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Playing minority game

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Abstract

We designed and conducted an experiment to see how people play the standard minority game. The experiment was conducted at the computer lab, using a special software, with a group of 15 students. Collected informations comprise individual choices and individual wealth after each time step. The results, in particular the coordination of the agents, measured by the volatility of the system, are compared for different amount of informations available to the agents.

Keywords: Minority game; Coordination; Laboratory experiment

1. Introduction

Across many scientific disciplines there is a growing interest in modelling complex adaptive systems of heterogeneous agents. One of the simplest paradigm for such complex adaptive systems is the minority game (MG), introduced by Challet and Zhang [1,2] as a mathematical model of the system of inductive agents, proposed by Arthur [3] and applied later e.g. to modelling markets, cf. e.g. [4–9]. There is a huge (and still growing) theoretical literature also on other aspects of the subject. Various kinds of variations and flavors of both settings and learning procedures are analyzed. Commonly accepted learning scheme is a stimulus–response procedure. For various aspects of the MG the reader is referred to the Econophysics web page [10].

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The original MG is an N-person game, where N is an odd number, played by heterogeneous, inductive rational agents, who interact through an aggregate, collective variable. The MG is constructed of a stage game repeated over time by agents using certain learning procedures. At each stage of the game each agent may choose one of two admissible actions: A or B (-1 or $+1$). Only one of two actions is successful—that chosen by minority. In each step all agents decide whether to use A or B. Once the actions are chosen, the game is played and all agents get their payoff. The MG is repeated over time. All agents know the sequence of the last M successful actions—this sequence is the only public information in the system. Decision on what action to play in the next step is based only on the public information. The link between public information and action is called a strategy. Parallelly to dynamics of the public information, there is adaptation in the system. Each agent has at least two strategies (the same number for each agent), and uses her best strategy. To decide which strategy is the best, each agent compares how many times each strategy would have been successful if used. The strategy with the highest "virtual" success is adopted. If there is more than one best strategy, the agent chooses randomly one of them.

From the game-theoretical point of view MG is an $N=2k+1$ person non-cooperative game with a unique symmetric mixed strategy equilibrium and many asymmetric Nash equilibria in pure strategies. It bears some resemblances to the market entry games, considered in experimental setting by many authors, cf. e.g. [11–18] and references therein. On the other hand, experimental investigations how real people play MG has been started only very recently, cf. [19]. Behavior of humans in a simplified MG-modelled market situations, in which individual humans play against computer-modelled agents, can be studied with the help of procedures developed in [20].

We designed an experiment to see if the actual human behavior is related in any way to the results of simulations (based on a stimulus–response learning scheme). We considered a standard MG. The experiment was designed as follows. The group of players consisted of 15 students. Each of the students observed the results of the game (history—public information) on a different monitor in a computer lab. At each time step of the game the only information displayed on the monitors consisted of a raw of M binary digits, 0 or 1, which indicated which option (-1 or 1) has been chosen by the minority of the players in the previous M games. At the beginning of the game this information has been supplied to the players randomly. There were several rounds, each with a different value of M (varying from 3 through 11). Each round consisted of 200 time steps. The students knew only the basic principles of the game (those who are at a given stage of the game in minority win the stage, those in majority lose), and the initial score (the same for all players). However, they could not control effectively their performance—actual score (current wealth)—during the game.

The results of the experiment indicate that humans do coordinate when playing the MG. The level of coordination, measured by the volatility of the system, does not seem to depend substantially on the length of the memory—the only public information available to the agents. An interesting finding is that the random signal does not seem to lead to an efficient correlated equilibrium.

2. Description of experiment

The group of players was composed of 15 students. Each of them sat in front of one of the monitors in a computer laboratory. At each unit of the game the only information displayed on their monitors comprised of M binary digits, 0 or 1, which indicated which option (-1 or 1) has been in minority in the previous M games. At the beginning of the game ($t = 0$) this information has been chosen randomly.

