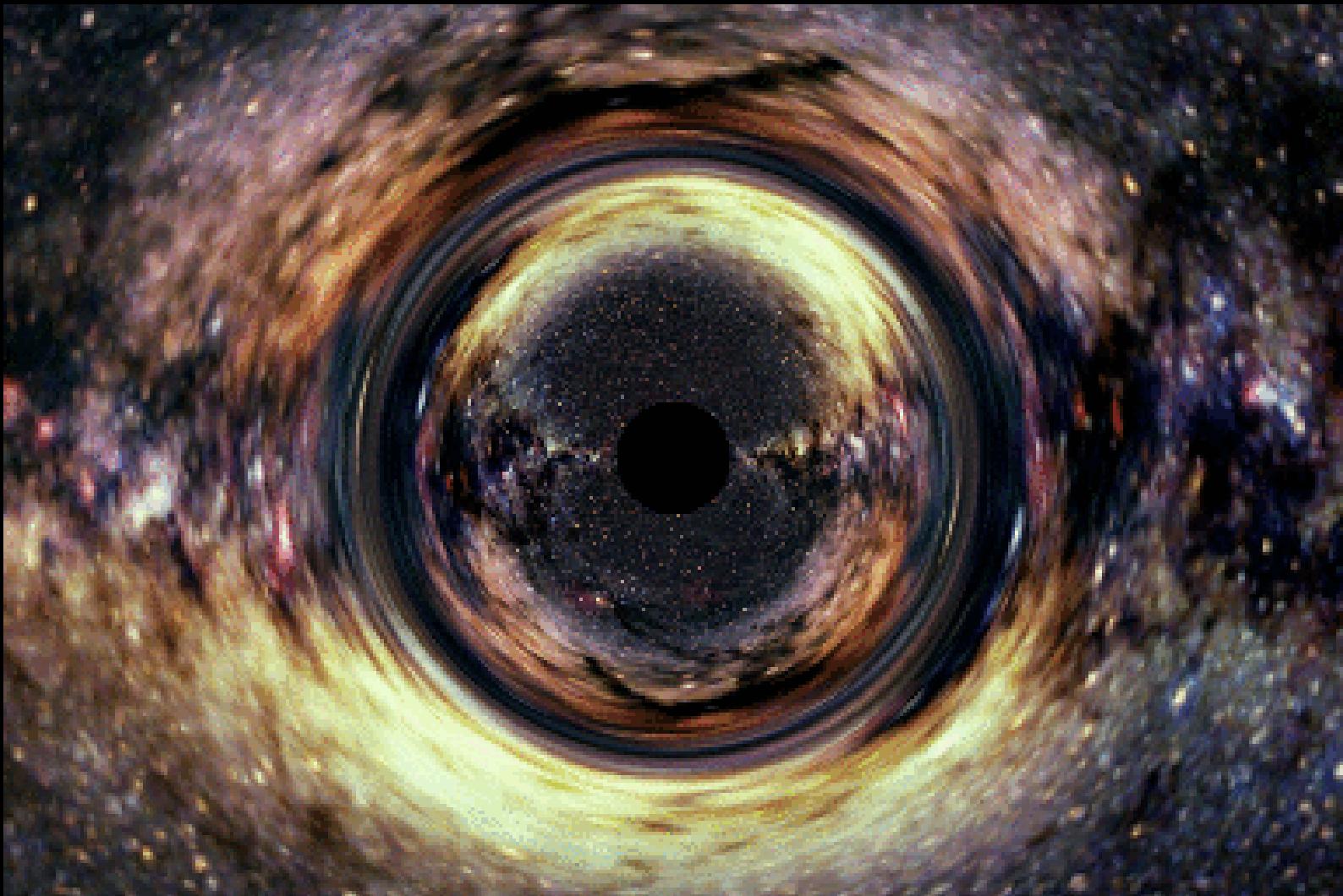


BLACK HOLES

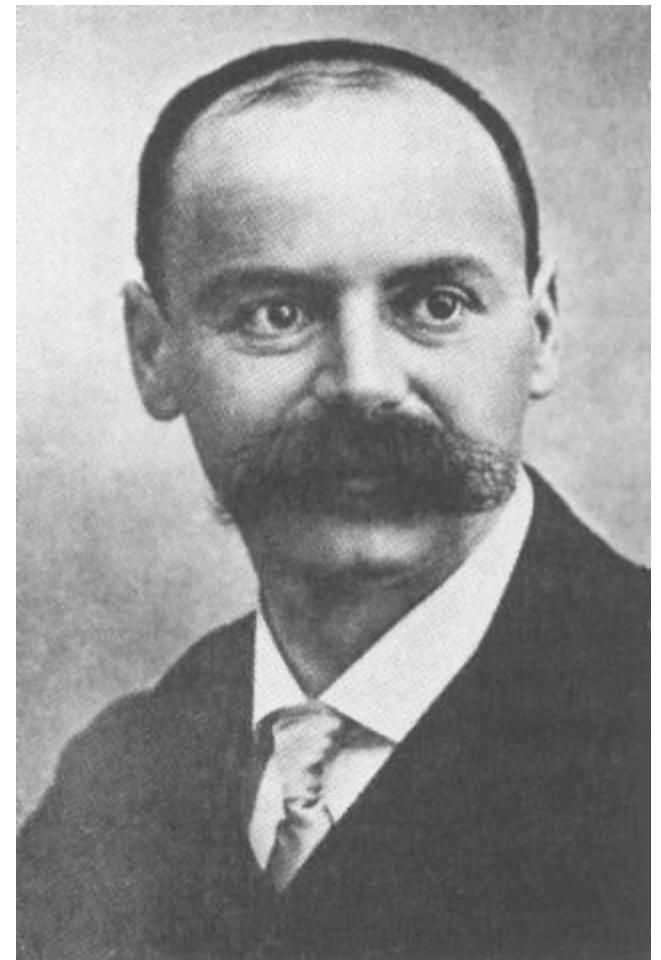


[Image by A. Hamilton]

Karl Schwarzschild's Work

In 1916 Schwarzschild read Einstein's paper on general relativity. He was interested in the physics of stars, and had a lot of spare time between battles on the Russian front, so he solved Einstein's field equation for the region outside a massive spherical object. His solution had many interesting features, including

