TÓPICOS EM CIÊNCIA DE DADOS PARA O ESPORTE

VARIÁVEIS ALEATÓRIAS

DIEGO RODRIGUES DSC

INFNET

CRONOGRAMA

DIA	NÚMERO	ÁREA	AULA	TRABALHOS
10/10/2023	1	Intro	Introdução a Disciplina e Organização do Ambiente	
17/10/2023	2	Dados	Coleta de Dados e Sensoriamento	
19/10/2023	3	Estatística	Variáveis Aleatórias	Grupos
24/10/2023	4		Análise Exploratória	
26/10/2023	5		Estatísticas para Ranqueamento	
31/10/2023	6		Ranqueamento Estatístico : ELO	Base de Dados
07/11/2023	7		Ranqueamento Estatístico : Glicko	
09/11/2023	8		Ranqueamento Estatístico : TrueSkill	
14/11/2023	9		Ranqueamento Estatístico : XELO	
16/11/2023	10	ML	Modelos de Aprendizado de Máquina	Pesquisa
21/11/2023	11		Machine Learning: Classificação	
23/11/2023	12		Machine Learning: Regressão	
28/11/2023	13		Machine Learning: Agrupamento	
30/11/2023	14		Machine Learning: Visão Computacional	Modelo
5/12/2023	15	Esportes	Aplicações & Artigos: Esportes Independentes	
7/12/2023	16		Aplicações & Artigos: Esportes de Combate	
12/12/2023	17		Aplicações & Artigos: Esportes de Objeto	
14/12/2023	18		Aplicações & Artigos : Betting	
19/12/2023	19	Workshop	Workshop	
21/12/2023	20		Apresentações de Trabalhos	Apresentação

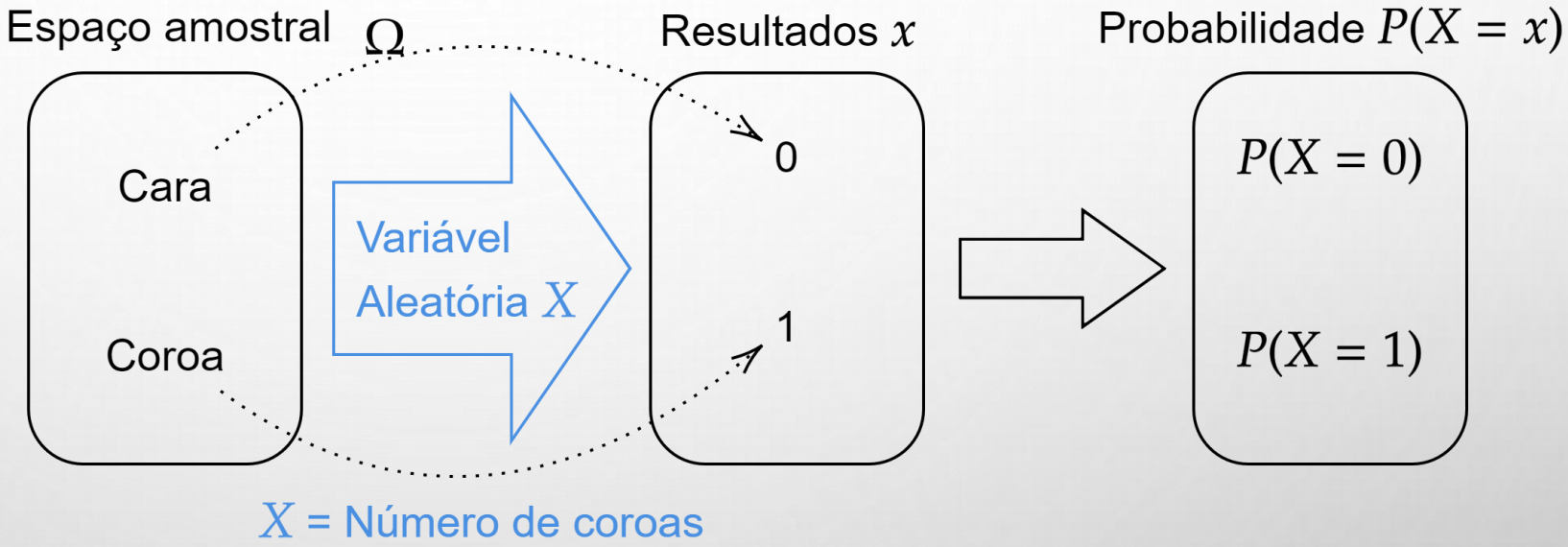
CRISP-DM

AGENDA

- PARTE 1 : TEORIA
  - VARIÁVEL ALEATÓRIA
  - DISTRIBUIÇÃO DE PROBABILIDADE
  - FUNÇÃO DENSIDADE DE PROBABILIDADE
  - ESTATÍSTICAS DE UMA DISTRIBUIÇÃO
  - DISTRIBUIÇÃO DE BERNOULLI E ESTIMATIVA DE PARÂMETROS
  - DISTRIBUIÇÃO UNIFORME DISCRETA
  - DISTRIBUIÇÃO CATEGÓRICA
  - DISTRIBUIÇÃO BINOMIAL
  - TEOREMA CENTRAL DO LIMITE
  - DISTRIBUIÇÃO NORMAL (GAUSSIANA)

