QCD-09 (pQCD @ finite T)

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Feynman Rules Review:

- Transform to Eucilidean:
  - In action: $i\int dt\, L \to \int_0^\beta d\tau\, L$; $\partial_0 \to i\partial_\tau \Rightarrow p^0 \sim i\partial_0 \to -\partial_\tau \sim i\omega_n$
  - $i\omega_n = 2n\pi iT$ (Bosonic: $i\omega_n\beta = 1 \Rightarrow G(\beta) = G(0)$) or $(2n+1)\pi iT$ (Fermionic: $i\omega_n\beta = -1 \Rightarrow S(\beta) = -S(0)$)
  - $\int \frac{dp^0}{2\pi} \to T\sum_n \square$
- Residue Magic
  - $\frac{1}{2}\tanh\frac{z}{2T}$ has poles at $(2n+1)\pi iT$, with residue equal to T
  - $\frac{1}{2}\coth\frac{z}{2T}$ has poles at $2n\pi iT$, with residue equal to T
  - $\int \frac{dp^0}{2\pi}\mathcal{F}(p^0) \to T\sum_n \mathcal{F}(i\omega_n) \to \frac{1}{2}\int_{C\downarrow+C\uparrow}\frac{dp^0}{2\pi i}\tanh\frac{p^0}{2T}\mathcal{F}(p^0)$ *or* $\frac{1}{2}\int_{C\downarrow+C\uparrow}\frac{dp^0}{2\pi i}\coth\frac{p^0}{2T}\mathcal{F}(p^0)$
  - As $T\to 0, \tanh x \to \coth x \to \operatorname{sgn} x$, reduce to Wick's rotation.
  - $\tanh\frac{z+2\pi niT}{2T} = \coth\frac{z+(2n+1)\pi iT}{2T} = \tanh\frac{z}{2T}$
  - $\coth\frac{z+2\pi niT}{2T} = \tanh\frac{z+(2n+1)\pi iT}{2T} = \coth\frac{z}{2T}$
- Propagator:
  - Quark:
    $\tilde{S}(p) = -(p^\mu\gamma_\mu - M)^{-1} = -\frac{p^\mu\gamma_\mu + M}{p^2 - M^2}$
  - *Gluon:*
    $\tilde{D}^{\mu\nu}(p) = p^{-2}\left[g^{\nu\mu} + (\xi - 1)\frac{p^\nu p^\mu}{p^2}\right]$
  - Ghost:
    $\tilde{G}(p) = p^{-2}$
- *Vertex*
  - Gluon-Quark Vertex
    $V_{\mu;a}^{ij;\alpha\beta} = -g\tau_{ij}^a\gamma_\mu^{\alpha\beta}$
  - 3-Gluon Vertex
    $V_{\alpha\beta\gamma}^{abc}(p,q,r) = -igf_{abc}\left[g_{\alpha\beta}(q-p)_\gamma + g_{\beta\gamma}(r-q)_\alpha + g_{\alpha\gamma}(p-r)_\beta\right]$ ← momentum of outgoing gluon
  - 4-Gluon Vertex
    $V_{\alpha\beta\gamma\delta}^{abcd} = -g^2\left[f_{abh}f_{cdh}(g_{\alpha\gamma}g_{\beta\delta} - g_{\alpha\delta}g_{\beta\gamma}) + f_{ach}f_{dbh}(g_{\alpha\delta}g_{\beta\gamma} - g_{\gamma\delta}g_{\alpha\beta}) + f_{adh}f_{bch}(g_{\alpha\beta}g_{\gamma\delta} - g_{\delta\beta}g_{\alpha\gamma})\right]$
  - Ghost-Gluon Vertex
    $V_{abc;\mu} = -igf_{abc}p_\mu$ ← momentum of outgoing ghost

QED Plasma

Photon Self-energy (General Properties)

$$\Pi_{\mu\nu}(q) = \int d^4x \langle \mathcal{T}_\tau\, j_\mu(x) j_\nu(0)\rangle e^{iq\cdot x}$$

