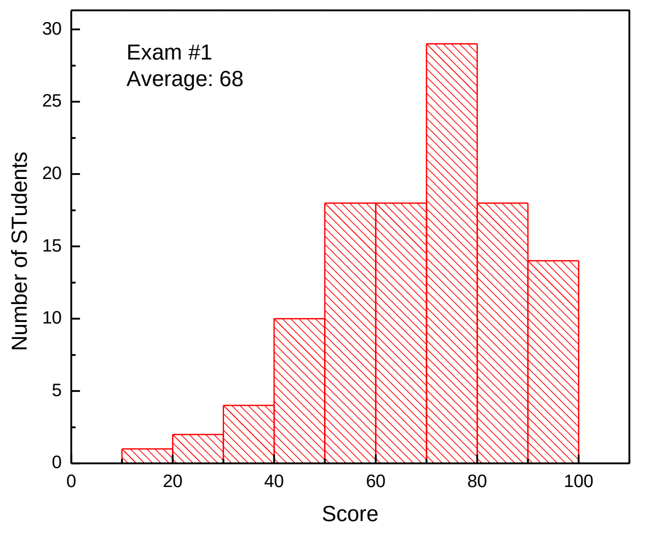
Homework

Reading: Chap. 26 and Chap. 27

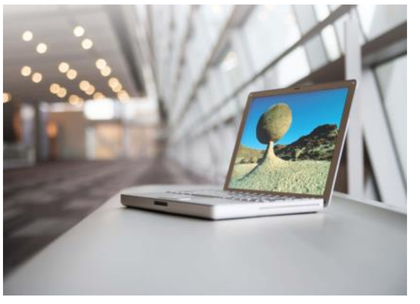
Suggested exercises: 26.3, 26.5, 26.7, 26.9, 26.10, 26.11, 26.13, 26.15, 26.17, 26.18, 26.20, 26.21, 26.24, 26.25, 26.26

Problems: 25.66, 25.74, 26.32, 26.33, 26.38, 26.40, 26.41, 26.44, 26.45, 26.50, 26.52, 26.57, 26.58, 26.63, 26.68 (due: Mon, Oct. 5)



Chapter 26. The Electric Field

Electric fields are responsible for the electric currents that flow through your computer and the nerves in your body. Electric fields also line up polymer molecules to form the images in a liquid crystal display (LCD).



Chapter Goal: To learn how to calculate and use the electric field.

Chapter 26. The Electric Field

Topics:

- Electric Field Models
- The Electric Field of Multiple Point Charges
- The Electric Field of a Continuous Charge Distribution
- The Electric Fields of Rings, Disks, Planes, and Spheres
- The Parallel-Plate Capacitor
- Motion of a Charged Particle in an Electric Field
- Motion of a Dipole in an Electric Field

Chapter 26. Basic Content and Examples

Find electric fields due to point charges

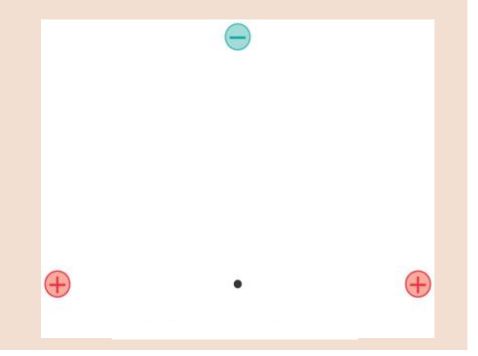
Use Coulomb's law to find the field

Superposition principle

Important: electric field is a vector, it has magnitude and direction.

In-Class Activity

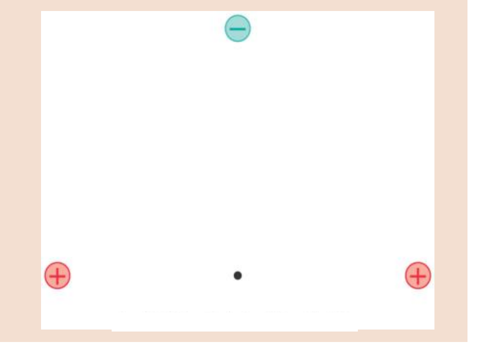
At the position of the dot, the electric field points



A. Up.
B. Down.
C. Left.
D. Right.
E. The electric field is zero.

In-Class Activity

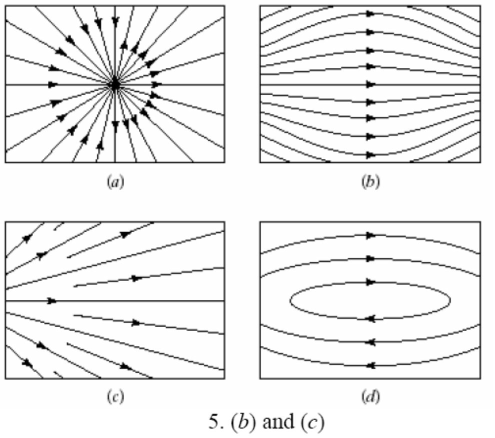
At the position of the dot, the electric field points



✔ **A. Up.**
B. Down.
C. Left.
D. Right.
E. The electric field is zero.

