$$p(x:\mu,\Sigma) = \frac{1}{2\pi^{n/2}|\Sigma|^{1/2}} \exp\left(-\frac{1}{2}(x-\mu)^T \Sigma^{-1}(x-\mu)\right)$$

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \qquad \mu = \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} \qquad \Sigma = \begin{bmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_2\sigma_1 & \sigma_2^2 \end{bmatrix}$$

$$p_{n=2} = \frac{1}{2\pi \begin{vmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_2\sigma_1 & \sigma_2^2 \end{vmatrix}^{1/2}} \cdot \exp\left(-\frac{1}{2}\begin{bmatrix} x_1-\mu_1 \\ x_2-\mu_2 \end{bmatrix}^T \begin{bmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_2\sigma_1 & \sigma_2^2 \end{bmatrix}^{-1} \begin{bmatrix} x_1-\mu_1 \\ x_2-\mu_2 \end{bmatrix}\right)$$

$$= \frac{1}{2\pi\sigma_1\sigma_2(1-\rho^2)^{1/2}} \cdot \exp\left(-\frac{1}{2}\begin{bmatrix} x_1-\mu_1 \\ x_2-\mu_2 \end{bmatrix} \frac{1}{\sigma_1^2\sigma_2^2(1-\rho^2)} \begin{bmatrix} \sigma_2^2 & -\rho\sigma_1\sigma_2 \\ -\rho\sigma_1\sigma_2 & \sigma_1^2 \end{bmatrix} \begin{bmatrix} x_1-\mu_1 \\ x_2-\mu_2 \end{bmatrix}\right)$$

$$= \frac{1}{2\pi\sigma_1\sigma_2(1-\rho^2)^{1/2}} \cdot \exp\left(-\frac{1}{2}\left[(x_1-\mu_1)(x_2-\mu_2)\right] \cdot \frac{1}{\sigma_1^2\sigma_2^2(1-\rho^2)} \begin{bmatrix} \sigma_2^2(x_1-\mu_1) - \rho\sigma_1\sigma_2(x_2-\mu_2) \\ -\rho\sigma_1\sigma_2(x_1-\mu_1) + \sigma_1^2(x_2-\mu_2) \end{bmatrix}\right)$$

$$= \frac{1}{2\pi\sigma_1\sigma_2(1-\rho^2)^{1/2}} \cdot \exp\left(\frac{1}{2\sigma_1^2\sigma_2^2(1-\rho^2)}\left[(x_1-\mu_1)\sigma_2^2(x_1-\mu_1) - \rho\sigma_1\sigma_2(x_2-x_1) + (x_2-\mu_2)\right]\right)$$

where

$$Z = (\sigma^2(x_2-\mu_2) - \rho\sigma_1\sigma_2(x_1-\mu_1))$$

$$= \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}} \cdot \exp\left[\frac{1}{2\sigma_1^2\sigma_2^2(1-\rho^2)} \cdot \left(\sigma_2^2(x_1-\mu_1)^2 - \rho\sigma_1\sigma_2(x_1-\mu_1)(x_2-\mu_2) + \sigma_1^2(x_2-\mu_2)^2 - \rho\sigma_1\sigma_2(x_1-\mu_1)(x_2-\mu_2)\right)\right]$$

$$= \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}} \cdot \exp\left(\frac{-1}{2\sigma_1^2\sigma_2^2(1-\rho^2)}\left(\sigma_2^2(x_1-\mu_1)^2 - 2\rho\sigma_1\sigma_2(x_1-\mu_1)(x_2-\mu_2) + \sigma_1^2(x_2-\mu_2)^2 - \rho\sigma_1\sigma_2(x_1-\mu_1)(x_2-\mu_2)\right)\right)$$

$$= \frac{1}{2\pi\sigma_1\sigma_2\sqrt{(1-\rho^2)}} \cdot \exp\left(\frac{-1}{2\sigma_1^2\sigma_2^2(1-\rho^2)}\left(\sigma_2^2(x_1-\mu_1)^2 - 2\rho\sigma_1\sigma_2(x_1-\mu_1)(x_2-\mu_2) + \sigma_1^2(x_2-\mu_2)^2\right)\right)$$

$$= \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}} \exp\frac{-1}{2(1-\rho^2)}\left[\frac{\sigma_2^2(x_1-\mu_1)^2}{\sigma_1^2\sigma_2^2} - \frac{2\rho\sigma_1\sigma_2(x_1-\mu_1)(x_2-\mu_2)}{\sigma_1^2\sigma_2^2} + \frac{\sigma_1^2(x_2-\mu_2)^2}{\sigma_1^2\sigma_2^2}\right]$$

$$= \frac{1}{2\pi\sigma_1\sigma_2\left(\sqrt{1-\rho^2}\right)^2} \exp\frac{-1}{2(1-\rho^2)} \cdot Z$$

where

$$Z = \frac{(x_1-\mu_1)^2}{\sigma_1^2} \cdot \frac{2\rho(x_1-\mu_1)(x_2-\mu_2)}{\sigma_1\sigma_2} + \frac{(x_2-\mu_2)^2}{\sigma_2^2}$$