8. Приближённое решение смешанной краевой задачи для уравнения теплопроводности

Варианты заданий

Найти приближённое решение смешанной краевой задачи для неоднородного уравнения теплопроводности при $0 \leqslant x \leqslant 1$. Для расчёта решения использовать симметричную схему с шагом $h = 0.05$ по переменной x. Предусмотреть возможность произвольного задания шага по переменной t и времени окончания расчёта (по умолчанию $\tau = 0.05$ и $T = 1$, соответственно). Для получения решения использовать граничные условия первого и второго порядка точности. Для сравнения приведено точное решение $u_0(x,t)$.

1. $$u_t = u_{xx} + \frac{5\big(1+\cos(x+t)-\sin(x+t)\big)}{4\big(1+\cos(x+t)\big)^2},$$
$$u_x(0,t) = \frac{5}{4(1+\cos t)}, \quad u(1,t)+u_x(1,t) = \frac{5\big(1+\sin(t+1)\big)}{4\big(1+\cos(t+1)\big)},$$
$$u(x,0) = \frac{5}{4}\operatorname{tg}\frac{x}{2}, \quad u_0(x,t) = \frac{5}{4}\operatorname{tg}\frac{x+t}{2};$$

2. $$u_t = u_{xx} + \frac{e^x}{2}\left(\frac{1}{\cos^2 t} - \operatorname{tg} t\right),$$
$$u(0,t)+u_x(0,t) = \operatorname{tg} t - \frac{1}{2}, \quad u(1,t) = \frac{e\operatorname{tg} t - 1}{2},$$
$$u(x,0) = -\frac{x}{2}, \quad u_0(x,t) = \frac{e^x\operatorname{tg} t - x}{2};$$

3. $$u_t = u_{xx} + \frac{2x(1+x+xt)+(1+t)^2}{4(1+x+xt)^{3/2}},$$
$$u(0,t)+2u_x(0,t) = 2+t, \quad u(1,t)+2u_x(1,t) = \frac{3+2t}{\sqrt{2+t}},$$
$$u(x,0) = \sqrt{1+x}, \quad u_0(x,t) = \sqrt{1+x+xt};$$

4. $$u_t = u_{xx} - 2(1+x-t) + \frac{x\,\mathrm{tg}(xt) - t^2\left(1 + 2\,\mathrm{tg}^2(xt)\right)}{2\cos(xt)},$$
$$u(0,t) = t^2 + \frac{1}{2}, \quad 3u(1,t) + u_x(1,t) = 5 - 8t + 3t^2 + \frac{3 + t\,\mathrm{tg}\,t}{2\cos t},$$
$$u(x,0) = x^2 + \frac{1}{2}, \quad u_0(x,t) = (x-t)^2 + \frac{1}{2\cos(xt)};$$

5. $$u_t = u_{xx} + \frac{3\left(3x^2 - t - 1\right)}{2\left(1 + t + x^2\right)^2},$$
$$u(0,t) - u_x(0,t) = \frac{3}{2}\ln(1+t), \quad u_x(1,t) = \frac{3}{2+t},$$
$$u(x,0) = \frac{3}{2}\ln\left(1 + x^2\right), \quad u_0(x,t) = \frac{3}{2}\ln\left(1 + t + x^2\right);$$

6. $$u_t = u_{xx} + \frac{x - 2t^2\,\mathrm{tg}(xt)}{2\cos^2(xt)},$$
$$u_x(0,t) = 1 + \frac{t}{2}, \quad u(1,t) + u_x(1,t) = 2 + \frac{1}{2}\left(\mathrm{tg}\,t + \frac{t}{\cos^2 t}\right),$$
$$u(x,0) = x, \quad u_0(x,t) = x + \frac{1}{2}\,\mathrm{tg}(xt);$$

