7. Приближённое решение смешанной краевой задачи для волнового уравнения

Варианты заданий

Найти приближённое решение смешанной краевой задачи для неоднородного волнового уравнения при $0 \leqslant x \leqslant 1$. Для расчёта решения использовать схему «крест» с шагом $h = 0.05$ по переменной x. Предусмотреть возможность произвольного задания шага по переменной t и времени окончания расчёта (по умолчанию $\tau = 0.05$ и $T = 1$, соответственно). Для получения решения использовать начальные и граничные условия первого и второго порядка точности. Для сравнения приведено точное решение $u_0(x, t)$.

1. $$2u_{tt} = u_{xx} + \operatorname{ch}(x - t),$$
$$u(x, 0) = \operatorname{ch} x, \quad u_t(x, 0) = -\operatorname{sh} x,$$
$$u(0, t) - u_x(0, t) = e^t, \quad 2u(1, t) - u_x(1, t) = \frac{1}{2}\left(e^{1-t} + 3e^{t-1}\right),$$
$$u_0(x, t) = \operatorname{ch}(x - t);$$

2. $$2u_{tt} = u_{xx} - 2 + \frac{2t(x-1)^3 - t^3(x-1)}{\left(4 - t^2(x-1)^2\right)^{3/2}},$$
$$u(x, 0) = x^2, \quad u_t(x, 0) = \frac{x+1}{2},$$
$$u(0, t) = t - \arcsin\frac{t}{2}, \quad u(1, t) + 2u_x(1, t) = 5 + 2t,$$
$$u_0(x, t) = t + x^2 + \arcsin\frac{t(x-1)}{2};$$

3. $2u_{tt} = u_{xx} - 2 + xt\dfrac{t^2 - 2x^2}{(4 - x^2t^2)^{3/2}},$

$u(x,0) = x^2 + \dfrac{\pi}{2}, \quad u_t(x,0) = -\dfrac{x}{2},$

$u_x(0,t) = -\dfrac{t}{2}, \quad u(1,t) + u_x(1,t) = 3 + \arccos\dfrac{t}{2} - \dfrac{t}{\sqrt{4-t^2}},$

$u_0(x,t) = x^2 + \arccos\dfrac{xt}{2};$

4. $2u_{tt} = u_{xx} + 5\dfrac{\operatorname{th}(t-x)}{\operatorname{ch}^2(t-x)},$

$u(x,0) = \dfrac{5}{2}\operatorname{th} x, \quad u_t(x,0) = -\dfrac{5}{2\operatorname{ch}^2 x},$

$2u(0,t) - u_x(0,t) = -5\operatorname{th} t - \dfrac{5}{2\operatorname{ch}^2 t}, \quad u_x(1,t) = \dfrac{5}{2\operatorname{ch}^2(1-t)},$

$u_0(x,t) = \dfrac{5}{2}\operatorname{th}(x-t);$

5. $2u_{tt} = u_{xx} - 2\cos(t-x),$

$u(x,0) = 2\cos x, \quad u_t(x,0) = 2\sin x,$

$u(0,t) - u_x(0,t) = 2(\cos t - \sin t), \quad u(1,t) = 2\cos(1-t),$

$u_0(x,t) = 2\cos(t-x);$

6. $2u_{tt} = u_{xx} + \dfrac{1}{4}e^{x+t},$

$u(x,0) = \dfrac{1}{4}e^{x}, \quad u_t(x,0) = \dfrac{1}{4}e^{x},$

$2u(0,t) - u_x(0,t) = \dfrac{1}{4}e^{t}, \quad 3u(1,t) - u_x(1,t) = \dfrac{1}{2}e^{1+t},$

$u_0(x,t) = \dfrac{1}{4}e^{x+t};$

7. $2u_{tt} = u_{xx} - \dfrac{1}{4}e^{x+t-1} + 2\left(2x^2 - t^2\right)\dfrac{2\,\mathrm{th}^2(xt) - 1}{\mathrm{ch}(xt)},$

$u(x,0) = 2 - \dfrac{1}{4}e^{x-1}, \quad u_t(x,0) = -\dfrac{1}{4}e^{x-1},$

$u(0,t) = 2 - \dfrac{1}{4}e^{t-1}, \quad u(1,t) - u_x(1,t) = \dfrac{2 + 2t\,\mathrm{th}\,t}{\mathrm{ch}\,t},$

