第 4 章

4.1

(1) $\dfrac{\alpha}{s(s+\alpha)}$

(2) $\dfrac{1}{(s+2)^2}$

(3) $\dfrac{1}{(s+\beta)}-\dfrac{s+\beta}{(s+\beta)^2+\alpha^2}$

(4) $2-\dfrac{3}{s+7}$

(5) $\dfrac{\beta}{(s+\alpha)^2-\beta^2}$

(6) $\dfrac{1}{(s+\alpha)(s+\beta)}$

(7) $\dfrac{(s+1)\mathrm{e}^{-a}}{(s+1)^2+\omega^2}$

(8) $\dfrac{(s+2)\mathrm{e}^{-(s-1)}}{(s+1)^2}$

(9) $aF(as+a^2)$

(10) $\dfrac{2s^3-24s}{(s^2+4)^3}$

(11) $-\ln\left(\dfrac{s}{s+a}\right)$

(12) $\ln\left(\dfrac{s+5}{s+3}\right)$

(13) $2\mathrm{e}^{-\frac{3}{2}t}$

(14) $\dfrac{4}{3}\left(1-\mathrm{e}^{-\frac{3}{2}t}\right)$

(15) $\dfrac{1}{5}\left[1-\cos\left(\sqrt{5}t\right)\right]$

(16) $6\mathrm{e}^{-4t}-3\mathrm{e}^{-2t}$

(17) $\sin t+\delta(t)$

(18) $1-2\mathrm{e}^{-\frac{t}{RC}}$

(19) $7\mathrm{e}^{-3t}-3\mathrm{e}^{-2t}$

(20) $\dfrac{1}{6}\left[\dfrac{\sqrt{3}}{3}\sin\left(\sqrt{3}t\right)-t\cos\sqrt{3}t\right]$

(21) $\dfrac{-a}{(\alpha-a)^2+\beta^2}\left\{\mathrm{e}^{-at}-\left[\cos(\beta t)+\dfrac{\alpha^2+\beta^2-a\alpha}{a\beta}\sin(\beta t)\mathrm{e}^{-\alpha t}\right]\right\}$

(22) $\dfrac{1}{t}\left(\mathrm{e}^{-9t}-1\right)$

4.2

(1) $\dfrac{1}{(s+1)}\mathrm{e}^{-2(s+1)}$

(2) $\dfrac{1}{s+1}\mathrm{e}^{-2s}$

(3) $\dfrac{\mathrm{e}^2}{s+1}$

(4) $\dfrac{2\cos 2 + s\sin 2}{s^2+4}\mathrm{e}^{-s}$

(5) $\dfrac{1}{s^2}\left[1-(1+s)\mathrm{e}^{-s}\right]\mathrm{e}^{-s}$

4.3

(1) $F(s)=\dfrac{\mathrm{e}^{-\frac{1}{2}s}}{s^2}+\dfrac{\mathrm{e}^{-\frac{1}{2}s}}{2s}$

(2) $F(s)=\dfrac{1}{s}\mathrm{e}^{-2s}$

(3) $F(s)=\dfrac{\pi\left(1+\mathrm{e}^{-s}\right)}{s^2+\pi^2}$

(4) $F(s)=\dfrac{2-s}{\sqrt{2}\left(s^2+4\right)}$

(5) $F(s)=\dfrac{s^2}{(s+1)^2+1}$

(6) $F(s)=\dfrac{\pi}{2}-\arctan\left(\dfrac{s}{a}\right)$

(7) $F(s)=\ln\dfrac{s+5}{s+3}$

4.4

(1) $f\left(0_+\right)=1$， $f\left(\infty\right)=0$

(2) $f\left(0_+\right)=1$, $f\left(\infty\right)=0$

(3) $f\left(0_+\right)=0$, $f\left(\infty\right)=\dfrac{1}{2}$

(4) $f(0_+)=0$， $f(\infty)=0$

(5) $f(0_+)=2$， $f(\infty)$ 不存在

(6) $f(0_+)=0$， $f(\infty)$ 不存在

4.5

(1) $\left[t-\dfrac{1}{2}\left(1-\mathrm{e}^{-2t}\right)\right]u(t)-\left[(t-1)-\dfrac{1}{2}\left(1-\mathrm{e}^{-2(t-1)}\right)\right]u(t-1)$

