**§10. Shock Discontinuity (Shock Wave)**

When mass flow occurs across a discontinuity (≠ 0, α = 1, 2), it follows from mass conservation equation (3.9.29) that the normal velocities *vknk* in discontinuity-fixed coordinates (i.e.,  and ) have the same sign on opposite sides of the surface. If the direction of the normal **n** at point P (from region *1* to region *2*, see Fig. 3.9.1) is chosen such that both  and  are negative, then the mass flow is from *1* to *2* (ξ > 0, see text around Eq. (3.9.27)). In other words, when the regions on opposite sides of a discontinuity are referred to by superscript (1) and (2) so that both  and  are negative, the medium in region *1* is ahead of the discontinuity while the medium in region *2* is behind it (i.e., the particles in region *2* have crossed the surface).

***Definition.*** A surface across which the normal velocity of the medium is discontinuous ([*vn*] ≠ 0, ξ ≠ 0), and therefore so is the normal stress ([σ*nn*] ≠ 0), is called a *shock wave*, a *shock*, or a *shock discontinuity*.

A shock discontinuity cannot be a contact one. Indeed, consider the component of momentum balance (3.9.30) along the normal **n**,

ξ = − [σ(*nn*)] or ρ(1) ( – *D*) [*vn*] = [σ(*nn*)]. (3.10.1)

It is clear from [*vn*] ≠ 0 that  ≠ . Then, using the mass balance ρ(1) = ρ(2) and noting that ρ′(α) ≠ 0 (α = 1, 2), we find that  ≠ 0, and hence  ≠ *D* (α = 1, 2). Therefore, [σ*nn*] ≠ 0 and there is always a flow of matter across a shock discontinuity.

A shock discontinuity is either a *compression shock* or a *rarefaction* *shock* (*expansion shock*), depending on whether density increases or decreases across the shock in the direction of motion of the medium. If  and  are negative, then ρ(2) > ρ(1) and ρ(2) < ρ(1) for compression and rarefaction shocks, respectively.

**Derivation of shock jump relations based on analysis of particle motion.** Balance of equations across a discontinuity can be derived not only from integral

conservation equations (as done in §9) but also by analyzing the motion of a particle in the neighborhood of the surface. Consider the one-dimensional shock schematized in Fig. 3.10.1, where particle motion is shown in an arbitrary coordinate system under conditions of uniaxial strain. The shock surface  is perpendicular to the velocities **v**(1) and **v**(2) on both sides of the discontinuity. Here, the unit normal **n** to the shock surface  is parallel to a principal stress direction, and directions perpendicular to **n** are also principal ones on both sides of :

 =  = 0,  =  = 0,

**v**(α) = *v*(α) **n**,  = σ(α) **n** (α = 1,2). (3.10.2)

***Definition.*** A shock discontinuity such that the velocity of the medium is parallel to the normal on both sides of it is called a *normal shock*.

Fig. 3.10.1. One-dimensional shock.

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If the propagation speed of  is *D* > 0, then the shock travels in the direction of **n**; if *D* < 0, then it travels in the opposite direction. The signs of velocity *v*(α)and traction σ(α) are determined analogously on either side of the shock (α = 1, 2). In particular, σ(α) > 0 and σ(α) < 0 correspond to tensile and compressive tractions in region α (α = 1, 2).

Therefore, balance equations (3.9.37) across a shock can be written in the laboratory frame *K*\* (where **v**(1) = 0, see (3.9.36)) as follows:

ρ(2) = ρ(1)  (*v*\* ≡ *v*\*(2)),

– σ(2) = – σ(1) + ρ(1) *Dv*\*,

*u*(2) = *u*(1) +  – , (3.10.3)

ρ(1)*D* (*s*(2) – *s*(1)) =  −  + ΨS, ΨS ≥ 0.

Fig. 3.10.2. Normal shock in the laboratory frame.

These equations can be obtained directly by using the schematic in Fig. 3.10.2, where a continuum particle is shown at closely spaced times *t* (lightly shaded area) and *t* + d*t* (more heavily shaded area). At time *t*, the particle is at rest in the laboratory frame *K*\* (see (3.9.36)), occupying a volume МА just ahead of the shock  (in region *1*) of length *D* d*t* along the normal **n**. At time *t* + d*t*, after the shock has just traversed the particle, its position  is at point *А*, while the particle occupies a volume М′А in region *2* and moves with velocity ≡ *v*(2) along the normal **n**. The particle's size has changed from *D* d*t* to (*D* − )d*t* over the time interval between *t* and *t* + d*t*. Accordingly, the conservation of particle mass is expressed as

ρ(1)*D* = ρ(2)(*D* – ). (3.10.4)

In Fig. 3.10.2, normal tractions ( and ) are replaced by pressures:

 ≡ −  (α = 1, 2). (3.10.5)

The normal component of momentum balance is determined by noting that the particle velocity has changed from zero to  because of the difference in pressure:

ρ(1)*D* =  − . (3.10.6)

Analogous calculations of the change in particle energy, due to the work done by the pressure  over the distance d*t* and the heat fluxes  and , and of the corresponding change in entropy yield

ρ(1) *D* (*u*(2) – *u*(1) + 1/2 (*v*\*)2) =  +  – , (3.10.7)

ρ(1)*D* (*s*(2) – *s*(1)) =  −  + ΨS, ΨS ≥ 0. (3.10.8)

It is obvious that Eqs. (3.10.4)–(3.10.8) are equivalent to (3.10.3).