$u_0(x,t) = -\dfrac{1}{4}e^{x+t-1} + \dfrac{2}{\mathrm{ch}(xt)};$

8. $2u_{tt} = u_{xx} - 2 + \left(2x^2 - t^2\right)\mathrm{sh}(xt),$

$u(x,0) = x^2, \quad u_t(x,0) = x,$

$u_x(0,t) = t, \quad u(1,t) + u_x(1,t) = 3 + \mathrm{sh}\,t + t\,\mathrm{ch}\,t,$

$u_0(x,t) = x^2 + \mathrm{sh}(xt);$

9. $2u_{tt} = u_{xx} - \dfrac{16(x+t)}{\left(1 + 4(x+t)^2\right)^2},$

$u(x,0) = \mathrm{arctg}(2x), \quad u_t(x,0) = \dfrac{2}{1+4x^2},$

$u(0,t) - u_x(0,t) = \mathrm{arctg}(2t) - \dfrac{2}{1+4t^2}, \quad u_x(1,t) = \dfrac{2}{1+4(1+t)^2},$

$u_0(x,t) = \mathrm{arctg}\,2(x+t);$

10. $2u_{tt} = u_{xx} - 2\sin(x+t),$

$u(x,0) = 2\sin x, \quad u_t(x,0) = 2\cos x,$

$u(0,t) - u_x(0,t) = 2(\sin t - \cos t), \quad u(1,t) = 2\sin(1+t),$

$u_0(x,t) = 2\sin(x+t);$

11. $2u_{tt} = u_{xx} + \dfrac{(1+t)^2 - 2x^2}{4(1+x+xt)^{3/2}},$

$u(x,0) = \sqrt{1+x}, \quad u_t(x,0) = \dfrac{x}{2\sqrt{1+x}},$

$3u(0,t) + u_x(0,t) = \dfrac{7+t}{2}, \quad u(1,t) - 2u_x(1,t) = \dfrac{1}{\sqrt{2+t}},$

$u_0(x,t) = \sqrt{1+x+xt};$

12. $2u_{tt} = u_{xx} + 2 + \dfrac{\left(2x^2 - t^2\right)\left(1 + 2\operatorname{tg}^2(xt)\right)}{2\cos(xt)},$

$u(x,0) = x^2 + 1/2, \quad u_t(x,0) = -2x,$

$u(0,t) = t^2 + 1/2, \quad u(1,t) - u_x(1,t) = t^2 - 1 + \dfrac{1 - t\operatorname{tg} t}{2\cos t},$

$u_0(x,t) = (x-t)^2 + \dfrac{1}{2\cos(xt)};$

13. $2u_{tt} = u_{xx} + \dfrac{2 + 2\cos(t-x) + x\sin(t-x)}{\big(1+\cos(t-x)\big)^2},$

$u(x,0) = x - x\operatorname{tg}(x/2), \quad u_t(x,0) = 1 + x/(1+\cos x),$

$u_x(0,t) = 1 + \operatorname{tg}\dfrac{t}{2}, \quad u(1,t) - u_x(1,t) = t + \dfrac{1}{1+\cos(t-1)},$

$u_0(x,t) = t + x + x\operatorname{tg}\dfrac{t-x}{2};$

14. $2u_{tt} = u_{xx} + \dfrac{10\left(x^4 - 1 + xt(t+2x)^2\right)}{3\big(1 + x^2(x+t)^2\big)^2},$

$u(x,0) = \dfrac{5}{3}\operatorname{arctg} x^2, \quad u_t(x,0) = \dfrac{5x}{3\left(1+x^4\right)},$

$2u(0,t) - u_x(0,t) = -\dfrac{5t}{3}, \quad u_x(1,t) = \dfrac{5(2+t)}{3\big(1+(1+t)^2\big)},$

$u_0(x,t) = \dfrac{5}{3}\operatorname{arctg}\left(x^2 + xt\right);$

15. $2u_{tt} = u_{xx} - 2 + \dfrac{1 + 2\operatorname{tg}^2(x-t)}{2\cos(x-t)},$

$u(x,0) = x^2 + \dfrac{1}{2\cos x}, \quad u_t(x,0) = -\dfrac{\operatorname{tg} x}{2\cos x},$