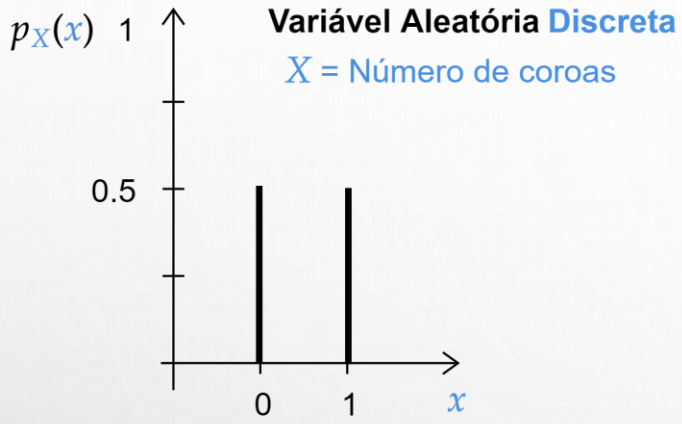
- PARTE 2 : PRÁTICA
  - PANDAS + MATPLOTLIB → VARIÁVEIS ALEATÓRIAS NO ESPORTE

VARIÁVEL ALEATÓRIA

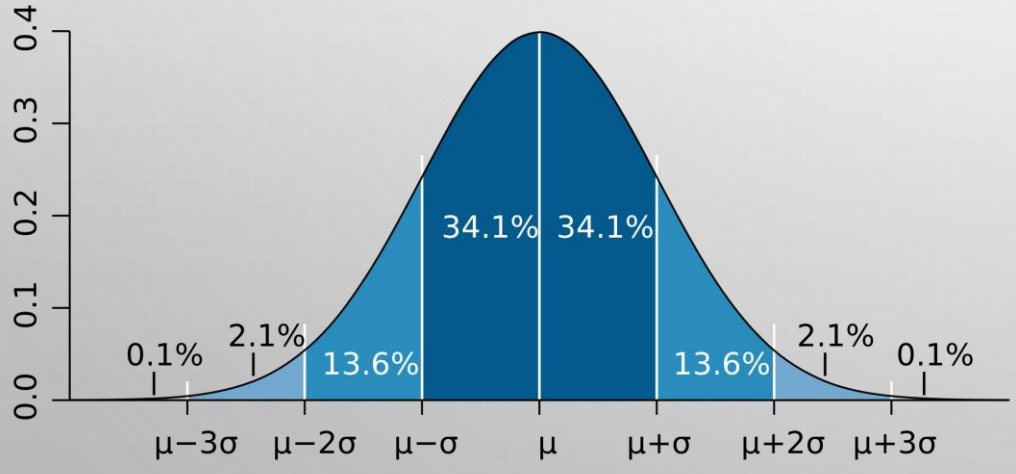


https://www.inf.ufsc.br/~andre.zibetti/probabilidade/variaveis_aleatorias.html[1]

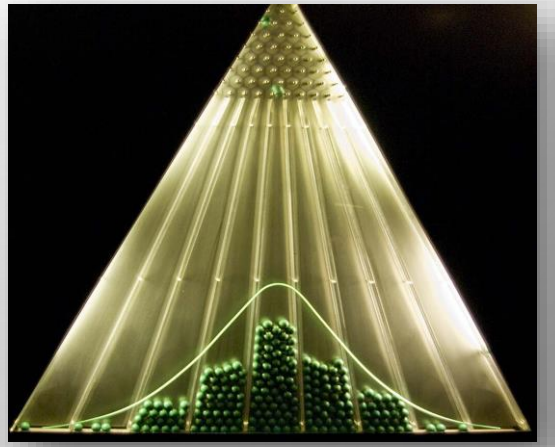
DISTRIBUIÇÃO DE PROBABILIDADE



Zibetti [1]

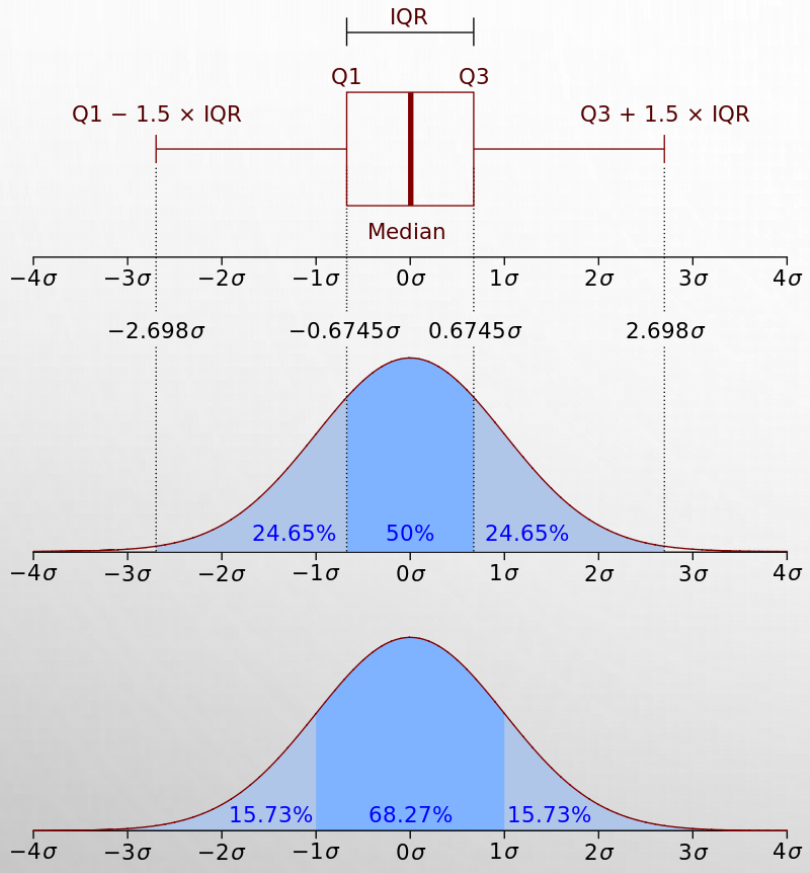


Wikipedia [2]



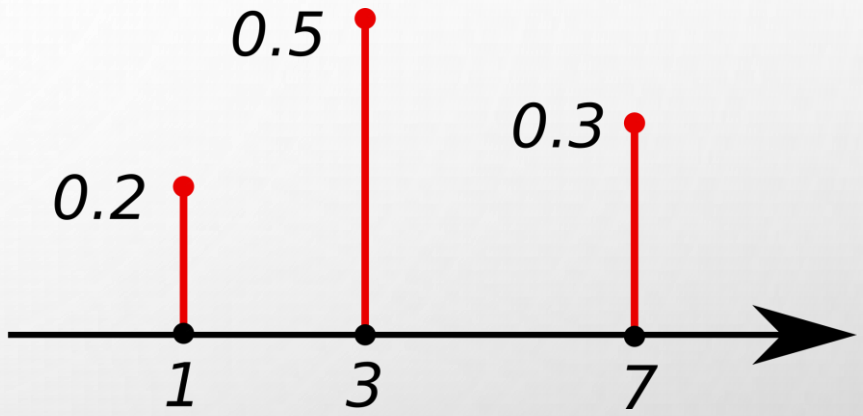
Distribuição teórica, paramétrica, e a realização experimental, não paramétrica.

