Chapter 11 Photon: quantum of energy

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11.1 Photoelectric Effect

When a surface is exposed to electromagnetic radiation above a certain threshold frequency (typically visible light for alkali metals, near ultraviolet for other metals, and extreme ultraviolet for non-metals), the radiation is absorbed and electrons are emitted. Light, and especially ultra-violet light, discharges negatively electrified bodies with the production of rays of the same nature as cathode rays. Under certain circumstances it can directly ionize gases. The first of these phenomena was discovered by Heinrich Hertz and Wilhelm Hallwachs in 1887. The second was announced first by Philipp Lenard in 1900.

The ultra-violet light to produce these effects may be obtained from an arc lamp, or by burning magnesium, or by sparking with an induction coil between zinc or cadmium terminals, the light from which is very rich in ultra-violet rays. Sunlight is not rich in ultra-violet rays, as these have been absorbed by the atmosphere, and it does not produce nearly so large an effect as the arc-light. Many substances besides metals discharge negative electricity under the action of ultraviolet light: lists of these substances will be found in papers by G. C. Schmidt and O. Knoblauch.

Experimental observations of photoelectric emission

Figure 11.1 shows the schematic of experimental apparatus to demonstrate the photoelectric effect. The light strikes the curved electrode (E), and electrons (e) are emitted to the collector (C). Electrons collected at C and passing through the ammeter are a current in the circuit. C is maintained at a positive potential by the power supply. The adjustable voltage can be increased until the current stops flowing. This "stopping voltage" is a function only of the electrode material and the frequency of the incident light, and is not affected by the intensity of the light.

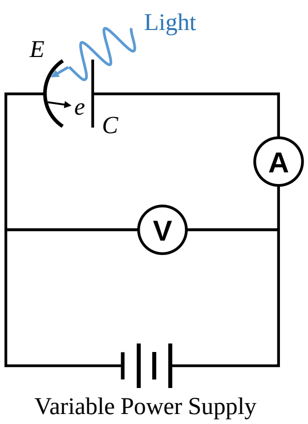

Figure 11.1 Photoelectric Effect Schematic

According to classical electromagnetic theory, the photoelectric effect can be attributed to the transfer of energy from the light to an electron. From this perspective, an alteration in the intensity of light induces changes in the kinetic energy of the electrons emitted from the metal. According to this theory, a sufficiently dim light is expected to show a time lag between the initial shining of its light and the subsequent emission of an electron. Also, the energy of the incident light can be increased by increasing the amplitude of waves.

But the experimental results did not correlate with either of the two predictions made by classical theory. Instead, experiments showed that electrons are dislodged only by the impingement of light when it reached or exceeded a threshold frequency. Below that threshold, no electrons are emitted from the material, regardless of the light intensity or the length of time of exposure to the light, as shown in Figure 11.2.

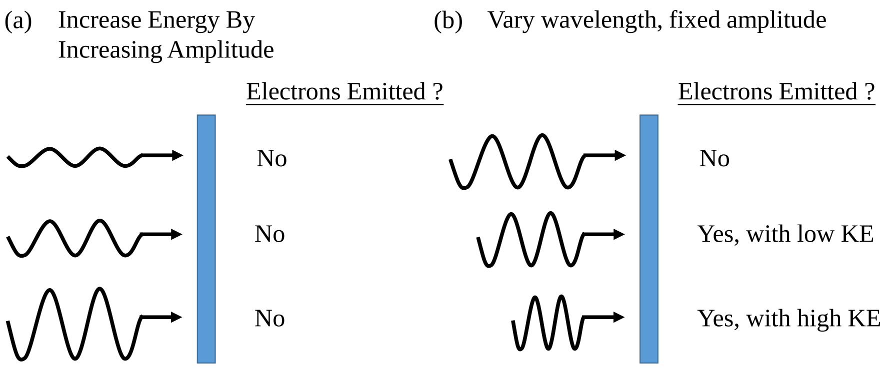

Figure 11.2 Observation of the Photoelectric Effect - a Quantum Phenomenon.