- Projectors:
  A new 4-vector involved $\Rightarrow u^\mu \Leftarrow$ flow velocity of the medium
  (a new frame is specified $\Rightarrow u = (1,\vec{0}) \Leftarrow$ medium rest frame)
  Further decompose the projector:
  $\boldsymbol{\Delta^{\mu\nu}(p) \equiv -g^{\mu\nu} + \frac{p^\mu p^\nu}{p^2} = \Delta_L^{\mu\nu}(p,u) + \Delta_T^{\mu\nu}(p,u)}$
  $\boldsymbol{\Delta_L^{\mu\nu} \equiv \frac{\Delta^\mu{}_\alpha\Delta^\nu{}_\beta u^\alpha u^\beta}{u\cdot\Delta\cdot u}}$; $\boldsymbol{\Delta_T = \Delta - \Delta_L}$
  $\Delta\cdot\Delta = -\Delta$; $\Delta\cdot\Delta_L = \Delta_L\cdot\Delta = -\Delta_L$; $\Delta_L\cdot\Delta_L = -\Delta_L$; $\Delta_L\cdot\Delta_T = 0$; $\Delta_T\cdot\Delta_T = -\Delta_T$
  In medium rest frame:
  $$\Delta_L^{\mu\nu}(p) = \frac{\Delta^\mu{}_0\Delta^\nu{}_0}{\Delta^{00}} = \frac{\left(-g_0^\mu + \frac{p^\mu p_0}{p^2}\right)\left(-g_0^\nu + \frac{p^\nu p_0}{p^2}\right)}{-1 + p_0^2/p^2} = \vec{p}^{-2}(-g_0^\mu p^2 + p^\mu p_0)\left(-g_0^\nu + \frac{p^\nu p_0}{p^2}\right)$$
  $$= \vec{p}^{-2}\left(g_0^\mu g_0^\nu p^2 - p_0 p^\mu g_0^\nu - p_0 p^\nu g_0^\mu + \frac{p_0^2}{p^2}p^\mu p^\nu\right) = \begin{pmatrix} \frac{\vec{p}^2}{p^2} & \frac{p^0 p^\nu}{p^2} \\ \frac{p^0 p^\mu}{p^2} & \frac{p_0^2}{p^2\vec{p}^2}p^\mu p^\nu \end{pmatrix}$$
  $$\Delta_T^{\mu\nu}(p) = \Delta^{\mu\nu} - \Delta_L^{\mu\nu} = -g^{\mu\nu} + \frac{p^\mu p^\nu}{p^2} - \vec{p}^{-2}\left(g_0^\mu g_0^\nu p^2 - p_0 p^\mu g_0^\nu - p_0 p^\nu g_0^\mu + \frac{p_0^2}{p^2}p^\mu p^\nu\right)$$
  $$= -g^{\mu\nu} - \vec{p}^{-2}\left(g_0^\mu g_0^\nu p^2 - p_0 p^\mu g_0^\nu - p_0 p^\nu g_0^\mu + p^\mu p^\nu\right) = \begin{pmatrix} 0 & 0 \\ 0 & \delta^{\mu\nu} - \frac{p^\mu p^\nu}{\vec{p}^2} \end{pmatrix}$$
  In M.R.F., Δ_T erases time component & takes the SPATIAL component perpendicular to $\vec{p}$
- Decomposition of $\Pi^{\mu\nu}$
  $\Pi^{\mu\nu}(p) = \Delta_L^{\mu\nu}(p)\Pi_L(p) + \Delta_T^{\mu\nu}(p)\Pi_T(p)$
  $\Pi_L(p) = \Delta_{\mu\nu}^L(p)\Pi^{\mu\nu}(p)$; $\Pi_T(p) = \frac{1}{2}\Delta_T^{\mu\nu}(p)\Pi_{\mu\nu}(p)$; $\Delta_{\mu\nu}(p)\Pi^{\mu\nu}(p) = \Pi_L(p) + 2\Pi_T(p)$

QED Plasma

Photon Self-energy (1-Loop Calc.)

Notation: $p_\pm \equiv p \pm \frac{k}{2}$; $E_\pm \equiv E_{p\pm\frac{k}{2}}$; $\tilde{p}_\pm \equiv \left(E_\pm, \vec{p} \pm \frac{\vec{k}}{2}\right)$; $\bar{p}_\pm \equiv \left(-E_\pm, \vec{p} \pm \frac{\vec{k}}{2}\right)$; $\widetilde{p_\pm^2} = M^2 = \bar{p}_\pm^2$

$$\Pi^{\mu\nu}\left(k^0 = 2\pi niT, \vec{k}\right) = -(-g)^2\int_{C\downarrow+C\uparrow}\frac{d^4p}{(2\pi)^4 i}\operatorname{tr}[\gamma^\mu S(p_-)\gamma^\nu S(p_+)]\frac{1}{2}\tanh\frac{p^0 + \frac{k^0}{2}}{2T}$$
$$= -g^2\int_{C\downarrow+C\uparrow}\frac{d^4p}{(2\pi)^4 i}\frac{\operatorname{tr}\left[\gamma^\mu((p_-)_\sigma\gamma^\sigma + M)\gamma^\nu((p_+)_\rho\gamma^\rho + M)\right]}{((p_-)^2 - M^2)((p_+)^2 - M^2)}\frac{1}{2}\tanh\frac{p^0 + \frac{k^0}{2}}{2T}$$
$$= -g^2\int_{C\downarrow+C\uparrow}\frac{d^4p}{(2\pi)^4 i}\frac{\operatorname{tr}[\gamma^\mu(p_-)_\sigma\gamma^\sigma\gamma^\nu(p_+)_\rho\gamma^\rho] + \operatorname{tr}[\gamma^\mu\gamma^\nu]M^2}{((p_-)^2 - M^2)((p_+)^2 - M^2)}\frac{1}{2}\tanh\frac{p^0 + \frac{k^0}{2}}{2T}$$
$$= -4g^2\int_{C\downarrow+C\uparrow}\frac{d^4p}{(2\pi)^4 i}\frac{p_-^\mu p_+^\nu + p_+^\mu p_-^\nu - g^{\mu\nu}p_+\cdot p_- + g^{\mu\nu}M^2}{\left(\left(p^0 - \frac{k^0}{2}\right)^2 - E_-^2\right)\left(\left(p^0 + \frac{k^0}{2}\right)^2 - E_+^2\right)}\frac{1}{2}\tanh\frac{p^0 + \frac{k^0}{2}}{2T}$$

