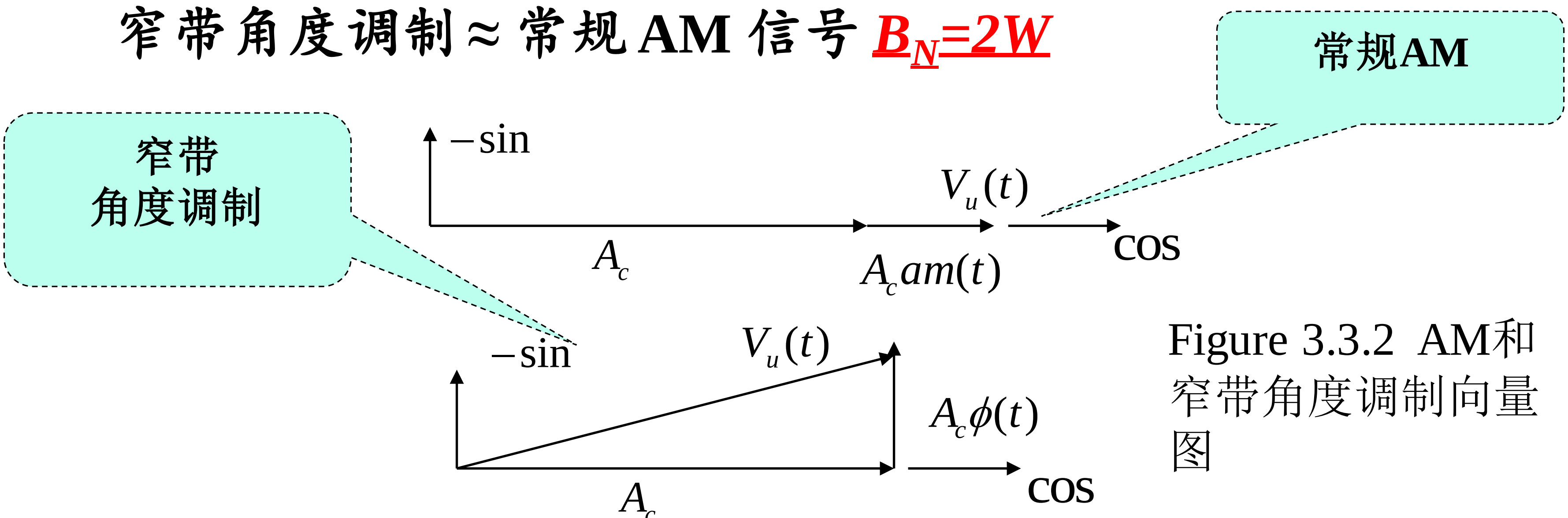


Figure 3.3.2 AM和窄带角度调制向量图

3.3.2 角度调制信号（宽带）的频谱特性

1. 正弦信号的角度调制.

$$m(t) = a\cos 2\pi f_m t$$

In PM, we have

$$\phi(t) = k_p m(t) = k_p a\cos 2\pi f_m t = \beta_p \cos 2\pi f_m t$$

In FM, we have

$$\phi(t) = 2\pi k_f \int_{-\infty}^{t} m(\tau)d\tau = \frac{k_f a}{f_m}\sin 2\pi f_c t = \beta_f \sin 2\pi f_m t$$

So we have

$$u(t) = \begin{cases} A_c\cos(2\pi f_c t + \beta_p\cos(2\pi f_m t)), & PM \\ A_c\cos(2\pi f_c t + \beta_f\sin(2\pi f_m t)), & FM \end{cases}$$

3.3.2 角度调制信号的频谱特性

以 FM 信号为例(PM signal is same as FM), 有

$$u(t)=A_c\cos(2\pi f_c t+\beta\sin 2\pi f_m t)=\mathrm{Re}\left[A_c e^{j2\pi f_c t}e^{j\beta\sin 2\pi f_m t}\right]$$