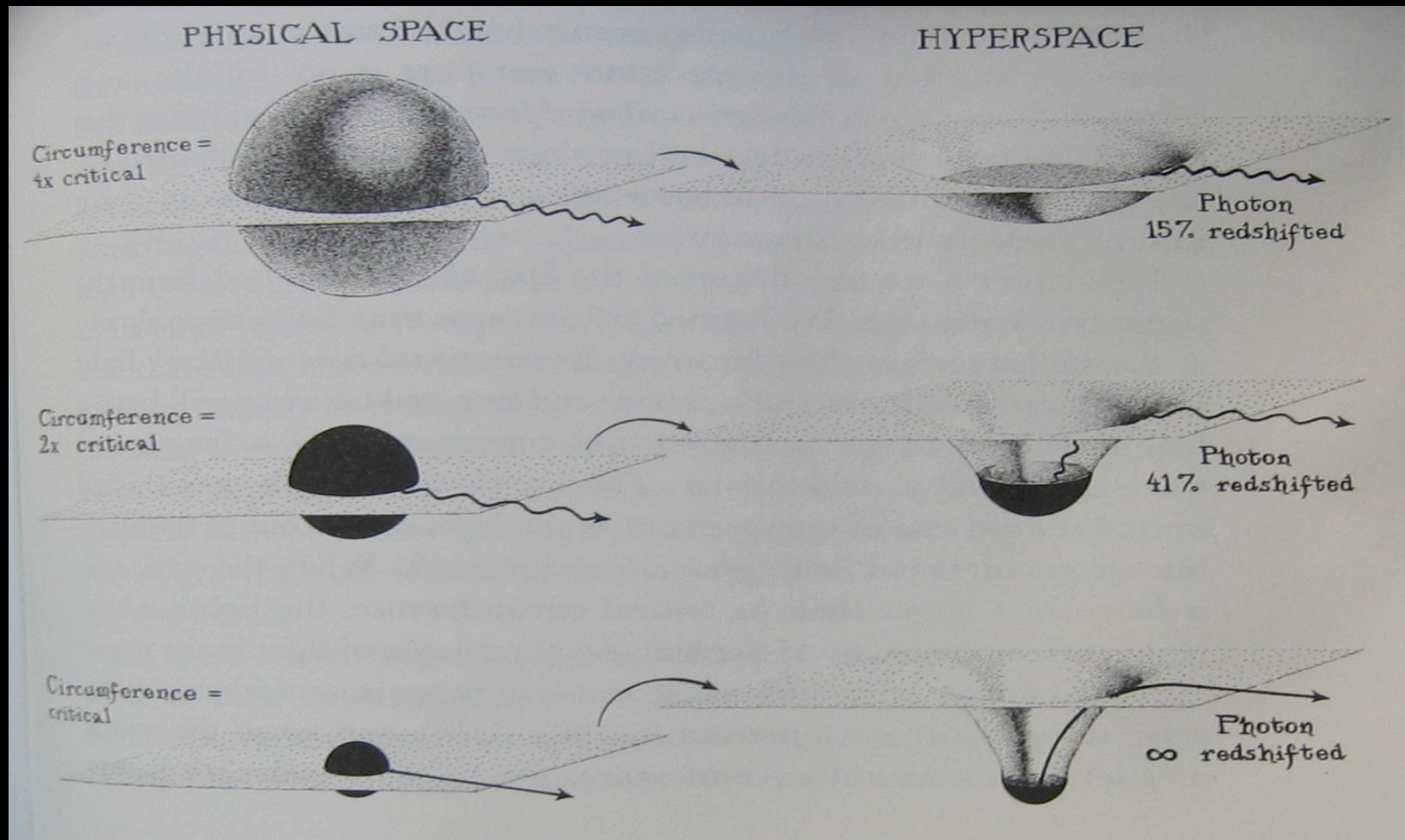
- prediction of space warping in strong gravity, and invention of embedding diagrams to visualize it.
- verification gravitational time dilation, just as Einstein had pictured it.
- prediction of black holes, though this was not recognized at the time.



[slide courtesy of D. Watson]

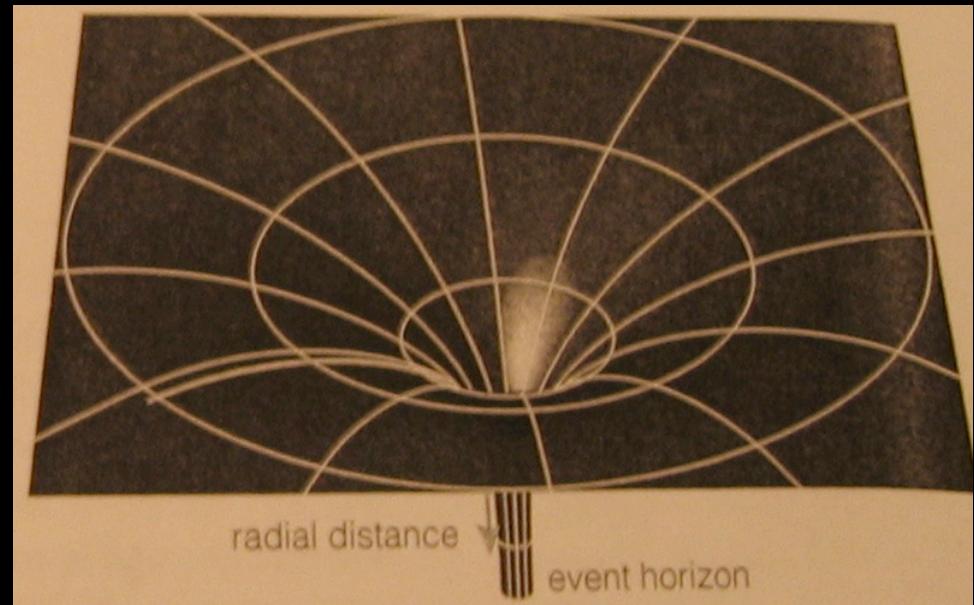
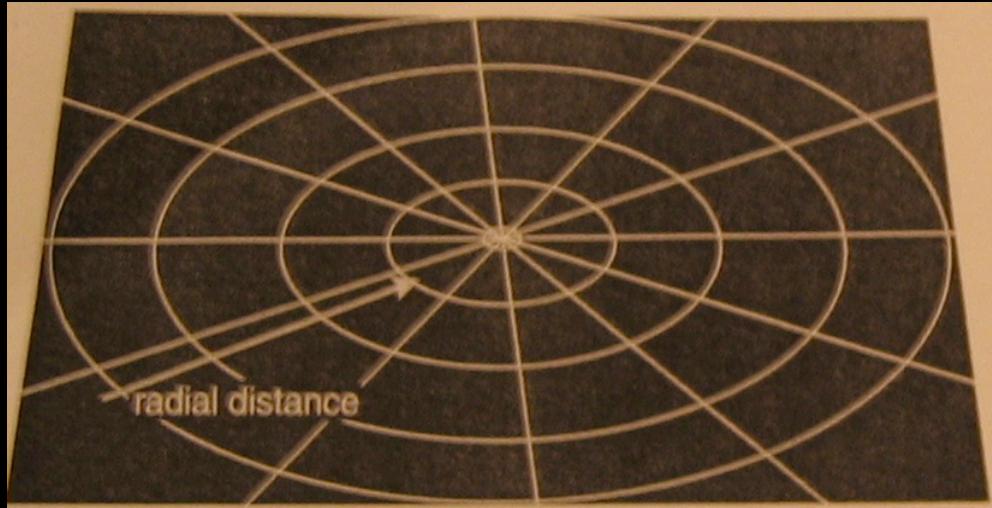
Schwarzschild's solution

Describes the spacetime curvature near a massive, spherically symmetric body. Solution ONLY depends on BH MASS.