7. $$u_t = u_{xx} + 2 + \frac{3}{2}e^{xt}\left(x\cos(xt) + \left(2t^2 - x\right)\sin(xt)\right),$$
$$u(0,t) - 2u_x(0,t) = \frac{3}{2} - 3t, \quad u(1,t) = \frac{3}{2}e^t\cos t - 1,$$
$$u(x,0) = \frac{3}{2} - x^2, \quad u_0(x,t) = \frac{3}{2}e^{xt}\cos(xt) - x^2;$$

8. $$u_t = u_{xx} + \left(2 - \frac{8x^2}{3}\right)e^{t-x^2},$$
$$u(0,t) - u_x(0,t) = \frac{2}{3}e^t, \quad u(1,t) + u_x(1,t) = -\frac{2}{3}e^{t-1},$$
$$u(x,0) = \frac{2}{3}e^{-x^2}, \quad u_0(x,t) = \frac{2}{3}e^{t-x^2};$$

9. $u_t = u_{xx} - \dfrac{2\Big(x\,\mathrm{th}(xt) + t^2\left(2\,\mathrm{th}^2(xt) - 1\right)\Big)}{\mathrm{ch}(xt)},$

$u(0,t) = 2 - \dfrac{1}{4}e^{t-1}, \quad u(1,t) - 2u_x(1,t) = \dfrac{1}{4}e^{t} + \dfrac{2(1+2t\,\mathrm{th}\,t)}{\mathrm{ch}\,t},$

$u(x,0) = 2 - \dfrac{1}{4}e^{x-1}, \quad u_0(x,t) = \dfrac{2}{\mathrm{ch}(xt)} - \dfrac{1}{4}e^{x+t-1};$

10. $u_t = u_{xx} + \dfrac{x + 2t^2\,\mathrm{th}(xt)}{\mathrm{ch}^2(xt)},$

$u(0,t) + u_x(0,t) = 1 + t, \quad u_x(1,t) = 1 + \dfrac{t}{\mathrm{ch}^2\,t},$

$u(x,0) = x, \quad u_0(x,t) = x + \mathrm{th}(xt);$

11. $u_t = u_{xx} + x - t^2\,\mathrm{ch}(xt) + x\,\mathrm{sh}(xt),$

$u_x(0,t) = t - 1, \quad u(1,t) - 3u_x(1,t) = 2(1-t) + \mathrm{ch}\,t - 3t\,\mathrm{sh}\,t,$

$u(x,0) = 1 - x, \quad u_0(x,t) = x(t-1) + \mathrm{ch}(xt);$

12. $u_t = u_{xx} + 2xt + \dfrac{1 + \mathrm{th}(x-t) - 2\,\mathrm{th}^2(x-t)}{\mathrm{ch}(x-t)},$

$u(0,t) + u_x(0,t) = t^2 + \dfrac{1 + \mathrm{th}\,t}{\mathrm{ch}\,t}, \quad u(1,t) = t^2 + \dfrac{1}{\mathrm{ch}(1-t)},$

$u(x,0) = \dfrac{1}{\mathrm{ch}\,x}, \quad u_0(x,t) = \dfrac{1}{\mathrm{ch}(x-t)} + xt^2;$

13. $u_t = u_{xx} - \mathrm{sh}(x-t) - \mathrm{ch}(x-t),$

$u(0,t) - u_x(0,t) = e^{t}, \quad u(1,t) + u_x(1,t) = e^{1-t},$

$u(x,0) = \mathrm{ch}\,x, \quad u_0(x,t) = \mathrm{ch}(x-t);$

14. $u_t = u_{xx} - \dfrac{3\left(1+\operatorname{ch}^2(1-x-t)+\operatorname{sh}(2-2x-2t)\right)}{8\big(\operatorname{ch}(1-x-t)\big)^{3/2}}$,

$u(0,t) = \dfrac{3}{2}\sqrt{\operatorname{ch}(1-t)}, \quad u(1,t) - 2u_x(1,t) = \dfrac{3(\operatorname{ch} t - \operatorname{sh} t)}{2\sqrt{\operatorname{ch} t}}$,