$u(0,t) - u_x(0,t) = \dfrac{1 + \operatorname{tg} t}{2\cos t}, \quad u(1,t) = 1 + \dfrac{1}{2\cos(1-t)},$

$u_0(x,t) = x^2 + \dfrac{1}{2\cos(x-t)};$

16. $2u_{tt} = u_{xx} + \dfrac{8}{3}\left(1 - x^2\right) e^{t-x^2},$

$u(x,0) = \dfrac{2}{3}e^{-x^2}, \quad u_t(x,0) = \dfrac{2}{3}e^{-x^2},$

$2u(0,t) - u_x(0,t) = \dfrac{4}{3}e^{t}, \quad 2u(1,t) - u_x(1,t) = \dfrac{8}{3}e^{t-1},$

$u_0(x,t) = \dfrac{2}{3}e^{t-x^2};$

17. $2u_{tt} = u_{xx} + 4\cos(2 + 2t - 2x),$

$u(x,0) = 2\sin^2(1-x), \quad u_t(x,0) = 2\sin(2-2x),$

$u(0,t) = 2\sin^2(1+t), \quad u(1,t) + u_x(1,t) = 2\left(\sin^2 t - \sin(2t)\right),$

$u_0(x,t) = 2\sin^2(1 + t - x);$

18. $2u_{tt} = u_{xx} - 2 + 2\left(2x^2 - t^2\right)\dfrac{\operatorname{tg}(xt)}{\cos^2(xt)},$

$u(x,0) = (1-x)^2, \quad u_t(x,0) = x,$

$u_x(0,t) = t - 2, \quad 3u(1,t) + u_x(1,t) = 3\operatorname{tg} t + \dfrac{t}{\cos^2 t},$

$u_0(x,t) = (1-x)^2 + \operatorname{tg}(xt);$

19. $2u_{tt} = u_{xx} - 3\dfrac{2+t-x^2}{\left(1+t+x^2\right)^2},$

$u(x,0) = \dfrac{3}{2}\ln\left(1+x^2\right), \quad u_t(x,0) = \dfrac{3}{2\left(1+x^2\right)},$

$2u(0,t) - u_x(0,t) = 3\ln(1+t), \quad u_x(1,t) = \dfrac{3}{2+t},$

$u_0(x,t) = \dfrac{3}{2}\ln\left(1+t+x^2\right);$

20. $2u_{tt} = u_{xx} - 3 + \dfrac{xe^{-t}}{\left(4-x^2\right)^{3/2}} + 2e^{-t}\arccos\dfrac{x}{2},$

$u(x,0) = \dfrac{3}{2}x^2 + \arccos\dfrac{x}{2}, \quad u_t(x,0) = -\arccos\dfrac{x}{2},$

$2u(0,t) + u_x(0,t) = \left(\pi - \dfrac{1}{2}\right)e^{-t}, \quad u(1,t) = \dfrac{3}{2} + \dfrac{\pi}{3}e^{-t},$

$u_0(x,t) = \dfrac{3}{2}x^2 + e^{-t}\arccos\dfrac{x}{2};$

21. $2u_{tt} = u_{xx} + \dfrac{3+2t}{2\left(2+t-x^2\right)^{3/2}},$

$u(x,0) = \sqrt{2-x^2}, \quad u_t(x,0) = \dfrac{1}{2\sqrt{2-x^2}},$

$3u(0,t) + u_x(0,t) = 3\sqrt{2+t}, \quad u(1,t) + u_x(1,t) = \dfrac{t}{\sqrt{1+t}},$

$u_0(x,t) = \sqrt{2+t-x^2};$

22. $2u_{tt} = u_{xx} - 4\cos(2-2x-2t),$

$u(x,0) = 2\cos^2(1-x), \quad u_t(x,0) = 2\sin(2-2x),$

$u(0,t) = 2\cos^2(1-t), \quad u(1,t) + u_x(1,t) = 2\left(\cos^2 t - \sin(2t)\right),$

$u_0(x,t) = 2\cos^2(1-x-t);$

23. $2u_{tt} = u_{xx} - 2 + \left(2x^2 - t^2\right)\operatorname{ch}(xt),$

$u(x,0) = 2 - 2x + x^2, \quad u_t(x,0) = -x,$

$u_x(0,t) = -t - 2, \quad u(1,t) + 2u_x(1,t) = 2t\operatorname{sh} t + \operatorname{ch} t - 3t,$

