Algebraic Causality Theory (ACT): From the Dirac Operator Spectrum to the Nature of Dark Energy and Fundamental Constants

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February 18, 2026

Abstract

This work presents a systematic exposition of the mathematical apparatus of Algebraic Causality Theory (ACT) — a candidate for a theory of everything in which causality emerges as the primary concept. The first part provides a detailed analysis of the Dirac operator spectrum within the octant model, its connection to fermion mass hierarchies and the number of generations through the topological index. The second part is devoted to cosmological consequences: topological modes between light cones are interpreted as dark energy, allowing us to derive its density and equation of state. The culmination of this work is the derivation of the fine structure constant $\alpha^{-1} = 136.7 \pm 0.9$ from the first principles of causal hypergraph dynamics. This value, coinciding with experiment to within 0.24% accuracy, demonstrates the predictive power of ACT and its ability to connect the geometry of spacetime, quantum field theory, and observed cosmology. Numerous correlations of α with other fundamental parameters are shown, opening pathways for experimental verification of the theory.

Contents

1 Definition of the Dirac Operator in the Octant Structure

In each octant i, the Dirac operator is given by:

$$D_i = \gamma^\mu \nabla_\mu^{(i)} + m_i, \tag{1}$$

where:

- γ^μ are gamma matrices (satisfying the Clifford algebra $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$);
- $\nabla_\mu^{(i)}$ is the covariant derivative in octant i, including gauge fields:

$$\nabla_\mu^{(i)} = \partial_\mu + ig_i A_\mu^a T^a + \Gamma_\mu, \tag{2}$$

- m_i is the mass in octant i (may be a function of coordinates);
- T^a are generators of the gauge group;
- Γ_μ is the spin connection (accounting for spacetime curvature).

2 Dirac Operator Spectrum

The spectrum of D_i is determined by eigenvalues $\lambda_k^{(i)}$ and eigenfunctions $\psi_k^{(i)}$ satisfying:

$$D_i \psi_k^{(i)} = \lambda_k^{(i)} \psi_k^{(i)}. \tag{3}$$

Key spectral properties:

- Discrete spectrum in compact octants (if octants are finite).
- Continuous spectrum for infinite octant sizes.
- Zero modes ($\lambda_k = 0$) correspond to massless fermions.
- Massive modes ($|\lambda_k| > 0$) yield massive fermions.

2.1 Eigenvalue Equation

Explicitly, the Dirac equation for octant i is:

$$(\gamma^\mu \nabla_\mu^{(i)} + m_i)\psi_k^{(i)} = \lambda_k^{(i)} \psi_k^{(i)}. \tag{4}$$

For free space (without gauge fields and curvature):

$$(i\gamma^\mu \partial_\mu - m_i)\psi_k^{(i)} = \lambda_k^{(i)} \psi_k^{(i)}. \tag{5}$$

2.2 Spectral Problem in a Compact Octant

If octant i has finite volume V_i, the spectrum becomes discrete. The eigenfunctions $\psi_k^{(i)}$ form a basis in the Hilbert space $\mathcal{H}_i$.

2.2.1 Example: Cubic Octant

Let the octant be a cube with side L. Boundary conditions:

- periodic: $\psi(x+L) = \psi(x)$;
- antiperiodic: $\psi(x+L) = -\psi(x)$.

Then the momentum spectrum is $p_n = \frac{2\pi n}{L}$ (periodic) or $p_n = \frac{(2n+1)\pi}{L}$ (antiperiodic), where $n \in \mathbb{Z}^3$.

Eigenvalues:

$$\lambda_k^{(i)} = \pm\sqrt{p_n^2 + m_i^2}. \tag{6}$$

2.3 Connection to Standard Model Fermions

SM fermion fields arise as Dirac zero modes ($\lambda_k \approx 0$). Three generations correspond to three configurations of connections between octants:

- First generation: lowest energy modes, weakly coupled between octants.
- Second generation: intermediate energy modes, moderate coupling between octants.
- Third generation: highest energy modes, strong coupling between octants.

Fermion mass matrix:

$$M_{ij} = \sum_k c_{ik} c_{jk}^* \lambda_k, \tag{7}$$

where c_{ik} are expansion coefficients in Dirac modes.

2.4 Influence of the Central Point (ϕ_0)

The condensate $\langle \phi_0 \rangle = v \neq 0$ modifies the Dirac operator:

$$D_i^{\text{cond}} = \gamma^\mu \nabla_\mu^{(i)} + (m_i + y_i v), \tag{8}$$

where y_i is the coupling constant to the condensate.

This leads to:

- spectrum splitting between octants;
- fermion mass generation via the Higgs mechanism;
- fixation of relative phases between octants.

3 Topological Aspects

The index of the Dirac operator $\text{ind}(D_i)$ determines the number of zero modes:

$$\text{ind}(D_i) = n_+ - n_-, \tag{9}$$

where:

- n_+ is the number of right zero modes;
- n_- is the number of left zero modes.

In the octant model:

- a nonzero index arises from nontrivial topology of connections between octants;
- the index is related to the number of fermion generations.

4 Renormalization Group and Spectral Flow

As the energy scale μ changes, the spectrum evolves according to RG equations:

$$\frac{d\lambda_k(\mu)}{d\ln\mu} = \beta(\lambda_k), \tag{10}$$

where $\beta(\lambda_k)$ is the beta function, depending on:

- gauge couplings;
- interactions with the condensate ϕ_0;
- topology of the octant network.

Fixed points of the RG flow correspond to observed SM coupling constants.

4.1 Example: Spectrum in Two Octants

Consider a simplified case of two octants ($i = 1, 2$) with masses m_1, m_2 and coupling ϵ between them. Matrix representation of the Dirac operator:

$$D = \begin{pmatrix} D_1 & \epsilon \\ \epsilon & D_2 \end{pmatrix}. \tag{11}$$

Eigenvalues:

$$\lambda_\pm = \frac{\lambda_1 + \lambda_2}{2} \pm \sqrt{\left(\frac{\lambda_1 - \lambda_2}{2}\right)^2 + \epsilon^2}, \tag{12}$$

where λ_1, λ_2 are eigenvalues of D_1, D_2.

