TM Big Research problem of Jo-Jo's bizarre adventure

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November 2022

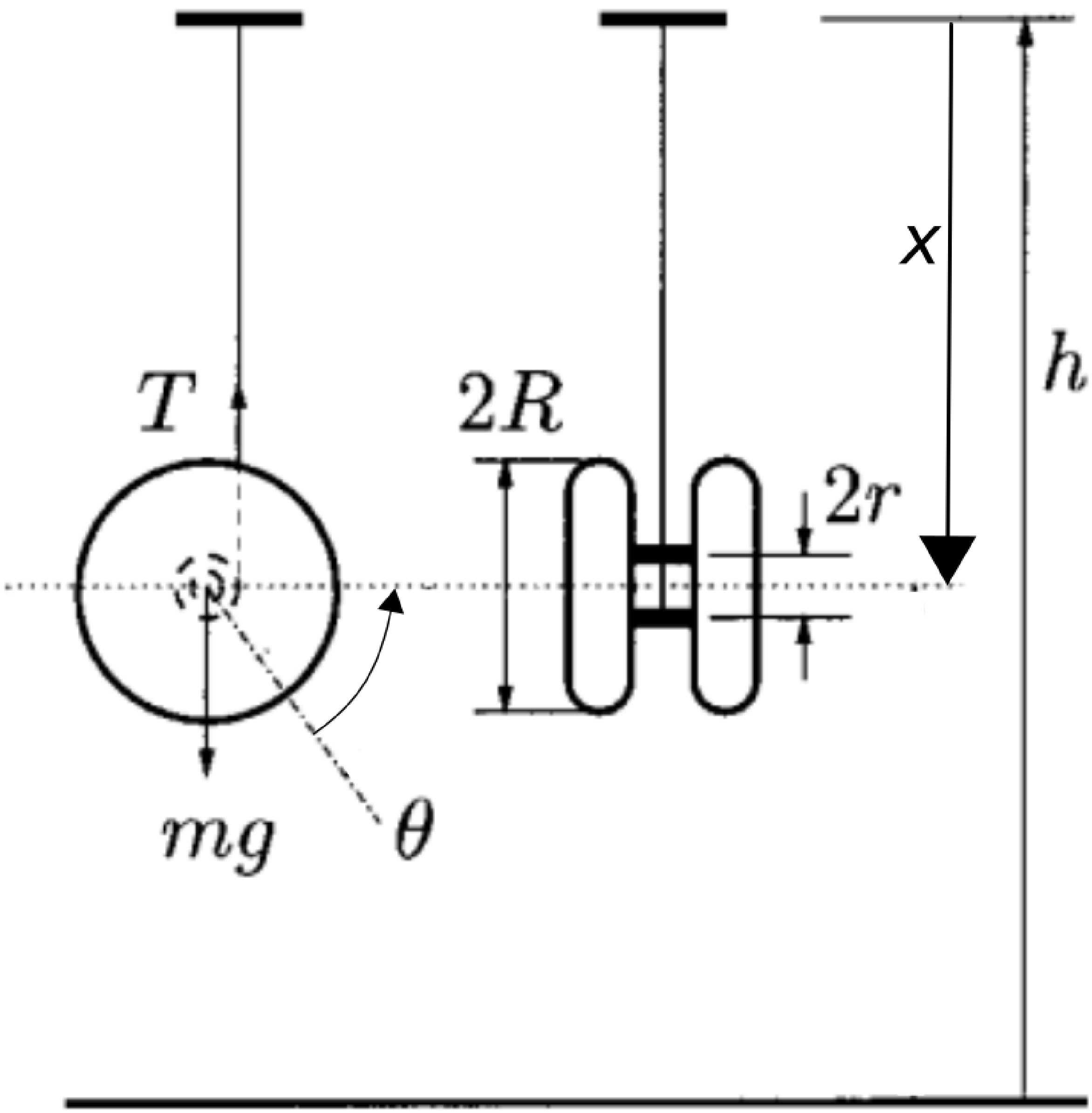


Figure 1: The Task

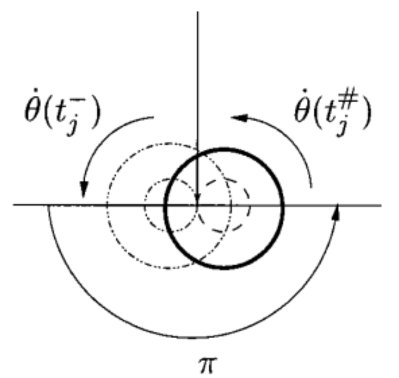
1 RO:

A Yo-yo moves planar
coordinate system : ϕ, y
In the model we make some assumptions:
1. The center of mass of yo-yo moves only in the vertical direction. The direction of the rotational axis is fixed and always orthogonal to the artical axis.
2. The string is flexible but not extensible. Its diameter and mass are negligible. 3. Friction between the string and the inner surfaces of the two disks is proportional to the rotational velocity of the yoyo.

