Chapter 8 Optical Properties

1. **Light waves in a homogeneous medium**
2. **Refractive index**
3. **Dispersion: refractive index – wavelength behavior**
4. **Snell's law and total internal reflection (TIR)**
5. Fresnel's equation
6. Light absorption and scattering
7. Luminescence, phosphors, and white LED

   Tutorial

   Course Review & Exam Briefing

Light waves in a homogeneous medium

Light exhibits wave-like properties such as interference and diffraction. Light is an electromagnetic (EM) wave (电磁波) with time-varying electric and magnetic fields $\mathbf{E_x}$ and $\mathbf{B_y}$, respectively, which propagate through space in a way that they are perpendicular to each other and the direction of propagation **z**.

The simplest mathematical form of a monochromatic (单色) plane wave:

$$E_x = E_0 \cos(\omega t - kz + \phi_0)$$

$\mathbf{E_x}$: the electric field at position **z** and time **t**

$\mathbf{E_o}$: the **amplitude（幅值）** of the wave

ω: the **angular frequency (ω=2πv)**（角频率）

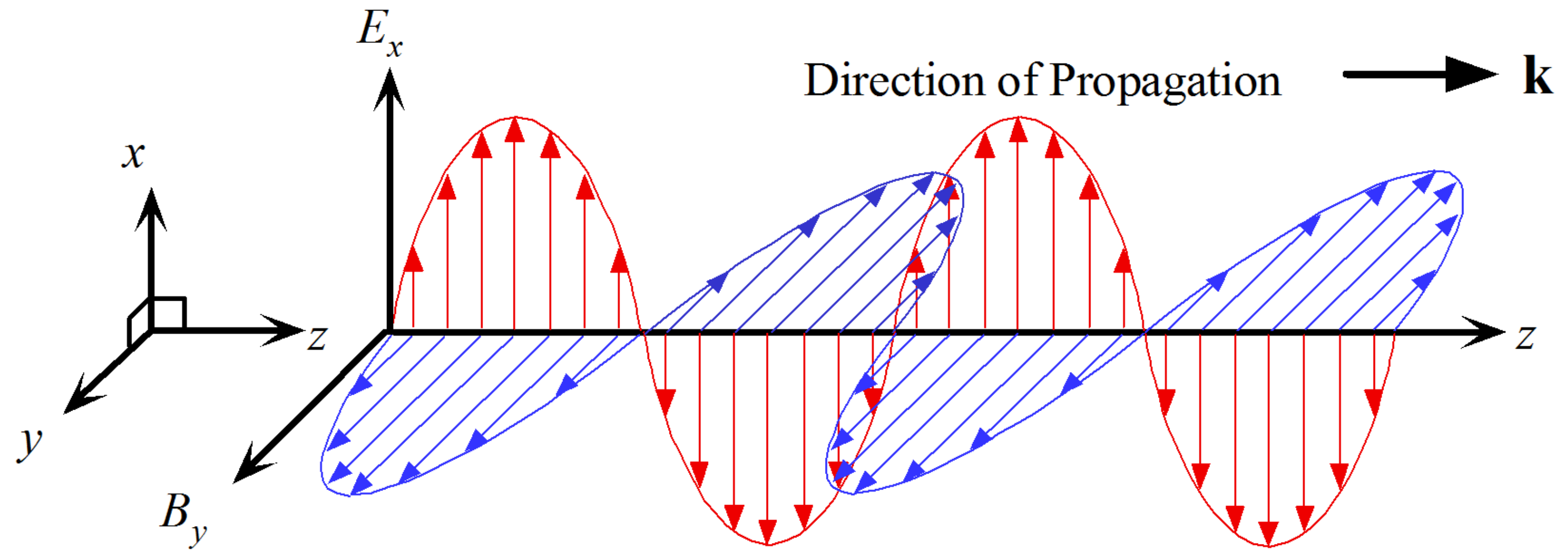
k: the **propagation constant**, **wavevector（波矢）**,