Conclusions:

- coupling ϵ splits degenerate levels;
- small ϵ yields nearly massless fermions;
- large ϵ corresponds to heavy states.

4.2 Table: Key Characteristics of the Dirac Spectrum

Parameter	Physical Interpretation	Impact on SM
$\lambda_k = 0$	Massless fermions	Neutrinos, electrons (as $m \to 0$)
$\mathrm{ind}(D_i) \neq 0$	Topological modes	Number of generations
Spectrum splitting	Symmetry breaking	Mass hierarchies
RG flow $\lambda_k(\mu)$	Evolution of couplings	Weinberg angle, α_s

4.3 Critical Tests of the Spectrum

The model is falsified if:

- RG coefficients do not match G_{oct} predictions (e.g., deviation of $\alpha_s(M_Z)$ from 0.118).
- New particles are not detected at the Λ_{ACT} scale (e.g., heavy fermions from high Dirac modes).
- Gravitational effects contradict predictions (e.g., absence of oscillations in the gravitational wave spectrum).

5 Topological Modes Between Light Cones and Dark Energy

5.1 Light Cones and Reality Slices

The light cone is a hypersurface in spacetime separating events into:

- timelike (inside the cone) — causally connected to the observer;
- spacelike (outside the cone) — not causally connected.

A reality slice is a spacelike hypersurface of constant time on which physical fields are defined. In ACT, this corresponds to:

- a fixed moment in the causal chronon network;
- a configuration of octants with a definite Dirac spectrum.

5.2 Topological Modes Between Light Cones

Topological modes are stable field configurations arising from nontrivial spacetime topology between light cones. They:

- cannot be eliminated by continuous deformations;
- are localized on boundaries of causal regions;
- connect remote regions through quantum correlations.

5.2.1 Mathematical Description

Let M be spacetime, C_p the light cone at point $p \in M$. Consider the region between two light cones C_{p_1} and C_{p_2}.

A topological mode Ψ_{top} is defined as a solution of the Dirac equation:

$$D\Psi_{\text{top}} = 0, \tag{13}$$

satisfying nontrivial boundary conditions on $\partial C_{p_1} \cup \partial C_{p_2}$.

The index of the Dirac operator in this region:

$$\text{ind}(D) = n_+ - n_- \neq 0, \tag{14}$$

where n_+ and n_- are the numbers of right and left zero modes. A nonzero index guarantees the existence of topological solutions.

5.3 Connection to Dark Energy

Dark energy is interpreted as vacuum energy associated with topological modes between light cones. Its density ρ_Λ is determined by:

- Horizon difference:
$$\Delta\mathcal{H} = \mathcal{H}_{\text{true}} - \mathcal{H}_{\text{visible}}. \tag{15}$$
- Energy of topological modes:
$$\rho_\Lambda \sim \sum_{k \in \Delta\mathcal{H}} |\lambda_k|^2. \tag{16}$$

- Topological contribution to the action:

$$S_{\text{top}} = \int_{\Delta\mathcal{H}} \text{Tr}(F \wedge \star F) + \text{CS}, \tag{17}$$

where F is the gauge field strength; CS is the Chern-Simons term.

5.3.1 Formation Mechanism

Stages of topological mode formation:

1. Chronon condensation at $\langle \phi_0 \rangle = v \neq 0$ creates macroscopic causal order.
2. Phase transition leads to the formation of light cones as boundaries of causal regions.
3. Nontrivial topology of the octant network between cones generates stable Dirac modes.
4. Quantum fluctuations amplify correlations between remote regions.
5. Energy accumulation in topological modes manifests as dark energy.

5.3.2 Equation of State

The equation of state parameter for dark energy:

$$w = \frac{p_\Lambda}{\rho_\Lambda}, \tag{18}$$

where pressure p_Λ is related to topological density:

$$p_\Lambda \sim -\frac{1}{V}\frac{\partial E_{\text{top}}}{\partial V}. \tag{19}$$

For stable topological configurations, $w \approx -1$, corresponding to a cosmological constant.

5.4 Mathematical Formulation in the Octant Model

In each octant i, the topological mode Ψ_i^{top} satisfies:

$$(\gamma^\mu \nabla_\mu^{(i)} + m_i + A_i)\Psi_i^{\text{top}} = 0, \tag{20}$$

where A_i is a topological potential arising from connections between octants.

The global topological mode is constructed as a superposition:

$$\Psi_{\text{top}} = \sum_{i=1}^{8} c_i \Psi_i^{\text{top}}, \tag{21}$$

with coefficients c_i determined by the network topology.

Energy density:

$$\rho_\Lambda = \frac{1}{8}\sum_{i=1}^{8} \left\langle \Psi_i^{\text{top}} | D_i^2 | \Psi_i^{\text{top}} \middle| \Psi_i^{\text{top}} | D_i^2 | \Psi_i^{\text{top}} \right\rangle. \tag{22}$$

5.5 Observational Consequences

5.5.1 Model Predictions

- **Dark energy anisotropies:** topological modes create fluctuations $\delta\rho_\Lambda/\rho_\Lambda \sim 10^{-5}$, observable in the CMB.
- **Vacuum oscillations:** topological modes may oscillate with frequencies $f \sim 10^{-18}$–10^{-16} Hz, affecting gravitational waves.
- **Large-scale correlations:** topological connections between light cones lead to anomalous correlations in galaxy distribution.
- **Evolution of $w(z)$:** dependence of the equation of state parameter on redshift z due to changing octant network topology.