FUNÇÃO DENSIDADE DE PROBABILIDADE



[2]

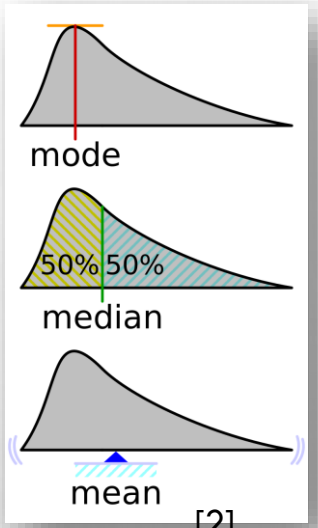
Função Densidade de Probabilidade (Contínua)



[2]

Função Massa de Probabilidade (Discreta)

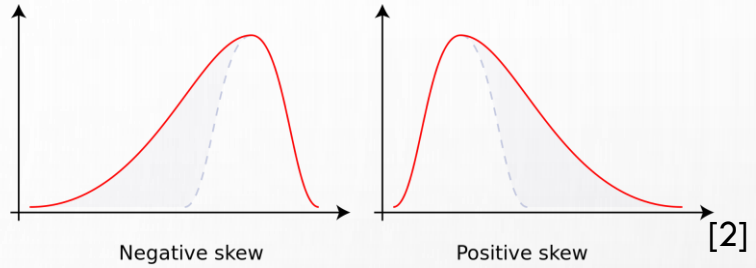
ESTATÍSTICAS DE UMA DISTRIBUIÇÃO



[2]

$$\bar{x} = \frac{x_1 + x_2 + \ldots + x_n}{n} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

MÉDIA – O MOMENTO CENTRAL

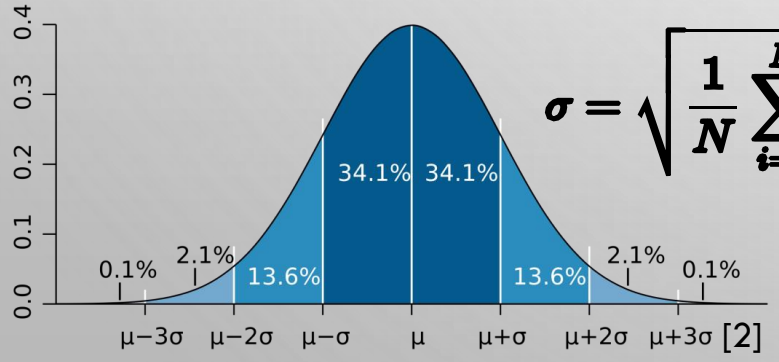


[2]

$$\tilde{\mu}_3 = \mathrm{E}\left[\left(\frac{X-\mu}{\sigma}\right)^3\right] = \frac{\mathrm{E}[X^3] - 3\mu\,\mathrm{E}[X^2] + 3\mu^2\,\mathrm{E}[X] - \mu^3}{\sigma^3} = \frac{\mathrm{E}[X^3] - 3\mu(\mathrm{E}[X^2] - \mu\,\mathrm{E}[X]) - \mu^3}{\sigma^3} = \frac{\mathrm{E}[X^3] - 3\mu\sigma^2 - \mu^3}{\sigma^3}.$$

ASSIMETRIA - DESEQUILIBRIO

O DESVIO PADRÃO – DISPERSÃO DOS DADOS



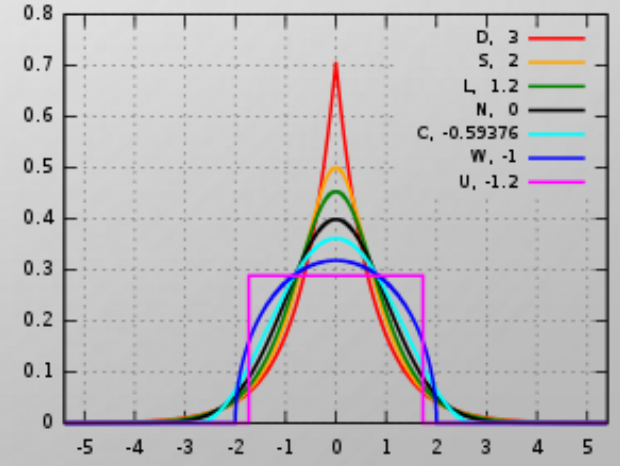
[2]

$$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \mu)^2}$$

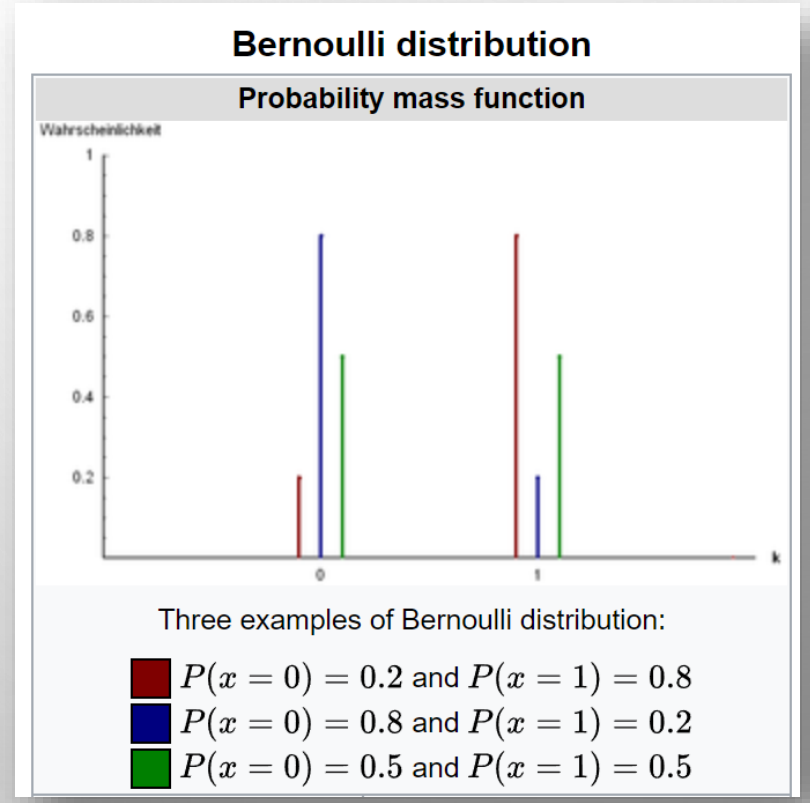
CURTOSE - HOMOGENEIDADE

$$g_2 = \frac{m_4}{m_2^2} - 3 = \frac{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^4}{\left[\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2\right]^2} - 3$$