Medium Modification:
$\Pi^{\mu\nu} = \Pi^{\mu\nu}(T=0) + \delta\,\Pi^{\mu\nu}$

$$\delta\,\Pi^{\mu\nu} = -4g^2\int_{C\downarrow+C\uparrow}\frac{d^4p}{(2\pi)^4 i}\frac{p_-^\mu p_+^\nu + p_+^\mu p_-^\nu - g^{\mu\nu}p_+\cdot p_- + g^{\mu\nu}M^2}{\left(p^0 - \frac{k^0}{2} - E_-\right)\left(p^0 - \frac{k^0}{2} + E_-\right)\left(p^0 + \frac{k^0}{2} - E_+\right)\left(p^0 + \frac{k^0}{2} + E_+\right)}\frac{1}{2}\left(\tanh\frac{p^0 + \frac{k^0}{2}}{2T} - \operatorname{sgn}\operatorname{Re}p^0\right)$$
$$= 2g^2\int\frac{d^3\vec{p}}{(2\pi)^3}\frac{2\tilde{p}_-^\mu\tilde{p}_-^\nu + \tilde{p}_-^\mu k^\nu + k^\mu\tilde{p}_-^\nu - g^{\mu\nu}k\cdot\tilde{p}_-}{2E_-(k^0 + E_- - E_+)(k^0 + E_- + E_+)}\left(\tanh\frac{E_-}{2T} - 1\right)$$
$$+2g^2\int\frac{d^3\vec{p}}{(2\pi)^3}\frac{2\bar{p}_-^\mu\bar{p}_-^\nu + \bar{p}_-^\mu k^\nu + k^\mu\bar{p}_-^\nu - g^{\mu\nu}k\cdot\bar{p}_-}{2E_-(k^0 - E_- - E_+)(k^0 - E_- + E_+)}\left(\tanh\frac{E_-}{2T} - 1\right)$$
$$+2g^2\int\frac{d^3\vec{p}}{(2\pi)^3}\frac{2\tilde{p}_+^\mu\tilde{p}_+^\nu - \tilde{p}_+^\mu k^\nu - k^\mu\tilde{p}_+^\nu + g^{\mu\nu}k\cdot\tilde{p}_+}{(E_+ - k^0 - E_-)(E_+ - k^0 + E_-)2E_+}\left(\tanh\frac{E_+}{2T} - 1\right)$$
$$+2g^2\int\frac{d^3\vec{p}}{(2\pi)^3}\frac{2\bar{p}_+^\mu\bar{p}_+^\nu - \bar{p}_+^\mu k^\nu - k^\mu\bar{p}_+^\nu + g^{\mu\nu}k\cdot\bar{p}_+}{(-E_+ - k^0 - E_-)(-E_+ - k^0 + E_-)(2E_+)}\left(\tanh\frac{E_+}{2T} - 1\right)$$

QED plasma

Photon Self-energy to 1-Loop (Transverse)

