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A laboratory experiment on the minority game

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Abstract

This work presents experimental results on a coordination game in which agents must repeatedly choose between two sides, and a positive fixed payoff is assigned only to agents who pick the minoritarian side. We conduct laboratory experiments in which stationary groups of five players play the game for 100 periods, and manipulate two treatment variables: the amount of 'memory' M that players have regarding the game history (i.e., the length of the string of past outcomes that players can see on the screen while choosing) and the amount of information about other players' past choices. Our results show that, at the aggregate level, a quite remarkable degree of coordination is achieved. Moreover, providing players with full information about other players' choice distribution does not appear to improve efficiency significantly.

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1. Introduction

In this work we present experiments on a very simple repeated game in which the payoff to each player is based on a minority rule. In the Minority Game, first introduced by Challet and Zhang [1,2], a fixed group of N players (where N is odd) must privately and independently choose each round between two actions available to them.[1] The players choosing the action that is chosen by the minority of the players earn a fixed, positive payoff, while players who end up on the majority side earn

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[1] See also the famous 'El Farol' bar problem developed in Ref. [3].

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nothing.[2] Formally, the model is an N-person non-cooperative game with multiple Nash equilibria in pure strategies, whereby agents repeatedly pick the same side, and a unique symmetric equilibrium in mixed strategies, requiring that all agents pick the two sides with equal probability.

This work is aimed at answering the following research questions: what is the degree of efficiency obtainable by groups of players interacting repeatedly in a minority game compared to the benchmark value offered by the mixed strategy equilibrium solution? What is the impact—in terms of aggregate efficiency—of varying the amount of information that players have regarding the game history and the past actions of the other players?

In order to answer these questions, we conducted experiments in which stationary groups of players played a minority game repeatedly and under different information conditions (see Section 3 for a description of the experimental design).

Our preliminary results can be roughly summarized as follows: first, efficiency—both allocative and informational—is higher on average than the benchmark value corresponding to the mixed strategy Nash equilibrium in all treatments, suggesting that a quite remarkable degree of coordination is achieved; second, providing players with more information does not appear to improve efficiency significantly.

The paper is organized as follows: Section 2 describes the game-theoretic framework. Section 3 describes the experimental design, and Section 4 analyzes the results. Finally, Section 5 offers some concluding remarks and directions for future research.

2. The minority game

The minority game is played repeatedly by a stationary set of N players, where N is an odd number. On each period of the stage game, each player must choose privately and independently between two actions or sides which will label 0 and 1. The payoff function, which is the same for all players, is given by

$$\pi_i = \begin{cases} 1 & \text{if } k_i \leqslant (N-1)/2 , \\ 0 & \text{otherwise} , \end{cases} \tag{1}$$

where $i \in \{0,1\}$ and k_i is the number of players choosing i. It is straightforward to see that the game has $N!/(((N-1)/2)!)^2$ Nash equilibria in pure strategies, in which exactly $(N-1)/2$ players choose either one of the two sides. The pure strategy equilibria imply a payoff-asymmetry between players belonging to the different sides, which can rule out the possibility that groups may develop a simple form of tacit coordination based on historical precedent, as it may occur in analogous games (e.g., [4]). The game also has a unique symmetric mixed-strategy Nash equilibrium, in which each player selects the two actions with equal probability. Finally, there are infinite asymmetric mixed strategy equilibria.

[2] For a large collection of papers concerning both the analytical and numerical explorations of the original minority game and various extensions see also http://www.unifr.ch/econophysics/.

3. The experiment: design and implementation

3.1. The information treatments

We conducted experiments in which stationary groups of five players played a minority game for 100 rounds, and we applied a $2X3$ factorial design in which the two treatment variables were the amount of 'memory' M that players could have regarding the game history and the amount of information that players had on the past choices of the other players. A total of 120 subjects participated in the experiment. Four groups of five players participated in the single treatments. Table 1 summarizes the experimental design.

Several previous simulation studies on the MG (e.g., [1,2]) have focussed on the role of the information available to players in determining aggregate efficiency. Indeed, these works have shown that the amount of agents' memory (and hence the amount of information on the 'history' of interaction) has a significant impact on the system's degree of self-organization when artificial agents are designed to behave according to experience-based strategies. In line with previous simulation studies, we also chose to focus on the role of information; moreover, in this study we want to jointly investigate the impact of both information about the series of past outcomes (i.e., the players' available memory capacity) and information (or lack thereof) about the behavior of single players in the population.