What happens if we vary the wavelength (frequency) of the incident light, with its amplitude fixed? Experimental observation shows that no electrons were emitted until the frequency of the light exceeded a critical frequency, at which point electrons were emitted from the surface! Recall that a smaller wavelength refers to a larger frequency.

In 1905, Einstein proposed an explanation of the photoelectric effect using a concept first put forward by Max Planck that light waves consist of tiny bundles or packets of energy known as photons or quanta. The electron energy as a function of frequency is plotted in Figure 11.3. The photon energy is the sum of the binding energy of electron and the electron kinetic energy, that is,

$$\hbar\omega = W + \frac{1}{2}mv^2 \tag{11.1}$$

where $h = 6.626\times10^{-34}\left[\mathrm{J}\cdot\mathrm{s}\right]$ is the Planck constant, $\hbar = h/2\pi$ is the reduced Planck constant, W is the binding energy of electron, $mv^2/2$ is the electron kinetic energy, ω is the angular frequency of light.

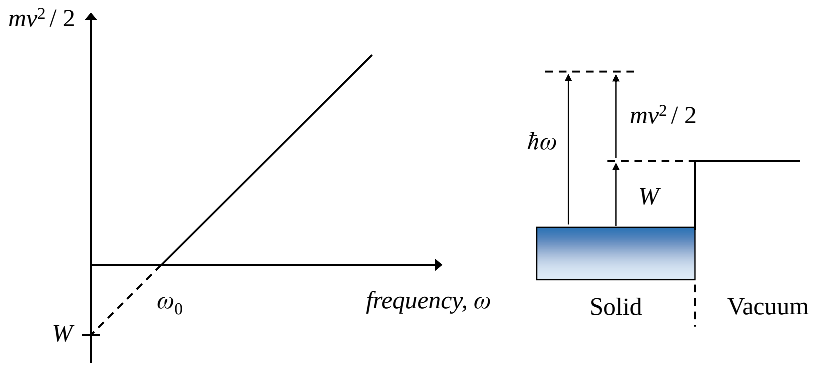

Figure 11.2 Electron Energy as a Function of Frequency

According to quantum theory, a photon has an energy given by

$$\boxed{E = \hbar\omega = \hbar c/\lambda} \tag{11.2}$$

What is the connection between Maxwell's equations and photons?

In the classical model, the intensity of light is described by the Poynting vector

$$\vec{S} = \vec{E} \times \vec{H} \tag{11.3}$$

with the unit [Watts/cm^2].

In terms of photons, the intensity of light is

$$|\vec{S}| = \frac{n\hbar\omega}{\tau A} \tag{11.4}$$

with the unit $\frac{\text{Photons}}{\text{sec}\cdot\text{cm}^2}\frac{\text{J}}{\text{Photon}} = \frac{\text{Watts}}{\text{cm}^2}$, in consistent to that of classical model.

In empty space, the photon moves at c (the speed of light) and its energy and momentum are related by $E = pc$, where p is the magnitude of the momentum vector p. This derives from the following relativistic relation, with m = 0:

$$E^2 = p^2c^2 + m^2c^4 \tag{11.5}$$

The momentum of a photon depends only on its frequency (ω) or inversely, its wavelength (λ):

$$\boxed{p = \frac{E}{c} = \frac{\hbar\omega}{c} = \frac{h}{\lambda}} \tag{11.6}$$

The momentum of a photon is in the direction of the photon's propagation ($\vec{k}$).

11.2 Compton Effect

Compton scattering, discovered by Arthur Holly Compton, is the scattering of a photon by a charged particle, usually an electron. It results in a decrease in energy (increase in wavelength) of the photon (which may be an X-ray or gamma ray photon), called the Compton effect. Part of the energy of the photon is transferred to the recoiling electron. Inverse Compton scattering occurs when a charged particle transfers part of its energy to a photon.

By the early 20th century, research into the interaction of X-rays with matter was well under way. It was observed that when X-rays of a known wavelength interact with atoms, the X-rays are scattered through an angle θ and emerge at a different wavelength related to θ. Although classical electromagnetism predicted that the wavelength of scattered rays should be equal to the initial wavelength, multiple experiments had found that the wavelength of the scattered rays was longer (corresponding to lower energy) than the initial wavelength.