5.5.2 Experimental Tests

- Cosmology: comparison of $\Omega_\Lambda^{\text{theory}}$ with Planck and DES data.
- Gravitational waves: search for oscillations in LIGO/Virgo spectra.
- Large-scale structure: analysis of correlation functions from SDSS and Euclid.
- Cosmic rays: anomalies in the ultra-high-energy particle spectrum (Pierre Auger Observatory).

5.6 Table: Connection Between Topological Modes and Dark Energy

Parameter	Mathematical Expression	Observed Value
Density ρ_Λ	$\sum_{k\in\Delta\mathcal{H}} \lambda_k^2$	$(2.3 \pm 0.1) \times 10^{-3}$ eV^4
Parameter w	$-\frac{1}{V}\frac{\partial E_{\text{top}}}{\partial V}/\rho_\Lambda$	-1.03 ± 0.03
Anisotropy	$\delta\rho_\Lambda/\rho_\Lambda$	$< 10^{-4}$
Correlation scale	$\xi_{\text{corr}} \sim L_{\text{octant}}$	~ 100 Mpc

6 Mathematics of the Dirac Operator Index

6.1 Basic Definitions

The Dirac operator D is a first-order elliptic differential operator acting on spinor fields ψ:

$$D\psi = \gamma^\mu \nabla_\mu \psi. \tag{23}$$

The index of the Dirac operator is defined as the difference between the dimensions of the kernels of the operator and its adjoint:

$$\text{ind}(D) = \dim\ker D - \dim\ker D^\dagger. \tag{24}$$

6.2 Zero Modes and Chirality

Zero modes are solutions of the Dirac equation with zero eigenvalue:

$$D\psi_0 = 0. \tag{25}$$

In chiral theories, zero modes are divided into:

- left-handed (ψ_0^+), satisfying $\gamma^5\psi_0^+ = +\psi_0^+$;
- right-handed (ψ_0^-), satisfying $\gamma^5\psi_0^- = -\psi_0^-$.

The index can then be rewritten as:

$$\mathrm{ind}(D) = n_+ - n_-. \tag{26}$$

6.3 Atiyah–Singer Index Theorem

The fundamental result connects the analytic index with the topology of the manifold:

$$\mathrm{ind}(D) = \int_M \hat{A}(TM) \wedge \mathrm{ch}(E), \tag{27}$$

where:

- $\hat{A}(TM)$ is the Euler-Pontryagin class (depends on the curvature of manifold M);
- $\mathrm{ch}(E)$ is the Chern class (depends on the gauge field on bundle E).

For 4-dimensional spacetime, this simplifies to:

$$\mathrm{ind}(D) = -\frac{1}{8\pi^2}\int_M \mathrm{tr}(F \wedge F), \tag{28}$$

where $F = dA + A \wedge A$ is the gauge field strength.

6.4 Index Calculation in the Octant Model

Within the octant model:

- manifold M is a network of octants with boundaries between light cones;
- gauge field A represents connections between octants;
- curvature F represents topological defects of the network.

6.4.1 Step-by-Step Calculation

1. **Zero mode localization:** For each octant i, solve $D_i\psi_0^{(i)} = 0$ with boundary conditions.
2. **Chiral mode counting:** Determine $n_+^{(i)}$ and $n_-^{(i)}$ for each octant.
3. **Global index:** Sum contributions from all octants:
$$\mathrm{ind}_{\mathrm{global}} = \sum_{i=1}^{8}(n_+^{(i)} - n_-^{(i)}). \tag{29}$$
4. **Topological contribution:** Account for connections between octants via the Chern-Simons integral:
$$\Delta\mathrm{ind} = \frac{1}{4\pi}\int_{\partial M} \mathrm{CS}(A), \tag{30}$$
where ∂M represents boundaries between light cones.

6.5 Example: Flat Case (2+1 Dimensions)

Consider a simplified case — 2 spatial dimensions + time. Let:

- octant be a square with side L;
- gauge field be constant strength $F_{12} = B$.

Then:

- number of zero modes: $n_+ = \frac{|B|L^2}{2\pi}, n_- = 0$;
- index: $\mathrm{ind}(D) = \frac{|B|L^2}{2\pi}$.

This corresponds to the Landau effect in a magnetic field.

6.6 General Formula for the Octant Network

For an octant network between light cones, the index is given by:

$$\mathrm{ind}(D) = \underbrace{\sum_{i=1}^{8} \mathrm{ind}_i}_{\text{octant contributions}} + \underbrace{\frac{1}{2\pi} \sum_{\langle ij \rangle} \oint_{\Gamma_{ij}} A}_{\text{connections between octants}}, \tag{31}$$

where ind_i is the index for a single octant, Γ_{ij} is the boundary between octants i and j, and the sum $\langle ij \rangle$ runs over all neighboring octants.

6.7 Connection to Dark Energy

Topological modes between light cones contribute to vacuum energy:

$$\rho_\Lambda \sim \frac{\hbar c}{L^4} \cdot |\mathrm{ind}(D)|, \tag{32}$$

where L is the characteristic scale of the octant network.

Key consequences:

- nonzero index $\Rightarrow$ nonzero ρ_Λ;
- index evolution during universe expansion $\Rightarrow$ possible change in $w(z)$;
- index anisotropies $\Rightarrow$ dark energy fluctuations.

6.8 Index Calculation Methods

6.8.1 Heat Kernel Method

Using the asymptotics of the kernel of $e^{-tD^\dagger D}$ as $t \to 0$:

$$\mathrm{ind}(D) = \lim_{t \to 0} \mathrm{tr}\Big(\gamma^5 e^{-tD^\dagger D}\Big). \tag{33}$$

This yields a local formula through curvature and field strength.

6.8.2 Getzler Method

Analytic derivation of the local index formula without topological tools:

$$\operatorname{ind}(D) = \int_M \left[\frac{(\hat{A}(R))}{(2\pi i)^n} \wedge \operatorname{ch}(F) \right]_{\text{top}}, \tag{34}$$

where the subscript "top" means taking the highest-degree differential form.

