TM Big Research problem of Jo-Jo's bizarre adventure

Anastasiia Stepanova, Vyacheslav Koshman, Egor Dmitriev

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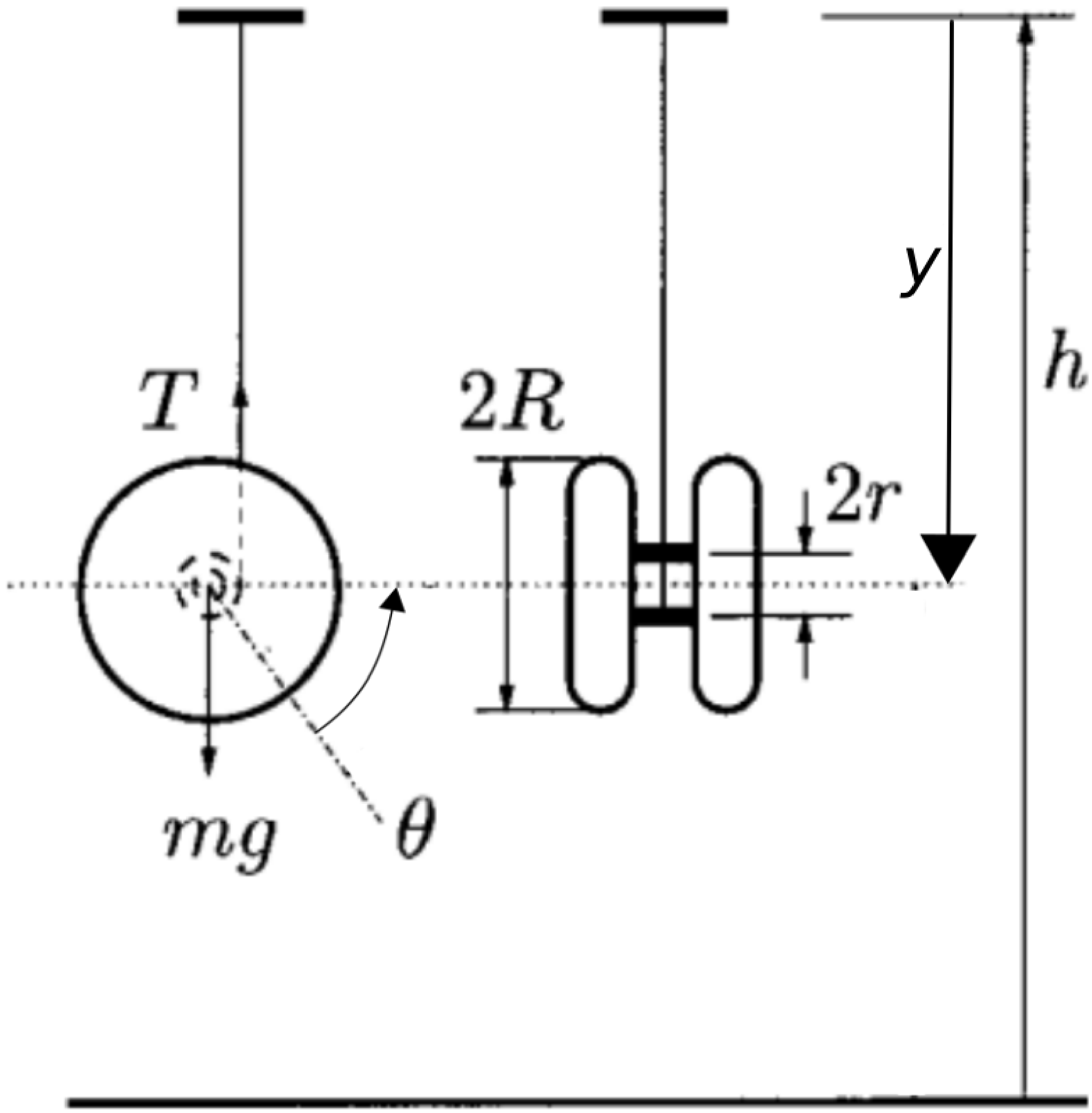