In-Class Activity

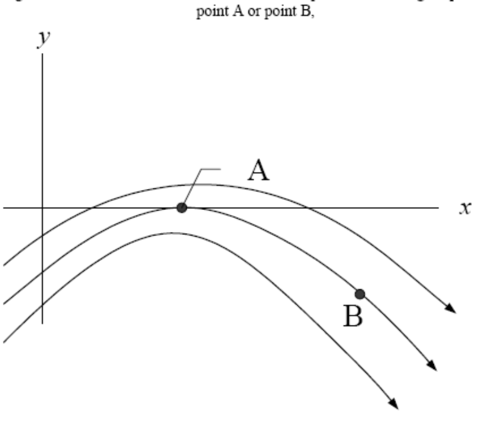
1. Consider the four field patterns shown. Assuming there are no charges in the regions shown, which of the patterns represent(s) a possible electrostatic field:



1. (*a*)
2. (*b*)
3. (*b*) and (*d*) combination
4. (*a*) and (*c*)
5. (*b*) and (*c*)
6. some other
7. None of the above.

Your answer is ________________. Explain.

4. The figure below shows three electric field lines. If a positive test charge is placed on either point A or point B,



a) Draw the direction of the electrostatic force at point A

b) Draw the direction of the electrostatic force at point A

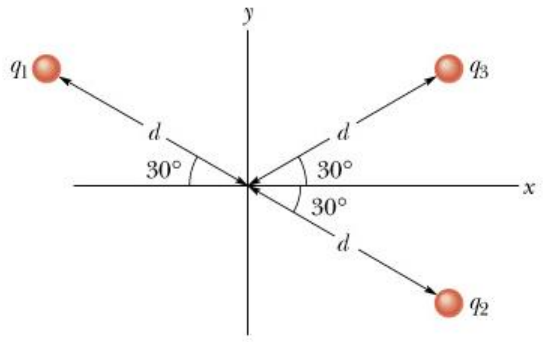
c) For the same test charge, if it is released, the acceleration at point A will be

Greater than
Less than
The same as
Cannot determine

the acceleration at point B

Example 1

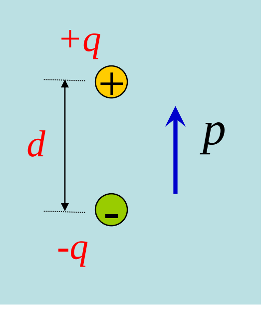
The following figure shows three particles with charge $q_1 = +2Q$, $q_2 = -2Q$, and $q_3 = -4Q$, each a distance d from the origin. What net electric field $\boldsymbol{E}$ is produced at the origin?



Electric Field Due to an Electric Dipole

Electric dipole: Definition

Two charged particles of the same magnitude q but of opposite sign, separated by a distance d



Electric dipole: Dipole moment

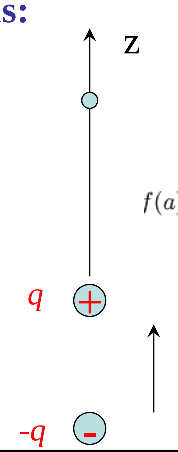
$$\vec{p} = qd\hat{r}$$

A vector !!!

Example 2

Electric Field Due to an Electric Dipole

Find the electric filed on z-axis:



Taylor expansion:

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n,$$

$$f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f^{(3)}(a)}{3!}(x-a)^3 + \cdots,$$

$$\frac{1}{\sqrt{1+x}} = 1 - \frac{1}{2}x + \frac{1\times 3}{2\times 4}x^2 - \frac{1\times 3\times 5}{2\times 4\times 6}x^3 + \ldots$$

The Electric Field of a Dipole

The electric field at a point on the axis of a dipole is

$$\vec{E}_{\text{dipole}} \approx \frac{1}{4\pi\epsilon_0}\frac{2\vec{p}}{r^3} \quad \text{(on the axis of an electric dipole)}$$

where r is the distance measured from the *center* of the dipole.

The electric field in the plane that bisects and is perpendicular to the dipole is

$$\vec{E}_{\text{dipole}} \approx -\frac{1}{4\pi\epsilon_0}\frac{\vec{p}}{r^3} \quad \text{(perpendicular plane)}$$

This field is opposite to the dipole direction, and it is only half the strength of the on-axis field at the same distance.