6.8.3 Atiyah–Patodi–Singer Theory

For manifolds with boundary (light cone boundaries):

$$\operatorname{ind}(D) = \int_M \hat{A}(R) \wedge \operatorname{ch}(F) + \eta(\partial M), \tag{35}$$

where $\eta(\partial M)$ is the eta invariant of the boundary, accounting for boundary conditions.

6.9 Physical Interpretation in ACT

Mathematical Expression	Physical Meaning
$\operatorname{ind}(D) = n_+ - n_-$	Number of fermion generations
$\Delta\text{ind} = \frac{1}{2\pi} \oint A$	Coupling strength between octants
$\rho_\Lambda \sim \frac{\hbar c}{L^4} \operatorname{ind}(D)$	Dark energy density
$w = -1 + \frac{d \ln \operatorname{ind}(D)}{d \ln a}$	Evolution of w parameter

6.10 Critical Tests

The model is falsified if:

- RG flow does not reproduce observed coupling constants for given $\operatorname{ind}(D)$.
- Number of generations differs from 3 (requires $\operatorname{ind}(D) = \pm 3$).
- Dark energy anisotropies exceed observational limits ($\delta\rho_\Lambda/\rho_\Lambda < 10^{-4}$).
- Evolution of $w(z)$ contradicts Supernova Cosmology Project data.

7 Algebraic Causality Theory (ACT)

ACT is a fundamental physical theory in which causality (the "earlier-later" relation) is the primary concept, while spacetime, quantum fields, and physical laws emerge as statistical regularities in a network of causal relations.

7.1 Basic Postulates

- **Primacy of causal structure:** the fundamental object is the chronon (elementary event); chronons are connected by causal relations $\prec$ ("earlier than"); the chronon network forms a causal hypergraph.
- **Algebraic description:** causality is formalized through an algebra of distinction (discrete structures, operations on events); spacetime and fields are emergent concepts arising from macroscopic averaging.

- **Discrete geometry:** no a priori spacetime; it emerges from the topology of the causal network; Planck scales set the minimum interval between chronons.
- **Emergence of physics:** the Standard Model (SM) and General Relativity (GR) are derived as low-energy approximations; SM symmetries ($SU(3)_C \times SU(2)_L \times U(1)_Y$) are not postulated but arise dynamically.

7.2 Key Mathematical Structures

7.2.1 Chronon τ

Described by a state in a 9-dimensional complex Hilbert space:

$$\mathcal{H}_\tau \cong \mathbb{C}^4_+ \otimes \mathbb{C}^4_- \otimes \mathbb{C}, \tag{36}$$

where:

- $\mathbb{C}^4_+$ is the future subspinor (causal connections);
- $\mathbb{C}^4_-$ is the past subspinor (gravitational memory);
- $\mathbb{C}$ is the scalar component (order parameter ϕ_0).

7.2.2 Causal Hypergraph $\mathcal{G}$

- vertices are chronons τ_i;
- edges are causal relations $\tau_i \prec \tau_j$;
- hypergraph topology determines spacetime geometry.

7.2.3 Fundamental Symmetry G_τ

$$G_\tau = \Gamma_{SU(3)\times SU(2)\times U(1)}, \tag{37}$$

where Γ is a discrete subgroup accounting for network structure.

7.3 Mathematics of the Causal Hypergraph in ACT

7.3.1 Definition of Causal Hypergraph

A causal hypergraph Γ is a pair (V, E), where:

- V is the set of vertices (elementary events, chronons);
- $E \subset \mathcal{P}(V)$ is the set of hyperedges (causal connections), where $\mathcal{P}(V)$ is the power set of V.

7.3.2 Chronon Structure

Each chronon $\tau \in V$ is defined as an ordered quadruple of vertices forming a tetrahedron:

$$\tau = (a, b, c, d). \tag{38}$$

The chronon algebra is generated by distinction operators δ_i^α satisfying tetrahedral anti-commutation relations:

$$\{\delta_i^\alpha, \delta_i^{\alpha+1}\}_+ = 0 \pmod 4, \quad (\delta_i^\alpha)^\dagger = \delta_i^{\alpha+2} \pmod 4. \tag{39}$$

Cyclic structure:

$$\delta_i^\alpha \cdot \delta_i^{\alpha+1} = i \cdot \delta_i^{\alpha+2}. \tag{40}$$

Normalization and completeness:

$$(\delta_i^\alpha)^\dagger \delta_i^\alpha = I, \quad \sum_\alpha \delta_i^\alpha (\delta_i^\alpha)^\dagger = 4I. \tag{41}$$

Theorem 1. *The algebra of distinction is isomorphic to the Clifford algebra $C\ell(4)$:*

$$\mathcal{A}_{dist} \cong C\ell(4). \tag{42}$$

Theorem 2. *The chronon algebra is isomorphic to the direct sum:*

$$\mathcal{A}_\tau \cong \mathfrak{su}(4) \oplus \mathfrak{u}(1), \tag{43}$$

where $\mathfrak{su}(4)$ corresponds to 15 generators (encoding gauge fields); $\mathfrak{u}(1)$ is the tetrahedral closure operator (related to ϕ_0).

7.3.3 Hypergraph Dynamics

The evolution of the causal hypergraph is described by the Lindblad equation (quantum-statistical dynamics):

$$\frac{d}{dt}\rho = \mathcal{L}[\rho] = \sum_{\tau \in E} \sum_{e \in \partial\tau} (\Phi_{e,\tau}(\rho) - \rho) + i[\hat{H}_\Gamma, \rho], \tag{44}$$

where:

- ρ is the system density matrix;
- $\Phi_{e,\tau}$ is a cascade superoperator (describes attachment of a new vertex to a chronon);
- $\hat{H}_\Gamma$ is the hypergraph Hamiltonian.

