第二章作业

2.1 $r_0(t) = e_0(t) * h_0(t)$. 以下图略.

1. $r_1(t) = 2r_0(t)$.
2. $r_2(t) = r_0(t) - r_0(t-2)$.
3. $r_3(t) = r_0(t-1)$.
4. $r_4(t) = e_0(-t) * h_0(t)$ 与信号和冲激响应有关, 故不能确定.
5. $r_5(t) = r_0(-t)$.
6. $r_6(t) = r_0''(t)$.

2.2 $f(t) = (t+1)u(t+1) - tu(t) - u(t-1)$.

1. $f'(t) = u(t+1) - u(t) - \delta(t-1)$.
2. $f''(t) = \delta(t+1) - \delta(t) - \delta'(t-1)$.

注 标答非本题.

2.3

1. $f''(t) = \delta(t) - \delta(t-1) - \delta(t-2)$.
2. $f(t) = tu(t) - (t-1)u(t-1) - (t-2)u(t-2)$.

注

- ⭐ 所有的积分默认为定积分, 从 $-\infty$ 积到 t.
- 标答有误.
- 图中冲激函数的值标为 (-1) 或 (1) 均可.

2.4

1. $y_{\text{zs}}(t) = \dfrac{t}{4}[u(t) - u(t-4)]$.
2. 思路一: 傅里叶变换
   1. $y_{\text{zs}}(t) = f(t) * h(t)$, 于是 $\mathcal{F}y_{\text{zs}}(t) = \mathcal{F}f(t)\mathcal{F}h(t)$.
   2. $$\begin{aligned}\mathcal{F}f(t) &= \frac{1}{2\pi}\mathrm{j}\pi\left[\delta(\omega+1) - \delta(\omega-1)\right] * \left[\pi\delta(\omega) + \frac{1}{\mathrm{j}\omega}\right] \\ &= \frac{\mathrm{j}\pi}{2}[\delta(\omega+1) - \delta(\omega-1)] + \frac{1}{2}\left[\frac{1}{\omega+1} - \frac{1}{\omega-1}\right],\end{aligned}$$
3. 思路二: 拉普拉斯变换

$$
\begin{aligned}
\mathcal{L}f(t) &= \int_0^{+\infty} \sin t \cdot u(t)\mathrm{e}^{-st}\,\mathrm{d}t = \frac{1}{1+s^2} \\
\mathcal{L}y_{\mathrm{zs}}(t) &= \frac{1}{4s^2} - \left(\frac{1}{4s^2} + \frac{1}{s}\right)\mathrm{e}^{-4s} \\
\mathcal{L}h(t) &= \frac{\mathcal{L}y_{\mathrm{zs}}(t)}{\mathcal{L}f(t)} = \frac{1}{4} + \frac{1}{4s^2} - \left(\frac{1}{4} + \frac{1}{4s^2} + \frac{1}{s} + s\right)\mathrm{e}^{-4s} \\
h(t) &= \frac{tu(t)+\delta(t)}{4} - \frac{tu(t-4)+\delta(t-4)+4\delta'(t-4)}{4} \\
&= \frac{\delta(t)-\delta(t-4)}{4} - \delta'(t-4) + \frac{t}{4}[u(t)-u(t-4)].
\end{aligned}
$$

注

1. 这样简写一般不会引起歧义.
2. 用傅里叶变换的话, 计算非常复杂.
3. 拉普拉斯变换我直接用计算机算的.
4. 标答最后两项丢了一个 t.

2.5

1. 无冲激信号及其高阶导数, 故无跳变. $r(0_+) = r(0_-) = 0$.
2. 有冲激信号, 有跳变.
    1. 思路一: $r'(t)$ 包含的最高阶冲激信号为 $\delta(t)$, 于是对微分方程两端积分, 得 $r(0_+) = r(0_-) + 3 = 3$.
    2. 思路二: $r(t)$ 包含的最高阶冲激信号为 $u(t)$ (即 $\delta^{(-1)}(t)$), 设系数为 a, 代入方程得 $a = 3$, 从而 $r(0_+) = r(0_-) + 3 = 3$.
    3. 思路三: $r(t) = \dfrac{3}{p+2}\delta(t) = 3\mathrm{e}^{-2t}u(t)$, 于是 $r(0_+) = 3$.
3. 有冲激信号, 有跳变.
    1. 思路一: $r''(t)$ 包含的最高阶冲激信号为 $\delta(t)$, 系数为 1,
        1. 对两端积分, 得 $r'(0_+) = r'(0_-) + 1$.
        2. 再次积分, 有 $r(0_+) = r(0_-) = 0$.