There were seven rounds: four for different M values, $M = 3, 5, 8, 11$, then the agents played one round with $M = 3$ and one with $M = 8$, however with faked (random) history, not knowing that the information has been faked. Finally, the last round has been played without any information at all, i.e., on the screen of the monitors the players had only the information whether their move has been sent to the server. Each round lasted 200 time steps. Each time step has been accomplished once all the players took the decision, which option they choose in the given stage of the game. This information has been sent by each player to the central unit by pressing one of the keys, 0 or 1 on his/her keyboard. Each round lasted about 20 min. Thus, on average each time step of the game lasted about 6 s, which seemed short enough time to assure independence of the decisions of the players. At the beginning of the game the students were asked not to communicate between themselves. A kind of "psychological" pression which furthered such a behavior has been maintained by the instructor, who supervised the procedure.

The students knew only the basic principles of the game—that those who are at a given stage of the game in minority win the stage, those in majority lose. Each of the winners collected a point, each member of the majority group lost a point. In order to motivate "psychologically" the students, each of them has been given the initial score of 100 points at $t = 0$. The students were aware of the initial score and were informed that the player with the highest score at the end of the tournament, would be announced the winner. They did not know however their scores during the subsequent time steps of the rounds. No particular prize has been awarded to the winner. The procedure worked smoothly during all the seven rounds of the game.

The experiment was conducted at the computer lab, using a special software. The program consists of the server and the clients, which communicate with the server by TCP pipe. First, the server is started. It opens a TCP pipe through which it receives all informations required to register clients. When a fixed number of players has registered, the server begins the first round of the game. During each round the server waits for the votes from all the players to come in. Having received all the votes, the server counts how many players voted for each option and calculates the payoff for each player. All the information about the current round of the game is stored in text files, separately for each player. Next the server sends the player a vector of winning strategies (public information). Following that the server closes all connections. The client process works in a loop. It receives the history from the server, reads the option chosen by the player, sends it to the server and wait for confirmation. After counting all the votes the server sends the updated history to all the players and the player may start the next round of the game. The program was written in the C programing language, under the Linux system.

3. Results

3.1. Description of figures

We present seven figures, each corresponding to a single round of the tournament. Each figure contains three pictures. Left one is a picture presenting price vs. time (gray lines). The thick line is a smoothed average. We denote price (or aggregate attendance) by $A(t)$ and define it to be

$$A(t) = \sum_{i=1}^{i=N} a_i(t) ,$$

where $a_i(t) \in \{-1, 1\}$ is an action played by ith player at time t.

One way of measuring a level of coordination of agents is based on the notion of the scaled volatility σ^2/N, where σ^2 is defined as

$$\sigma^2 = \lim_{t\to\infty} \frac{1}{t} \sum_{l=1}^{t} A^2(l) . \tag{1}$$

Middle picture presents volatility and how it changes over time in our experiments. The gray line is volatility computed from the five consecutive prices. The thick black line is volatility computed from all prices up to the current step, hence the very last point is just volatility computed from the whole sample. The dashed straight line corresponds to the situation in which all the players make their choices completely at random, which results in $\sigma^2/N = 1$.

Finally, the right picture presents individual wealths (gray lines) and an average wealth (thick black line) in population vs. time.

For any player i we defined wealth $W_i(t)$ earned by that player up to time t as

$$W_i(t) = \sum_{l=0}^{l=t} w_i(l) ,$$

where $w_i(t)$ is the payoff of ith player playing at time t, defined as

$$w_i(t) = -a_i(t)A(t) .$$

Additionally there is (straight black line) a linear regression plotted. The black dots are final wealth levels of players, i.e., the values $W_i(200)$.