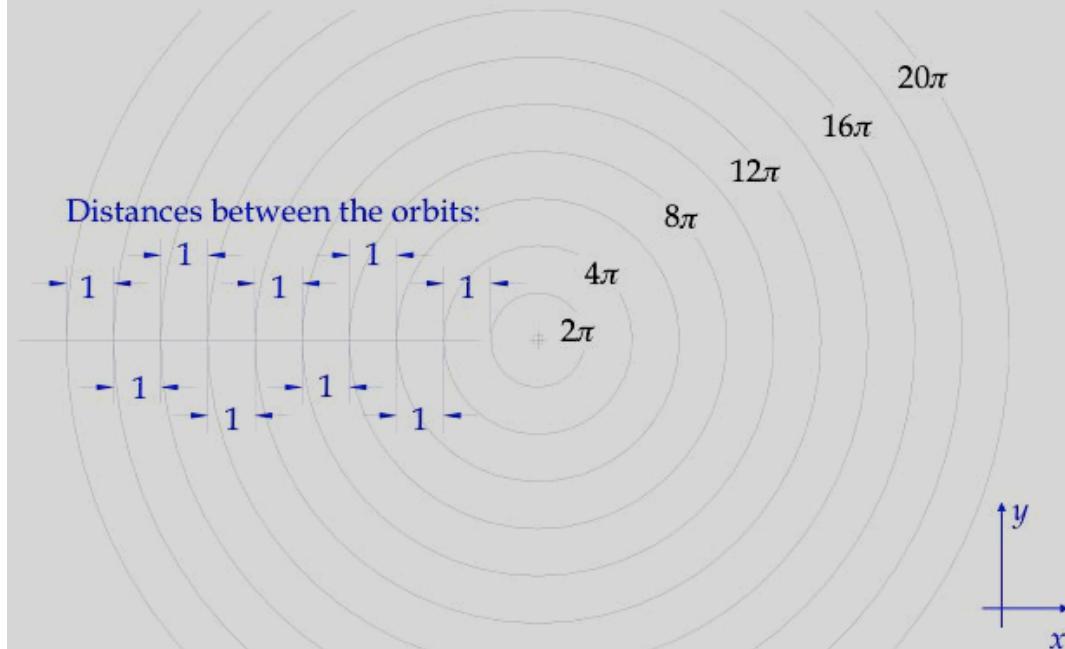


[Figure from Thorne's "Black Holes and Time warps"]

In curved space, rules of geometry different than Euclidean.....



Circular orbits in flat space (all in the same plane)

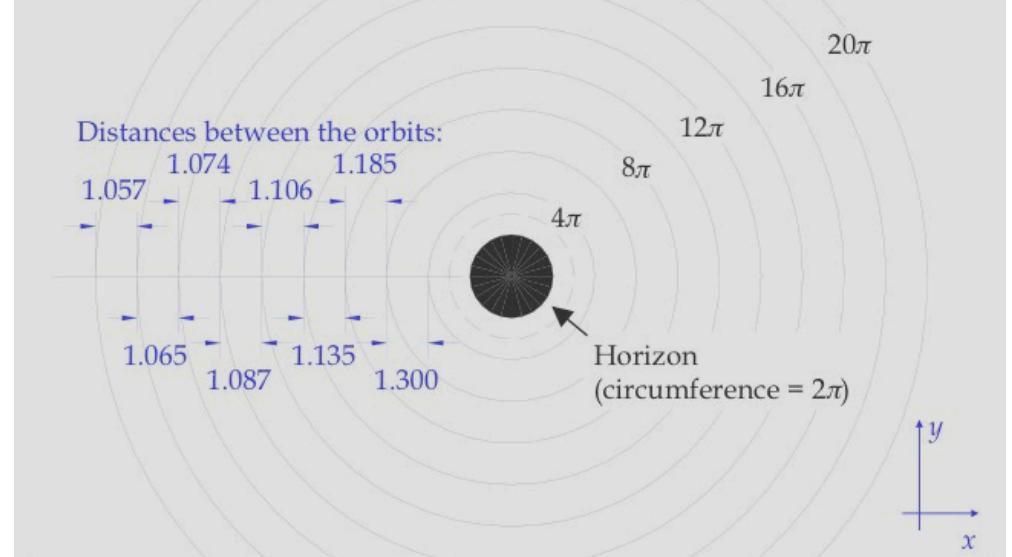


In flat space, the distance between any two circumferences in the figure is 1.

In the curved space around a BH, the distance between any two circumferences is greater than 1.