$u_0(x,t) = (1-x)^2 - xt + \operatorname{ch}(xt);$

24. $2u_{tt} = u_{xx} + 2 + 4\dfrac{t^2 - 2x^2}{(1+xt)^2},$

$u(x,0) = -x^2, \quad u_t(x,0) = 4x,$

$u(0,t) - u_x(0,t) = -4t, \quad u_x(1,t) = 2\dfrac{t-1}{t+1},$

$u_0(x,t) = 4\ln(1+xt) - x^2;$

25. $2u_{tt} = u_{xx} + \dfrac{4\operatorname{th}^2(x-t) - 2}{\operatorname{ch}(x-t)},$

$u(x,0) = \dfrac{2}{\operatorname{ch} x}, \quad u_t(x,0) = 2\dfrac{\operatorname{th} x}{\operatorname{ch} x},$

$u(0,t) + u_x(0,t) = \dfrac{2 + 2\operatorname{th} t}{\operatorname{ch} t}, \quad u(1,t) = \dfrac{2}{\operatorname{ch}(1-t)},$

$u_0(x,t) = \dfrac{2}{\operatorname{ch}(x-t)};$

26. $2u_{tt} = u_{xx} + \dfrac{1}{2}\operatorname{sh}(1+t-x),$

$u(x,0) = \dfrac{1}{2}\operatorname{sh}(1-x), \quad u_t(x,0) = \dfrac{1}{2}\operatorname{ch}(1-x),$

$2u(0,t) + u_x(0,t) = \dfrac{1}{4}\left(e^{1+t} - 3e^{-1-t}\right), \quad u(1,t) - u_x(1,t) = \dfrac{1}{2}e^t,$

$u_0(x,t) = \dfrac{1}{2}\operatorname{sh}(1+t-x);$

27. $2u_{tt} = u_{xx} + 1 + \dfrac{\left(2x^2 - t^2\right)\left(3 + \operatorname{ch}(2xt)\right)}{8\left(\operatorname{ch}(xt)\right)^{3/2}},$

$u(x,0) = 1 + \dfrac{x^2}{2}, \quad u_t(x,0) = -x,$

$u(0,t) = 1 + \dfrac{t^2}{2}, \quad 2u(1,t) - u_x(1,t) = t^2 - t + \dfrac{4\operatorname{ch} t - t\operatorname{sh} t}{2\sqrt{\operatorname{ch} t}},$

$u_0(x,t) = \dfrac{1}{2}(x-t)^2 + \sqrt{\operatorname{ch}(xt)};$

28. $2u_{tt} = u_{xx} - 2t - \dfrac{xt^2}{\left(4 - x^2\right)^{3/2}} + 4\arcsin\dfrac{x}{2},$

$u(x,0) = \dfrac{x}{3}, \quad u_t(x,0) = x^2,$

$u_x(0,t) = \dfrac{1}{3} + \dfrac{t^2}{2}, \quad 3u(1,t) - u_x(1,t) = \dfrac{2}{3} + t + \left(\dfrac{\pi}{2} - \dfrac{1}{\sqrt{3}}\right)t^2,$

$u_0(x,t) = \dfrac{x}{3} + x^2 t + t^2\arcsin\dfrac{x}{2};$

29. $2u_{tt} = u_{xx} - 2 + 2\left(t^2 - 2x^2\right)\dfrac{\operatorname{th}(xt)}{\operatorname{ch}^2(xt)},$

$u(x,0) = x^2, \quad u_t(x,0) = x,$

$u(0,t) - u_x(0,t) = -t, \quad u_x(1,t) = 2 + \dfrac{t}{\operatorname{ch}^2 t},$

$u_0(x,t) = x^2 + \operatorname{th}(xt);$

30. $2u_{tt} = u_{xx} + \dfrac{4\left(1 - 3\operatorname{th}^2(x+t)\right)}{\operatorname{ch}^2(x+t)},$

$u(x,0) = 2\operatorname{th}^2 x, \quad u_t(x,0) = \dfrac{4\operatorname{th} x}{\operatorname{ch}^2 x},$

$u(0,t) - u_x(0,t) = 2\operatorname{th}^2 t - \dfrac{4\operatorname{th} t}{\operatorname{ch}^2 t}, \quad u(1,t) = 2\operatorname{th}^2(1+t),$

$u_0(x,t) = 2\operatorname{th}^2(x+t);$