$e^{j\beta\sin 2\pi f_m t}$ 是周期函数，可以展成傅立叶级数的形式，其傅立叶级数的系数为

$$c_n=f_m\int_0^{\frac{1}{f_m}}e^{j\beta\sin 2\pi f_m t}e^{-jn2\pi f_m t}dt\;\underline{\underline{\;u=2\pi f_m t\;}}\;\frac{1}{2\pi}\int_0^{2\pi}e^{j(\beta\sin u-nu)}du=J_n(\beta)$$

so

$$e^{j\beta\sin 2\pi f_m t}=\sum_{n=-\infty}^{\infty}J_n(\beta)e^{j2\pi n f_m t}$$

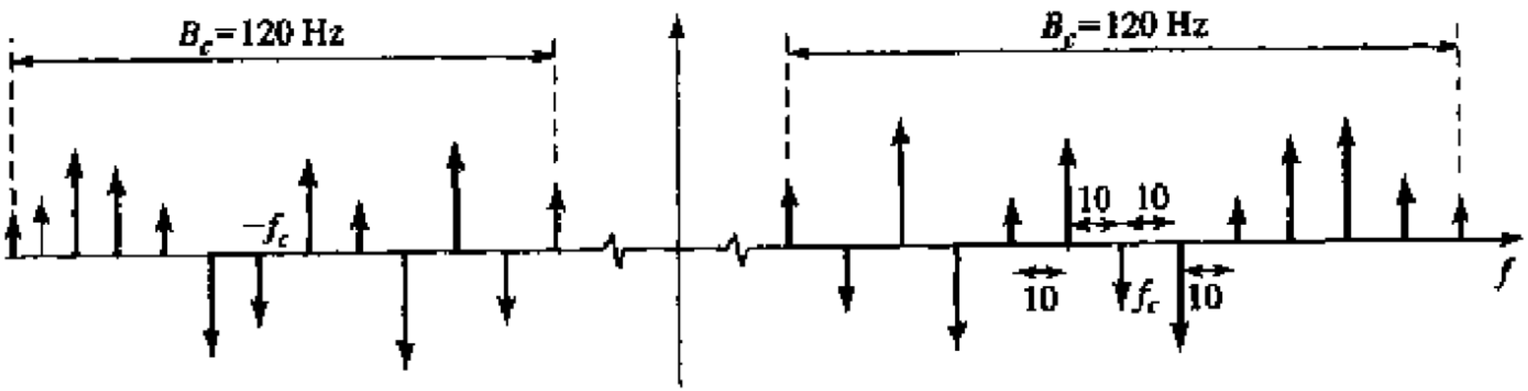
$$u(t)=\mathrm{Re}\left(A_c\sum_{n=-\infty}^{\infty}J_n(\beta)e^{j2\pi n f_m t}e^{j2\pi f_c t}\right)=\sum_{n=-\infty}^{\infty}A_c J_n(\beta)\cos(2\pi(f_c+nf_m)t)$$

3.3.2 角度调制信号的频谱特性

- $J_n(\beta)$ 是第一类n阶贝塞尔（Bessel）函数

$$J_n(\beta)=\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{\beta}{2}\right)^{n+2k}}{k!(k+n)!}\approx\frac{\beta^n}{2^n n!},\quad J_{-n}(\beta)=\begin{cases}J_n(\beta), & n\quad even\\ -J_n(\beta), & n\quad odd\end{cases}$$