The first four figures (Figs. 1–4) present the results for different lengths of the "memory" (public bit string length): $m = 3, 5, 8, 11$. Fig. 5 shows the results for the case of no public information ($m = 0$)—the agents had to make their choices for each stage of the game without any information at all. Figs. 6 and 7 depict a case of the public string memory length m, respectively, $m = 3, 8$, however with the faked information—the server provided random strings, however the agents did not know that the information has been faked.

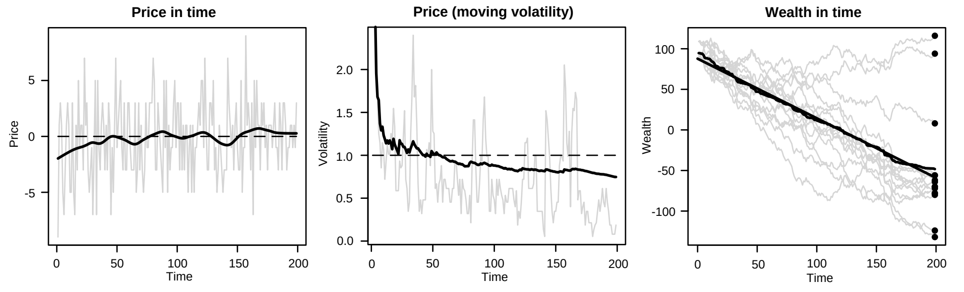

Fig. 1. Public information length is $M = 3$ and the information is real.

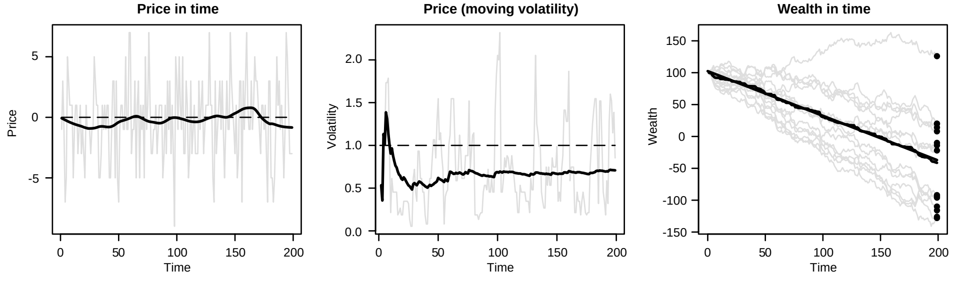

Fig. 2. Public information length is $M = 5$ and the information is real.

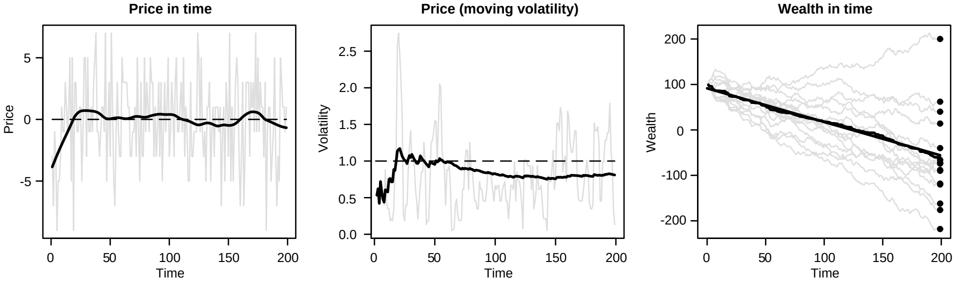

Fig. 3. Public information length is $M = 8$ and the information is real.

3.2. Discussion

There is a positive message that comes from Figs. 1 to 4—people do coordinate! Table 1 contains summary statistics for first four samples, including mean (μ), standard deviation (σ), scaled volatility (σ^2/N), and quantiles. We notice that both mean and median are very close to 0, hence on average the market is close to the equilibrium—50% of observed prices fall into the $[-3, 3]$ interval with exception for $M = 11$ with the interval $[-1, 3]$.