$$\delta\Pi_T = \frac{1}{2}\Delta_T^{\mu\nu}(k)\Pi_{\mu\nu}(k)$$
$$= 2g^2\int\frac{d^3\vec{p}}{(2\pi)^3}\frac{\left(|\overrightarrow{p_-}|^2 - (\vec{p}_-\cdot\hat{k})^2\right) - \vec{k}\cdot\overrightarrow{p_-} + k^0E_-}{2E_-(k^0 + E_- - E_+)(k^0 + E_- + E_+)}\left(\tanh\frac{E_-}{2T} - 1\right)$$
$$+2g^2\int\frac{d^3\vec{p}}{(2\pi)^3}\frac{\left(|\overrightarrow{p_-}|^2 - (\vec{p}_-\cdot\hat{k})^2\right) - \vec{k}\cdot\overrightarrow{p_-} - k^0E_-}{2E_-(k^0 - E_- - E_+)(k^0 - E_- + E_+)}\left(\tanh\frac{E_-}{2T} - 1\right)$$
$$+2g^2\int\frac{d^3\vec{p}}{(2\pi)^3}\frac{\left(|\overrightarrow{p_+}|^2 - (\vec{p}_+\cdot\hat{k})^2\right) + \vec{k}\cdot\overrightarrow{p_+} - k^0E_+}{(E_+ - k^0 - E_-)(E_+ - k^0 + E_-)2E_+}\left(\tanh\frac{E_+}{2T} - 1\right)$$
$$+2g^2\int\frac{d^3\vec{p}}{(2\pi)^3}\frac{\left(|\overrightarrow{p_+}|^2 - (\vec{p}_+\cdot\hat{k})^2\right) + \vec{k}\cdot\overrightarrow{p_+} + k^0E_+}{(-E_+ - k^0 - E_-)(-E_+ - k^0 + E_-)(2E_+)}\left(\tanh\frac{E_+}{2T} - 1\right)$$
$$= -4g^2\int\frac{d^3\vec{p}}{(2\pi)^3}\frac{\vec{p}^2 - (\vec{p}\cdot\hat{k})^2 + \frac{\vec{k}^2}{2} - \vec{p}\cdot\vec{k} + k^0E_-}{2E_-(k^0 + E_- - E_+)(k^0 + E_- + E_+)}f\left(\frac{E_-}{T}\right)$$
$$-4g^2\int\frac{d^3\vec{p}}{(2\pi)^3}\frac{\vec{p}^2 - (\vec{p}\cdot\hat{k})^2 + \frac{\vec{k}^2}{2} - \vec{p}\cdot\vec{k} - k^0E_-}{2E_-(k^0 - E_- - E_+)(k^0 - E_- + E_+)}f\left(\frac{E_-}{T}\right)$$
$$-4g^2\int\frac{d^3\vec{p}}{(2\pi)^3}\frac{\vec{p}^2 - (\vec{p}\cdot\hat{k})^2 + \frac{\vec{k}^2}{2} + \vec{k}\cdot\vec{p} - k^0E_+}{(E_+ - k^0 - E_-)(E_+ - k^0 + E_-)2E_+}f\left(\frac{E_+}{T}\right)$$
$$-4g^2\int\frac{d^3\vec{p}}{(2\pi)^3}\frac{\vec{p}^2 - (\vec{p}\cdot\hat{k})^2 + \frac{\vec{k}^2}{2} + \vec{k}\cdot\vec{p} + k^0E_+}{(-E_+ - k^0 - E_-)(-E_+ - k^0 + E_-)(2E_+)}f\left(\frac{E_+}{T}\right)$$

Substitution:
$\varepsilon \equiv \frac{1}{2}(E_+ + E_-)$; $\omega \equiv E_+ - E_-$; $E_\pm = \varepsilon \pm \frac{1}{2}\omega$; $\varepsilon\omega = \frac{1}{2}(E_+^2 - E_-^2) = \vec{p}\cdot\vec{k}$; $\varepsilon^2 + \frac{1}{4}\omega^2 = \frac{1}{2}(E_+^2 + E_-^2) = \vec{p}^2 + \frac{\vec{k}^2}{4} + M^2$;
$\int d^3\vec{p} = 2\pi\int dp\vec{p}^2\int_{-1}^1 dz = 2\pi\int d\varepsilon d\omega\ \vec{p}^2\left|\frac{\partial(\varepsilon,\omega)}{\partial(p,z)}\right|^{-1} = \frac{2\pi}{|\vec{k}|}\int d\varepsilon d\omega\, E_+E_- = \frac{2\pi}{|\vec{k}|}\int_{\varepsilon_0}^\infty d\varepsilon\int_{-\omega_0}^{\omega_0}d\omega\left(\varepsilon^2 - \frac{1}{4}\omega^2\right)$
$\varepsilon_0 \equiv \sqrt{|\vec{k}|^2 + \frac{M^2}{4}}$; $\omega_0 \equiv |\vec{k}|\sqrt{1 - \frac{4M^2}{4\varepsilon^2 - |\vec{k}|^2}}$