As shown in Figure 11.3, The interaction between an electron and a photon results in the electron being given part of the energy (making it recoil), and a photon of the remaining energy being emitted in a different direction from the original, so that the overall momentum of the system is also conserved.

In 1923, Compton published a paper in the Physical Review that explained the X-ray shift by attributing particle-like momentum to light quanta (Einstein had proposed light quanta in 1905 in explaining the photo-electric effect, but Compton did not build on Einstein's work). The energy of light quanta depends only on the frequency of the light. In his paper, Compton derived the mathematical relationship between the shift in wavelength and the scattering angle of the X-rays by assuming that each scattered X-ray photon interacted with only one electron. His paper concludes by reporting on experiments which verified his derived relation:

$$\boxed{\Delta\lambda = \lambda_2 - \lambda_1 = \frac{h}{m_0 c}(1-\cos\theta)} \tag{11.7}$$

where λ_1 is the initial wavelength, λ_2 is the wavelength after scattering, m_0 is the electron rest mass, c is the speed of light, θ is the scattering angle. The quantity of h / m_0c is known as the Compton wavelength of the electron, it is equal to 2.43×10^{-12} m. The wavelength shift $\lambda_2 - \lambda_1$ is at least zero (for $\theta = 0°$) and at most twice the Compton wavelength of the electron (for $\theta = 180°$)

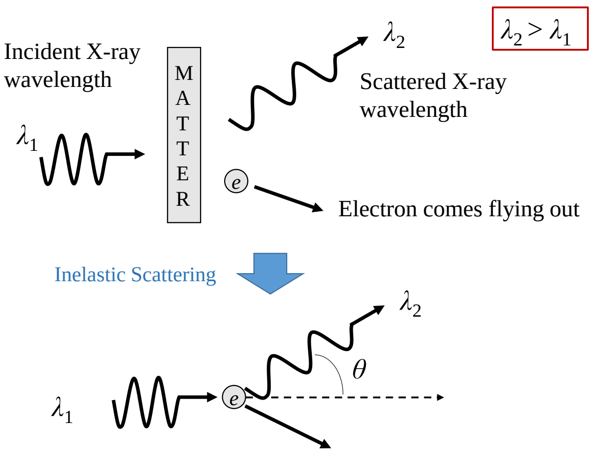

Figure 11.3 The Compton Effect

Compton scattering is an example of inelastic scattering of light by a free charged particle, where the wavelength of the scattered light is different from that of the incident radiation. In Compton's original experiment, the energy of the X ray photon ($\approx$17 keV) was very much larger than the binding energy of the atomic electron, so the electrons could be treated as being free. The amount by which the light's wavelength changes is called the Compton shift. Although nuclear Compton scattering exists, Compton scattering usually refers to the interaction involving only the electrons of an atom. The Compton effect was observed by Arthur Holly Compton in 1923 at Washington University in St. Louis and further verified by his graduate student Y. H. Woo in the years following. Compton earned the 1927 Nobel Prize in Physics for the discovery.

The effect is significant because it demonstrates that light cannot be explained purely as a wave phenomenon. Thomson scattering, the classical theory of an electromagnetic wave scattered by charged particles, cannot explain shifts in wavelength at low intensity: classically, light of sufficient intensity for the electric field to accelerate a charged particle to a relativistic speed will cause radiation-pressure recoil and an associated Doppler shift of the scattered light, but the effect would become arbitrarily small at sufficiently low light intensities regardless of wavelength. Thus, light behaves as if it consists of particles, if we are to explain low-intensity Compton scattering. Or the assumption that the electron can be treated as free is invalid resulting in the effectively infinite electron mass equal to the nuclear mass (see e.g. the comment below on elastic scattering of X-rays being from that effect). Compton's experiment convinced physicists that light can be treated as a stream of particle-like objects (quanta called photons), whose energy is proportional to the light wave's frequency.

As a conclusion, photons, like all quantum objects, exhibit wave-like and particle-like properties. Their dual wave–particle nature can be difficult to visualize. The photon displays clearly wave-like phenomena such as diffraction and interference on the length scale of its wavelength. For example, a single photon passing through a double-slit experiment exhibits interference phenomena but only if no measure was made at the slit.

