Problem Wk.11.1.4: Simulating Hallways

Below are some hand simulations of state estimation.

Part 1: Perfect sensor, perfect action

Refer to the course notes (Section 7.7). Assume the robot is in a hallway with 3 rooms, with one green room in the middle and a white room on either side. The robot is trying to estimate which room it is in. For this, we will use state estimation with the states being the rooms -- represented as the indices 0 through 2.

We will assume that initially we don't know where the robot is and all states are equally likely.

Let's also assume perfect sensor and motion models, that is, there is no uncertainty in motion or sensing. If the commanded action (one of -1, 0, or 1) would take the robot off the edges of the hallway, then the robot moves just as far as it can and then stops.

Recall that the uppercase S_t, I_t, O_t refer to random variables for the state at time *t*, action at time *t*, and observation at time *t*. Lowercase symbols, *s, i, o* are normal (non-random) variables that denote any value in the domain of states D_s, actions and observations, thus $\sum_{s \in D_s} Pr(S_0 = s) = 1.0$.

We encourage you to do your computations using fractions; you can enter fractions, e.g. `5/27`, in the boxes below. If you enter decimals, they need to be accurate to within 0.001.

1. What is the robot's prior belief $B_0(s) = Pr(S_0 = s)$ for each of the states *s*?
   $B_0(s) = Pr(S_0 = s)$

2. First, the robot makes an observation. Let's assume it sees 'white', because it is in a white room and there's no sensor noise. So, $O_0 = white)$. We want to know what is the new belief state after this observation?
   - First, figure out $Pr(O_0 = white|S_0 = s)$ for each state *s*
   - Then compute $Pr(O_0 = white|S_0 = s)Pr(S_0 = s)$ for each state *s*. Note that this is the same as $Pr(O_0 = white, S_0 = s)$
   - Now, compute $Pr(O_0 = white)$.
   - Compute the new belief state after the observation; using the definition of conditional probability and the previous two results:
     $B'_0(s) = Pr(S_0 = s|O_0 = white)$.

3. If we told the robot to go right one room with action $I_0 = 1$, what would the belief state be after taking the state transition into account?
   $B_1(s) = Pr(S_1 = s|O_0 = white, I_0 = 1)$

4. Now, assume the robot observes 'green' because it's in a green room and there's no noise, $O_1 = green$). What will the belief state be after this?
   $B'_1(s) = Pr(S_1 = s|O_0 = white, I_0 = 1, O_1 = green)$

5. If we told the robot to go right with action $I_1 = 1$, what would the belief state be after taking the state transition into account?
   $B_2(s) = Pr(S_2 = s|O_0 = white, I_0 = 1, O_1 = green, I_1 = 1)$

Part 2: Noisy sensor, Perfect action

Refer to the course notes (Section 7.7).

Assume the robot is in a hallway with 3 rooms, with one green room in the middle and a white room on either side. The robot is trying to estimate which room it is in. For this, we will use state estimation with the states being the rooms -- represented as the indices 0 through 2.

We will assume that initially we don't know where the robot is and all states are equally likely.

We'll assume that there are 5 possible colors:

`('black', 'white', 'red', 'green', 'blue')`

We assume a noisy sensor which has a probability of 0.8 of seeing the correct color for the current room and a probability of 0.05 of seeing each of the other posible colors

We assume perfect motion, with actions -1, 0, 1. If the commanded action would take the robot off the edges of the hallway, then the robot moves as far as it can, then stops.

Recall that the uppercase S_t, I_t, O_t refer to random variables for the state at time *t*, action at time *t*, and observation at time *t*. Lowercase symbols, *s, i, o* are normal (non-random) variables that denote any value in the domain of states, actions and observations, thus $\sum_{s \in D_s} Pr(S_0 = s) = 1.0$.

We encourage you to do your computations using fractions; you can enter fractions, e.g. 5/27, in the boxes below. If you enter decimals, they need to be accurate to within 0.001.

1. What is the robot's prior belief for each of the states *s*?
   $B_0(s) = Pr(S_0 = s)$

2. What is the distribution over what the robot sees? That is, what is $Pr(O_0 = o)$ for all possible colors *o*?
   $P(O_0 = black) =$ $P(O_0 = white) =$ $P(O_0 = red) =$
   $P(O_0 = green) =$ $P(O_0 = blue) =$

3. First, the robot makes an observation. Let's assume it sees 'white'. So, $O_0 = white)$. We want to know the new belief state after the observation.
   $B'_0(s) = Pr(S_0 = s|O_0 = white)$.

4. If we told the robot to go right $I_0 = 1$, what would the belief state be after taking the state transition into account? Recall that motion is perfect.
   $B_1(s) = Pr(S_1 = s|O_0 = white, I_0 = 1)$

5. Now, what is the distribution over what the robot sees? That is, what is $Pr(O_1 = o)$ for all possible colors *o*?
   $P(O_1 = black) =$ $P(O_1 = white) =$ $P(O_1 = red) =$
   $P(O_1 = green) =$ $P(O_1 = blue) =$

6. Now, assume the robot sees 'white' again, $O_1 = white)$. What will the belief state be after this?
   $B'_1(s) = Pr(S_1 = s|O_0 = white, I_0 = 1, O_1 = white)$

7. If we told the robot to go right $I_1 = 1$, what would the belief state be after taking the state transition into account?
   $Pr(S_2 = s|O_0 = white, I_0 = 1, O_1 = white, I_1 = 1)$

8. If instead of having seen 'white' and gone right (as above), the robot had seen 'green' and gone right, what would the belief state be? That is, what is
   $Pr(S_2 = s|O_0 = white, I_0 = 1, O_1 = green, I_1 = 1)$