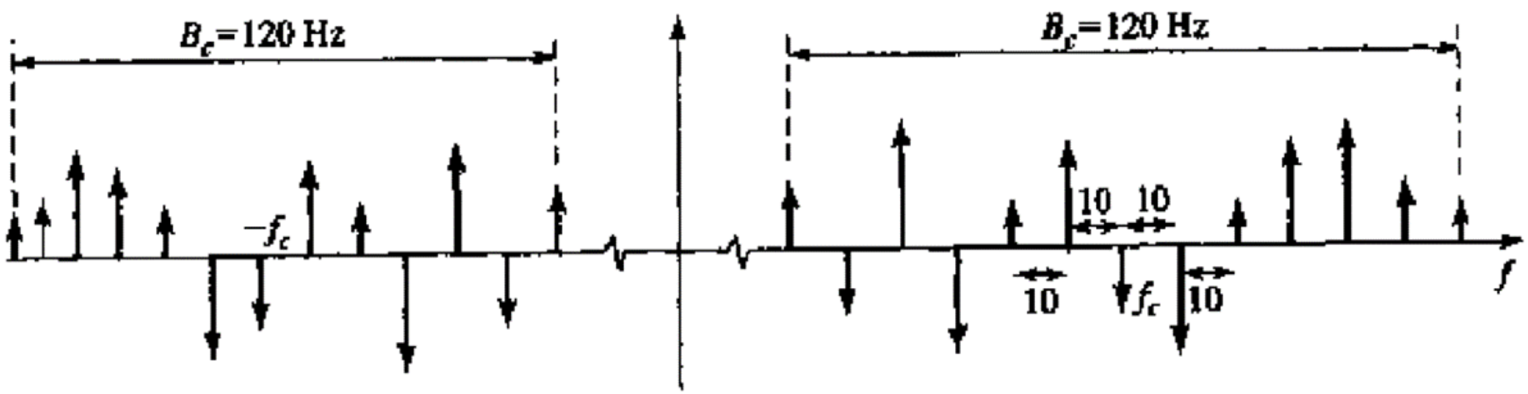
下图为$k_f=50, f_m=10Hz$时的调频信号频谱。可见单频信号经过FM之后，产生了一些列频率分量。



3.3.2 角度调制信号的频谱特性

一般情况下，已调角信号的有效带宽应包含至少98% 的信号功率，因此单频已调角信号的带宽由下式给出

$$B_c = 2(\beta + 1)f_m = \begin{cases} 2(k_p a + 1)f_m = 2(k_p a + 1)f_m & PM \\ 2(\dfrac{k_f a}{f_m} + 1)f_m = 2(k_f a + f_m) & FM \end{cases}$$



3.3.2 角度调制信号的频谱特性

- 将带宽内的谐波数量用M_c来表示，则

$$M_c = 2\lfloor\beta\rfloor + 3 = \begin{cases} 2\lfloor k_p a \rfloor + 3 & PM \\ 2\left\lfloor \dfrac{k_f a}{f_m} \right\rfloor + 3, & FM \end{cases}$$

1. 带宽与谐波数量紧密相关。相邻谐波间隔f_m。
2. a增加，带宽增加。
3. FM信号的带宽对消息信号的频率的敏感度较低
4. FM, $f_m\uparrow$, $M_c\downarrow$, 谐波之间的间隔↑，B_c略微增加.
5. PM, $f_m\uparrow$, M_c 不变,谐波之间的间隔↑，B_c线性增大.

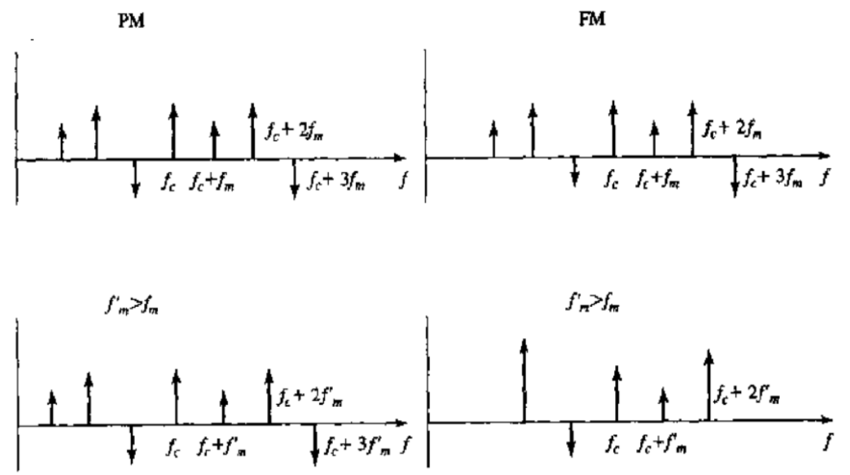


图 3.30 在 FM 和 PM 中增加消息带宽的影响

3.3.2 角度调制信号的频谱特性

- 非正弦信号的角度调制
- Carson's rule (卡森公式)

$$B_c = 2(\beta + 1)W$$

W 是消息信号 *m(t)* 的带宽，且定义

$$\beta = \begin{cases} k_p \max\left[\left|m(t)\right|\right], & PM \\ \dfrac{k_f \max\left[\left|m(t)\right|\right]}{W}, & FM \end{cases}$$