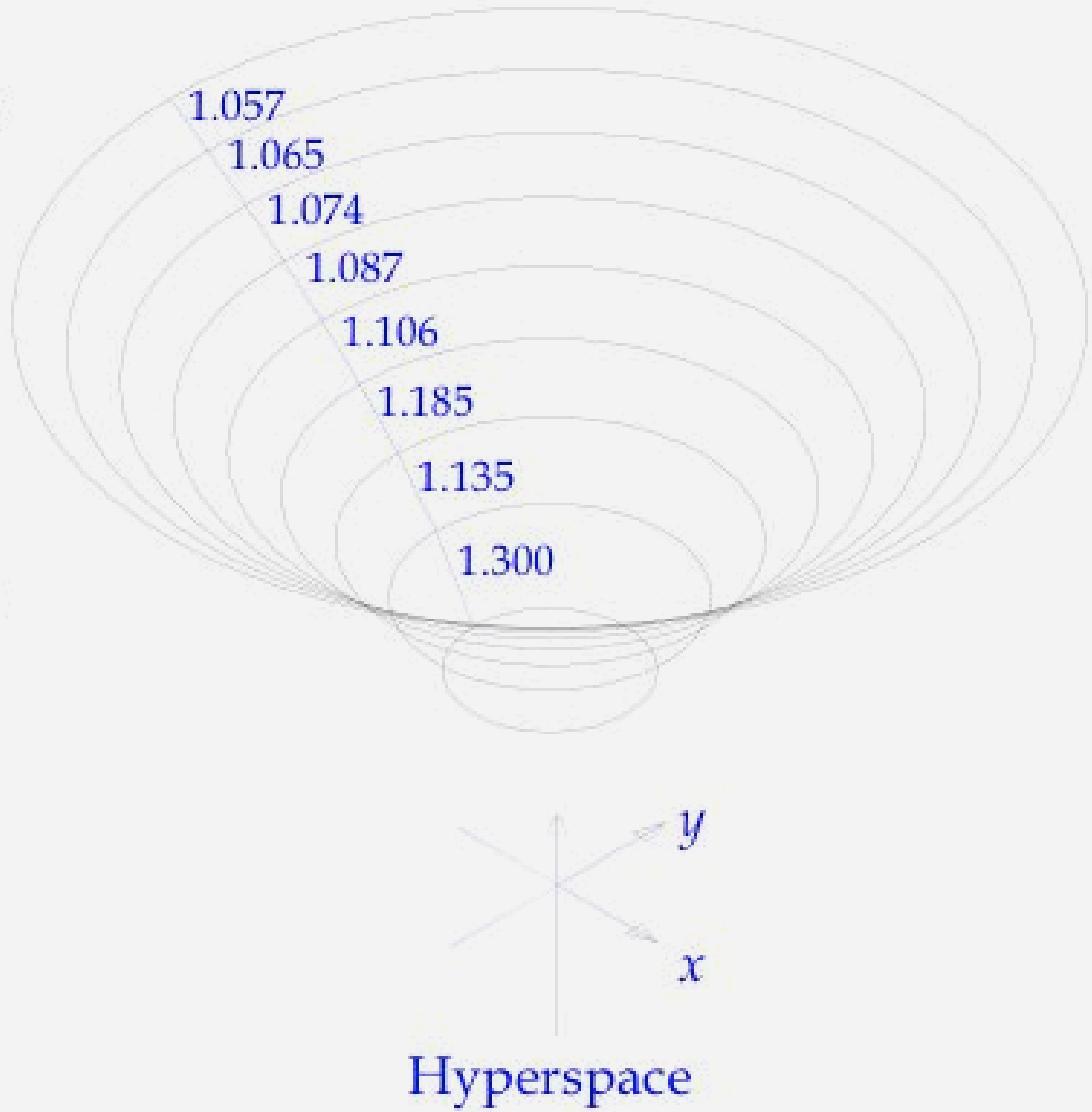
[Images by D. Watson]

Circular orbits in space warped by a black hole, same circumferences as before (still all in same plane)



One way to visualize warped space: “hyperspace”

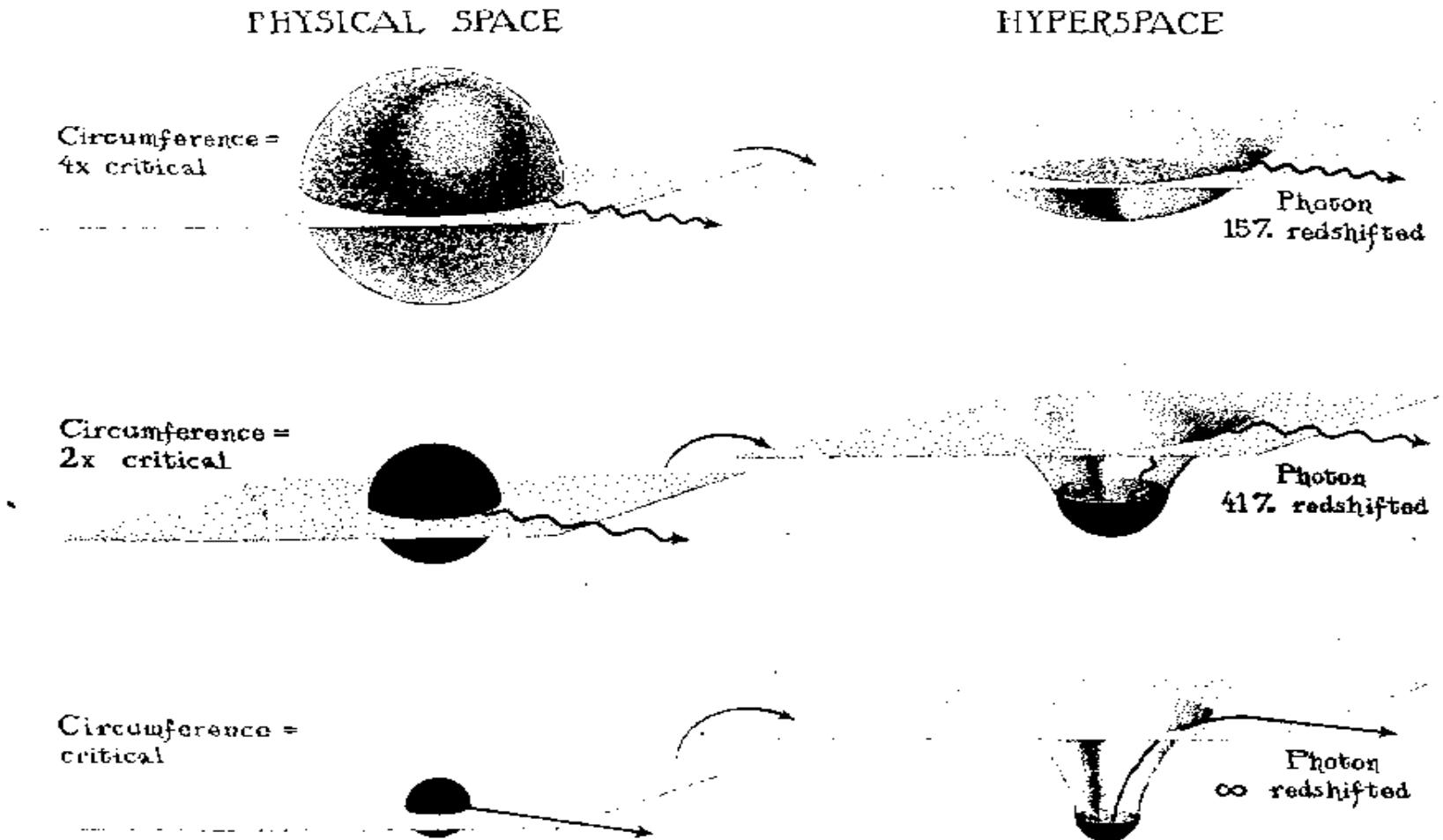
To connect these circles with segments of these “too long” lengths, one can consider them to be offset from one another along some **imaginary** dimension that is perpendicular to x and y but is not z . (If it were z , the circles wouldn’t appear to lie in a plane!). Such additional dimensions comprise **hyperspace**.