[2]



DISTRIBUIÇÃO DE BERNOULLI E ESTIMATIVA DE PARÂMETROS

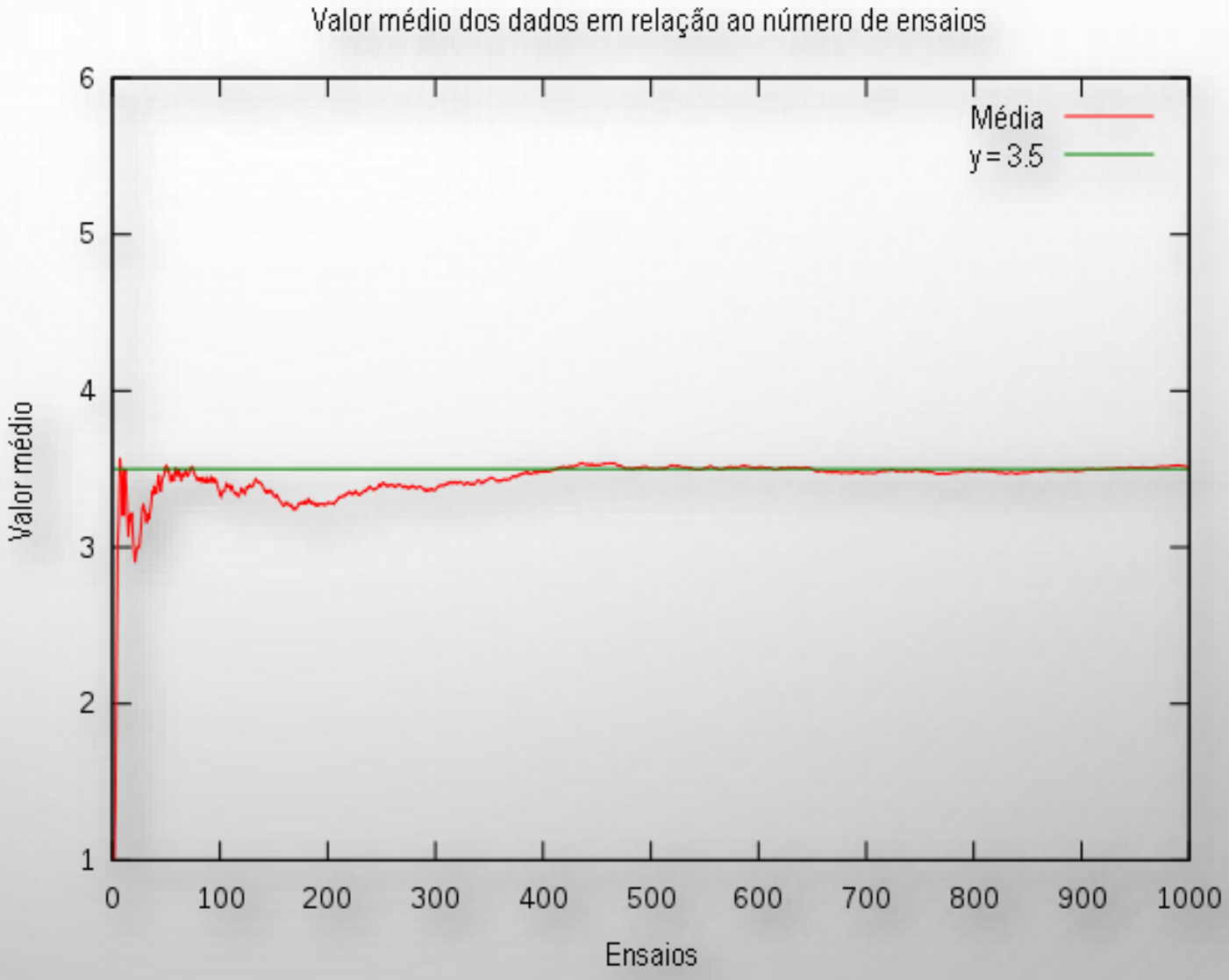


[2]

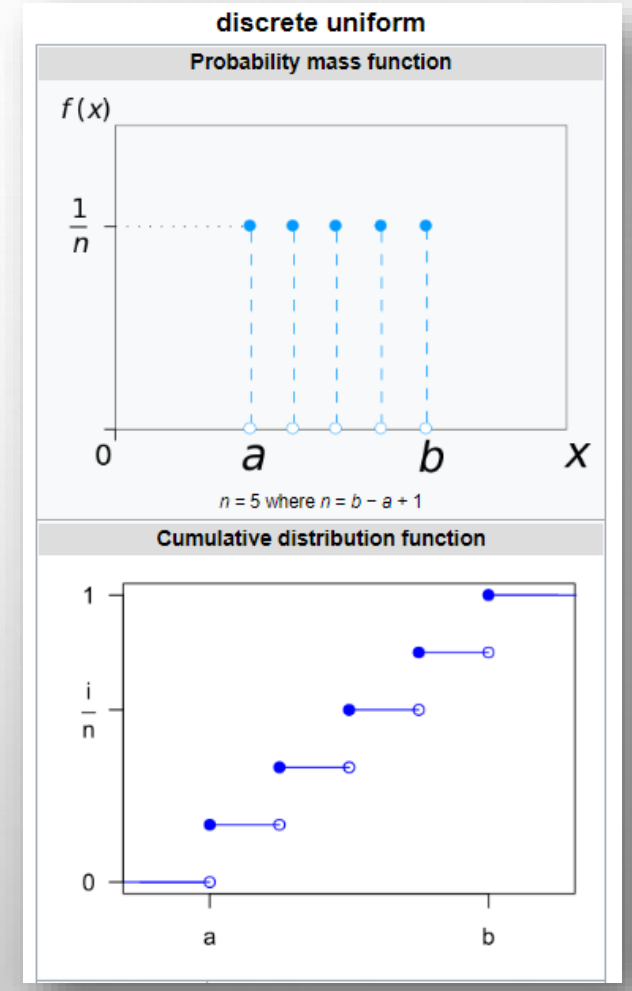
Parameters	$0 \le p \le 1$ $q = 1 - p$
Support	$k \in \{0, 1\}$
PMF	$\begin{cases} q = 1 - p & \text{if } k = 0 \\ p & \text{if } k = 1 \end{cases}$
CDF	$\begin{cases} 0 & \text{if } k < 0 \\ 1 - p & \text{if } 0 \le k < 1 \\ 1 & \text{if } k \ge 1 \end{cases}$
Mean	p
Median	$\begin{cases} 0 & \text{if } p < 1/2 \\ [0, 1] & \text{if } p = 1/2 \\ 1 & \text{if } p > 1/2 \end{cases}$
Mode	$\begin{cases} 0 & \text{if } p < 1/2 \\ 0, 1 & \text{if } p = 1/2 \\ 1 & \text{if } p > 1/2 \end{cases}$
Variance	$p(1 - p) = pq$
MAD	$\frac{1}{2}$
Skewness	$\frac{q - p}{\sqrt{pq}}$
Ex. kurtosis	$\frac{1 - 6pq}{pq}$
Entropy	$-q \ln q - p \ln p$
MGF	$q + pe^t$
CF	$q + pe^{it}$
PGF	$q + pz$
Fisher information	$\frac{1}{pq}$

