Animation

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1. Physics simulation.

Model: Particle Spring Model

$$f_{a\to b} = k_s \frac{\vec{b}-\vec{a}}{\|\vec{b}-\vec{a}\|}\left(\|\vec{b}-\vec{a}\|-l\right)$$

rest length

Damping force (阻尼力)

$$f_b = -k_d \boxed{\frac{\vec{b}-\vec{a}}{\|\vec{b}-\vec{a}\|}(\dot{\vec{b}}-\dot{\vec{a}})}\,\frac{\vec{b}-\vec{a}}{\|\vec{b}-\vec{a}\|}$$

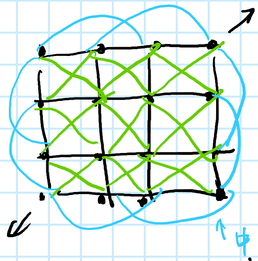
速度向连线投影

direction.

2. Structures from Springs

eg. cloth:

- structure resists shearing 切变(拉伸)
- structure resists out-of-plane bending



shearing force.

green spring: resist diagnal bending

blue springs: resist horizontal/vertical bending. (weak)

中间点同理.

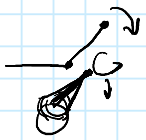
3. FEM = finite element Method.

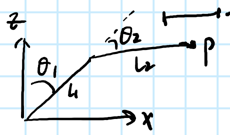
4. Particle Systems: Attraction, Collisions, Damping forces "ODE"

5. Forward Kinematics

Joint types

- Pin 1D rotation
- Ball 2D rotation
- Prismatic Joint (translation)





$p_x = \cdots f(\theta_1, \theta_2)$

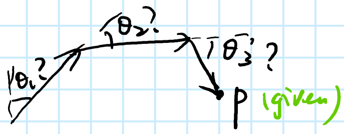
$p_z = \cdots g(\theta_1, \theta_2)$

6. Inverse Kinematics

定义终端位置，解出 θ_1, θ_2

Multiple solutions in a large dimensional equation

Or, no solutions



Numerical solution to general N-link IK problem

- choose initial setup
- define error metric
- gradient descent

7. Rigging (绑定)

define "controlling point"

8. Blend shapes

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How to ~~interpolate~~ between 2 controlling points.

9. Motion Capture.

10. Single Particle Simulation

$v(x,t)$ velocity field.

$\frac{dx}{dt} = v(x,t)$,

first-order differential equation.

Euler Method:

- Simple iterative method.
- inaccurate
- Most times, unstable.

$$\boxed{\begin{aligned} x^{t+\Delta t} &= x^{t} + \Delta t \cdot \dot{x}^{t} \\ \dot{x}^{t+\Delta t} &= \dot{x}^{t} + \Delta t\, \ddot{x}^{t} \end{aligned}}$$

inaccuracies increases with time

11. Combating Instability

① Midpoint Method

② Adapative Step size

③ Implicit Methods

④ Position-based / Verlet Integration.

Midpoint Method

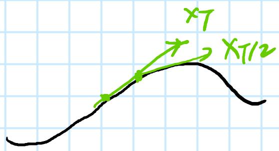
$x_{mid} = x(t) + \Delta t/2 \cdot v(x(t), t)$

$x(t+\Delta t) = x(t) + \Delta t \cdot v(x_{mid}, t)$

Adaptive Step Size

Repeat until error is below threshold.

- compute x_T. with Euler step T
- Compute $x_{T/2}$ two steps, size $T/2$
- Compute error $\|x_T - x_{T/2}\|$
- If error > threshold, reduce step size and try



Implicit Euler Method

Use derivatives in the future.

$x^{t+\Delta t} = x^{t} + \Delta t \cdot \dot{x}^{t+\Delta t}$

$\dot{x}^{t+\Delta t} = \dot{x}^{t} + \Delta t \cdot \ddot{x}^{t+\Delta t}$

Stable!

Solve non linear equation for $x(t+\Delta t)$ & $v(t+\Delta t)$

12. determine stability

① local truncation error (every step)

② total error (overall).

Implicit Euler: order 1 ⇒ local error: $O(h^2)$ h: step

Global : $O(h)$

13. Runge-Kutta Families

advanced methods for ODE

RK4 (order-4) is most widely used.

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Initial: $\frac{dy}{dt} = f(t, y)$, $y(t_0) = y_0$

Solution: $\begin{cases} y_{n+1} = y_n + \frac{1}{6} h (k_1 + 2k_2 + 2k_3 + k_4) \\ t_{n+1} = t_n + h \end{cases}$

$\begin{cases} k_1 = f(t_n, y_n) \\ k_2 = f(t_n + \frac{h}{2}, y_n + h\frac{k_1}{2}) \\ k_3 = f(t_n + \frac{h}{2}, y_n + h\frac{k_2}{2}) \\ k_4 = f(t_n + h, y_n + hk_3) \end{cases}$ ⇒ reference: Numerical Analysis

14. Position based Integration

not physically based.. in Assignment 8

15. Rigid Body Simulation.

$$\frac{d}{dt}\begin{pmatrix} X \\ \theta \\ \dot{X} \\ \omega \end{pmatrix} = \begin{pmatrix} \dot{X} \\ \omega \\ F/M \\ \Gamma/I \end{pmatrix}$$

Γ: torque.

I = momentum of inertia

16. Fluid Simulation.

Position-based Method

- Assume water is composed of rigid-body spheres
- water can't be compressed.
- Thus: when density changes particles move positions to keep density the same.
- Gradient of density (particle's position) How? Gradient descent

17. Lagrangian perspective.

Euler perspective.