[slide courtesy of D. Watson]

Predictions of the Schwarzschild's solution

For an object of a given mass, the spacetime curvature increases as its size decreases

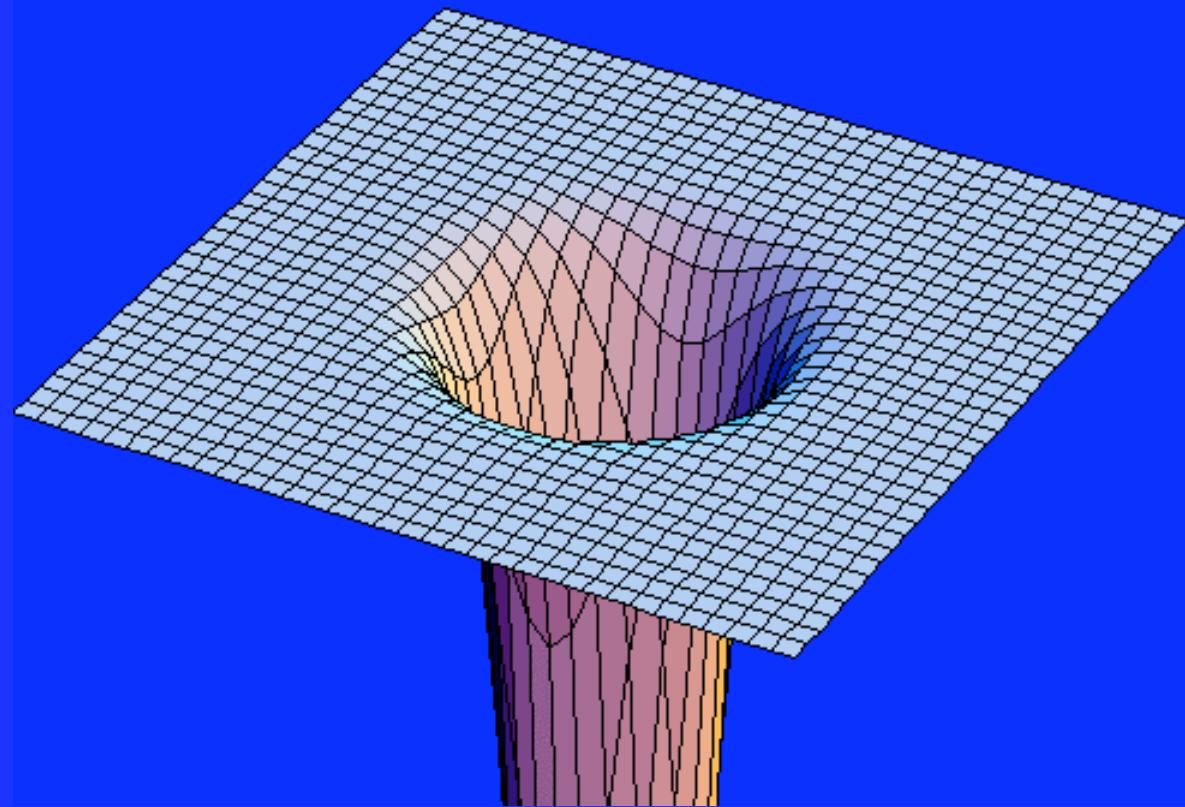


[Figure from Thorne's "Black Holes and time warps"]

As the radius of the object becomes smaller than a critical value, the spacetime curvature becomes infinite



BLACK HOLE



What characterizes a Black Hole?

Presence of a “surface” called **EVENT HORIZON**, from which nothing, not even light can escape.

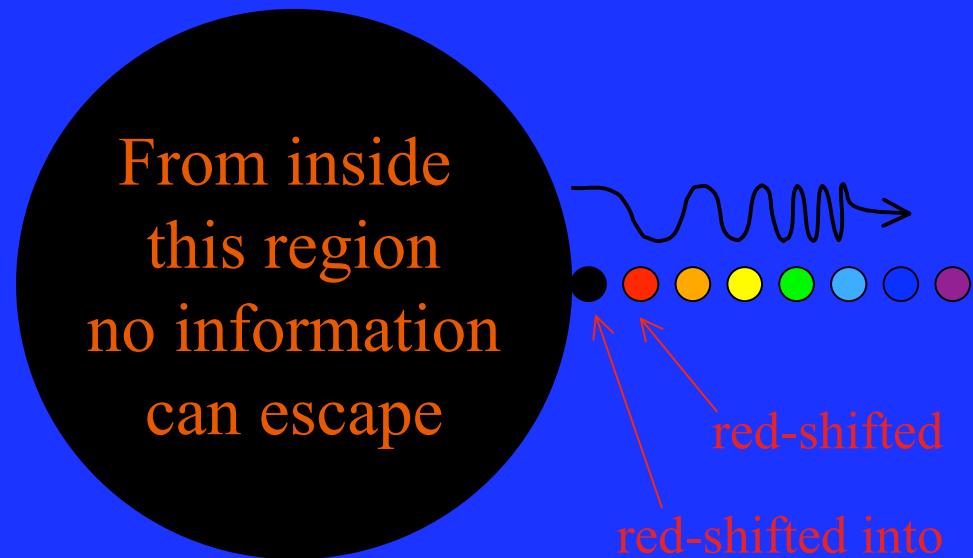
The radius of this surface is called the **SCHWARZCHILD RADIUS**

$$R_S = \frac{2GM}{c^2} = 3(M/M_{Sun}) \text{ kilometers}$$

The diagram illustrates the formula for the Schwarzschild radius. A dark blue rectangular box contains the equation $R_S = \frac{2GM}{c^2} = 3(M/M_{Sun}) \text{ kilometers}$. Three arrows point from circles containing text to the corresponding terms in the equation: an arrow from the circle labeled "Schwarzschild radius" points to the term R_S ; an arrow from the circle labeled "gravitational constant" points to the term GM ; and an arrow from the circle labeled "speed of light" points to the term c^2 .

Black Holes

- To a stationary observer far away, *time* is dilated near a black hole, and at the critical surface (at R_S), it is slowed down *infinitely*.
- Light emitted close to the critical surface is severely red-shifted (the frequency is lower) and at the critical surface, the *redshift* is *infinite*.