$u(x,0) = \dfrac{3}{2}\sqrt{\operatorname{ch}(1-x)}, \quad u_0(x,t) = \dfrac{3}{2}\sqrt{\operatorname{ch}(1-x-t)}$;

15. $u_t = u_{xx} + \dfrac{5\big(2\operatorname{th}(x-t)-1\big)}{2\operatorname{ch}^2(x-t)}$,

$u(0,t) - u_x(0,t) = -\dfrac{5}{2}\left(\operatorname{th} t + \dfrac{1}{\operatorname{ch}^2 t}\right), \quad u_x(1,t) = \dfrac{5}{2\operatorname{ch}^2(t-1)}$,

$u(x,0) = \dfrac{5}{2}\operatorname{th} x, \quad u_0(x,t) = \dfrac{5}{2}\operatorname{th}(x-t)$;

16. $u_t = u_{xx} - 1 + x\operatorname{ch}(xt) - t^2\operatorname{sh}(xt)$,

$u_x(0,t) = t, \quad 2u(1,t) + u_x(1,t) = 2 + 2\operatorname{sh} t + t\operatorname{ch} t$,

$u(x,0) = \dfrac{x^2}{2}, \quad u_0(x,t) = \dfrac{x^2}{2} + \operatorname{sh}(xt)$;

17. $u_t = u_{xx} - 2 - \dfrac{1+\operatorname{tg}(x-t)+2\operatorname{tg}^2(x-t)}{2\cos(x-t)}$,

$3u(0,t) - u_x(0,t) = \dfrac{3+\operatorname{tg} t}{2\cos t}, \quad u(1,t) = 1 + \dfrac{1}{2\cos(1-t)}$,

$u(x,0) = x^2 + \dfrac{1}{2\cos x}, \quad u_0(x,t) = x^2 + \dfrac{1}{2\cos(x-t)}$;

18. $u_t = u_{xx} + \dfrac{6+3t-x^2}{2\left(2+t-x^2\right)^{3/2}}$,

$u(0,t) - 3u_x(0,t) = \sqrt{2+t}, \quad u(1,t) + 3u_x(1,t) = \dfrac{t-2}{\sqrt{1+t}}$,

$u(x,0) = \sqrt{2-x^2}, \quad u_0(x,t) = \sqrt{2+t-x^2}$;

19. $u_t = u_{xx} + 3 + 2x\cos(xt) + 2t^2\sin(xt),$
$u(0,t) = t, \quad u(1,t) + u_x(1,t) = t - 3 + 2(\sin t + t\cos t),$
$u(x,0) = -x^2, \quad u_0(x,t) = 2\sin(xt) + t - x^2;$

20. $u_t = u_{xx} + \dfrac{2}{3} + \dfrac{3\left(x + t^2 + x^2 t\right)}{(1+xt)^2},$
$5u(0,t) - u_x(0,t) = -3t, \quad u_x(1,t) = \dfrac{3t}{1+t} - \dfrac{2}{3},$
$u(x,0) = -\dfrac{x^2}{3}, \quad u_0(x,t) = 3\ln(1+xt) - \dfrac{x^2}{3};$

21. $u_t = u_{xx} + 1 - \dfrac{2xe^t}{(4-x^2)^{3/2}} + 2e^t \arcsin\dfrac{x}{2},$
$u_x(0,t) = e^t, \quad u(1,t) + u_x(1,t) = \left(\dfrac{\pi}{3} + \dfrac{2}{\sqrt{3}}\right)e^t - \dfrac{3}{2},$
$u(x,0) = 2\arcsin\dfrac{x}{2} - \dfrac{x^2}{2}, \quad u_0(x,t) = 2e^t\arcsin\dfrac{x}{2} - \dfrac{x^2}{2};$

22. $u_t = u_{xx} + 2\cos(x+t) + 2\sin(x+t),$
$u(0,t) + 4u_x(0,t) = 2\sin t + 8\cos t, \quad u(1,t) = 2\sin(1+t),$
$u(x,0) = 2\sin x, \quad u_0(x,t) = 2\sin(x+t);$