2 Force analysis:

$G = mg$
$J = m^2$
Thus, $\dot{\theta}$ is negative when the yoyo is unwinding and positive when it is winding up the string, even though the direction of rotation does not change
$J\ddot{\phi} + R\epsilon\dot{\theta} = -rT$
$m\ddot{y} = -mg + T$
$f(y, \theta, t) = L - r\theta + y - h(t) \geq 0$
where T is the tension in the string, m and l are the mass and the inertia of the yoyo, respectively, and e is the friction coefficient.

Figure 2: The transition of ϕ at the bottom



2.1 Motion Phases:

There are 3 motion phases:
1 - The string is tight and the system in fact has only one degree of freedom.
$-r\dot{\theta} + \dot{y} - \dot{h} = 0$
2 - Constrained motion phase
$(I + m^2)\ddot{\theta} + R\epsilon\dot{\theta} = -mr(g + \ddot{h}$
3 - Free motion phase
Occurs after the bottom position. $I\ddot{\theta} = 0$
$m\ddot{y} = -mg$
$f(y, \theta, t) = L - r\theta + y - H(t)$ 4 -Bottom phase
The yoyo eventually reaches the end of the string. Before it starts winding up again, the yoyo must rotate by π. No string is wrapped around the axle during this rotation of π, see Fig. 2. Both the rotational and translational velocities tend to keep their initial values because of inertia, so an impact must occur.
Assumption 4. The bottom phase consists of a kinematic rotation by π and a dynamic impact such that
1. The time needed for the rotation by π is negligible.
2. After the rotation by π, both the rotational and translational velocities retain their initial values, respectively.
3. The impact happens immediately after the rotation by π.

3 Kinematics analysis

1. When $T \neq 0$:
$\dot{\phi} = \frac{\dot{y}}{r}$
$h = y - l \geq 0$, where l is the length of the string

2. When $T = 0$:
The values of ϕ and y are independent.

4 Kinematic energy:

$T = \frac{1}{2}m\dot{y}^2 + \frac{1}{2}J\dot{\phi}^2 = \frac{m(\dot{y}^2+\rho^2)}{2r^2}$

5 Solution:

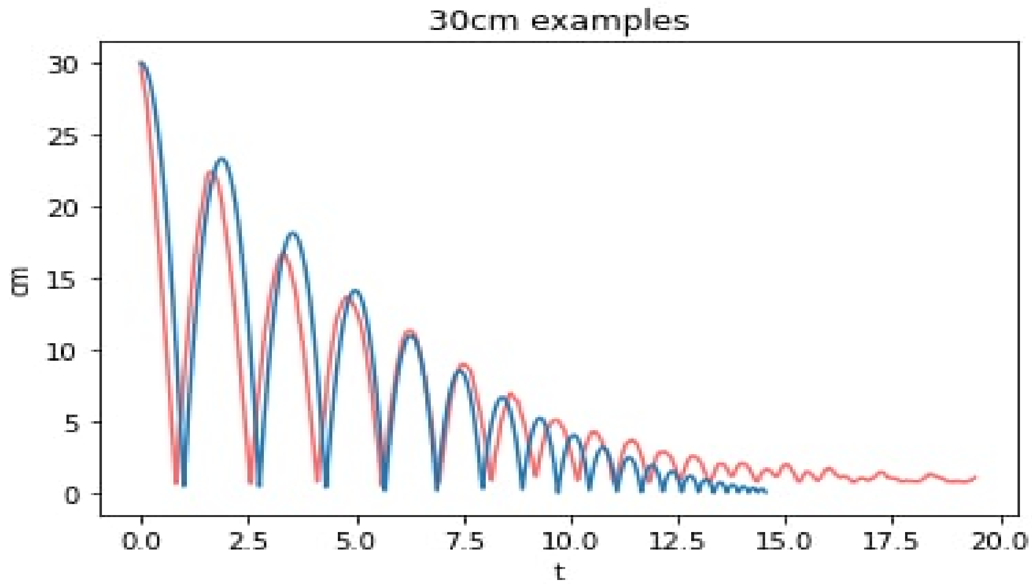
Use Lagrange II order equation:

$$\begin{bmatrix} m & 0 \\ 0 & J \end{bmatrix} \begin{bmatrix} \ddot{y} \\ \ddot{\phi} \end{bmatrix} = \begin{bmatrix} G - T \\ Tr \end{bmatrix}$$