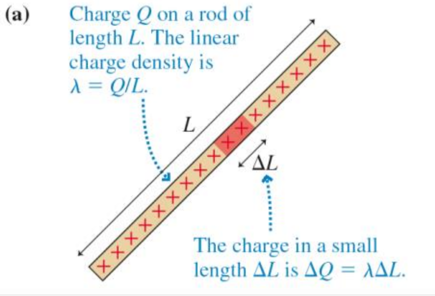
Continuous Charge Distribution

Name	Symbol	SI Unit
Point charge	q	C
Linear charge density	λ	C/m
Surface charge density	η	C/m^2
Volume charge density	ρ	C/m^3

The Electric Field of a Continuous Charge Distribution

The linear charge density of an object of length *L* and charge *Q*, is defined as

$$\lambda = \frac{Q}{L}$$



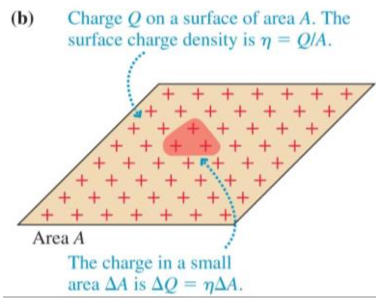
Linear charge density, which has units of C/m, is the amount of charge *per meter* of length.

The Electric Field of a Continuous Charge Distribution

The surface charge density of a two-dimensional distribution of charge across a surface of area A is defined as

$$\eta = \frac{Q}{A}$$

Surface charge density, with units C/m^2, is the amount of charge *per square meter*.

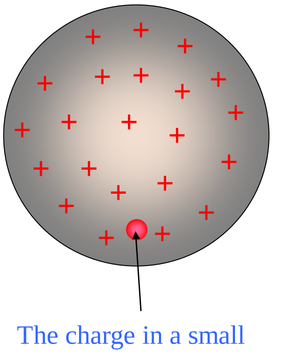


The Electric Field of a Continuous Charge Distribution

The bulk charge density of a three-dimensional distribution of charge in a volume of V is defined as

$$\rho = \frac{Q}{V}$$

Volume charge density, with units C/m^3, is the amount of charge *per cubic meter*.



Find electric fields due to charge distribution

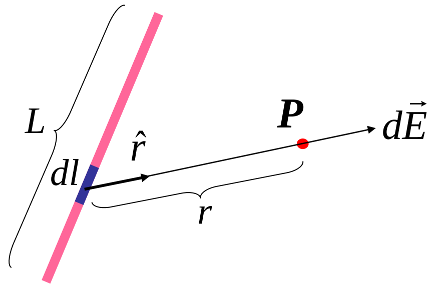
Use Coulomb's law to find the field: divide charge distribution into small sections

Superposition principle: integrate different components togther

Use symmetry principle if there is any

Important: electric field is a vector, it has magnitude and direction.

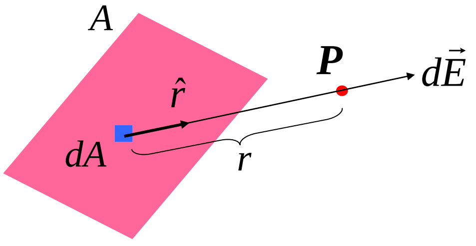
For Line Charge Distribution



$$d\vec{E} = \frac{dQ}{4\pi\varepsilon_0 r^2}\hat{r} = \frac{\lambda dl}{4\pi\varepsilon_0 r^2}\hat{r}$$

$$\vec{E} = \int_L \frac{\lambda dl}{4\pi\varepsilon_0 r^2}\hat{r}$$

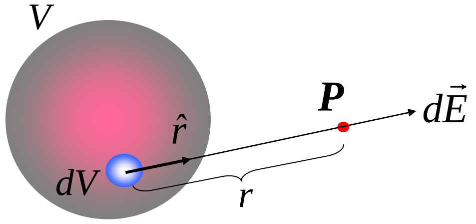
For Surface Charge Distribution



$$d\vec{E} = \frac{dQ}{4\pi\varepsilon_0 r^2}\hat{r} = \frac{\eta dA}{4\pi\varepsilon_0 r^2}\hat{r}$$

$$\vec{E} = \iint_A \frac{\eta dA}{4\pi\varepsilon_0 r^2}\hat{r}$$

For Volume Charge Distribution

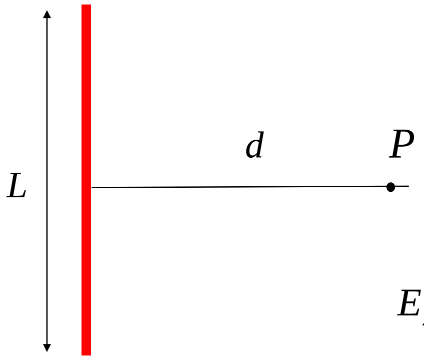


$$d\vec{E} = \frac{dQ}{4\pi\varepsilon_0 r^2}\hat{r} = \frac{\rho dV}{4\pi\varepsilon_0 r^2}\hat{r}$$