or **wavenumber (k = 2π/λ)** （波数）

(ωt – kz +ϕ_0): is called the phase, ϕ（相位）

$\mathbf{\Phi_0}$: a phase constant（相位常数）

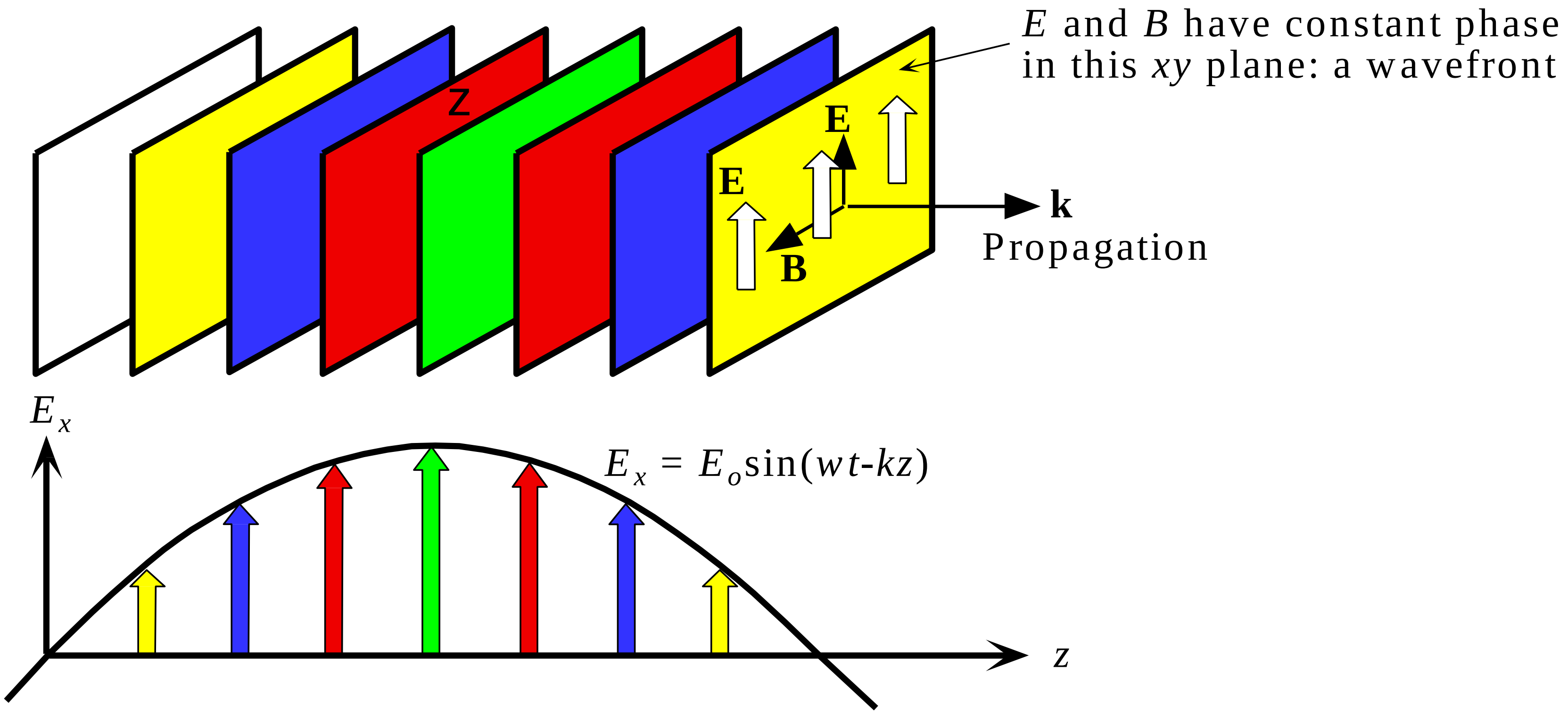
Light waves： EM wave



An electromagnetic wave is a travelling wave which has time varying electric and magnetic fields which are perpendicular to each other and the direction of propagation, z.

A traveling electric field $\mathbf{E}_x$ is accompanied by a traveling magnetic field $\mathbf{B}_y$ with the same wave frequency and propagation constant (**ω and k**).
We describe the interaction of a light wave with a matter through the electric field component $\mathbf{E}_x$ rather than $\mathbf{B}_y$ because it is the electric field that displaces the electrons in ions or molecules in the crystal. The **optical field (光学场)** refers to the electric field $\mathbf{E}_x$.

Wavefront（波前）



A plane EM wave travelling along z, has the same E_x (or B_y) at any point in a given xy plane. All electric field vectors in a given xy plane are therefore **in phase**.

Phase velocity （相速度）

The time and space evolution of a given phase ϕ, is described by

$$\phi = \omega t - kz + \phi_o = \text{constant}$$

During a time interval δt, this constant phase moves a distance δz.
The phase velocity of this wave is therefore $\delta z/\delta t$.
From $\phi = \omega t - kz + \phi_0$ = constant, we obtain: $\omega\,\delta t - k\,\delta z = 0$

The **phase velocity** （相速度）v is:

$$v = \frac{dz}{dt} = \frac{\omega}{k} = \upsilon\lambda$$

Refractive index（折射率）

EM traveling in a medium: the oscillating **electric field** polarizes the molecules of the medium at the frequency of the wave.

The field and the induced molecular dipoles become coupled. The polarization mechanism **delays** the propagation of the EM wave.

The stronger the interaction between the field and the dipoles, the slower is the propagation of the wave.

For an EM wave traveling in a non-magnetic medium, the phase velocity is given by:

$$v = \frac{1}{\sqrt{\varepsilon_r \varepsilon_0 \mu_0}}$$

$\varepsilon_0 = 8.8542 \times 10^{-12}\ CV^{-1}M^{-1}$ 真空介电常数

$\mu_0 = 4\pi \times 10^{-7}\ Hm^{-1}$ 真空磁导率

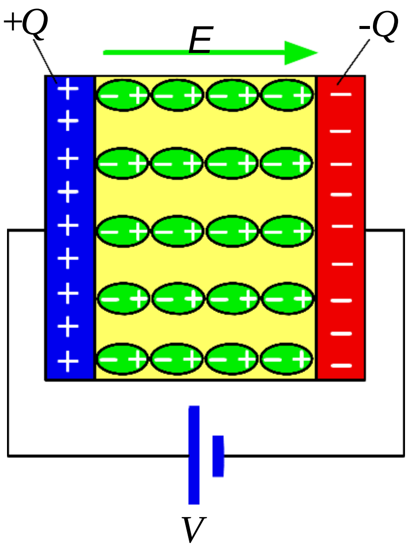
optical frequency range

In free space, $\varepsilon_r = 1$ and $v_{vacuum} = 1/\sqrt{(\varepsilon_0\mu_0)} = c = 3 \times 10^8$ m/s

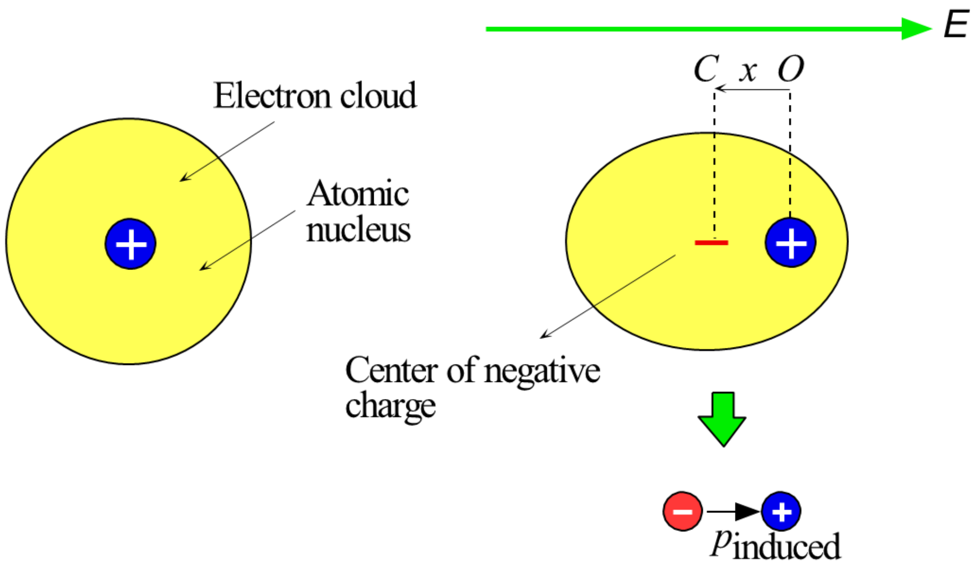
$$v = \frac{c}{\sqrt{\varepsilon_r}}$$

When a dielectric slab is inserted into the parallel plate capacitor, the charge on the electrodes increases from Q_0 to Q. The relative permittivity (or the dielectric constant) ε_r of the dielectric is defined as:

$$\varepsilon_r = \frac{Q}{Q_0} = \frac{C}{C_0}$$



The increase in the stored charge is due to the polarization of the dielectric by the applied field.