23. $u_t = u_{xx} + \dfrac{1}{2}\,\mathrm{ch}(1+t-x) - \dfrac{1}{2}\,\mathrm{sh}(1+t-x),$
$u(0,t) - u_x(0,t) = \dfrac{1}{2}e^{1+t}, \quad u(1,t) + 2u_x(1,t) = -\dfrac{1}{4}\left(e^t + 3e^{-t}\right),$
$u(x,0) = \dfrac{1}{2}\,\mathrm{sh}(1-x), \quad u_0(x,t) = \dfrac{1}{2}\,\mathrm{sh}(1+t-x);$

24. $u_t = u_{xx} - 1 + \dfrac{(x-1)\left(4 - t^3 - t^2(x-1)^2\right)}{\left(4 - t^2(x-1)^2\right)^{3/2}},$

$u(0,t) = t - \arcsin\dfrac{t}{2}, \quad u(1,t) + 4u_x(1,t) = 3(3+t),$

$u(x,0) = x^2, \quad u_0(x,t) = t + x^2 + \arcsin\dfrac{t(x-1)}{2};$

25. $u_t = u_{xx} + \dfrac{8(x+t)(x+t+2)+2}{\left(1+4(x+t)^2\right)^2},$

$u(0,t) - u_x(0,t) = \operatorname{arctg}(2t) - \dfrac{2}{1+4t^2}, \quad u_x(1,t) = \dfrac{2}{1+4(1+t)^2},$

$u(x,0) = \operatorname{arctg}(2x), \quad u_0(x,t) = \operatorname{arctg}(2x+2t);$

26. $u_t = u_{xx} + x + \dfrac{8t + x^2\left(x^2t^2 - t^3 - 4\right)}{(4 - x^2t^2)^{3/2}},$

$u_x(0,t) = t + \dfrac{\pi}{2}, \quad u(1,t) + u_x(1,t) = 2t + 2\arccos\dfrac{t}{2} - \dfrac{t}{\sqrt{4-t^2}},$

$u(x,0) = \dfrac{\pi x}{2}, \quad u_0(x,t) = xt + x\arccos\dfrac{xt}{2};$

27. $u_t = u_{xx} - 3 + \dfrac{xe^{-t}}{(4-x^2)^{3/2}} - e^{-t}\arccos\dfrac{x}{2},$

$u(0,t) - u_x(0,t) = \dfrac{1+\pi}{2}e^{-t}, \quad u(1,t) = \dfrac{3}{2} + \dfrac{\pi}{3}e^{-t},$

$u(x,0) = \dfrac{3x^2}{2} + \arccos\dfrac{x}{2}, \quad u_0(x,t) = \dfrac{3x^2}{2} + e^{-t}\arccos\dfrac{x}{2};$

28. $u_t = u_{xx} + \left(1 + xt - t^3\right)e^{xt},$

$u(0,t) - u_x(0,t) = 1 + t - t^2, \quad u(1,t) + u_x(1,t) = \left(t + t^2\right)e^t - 2,$

$u(x,0) = -x, \quad u_0(x,t) = te^{xt} - x;$

29. $u_t = u_{xx} + 2 + 2\cos(1 - x - t) + 2\sin(1 - x - t),$
$u(0,t) = 2\cos(1-t), \quad 2u(1,t) + u_x(1,t) = 4\cos t - 2\sin t - 4,$
$u(x,0) = 2\cos(1-x) - x^2, \quad u_0(x,t) = 2\cos(1-x-t) - x^2;$

30. $u_t = u_{xx} + \dfrac{4x\left(1 + x^2t^2 + 2t^3\right)}{\left(1 + x^2t^2\right)^2},$
$u(0,t) - u_x(0,t) = 1 - 4t, \quad u_x(1,t) = \dfrac{4t}{1+t^2} - 1,$
$u(x,0) = -x, \quad u_0(x,t) = 4\,\mathrm{arctg}(xt) - x;$