Prerequisites: Symbols and Logic

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1 Math shorthand

Symbol	Meaning
$\forall$	for all, for each, for every, for any, etc.
$\exists$	there exists (at least one), for some
$a \in A$	a is an element of A, a is in A
$\neg$, $\sim$	not
$P \to Q$	if P then Q (as proposition)
$P \Rightarrow Q$	P implies Q (as conclusion or theorem)
$P \Leftrightarrow Q$	P is logically equivalent to Q: (P implies Q) and (Q implies P)
iff	if and only if

2 Logical Operations

There are four basic logical operations: AND, OR, NOT, XOR. The meaning of AND and NOT is the same as in everyday language, but OR and XOR require some explanation. OR (inclusive OR) is the operation that returns TRUE if at least one side holds (is true). This means that for A or B holds if A holds or if B holds or if *both* A and B hold. For example, when I can say »people who speak German or English«, I also include people who speak both languages. On the other hand, if I want to emphasize that I want people who speak *either* German or English, but not both, then I need an XOR. XOR (exclusive OR) is the operation that returns TRUE if exactly one side holds (is true). This means that for A xor B holds if A holds or if B holds but *not* if *both* A and B hold. This is summarized in the following table ('T' for 'True', 'F' for 'False').

P	Q	$\sim P$	$\sim Q$	P and Q	P or Q	P xor Q
F	F	T	T	F	F	F
F	T	T	F	F	T	T
T	F	F	T	F	T	T
T	T	F	F	T	T	F

Comprehension Check: Restate A xor B in terms of the other logical operations (and, or, not).

3 Logical Propositions and their Negation

Given a logical proposition $P \to Q$, we can derive the following three propositions:

Inverse $\sim P \to\ \sim Q$

Converse $Q \to P$

Contrapositive $\sim Q \to\ \sim P$.

The contrapositive is the »inverse of the converse« or »the converse of the inverse«.

Note that $P \to Q \Leftrightarrow \sim Q \to\ \sim P$. However, the inverse and the converse are not implied by the original proposition (although they may still be true).

Now, we wish to consider the opposite of propositions with quantifiers. For universal quantification, negation is achieved by at least one counterexample, i.e. existential quantification of the inverse.

Proposition	Meaning	Negation	Negated Meaning
$\forall x \in X : R$	»for all x in X such that R holds«	$\exists x \in X :\ R$	»there exists an x in X such that R does not hold«
$\exists x \in X : R$	»there exists x in X such that R holds«	$\forall x \in X :\ R$	»for all x in X, R does not hold«
A and B	»A and B both hold«	$\sim A$ or $\sim B$	»At least one of A, B does not hold.«
A or B	»At least one of A, B holds.«	$\sim A$ and $\sim B$	»Both A and B do not hold.«
A xor B	»Exactly one of A, B holds.«	(A and B) or ($\sim A$ and $\sim B$)	»Both A and B hold OR both do not hold: A and B have the same truth value.«