6. Приближённое решение краевой задачи для обыкновенного дифференциального уравнения

Варианты заданий

Найти приближённое решение краевой задачи для обыкновенного дифференциального уравнения на отрезке $[0,1]$ с шагом $h = 0.05$. Для вычисления решения использовать метод прогонки с краевыми условиями первого и второго порядка точности. Для сравнения приведено точное решение $u_0(x)$.

1. $u'' + \dfrac{1}{2(1+x)}u' - \dfrac{1+2x}{2(1+x)}u = \dfrac{3\cos x - (3+4x)\sin x}{2\sqrt{1+x}}$,
$u(0) = 1, \quad u(1) - 2u'(1) = 0.1704$,
$u_0(x) = \sqrt{1+x}\sin x + e^{-x}$;

2. $u'' - (\operatorname{th} x)u' + \left(\operatorname{ch}^2 x\right)u = \dfrac{x\operatorname{ch}^2 x - \operatorname{th} x}{3}$,
$u(0) + u'(0) = 1.3333, \quad u'(1) = 0.9280$,
$u_0(x) = \sin(\operatorname{sh} x) + \dfrac{x}{3}$;

3. $u'' + (\cos x)u' + (\sin x)u = 1 - \cos x - \sin x$,
$u(0) - u'(0) = 0, \quad u(1) = 1.3818$,
$u_0(x) = \sin x + \cos x$;

4. $u'' + \dfrac{1}{1+x}u' + \dfrac{1}{(1+x)^2}u = \dfrac{2+6x+5x^2}{(1+x)^2}$,
$2u(0) - u'(0) = -1, \quad 3u(1) + u'(1) = 7.3015$,
$u_0(x) = x^2 + \sin\bigl(\ln(1+x)\bigr)$;

5. $u'' + (\operatorname{ch} x)u' + (\operatorname{sh} x)u = \operatorname{ch} x + x\operatorname{sh} x$,
$u'(0) = 0, \quad 6u(1) + u'(1) = 8.3761$,
$u_0(x) = \exp(-\operatorname{sh} x) + x$;

6. $u'' + \dfrac{2x}{1+x^2}u' + \dfrac{2x\,\mathrm{tg}\,x}{1+x^2}u = \dfrac{2x\,\mathrm{tg}\,x}{1+x^2}\,\mathrm{arctg}\,x - \cos x,$
$u(0) = 1, \quad u(1) + 2u'(1) = 0.6428,$
$u_0(x) = \cos x + \mathrm{arctg}\,x;$

7. $u'' - \dfrac{\mathrm{tg}\,x}{2}u' - \left(1 + \dfrac{\mathrm{tg}\,x}{2}\right)u = -\dfrac{\sqrt{\cos x}}{2}(3 + \mathrm{tg}\,x),$
$4u(0) + u'(0) = 7, \quad u'(1) = -0.9403,$
$u_0(x) = \sqrt{\cos x} + e^{-x};$

8. $u'' + \dfrac{1}{1+x}u' + \dfrac{\mathrm{tg}\,x}{1+x}u = \dfrac{2\,\mathrm{tg}\,x}{1+x}\ln(1+x) - \cos x,$
$u(0) - u'(0) = -1, \quad u(1) = 1.9266,$
$u_0(x) = \cos x + 2\ln(1+x);$

9. $u'' + (\mathrm{tg}\,x)u' - \dfrac{2x}{\cos x}u = 2 - \dfrac{2x^3}{\cos x},$
$2u(0) - u'(0) = -1, \quad 3u(1) + u'(1) = 8.0647,$
$u_0(x) = \sin x + x^2;$

10. $u'' + \dfrac{2x}{1+x^2}u' - \dfrac{2}{1+x^2}u = \dfrac{2 + \left(1 - 2x - x^2\right)e^x}{1+x^2},$
$u'(0) = -1, \quad 4u(1) + u'(1) = -9.1644,$
$u_0(x) = x\,\mathrm{arctg}\,x - e^x;$

11. $u'' + \dfrac{1}{1+x}u' + \dfrac{2}{\mathrm{ch}^2 x}u = \dfrac{1}{\mathrm{ch}^2 x}\left(\dfrac{1}{1+x} + 2\ln(1+x)\right),$
$u(0) = 0, \quad u(1) - u'(1) = 0.5348,$
$u_0(x) = \ln(1+x) + \mathrm{th}\,x;$

12. $u'' + \left(\operatorname{th}\dfrac{x}{2}\right) u' - (\cos x)u = -\sin x - \dfrac{1}{2}\sin 2x,$
$u(0) - u'(0) = -1.5, \quad u'(1) = 0.9335,$
$u_0(x) = \sin x + \operatorname{th}\dfrac{x}{2};$

13. $u'' + \dfrac{x}{1+x^2}u' - \dfrac{1}{1+x^2}u = \dfrac{3-2x+4x^2}{1+x^2}e^{-2x},$
$2u(0) - u'(0) = 6, \quad u(1) = 1.5495,$
$u_0(x) = \sqrt{1+x^2} + e^{-2x};$

14. $u'' + (\cos x)u' + (\sin x)u = x\sin x,$
$3u(0) - u'(0) = 2, \quad 2u(1) + u'(1) = 3.2391,$
$u_0(x) = x + \cos x;$

15. $u'' + \dfrac{1}{1+x}u' - \dfrac{x}{1+x}u = -\dfrac{x\ln(1+x)}{1+x},$
$u'(0) = -1, \quad 6u(1) + u'(1) = 8.3377,$
$u_0(x) = \ln(1+x) + 2e^{-x};$

