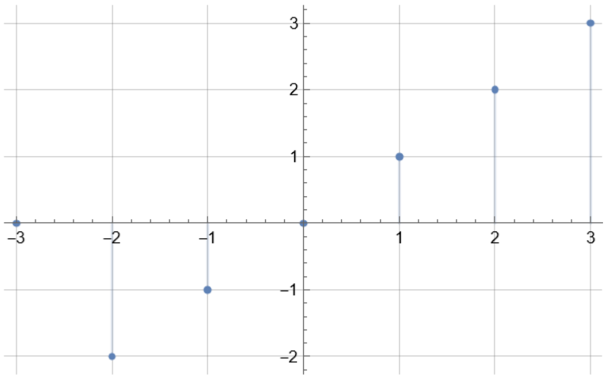
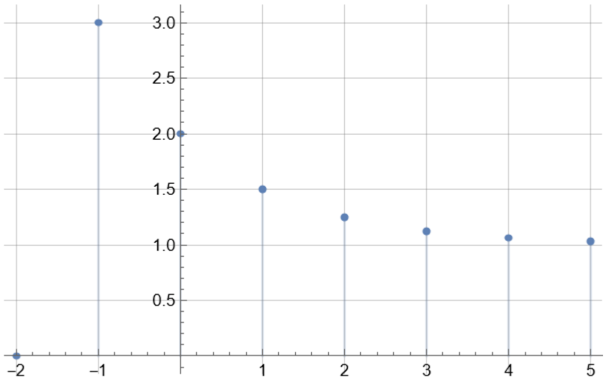
第五章作业

5.1

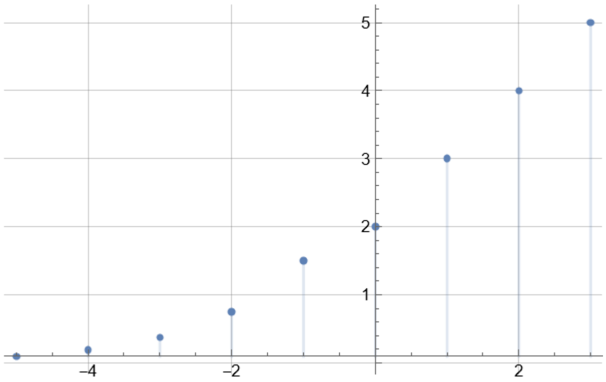
1. $x_1(n) = n \cdot u(n+2)$.



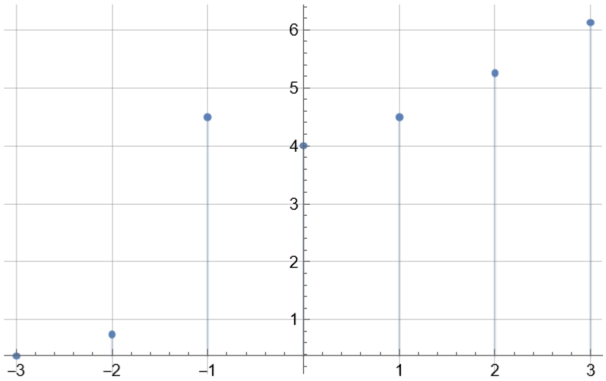
2. $x_2(n) = (2^{-n}+1)u(n+1)$.



3. $x_3(n) = \begin{cases} n+2, & n \geq 0, \\ 3 \cdot 2^n, & n < 0. \end{cases}$



4. $x_4(n) = x_2(n) + x_3(n)$.



附　绘图 mathematica 代码如下：

```
x1 = n UnitStep[n + 2];
x2 = (2^-n + 1) UnitStep[n + 1];
x3 = Which[n < 0, 3*2^n, n >= 0, n + 2];
x4 = x2 + x3;

DiscretePlot[x1, {n, -3, 3}, PlotRange -> All, GridLines -> Automatic]
DiscretePlot[x2, {n, -2, 5}, PlotRange -> All, GridLines -> Automatic]
DiscretePlot[x3, {n, -5, 3}, PlotRange -> All, GridLines -> Automatic]
DiscretePlot[x4, {n, -3, 3}, PlotRange -> All, GridLines -> Automatic]
```

5.2

1. $(n-1)\left[u(n-1) - u(n-5)\right]$.
2. $u(n-3) - u(n-6)$.
3. $(-1)^n u(n-1)$.
4. $-u(n) + 2u(n-3) - u(n-6)$.

5.3

1. 周期性，$N = 14$.
2. 非周期.

备注　对于 $\sin(\omega n + \varphi)$ 或 $\mathrm{e}^{\mathrm{j}\omega n}$，若 $\dfrac{2\pi}{\omega} \in \mathbb{Q}$，则为周期序列，否则为非周期序列.