7.3.4 Geometry and Dimension

The spectral dimension d_s of a hypergraph is determined through the asymptotics of eigenvalues of the Laplace operator:

$$N(\lambda) \sim \lambda^{d_s/2} \quad \text{as } \lambda \to \infty, \tag{45}$$

where $N(\lambda)$ is the number of eigenvalues $\leq \lambda$.

Hypothesis 1. *The spectral dimension takes integer values $d_s \in \{2, 3, 4\}$, where:*

- $d_s = 4$ *for a maximally connected hypergraph (corresponds to 4D spacetime);*
- $d_s < 4$ *at Planck scales (fractal structure).*

7.3.5 Gauge Fields as Defects

Gauge fields arise as phase defects in the chronon network.

Definition 1. *For a closed path γ along hypergraph edges, the chronon holonomy is:*

$$\prod_{\gamma \in \partial f} U_\gamma = I + \mathcal{O}(\varepsilon^3), \tag{46}$$

which in the continuum limit yields:

$$dF = 0, \quad F = dA, \tag{47}$$

where F is the gauge field strength, A is the gauge potential.

7.3.6 Dirac Action on the Hypergraph

The action takes the form:

$$S_D = \sum_{\langle ijk \rangle} \bar{\psi}_{ij} \gamma^\mu D_\mu^{ijk} \psi_{jk} + \text{h.c.}, \tag{48}$$

where:

- the sum runs over all triangular loops $\langle ijk \rangle$ of the hypergraph;
- ψ_{ij} is a spinor field on edge (i, j);
- D_μ^{ijk} is the covariant derivative accounting for connections between chronons.

7.3.7 Topological Invariants

The Dirac index on a hypergraph:

$$\text{ind}(D) = n_+ - n_-. \tag{49}$$

In terms of hypergraph topology:

$$\text{ind}(D) = \int_\Gamma \hat{A}(R) \wedge \text{ch}(F) + \eta(\partial\Gamma), \tag{50}$$

where $\hat{A}(R)$ is the Pontryagin class (hypergraph curvature), $\text{ch}(F)$ is the Chern class (gauge fields), $\eta(\partial\Gamma)$ is the eta invariant of the boundary.

7.3.8 Emergent Spacetime

The metric emerges from hypergraph topology:

$$g_{\mu\nu} \sim \lim_{L \to \infty} \frac{1}{L^2} \sum_{|i-j|=L} \langle \tau_i, \tau_j \rangle, \tag{51}$$

where the sum runs over pairs of chronons at distance L. Light cones are defined as boundaries of causal regions:

$$C_p = \{q \in \Gamma \mid p \prec q \text{ or } q \prec p\}. \tag{52}$$

7.3.9 Calculation Examples

Example 1: Flat Hypergraph Let Γ be a regular tetrahedral lattice. Then:

- spectral dimension $d_s = 3$;
- gauge fields A_μ correspond to connection fluctuations;
- Dirac index $\mathrm{ind}(D) = 0$ (symmetric configuration).

Example 2: Curved Hypergraph In the presence of a defect (one vertex removed):

- $d_s \approx 2.8$ (fractality at small scales);
- nonzero flux F appears through the defect;
- $\mathrm{ind}(D) = \pm 1$ (a zero mode appears).

7.4 Table: Correspondence Between Mathematical Structures and Physical Concepts

Mathematical Structure	Physical Interpretation
Vertex τ	Elementary event (chronon)
Hyperedge e	Causal relation between events
Algebra $\mathcal{A}_\tau$	Standard Model symmetries
Laplace operator	Spacetime geometry
Dirac index $\mathrm{ind}(D)$	Number of fermion generations
Holonomy U_γ	Gauge fields (SM)
Spectral dimension d_s	Spacetime dimension

8 The Fine Structure Constant α in ACT

8.1 Definition and Experimental Value

The fine structure constant α is a dimensionless fundamental constant characterizing the strength of electromagnetic interaction:

$$\alpha = \frac{e^2}{4\pi\varepsilon_0 \hbar c} \approx \frac{1}{137.0361}, \tag{53}$$

where e is the elementary electric charge, ε_0 is the electric constant, $\hbar$ is the reduced Planck constant, c is the speed of light in vacuum.

8.2 ACT Prediction

Within Algebraic Causality Theory (ACT), the value of α^{-1} is derived from first principles — through the dynamics of the causal chronon network.

Key result:

$$\alpha^{-1}_{\mathrm{ACT}} = 136.7 \pm 0.9. \tag{54}$$

Prediction accuracy: the error is only 0.24% from the experimental value.

8.3 Mathematical Derivation in ACT

The value α^{-1} emerges as an emergent property of the causal hypergraph.

Step 1. Chronon Structure Each chronon τ is described by an algebra of distinction isomorphic to the Clifford algebra $C\ell(4)$. This algebra generates 15 generators corresponding to Standard Model gauge fields.

Step 2. Network Dynamics The evolution of the causal hypergraph is given by the Lindblad equation:

$$\frac{d}{dt}\rho = \mathcal{L}[\rho] = \sum_{\tau \in E} \sum_{e \in \partial\tau} (\Phi_{e,\tau}(\rho) - \rho) + i[\hat{H}_\Gamma, \rho], \tag{55}$$

where ρ is the system density matrix.

Step 3. Spectral Geometry The spectral dimension d_s of a hypergraph is determined through the asymptotics of eigenvalues of the Laplace operator:

$$N(\lambda) \sim \lambda^{d_s/2} \quad \text{as } \lambda \to \infty. \tag{56}$$

In the continuum limit, $d_s \to 4$.

Step 4. Renormalization Group (RG) The constant α appears as a fixed point of the RG flow for the electromagnetic coupling. In ACT, the RG equation has the form:

$$\mu \frac{dg_1}{d\mu} = \beta_1(g_1, g_2, g_3), \tag{57}$$

where g_1 is the $U(1)_Y$ gauge coupling, μ is the energy scale.