(a) A neutral atom in $E = 0$.

(b) Induced dipole moment in a field

The origin of electronic polarization.

The ratio of the speed of a light in free space to its speed in a medium is called the **refractive index** of the medium:

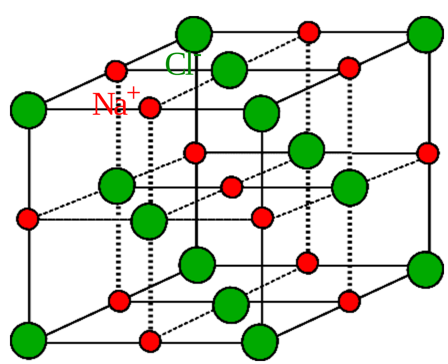
$$\boldsymbol{n} = \frac{c}{v} = \sqrt{\boldsymbol{\varepsilon}_{\boldsymbol{r}}}$$

Suppose that in free space k_0 is the wavevector ($2\pi/\lambda_0$), the wavevector k in the medium will be **nk_0** and **$\lambda=\lambda_0/n$**, if the medium is **isotropic** (各向同性).

$$n = \frac{k}{k_0}$$

For an anisotropic crystal, $\varepsilon_x \neq \varepsilon_y \neq \varepsilon_z$ in general, the refractive index n is **orientation dependent**.

Noncrystalline materials such as glasses and liquids, are optically isotropic.



The frequency dependence of the real and imaginary parts of the dielectric constant in the presence of interfacial, orientational, ionic and electronic polarization mechanisms.

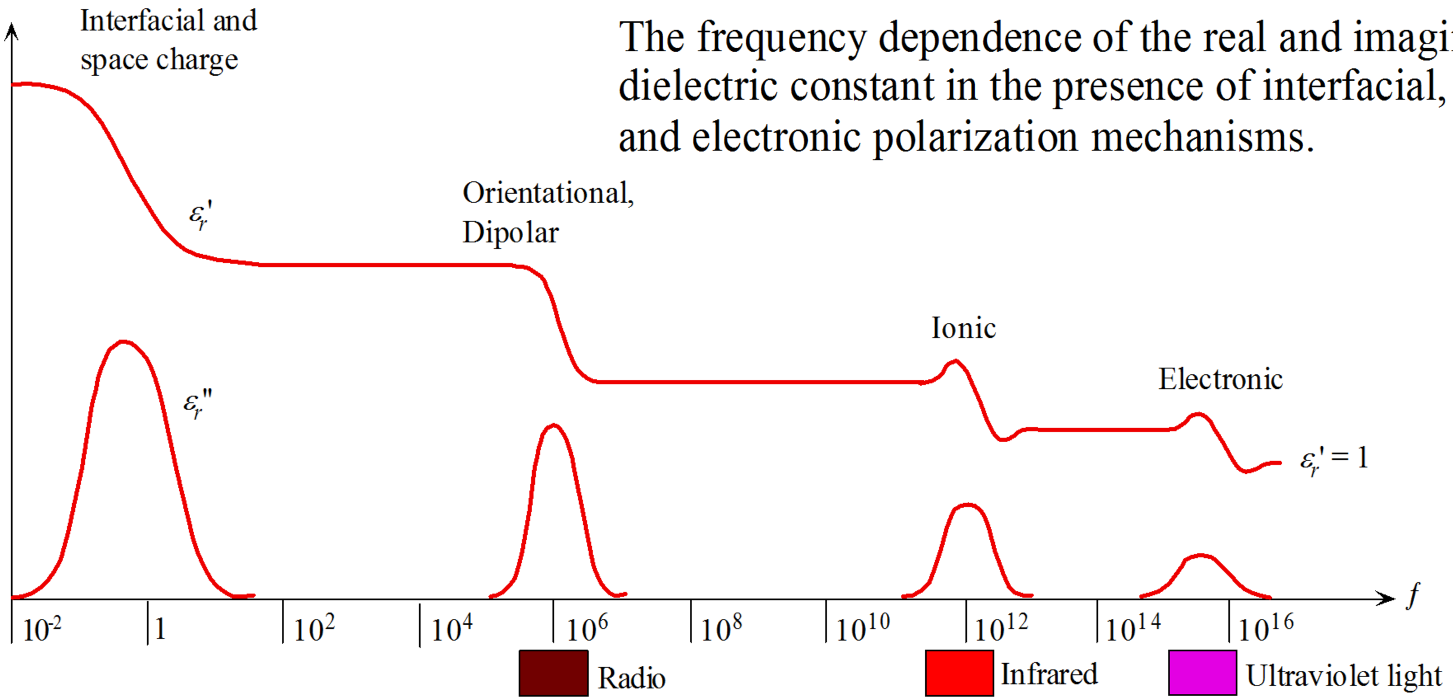
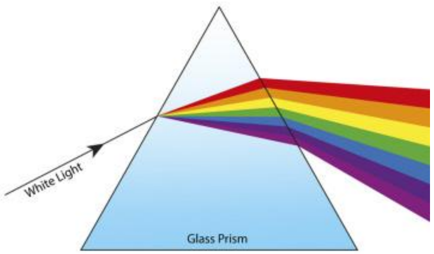