$$\vec{E} = \iiint_V \frac{\rho dV}{4\pi\varepsilon_0 r^2}\hat{r}$$

Example 3

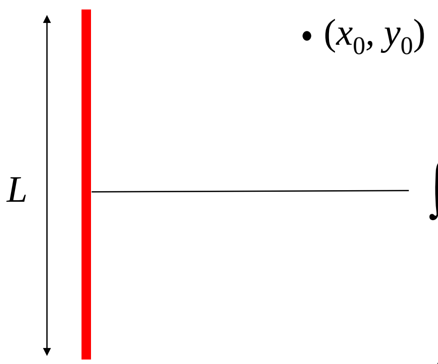
Electric Field by a Uniformly Charged Rod



$$E_x = \frac{1}{4\pi\varepsilon_0}\frac{\lambda L}{d\sqrt{d^2+(L/2)^2}}$$

$$E_y = 0$$

Electric Field by a Uniformly Charged Rod

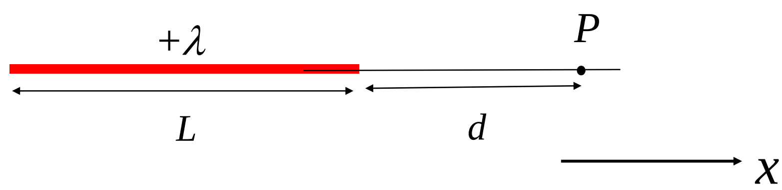


$$\int \frac{dy}{\left(a^2+y^2\right)^{3/2}} = \frac{y}{a^2\sqrt{a^2+y^2}}$$

$$\int \frac{dy}{\left(a+y\right)^{3/2}} = -\frac{2}{\sqrt{a+y}}$$

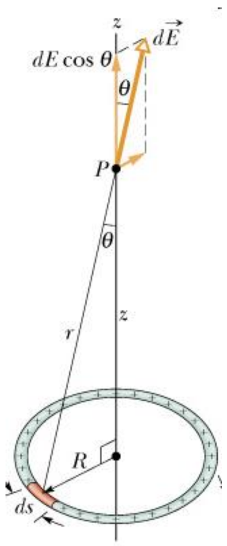
In-Class

Electric Field by a Uniformly Charged Rod



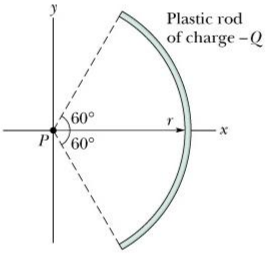
(1)Direction of the field
(2) Integration expression of the field

E-field of a Uniformly Charged Ring



Example 4

Figure shows a plastic rod has a uniformly distributed charge –*Q*. The rod has been bent in a 120º circular arc of radius *r*. We place coordinate axes such that the axis of symmetry of the rod lies along the x-axis and the origin is at the center of the curvature P of the rod. In terms of *Q* and *r*, what is the electric field **E** due to the rod at point *P*?



(*a*)

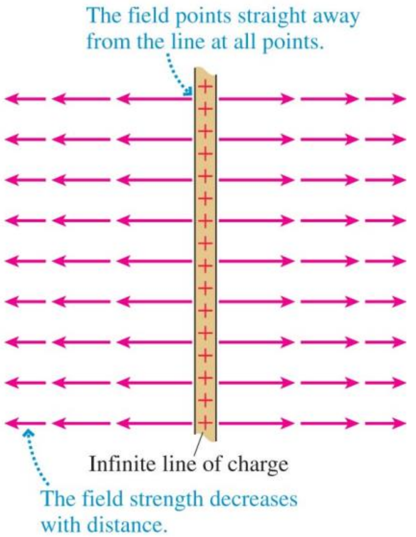
An Infinite Line of Charge

A very long, thin rod, with linear charge density λ, has an electric field

$$E_{\text{line}} = \lim_{L\to\infty} \frac{1}{4\pi\epsilon_0} \frac{|Q|}{r\sqrt{r^2 + (L/2)^2}} = \frac{1}{4\pi\epsilon_0} \frac{|Q|}{rL/2} = \frac{1}{4\pi\epsilon_0} \frac{2|\lambda|}{r}$$

Where *r* is the radial distance away from the rod.

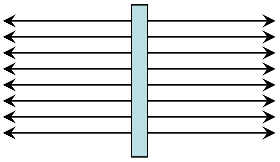
FIGURE 27.14 The electric field of an infinite line of charge.



A Plane of Charge

The electric field of an infinite plane of charge with surface charge density η is:

$$E_{\text{plane}} = \frac{\eta}{2\epsilon_0} = \text{constant}$$



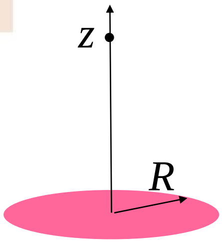
For a positively charged plane, with $\eta > 0$, the electric field points *away from* the plane on both sides of the plane.
For a negatively charged plane, with $\eta < 0$, the electric field points *towards* the plane on both sides of the plane.