角度调制信号的带宽远大于其它各种振幅调制信号的带宽。

主观题 10分

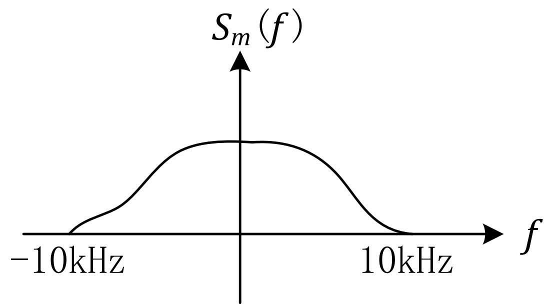
设置

信号$m(t)$的功率谱$S_m(f)$如下图所示，信号从A点发送到B点。已知信号已经归一化，即$-1 \leq m(t) \leq 1$。

1、如果采用USSB调制，已调信号带宽是多少?

2、如果采用$\alpha = 0.8$的AM，已调信号带宽是多少?

3、如果采用$k_f = 60kHz$的FM，已调信号带宽是多少?



正常使用主观题需2.0以上版本雨课堂

作答

- 解：

1、USSB信号带宽为10kHz;

2、$\alpha=0.8$的AM信号带宽为20kHz;

3、$k_f=60kHz$的FM信号

$$\beta_{\mathrm{FM}}=k_f\frac{max|m(t)|}{W}=60k\frac{1}{10k}=6$$

$$B_c=2(\beta_{\mathrm{FM}}+1)\mathrm{W}=140\mathrm{kHz}$$

3.3.3 角度调制器和解调器的实现
（自学或在高频电路课程中学习）

调制器

直接法：压控振荡器-变容二极管或电抗管.

间接法：窄带调制→宽带调制.