(2) $(1+2t)u(t)$

4.6 $\left(1-\mathrm{e}^{-2t}+2\mathrm{e}^{-3t}\right)u(t)$

4.7 $0.5\left(1+\mathrm{e}^{-2t}\right)u(t)$

4.8

(1) $\left[1-\mathrm{e}^{-2(t-2)}\right]u(t-2)$

(2) $2\left(\mathrm{e}^{-t}-\mathrm{e}^{-2t}\right)u(t)$

(3) $\left[t-\dfrac{1}{2}\left(1-\mathrm{e}^{-2t}\right)\right]u(t)$

4.9 $\left(1-\dfrac{1}{2}\mathrm{e}^{-2t}\right)u(t)$

4.10 设 $\alpha=\dfrac{1}{2RC}$， $\omega_0=\dfrac{1}{\sqrt{LC}}$， $\omega_\mathrm{d}^2=\omega_0^2-\alpha^2$， $i(t)=\dfrac{E}{R}\left[1-\dfrac{2\alpha}{\omega_\mathrm{d}}\mathrm{e}^{-\alpha t}\sin(\omega_\mathrm{d}t)\right]u(t)$

4.11

(1) 设 $\alpha=\dfrac{R+R_0}{2RR_0C}$， $\omega_0=\dfrac{1}{\sqrt{LC}}$， $\omega_\mathrm{d}^2=\omega_0^2-\alpha^2$，且假设 $\alpha<\omega_0$

$$h(t)=\frac{1}{RC}\mathrm{e}^{-\alpha t}\left[\cos(\omega_\mathrm{d}t)-\frac{\alpha}{\omega_\mathrm{d}}\sin(\omega_\mathrm{d}t)u(t)\right]$$

$$H(s)=\frac{R(s)}{E(s)}=\frac{1}{RC}\frac{s}{s^2+\frac{R+R_0}{RCR_0}s+\frac{1}{LC}}$$

(2) 设 $\alpha=\frac{1}{R_1R_2C_1C_2}$，$\beta=R_1C_1+R_1C_2+R_2C_2$，$p_1=\frac{\alpha}{2}\left(-\beta+\sqrt{\beta^2-\frac{4}{\alpha}}\right)$ $p_2=\frac{\alpha}{2}\left(-\beta-\sqrt{\beta^2-\frac{4}{\alpha}}\right)$

$$H(s)=1+\frac{1}{p_2-p_1}\left(\frac{p_1\alpha\beta+\alpha}{s-p_1}-\frac{p_2\alpha\beta+\alpha}{s-p_2}\right)$$

$$h(t)=\delta(t)+\frac{1}{p_2-p_1}\left[(p_1\alpha\beta+\alpha)\mathrm{e}^{p_1t}-(p_2\alpha\beta+\alpha)\mathrm{e}^{p_2t}\right]u(t)$$

4.12 设 $\omega_0=\frac{1}{\sqrt{LC}}$，$i(t)=\frac{E}{2L\omega_0}\sin(\omega_0t)u(t)$

4.13 $\frac{E}{2}\mathrm{e}^{-20t}u(t)-\frac{E}{40T}\left\{(1-\mathrm{e}^{-20t})u(t)-\left[1-\mathrm{e}^{-20(t-T)}\right]u(t-T)\right\}$

4.14

(1) $H(s)=\frac{K}{s^2+(3-K)s+1}$

(2) 当 $K=2$ 时，$h(t)=\frac{4}{\sqrt{3}}\mathrm{e}^{-\frac{1}{2}t}\sin(\frac{\sqrt{3}}{2}t)u(t)$

4.15

(1) $\frac{1}{s\left(1+\mathrm{e}^{-\frac{sT}{2}}\right)}$

(2) $\frac{\omega}{s^2+\omega^2}\frac{1+\mathrm{e}^{-\frac{sT}{2}}}{1-\mathrm{e}^{-\frac{sT}{2}}}$

4.16

(1) $\sum_{n=0}^{\infty}f(nT)\,\mathrm{e}^{-nsT}$

(2) $\frac{1}{1-\mathrm{e}^{-(a+s)T}}$

4.17

(1) $\left(1+\mathrm{e}^{-t}-\mathrm{e}^{-2t}\right)u(t)$

(2) $\left(\frac{1}{2}+t-\mathrm{e}^{-t}+\frac{1}{2}\mathrm{e}^{-2t}\right)u(t)$

4.18

(1) $u(t)-u(t-2)$

(2) $\frac{1}{2}t^2u(t)-\frac{1}{2}(t-2)^2u(t-2)$

4.19

(1) $H(s)=\frac{C_1}{C_1+C_2}\cdot\frac{S+\frac{1}{C_1R}}{S+\frac{1}{(C_1+C_2)R}}$，$v_2(t)=\frac{C_1}{C_1+C_2}\left[\delta(t)+\frac{C_2}{C_1(C_1+C_2)R}\mathrm{e}^{-\frac{t}{R(C_1+C_2)}}u(t)\right]$