A Disk of Charge

The on-axis electric field of a charged disk of radius R, centered on the origin with axis parallel to z, and surface charge density $\eta = Q/\pi R^2$ is

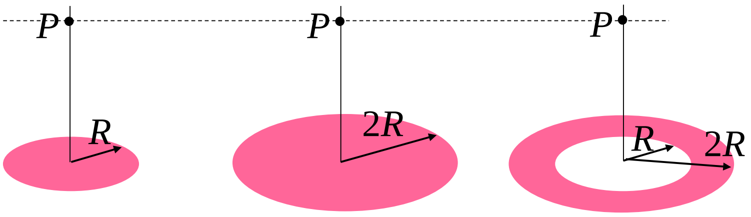
$$(E_{\text{disk}})_z = \frac{\eta}{2\epsilon_0}\left[1 - \frac{z}{\sqrt{z^2 + R^2}}\right]$$

NOTE: This expression is only valid for $z > 0$. The field for $z < 0$ has the same magnitude but points in the opposite direction.



In-Class Activity

The following figure shows two disks and a flat ring, each with the same uniform charge Q. Rank the objects according to the magnitude of the electric field they create at points P (which are at the same vertical heights), greatest first.



A Sphere of Charge

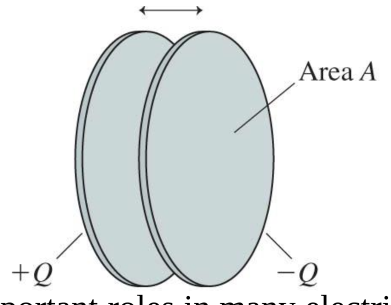
A sphere of charge Q and radius R, be it a uniformly charged sphere or just a spherical shell, has an electric field *outside* the sphere that is exactly the same as that of a point charge Q located at the center of the sphere:

$$\vec{E}_{\text{sphere}} = \frac{Q}{4\pi\epsilon_0 r^2}\hat{r} \qquad \text{for } r \geq R$$

The Parallel-Plate Capacitor

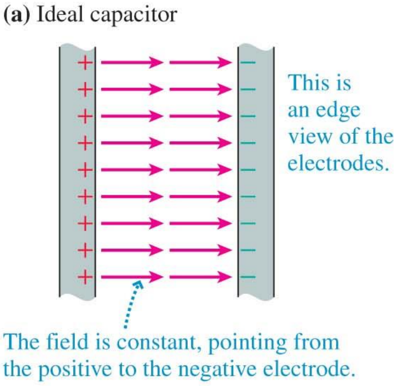
- The figure shows two electrodes, one with charge $+Q$ and the other with $-Q$ placed face-to-face a distance d apart.

•This arrangement of two electrodes, charged equally but oppositely, is called a **parallel-plate capacitor.**



- Capacitors play important roles in many electric circuits.

FIGURE 27.22 The electric field of a capacitor.



The Parallel-Plate Capacitor

The electric field inside a capacitor is

$$\vec{E}_{\text{capacitor}} = \vec{E}_{+} + \vec{E}_{-} = \left(\frac{\eta}{\epsilon_0}, \text{ from positive to negative}\right)$$

$$= \left(\frac{Q}{\epsilon_0 A}, \text{ from positive to negative}\right)$$

where A is the surface area of each electrode. Outside the capacitor plates, where E_+ and E_- have equal magnitudes but *opposite* directions, the electric field is zero.

Motion of a Charged Particle in an Electric Field

The electric field exerts a force

$$\vec{F}_{\text{on } q} = q\vec{E}$$

on a charged particle. If this is the only force acting on q, it causes the charged particle to accelerate with

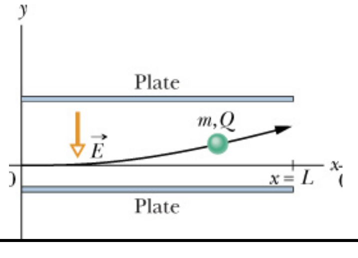
$$\vec{a} = \frac{\vec{F}_{\text{on } q}}{m} = \frac{q}{m}\vec{E}$$

In a uniform field, the acceleration is constant:

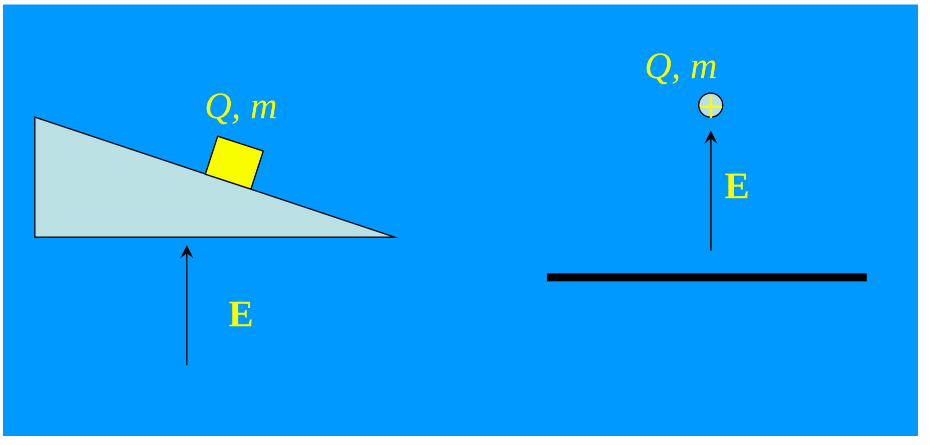
$$a = \frac{qE}{m} = \text{constant}$$

Example 5

The following figure shows the deflecting plates of an ink-jet printer, with superimposed coordinate axes, An ink drop with a mass m of 1.3×10^{-10} kg and a negative charge of magnitude $Q = 1.5\times10^{-13}$ C enters the region between the plates, initially moving along the x-axis with speed $v_x = 18$ m/s. The length L of the plates is 1.6 cm. The plates are charged and thus produce an electric field at all points between them. Assume that field ***E*** is downward directed, uniform, and has a magnitude of 1.4×10^{6} N/C. What is the vertical deflection of the drop at the far edge of the plate? (The gravitational force on the drop is small relative to the electrostatic force acting on the drop and can be neglected.)



Motion of Charged Object in a Uniform Electric Field



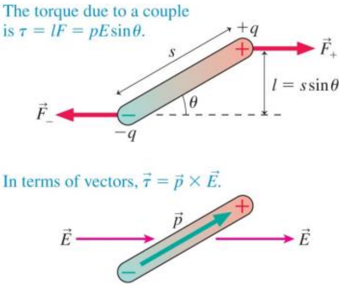
Dipoles in an Electric Field

The torque on a dipole in an electric field is

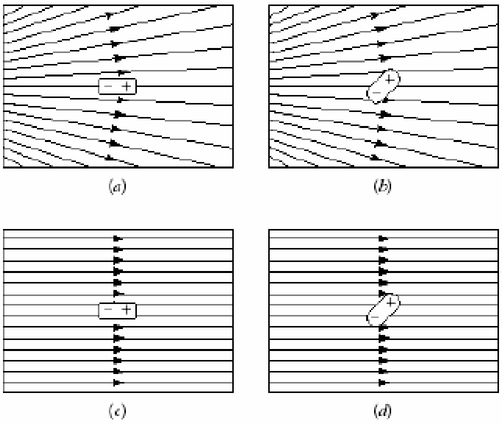
$$\tau = lF = (s\sin\theta)(qE) = pE\sin\theta$$

where θ is the angle the dipole makes with the electric field.

FIGURE 27.30 The torque on a dipole.



3. An electrically neutral dipole is placed in an external field. In which situation(s) is the net force on the dipole zero?



1. (*a*)
2. (*c*)
3. (*b*) and (*d*) combination
4. (*a*) and (*c*)
5. (*c*) and (*d*)
6. some other
7. none of the above

Your answer is _______________. Explain.

Chapter 26. Summary Slides

General Principles

Sources of $\vec{E}$

Electric fields are created by charges.

Two major tools for calculating $\vec{E}$ are

- The field of a point charge:

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\,\hat{r}$$

- The principle of superposition

Multiple point charges

Use superposition: $\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \cdots$

Continuous distribution of charge

- Divide the charge into segments ΔQ for which you already know the field.
- Find the field of each ΔQ.
- Find $\vec{E}$ by summing the fields of all ΔQ.

The summation usually becomes an integral. A critical step is replacing ΔQ with an expression involving a **charge density** (λ or η) and an integration coordinate.

General Principles

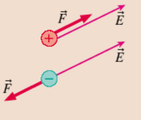
Consequences of $\vec{E}$

The electric field exerts a force on a charged particle:

$$\vec{F} = q\vec{E}$$

The force causes acceleration:

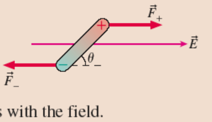
$$\vec{a} = (q/m)\vec{E}$$



Trajectories of charged particles are calculated with kinematics.

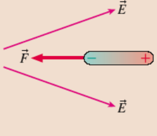
The electric field exerts a torque on a dipole:

$$\tau = pE\sin\theta$$



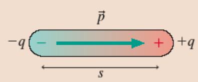
The torque tends to align the dipoles with the field.

In a nonuniform electric field, a dipole has a net force in the direction of increasing field strength.