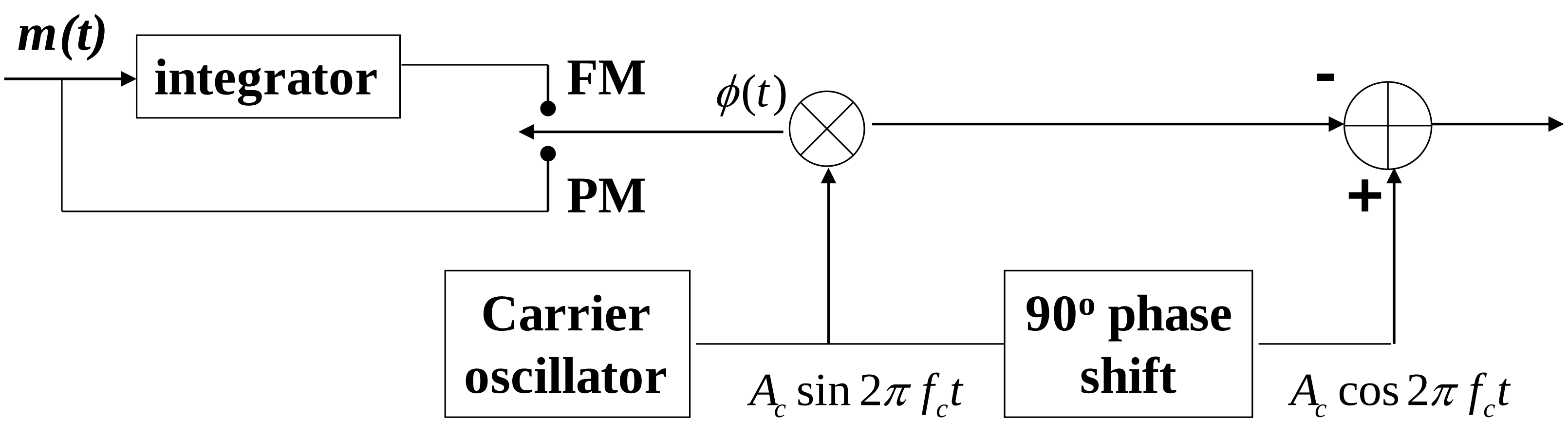


Figure 3.3.3 窄带调制信号的产生

3.3.3 角度调制器和解调器的实现

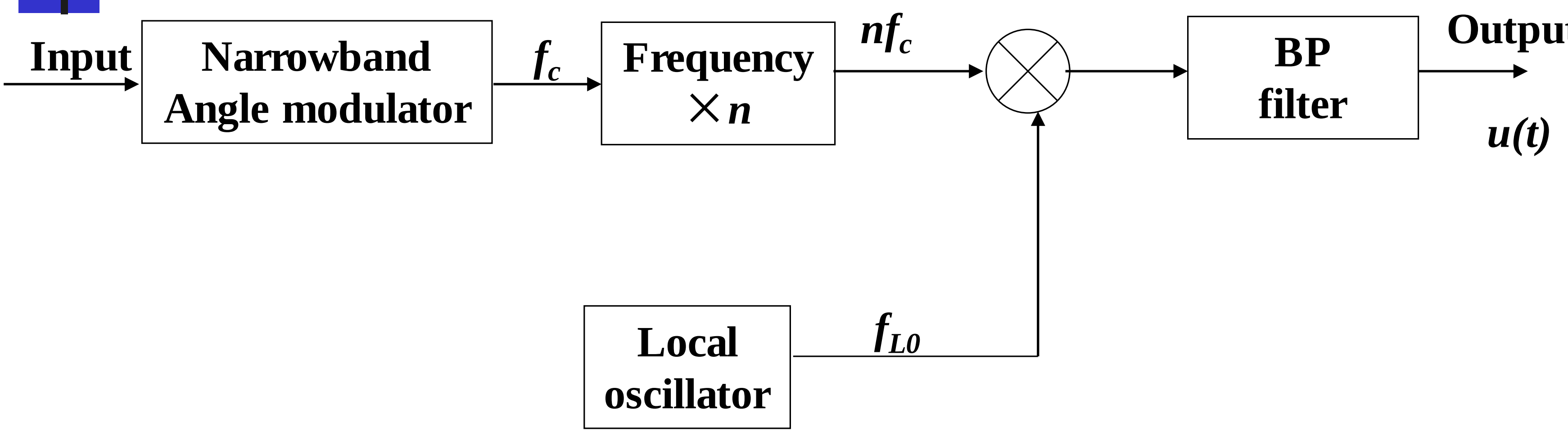


Figure 3.3.4 角度调制信号的间接法产生

$$u(t) = A_c \cos\left(2\pi(nf_c - f_{L0})t + n\phi(t)\right)$$

3.3.3 角度调制器和解调器的实现

解调器

Differentiator(微分器)
or balance discriminator (鉴频器)