Figure 1: The Task

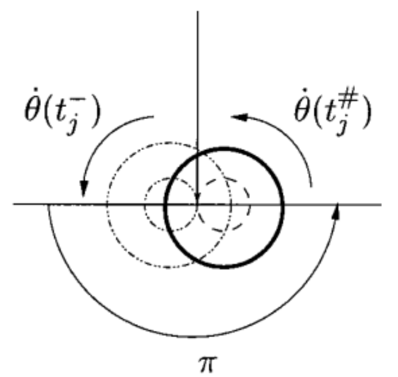
1 RO:

A Yo-yo moves planar
coordinate system : θ, y
The model is based on the article Yoyo Dynamics: Sequence of Collisions Captured by a Restitution Effect [DOI: 10.1115/1.1485750]. In the model they make some assumptions:
1. The center of mass of yo-yo moves only in the vertical direction. The direction of the rotational axis is fixed and always orthogonal to the artical axis.
2. The string is flexible but not extensible. Its diameter and mass are negligible. 3. Friction between the string and the inner surfaces of the two disks is proportional to the rotational velocity of the yoyo.
4. The bottom phase consists of a kinematic rotation by π and a dynamic impact such that
1.The time needed for the rotation by π is negligible.
2.After the rotation by π, both the rotational and translational velocities retain their initial values, respectively.
3.The impact happens immediately after the rotation by p.

2 Kinematics analysis

1. When $T \neq 0$:
$\dot{\theta} = \frac{\dot{y}}{r}$
2. When $T = 0$:
The values of θ and y are independent.

Figure 2: The transition of θ at the bottom



3 Force analysis:

r_g - radius of yo-yo Gyration
$G = mg$
$J = mr_g^2$
Thus, $\dot{\theta}$ is negative when the yoyo is unwinding and positive when it is winding up the string, even though the direction of rotation does not change
$J\ddot{\theta} + R\epsilon\dot{\theta} = -rT$
$m\ddot{y} = -mg + T$
$f(y, \theta, t) = L - r\theta + y - h(t) \geq 0$
where T is the tension in the string, m and l are the mass and the inertia of the yoyo, respectively, and e is the friction coefficient.

3.1 Motion Phases:

There are 3 motion phases:
1 - The string is tight and the system in fact has only one degree of freedom.
$-r\dot{\theta} + \dot{y} - \dot{h} = 0$
2 - Constrained motion phase
$(I + mr_g^2)\ddot{\theta} + R\epsilon\dot{\theta} = -mr(g + \ddot{h})$
3 - Free motion phase
Occurs after the bottom position. $I\ddot{\theta} = 0$
$m\ddot{y} = -mg$
$f(y, \theta, t) = L - r\theta + y - H(t)$ 4 -Bottom phase
The yoyo eventually reaches the end of the string. Before it starts winding up again, the yoyo must rotate by π. No string is wrapped around the axle during this rotation of π, see Fig. 2. Both the rotational and translational velocities tend to keep their initial values because of inertia, so an impact must occur.

Assumption 4. The bottom phase consists of a kinematic rotation by π and a dynamic impact such that
1. The time needed for the rotation by π is negligible.
2. After the rotation by π, both the rotational and translational velocities retain their initial values, respectively.
3. The impact happens immediately after the rotation by π.

4 Solution:

Use Lagrange II order equation:
$\begin{bmatrix}\dot{\theta}(t_k^+)\\ \dot{y}(t_k^+)\end{bmatrix} = \frac{1}{J+mr^2}\begin{bmatrix}J-emr^2 & (1+e)Jr\\ (1+e)Jr & mr^2-eJ\end{bmatrix}\begin{bmatrix}\dot{\theta}(t_k^-)\\ \dot{y}(t_k^-)\end{bmatrix} + \frac{1+e}{J+mr^2}\begin{bmatrix}-mr\\ J\end{bmatrix}\dot{h}(t_k^+)$
That is further simplified to:

$$\begin{aligned}\ddot{\theta} &= -\gamma(g+\ddot{h}), &&\text{for } \theta(t)>0\\ \dot{\theta}(t_j^+) &= -e_{eq}\dot{\theta}(t_j^-), &&\text{for } \theta(t_j)=0\end{aligned}$$

$$y(t) = h(t) - L + r\theta(t)$$

With assumptions that:
1. The transition phases are always complete, i.e., every transition phase ends before the yoyo enters another bottom phase.
2. The restitution coefficient $e = 0$.
3. $\dot{h}(t)$ is always continuous, so that $\sigma_{\dot{h}} = 0$.
Where:
Directly measurable values
1. L - total length of the string
2. r - inner radius of the yoyo
3. m - mass of the yoyo
4. J - moment of inertia of the yoyo
5. $h(t) = 0$ - motion of the hand
Proxy values
1. $\eta \triangleq \frac{mr^2}{I+mr2}$ - characteristic parameter of the yoyo
2. $\gamma \triangleq \frac{\eta}{r}$
3. $e_{eq} = 1 - 2\eta$ - equivalent restitution coefficient

5 The experiment:

Equipment:
- camera 120 fps,
- a beam to which a rope from a yo-yo is attached
- improvised camera stand
- a computer.

Experiments with different yo-yo were not carried out because we did not have enough time, so for simplicity and ease of testing our simulation, we used the yo-yo that seemed to us the most suitable for our purposes (what was described in the task itself, not Egor's Yo-Yo, really sorry)

6 Simulation analysis

6.1 Our model improvement based on real data analysis:

As we can see on Fig.3 this model yields poor results closer to the end of the simulation. It does not account for the time needed to make the turn at the end of the string and the error accumulates. It becomes more explicit when yoyo makes shorter trips to the top and spends relatively more time at

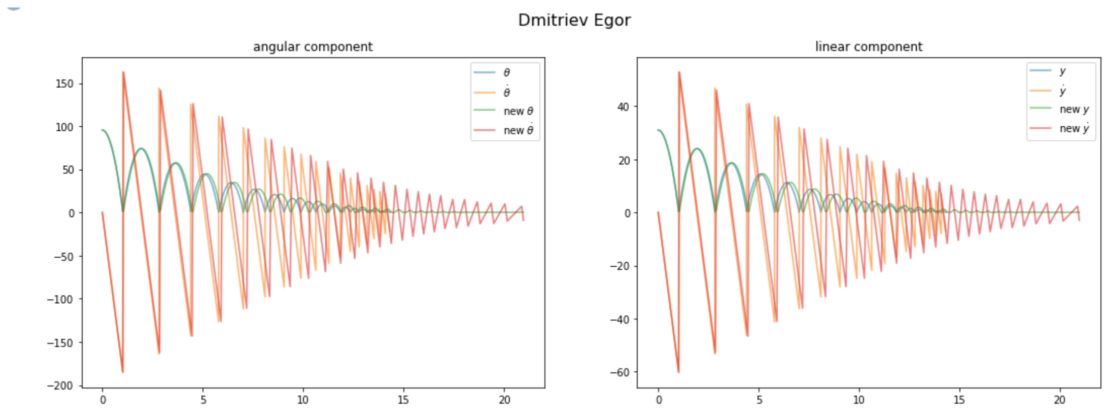


Figure 3: The first simulation model results

the bottom making a turn. Let's propose a correction to the **Assumption 4.1**: let the time needed for the rotation by π be $\frac{\pi}{\dot{\theta}}$ instead of 0. By doing so we will account for the time yoyo makes a turn and reduce the error of our model in later stages of the simulation. By doing so we'll have to correct our equations to the following:

$$\begin{aligned} \ddot{\theta} &= -\gamma(g + \ddot{h}), && \text{for } \theta(t) > 0 \\ \dot{\theta}(t^{+}_{j+\frac{\pi}{|\dot{\theta}|}}) &= -e_{eq}\dot{\theta}(t^{-}_{j}), && \text{for } \theta(t_j) = 0 \end{aligned}$$

$$y(t) = h(t) - L + r\theta(t)$$

The results are on the figure 4.