[2]

LEI DOS GRANDES NÚMEROS



DISTRIBUIÇÃO DE MASSA UNIFORME

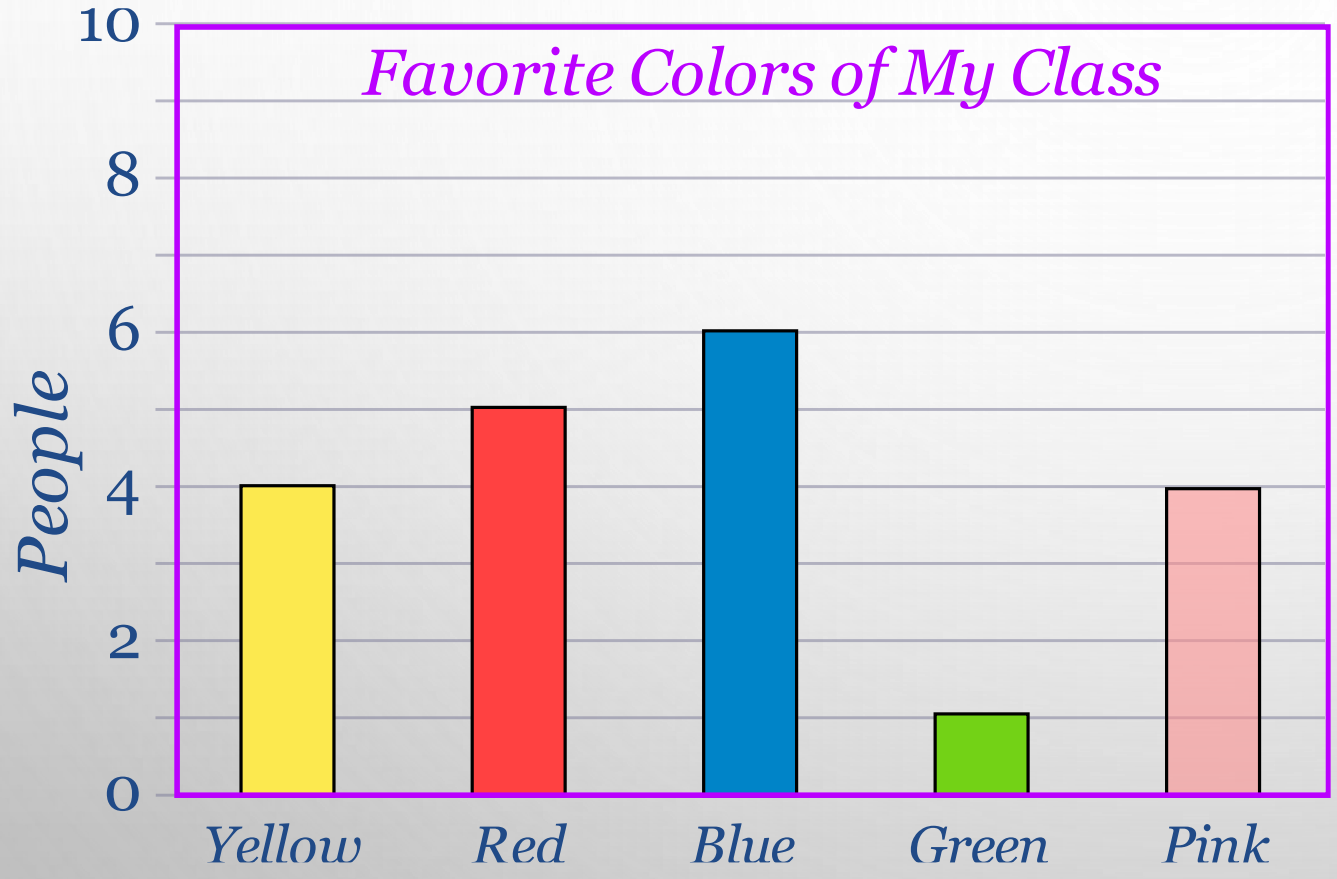


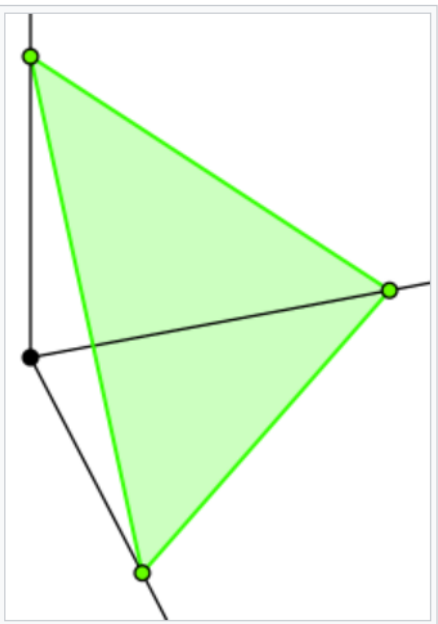
[2]