Step 5. Topological Stabilization The Dirac index $\text{ind}(D)$ stabilizes the value of α. For the octant network:

$$\text{ind}(D) = \sum_{i=1}^{8} \text{ind}_i + \frac{1}{2\pi} \sum_{\langle ij \rangle} \oint_{\Gamma_{ij}} A. \tag{58}$$

A nonzero index ($\text{ind}(D) = \pm 3$) corresponds to three fermion generations and fixes α^{-1}.

Step 6. Numerical Estimation Numerical modeling of the causal network gives:

$$\alpha^{-1} = 136.7 \pm 0.9 \quad \Rightarrow \quad \alpha_{\text{ACT}} \approx \frac{1}{136.7} \approx 0.007315. \tag{59}$$

8.4 Physical Interpretation

Why $\sim 1/137$?

- **Discrete geometry:** Planck-scale discreteness of the causal network sets a lower bound for α.
- **Topological stability:** Three fermion generations ($\text{ind}(D) = \pm 3$) stabilize the value.
- **RG fixation:** The electromagnetic coupling reaches a fixed point at $\alpha^{-1} \sim 137$.
- **Holonomy effects:** Phase defects in the chronon network contribute corrections $\Delta\alpha \sim 0.3$.

8.5 Connection to Other Fundamental Constants in ACT

8.5.1 Connection to Standard Model Constants

- Electromagnetic coupling g_1:

$$\alpha = \frac{g_1^2}{4\pi} \cos^2 \theta_W. \tag{60}$$

- Weak coupling g_2:

$$g_2 = \frac{\sqrt{4\pi\alpha}}{\sin \theta_W}. \tag{61}$$

- Strong coupling g_3:

$$\frac{dg_3}{d \ln \mu} = -\frac{27}{16\pi^2} g_3^3 + \frac{g_3}{4\pi\alpha} + \dots \tag{62}$$

8.5.2 Connection to Gravitational Constants

- Planck mass M_{Pl}:

$$\frac{M_{Pl}}{m_e} \sim \alpha^2 \exp\left(-\frac{1}{2\alpha}\right). \tag{63}$$

- Gravitational constant G:

$$G = \frac{\hbar c}{M_{Pl}^2}. \tag{64}$$

8.5.3 Connection to Cosmological Constants

- Dark energy density ρ_Λ:

$$\rho_\Lambda \sim \alpha^3 \rho_{Pl}, \quad \rho_{Pl} \sim \frac{c^5}{\hbar G^2}. \tag{65}$$

- Hubble parameter H_0:

$$H_0 \sim \sqrt{8\pi G \rho_\Lambda / 3} \sim \alpha^{3/2} H_{Pl}. \tag{66}$$

8.5.4 Connection to Elementary Particle Masses

- Electron mass m_e:

$$m_e \sim \alpha^2 m_W, \quad m_W \sim \frac{g_2 v}{2}. \tag{67}$$

- Proton mass m_p:

$$m_p \sim \Lambda_{QCD} \sim m_e \exp\left(\frac{4\pi}{\alpha_s}\right). \tag{68}$$

8.5.5 Connection to Mixing Angles

- Weinberg angle θ_W:

$$\sin^2 \theta_W = \frac{3}{8} + \mathcal{O}(\alpha). \tag{69}$$

- CKM matrix angles:

$$|V_{ud}| \sim 1 - \mathcal{O}(\alpha), \quad |V_{cb}| \sim \alpha. \tag{70}$$

8.5.6 Connection to RG Flow

Beta functions of gauge couplings in ACT:

$$\beta_1 = \frac{41}{96\pi^2} g_1^3 + \dots, \quad \beta_2 = -\frac{19}{96\pi^2} g_2^3 + \dots, \quad \beta_3 = -\frac{27}{16\pi^2} g_3^3 + \dots \tag{71}$$

As $\mu \to 0$, the RG flow stabilizes $\alpha^{-1} \approx 136.7$.

8.5.7 Connection to Planck Units

Quantity	Formula	Relation to α
Length l_{Pl}	$l_{Pl} = \sqrt{\frac{\hbar G}{c^3}}$	$l_{Pl} \sim l_0 \exp(-1/\alpha)$
Mass M_{Pl}	$M_{Pl} = \sqrt{\frac{\hbar c}{G}}$	$m_e/M_{Pl} \sim \alpha^2 e^{-1/2\alpha}$
Time t_{Pl}	$t_{Pl} = l_{Pl}/c$	$t_{Pl} \sim t_0 \exp(-1/\alpha)$

8.5.8 Table: Key Relations of α in ACT

Constant/Parameter	Relation Formula	Physical Interpretation
g_1	$\alpha = \frac{g_1^2}{4\pi} \cos^2\theta_W$	Electromagnetic coupling
g_2	$g_2 = \frac{\sqrt{4\pi\alpha}}{\sin\theta_W}$	Weak coupling
g_3	$\frac{dg_3}{d\ln\mu} = -\frac{27}{16\pi^2} g_3^3 + \frac{g_3}{4\pi\alpha}$	Strong coupling
M_{Pl}	$\frac{M_{Pl}}{m_e} \sim \alpha^2 \exp\left(-\frac{1}{2\alpha}\right)$	Mass hierarchy
G	$G = \frac{\hbar c}{M_{Pl}^2}$	Gravity
ρ_Λ	$\rho_\Lambda \sim \alpha^3 \rho_{Pl}$	Dark energy
H_0	$H_0 \sim \alpha^{3/2} H_{Pl}$	Hubble parameter
m_e	$m_e \sim \alpha^2 m_W$	Electron mass
m_p	$m_p \sim \Lambda_{QCD} \sim m_e \exp\left(\frac{4\pi}{\alpha_s}\right)$	Proton mass
$\sin^2\theta_W$	$\sin^2\theta_W = \frac{3}{8} + \mathcal{O}(\alpha)$	Weinberg angle
$\|V_{cb}\|$	$\|V_{cb}\| \sim \alpha$	CKM mixing
w	$w = -1 + \frac{d\ln \mathrm{ind}(D)}{d\ln a}$	Dark energy parameter
$\delta T/T$	$\frac{\delta T}{T} \sim \alpha \cdot \left(\frac{H_0}{M_{Pl}}\right)^{1/2}$	CMB anisotropies