11.3 Matter waves

In above discussions, we have realized that light exhibit both wave-like and particle-like properties. What about matters?

Matter waves are a central part of the theory of quantum mechanics, being an example of wave–particle duality. All matter exhibits wave-like behavior. For example, a beam of electrons can be diffracted just like a beam of light or a water wave. In most cases, however, the wavelength is too small as to have a practical impact on day-to-day activities. Hence in our day-to-day lives with objects the size of tennis balls and people, matter waves are not relevant.

The concept that matter behaves like a wave was proposed by Louis de Broglie in 1924. It is also referred to as the *de Broglie hypothesis*. Matter waves are referred to as *de Broglie waves*.

De Broglie, in his 1924 PhD thesis, proposed that just as light has both wave-like and particle-like properties, electrons also have wave-like properties. By rearranging the momentum equation, we find a relationship between the wavelength, λ associated with an electron and its momentum, p, through the Planck constant, h:

$$\boxed{\lambda = \frac{h}{p}} \tag{11.8}$$

The relationship is now known to hold for all types of matter: all matter exhibits properties of both particles and waves.

When I conceived the first basic ideas of wave mechanics in 1923–24, I was guided by the aim to perform a real physical synthesis, valid for all particles, of the coexistence of the wave and of the corpuscular aspects that Einstein had introduced for photons in his theory of light quanta in 1905. - De Broglie.

Wave-like behavior of matter was first experimentally demonstrated by George Paget Thomson's thin metal diffraction experiment, and independently in the Davisson–Germer experiment both using electrons, and it has also been confirmed for other elementary particles, neutral atoms and even molecules.

In 1926, Erwin Schrödinger published an equation describing how a matter wave should evolve—the matter wave analogue of Maxwell's equations—and used it to derive the energy spectrum of hydrogen.

Special relativity

Using two formulas from special relativity, one for the relativistic momentum and one for the relativistic mass energy

$$E = mc^2 = m_0 c^2 \Big/ \sqrt{1 - v^2/c^2} \tag{11.9}$$

$$\vec{p} = m\vec{v} = m_0 \vec{v} \Big/ \sqrt{1 - v^2/c^2} \tag{11.10}$$

allows the de Broglie relation to be written as

$$\boxed{\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{m_0 v}\sqrt{1 - \frac{v^2}{c^2}}} \tag{11.11}$$

where m_0 denotes the particle’s rest mass, v its velocity, and c the speed of light.

11.4 Double-slit experiment of electrons

In modern physics, the double-slit experiment is a demonstration that light and matter can display characteristics of both classically defined waves and particles; moreover, it displays the fundamentally probabilistic nature of quantum mechanical phenomena. The experiment was first performed with light by Thomas Young in 1801. In 1927, Davisson and Germer demonstrated that electrons show the same behavior, which was later extended to atoms and molecules. In 2012, Stefano Frabboni and co-workers eventually performed the double-slit experiment with electrons and real slits, following the original scheme proposed by Feynman. They sent single electrons onto nanofabricated slits (about 100 nm wide) and, by collecting the transmitted electrons with a single-electron detector, they could show the build-up of a double-slit interference pattern. In 2019, single particle interference was demonstrated for Antimatter by Marco Giammarchi and coworkers.

Let us now discuss the double-slit experiment with quantum material particles such as electrons. Figure 11.4 shows three different experiments where the source S shoots a stream of electrons, first with only S_1 open, then with only S_2 open, and finally with both slits open. In the first two cases, the distributions of the electrons on the screen are smooth; the sum of these distributions is also smooth, a bell-shaped curve like the one obtained for classical particles.

But when both slits are open, we see a rapid variation in the distribution, an interference pattern. So in spite of their discreteness, the electrons seem to interfere with themselves; this means that each electron seems to have gone through both slits at once! One might ask, if an electron cannot be split, how can it appear to go through both slits at once? Note that this interference pattern has nothing to do with the intensity of the electron beam. In fact, experiments were carried out with beams so weak that the electrons were sent one at a time (i.e., each electron was sent only after the previous electron has reached the screen). In this case, if both slits were open and if we wait long enough so that sufficient impacts are collected on the screen, the interference pattern appears again.

