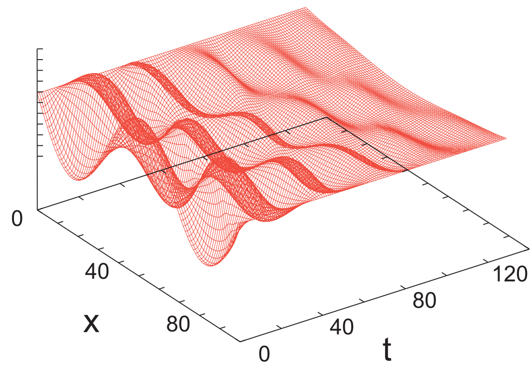
Figure 18.3 The vertical displacement as a function of position x and time t for a string initially plucked near its right end. Notice how a pulse forms and divides into waves traveling to the right and to the left. (Courtesy of J. Wiren.)



4. Use the plotted time dependence to estimate the peak's propagation velocity c. Compare the deduced c to (18.3).
5. Our solution of the wave equation for a plucked string leads to the formation of a wave packet that corresponds to the sum of multiple normal modes of the string. On the right in Figure 18.3 we show the motion resulting from the string initially placed in a single normal mode (standing wave),

$$y(x, t = 0) = 0.001 \sin 2\pi x, \qquad \frac{\partial y}{\partial t}(x, t = 0) = 0.$$

Modify the program to incorporate this initial condition and see if a normal mode results.
6. Observe the motion of the wave for initial conditions corresponding to the sum of two adjacent normal modes. Does beating occur?
7. When a string is plucked near its end, a pulse reflects off the ends and bounces back and forth. Change the initial conditions of the model program to one corresponding to a string plucked exactly in its middle and see if a traveling or a standing wave results.
8. ⊙ Figure 18.4 shows the wave packets that result as a function of time for initial conditions corresponding to the double pluck indicated on the left in the figure. Verify that initial conditions of the form

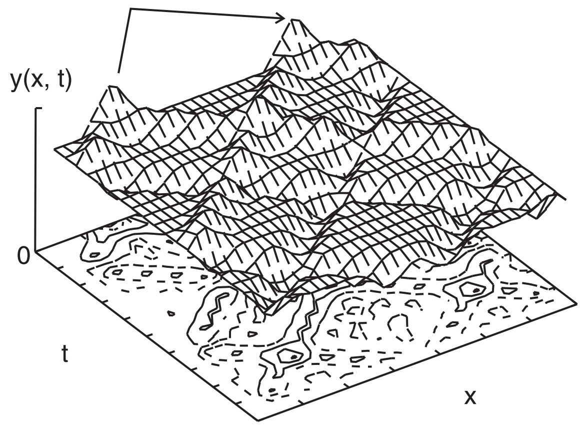
$$\frac{y(x, t = 0)}{0.005} = \begin{cases} 0, & 0.0 \le x \le 0.1, \\ 10x - 1, & 0.1 \le x \le 0.2, \\ -10x + 3, & 0.2 \le x \le 0.3, \\ 0, & 0.3 \le x \le 0.7, \\ 10x - 7, & 0.7 \le x \le 0.8, \\ -10x + 9, & 0.8 \le x \le 0.9, \\ 0, & 0.9 \le x \le 1.0 \end{cases}$$

lead to this type of a repeating pattern. In particular, observe whether the pulses move or just oscillate up and down.

18.3 WAVES WITH FRICTION (EXTENSION)

The string problem we have investigated so far can be handled by either a numerical or an analytic technique. We now wish to extend the theory to include some more realistic physics.

Figure 18.4 The vertical displacement as a function of position and time of a string initially plucked simultaneously at two points, as shown by arrows. Note that each initial peak breaks up into waves traveling to the right and to the left. The traveling waves invert on reflection from the fixed ends. As a consequence of these inversions, the $t \simeq 12$ wave is an inverted $t = 0$ wave.



These extensions have only numerical solutions.

Real plucked strings do not vibrate forever because the real world contains friction. Consider again the element of a string between x and $x + dx$ (Figure 18.1 right) but now imagine that this element is moving in a viscous fluid such as air. An approximate model has the frictional force pointing in a direction opposite the (vertical) velocity of the string and proportional to that velocity, as well as proportional to the length of the string element:

$$F_f \simeq -2\kappa\, \Delta x\, \frac{\partial y}{\partial t}, \tag{18.24}$$

derivada parcial y/t
velocidad

where κ is a constant that is proportional to the viscosity of the medium in which the string is vibrating. Including this force in the equation of motion changes the wave equation to

Se le resta una proporción de la derivada parcial de y con respecto a t

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2} - \frac{2\kappa}{\rho}\frac{\partial y}{\partial t}. \tag{18.25}$$

In Figure 18.3 we show the resulting motion of a string plucked in the middle when friction is included. Observe how the initial pluck breaks up into waves traveling to the right and to the left that are reflected and inverted by the fixed ends. Because those parts of the wave with the higher velocity experience greater friction, the peak tends to be smoothed out the most as time progresses.

Usar los parámetros dados en el ejercicio

Exercise: Generalize the algorithm used to solve the wave equation to now include friction and check if the wave's behavior seems physical (damps in time). Start with $T = 40$ N and $\rho = 10$ kg/m and pick a value of κ large enough to cause a noticeable effect but not so large as to stop the oscillations. As a check, reverse the sign of κ and see if the wave grows in time (which would eventually violate our assumption of small oscillations). ▮

18.4 WAVES FOR VARIABLE TENSION AND DENSITY (EXTENSION)

We have derived the propagation velocity for waves on a string as $c = \sqrt{T/\rho}$. This says that waves move slower in regions of high density and faster in regions of high tension. If the density of the string varies, for instance, by having the ends thicker in order to support the weight of the middle, then c will no longer be a constant and our wave equation will need to be extended. In addition, if the density increases, then so will the tension because it takes greater