$$\delta\Pi_T = -\frac{g^2}{2\pi^2|\vec{k}|}\int_{\varepsilon_0}^\infty d\varepsilon\int_{-\omega_0}^{\omega_0}d\omega\left(\varepsilon^2 - \frac{1}{4}\omega^2\right)\frac{\varepsilon^2 + \frac{1}{4}(\omega^2 + \vec{k}^2) - M^2 - \left(\frac{\varepsilon\omega}{|\vec{k}|}\right)^2 - \varepsilon\omega + k^0\varepsilon - \frac{1}{2}k^0\omega}{\left(\varepsilon - \frac{1}{2}\omega\right)(k^0 - \omega)(k^0 + 2\varepsilon)}f\left(\frac{\varepsilon - \frac{1}{2}\omega}{T}\right)$$
$$-\frac{g^2}{2\pi^2|\vec{k}|}\int_{\varepsilon_0}^\infty d\varepsilon\int_{-\omega_0}^{\omega_0}d\omega\left(\varepsilon^2 - \frac{1}{4}\omega^2\right)\frac{\varepsilon^2 + \frac{1}{4}(\omega^2 + \vec{k}^2) - M^2 - \left(\frac{\varepsilon\omega}{|\vec{k}|}\right)^2 - \varepsilon\omega - k^0\varepsilon + \frac{1}{2}k^0\omega}{\left(\varepsilon - \frac{1}{2}\omega\right)(k^0 - 2\varepsilon)(k^0 + \omega)}f\left(\frac{\varepsilon - \frac{1}{2}\omega}{T}\right)$$
$$-\frac{g^2}{2\pi^2|\vec{k}|}\int_{\varepsilon_0}^\infty d\varepsilon\int_{-\omega_0}^{\omega_0}d\omega\left(\varepsilon^2 - \frac{1}{4}\omega^2\right)\frac{\varepsilon^2 + \frac{1}{4}(\omega^2 + \vec{k}^2) - M^2 - \left(\frac{\varepsilon\omega}{|\vec{k}|}\right)^2 + \varepsilon\omega - k^0\varepsilon - \frac{1}{2}k^0\omega}{\left(\varepsilon + \frac{1}{2}\omega\right)(k^0 - \omega)(k^0 - 2\varepsilon)}f\left(\frac{\varepsilon + \frac{1}{2}\omega}{T}\right)$$
$$-\frac{g^2}{2\pi^2|\vec{k}|}\int_{\varepsilon_0}^\infty d\varepsilon\int_{-\omega_0}^{\omega_0}d\omega\left(\varepsilon^2 - \frac{1}{4}\omega^2\right)\frac{\varepsilon^2 + \frac{1}{4}(\omega^2 + \vec{k}^2) - M^2 - \left(\frac{\varepsilon\omega}{|\vec{k}|}\right)^2 + \varepsilon\omega + k^0\varepsilon + \frac{1}{2}k^0\omega}{\left(\varepsilon + \frac{1}{2}\omega\right)(k^0 + 2\varepsilon)(k^0 + \omega)}f\left(\frac{\varepsilon + \frac{1}{2}\omega}{T}\right)$$
$$= -\frac{g^2}{\pi^2|\vec{k}|}\int_{\varepsilon_0}^\infty d\varepsilon\int_{-\omega_0}^{\omega_0}d\omega\left(\varepsilon^2 - \frac{1}{4}\omega^2\right)\frac{\varepsilon^2 + \frac{1}{4}(\omega^2 + \vec{k}^2) - M^2 - \left(\frac{\varepsilon\omega}{|\vec{k}|}\right)^2 - \frac{1}{2}k^0\omega - \varepsilon\omega + k^0\varepsilon}{\left(\varepsilon - \frac{1}{2}\omega\right)(k^0 - \omega)(k^0 + 2\varepsilon)}f\left(\frac{\varepsilon - \frac{1}{2}\omega}{T}\right)$$
$$-\frac{g^2}{\pi^2|\vec{k}|}\int_{\varepsilon_0}^\infty d\varepsilon\int_{-\omega_0}^{\omega_0}d\omega\left(\varepsilon^2 - \frac{1}{4}\omega^2\right)\frac{\varepsilon^2 + \frac{1}{4}(\omega^2 + \vec{k}^2) - M^2 - \left(\frac{\varepsilon\omega}{|\vec{k}|}\right)^2 - \frac{1}{2}k^0\omega + \varepsilon\omega - k^0\varepsilon}{\left(\varepsilon + \frac{1}{2}\omega\right)(k^0 - \omega)(k^0 - 2\varepsilon)}f\left(\frac{\varepsilon + \frac{1}{2}\omega}{T}\right)$$
$$= -\frac{g^2}{\pi^2|\vec{k}|}\int_{\varepsilon_0}^\infty d\varepsilon\int_{-\omega_0}^{\omega_0}d\omega\left(\varepsilon^2 - \frac{1}{4}\omega^2\right)\left[\frac{\left(\varepsilon^2 + \frac{\vec{k}^2}{4}\right)\left(1 - \frac{\omega^2}{\vec{k}^2}\right) - M^2}{k^0 - \omega} - \frac{\omega}{2}\right]\left[\frac{f\left(\frac{\varepsilon - \frac{1}{2}\omega}{T}\right)}{\left(\varepsilon - \frac{1}{2}\omega\right)(k^0 + 2\varepsilon)} + \frac{f\left(\frac{\varepsilon + \frac{1}{2}\omega}{T}\right)}{\left(\varepsilon + \frac{1}{2}\omega\right)(k^0 - 2\varepsilon)}\right]$$
$$+\frac{g^2}{\pi^2|\vec{k}|}\int_{\varepsilon_0}^\infty d\varepsilon\,\varepsilon\int_{-\omega_0}^{\omega_0}d\omega\left(\varepsilon^2 - \frac{1}{4}\omega^2\right)\left[\frac{f\left(\frac{\varepsilon - \frac{1}{2}\omega}{T}\right)}{\left(\varepsilon - \frac{1}{2}\omega\right)(k^0 + 2\varepsilon)} - \frac{f\left(\frac{\varepsilon + \frac{1}{2}\omega}{T}\right)}{\left(\varepsilon + \frac{1}{2}\omega\right)(k^0 - 2\varepsilon)}\right]$$

