Coordination in the El Farol Bar problem

The Role of Social Preferences and Social Networks

Shu-Heng Chen and Umberto Gostoli

Abstract

- Self Coordination Mechanism
- Efficiency (optimal use of public facility) and distribution of the public resources among all agents
- Self-coordinated solutions to optimally use public resources ("good society") requires social networks and social preferences
- Hope that resources can be utilized without too much idle capacity left, but don't want them to be overused (congestion)

El Farol Concepts

- Two kinds of learning mechanisms
- Best responsive learning: Keep track of numerous beliefs and models. Fluctuation around the threshold (switch between idle and congestion), difficult to reach steady state
- Reinforcement learning: Stimulus-response learning. System asymptotically converges to perfect coordination (complete segregation - Inequity issue)
- Social networks: Assumptions on which interactions are based, structure of agents' interactions. Use of local information instead of global information
- Social preferences: Fair share, minimum attendance frequency (threshold) - Average of their neighbors' attendance frequencies
- Not every agent is needed to be inequity-averse (20 % - 25 %)

Original problem

In the original El Farol Bar problem (Arthur, 1994), N people decide independently, without collusion or prior communication, whether to go to a bar. Going is enjoyable only if the bar is not crowded, otherwise the agents would prefer to stay home. The bar is crowded if more than B people show up, whereas it is not crowded, and thus enjoyable, if attendees are B or fewer. If we denote the agent's decision "to go" by "1" and "not to go" by "0", and the actual number of attendees by n ($n \leq N$), then the agent's payoff function has the general form (1).

$$U(x,n) = \begin{cases} u_1, & \text{if } x = 0 \text{ and } n > B, \\ u_2, & \text{if } x = 0 \text{ and } n \leq B, \\ u_3, & \text{if } x = 1 \text{ and } n > B, \\ u_4, & \text{if } x = 1 \text{ and } n \leq B. \end{cases} \tag{1}$$

Best responsive learning

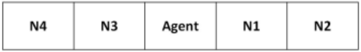
The payoffs have either the order $u_4 = u_1 > u_2 = u_3$ or the order $u_4 > u_1 = u_2 > u_3$.[7] Reinforcement learning

Historical Results

The learning process will asymptotically lead to a state of perfect coordination with complete segregation, where a fraction $\frac{B}{N}$ of the population will always go and the fraction $1 - \frac{B}{N}$ will always stay at home

Proposed Model

- Locally-based cellular automata
- Learning through imitation from neighbours
- $N = 100, \frac{B}{N} = 0{,}6$
- Network typologies: Circular network (ring) and Von Neumann network (grid, torus)



Proposed Model

Rule	N1	N2	N3	N4	D
1	0	0	0	0	**0**
2	0	0	0	1	**0**
3	0	0	1	0	**1**
4	0	0	1	1	**0**
5	0	1	0	0	**0**
6	0	1	0	1	**0**
7	0	1	1	0	**1**
8	0	1	1	1	**1**

Rule	N1	N2	N3	N4	D
9	1	0	0	0	**1**
10	1	0	0	1	**0**
11	1	0	1	0	**1**
12	1	0	1	1	**0**
13	1	1	0	0	**1**
14	1	1	0	1	**1**
15	1	1	1	0	**1**
16	1	1	1	1	**0**

- 1 strategy = 16 rules
- $d_i(t)$:=Action taken by agent i at period t (1 or 0)
- $s_i(t)$:= Outcome of agent i's decision at time t (1 or 0)
- Memory: Vectors **d** and **s** of length $m = 10$
- $a_i = \frac{1}{m} \sum_{j=t}^{t+1-m} d_i(j)$:= Agent i's attendance frequency over most recent m periods
- $f_i = \frac{1}{m} \sum_{j=t}^{t+1-m} s_i(j)$:= Agent i's decision accuracy rate

Proposed Model

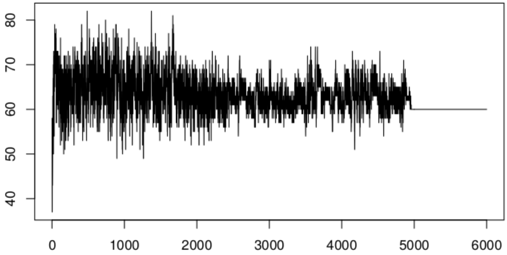
- $r_i :=$ Duration of agent i's current strategy
- $\alpha_i \in [0, 0{,}6] :=$ Minimum attendance treshold, Expected attendance frequency

Learning (i imitates j) conditions:

- $f_i < 1$ or $a_i < \alpha_i$
- $r_i \geq m_i$
- $f_j > f_i$
- $a_j \geq \alpha_i$
- $r_j \geq m_j$
- $z_j \neq z_i$ (Avoid strategy repetition)
- Agents may randomly mutate if first two conditions are met (probability $p << 1$)

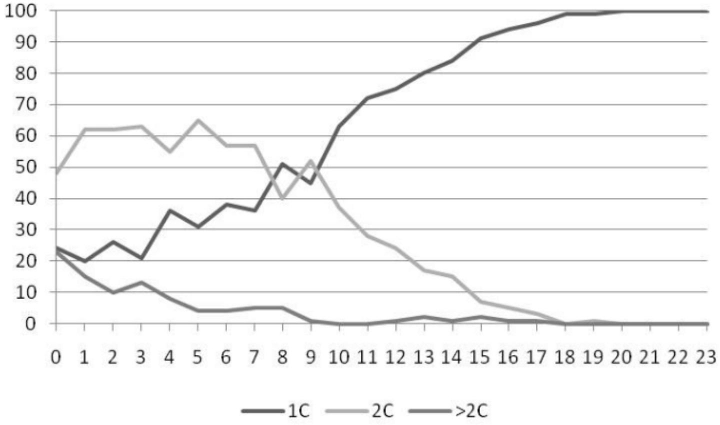
Stop learning conditions: $f_i = 1$ and $a_i \geq \alpha_i$

Results



- Always reaches equilibrium
- Usual (segregated) equilibrium: 0.6(B) always go, 0.4(1-B) never goes
- Fair equilibrium: Everyone goes with a 0.6(B) frequency (Notation: Ξ_G)
- In Von Neumann network: $P(\Xi_G) = 18\,\%$. In Circular network: $P(\Xi_G) = 2\,\%$
- Increasing α increases probability of getting Ξ_G

Results



100
90
80
70
60
50
40
30
20
10
0
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23
1C
2C
>2C