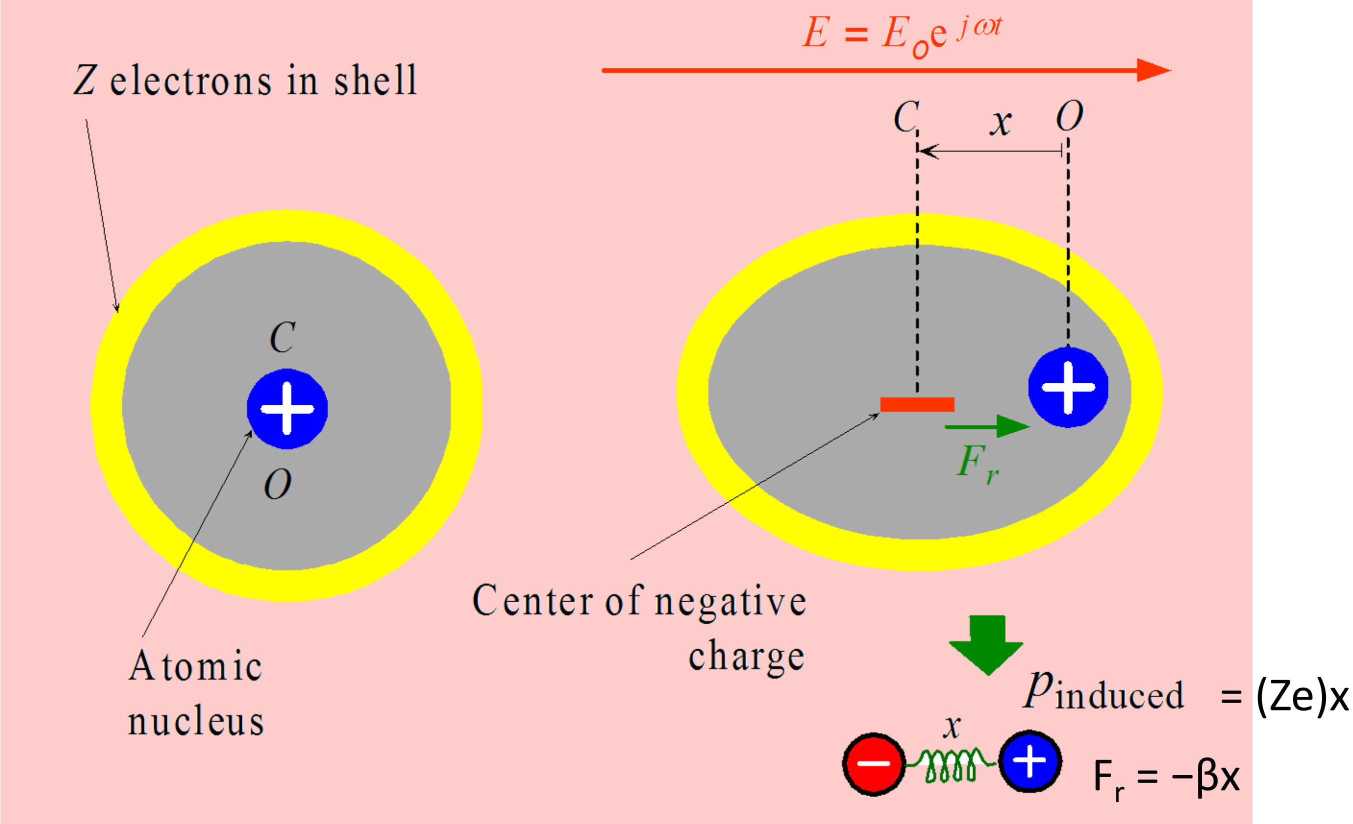
Table 9.1 Low-frequency (LF) relative permittivity ε_r(LF) and refractive index n

Material	ε_r (LF)	$\sqrt{\varepsilon_r(\mathrm{LF})}$	n (optical)	Comments
Diamond	5.7	2.39	2.41 (at 590 nm)	Electronic bond polarization up to UV light
Si	11.9	3.44	3.45 (at 2.15 μm)	Electronic bond polarization up to optical frequencies
AgCl	11.14	3.33	2.00 (at 1–2 μm)	Ionic polarization contributes to ε_r(LF)
SiO_2	3.84	2.00	1.46 (at 600 nm)	Ionic polarization contributes to ε_r(LF)
Water	80	8.9	1.33 (at 600 nm)	Dipolar polarization contributes to ε_r(LF), which is large

Dispersion（色散）: refractive index – wavelength behavior



The refractive index in general depends on the frequency, or the wavelength.



(a) A neutral atom in $E = 0$. (b) Induced dipole moment in a field

Electronic polarization of an atom

Dispersion relation（色散关系）: relationship between **n** and λ (or ω)

In equilibrium under a E field, the net force on the negative charge is zero or ZeE = βx:

$$p_{induced} = (Ze)x = \frac{Z^2 e^2}{\beta} E$$

When the E field is removed:
restoring force = mass × acceleration

$$-\beta x = Z m_e \frac{d^2 x}{dt^2}$$

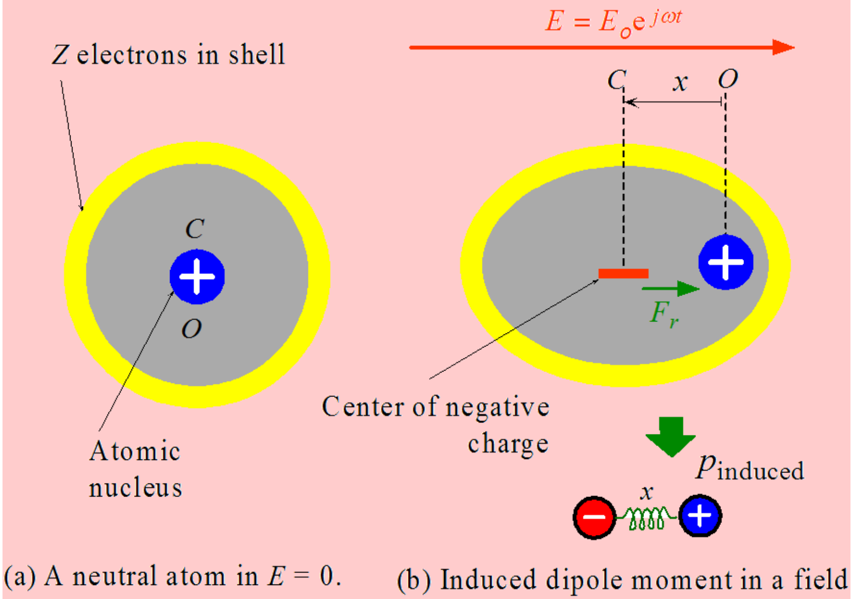
A simple harmonic motion:

$$x(t) = x_0 \cos(\omega_0 t)$$

The solution of the oscillation ω_0:

$$\omega_0 = \left(\frac{\beta}{Z m_e}\right)^{1/2}$$

ω_0 is called the resonance frequency



Electronic polarization of an atom

In equilibrium under a E field, the net force on the negative charge is zero or ZeE = βx. The induced electronic dipole:

$$p_{induced} = (Ze)x = \frac{Z^2 e^2}{\beta} E$$

When an ac field is applied $E = E_0 \exp(j\omega t)$:

$$Zm_e \frac{d^2 x}{dt^2} = -Ze \cdot E_0 \exp(j\omega t) - \beta x$$

The solution of the above equation:

$$x = x(t) = -\frac{eE_0 \exp(j\omega t)}{m_e(\omega_0^2 - \omega^2)}$$

The electronic polarizability（极化率）α_e:

$$\alpha_e = \frac{p_{induced}}{E} = \frac{Ze^2}{m_e(\omega_0^2 - \omega^2)}$$

$$\varepsilon_r = 1 + \frac{N}{\varepsilon_o}\alpha_e$$

From $n^2 = \varepsilon_r$ and $\lambda_0 = 2\pi c/\omega_0$:

Dispersion relation: relationship between n and λ (or ω)

$$n^2 = \varepsilon_r = 1 + \frac{N}{\varepsilon_0}\alpha_e = 1 + \left(\frac{NZe^2}{\varepsilon_0 m_e}\right)\frac{1}{\omega_0^2 - \omega^2} = 1 + \left(\frac{NZe^2}{\varepsilon_0 m_e}\right)\left(\frac{\lambda_0}{2\pi c}\right)^2 \frac{\lambda^2}{\lambda^2 - \lambda_0^2}$$

N is the number of atoms per unit volume

Dispersion relation（色散关系）

The relationship between **n and λ (or ω)** is called the **dispersion relation**.

In a solid, a series of **resonance frequencies** may exist, Sellmeier dispersion 塞梅尔色散：

$$n^2 = 1 + \frac{A_1\lambda^2}{\lambda^2 - \lambda_1^2} + \frac{A_2\lambda^2}{\lambda^2 - \lambda_2^2} + \frac{A_3\lambda^2}{\lambda^2 - \lambda_3^2}$$

Where A_1, A_2, A_3 and λ_1, λ_2 and λ_3 are constant, called Sellmeier coefficients.

There is another well-known useful **n-λ** dispersion by Cauchy (1836):

$$n^2 = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4}$$

Cauchy equation
柯西方程:

$$n = n_{-2}(h\upsilon)^{-2} + n_0 + n_2(h\upsilon)^2 + n_4(h\upsilon)^4$$

where hv is the photon energy, and n_0, n_{-2}, n_2, and n_4 are constants

Table 9.2 Sellmeier and Cauchy coefficients

	Sellmeier					
	A_1	A_2	A_3	λ_1 (μm)	λ_2 (μm)	λ_3 (μm)
SiO_2 (fused silica)	0.696749	0.408218	0.890815	0.0690660	0.115662	9.900559
86.5% SiO_2–13.5% GeO_2	0.711040	0.451885	0.704048	0.0642700	0.129408	9.425478
GeO_2	0.80686642	0.71815848	0.85416831	0.068972606	0.15396605	11.841931
Sapphire	1.023798	1.058264	5.280792	0.0614482	0.110700	17.92656
Diamond	0.3306	4.3356	—	0.1750	0.1060	

	Cauchy				
	Range of $h\nu$ (eV)	n_{-2} (eV^2)	n_0	n_2 (eV^{-2})	n_{-4} (eV^{-4})
Diamond	0.05–5.47	-1.07×10^{-5}	2.378	8.01×10^{-3}	1.04×10^{-4}
Silicon	0.002–1.08	-2.04×10^{-8}	3.4189	8.15×10^{-2}	1.25×10^{-2}
Germanium	0.002–0.75	-1.0×10^{-8}	4.003	2.2×10^{-1}	1.4×10^{-1}

SOURCE: Sellmeier coefficients combined from various sources. Cauchy coefficients from D. Y. Smith *et al.*, *J. Phys. CM* **13**, 3883, 2001.

Example (GaAs dispersion relation): *For GaAs, from λ = 0.89 to 4.1 μm, the refractive index is given as the following dispersion relation:*

$$n^2 = 7.10 + \frac{3.78\lambda^2}{\lambda^2 - 0.2767}$$

Where λ is in microns (μm). What is the refractive index of GaAs for light with a photon energy of 1 eV.

At hν = 1 eV:

$$\lambda = \frac{hc}{h\upsilon} = \frac{6.62x10^{-34} \cdot 3x10^{8}}{1x1.6x10^{-19}} = 1.24\mu m$$