QED Plasma

Photon Self-energy to 1-Loop (Longitudinal)

$$\delta\Pi_L(k) = \Delta_{\mu\nu}^L(k)\delta\Pi^{\mu\nu}(k)$$
$$= \frac{2g^2}{k^2}\int\frac{d^3\vec{p}}{(2\pi)^3}\frac{(2E_-^2 + E_-k_0 + k_0^2)\vec{k}^2 - (4E_-k^0 + 3k_0^2 - \vec{k}^2)(\vec{k}\cdot\vec{p}) + 2(\vec{k}\cdot\vec{p})^2\frac{k_0^2}{\vec{k}^2} + k_0^3E_- - \frac{\vec{k}^4}{2}}{2E_-(k^0 + E_- - E_+)(k^0 + E_- + E_+)}\left(\tanh\frac{E_-}{2T} - 1\right)$$
$$+\frac{2g^2}{k^2}\int\frac{d^3\vec{p}}{(2\pi)^3}\frac{(2E_-^2 - E_-k^0 + k_0^2)\vec{k}^2 + (4E_-k^0 - 3k_0^2 + \vec{k}^2)(\vec{k}\cdot\vec{p}) + 2(\vec{k}\cdot\vec{p})^2\frac{k_0^2}{\vec{k}^2} - k_0^3E_- - \frac{\vec{k}^4}{2}}{2E_-(k^0 - E_- - E_+)(k^0 - E_- + E_+)}\left(\tanh\frac{E_-}{2T} - 1\right)$$
$$+\frac{2g^2}{k^2}\int\frac{d^3\vec{p}}{(2\pi)^3}\frac{(2E_+^2 - E_+k^0 + k_0^2)\vec{k}^2 - (4E_+k^0 - 3k_0^2 + \vec{k}^2)(\vec{k}\cdot\vec{p}) + 2(\vec{k}\cdot\vec{p})^2\frac{k_0^2}{\vec{k}^2} - k_0^3E_+ - \frac{\vec{k}^4}{2}}{(E_+ - k^0 - E_-)(E_+ - k^0 + E_-)2E_+}\left(\tanh\frac{E_+}{2T} - 1\right)$$
$$+\frac{2g^2}{k^2}\int\frac{d^3\vec{p}}{(2\pi)^3}\frac{(2E_+^2 + E_+k_0 + k_0^2)\vec{k}^2 + (4E_+k^0 + 3k_0^2 - \vec{k}^2)(\vec{k}\cdot\vec{p}) + 2(\vec{k}\cdot\vec{p})^2\frac{k_0^2}{\vec{k}^2} + k_0^3E_+ - \frac{\vec{k}^4}{2}}{(-E_+ - k^0 - E_-)(-E_+ - k^0 + E_-)(2E_+)}\left(\tanh\frac{E_+}{2T} - 1\right)$$

$$= -\frac{g^2}{\pi^2k^2}\frac{1}{|\vec{k}|}\int_{\varepsilon_0}^\infty d\varepsilon\int_{-\omega_0}^{\omega_0}d\omega\left(\varepsilon^2 - \frac{1}{4}\omega^2\right)\frac{\left(2\left(\varepsilon^2 + \frac{\omega^2}{4} - \varepsilon\omega\right) + k_0^2 - \frac{\vec{k}^2}{2}\right)\vec{k}^2 - (3k_0^2 - \vec{k}^2)\varepsilon\omega + 2(\varepsilon\omega)^2\frac{k_0^2}{\vec{k}^2} + (k_0^2 + \vec{k}^2 - 4\varepsilon\omega)k^0\left(\varepsilon - \frac{\omega}{2}\right)}{\left(\varepsilon - \frac{\omega}{2}\right)(k^0 - \omega)(k^0 + 2\varepsilon)}f\left(\frac{\varepsilon - \frac{\omega}{2}}{T}\right)$$
$$-\frac{g^2}{\pi^2k^2}\frac{1}{|\vec{k}|}\int_{\varepsilon_0}^\infty d\varepsilon\int_{-\omega_0}^{\omega_0}d\omega\left(\varepsilon^2 - \frac{1}{4}\omega^2\right)\frac{\left(2\left(\varepsilon^2 + \frac{\omega^2}{4} + \varepsilon\omega\right) + k_0^2 - \frac{\vec{k}^2}{2}\right)\vec{k}^2 + (3k_0^2 - \vec{k}^2)\varepsilon\omega + 2(\varepsilon\omega)^2\frac{k_0^2}{\vec{k}^2} - (k_0^2 + \vec{k}^2 + 4\varepsilon\omega)k^0\left(\varepsilon + \frac{\omega}{2}\right)}{\left(\varepsilon + \frac{\omega}{2}\right)(k^0 - \omega)(k^0 - 2\varepsilon)}f\left(\frac{\varepsilon + \frac{\omega}{2}}{T}\right)$$
$$= -\frac{g^2}{\pi^2k^2}\frac{1}{|\vec{k}|}\int_{\varepsilon_0}^\infty d\varepsilon\int_{-\omega_0}^{\omega_0}d\omega\left(\varepsilon^2 - \frac{1}{4}\omega^2\right)\frac{\left(2\left(\varepsilon^2 + \frac{\omega^2}{4}\right) + k_0^2 - \frac{\vec{k}^2}{2}\right)\vec{k}^2 + 2(\varepsilon\omega)^2\frac{k_0^2}{\vec{k}^2} - (k_0^2 + \vec{k}^2)k^0\frac{\omega}{2} - 4\varepsilon^2\omega k^0}{(k^0 - \omega)}\left[\frac{f\left(\frac{\varepsilon - \frac{\omega}{2}}{T}\right)}{\left(\varepsilon - \frac{\omega}{2}\right)(k^0 + 2\varepsilon)} + \frac{f\left(\frac{\varepsilon + \frac{\omega}{2}}{T}\right)}{\left(\varepsilon + \frac{\omega}{2}\right)(k^0 - 2\varepsilon)}\right]$$
$$-\frac{g^2}{\pi^2k^2}\frac{1}{|\vec{k}|}\int_{\varepsilon_0}^\infty d\varepsilon\int_{-\omega_0}^{\omega_0}d\omega\left(\varepsilon^2 - \frac{1}{4}\omega^2\right)\frac{(k_0^2 + \vec{k}^2)k^0\varepsilon + 2\varepsilon\omega^2k^0 - (3k_0^2 + \vec{k}^2)\varepsilon\omega}{(k^0 - \omega)}\left[\frac{f\left(\frac{\varepsilon - \frac{\omega}{2}}{T}\right)}{\left(\varepsilon - \frac{\omega}{2}\right)(k^0 + 2\varepsilon)} - \frac{f\left(\frac{\varepsilon + \frac{\omega}{2}}{T}\right)}{\left(\varepsilon + \frac{\omega}{2}\right)(k^0 - 2\varepsilon)}\right]$$