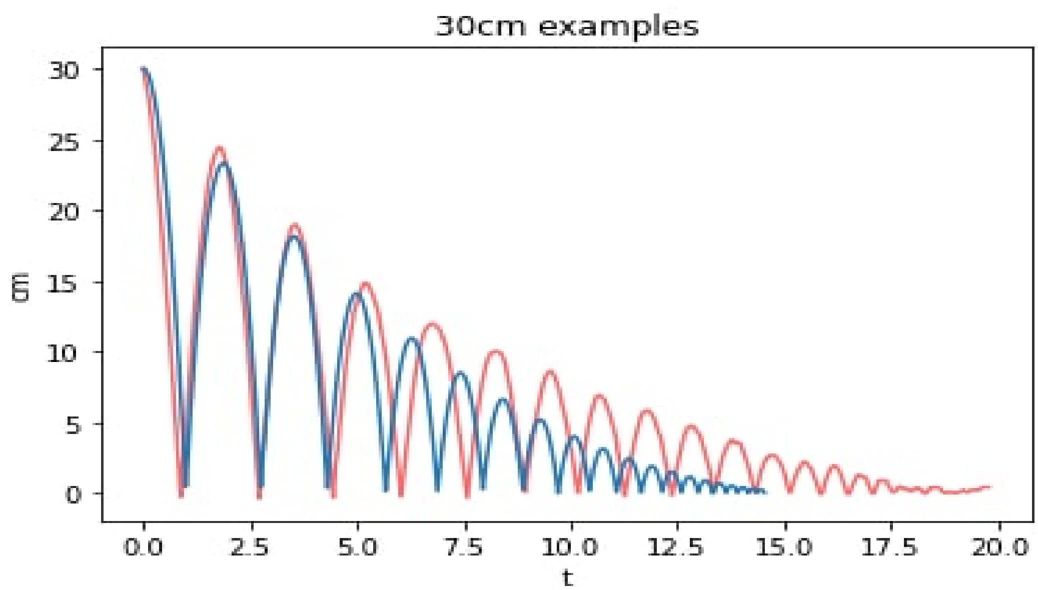
$$\begin{bmatrix} \dot{\theta}(t_k^+) \\ \dot{y}(t_k^+) \end{bmatrix} = \frac{1}{J+mr^2} \begin{bmatrix} J - emr^2 & (1+e)Jr \\ (1+e)Jr & mr^2 - eJ \end{bmatrix} \begin{bmatrix} \dot{\theta}(t_k^-) \\ \dot{y}(t_k^-) \end{bmatrix} + \frac{1+e}{J+mr^2} \begin{bmatrix} -mr \\ J \end{bmatrix} \dot{h}(t_k^+)$$

6 Simulation analysis

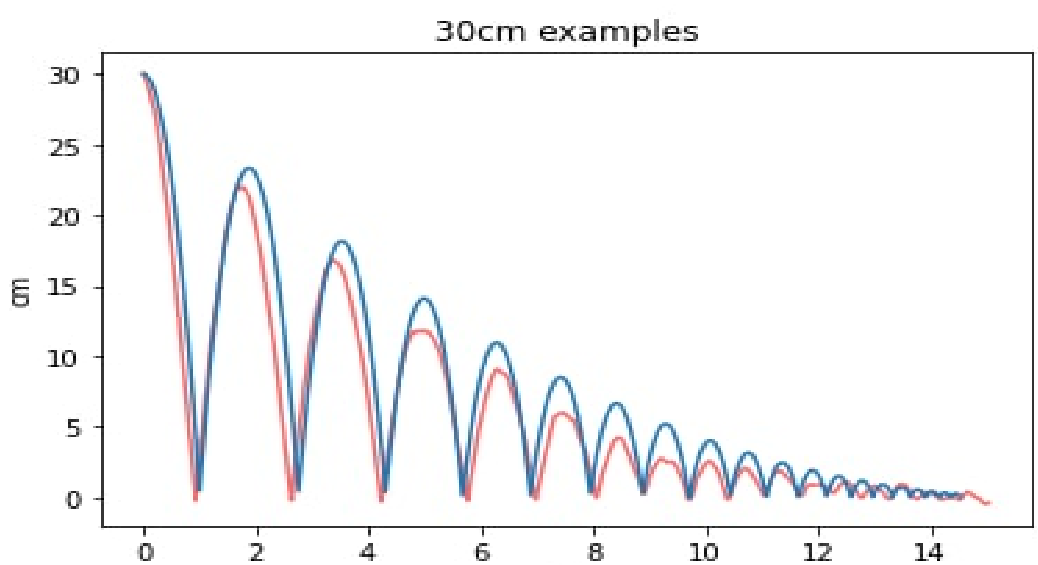
The plots shows that the simulation doesn't perfectly mach the experiment.
So, our experiments shows that the simulation is not perfect. We do not consider energy losses due to the damping effect created in the installation. Part of the energy goes through the rope into the beam on which it is fixed, and then into the closet.
Also, part of the energy goes into rotation around y axis, which occurs because of winding of the string is not ideal, since our model implies that at each moment of time the point of application of the elastic force is in the center, and in the real model it is shifted relative to y axis. Therefore, the rotation occurs in new axes.



(a) First experiment



(b) Second experiment



(c) Third experiment