注 题目中忘了给出 $r'(0_-)$.

2.6

1. 特征方程为 $p^3 + 7p^2 + 15p + 9 = (p+1)(p+3)^2$.
2. 于是齐次通解为 $y_{\mathrm{zi}}(t) = C_1\mathrm{e}^{-t} + (C_2 + C_3 t)\mathrm{e}^{-3t}$.
3. 代入初值解得 $y_{\mathrm{zi}}(t) = 6\mathrm{e}^{-t} - 4\mathrm{e}^{-3t} - 5t\mathrm{e}^{-3t}$.

注 若要计算零状态响应, 可用传输算子 $H(p) = -\dfrac{3}{4}\dfrac{1}{p+1} + \dfrac{11}{4}\dfrac{1}{p+3} + \dfrac{3}{2}\dfrac{1}{(p+3)^2}$.

2.7

$$
\begin{aligned}
&\mathrm{e}^{-2t}u(t) * t^n u(t) * \big[\delta''(t) + 3\delta'(t) + 2\delta(t)\big] * \mathrm{e}^{-t}u(t) \\
={}& \big(\mathrm{e}^{-t} - \mathrm{e}^{-2t}\big)u(t) * \big[\delta''(t) + 3\delta'(t) + 2\delta(t)\big] * t^n u(t) \\
={}& 0
\end{aligned}
$$

注

- 标答有误 (已用计算机验证).
- 计算诸如 $\dfrac{\mathrm{d}}{\mathrm{d}t}\big(\mathrm{e}^{-t} - \mathrm{e}^{-2t}\big)u(t)$ 的式子时, 完全不需要全部展开. 这是连续的, 忽略 $u(t)$ 直接导就行了. 之前一个个展开真的是憨憨做法.

2.8 首先解得零状态响应与零输入响应:

$$
\begin{cases} y_{x(t)} + y_{x(0)} = (2\mathrm{e}^{-3t} + \sin 2t)u(t), \\ 2y_{x(t)} + y_{x(0)} = (\mathrm{e}^{-3t} + 2\sin 2t)u(t). \end{cases} \quad \Rightarrow \quad \begin{cases} y_{x(t)} = (\sin 2t - \mathrm{e}^{-3t})u(t), \\ y_{x(0)} = 3\mathrm{e}^{-3t}u(t). \end{cases}
$$

1. $y_1(t) = (5.5e^{-3t} + 0.5\sin 2t)u(t)$.
2. $y_2(t) = 3\mathrm{e}^{-3t}u(t) + (\sin 2(t-t_0) - \mathrm{e}^{-3(t-t_0)})u(t-t_0)$.

注 标答漏写了 $u(t-t_0)$.

2.9

$$
\begin{aligned}
&\mathrm{e}^{-t}u(t) * t^n u(t) * \delta''(t) * \mathrm{e}^{2t}u(-t) \\
={}& t^n u(t) * \delta''(t) * \int_{-\infty}^{+\infty} \mathrm{e}^{3x-t} I_{\{x<0, x<t\}}\,\mathrm{d}x \\
={}& t^n u(t) * \delta''(t) * \frac{\mathrm{e}^{-t}u(t) + \mathrm{e}^{2t}u(-t)}{3} \\
={}& t^n u(t) * \delta''(t) * \frac{\mathrm{e}^{2t} + (\mathrm{e}^{-t} - \mathrm{e}^{2t})u(t)}{3} \\
={}& t^n u(t) * \frac{\mathrm{e}^{-t}u(t) + 4\mathrm{e}^{2t}u(-t)}{3}
\end{aligned}
$$

注

- 下面这个积分很容易展开, 但是挺麻烦的. 可以用下不完全伽马函数表示, 但是没必要.

$$
\begin{aligned}
\mathrm{e}^{-\alpha t}u(t) * t^n u(t) &= \int_{-\infty}^{+\infty} \mathrm{e}^{-\alpha(t-x)} x^n I_{\{0<x<t\}}\,\mathrm{d}x \\
&= \mathrm{e}^{-\alpha t}u(t) \int_0^t \mathrm{e}^{\alpha x} x^n\,\mathrm{d}x
\end{aligned}
$$

- 我没有继续计算了, 但我不相信可以化成标答的形式.