(2) $H(s)=\frac{L_2}{L_1+L_2}\cdot\frac{S}{S+\frac{R}{(L_1+L_2)}}$，$v_2(t)=\frac{L_2}{L_1+L_2}\left[\delta(t)-\frac{R}{L_1+L_2}\mathrm{e}^{-\frac{R}{(L_1+L_2)}}u(t)\right]$

(3) $H(s)=\frac{s}{10s^2+s+10}$，$v_2(t)=\frac{1}{10}\mathrm{e}^{-\frac{t}{20}}\left[\cos\left(\frac{\sqrt{399}}{20}t\right)-\frac{1}{\sqrt{399}}\sin\left(\frac{\sqrt{399}}{20}t\right)\right]u(t)$

(4) $H(s)=\dfrac{0.1s}{s+1}$，　　$v_2(t)=0.1\left[\delta(t)-\mathrm{e}^{-t}u(t)\right]$

4.20

(1) $H(s)=\dfrac{5}{s^2+s+5}$

(2) 极点　$p_{1,2}=\dfrac{-1\pm\mathrm{j}\sqrt{19}}{2}$

(3) $h(t)=\dfrac{10}{\sqrt{19}}\mathrm{e}^{-\frac{t}{2}}\sin\left(\dfrac{\sqrt{19}}{2}t\right)u(t)$，　$g(t)=1-\mathrm{e}^{-\frac{t}{2}}\left[\cos\left(\dfrac{\sqrt{19}}{2}t\right)+\dfrac{1}{\sqrt{19}}\sin\left(\dfrac{\sqrt{19}}{2}t\right)\right]u(t)$

4.21　$v_2(t)=\dfrac{5}{2}\left\{-\dfrac{48}{37}\cos t+\dfrac{8}{37}\sin t+\mathrm{e}^{-\frac{t}{16}}\left[\dfrac{48}{37}\cos\left(\dfrac{\sqrt{63}}{16}t\right)-\dfrac{80}{37\sqrt{63}}\sin\left(\dfrac{\sqrt{63}}{16}t\right)\right]\right\}u(t)$

其中前两项为强迫响应，后两项为自由响应

4.22

(1) $H(s)=\dfrac{s^2+\dfrac{1}{LC}}{s^2+\dfrac{1}{RC}s+\dfrac{1}{LC}}$

(2) $LC=\dfrac{1}{4}$

(3) $(1-2t)\mathrm{e}^{-2t}u(t)$

4.23　$v_2(t)=\underbrace{2\mathrm{e}^{-t}}_{\text{自由}}+\underbrace{\dfrac{1}{2}\mathrm{e}^{-3t}}_{\text{强迫}}$，稳态响应为零，完全响应中只有即瞬态响应

4.24　$H(s)=\dfrac{5(s^3+4s^2+5s)}{s^3+5s^2+16s+30}$

4.25

(1) $\dfrac{s}{s^2+s+1}$

(2) $\dfrac{s^2}{s^2+s+1}$

4.26

(1) $\dfrac{1}{s^2+\sqrt{2}s+1}$

(2) $\dfrac{1}{(1-\omega_2)+\sqrt{2}\omega j}$，为低通滤波器

(3) 当 $\omega=0$ 时，$\left|H(\mathrm{j}\omega)\right|$ 出现最大值 $\left|H(\mathrm{j}0)\right|=1$，　$\omega_{\mathrm{c}}=1\mathrm{rad/s}$

4.27

(1) $\dfrac{s-2}{s+2}$

(2) $\dfrac{s-1}{(s+2)(s+1)}$

4.28　略

4.29

(1) 低通

(2) 带通

(3) 高通

(4) 带通

(5) 带通

(6) 带阻

(7) 高通

(8) 带通−带阻

4.30

(1) $H(s)=\dfrac{L_1L_2Cs^3+L_1s}{L_1L_2Cs^3+RC(L_1+L_2)s^2+L_1s+R}$

(2) $H(s)=\dfrac{L_1L_2C_1s^2+L_2}{L_1L_2(C_1+C_2)s^2+L_1+L_2}$

(3) $H(s)=\dfrac{L_2C_1s^2}{L_1L_2C_1C_2s^4+(L_1C_1+L_2C_2+L_2C_1)s^2+1}$

4.31

(1) $H_1(s)=\dfrac{s+3+K}{2(s+3-K)}$

(2) $K<3$

4.32

(1) $H(s)=\dfrac{Ks}{s^2+(4-K)s+4}$

(2) $K\le 4$

(3) $h(t)=4\cos(2t)u(t)$

4.33

(1) $\dfrac{s^2+4s+3}{s^2+2s+2-K}$

(2) $K<2$