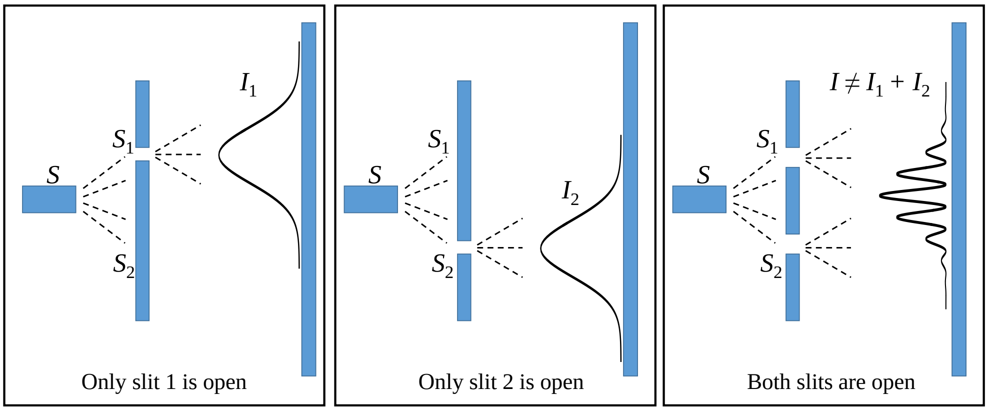

Figure 11.4 The double-slit experiment: S is a source of electrons, I_1 and I_2 are the intensities recorded on the screen when only S_1 is open, and then when only S_2 is open, respectively. When both slits are open, the total intensity is equal to the sum of I_1, I_2 and an oscillating term.

11.5 Heisenberg's Uncertainty Principle

According to classical physics, given the initial conditions and the forces acting on a system, the future behavior (unique path) of this physical system can be determined exactly. That is, if the initial coordinates $\vec{r}_0$, velocity $\vec{v}_0$, and all the forces acting on the particle are known, the position $\vec{r}(t)$ and velocity $\vec{v}(t)$ are uniquely determined by means of Newton' s second law. Classical physics is thus completely deterministic.

Does this deterministic view hold also for the microphysical world? Since a particle is represented within the context of quantum mechanics by means of a wave function corresponding to the particle's wave, and since wave functions cannot be localized, then a microscopic particle is somewhat spread over space and, unlike classical particles, cannot be localized in space. In addition, we have seen in the double-slit experiment that it is impossible to determine the slit that the electron went through without disturbing it. The classical concepts of exact position, exact momentum, and unique path of a particle therefore make no sense at the microscopic scale. This is the essence of Heisenberg's uncertainty principle. In its original form, Heisenberg's uncertainty principle states that:

If the x-component of the momentum of a particle is measured with an uncertainty Δp_x*, then its x-position cannot, at the same time, be measured more accurately than* $\Delta x = \hbar / (2\Delta p_x)$.

The three-dimensional form of the uncertainty relations for position and momentum can be written as follows:

$$\boxed{\Delta x \Delta p_x \geq \frac{\hbar}{2}, \quad \Delta y \Delta p_y \geq \frac{\hbar}{2}, \quad \Delta z \Delta p_z \geq \frac{\hbar}{2}} \tag{11.12}$$