[slide courtesy of M.Begelman]

Time Dilation near a Black Hole (here $M=6M_{\text{sun}}$)

Time is warped in strong gravity.



On time.

Unchanged

Very slightly
(factor of
 1.000002)
slower.

Very slow.

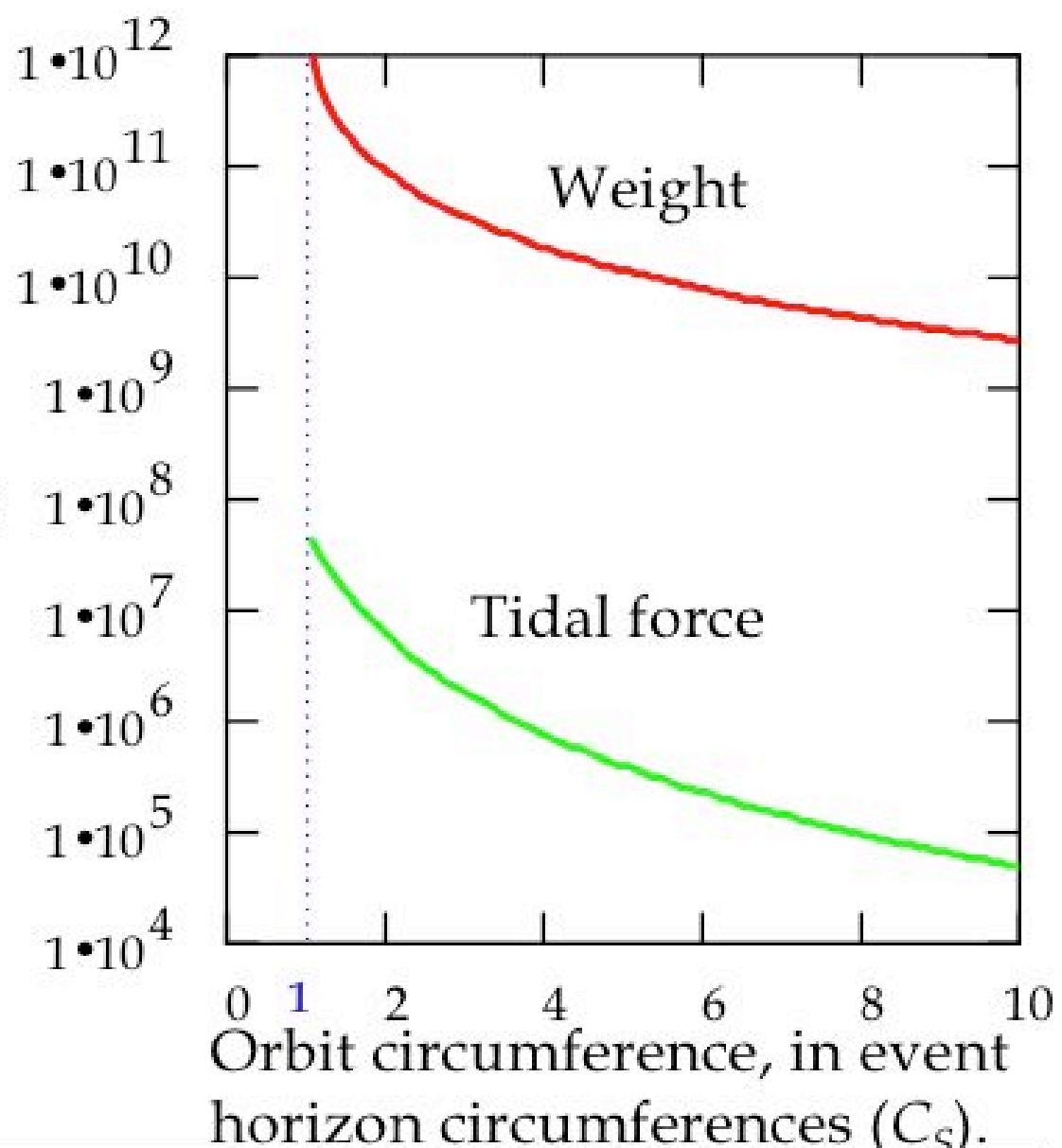
$9 \times 10^6 \text{ km}$

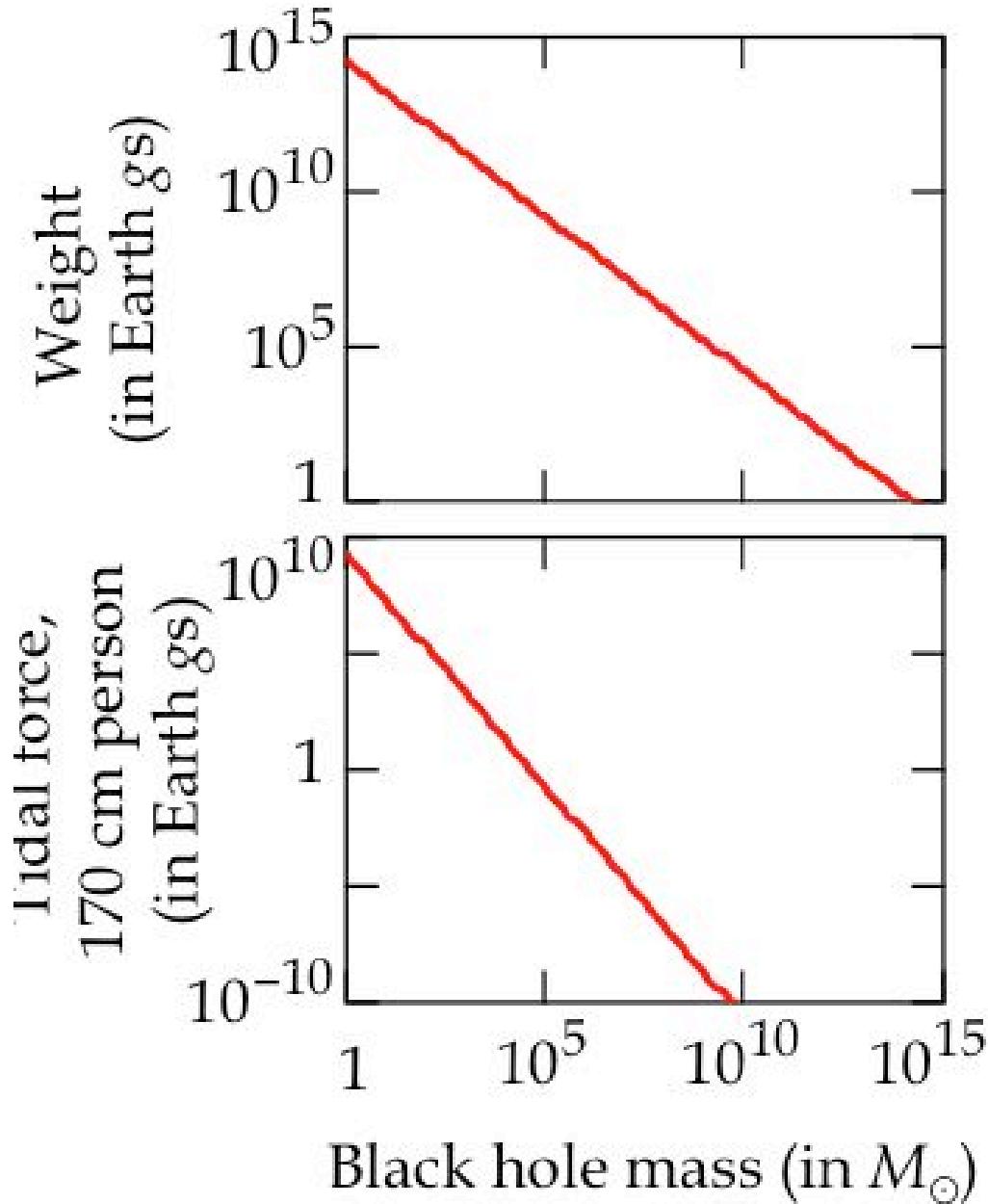
[This and next three slides courtesy of D. Watson]