1. Discriminator (鉴频器):

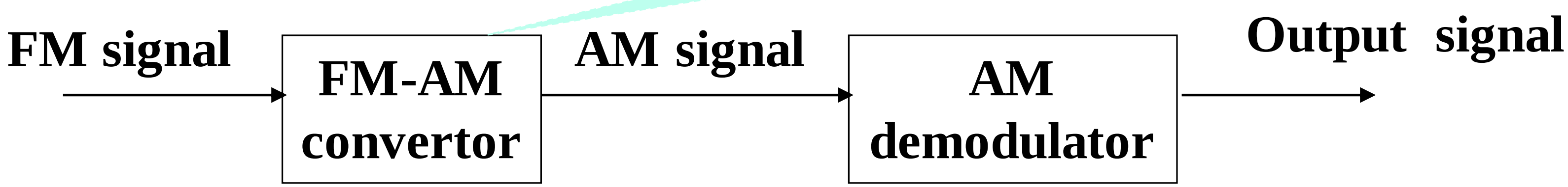


Figure 3.3.5 FM解调器

2. FM demodulator with feedback (FMFB，调频负反馈)

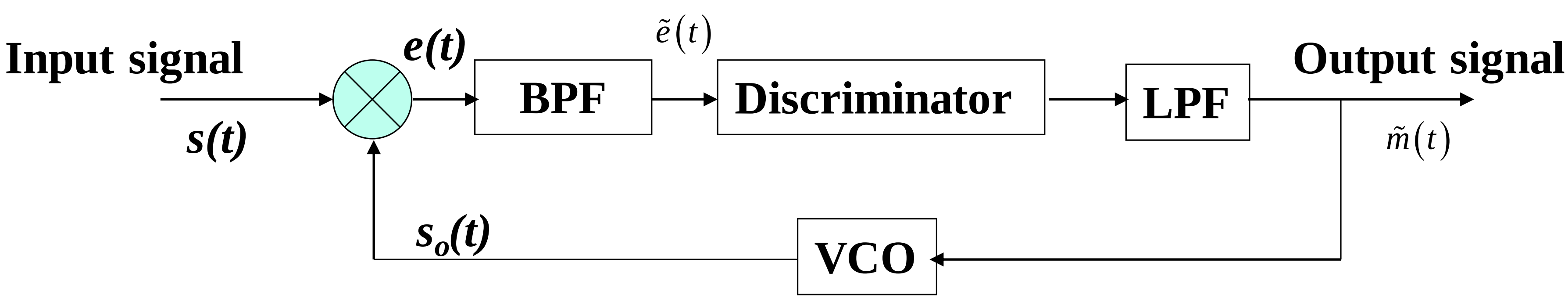


Figure 3.3.6 FMFB 解调器方框图

3.3 Angle Modulation

输入信号为

$$s(t)=A_c\cos\left(2\pi f_c t+\varphi(t)\right)=A_c\cos\left(2\pi f_c t+2\pi k_f\int_{-\infty}^{t}m(\tau)d\tau\right)$$

VCO 的输出信号为

$$s_o(t)=A_o\cos\left(2\pi f_o t+\varphi_o(t)\right)=A_o\cos\left(2\pi f_o t+2\pi k_v\int_{-\infty}^{t}\tilde{m}(\tau)d\tau\right)$$

则

$$\begin{aligned}e(t)&=A_cA_o\cos\left(2\pi f_c t+\varphi(t)\right)\cos\left(2\pi f_o t+\varphi_o(t)\right)\\&=\frac{1}{2}A_cA_o\cos\left(2\pi(f_c-f_o)t+\varphi(t)-\varphi_o(t)\right)\\&+\frac{1}{2}A_cA_o\cos\left(2\pi(f_c+f_o)t+\varphi(t)+\varphi_o(t)\right)\end{aligned}$$

3.3 Angle Modulation

假设 BPF 的中心频率为 f_c-f_o, 则

$$\tilde{e}(t)=\frac{1}{2}A_cA\cos\left(2\pi(f_c-f_o)t+\varphi(t)-\varphi_o(t)\right)$$

鉴频器测的输出为瞬时频率偏移，故输出信号为误差信号相位偏移的微分，所以

$$\tilde{m}(t)=\frac{1}{2\pi}\frac{d\left[\varphi(t)-\varphi_o(t)\right]}{dt}$$

$$=\frac{1}{2\pi}\frac{d\left[\left(\int_{-\infty}^{t}2\pi k_f m(\tau)-2\pi k_v\tilde{m}(\tau)\right)d\tau\right]}{dt}$$

$$=k_f m(t)-k_v\tilde{m}(t)$$

$$\tilde{m}(t)=\frac{k_f}{1+k_v}m(t)$$

3.3 Angle Modulation

- 同时还有

$$\tilde{e}(t)=\frac{1}{2}A_cA\cos\left[2\pi(f_c-f_o)t+\left(\frac{1}{1+k_v}\right)2\pi k_f\int_{-\infty}^{t}m(\tau)d\tau\right]$$

是一个调制指数为 [1/(1+k_v)]的调频信号.

由于 k_v>>1, 上述信号是一个窄带调频信号.