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Chiral and High-T limit

- $M \to 0$; $\Rightarrow$ $\varepsilon_0 \sim \omega_0 \to |\vec{k}|$
- $T \gg |\vec{k}| \sim \omega_0 \Rightarrow \int_{-\omega_0}^{\omega_0}d\omega\ \mathcal{F}(\omega)f\left(\frac{??-\omega}{T}\right) \approx \omega_0\left[\mathcal{F}(\omega_0)f\left(\frac{??-\omega_0}{T}\right) + \mathcal{F}(-\omega_0)f\left(\frac{??+\omega_0}{T}\right)\right]$
- Transverse Self-energy:
  $$\delta\Pi_T \approx -\frac{g^2}{2\pi^2}\int_{\varepsilon_0}^\infty d\varepsilon\frac{\left(4\varepsilon^2 - |\vec{k}|^2\right)}{(4\varepsilon^2 - k_0^2)}\varepsilon|\vec{k}|\left[\frac{f\left(\frac{\varepsilon + \frac{1}{2}|\vec{k}|}{T}\right)}{\left(\varepsilon + \frac{1}{2}|\vec{k}|\right)} - \frac{f\left(\frac{\varepsilon - \frac{1}{2}|\vec{k}|}{T}\right)}{\left(\varepsilon - \frac{1}{2}|\vec{k}|\right)}\right] + \frac{g^2}{\pi^2}\int_{\varepsilon_0}^\infty d\varepsilon\,\varepsilon^2\frac{\left(4\varepsilon^2 - |\vec{k}|^2\right)}{(4\varepsilon^2 - k_0^2)}\left[\frac{f\left(\frac{\varepsilon + \frac{1}{2}|\vec{k}|}{T}\right)}{\left(\varepsilon + \frac{1}{2}|\vec{k}|\right)} + \frac{f\left(\frac{\varepsilon - \frac{1}{2}|\vec{k}|}{T}\right)}{\left(\varepsilon - \frac{1}{2}|\vec{k}|\right)}\right]$$
  $$\approx \frac{g^2}{\pi^2}\int_{\varepsilon_0}^\infty d\varepsilon\ \vec{k}^2\left(\frac{f[\varepsilon]}{\varepsilon} - f'[\epsilon]\right) + \frac{g^2}{\pi^2}\int_{\varepsilon_0}^\infty d\varepsilon\left(2T^2\varepsilon f[\varepsilon] + \frac{k^2}{2}\frac{f[\varepsilon]}{\varepsilon} + \frac{1}{4}\varepsilon f''[\epsilon]\vec{k}^2\right)$$
  $$\approx \frac{g^2}{6}T^2 + \frac{g^2}{4\pi^2}\ln\frac{T}{|\vec{k}|}\left\{k_0^2 + \vec{k}^2\right\} + \frac{5}{8}\frac{g^2}{\pi^2}\vec{k}^2$$
- Longitudinal Self-energy:
  $$\delta\Pi_L \approx \frac{g^2}{2\pi^2k^4}\int_{\varepsilon_0}^\infty d\varepsilon\frac{(4\varepsilon^2 - \vec{k}^2)}{(4\varepsilon^2 - k_0^2)}\left(2\varepsilon^2(\vec{k}^2 + k_0^2) + k_0^2\vec{k}^2\right)\left[\frac{(k_0^2 - 2\varepsilon|\vec{k}|)f\left(\frac{\varepsilon - \frac{|\vec{k}|}{2}}{T}\right)}{\left(\varepsilon - \frac{|\vec{k}|}{2}\right)} + \frac{(k_0^2 + 2\varepsilon|\vec{k}|)f\left(\frac{\varepsilon + \frac{|\vec{k}|}{2}}{T}\right)}{\left(\varepsilon + \frac{|\vec{k}|}{2}\right)}\right] + \frac{g^2k_0^2}{\pi^2k^4}|\vec{k}|\int_{\varepsilon_0}^\infty d\varepsilon\frac{(4\varepsilon^2 - \vec{k}^2)}{(4\varepsilon^2 - k_0^2)}\left((k_0^2 + \vec{k}^2)\frac{1}{2} + 4\varepsilon^2\right)\left[f\left(\frac{\varepsilon - \frac{|\vec{k}|}{2}}{T}\right) - f\left(\frac{\varepsilon + \frac{|\vec{k}|}{2}}{T}\right)\right]$$