2.10

1. $e(t) = tu(t) - (t-1)u(t-1) - (t-3)u(t-3) + (t-4)u(t-4)$.
2. 暂记 $r_1(t) = \mathrm{e}^{-t}u(t) * tu(t) = (\mathrm{e}^{-t} + t - 1)u(t)$.
3. 于是 $r(t) = r_1(t) - r_1(t-1) - r_1(t-3) + r_1(t-4)$.

2.11

1. 思路一: 传输算子法.
    1. $iR+L\dfrac{\mathrm{d}i}{\mathrm{d}t}=\dfrac{1}{C}\displaystyle\int_{-\infty}^{t}i\,\mathrm{d}t=f(t).$
    2. $(p^2+5p+6)i(t)=\delta(t).$
    3. $i_{\text{zs}}(t)=\dfrac{\delta(t)}{p+2}-\dfrac{\delta(t)}{p+3}=(\mathrm{e}^{-2t}-\mathrm{e}^{-3t})u(t).$
2. 思路二: 拉普拉斯变换法.
    1. $i(s)=\dfrac{\frac{1}{s}}{5+s+\frac{6}{s}}=\dfrac{1}{s^2+5s+6}=\dfrac{1}{s+2}-\dfrac{1}{s+3}.$
    2. $i_{\text{zs}}(t)=(\mathrm{e}^{-2t}-\mathrm{e}^{-3t})u(t).$

注　标答有误 (甚至方向都错了).

2.12

1. $r(t)=\dfrac{2p}{p+3}\delta(t)=2\delta(t)-\dfrac{6}{p+3}\delta(t)=2\delta(t)--6\mathrm{e}^{-3t}u(t).$
2. $r(t)=\dfrac{p^2+3p+3}{p+2}=\left(p+1+\dfrac{1}{p+2}\right)\delta(t)=\delta'(t)+\delta(t)+\mathrm{e}^{-2t}u(t).$

2.13

$$\begin{aligned}&[\delta(t)+b\delta(t)]*\mathrm{e}^{-bt}u(t)*t^nu(t)\\=&(b+1)\mathrm{e}^{-bt}u(t)\int_0^t\mathrm{e}^{bx}x^n\,\mathrm{d}x\\=&\frac{(b+1)\mathrm{e}^{-bt}u(t)}{(-b)^{n+1}}\gamma\left(n+1,-bt\right)\end{aligned}$$

注

- 我不理解为什么这么喜欢出不完全伽马函数的题, 表示起来枯燥无趣.
- 关系式: $\gamma(s,x)+\Gamma(s,x)=\Gamma(s).$

2.14

$$\begin{aligned}t^au(t)*t^bu(t)=u(t)&\int_0^t x^a(t-x)^b\,\mathrm{d}x\\&=\mathrm{B}(a+1,b+1)t^{a+b+1}u(t),\end{aligned}$$

代入 $a=2,b=3$ 即得原式 $=\dfrac{\Gamma(3)\Gamma(4)}{\Gamma(7)}t^6u(t)=\dfrac{t^6u(t)}{60}.$

注　好, 这下欧拉积分都齐了.

2.15

$$f(t)*\delta_T(t)=\sum_{n=-\infty}^{+\infty}f(t-nT),$$

具体就不展开了, 参考第二次翻转课堂练习题第五题.

2.16

$$\begin{aligned}u(t) * \mathrm{e}^{-\lambda t}u(t) &= u(t)\int_0^t \mathrm{e}^{-\lambda x}\,\mathrm{d}x \\ &= \frac{1-\mathrm{e}^{-\lambda t}}{\lambda}u(t).\end{aligned}$$

注 我是显微镜, 标答把 t 写成 τ 了.

2.17

1. 当 $t = 0_-$ 时, 电流源被电感短路.
2. 当 $t > 0$ 时,
    1. 求 u_{C}.
        1. 思路一: 解 $2 = \dfrac{u_{\mathrm{C}}}{3} + \dfrac{\mathrm{d}u_{\mathrm{C}}}{\mathrm{d}t}$.
        2. 思路二: 由三要素法, $u_{\mathrm{C}} = 6\left(1-\mathrm{e}^{-\frac{t}{3}}\right)u(t)$.
    2. 求 u_{L}.
        1. 思路一: 解微分方程.
        2. 思路二: 由三要素法, $u_{\mathrm{L}} = -\mathrm{e}^{-\frac{t}{4}}u(t)$.
    3. 于是 $u_{\mathrm{ac}}(t) = 6\left(1-\mathrm{e}^{-\frac{t}{3}}\right)u(t) + \mathrm{e}^{-\frac{t}{4}}u(t)$.

