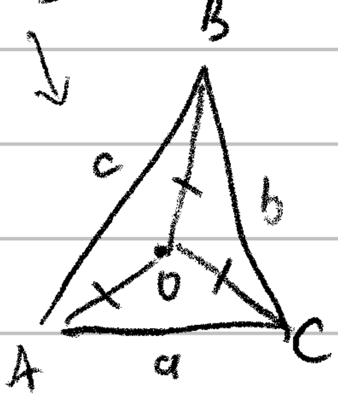
Weighted average of vertices

Barycentric Prerequisites

opposite side naming



O - circumcenter (origin)

↓

all Vectors are equal

i.e.

$|\vec{A}| = R$ (circum radius)

$\vec{u} \cdot \vec{v} = |u||v| \cos \theta$ 

(dot product)

$\vec{A} \cdot \vec{A} = |\vec{A}| \cdot |\vec{A}| \cdot \cos 0 = \ldots$

↓ angle between vector and itself

$= |\vec{A}| \cdot |\vec{A}| \cdot 1 = R^2$

$\cos(0) = 1$ $\quad \cos(90^\circ) = \frac{\pi}{2} = 0$

$(0,1)$
$x \; y$

$\cos \theta = \frac{adj}{hyp}$

$\vec{A} \cdot \vec{B} = |\vec{A}| \cdot |\vec{B}| \cos \theta =$

$= R^2 \cos \angle AOB$ ①

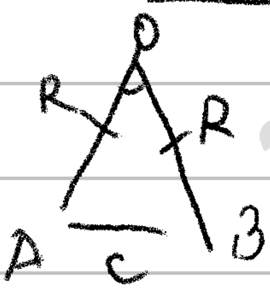
↓

$R^2 \left(\frac{2R^2 - c^2}{2R^2} \right) = \frac{2R^2 - c^2}{2} =$

$= R^2 - \frac{c^2}{2}$

i.e. the smaller is angle ~~the~~ less is $\cos \theta$

when angle is 0 it's 1



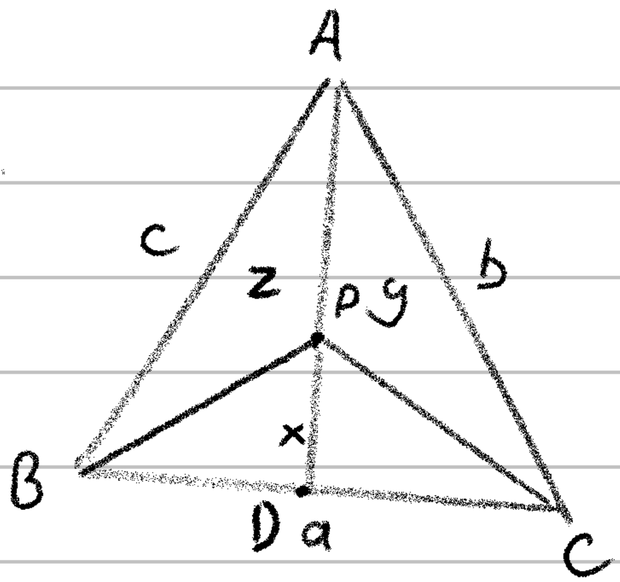
$c^2 = R^2 + R^2 - 2R \cdot R \cdot \cos \theta$

$\cos \theta = \frac{2R^2 - c^2}{2R^2}$ ①

$\cos(\theta) = \cos(2\pi - \theta)$

Names:

Barycentric / Trilinear / Areal coordinates



Counter-clock wise!

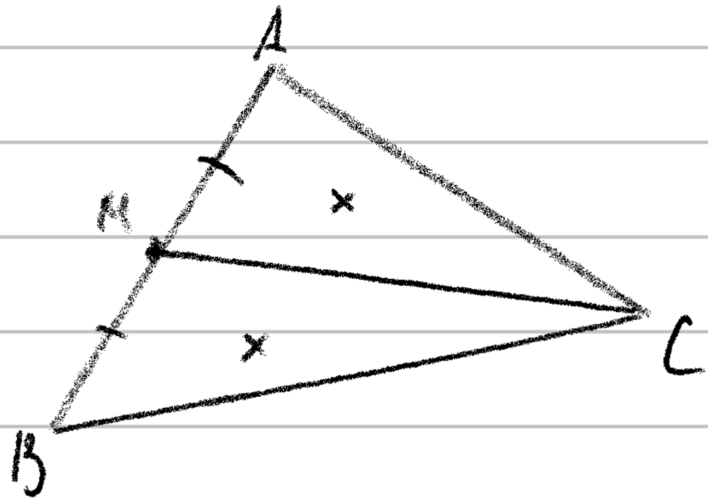
$x = \frac{[PBC]}{[ABC]}$ ([ ] - signed area)