Other effects of spacetime curvature: weight and tides

Force, in Earth gs

The tides are for a 170 cm person lying along the direction toward the black hole.





How weight and tides depend upon black hole mass

For comparison: the weight and tidal force you're feeling right now are respectively $1g$ and $5 \times 10^{-7}g$.

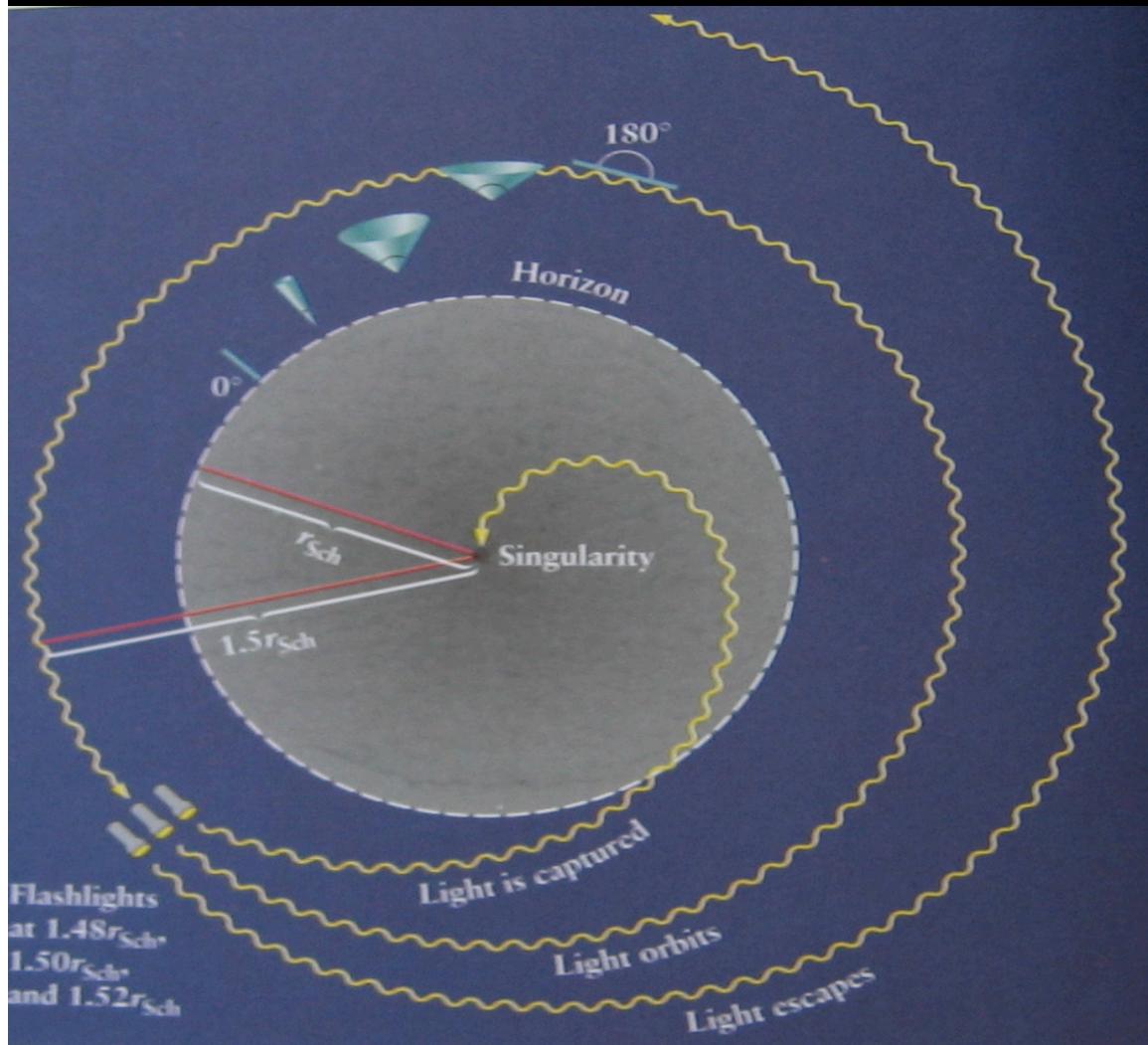
The dangers of getting too close to a small black hole.... .

A body would be stretched along the direction toward the black hole, and squeezed in the perpendicular directions.