2.18

以下只考虑 $t > 0$ 的情况.

1. 齐次通解为 $y(t) = C\mathrm{e}^{-3t}$, 代入初值 $y(0_-) = 1.5$, 得 $y_{\mathrm{zi}}(t) = 1.5\mathrm{e}^{-3t}$.
2. 全解为 $y(t) = C\mathrm{e}^{-3t} + 1$, 代入初值 $y(0_-) = 0$, 得 $y_{\mathrm{zs}}(t) = 1 - \mathrm{e}^{-3t}$.
3. 全响应为 $y(t) = 1 + 0.5\mathrm{e}^{-3t}$.

2.19

$$\begin{aligned}f_1(t) &= \mathbb{I}\{-2<t<2\} \\ f_2(t) &= \frac{1}{2}\mathbb{I}\{0<t<2\} \\ f_1(t) * f_2(t) &= \frac{1}{2}\int_{-\infty}^{+\infty}\mathbb{I}\{-2<x<2, t-2<x<t\}\,\mathrm{d}x \\ &= \begin{cases}0, & t<-2, \\ \dfrac{t+2}{2}, & -2\le t<0, \\ 1, & 0\le t<2, \\ \dfrac{4-t}{2}, & 2\le t<4, \\ 0, & 4\le t.\end{cases}\end{aligned}$$

2.20

$$
\begin{aligned}
h(t) &= \frac{Mp}{(L^2-M^2)p^2+2RLp+R^2}\delta(t) \\
&= \frac{1}{2}\frac{1}{(L-M)p+R}\delta(t) - \frac{1}{2}\frac{1}{(L+M)p+R}\delta(t) \\
&= \frac{1}{2}\left(\frac{\mathrm{e}^{-\frac{R}{L-M}t}}{L-M} - \frac{\mathrm{e}^{-\frac{R}{L+M}t}}{L+M}\right)u(t).
\end{aligned}
$$

2.21

1. $0 < t < 3$,
   1. $u_{\mathrm{C}}(t) = 10\left(1-\mathrm{e}^{-\frac{t}{5}}\right)u(t)\ \mathrm{V}.$
   2. $i_{\mathrm{C}}(t) = \mathrm{e}^{-\frac{t}{5}}u(t)\ \mathrm{A}.$
2. $t = 3$,
   1. $u_{\mathrm{C}}(t) \approx 4.51188\ \mathrm{V}.$
   2. $i_{\mathrm{C}}(t) \approx 0.548812\mathrm{A}.$
3. $t \geq 3$,
   1. $u_{\mathrm{C}}(t) \approx \left(\frac{10}{3} + 1.17855\mathrm{e}^{-\frac{3(t-3)}{5}}\right)u(t-3)\ \mathrm{V}.$
   2. $i_{\mathrm{C}}(t) \approx -0.353565\mathrm{e}^{-\frac{3(t-3)}{5}}u(t-3).$

注

- 标答只给了 $t \geq 3$ 时的结果.
- $t \geq 3$ 时不是直接对 $u_{\mathrm{C}}(t)$ 的表达式求导, 那样会多出一项 $4.51188\delta(t-3)$.

2.22

$\sin(t)u(t) * h(t) = 2tu(t) - 4(t-1)u(t-1).$

1. 思路一: 傅里叶变换
2. 思路二: 拉普拉斯变换
   1. $\mathcal{L}\left[\sin(t)\right](s) = \int_0^{+\infty}\sin(t)\mathrm{e}^{-st}\,\mathrm{d}t = \frac{1}{s^2+1}.$
   2. $\mathcal{L}\left[y_{\mathrm{zs}}(t)\right](s) = \frac{2-4\mathrm{e}^{-s}}{s^2}.$
   3. $\mathcal{L}\left[h(t)\right](s) = 2-4\mathrm{e}^{-s} + \frac{2-4\mathrm{e}^{-s}}{s^2}.$
   4. $h(t) = 2\delta(t) - 4\delta(t-1) + 2tu(t) - 4(t-1)u(t-1).$

提问　标答的思路是什么? ⭐

注　恼恼, 怎么又要求卷积的逆. 好, 我推一下.