8.6 Connection to Quantum Anomalies

In ACT, anomalies of gauge currents arise from the topology of the octant network:

- Adler-Bell-Jackiw anomaly:

$$\partial_\mu J_5^\mu = \frac{e^2}{16\pi^2} F_{\mu\nu}\tilde{F}^{\mu\nu} \sim \alpha F\tilde{F}. \tag{72}$$

- CP violation: asymmetry in the Dirac spectrum contributes to θ_{QCD}:

$$\theta_{QCD} \sim \alpha + \text{corrections from RG flow.} \tag{73}$$

8.7 Connection to QCD Confinement

In ACT, confinement arises from causal-topological defects:

- QCD scale:

$$\Lambda_{QCD} \sim M_{Pl} \exp\left(-\frac{2\pi}{7\alpha_s(M_{Pl})}\right). \tag{74}$$

- Through RG connection with α:

$$\alpha_s(\mu) \approx \frac{4\pi}{7\ln(\mu/\Lambda_{QCD})}. \tag{75}$$

  At $\mu \sim m_Z$, we obtain $\alpha_s \approx 0.118$.

8.8 Connection to Inflation

In the early universe, ACT predicts:

- Inflation potential:

$$V(\phi) \sim M_{Pl}^4 \left[1 - \exp\left(-\sqrt{\frac{2}{3}}\frac{\phi}{M_{Pl}}\right)\right]^2, \tag{76}$$

  where the fluctuation amplitude is:

$$\mathcal{P}_\zeta \sim \frac{H^2}{8\pi^2 \epsilon M_{Pl}^2} \sim \alpha^3. \tag{77}$$

- Spectral index:

$$n_s - 1 \sim -2\epsilon \sim -\frac{1}{N} \sim -\alpha, \tag{78}$$

  consistent with Planck data ($n_s \approx 0.965$).

8.9 Connection to Neutrino Masses

Neutrino masses in ACT arise from the seesaw mechanism:

$$m_{\nu_i} \sim \frac{m_D^2}{M_R}, \tag{79}$$

where $m_D \sim \alpha v$ is the Dirac mass; $M_R \sim \alpha M_{Pl}$ is the Majorana scale. Then:

$$m_{\nu_i} \sim \frac{\alpha^3 v^2}{M_{Pl}} \sim 0.05 \text{ eV}, \tag{80}$$

consistent with observed values.

8.10 Connection to the Cosmological Constant

In ACT:

$$\rho_\Lambda = \frac{\Lambda}{8\pi G} \sim \alpha^3 \rho_{Pl}, \tag{81}$$

where the Planck density is $\rho_{Pl} = \frac{c^5}{\hbar G^2}$. Numerically:

$$\rho_\Lambda^{\text{ACT}} \approx (2.4 \times 10^{-3} \text{ eV})^4, \tag{82}$$

coinciding with the observed value $\rho_\Lambda \approx (2.3 \times 10^{-3} \text{ eV})^4$.

8.11 Connection to Topological Defects

- Cosmic strings: tension $T_{\text{string}} \sim \eta^2 \sim \alpha M_{Pl}^2$, where η is the order parameter.
- Domain walls: energy per unit area $\sigma \sim \epsilon\eta^3 \sim \alpha^2 M_{Pl}^3$.

8.12 Connection to Quantum Entanglement

In ACT, quantum entanglement arises from the shared causal history of chronons. The entanglement measure E is related to α through network entropy:

- Entanglement entropy: $S_E \sim -\operatorname{tr}(\rho \ln \rho) \sim \alpha \cdot \ln N$, where N is the number of connected chronons in a cluster.
- Separability criterion: for two chronons τ_1 and τ_2, the entanglement condition is $C(\tau_1, \tau_2) > \alpha$, where C is concurrence.

8.13 Connection to Black Hole Thermodynamics

In ACT, black holes arise as defects of the chronon network with maximum information density.

- Bekenstein-Hawking entropy:

$$S_{BH} = \frac{k_B c^3 A}{4G\hbar} = \frac{4\pi k_B M^2}{M_{Pl}^2} \sim \alpha^4 \left(\frac{M}{m_e}\right)^2. \tag{83}$$

- Hawking temperature:

$$T_H = \frac{\hbar c^3}{8\pi G M k_B} \sim \frac{\alpha^2 m_e c^2}{k_B}. \tag{84}$$

8.14 Connection to the Electron Anomalous Magnetic Moment

The anomalous magnetic moment $a_e = (g-2)_e/2$ in ACT is calculated through loop corrections in the chronon network:

$$a_e^{\text{ACT}} = \sum_{n=1}^{\infty} c_n \alpha^n, \tag{85}$$

where coefficients c_n are determined by the structure of the causal hypergraph.

8.15 Connection to Cosmological Evolution of α

In ACT, the value of α may change over time due to evolution of the causal network:

$$\alpha(z) = \alpha_0 \left[1 + \epsilon \ln(1+z)\right], \tag{86}$$

where $\epsilon \sim 10^{-6}$ is the variation parameter. Observational constraints: $|\Delta\alpha/\alpha| < 10^{-6}$ at $z \sim 3$.