3. PLL-FM demodulator (工作原理自学，见pp89—90)

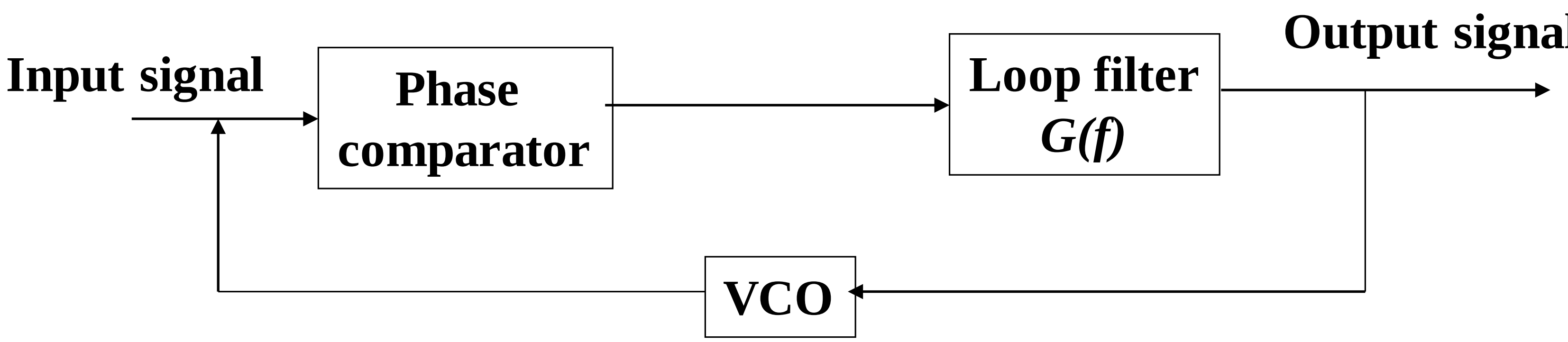


Figure 3.3.7 Block diagram of PLL-FM demodulator

3.4 无线电广播和电视广播（自学或高频电路课程中学习）

3.4.1 Superheterodyne AM receiver (超外差接收机)

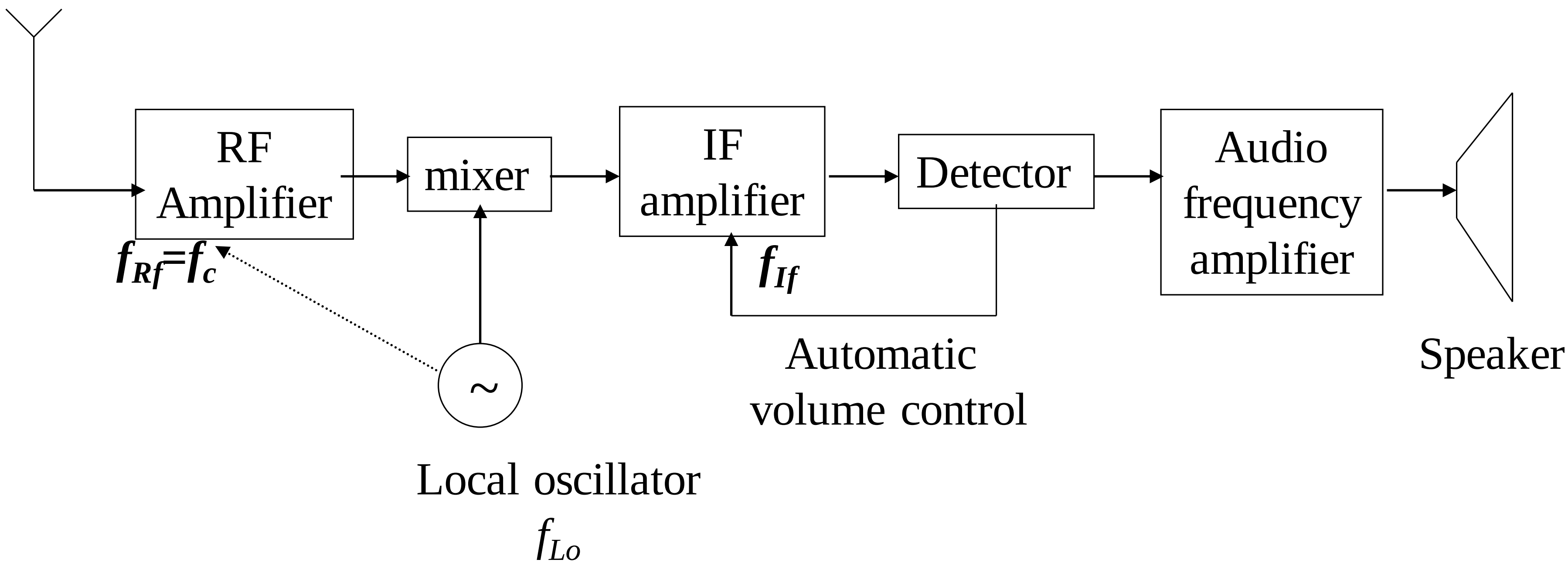


Figure 3.4.1 超外差 AM 接收机

$$f_{LO} = f_{Rf} + f_{If} = f_c + f_{If}$$

3.4 Radio and Television Broadcasting

f_{RF}=535~1605kHz, f_{IF}=455kHz where is chosen to be below the lowest transmitted signal（<535kHz） but high enough to avoid radio frequency interference from image signals, whose frequency is $f_{image}=f_{LO}+f_{If}$

The function of the mixer (混频器) is as following:

$$u_o(t) = u_i(t)\cos 2\pi f_{LO}t = m(t)\cos 2\pi f_c t \cos 2\pi f_{LO}t$$