Notation	$\mathcal{U}\{a,b\}$ or $\text{unif}\{a,b\}$
Parameters	a, b integers with $b \geq a$ $n = b - a + 1$
Support	$k \in \{a, a+1, \ldots, b-1, b\}$
PMF	$\frac{1}{n}$
CDF	$\frac{\lfloor k \rfloor - a + 1}{n}$
Mean	$\frac{a+b}{2}$
Median	$\frac{a+b}{2}$
Mode	N/A
Variance	$\frac{n^2-1}{12}$
Skewness	0
Ex. kurtosis	$-\frac{6(n^2+1)}{5(n^2-1)}$
Entropy	$\ln(n)$
MGF	$\frac{e^{at} - e^{(b+1)t}}{n(1-e^t)}$
CF	$\frac{e^{iat} - e^{i(b+1)t}}{n(1-e^{it})}$
PGF	$\frac{z^a - z^{b+1}}{n(1-z)}$

[2]

DISTRIBUIÇÃO CATEGÓRICA





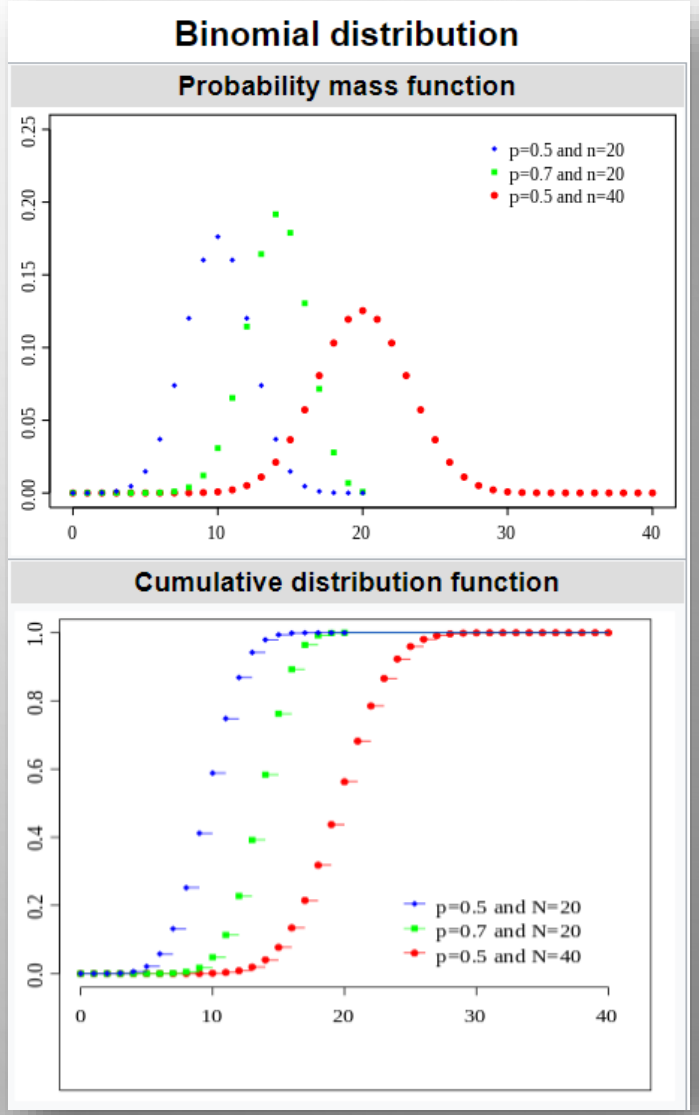
The possible probabilities for the categorical distribution with $k = 3$ are the 2-simplex $p_1 + p_2 + p_3 = 1$, embedded in 3-space.

[2]

Categorical

Parameters	$k > 0$ number of categories (integer) $p_1, \ldots, p_k$ event probabilities $(p_i \geq 0,\ \Sigma p_i = 1)$
Support	$x \in \{1, \ldots, k\}$
PMF	(1) $p(x = i) = p_i$ (2) $p(x) = p_1^{[x=1]} \cdots p_k^{[x=k]}$ (3) $p(x) = [x = 1] \cdot p_1 + \cdots + [x = k] \cdot p_k$ where $[x = i]$ is the Iverson bracket
Mode	i such that $p_i = \max(p_1, \ldots, p_k)$