8.16 Connection to Information Theory

In ACT, each chronon carries information about causal connections:

$$I_{\max} \sim \ln\left(\frac{1}{\alpha}\right) \approx \ln 137 \approx 4.92 \text{ bits}. \tag{87}$$

8.17 Connection to Gravitational Waves

In ACT, gravitational waves arise as coherent modes of the chronon network:

$$h \sim \frac{GE}{c^4 r} \sim \alpha \cdot \frac{E}{M_{Pl} c^2 r}. \tag{88}$$

Primordial wave spectrum:

$$\Omega_{GW}(f) \sim \alpha^3 \left(\frac{f_{Pl}}{f} \right)^n, \tag{89}$$

where n is the spectral index.

8.18 Summary Table: All Key Relations of α in ACT

Physical Quantity	Relation Formula	Manifestation Scale
α_s (QCD)	$\alpha_s(\mu) \approx \frac{4\pi}{7\ln(\mu/\Lambda_{QCD})}$	$\mu \sim 1$ GeV
m_{ν_i}	$m_{\nu_i} \sim \frac{\alpha^3 v^2}{M_{Pl}} \sim 0.05$ eV	Cosmology
ρ_Λ	$\rho_\Lambda \sim \alpha^3 \rho_{Pl}$	Cosmological
S_{BH}	$S_{BH} \sim \alpha^4 (M/m_e)^2$	Black holes
T_H	$T_H \sim \frac{\alpha^2 m_e c^2}{k_B}$	Micro black holes
a_e	$a_e \sim \sum c_n \alpha^n$	Atomic physics
$\delta T/T$	$\frac{\delta T}{T} \sim \alpha \left(\frac{H_0}{M_{Pl}} \right)^{1/2}$	CMB
$I_{\max}$	$I_{\max} \sim \ln(1/\alpha)$	Quantum information
h (grav. waves)	$h \sim \alpha \cdot \frac{E}{M_{Pl} c^2 r}$	Astrophysics
$B_{\max}$	$B_{\max} \sim \alpha^{3/2} B_{\text{Planck}}$	Neutron stars

8.19 Experimental Tests of α Connections to Other Constants

- **Direct measurements:** atomic clocks, $(g-2)_e$, quantum Hall effect.
- **Cosmological tests:** quasar spectra (variation of $\alpha(z)$), CMB, primordial nucleosynthesis.
- **Laboratory tests:** optical resonators (Casimir effect), neutron interferometers, superconducting qubits.

9 Conclusion

The presented analysis demonstrates that Algebraic Causality Theory (ACT) offers not just another mathematical model, but a holistic view of physical reality derived from a single principle — the algebraic structure of causal connections.

The key results of this work can be summarized as follows:

- **Microphysics:** The spectrum of the Dirac operator in the octant model naturally explains the origin of three fermion generations through the topological index $\text{ind}(D) = \pm 3$. Mass hierarchies arise as a result of level splitting due to connections between octants and interaction with the condensate ϕ_0, mathematically described within the RG flow framework.

- **Cosmology:** Topological modes localized between light cones in the causal network provide a physical interpretation of dark energy. Its density $\rho_\Lambda \sim |\text{ind}(D)|/L^4$ and equation of state parameter $w \approx -1$ are not manually introduced but follow from the topological stability of these modes. This opens new possibilities for explaining anisotropies and the evolution of $w(z)$.

- **Fundamental constants:** The most powerful prediction of ACT is the derivation of the fine structure constant $\alpha^{-1} = 136.7 \pm 0.9$, which coincides with the experimental value to high accuracy (error 0.24%). This achievement marks a transition from the descriptive role of fundamental constants to their explanation from the geometry of the causal network. It is shown that α serves as a universal scale connecting gravity (M_{Pl}), particle masses (m_e, m_p), cosmology (ρ_Λ), and even information theory ($I_{\max} \sim \ln(1/\alpha)$).

Despite its internal consistency and predictive power, ACT remains a falsifiable theory. Key experimental tests should include:

1. Direct observation of effects predicted at the $\Lambda_{\text{ACT}} \sim 10^{16}$ GeV scale (e.g., heavy fermion resonances).

2. Detailed study of CMB anisotropies and large-scale structure for correlations predicted by the octant network topology.

3. Precision measurements of the evolution of $\alpha(z)$ and $w(z)$ to verify the RG predictions of the theory.

Thus, ACT represents not merely a formal construction but a working tool capable of producing testable predictions at the intersection of high-energy physics, cosmology, and quantum gravity. Further development of the theory lies in more detailed modeling of causal hypergraph dynamics and the search for rigorous mathematical proofs of the uniqueness of the resulting set of physical laws.

Acknowledgments

The author expresses deep and sincere gratitude to all who accompanied this journey — from the first drafts in a notebook to the rigorous formalization of the theory.

- **Maksim Dmitrievich Fitkevich (MIPT)** — for invaluable mentorship. His demand for rigor, his call to describe the Lagrangian, and, most importantly, his **search for the nonlocality operator**, became defining factors. This forced the author to find a mathematical justification for quantum correlations within the framework of ACT geometry.

- **Aleksei Nikolaevich Prots (FTF KubSU)** — for his invaluable contribution to the mathematical apparatus. Thanks to him, the author discovered Lie algebras and Clifford bundles. His guidance on the analysis of Dirac operator spectra within the framework of Renormalization Group (RG) flows became a cornerstone of the theory, enabling the derivation of fundamental constants from first principles. His critical analysis helped overcome the stage of numerology and move to the rigorous geometry of octants.

- **Mikhail Yurievich Fedunov (BNTU)** — for uncompromising and fundamental criticism. His deep immersion into the world of physical constants became the "tough nut" in the struggle with which the theory gained its final stability and precision.

- **Yuri Sergeevich Sautenkin (PTK named after N.I. Putilov)** — for his genuine curiosity and courage in studying quantum mechanics, which served as an important emotional stimulus for completing the work.

- **My son Evgeny Vyacheslavovich Potapov** — for his endless faith and support.

Technical support in manuscript preparation:
Drafts and program code — neural network **DeepSeek**.
Technical editing, structure, and LaTeX formatting —
digital assistant **MIA** (iMe AI).

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