16. $u'' - \dfrac{x}{4-x^2}u' - \dfrac{x\operatorname{tg} x}{4-x^2}u = -2\cos x - \dfrac{x\operatorname{tg} x}{4-x^2}\arcsin\dfrac{x}{2},$
$u(0) = 2, \quad u(1) - 2u'(1) = 3.8154,$
$u_0(x) = \arcsin\dfrac{x}{2} + 2\cos x;$

17. $u'' - \dfrac{2x}{1+x^2}u' - \dfrac{2\left(1-x^2\right)}{\left(1+x^2\right)^2}u = -\dfrac{5\left(x^5+2x^3+3x\right)}{2\left(1+x^2\right)^2},$
$2u(0) + u'(0) = 2, \quad u'(1) = 1.3916,$
$u_0(x) = 2(1+x^2)\operatorname{arctg} x - \dfrac{5}{4}x^3;$

18. $u'' + (\operatorname{th} x)u' + \dfrac{1}{\operatorname{ch}^2 x}u = -2\operatorname{th} x - \dfrac{2x}{\operatorname{ch}^2 x}$,
$u(0) - u'(0) = 3, \quad u(1) = -1.3519,$
$u_0(x) = \dfrac{1}{\operatorname{ch} x} - 2x;$

19. $u'' + (\cos x)u' + (1 + \sin x)u = \dfrac{e^x}{2}(2 + \sin x + \cos x),$
$2u(0) - u'(0) = 2.5, \quad 2u(1) - u'(1) = 3.2812,$
$u_0(x) = \cos x + \dfrac{e^x}{2};$

20. $u'' + (\operatorname{tg} x)u' + \dfrac{\cos^2 x}{4(1 + \sin x)^2}u = 1 + x\operatorname{tg} x + \dfrac{x^2\cos^2 x}{8(1 + \sin x)^2},$
$u'(0) = 0.5, \quad 6u(1) - u'(1) = 9.9430,$
$u_0(x) = \dfrac{x^2}{2} + \sqrt{1 + \sin x};$

21. $u'' + (\operatorname{tg} x)u' + (\cos^2 x)u = \operatorname{tg} x + x\cos^2 x,$
$u(0) = 1, \quad u(1) - u'(1) = 1.0692,$
$u_0(x) = \cos(\sin x) + x;$

22. $u'' - \dfrac{x}{4 - x^2}u' + \dfrac{1}{4 - x^2}u = -\dfrac{2x}{4 - x^2},$
$u(0) + u'(0) = 2, \quad u'(1) = 1.6977,$
$u_0(x) = x + 2\sqrt{1 - \dfrac{x^2}{4}}\arcsin\dfrac{x}{2};$

23. $u'' - 2\,(\operatorname{tg} x)\,u' + \dfrac{1}{\cos^4 x}u = \dfrac{\operatorname{tg} x}{\cos^4 x},$
$u(0) - u'(0) = 0, \quad u(1) = 1.5708,$
$u_0(x) = \operatorname{tg} x + \cos(\operatorname{tg} x);$

24. $u'' + 2(\operatorname{th} x)u' + 2(\sin x)u = \sin 2x - \cos x,$
$3u(0) - u'(0) = 2, \quad 2u(1) - u'(1) = 3.0253,$
$u_0(x) = \cos x + \operatorname{th} x;$

25. $u'' + 3(\operatorname{th} 2x)u' + (1 - \operatorname{th} 2x)u = \dfrac{1}{2}(3\cos x - \sin x)\operatorname{th} 2x,$
$u'(0) = 1.5, \quad 3u(1) - u'(1) = 5.1460,$
$u_0(x) = \sqrt{1 + \operatorname{th} 2x} + \dfrac{1}{2}\sin x;$

26. $u'' + \dfrac{1}{\cos^2 x}u' + 2\dfrac{\operatorname{tg} x}{\cos^2 x}u = 2 + 2x\dfrac{1 + x\operatorname{tg} x}{\cos^2 x},$
$u(0) = 1, \quad u(1) - u'(1) = -0.0676,$
$u_0(x) = x^2 + \exp(-\operatorname{tg} x);$

27. $u'' + 2(\operatorname{th} x)u' + \dfrac{1}{\operatorname{ch}^4 x}u = -4\operatorname{th} x - \dfrac{2x}{\operatorname{ch}^4 x},$
$u(0) + u'(0) = -1, \quad u'(1) = -1.6960,$
$u_0(x) = \sin(\operatorname{th} x) - 2x;$

28. $u'' + (\operatorname{tg} x)u' + xu = (1 + x)\cos x + x^2 \sin x,$
$u(0) - u'(0) = 1, \quad u(1) = 1.3818,$
$u_0(x) = x\sin x + \cos x;$

29. $u'' + (2\operatorname{th} x)u' + (1 - \operatorname{th} x)u = (1 - \operatorname{th} x)\arcsin(\operatorname{th} x),$
$3u(0) - u'(0) = 2, \quad 2u(1) + u'(1) = 3.1821,$
$u_0(x) = \arcsin(\operatorname{th} x) + \dfrac{1}{\operatorname{ch} x};$

30. $u'' - \dfrac{1}{1 + x}u' + \dfrac{1}{(1 + x)^2}u = -1 - \dfrac{1}{(1 + x)^2},$
$u'(0) = 2, \quad u(1) - u'(1) = 0.3863,$
$u_0(x) = 2(1 + x)\ln(1 + x) - x^2;$