1. 三角函数的拉普拉斯变换

$$\int \mathrm{e}^{(a+b\mathrm{i})x}\,\mathrm{d}x = \frac{a-b\mathrm{i}}{a^2+b^2}\mathrm{e}^{(a+b\mathrm{i})x}+C,$$
$$\int \mathrm{e}^{ax}\cos bx\,\mathrm{d}x = \frac{a\cos bx+b\sin bx}{a^2+b^2}\mathrm{e}^{ax}+C,$$
$$\int \mathrm{e}^{ax}\sin bx\,\mathrm{d}x = \frac{a\sin bx-b\cos bx}{a^2+b^2}\mathrm{e}^{ax}+C.$$

于是有

$$\mathcal{L}\left[\sin\omega t\right](s) = \frac{\omega}{s^2+\omega^2},$$
$$\mathcal{L}\left[\cos\omega t\right](s) = \frac{s}{s^2+\omega^2}.$$

2. 幂函数的拉普拉斯变换

$$\mathcal{L}\left[t^n\right](s) = \int_0^{+\infty} t^n\mathrm{e}^{-st}\,\mathrm{d}t = \frac{n!}{s^{n+1}}.$$

3. 时移性质

若 $F(s)=\mathcal{L}[f(t)u(t)]$, 则

$$\mathcal{L}\left[f(t-t_0)u(t-t_0)\right](s) = \int_0^{+\infty} f(t-t_0)u(t-t_0)\mathrm{e}^{-st}\,\mathrm{d}t = F(s)\mathrm{e}^{-st_0}.$$

4. 频移性质

$$\mathcal{L}\left[f(t)\mathrm{e}^{-s_0t}\right](s) = F(s+s_0).$$

2.23

1. $t=0_-$,
   1. $i_\mathrm{L}(0_-)=1\ \mathrm{A}$.
   2. $u_\mathrm{C}(0_-)=6\ \mathrm{V}$.
2. $t>0$,
   1. $10=u_\mathrm{C}+4\left(\frac{1}{5}u_\mathrm{C}'+i_\mathrm{L}\right)=i_\mathrm{L}'+6i_\mathrm{L}+\frac{4}{5}\left(i_\mathrm{L}''+6i_\mathrm{L}'+5i_\mathrm{L}\right)$.

      $4i''+29i'+50i=50$.
   2.

注　这里算的有问题了. ⭐

2.24

1. $h(t)=2\mathrm{e}^{-2t}u(t)$.

   $y_\mathrm{zs}(t)=x_1(t)*h(t)=2\left(\mathrm{e}^{-t}-\mathrm{e}^{-2t}\right)u(t)$.

   $y_\mathrm{zi}(t)=y_1(t)-y_\mathrm{zs}(t)=2\mathrm{e}^{-2t}u(t)$.
2. $y_\mathrm{zi}(t)=4\mathrm{e}^{-2t}u(t)$.

   $y_\mathrm{zs}(t)=2\delta(t)-4\mathrm{e}^{-2t}u(t)$.

   $y_2(t)=2\delta(t)$.

2.25

1. $r_f(t) = \big(\cos 2t - \mathrm{e}^{-t}\big)u(t)$.
2. $r(t) = \big(4\cos 2t - \mathrm{e}^{-t}\big)u(t)$.

2.26

1. $g'(t) + r_{\mathrm{zi}}(t) = \delta(t) + \mathrm{e}^{-t}u(t)$.
2. $g(t) + r_{\mathrm{zi}}(t) = 3\mathrm{e}^{-t}u(t)$.
3. $g'(t) - g(t) = \delta(t) - 2\mathrm{e}^{-t}u(t)$.
4. $g(t) = \mathrm{e}^{-t}u(t)$. (一般方法?)
5. $h(t) = \delta(t) - \mathrm{e}^{-t}u(t)$.
6. $r_{\mathrm{zi}}(t) = 2\mathrm{e}^{-t}u(t)$.
7. $x_3(t) = t\,(u(t) - u(t-1))$.
8. $y_3(t) = x_3(t) * h(t) + r_{\mathrm{zi}}(t)$.
9. $y_3(t) = (1 + \mathrm{e}^{-t})u(t) - u(t-1)$.