  $$-\frac{g^2(k_0^2 + 3\vec{k}^2)k_0^2}{\pi^2k^4}\int_{\varepsilon_0}^\infty d\varepsilon\,\varepsilon\frac{(4\varepsilon^2 - \vec{k}^2)}{(4\varepsilon^2 - k_0^2)}\left[f\left(\frac{\varepsilon - \frac{|\vec{k}|}{2}}{T}\right) + f\left(\frac{\varepsilon + \frac{|\vec{k}|}{2}}{T}\right)\right] - \frac{g^2(3k_0^2 + \vec{k}^2)}{2\pi^2k^4}|\vec{k}|\int_{\varepsilon_0}^\infty d\varepsilon\,\varepsilon\frac{(4\varepsilon^2 - \vec{k}^2)}{(4\varepsilon^2 - k_0^2)}\left[\frac{(k_0^2 - 2\varepsilon|\vec{k}|)f\left(\frac{\varepsilon - \frac{|\vec{k}|}{2}}{T}\right)}{\left(\varepsilon - \frac{|\vec{k}|}{2}\right)} - \frac{(k_0^2 + 2\varepsilon|\vec{k}|)f\left(\frac{\varepsilon + \frac{|\vec{k}|}{2}}{T}\right)}{\left(\varepsilon + \frac{|\vec{k}|}{2}\right)}\right]$$
  $$\approx \frac{g^2}{2\pi^2k^4}\int_{\varepsilon_0}^\infty d\varepsilon(\vec{k}^2 + k_0^2)\left[\varepsilon k_0^2\left(8f[\epsilon]T^2 + f''[\epsilon]\vec{k}^2\right) + 4\varepsilon\vec{k}^2\left(T^2 - \frac{\vec{k}^2 + k_0^2}{4\varepsilon^2} + \frac{k_0^2\vec{k}^2}{2\varepsilon^2(\vec{k}^2 + k_0^2)}\right)(-f[\epsilon] + \epsilon f'[\epsilon])\right] + \frac{g^2k_0^2}{\pi^2k^4}\vec{k}^2\int_{\varepsilon_0}^\infty d\varepsilon\,4\varepsilon^2\left(T^2 + \frac{k^2}{4\varepsilon^2} + \frac{(k_0^2 + \vec{k}^2)\frac{1}{2}}{4\varepsilon^2}\right)f'(\varepsilon)$$
  $$-\frac{g^2(k_0^2 + 3\vec{k}^2)k_0^2}{\pi^2k^4}\int_{\varepsilon_0}^\infty d\varepsilon\,\varepsilon\left(2f[\epsilon]T^2 + \frac{k^2}{4\varepsilon^2}2f[\epsilon] + \frac{1}{4}f''[\epsilon]\vec{k}^2\right) - \frac{g^2(3k_0^2 + \vec{k}^2)}{2\pi^2k^4}\vec{k}^2\int_{\varepsilon_0}^\infty d\varepsilon\left[(k_0^2 + \vec{k}^2)\left[\frac{(-f[\epsilon] + \epsilon f'[\epsilon])}{\varepsilon}\right] - \left(4\varepsilon f[\epsilon]T^2 + k^2\frac{f[\epsilon]}{\epsilon} + \frac{\epsilon f''[\epsilon]\vec{k}^2}{2}\right)\right]$$
  $$\approx \frac{g^2}{6k^4}\left(k_0^4 - 5k_0^2\vec{k}^2 - 2\vec{k}^4\right)T^2 + \frac{g^2}{8\pi^2k^4}\vec{k}^2\left[k_0^4 + 12\vec{k}^2k_0^2 + 3\vec{k}^4\right] + \frac{g^2}{4\pi^2k^4}\left[\vec{k}^6 - k_0^6 + 5\vec{k}^2k_0^2(\vec{k}^2 + k_0^2)\right]\ln\frac{T}{|\vec{k}|}$$

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Linear Response Thoery (see lecture notes on plasma)