$P = (x, y, z) \rightarrow x + y + z = 1$ (or a,b,c which are weights)

$\vec{P} = x\vec{A} + y\vec{B} + z\vec{C}$ (for arbitrary origin)

$A = (1,0,0) \quad B = (0,1,0) \quad C = (0,0,1)$

$\frac{BD}{DC} = \frac{z}{y}$ i.e. ray projected from A to a divides a to parts with ratio equal to $\frac{z}{y}$



Find mid point

$M = (x, x, 0)$

$x + x = 1$

$x = \frac{1}{2}$

$(\frac{1}{2}, \frac{1}{2}, 0) \rightarrow$ coordinates of a mid point M

> If a point lies on a side - one of the triangles it makes a triangle degenerate so it will have 0 coord

> If a point is outside of a triangle then it must have negative coordinate

T1

Equation of a line is $ux + vy + wz = 0$

for constants u, v, w

every line in a projective plane has a linea equation of this form

<u>Find BC</u>

1. $x = 0$ (because the point is on the line)

2. $B = (0, 1, 0)$ (see before)

$u \cdot 0 + v \cdot 1 + w \cdot 0 = 0 \quad v = 0$

$C = (0, 0, 1) \quad w = 0$

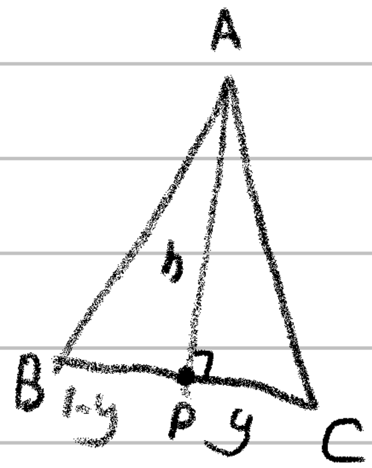
$ux = 0$ (because above here in B, C x always 0)

$x = 0 \rightarrow y + z = 1$ (because $x + y + z = 1$)

3. $P \in BC \quad P = (0, y, 1-y)$

$\boxed{\frac{PC}{BC} = y}$

↓ because



$y = \frac{[PAC]}{[ABC]}$

AP is h for both PAC and ABC

$y = \frac{(h/2) PC}{(h/2) BC} = \frac{PC}{BC}$

ℓ (line) passes through A $\begin{matrix} 1 & 0 & 0 \end{matrix}$

$u = 0$

$vy + wz = 0$

$y = -\frac{w}{v} z$

$y = kz$ (k is just constant for $-\frac{w}{v}$)
(which is actually slope intercept
$y = mx + b$ for cartesian)

i.e. equation of any line through
A is $y = kz$
in other words y and z
are directly proportional

[tʃ]

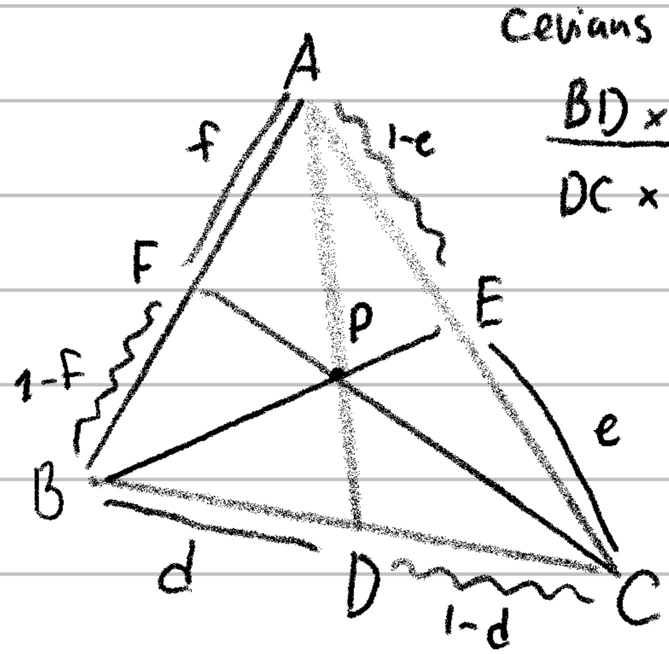
Ceva's Theorem

if and only if

AD, BE, CF - cevians of ABC

cevians concur iff:

$$\frac{BD \times CE \times AF}{DC \times EA \times FB} = 1$$



AD: $y = \frac{1-d}{d} z$ (see $y = \frac{DC}{BC}$)

k

Plan:

- Finalize this sequence in a separate sheet (mark video)
- Write better explanation from chat GPT + websites