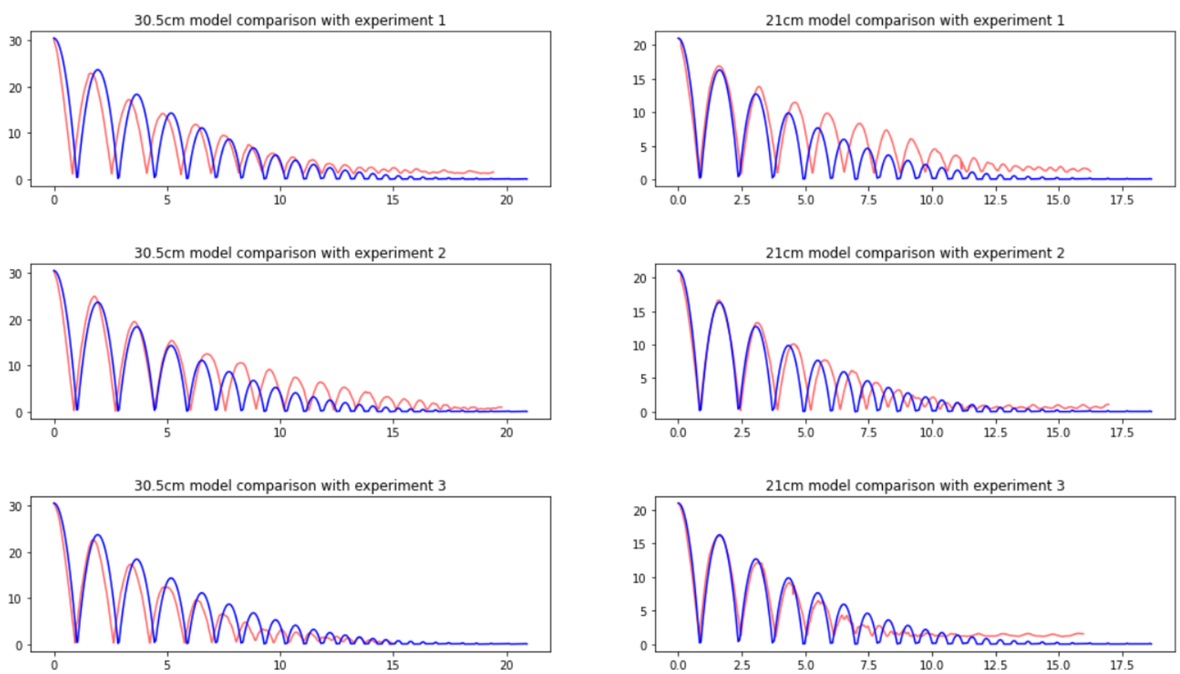


Figure 4: The y coordinate comparison between simulation and experiments

6.2 Why the final simulation is not perfectly match the experimental results:

The plots Fig.4 shows that the simulation doesn't perfectly match with "30.5 cm of rope" experiment. However, the mean value of mean absolute error for all experiments is less then 6 cm, which is good enough for the model. The biggest error appears at last moments of movement because the simulation shows that it continuous without extra energy loss.
So, our experiments shows that the simulation is not perfect, it happens because:
- We do not consider energy losses due to the damping effect created in the installation. Part of the energy goes through the rope into the beam on which it is fixed, and then into the closet.
- Part of the energy goes into rotation around y and x axis, which occurs because of winding of the string is not ideal, since our model implies that at each moment of time the point of application of the elastic force is in the center, when in the real model it is shifted relative to y axis. Therefore, the rotation occurs in new axes.
- Small air resistance takes some of the energy from yo-yo movement.

