Problem Wk.11.1.5: Simulating Hallways: The Noisy-Noisy Case

Below are some hand simulations of state estimation.
Don't check every single number you type in! You'll run out of checks and overload the server.

Refer to the course notes (Section 7.7).

Assume the robot is in a one-dimensional grid with 3 squares, with one green square in the middle and white squares on either side. The state indices are 0-based.

We will assume that initially we don't know where the robot is and all states are equally likely.

We'll assume that there are 5 possible colors:

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('black', 'white', 'red', 'green', 'blue')
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We assume a noisy sensor which has a probability of 0.8 of seeing the correct color for the current square and a probability of 0.05 of seeing each of the other posible colors

We assume noisy motion (actions are -1, 0 or +1) as well. The 'nominal' result of taking an action is to move that many spaces in the appropriate direction. If that nominal resulting location is out of bounds, then the nominal location is 'clipped' to be in bounds (that is, it will either be one end or the other of the hallway). With noise, there's a 0.8 probability of moving to the nominal square, and there's a 0.1 probability of being on either side of that square, that is, not moving far enough or going one square too far. If the noisy resulting location is off one end of the hallway, then the the probability associated with that result is assigned to the appropriate hallway end.

Recall that the uppercase S_t, I_t, O_t refer to random variables for the state at time *t*, action at time *t*, and observation at time *t*. Lowercase symbols, *s, i, o* are normal (non-random) variables that denote any value in the domain of states, actions and observations, thus $\sum_{s \in D_s} Pr(S_0 = s) = 1.0$.

We encourage you to do your computations using fractions; you can enter fractions, e.g. `5/27`, in the boxes below. If you enter decimals, they need to be accurate to within 0.001.

1. What is the robot's prior belief $B_0(s) = Pr(S_0 = s)$ for each of the states *s*?
   $B_0(s) = Pr(S_0 = s)$

2. First, the robot makes an observation. Let's assume it sees 'white'. So, $O_0 = white$. We want to know the new belief state after the observation $B'_0(s) = Pr(S_0 = s | O_0 = white)$.

3. If we told the robot to go right $I_0 = 1$, what would the belief state be after taking the state transition into account?
   $B_1(s) = Pr(S_1 = s | O_0 = white, I_0 = 1)$

4. Now, assume the robot sees 'white' again, $O_1 = white$. What will the belief state be after this?
   $B_1'(s) = Pr(S_1 = s | O_0 = white, I_0 = 1, O_1 = white)$

5. If we told the robot to go right $I_1 = 1$, what would the belief state be after taking the state transition into account?
   $B_2(s) = Pr(S_2 = s | O_0 = white, I_0 = 1, O_1 = white, I_1 = 1)$