5.4

$$y(n)-\frac{1}{3}y(n-1)=x(n),\quad y(-1)=0.$$

1. 迭代得 $y(0)=1$，齐次通解为 $y(n)=\dfrac{C}{3^n}u(n)$，代入有 $y_1(n)=h(n)=\dfrac{1}{3^n}u(n)$.
2. $y_2(n)=x(n)*h(n)=\dfrac{3-3^{-n}}{2}u(n)$.
3. 输出信号为

$$\begin{aligned}y_3(n)&=\frac{3-3^{-n}}{2}u(n)-\frac{3-3^{5-n}}{2}u(n-5)\\&=\frac{3-3^{-n}}{2}[u(n)-u(n-5)]+\frac{121}{3^n}u(n-5).\end{aligned}$$

波形图略.

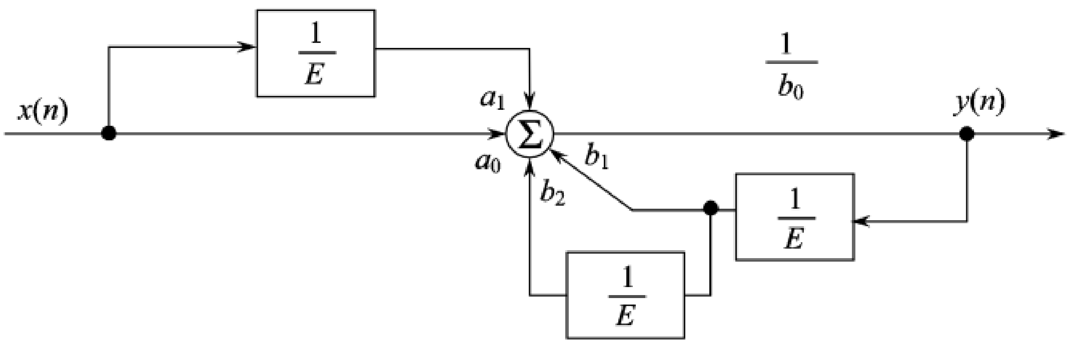
5.5

1. $y(n)-ay(n-1)+by(n-2)=x(n)$.
2. 二阶差分方程（二阶系统）.

5.6

1. $y(n)-b_1y(n-2)-b_2y(n-2)=a_0x(n)+a_1x(n-1)$.
2. 二阶差分方程（二阶系统）.

5.7



备注 不知道有什么便捷的工具可以绘制系统框图；上图直接截的标答.

5.8

1. $y(n)=2^{-n}$.
2. $y(n)=2^{n+1}$.
3. $y(n)=(-3)^{n-1}$.
4. $y(n)=\dfrac{(-3)^{-n}}{3}=-(-3)^{-n-1}$.

5.9

1. $y(n) = 4(-1)^n - 12(-2)^n$.
2. $y(n) = (2n+1)(-1)^n$.
3. $\cos\dfrac{n\pi}{2} + 2\sin\dfrac{n\pi}{2}$。

5.10

1. 特征根为 $2, 2, 3$，设 $y(n) = (an+b)2^n + c\cdot 3^n$，代入得

$$\begin{cases} b+c=0, \\ 2a+2b+3c=-1, \\ 8a+4b+9c=-3. \end{cases} \quad \Rightarrow \quad \begin{cases} a=-1, \\ b=-1, \\ c=1. \end{cases}$$

于是 $y(n) = 3^n - (n+1)2^n, n \geq 0$.

2. 大概是题目打错了吧...这个计算量挺离谱的：

```
RSolveValue[{
    y[n] - 2 y[n - 1] + 26 y[n - 2] - 2 y[n - 3] + y[n - 4] == 0,
    y[0] == 0, y[1] == 1, y[2] == 2, y[3] == -3
}, y[n], n]
```

5.11

1. 齐次通解为 $y(n) = C(-2)^n$.
2. 设特解为 $y_0(n) = an+b$，代入得 $a = \dfrac{1}{3}, b = -\dfrac{4}{9}$.
3. 于是全解为 $y(n) = C(-2)^n + \dfrac{n}{3} - \dfrac{4}{9}$.
4. 代入初值，得 $y(n) = \dfrac{13}{9}(-2)^n + \dfrac{n}{3} - \dfrac{4}{9}$.

5.12

1. 齐次通解为 $y(n) = (an+b)(-1)^n$.
2. 设特解为 $y_0(n) = A\cdot 3^n$，代入得 $A = \dfrac{9}{16}$.
3. 于是全解为 $y(n) = (an+b)(-1)^n + \dfrac{9}{16}\cdot 3^n$.
4. 代入初值，得 $y(n) = \left(-\dfrac{3}{4}n - \dfrac{9}{16}\right)(-1)^n + \dfrac{9}{16}\cdot 3^n$.

5.13

1. 全解为 $y(n) = a\cos\dfrac{n\pi}{2} + b\sin\dfrac{n\pi}{2} + A\sin n + B\cos n$.

   代入即可解得.

2.