$$= \frac{1}{2}m(t)[\cos 2\pi f_{IF}t + \cos 2\pi(2f_c + f_{IF})t]$$

3.4 Radio and Television Broadcasting

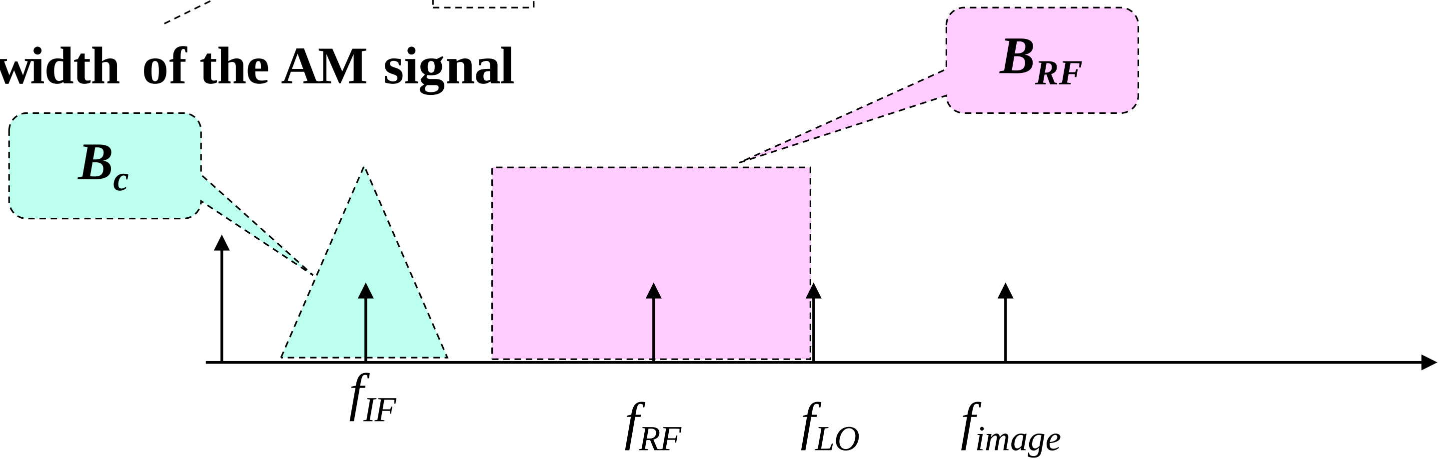
$$u_o(t) = u_i(t)\cos 2\pi f_{LO}t = m(t)\cos 2\pi f_{image}t \cos 2\pi f_{LO}t$$

$$= \frac{1}{2}m(t)[\cos 2\pi f_{IF}t + \cos 2\pi(2f_c + 3f_{IF})t)$$

So the bandwidth of the RF amplifier to the range

$\boldsymbol{B_c < B_{RF} < 2f_{IF}}$

Bandwidth of the AM signal



3.4 Radio and Television Broadcasting

- **3.4.2 FM Radio Broadcasting (studying by yourself)**
- **3.4.3 Television Broadcasting (studying by yourself)**

Summarizing

- **Angle modulation**

Including: basic idea and implementation

问题: 角度调制信号的带宽是多少?

$$B_c = 2(\beta+1)W$$

Chapter 3

Thank you for your attention!

Any question?