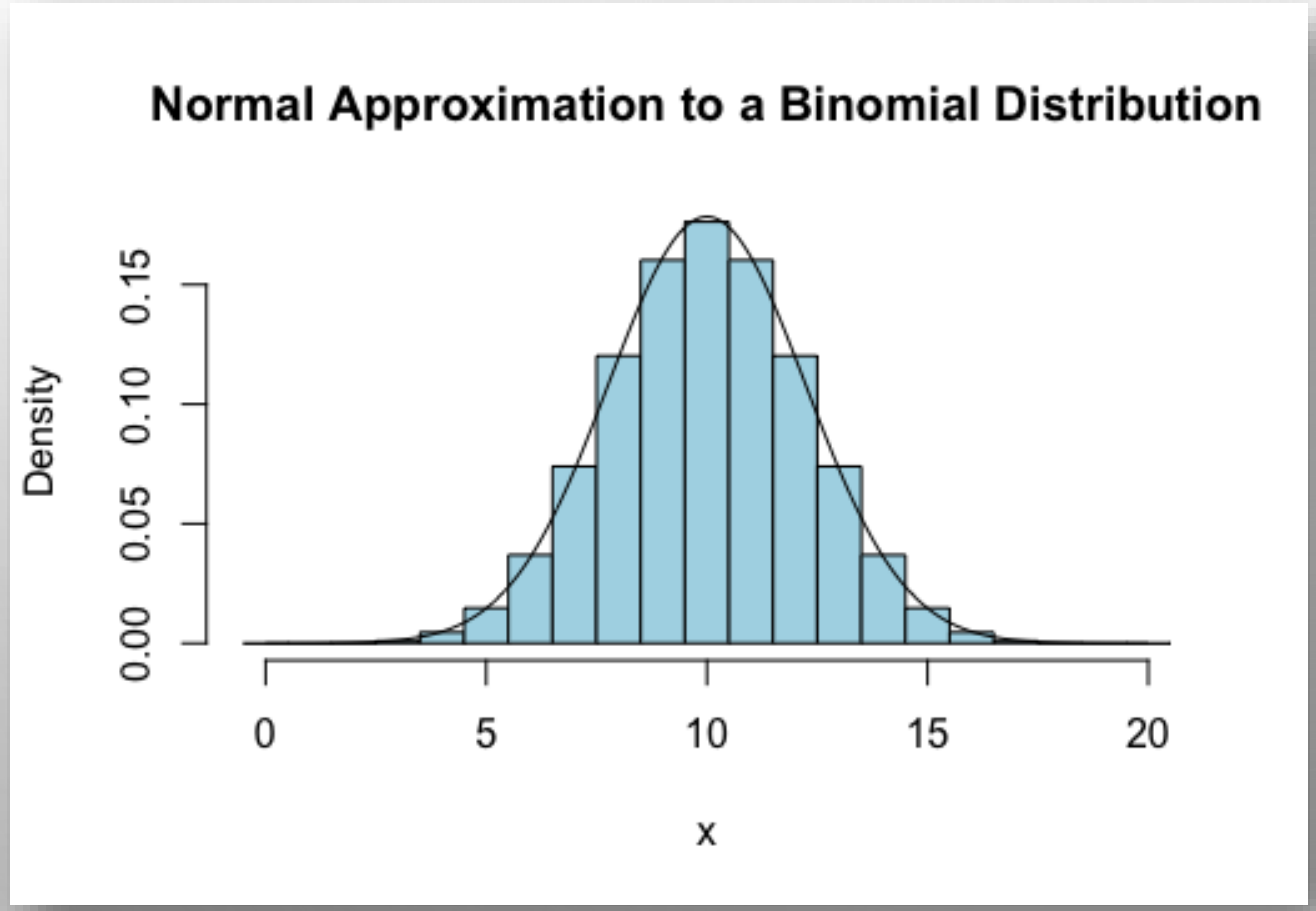
DISTRIBUIÇÃO BINOMIAL



Notation	$B(n, p)$
Parameters	$n \in \{0, 1, 2, \ldots\}$ – number of trials $p \in [0, 1]$ – success probability for each trial $q = 1 - p$
Support	$k \in \{0, 1, \ldots, n\}$ – number of successes
PMF	$\binom{n}{k} p^k q^{n-k}$
CDF	$I_q(n-k, 1+k)$ (the regularized incomplete beta function)
Mean	np
Median	$\lfloor np \rfloor$ or $\lceil np \rceil$
Mode	$\lfloor (n+1)p \rfloor$ or $\lceil (n+1)p \rceil - 1$
Variance	npq
Skewness	$\frac{q-p}{\sqrt{npq}}$
Ex. kurtosis	$\frac{1-6pq}{npq}$
Entropy	$\frac{1}{2}\log_2(2\pi enpq) + O\left(\frac{1}{n}\right)$ in shannons. For nats, use the natural log in the log.
MGF	$(q + pe^t)^n$
CF	$(q + pe^{it})^n$
PGF	$G(z) = [q + pz]^n$
Fisher information	$g_n(p) = \frac{n}{pq}$ (for fixed n)

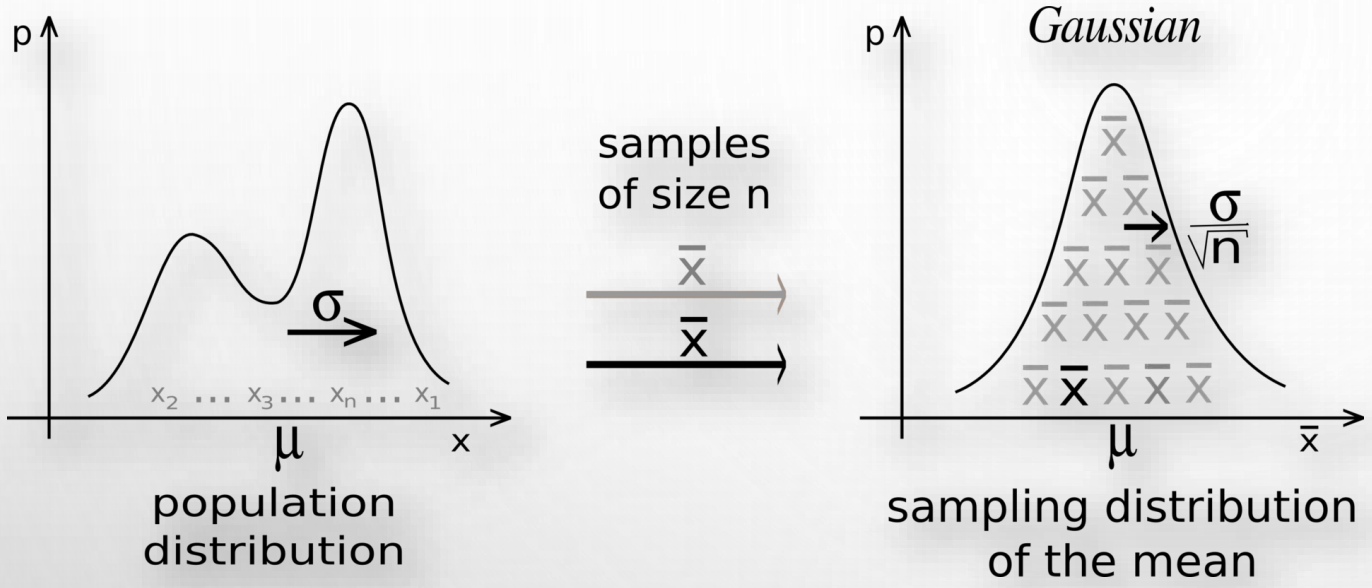
[2]