Applications

Electric dipole



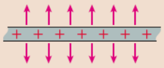
The electric dipole moment is

$\vec{p} = (qs$, from negative to positive$)$

Field on axis: $\vec{E} = \dfrac{1}{4\pi\epsilon_0}\dfrac{2\vec{p}}{r^3}$

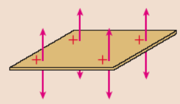
Field in bisecting plane: $\vec{E} = -\dfrac{1}{4\pi\epsilon_0}\dfrac{\vec{p}}{r^3}$

Infinite line of charge with linear charge density λ



$$\vec{E} = \left(\frac{1}{4\pi\epsilon_0}\frac{2\lambda}{r}, \text{ perpendicular to line}\right)$$

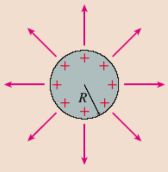
Infinite plane of charge with surface charge density η



$$\vec{E} = \left(\frac{\eta}{2\epsilon_0}, \text{ perpendicular to plane}\right)$$

Sphere of charge

Same as a point charge Q for $r > R$

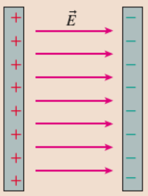


Applications

Parallel-plate capacitor

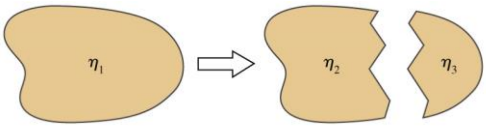
The electric field inside an ideal capacitor is a **uniform electric field:**

$$\vec{E} = \left(\frac{\eta}{\epsilon_0}, \text{ from positive to negative}\right)$$



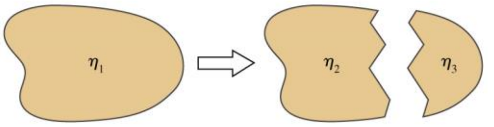
A real capacitor has a weak **fringe field** around it.

Chapter 26. Clicker Questions



A piece of plastic is uniformly charged with surface charge density η_1. The plastic is then broken into a large piece with surface charge density η_2 and a small piece with surface charge density η_3. Rank in order, from largest to smallest, the surface charge densities η_1 to η_3.

A. $\eta_2 = \eta_3 > \eta_1$
B. $\eta_1 > \eta_2 > \eta_3$
C. $\eta_1 > \eta_2 = \eta_3$
D. $\eta_3 > \eta_2 > \eta_1$
E. $\eta_1 = \eta_2 = \eta_3$



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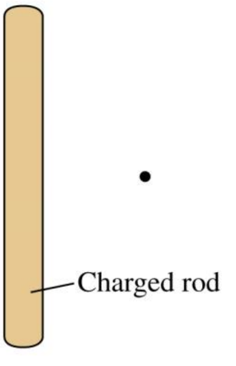
✔ **E. $\eta_1 = \eta_2 = \eta_3$**

Which of the following actions will increase the electric field strength at the position of the dot?



A. Make the rod longer without changing the charge.
B. Make the rod fatter without changing the charge.
C. Make the rod shorter without changing the charge.
D. Remove charge from the rod.
E. Make the rod narrower without changing the charge.

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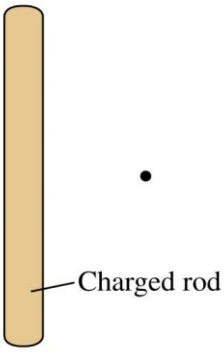

A. Make the rod longer without changing the charge.
B. Make the rod fatter without changing the charge.
✔ **C. Make the rod shorter without changing the charge.**
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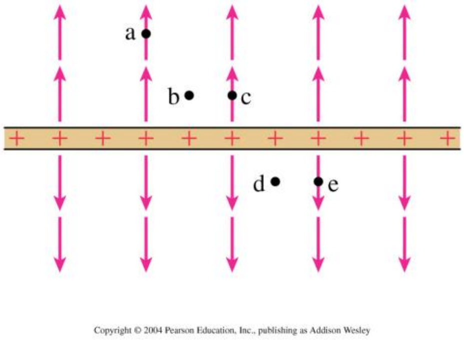
Rank in order, from largest to smallest, the electric field strengths E_a to E_e at these five points near a plane of charge.