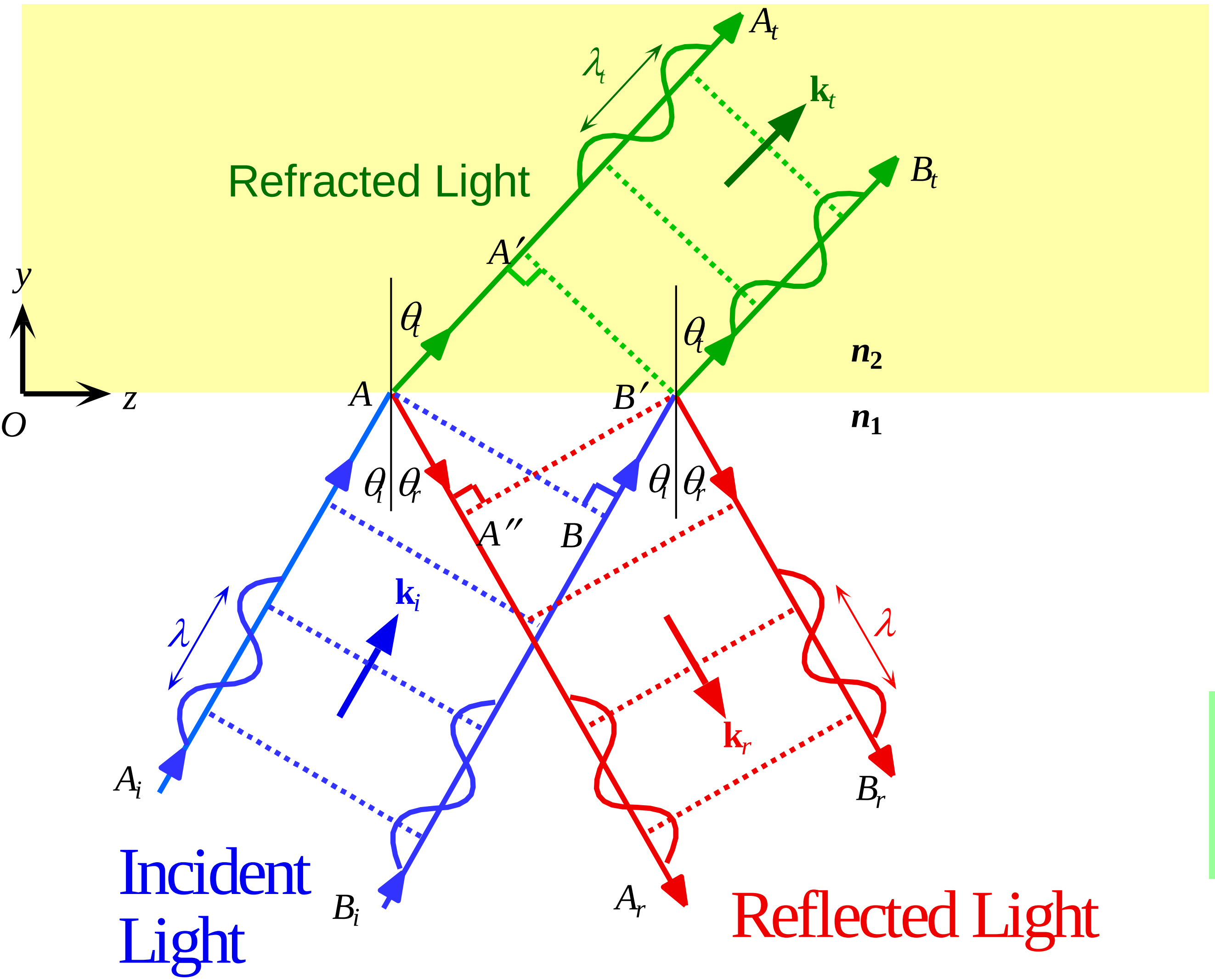
Thus

$$n^2 = 7.10 + \frac{3.78\lambda^2}{\lambda^2 - 0.2767} = 7.10 + \frac{3.78 \cdot (1.24)^2}{(1.24)^2 - 0.2767} = 11.71$$

So that

$$n = 3.42$$

Snell's law 斯涅尔定律 and total internal reflection (TIR)



A traveling plane EM wave in a medium (1) with the refractive index n_1 propagating toward a medium (2) with the refractive index n_2.

$$BB' = v_1 t = ct / n_1$$

$$AA' = v_2 t = ct / n_2$$

$\Rightarrow$

$$AB' = \frac{v_1 t}{\sin\theta_i} = \frac{v_2 t}{\sin\theta_t}$$

A light wave travelling in a medium with a greater refractive index $(n_1 > n_2)$ suffers reflection and refraction at the boundary.

Snell's law:

$$\frac{\sin\theta_i}{\sin\theta_t}=\frac{v_1}{v_2}=\frac{n_2}{n_1}$$

Consider the reflected wave: BB'=AA''=v_1t:

$$AB'=\frac{v_1 t}{\sin\theta_i}=\frac{v_1 t}{\sin\theta_r}$$

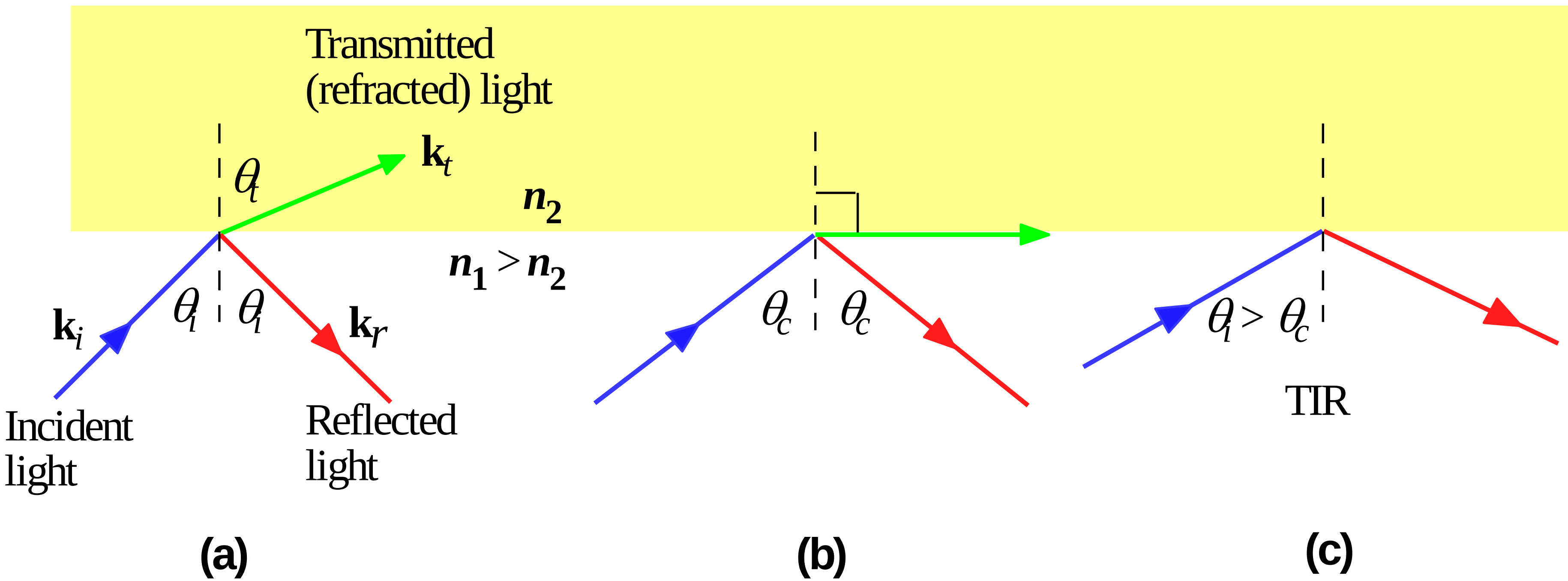
$\Rightarrow$ $$\sin\theta_i=\sin\theta_r$$ and $$\theta_i=\theta_r$$

When $n_1 > n_2$, $\theta_t > \theta_i$ from the Snell's law.