DISTRIBUIÇÃO BINOMIAL



https://anydice.com/

TEOREMA CENTRAL DO LIMITE

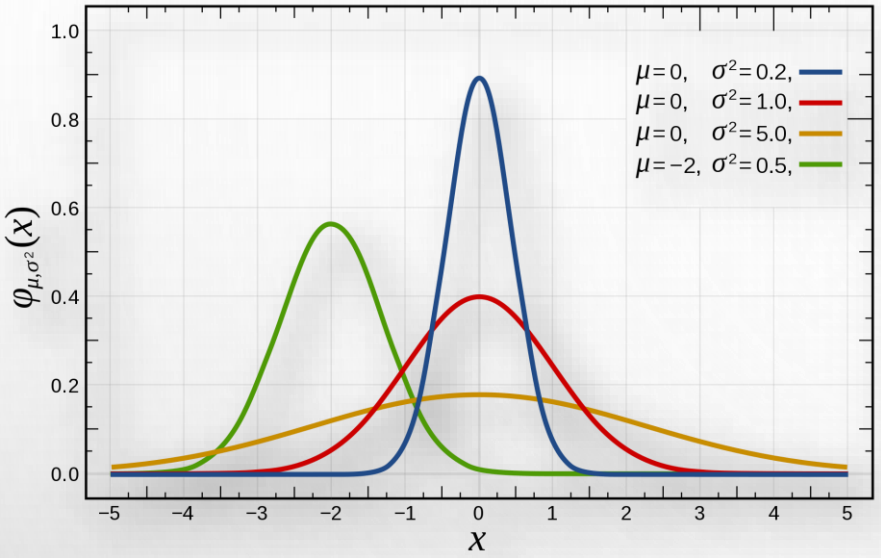


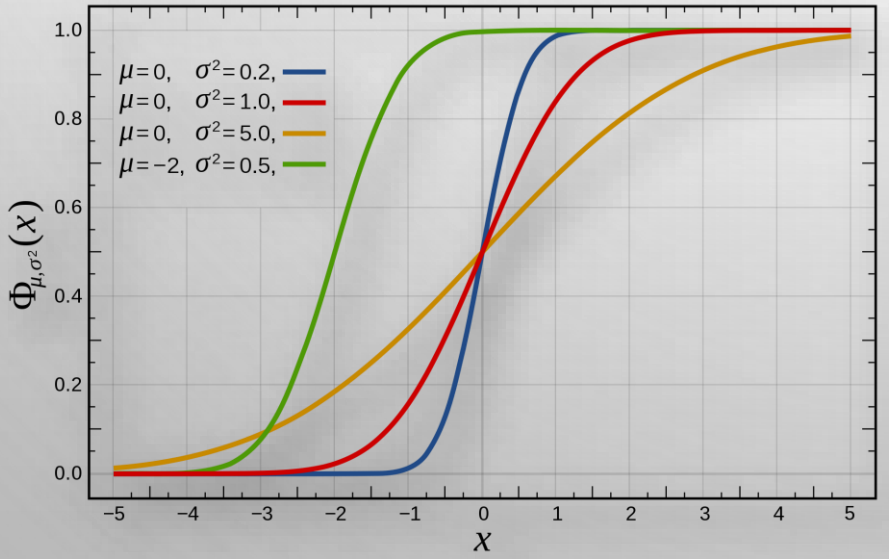
Lindeberg–Lévy CLT — Suppose $\{X_1, \ldots, X_n\}$ is a sequence of i.i.d. random variables with $\mathrm{E}[X_i] = \mu$ and $\mathrm{Var}[X_i] = \sigma^2 < \infty$. Then, as n approaches infinity, the random variables $\sqrt{n}(\bar{X}_n - \mu)$ converge in distribution to a normal $\mathcal{N}(0, \sigma^2)$:[4]

$$\sqrt{n}\left(\bar{X}_n - \mu\right) \xrightarrow{a} \mathcal{N}\left(0, \sigma^2\right).$$

[2]

DISTRIBUIÇÃO NORMAL (GAUSSIANA)





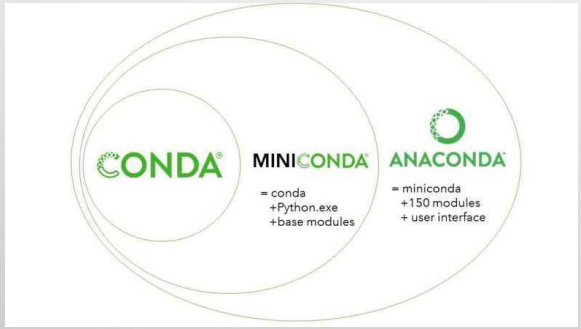
Notation	$\mathcal{N}(\mu, \sigma^2)$
Parameters	$\mu \in \mathbb{R}$ = mean (location) $\sigma^2 \in \mathbb{R}_{>0}$ = variance (squared scale)
Support	$x \in \mathbb{R}$
PDF	$\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$
CDF	$\Phi(\frac{x-\mu}{\sigma}) = \frac{1}{2}\left[1 + \operatorname{erf}\left(\frac{x-\mu}{\sigma\sqrt{2}}\right)\right]$
Quantile	$\mu + \sigma\sqrt{2}\,\operatorname{erf}^{-1}(2p-1)$
Mean	μ
Median	μ
Mode	μ
Variance	σ^2
MAD	$\sigma\sqrt{2}\,\operatorname{erf}^{-1}(1/2)$
Skewness	0
Ex. kurtosis	0
Entropy	$\frac{1}{2}\ln(2\pi\sigma^2) + \frac{1}{2}$
MGF	$\exp(\mu t + \sigma^2 t^2/2)$
CF	$\exp(i\mu t - \sigma^2 t^2/2)$
Fisher information	$\mathcal{I}(\mu, \sigma) = \begin{pmatrix} 1/\sigma^2 & 0 \\ 0 & 2/\sigma^2 \end{pmatrix}$ $\mathcal{I}(\mu, \sigma^2) = \begin{pmatrix} 1/\sigma^2 & 0 \\ 0 & 1/(2\sigma^4) \end{pmatrix}$
Kullback–Leibler divergence	$\frac{1}{2}\left\{\left(\frac{\sigma_0}{\sigma_1}\right)^2 + \frac{(\mu_1-\mu_0)^2}{\sigma_1^2} - 1 + \ln\frac{\sigma_1^2}{\sigma_0^2}\right\}$

SETUP INICIAL DO AMBIENTE PYTHON

4. Variáveis Aleatórias

5. Visualização

1. Editor de Código



2. Gestor de Ambiente

3. Ambiente Python do Projeto

3. Notebook Dinâmico

PRÓXIMA AULA LEITURA: ANÁLISE EXPLORATÓRIA (LIVRO OU ARTIGOS)