A. $E_a > E_c > E_b > E_e > E_d$
B. $E_a = E_b = E_c = E_d = E_e$
C. $E_a > E_b = E_c > E_d = E_e$
D. $E_b = E_c = E_d = E_e > E_a$
E. $E_e > E_d > E_c > E_b > E_a$

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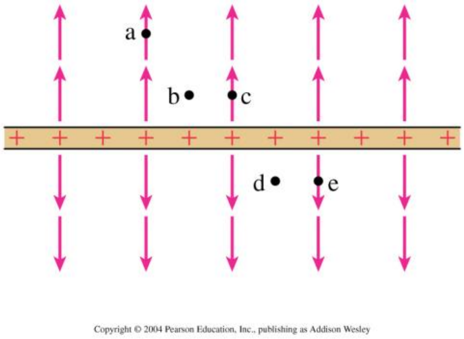

A. $E_a > E_c > E_b > E_e > E_d$
✔ **B. $E_a = E_b = E_c = E_d = E_e$**
C. $E_a > E_b = E_c > E_d = E_e$
D. $E_b = E_c = E_d = E_e > E_a$
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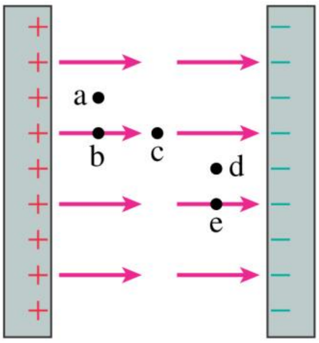
Rank in order, from largest to smallest, the forces F_a to F_e a proton would experience if placed at points a – e in this parallel-plate capacitor.



A. $F_a = F_b = F_d = F_e > F_c$
B. $F_a = F_b > F_c > F_d = F_e$
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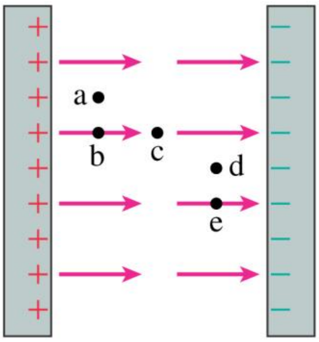

A. $F_a = F_b = F_d = F_e > F_c$
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✔ **C. $F_a = F_b = F_c = F_d = F_e$**
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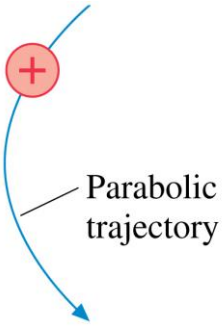
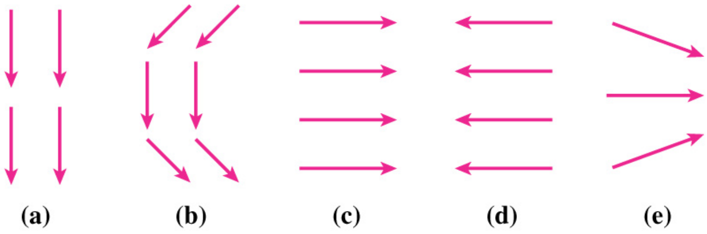
Which electric field is responsible for the trajectory of the proton?





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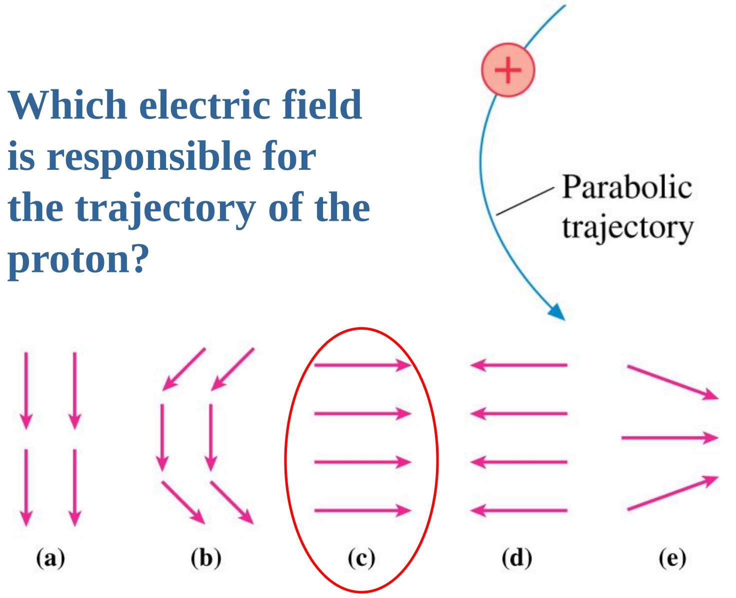

Chapter 26. Reading Quizzes

What device provides a practical way to produce a uniform electric field?

A. A long thin resistor
B. A Faraday cage
C. A parallel plate capacitor
D. A toroidal inductor
E. An electric field uniformizer

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For charged particles, what is the quantity *q*/*m* called?

A. Linear charge density
B. Charge-to-mass ratio
C. Charged mass density
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Which of these charge distributions did *not* have its electric field determined in Chapter 26?

A. A line of charge
B. A parallel-plate capacitor
C. A ring of charge
D. A plane of charge
E. They were *all* determined

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A. A line of charge
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D. A plane of charge
✔ **E. They were *all* determined**

The worked examples of charged-particle motion are relevant to

A. a transistor.
B. a cathode ray tube.
C. magnetic resonance imaging.
D. cosmic rays.
E. lasers.

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