This effect is colloquially known as “**spaghettification**”

Light close to a Black Hole

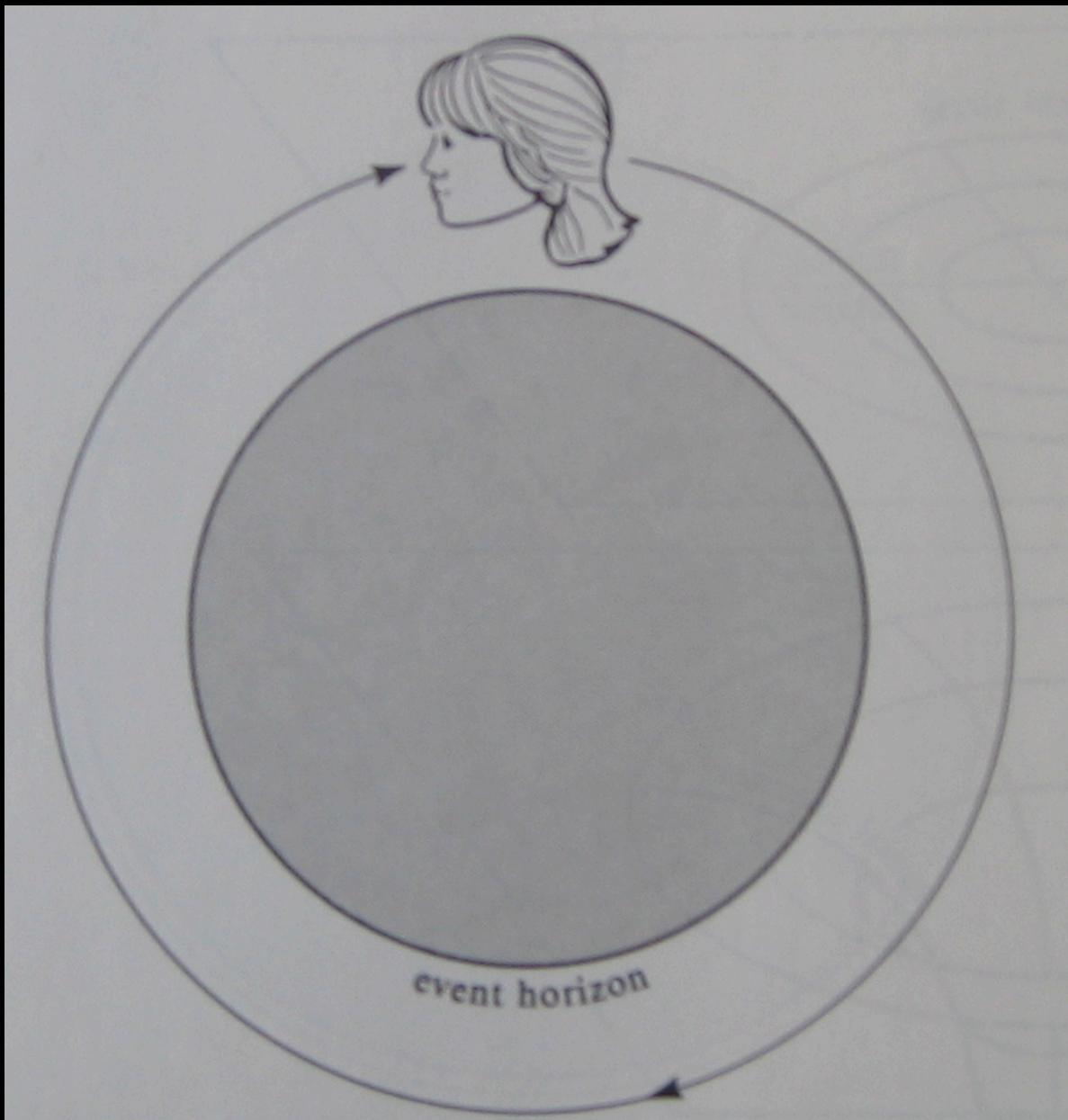


[Image from “Gravity’s fatal attraction” by M. Begelman & M. Rees]

In the vicinity of a black hole, light is severely deflected.

The amount of deflection becomes larger closer to the black hole. When the distance is $D = 1.5 R_s$, light goes around in a circular orbit. A light ray emitted tangentially to the horizon at a distance $< 1.5 R_s$ would be captured. In order to escape it would have to be emitted at an angle $< 180^\circ$. As the radius is decreased from $1.5 R_s$ to R_s , the range of angles from which light can escape decreases, to become 0° at the horizon, at which point light can no longer escape.

An observer at $D = 1.5 R_s$ would be able to see the back of her head!



[Image from "The Physical Universe" by F. Shu]

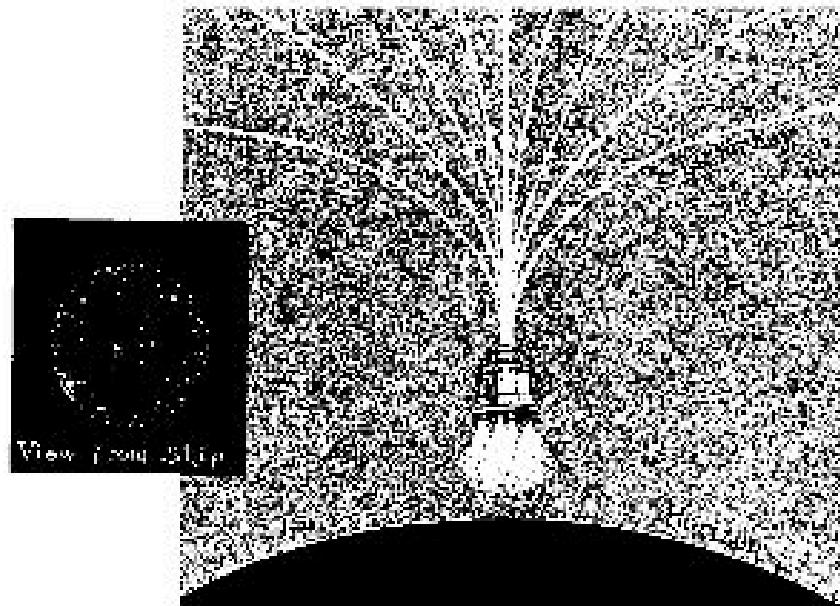
“Space and time are stuck at black hole horizons”

Time is stuck at the event horizon.

- From the viewpoint of a distant observer, time appears to stop there (infinite gravitational time dilation).

Space is stuck at the event horizon.

- Within $r = 1.5 R_S$, all **geodesics** (paths of light or freely-falling masses) terminate at the horizon, because the orbital speed is equal to the speed of light at $r = 1.5 R_S$: nothing can be in orbit.



[This and following slide courtesy of D. Watson - Image from Thorne's “Black Holes and time Warps”]

Space and time are stuck at black hole horizons (continued)

- Thus: from near the horizon, the sky appears to be compressed into a small range of angles directly overhead; the range of angles is smaller the closer one is to the horizon, and vanishes at the horizon. (The objects in the sky appear bluer than their natural colors as well, because of the gravitational Doppler shift).
- Thus space itself is stuck to the horizon, since one end of each geodesic is there.

If the horizon were to move or rotate, the ends of the geodesics would move or rotate with it. Black holes can drag space and time around.