**Relationship between discontinuity modeling and continuous representation.** There are several modeling frameworks used to describe motion in continuum mechanics. In the most comprehensive one, all field quantities (density, velocity, stress, strain, etc.) are represented by functions that are continuously differentiable at each point and satisfy mass, momentum, energy, and other balance equations, as well as equations of state and constitutive equations characterizing the medium in question. Regions where these functions are well defined are referred to as *continuous regions*.

In discontinuity modeling, continuous regions are separated by weak or strong discontinuities, and field quantities are represented in terms of piecewise continuously differentiable functions. Their values on opposite sides of a discontinuity and the discontinuity's propagation speed are related by algebraic balance equations.

Note also that continuous regions may contain singular points or curves where the values of field quantities (such as density, velocity, stress, or strain) go to infinity.

Since matter crosses a discontinuity from one continuous region to another, the gradients of field quantities, or their rates of change within a particle, are formally infinite. A detailed analysis taking into account molecular structure and interactions shows that shocks are thin layers whose thickness δ(sh) (Fig. 3.10.3) is determined by a molecular length scale . In particular, the shock thickness is similar in order of magnitude to the molecular mean free path  in gases and to the molecular spacing  in condensed matter, the latter being comparable to molecular size *d*mol (*d*mol ~ ). The processes inside such a layer are dominated by dissipation and relaxation due to intramolecular and intermolecular interactions[[1]](#footnote-1), which are inconspicuous in continuous regions.

Fig. 3.10.3. One-dimensional distribution of normal velocity *vn* ≡ **v⋅n** along the *x* axis parallel to **n**. Curves АS(1)S(2)B and АC(1)C(2)B correspond to discontinuity modeling and continuous representation, respectively.

The point *x* = *x*(sh) in Fig. 3.10.3 is the position of a shock modeled as a step S(1)S(2), with normal velocities  and  on its opposite sides. In continuous representation (with dissipation and relaxation taken into account), the step distribution of *vn*, АS(1)S(2)B, is replaced by a continuous distribution, АC(1)C(2)B, with shock transition layer C(1)C(2) of finite thickness δ(sh) instead of zero-thickness step S(1)S(2).

Dissipation effects are also significant in contact layers (Fig. 3.10.4).

Thus, a detailed study of the structure of transition layers corresponding to shocks and other surfaces of discontinuity requires the use of generalized equations of continuum mechanics.

Studies of motions of fluids and solids have shown that a comprehensive description is not always possible if analysis is restricted to continuous fields governed by differential equations, and surfaces of discontinuity have to be introduced.

**Fig. 3.10.4.** Velocity distributions across contact discontinuity *АВ* in discontinuity-fixed coordinates: (*а*) inviscid approximation; (*b*) more detailed structure taking into account molecular interactions, with a continuous velocity distribution across a viscous layer of thickness δ (**v** = **v**τ, *vn* = 0).

This has to be done when the differential equations used to model a continuous motion cannot adequately account for the effects of internal friction, viscosity, or heat conduction. These dissipative phenomena always manifest themselves when strain rates are high, temperature gradients are steep, or the system is in a strongly nonequilibrium state[[2]](#footnote-2), requiring the use of molecular physics models. However, dissipative processes of this kind are negligible in regions where strain rates, temperature gradients, and deviations from equilibrium are relatively weak.

Taking due account of the dissipative phenomena mentioned above leads to much more complex equations, while the continuous regions obtained instead of surfaces of discontinuity make mathematical and logical aspects of modeling based on conservation equations less complicated. As a result, the surfaces representing discontinuities in the nondissipative approximation are replaced by thin, yet finite, regions of rapidly varying field quantities (velocity, stress, temperature etc.), as illustrated by Figs. 3.10.3 and 3.10.4.

The state of a particle passing through a shock wave changes in a very short time *t*(sh) determined by the residence time in a shock layer of thickness δ(sh)~ . The particle velocity relative to the shock layer is comparable to the speed of sound *С*. Therefore,

*t*(sh) ~ /*С*. (3.10.9)

Since *С* ≥102 m/s, the residence time is estimated as *t*(sh) ~ 10-9 s. It is too short for the heat fluxes ahead of and behind the shock ( and ) to contribute significantly to the change in internal energy, kinetic energy, or entropy as the particle crosses the shock.

Processes that do not involve heat transfer between the system and its surroundings are called *adiabatic* (see Chapter 5). Thus, the deformation of a particle crossing a shock is adiabatic because the contributions of heat gain and loss to energy balance within the shock layer are negligible:

 =  = 0. (3.10.10)

Deviations from adiabatic behavior can occur at very high temperatures because of electromagnetic emission from the shock wave.

It should be kept in mind that heat fluxes can be significant in continuous regions (i.e., outside the shock layer).

1. These effects may be due to molecular vibration, rotation, dissociation, ionization, emission, etc. [↑](#footnote-ref-1)
2. The concepts of equilibrium and nonequilibrium states are introduced in Chapter 5. [↑](#footnote-ref-2)