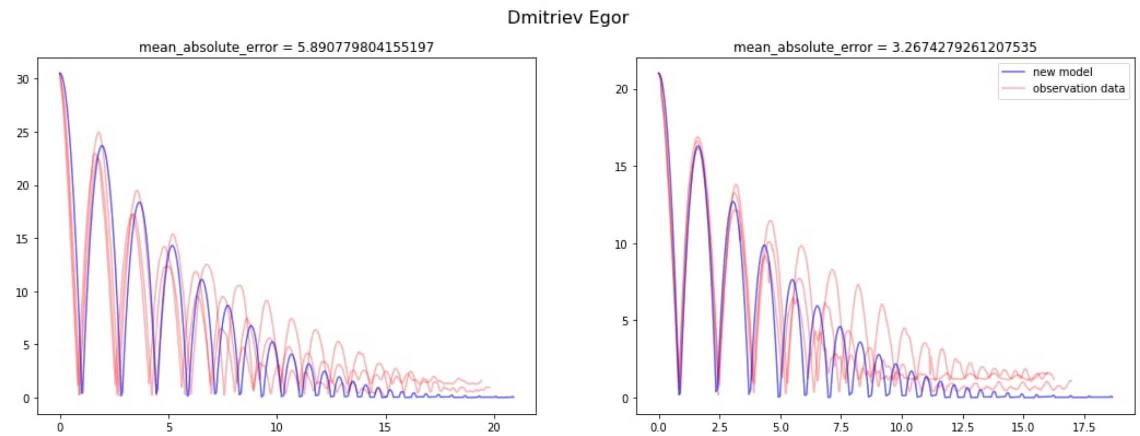


Figure 5: The MAE comparison for different length of the rope

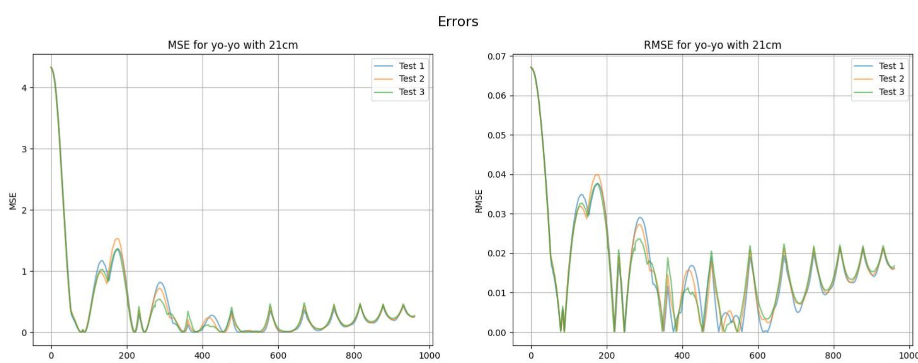


Figure 6: The MSE and RMSE comparison for different length of the rope

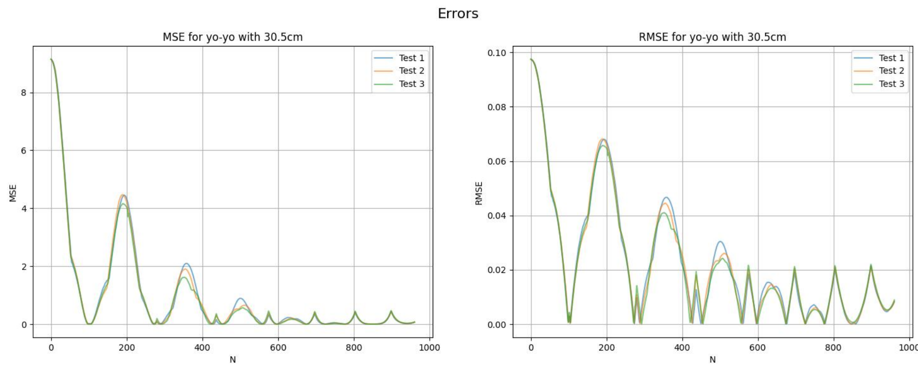


Figure 7: The MSE and RMSE comparison for different length of the rope

Figure 8: Conclusion: No Spin today. Only sleep