If $\theta_t = 90^o$, $\Rightarrow$ the critical angle θ_i

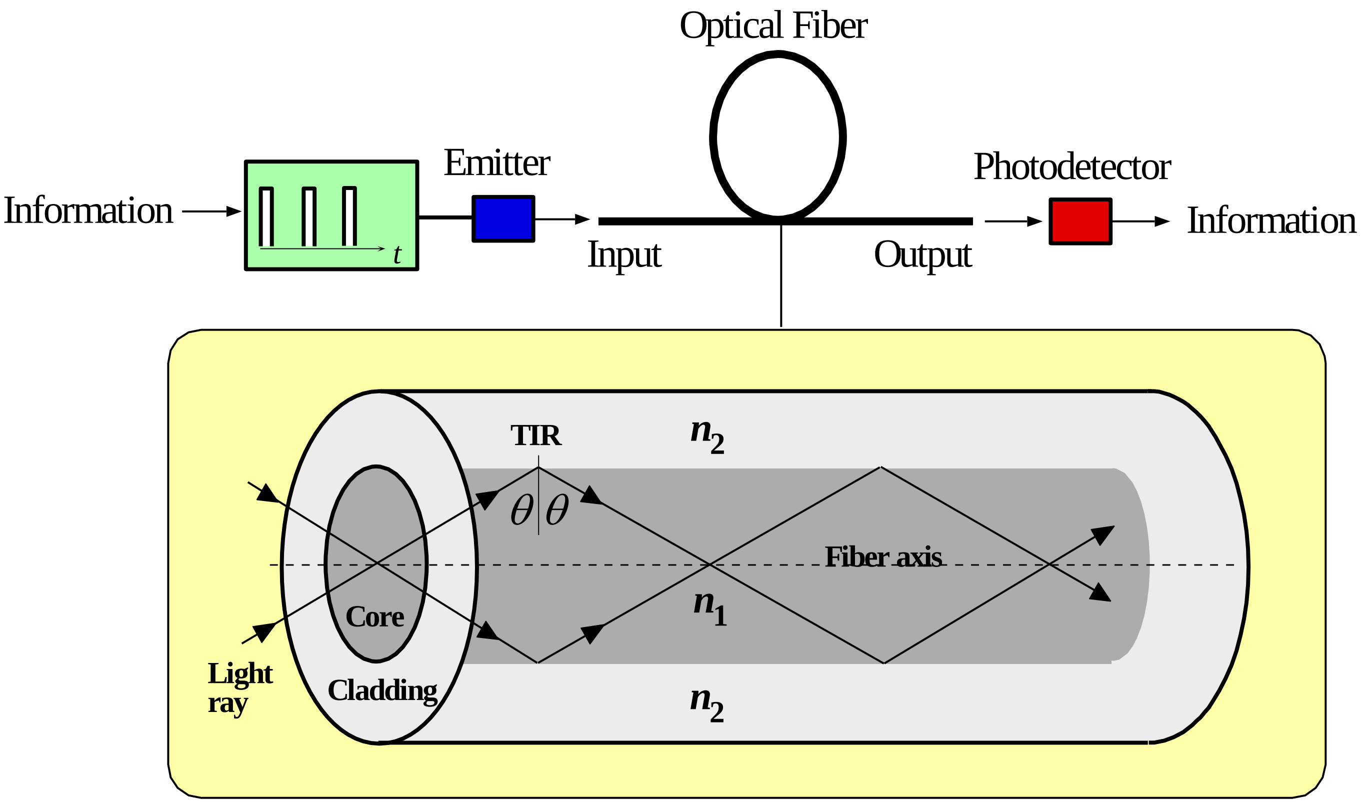
$$\sin\theta_c=\frac{n_2}{n_1}$$

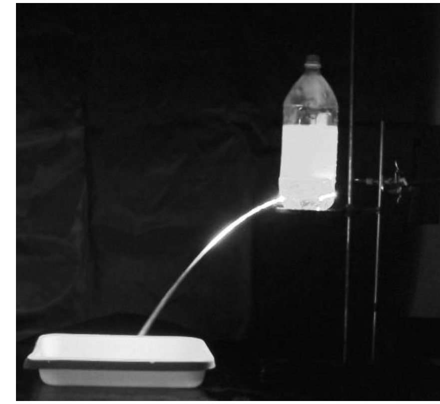
Total internal reflection (TIR)



Light wave travelling in a more dense medium strikes a less dense medium. Depending on the incidence angle with respect to q_C, which is determined by the ratio of the refractive indices, the wave may be transmitted (refracted) or reflected.

(a) $\theta_i < \theta_C$ (b) $\theta_i = \theta_C$ (c) $\theta_i > \theta_C$, total internal reflection (TIR).





The light guiding by a water jet was demonstrated by John Tyndall in 1854 to the Royal Institution.

An optical fiber link for transmitting digital information in communications. The fiber core has a higher refractive index so that the light travels along the fiber inside the fiber core by total internal reflection at the core-cladding interface.

If n_1(core) = 1.455 and n_2 (cladding) = 1.440

$$\theta_c = \arcsin(\frac{n_2}{n_1}) = \arcsin(\frac{1.440}{1.455}) = 81.8^\circ$$

PART B

HOME SPORT MONEY

THE STRAITS TIMES TUESDAY, APRIL 14 2009 B1

HOME

High-end broadband to be cheaper soon

Price competition likely as more ISPs start offering 100Mbps plans

By Chua Hian Hou

CONSUMERS can expect more Internet service providers (ISPs) to launch 100Mbps consumer broadband services next year at about $75 a month or less.

Two such high-end offerings now available cost almost $90 a month.

The impetus to make available more choices in high-end broadband services is Singapore's upcoming broadband network, portions of which will be up by the middle of next year.

StarHub, awarded the tender to install the hardware for this network, will sell ISPs access to it at a wholesale rate of $21 per 100Mbps for residential lines.