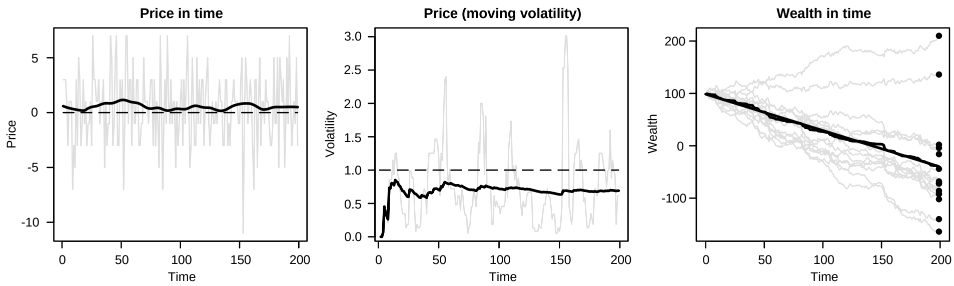

Fig. 4. Public information length is $M = 11$ and the information is real.

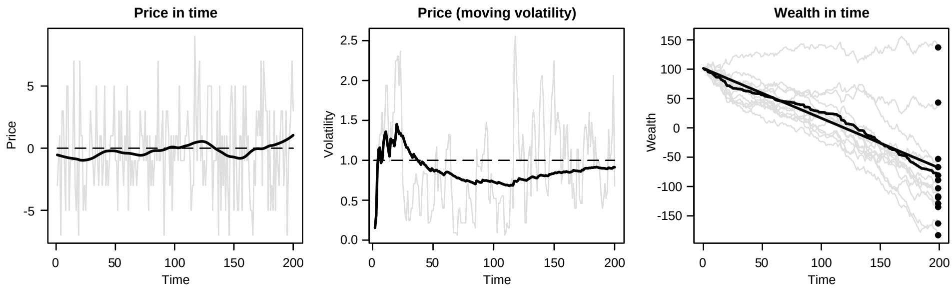

Fig. 5. Public information length is $M = 0$—no public information.

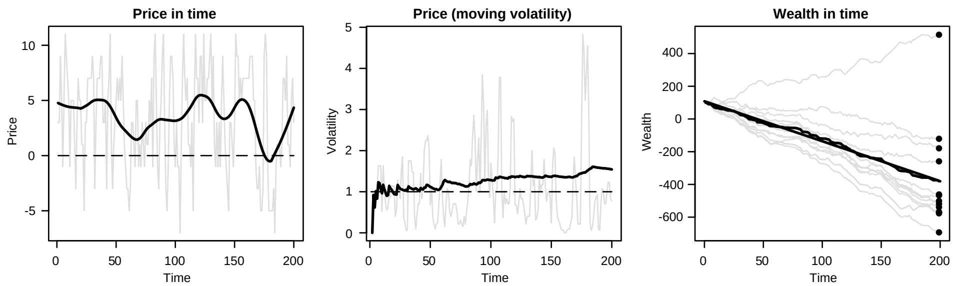

Fig. 6. Public information length is $M = 3$ and the information is fake.

The same conclusion can be drawn from the average attendance, attendance being defined as the number of agents choosing one of the options (e.g. A). The relevant results are not shown, because the smoothed plots for both attendance and prices are very similar since price is just a linear transformation of attendance. Thus, on average, the number of sellers is approximately $\frac{1}{2}$ of the whole population.

We can also look at the volatility (black line), it is well below 1 (theoretical random case), which means people play better than just at random. For all first four samples the stable level of volatility is between 0.6 and 0.8. The other interesting thing is