This principle indicates that, although it is possible to measure the momentum or position of a particle accurately, it is not possible to measure these two observables simultaneously to an arbitrary accuracy. That is, we cannot localize a microscopic particle without giving to it a rather large momentum. We cannot measure the position without disturbing it; there is no way to carry out such a measurement passively as it is bound to change the momentum. To understand this, consider measuring the position of a macroscopic object (e.g., a car) and the position of a microscopic system (e.g., an electron in an atom). On the one hand, to locate the position of a macroscopic object, you need simply to observe it; the light that strikes it and gets reflected to the detector (your eyes or a measuring device) can in no measurable way affect the motion of the object. On the other hand, to measure the position of an electron in an atom, you must use radiation of very short wavelength (the size of the atom). The energy of this radiation is high enough to change tremendously the momentum of the electron; the mere observation of the electron affects its motion so much that it can knock it entirely out of its orbit. It is therefore impossible to determine the position and the momentum simultaneously to arbitrary accuracy. If a particle were localized, its wave function would become zero everywhere else and its wave would then have a very short wavelength. According to de Broglie's relation $p = h / \lambda$, the momentum of this particle will be rather high. Formally, this means that if a particle is accurately localized (i.e., $\Delta x \to 0$), there will be total uncertainty about its momentum (i.e., $\Delta p_x \to \infty$). To summarize, since all quantum phenomena are described by waves, we have no choice but to accept limits on our ability to measure simultaneously any two complementary variables.

Heisenberg's uncertainty principle can be generalized to any pair of complementary, or canonically conjugate, dynamical variables: *it is impossible to devise an experiment that can measure simultaneously two complementary variables to arbitrary accuracy* (if this were ever achieved, the theory of quantum mechanics would collapse).

Energy and time, for instance, form a pair of complementary variables. Their simultaneous measurement must obey the time–energy uncertainty relation:

$$\boxed{\Delta E \Delta t \geq \frac{\hbar}{2}} \tag{11.13}$$

This relation states that if we make two measurements of the energy of a system and if these measurements are separated by a time interval Δt, the measured energies will differ by an amount ΔE which can in no way be smaller than $\hbar / \Delta t$. If the time interval between the two measurements is large, the energy difference will be small. This can be attributed to the fact that, when the first measurement is carried out, the system becomes perturbed and it takes it a long time to return to its initial, unperturbed state. This expression is particularly useful in the study of decay processes, for it specifies the relationship between the mean lifetime and the energy width of the excited states.

We see that, in sharp contrast to classical physics, *quantum mechanics* is a completely *indeterministic* theory. Asking about the position or momentum of an electron, one cannot get a definite answer; only a *probabilistic* answer is possible. According to the uncertainty principle, if the position of a quantum system is well defined, its momentum will be totally undefined. In this context, the uncertainty principle has clearly brought down one of the most sacrosanct concepts of classical physics: the deterministic nature of Newtonian mechanics.

Example 11.1 Uncertainties for microscopic and macroscopic systems

Estimate the uncertainty in the position of (a) a neutron moving at 5×10^6 m/s and (b) a 50 kg person moving at 2 m/s.

Solution:

(a) Using the uncertainty principle, we can write the position uncertainty as

$$\Delta x \geq \frac{\hbar}{2\Delta p_x} \approx \frac{\hbar}{2m_n v} = \frac{1.05\times10^{-34}\left[\mathrm{J\cdot s}\right]}{2\times1.65\times10^{-27}[\mathrm{kg}]\times5\times10^{-6}\left[\mathrm{m/s}\right]} = 6.4\times10^{-15}\,\mathrm{m}$$

This distance is comparable to the size of a nucleus.

(b) The position uncertainty for the person is

$$\Delta x \geq \frac{\hbar}{2\Delta p_x} \approx \frac{\hbar}{2mv} = \frac{1.05\times 10^{-34}\left[\mathrm{J}\cdot\mathrm{s}\right]}{2\times 50[\mathrm{kg}]\times 2\left[\mathrm{m}/\mathrm{s}\right]} = 0.5\times 10^{-36}\,\mathrm{m}$$

An uncertainty of this magnitude is beyond human detection; therefore, it can be neglected. The accuracy of the person's position is limited only by the uncertainties induced by the device used in the measurement. So the position and momentum uncertainties are important for microscopic systems, but negligible for macroscopic systems.

11.6 Additional Problems

See References [1-4].

Reference:

[1] MIT Course 6.013: <<Electromagnetics and Applications>>, by David Staelin.

[2] << Quantum Mechanics: Concepts and Applications>>, Nouredine Zettili, Wiley, Second Edition, 2008.

[3] << Introduction to Quantum Mechanics >>, David J. Griffiths, Third Edition, 2018.

[4] https://www.wikipedia.org/

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