Kovalev

1-3

№1

$$\min_{x \in R} \|Ax-b\|_2^2 + \gamma \|x-u\|_2^2$$

$$\frac{\partial f}{\partial x} = A^T(Ax-b) + \gamma(x-u) = 0.$$

$$A^TAx - A^Tb + \gamma x - \gamma u = 0$$

$$(A^TA + \gamma I)x = A^Tb + \gamma u$$

$$x^* = (A^TA+\gamma I)^{-1}(A^Tb+\gamma u)$$

№2 ~~min~~ $\max(Ax-b) \le t$

$$\Downarrow$$

$$\begin{cases} \min t \\ \text{s.t } Ax-b \le t \\ \quad (Ax-b) \ge -t \end{cases}$$

LP problem:

$$\min t$$
$$\text{s.t} \quad Ax-b \le t \cdot 1$$
$$\quad b-Ax \le t \cdot 1$$

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or $\min 1^T \bar{t}$

$$\text{s.t} \quad -I\bar{t} \le b-Ax$$
$$\quad -I\bar{t} \le Ax-b.$$

$$\max -1^T \begin{bmatrix} \bar{t} \\ \bar{t} \end{bmatrix}$$

$$\text{s.t} \quad -I \cdot \begin{bmatrix} \bar{t} \\ \bar{t} \end{bmatrix} \le \begin{bmatrix} b-Ax \\ Ax-b \end{bmatrix}$$

№2 continue

The dual

$$\min_{y} \begin{bmatrix} b - Ax \\ Ax - b \end{bmatrix}^T \cdot y$$

s.t.

$$-I \cdot y = -1$$

$$y \geq 0$$

№3 $\min \|x\|_1$ s.t. $Ax = b$

$$L = \|x\|_1 + y^T(Ax - b)$$

$$g = \inf_x L = \inf(y^T A x + \|x\|_1) - y^T b =$$

$$= -\sup(\underbrace{\overset{v}{(-y^T A)} x - \|x\|_1}_{r^*}) - y^T b \;=$$

$$r^*(v) = \begin{cases} 0 & \text{if } \|v\|_\infty \leq 1 \\ \infty & \text{else} \end{cases}$$

$$= -r^*(-y^T A) - y^T b = -y^T b$$

The dual

$$\max_{y} g(y) = -y^T b$$

s.t. $\|-y^T A\|_\infty \leq 1$

Kovalev

№ 1.

$$\min \sup \|Ax - b\|_2$$

$$A \in \{A_0 + U \mid \|U\|_F \le \alpha\}$$

$$\sup_A (Ax - b) = \sup_U ((A_0 + U)x - b) = \sup_U (A_0 x + (Ux - b))$$

$$\sup_U (Ux - b)$$

$$Ux = \mu \frac{b}{\|b\|_F}$$

$$\|Ux\|_F^2 \le \|U\|_F^2 \|x\|_F^2 \Rightarrow \frac{\|Ux\|_F^2}{\|x\|_F^2} \le \|U\|_F^2 = \alpha$$

$$\left\| \frac{\mu b}{\|b\|_F} \right\|_F^2 = \mu^2$$

$$\frac{\mu^2}{\|x\|_F^2} \le \alpha \Rightarrow \mu \le \sqrt{\alpha} \cdot \|x\|_F$$

μ must be max $\Rightarrow$ to make $(Ux - b)$ - max

$$\Rightarrow \mu = \sqrt{\alpha} \|x\|_F$$

$$\boxed{\begin{aligned} &\min_x \|A_0 x + Ux - b\| \\ &\text{s.t. } Ux = \frac{\sqrt{\alpha} \cdot \|x\|_F \cdot b}{\|b\|_F} \end{aligned}}$$

№8 $W = U^T V$

$$f(U,V) = \frac{1}{2}\left(\|U\|_F^2 + \|V\|_F^2\right)$$

$$L = \frac{1}{2}\|W\|_F^2 - \sum \gamma_k \left(y_k\left(tr(W^T X_k) + b\right) - 1\right)$$

$$\frac{\partial L}{\partial W} = W - \sum \gamma_k y_k X_k = 0$$

$$W^* = \sum \gamma_k y_k X_k$$

$$\frac{\partial L}{\partial b} = \sum \gamma_k y_k = 0$$

$$L = \frac{1}{2}\|W^*\|_F^2 - \sum \gamma_k y_k tr(W^{*T} X_k) + \overset{0}{\overset{\|}{\sum \gamma_k y_k}} b -$$

$$- \sum \gamma_k =$$

$$L^* = \frac{1}{2}\|W^*\|_F^2 - \sum \gamma_k y_k tr(W^{*T} X_k) - \sum \gamma_k.$$

$$\gamma_k^{k+1} = \arg\max_{\gamma_k} L^*(W^{*k}, \gamma_k^k)$$

$$W^{*k+1} = \sum \gamma_k^{k+1} y_k X_k$$

$$b^{k+1} = \arg\min_b L(W^{*k+1}, \gamma_k^{k+1}, b)$$