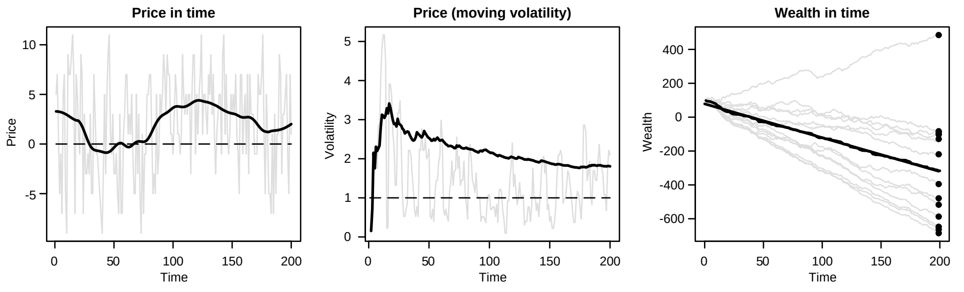

Fig. 7. Public information length is $M = 8$ and the information is fake.

Table 1
Summary statistics for first four samples (real history)

M	μ	σ	σ^2/N	$q_{0.25}$	$q_{0.50}$	$q_{0.75}$
3	0.16	3.024	0.609	−3	1	3
5	−0.04	3.308	0.729	−3	0	3
8	−0.18	3.459	0.797	−3	−1	3
11	0.42	3.156	0.664	−1	1	3

All statistics are based on the last 100 steps (hence include only a "stable situation").

the dynamics of convergence of (scaled) volatility. In all samples the stable level of volatilty is reached after roughly 100 steps.

The above observations lead to the striking conclusion: the length M of the public information seems to have no influence on the summary statistics of prices and the dynamics of volatility. This fact is obviously at odds with what can be observed in simulated samples.

The wealth plots provide a new structure of players. There are always some players gaining a lot (one or two) and the rest loosing even more, so the wealth seems to be clustered. Per capita wealth seems to be well approximated by a linear trend, however such conclusion would need more experimental evidences.

The most surprising fact comes from Figs. 5–7. Fig. 5 describes behavior of people playing without any information, while Figs. 6 and 7 summarize behavior of people playing a game with a fake history not knowing this!

Table 2 contains the same summary statistics as Table 1 for the last three samples. The surprising result is the following: it seems that people coordinate far more better without any information than if provided with an external fake signal. Although in Fig. 5 the volatility seems to be increasing (which is not surprising), but is below 1 (if only "stable" situation is included—last 100 steps—the scaled volatility is approximately 1). On Figs. 6 and 7 the volatility is well above 1, hence it is much worse than the theoretical random case. This fact is interesting since it suggests that people will not use a random signal to reach an efficient correlated equilibrium. That would

Table 2
Summary statistics for the last three samples (no history or faked history)

M	μ	σ	σ^2/N	$q_{0.25}$	$q_{0.50}$	$q_{0.75}$
0	0.04	3.76	1.09	−3	−1	3
3	3.22	4.87	1.82	−1	3	7
8	2.86	4.16	1.33	1	3	5

All statistics are based on the last 100 steps.

suggest that any learning procedure leading to such an equilibrium is not appropriate description of actual human behavior in this game.

4. Open problems

There are various interesting problems which could be studied in the considered experimental setting. We mention some of them:

- How would the information about the actual price (i.e., difference between the number of winners and losers) in each round influence the level of coordination. Since this kind of signal contains some crucial information one may hope for better coordination. However, it may very well turn out that this information will not increase coordination level because of its symmetry with respect to players. This rises another question about strategic asymmetries (i.e., only part of the players may access precise price information).
- In the considered experiment the players spent on average about 6 s to decide which option to choose (the play of 200 rounds lasted on average 20 min). How would the results change if the players would have more time for the decision, in particular whether in this case the results would depend on the length of the memory.
- How the results would depend on the social structure of the group (e.g. sex ratio, social and age structure of the group etc.).
- One may also study the dependence of the level of coordination on the information supplied to the group. In particular, what happens when the true signal is replaced by a faked one during the game or the opposite, when the faked signal is replaced by the true one. The information about possibilities of such a manipulation could be given or not to the players. One could also provide an information that the commonly known strings are faked with a certain probability, to see how the players react in such a fuzzy setting.

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