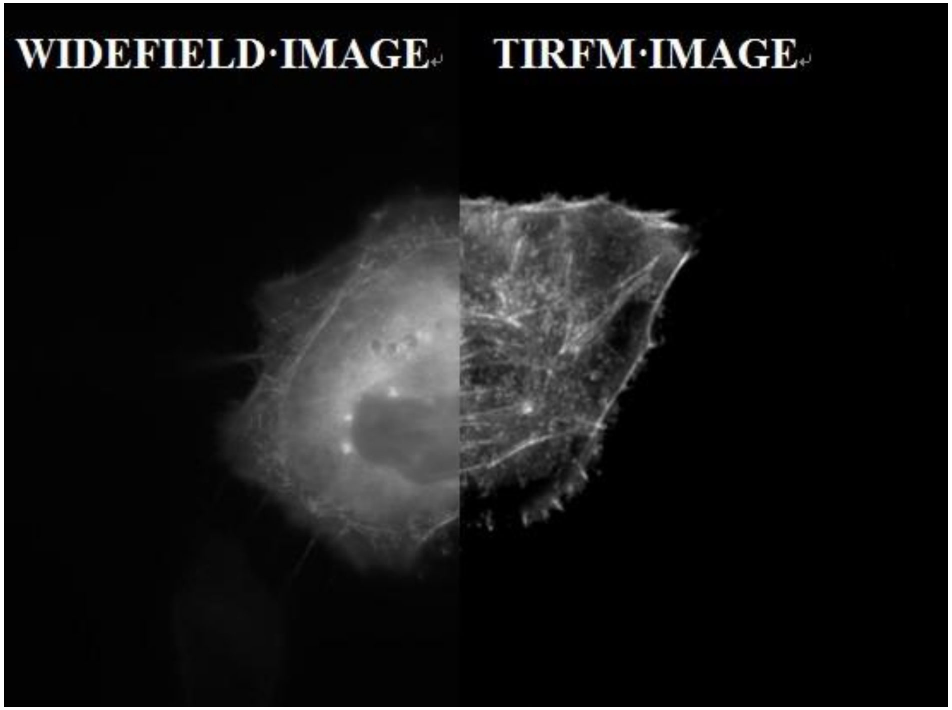
This price is much lower than what ISPs offering consumer broadband services now pay StarHub or SingTel to piggyback on their networks: StarHub charges $21.56 for a 2Mbps line and $35.71 for a 100Mbps line; SingTel's charges range from $15 for a 512kbps line to $34 for a 10Mbps line.

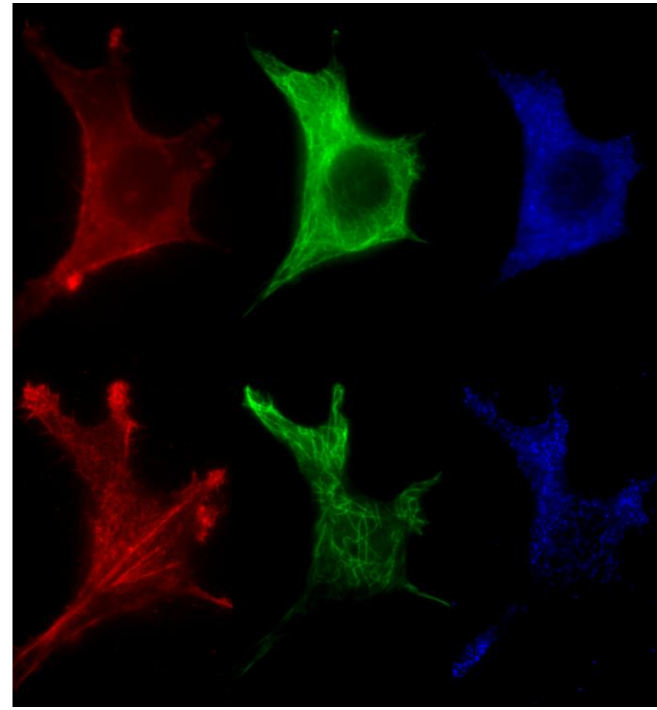
Aside from these charges, ISPs also have to factor their other overheads – billing, customer service, overseas bandwidth charges and their profits – into the final price they now charge their customers for Internet access.

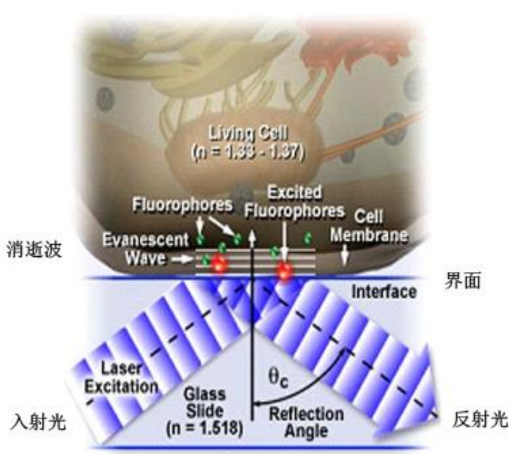
Mr Foong King Yew, consulting firm Gartner's research director for carrier operations and strategies, said the lower price was likely to lure new players into launching 100Mbps packages.

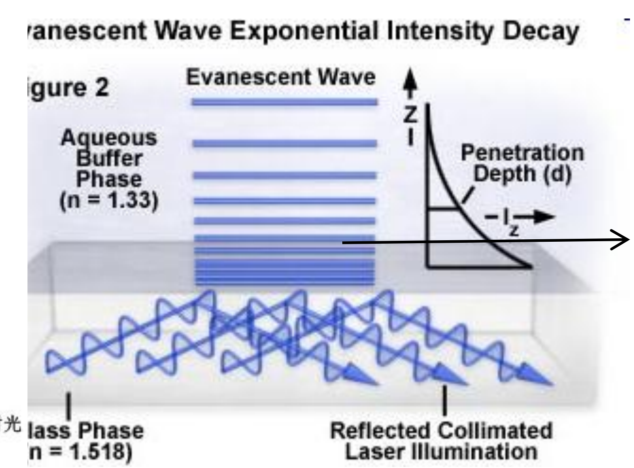
These are now available only from StarHub and M1 at almost $90 a month.

Total internal reflection fluorescence microscope (TIRF)
全内反射显微技术









倏逝波（Evanescent wave）

完成 **Assignment 8.1**

提交时间：**6月5日**（周四）前，提交**Assignment 8.1**

提交方式：电子版（写明姓名、学号），通过本班课代表统一提交