As far as the first parameter is concerned, in the case of human agents memory itself cannot be directly controlled for; hence, a rough 'proxy' for it was devised by varying the number of past outcomes for which information was disclosed and remained visible on the subjects' computer screens while playing. We chose three levels of memory: short ($M = 1$), medium ($M = 4$), and long ($M = 16$). The second treatment variable was the type of information provided to subjects at each round of play. In the *aggregate* information treatment, subjects were only allowed to know which side (0 or 1) was the winning side at each round (i.e., they only knew the aggregate outcome of interaction at every time step), while in the *full* information treatment, subjects could also see the entire distribution of individual choices in their group. The choices of players always appeared in the same order, and this fact was explicitly stated in the instructions, so subjects were able to keep track of every individual history of choices in their group.

Table 1
The $3X2$ factorial design of the experiments. Numbers in the cells indicate the number of groups that were assigned to each treatment

	Memory (M)		
	$M = 1$	$M = 4$	$M = 16$
Information			
Aggregate (A)	4	4	4
Full (F)	4	4	4

3.2. Implementation

Subjects on each round had to choose between two actions that were labelled A and B in the experiment. The information that each player received at the end of every round was the visualization of the action chosen in that round (A or B), the payoff gained (1 or 0), the additional information prescribed by the experimental treatment, and his or her cumulative payoff, expressed as a percentage of successful rounds over the total at that point.[3]

The experiment was ran at the Computable and Experimental Economics Laboratory of the University of Trento using undergraduate students from various departments. The experiment was conducted in six sessions of 20 players each, and it was computerized. Subjects were randomly assigned a seat at the lab computer room upon arrival, and were given written instructions.[4] They could see each other, but no form of communication was allowed during the entire session.

Subjects were told that they would be randomly divided in four groups of five players at the beginning of the experiment and that such division would be known only to the computer program. Then they would participate in a repeated decision-making experiment with the same group of people for a total of 100 periods. Each subject received a fixed show up fee plus anything he or she could earn in the experiment. Payoffs in the game were expressed in 'experimental points', and the monetary exchange rate was fixed at E.18 for each point, for a theoretical maximum of E 25.82 per subject (show up fee included). When all members of a group entered their choice, the computer calculated each player's payoff and disclosed information about the round.

4. Results

4.1. Allocative efficiency

We first define the system *allocative efficiency* Σ as

$$\Sigma = \sum_t \left(N_0(t) - \frac{N}{2} \right)^2 , \tag{2}$$

where $N_0(t)$ is the number of players choosing party 0 at time t and N is the total number of players participating in the game. The value of Σ can be comprised between 0.25 and 6.25. Clearly, the lower the value, the higher the efficiency.

Table 2 reports the values of σ over time separately by the value of M and averaged across groups in both the aggregate and full information treatments.

The 'benchmark' value for σ in the case of random symmetric players is equal to 1.25. Note that the degree of allocative efficiency in all treatments is significantly

[3] Subjects were explicitly told that in case all five members chose the same action, they would all earn a payoff of zero, and the computer would display the action not chosen by anyone as the winning action.

[4] The complete set of instructions is available from the authors upon request.

Table 2
Values of σ averaged over successive 25 time steps and across groups, reported separately for values of M and for the aggregate (A) and full (F) information treatments

Time steps	$M=1$		$M=4$		$M=16$	
	A	F	A	F	A	F
25	1.31	1.05	0.99	1.45	1.25	0.91
50	1.25	1.29	0.83	1.13	1.29	0.71
75	1.19	1.31	0.93	0.85	0.89	0.97
100	0.99	1.03	0.91	0.77	0.89	0.61

Table 3
Values of σ over time averaged across values of M for the aggregate and full information treatments

Time steps	A	F
25	1.18	1.14
50	1.12	1.04
75	1.00	1.04
100	0.93	0.80

Table 4
Values of σ over time averaged across information treatments for different values of M

Time	$M=1$	$M=4$	$M=16$
25	1.18	1.22	1.08
50	1.27	0.98	1
75	1.25	0.89	0.93
100	1.01	0.84	0.75

higher than the benchmark value. Furthermore, a general increasing trend in efficiency is clearly detectable (see also Tables 3 and 4).

Hence, all groups in the experiment are able to achieve a quite remarkable degree of self-organization, with a higher efficiency than the one attainable through purely random play. Recall that the highest degree of efficiency would be obtained in correspondence of any of the pure strategy Nash equilibria, whereby players repeatedly choose the same side. However, as such equilibria are payoff-asymmetric, they were in fact never observed.

Given the high degree of allocative efficiency obtained by all groups, and hence the high level of dynamic coordination between players, one might ask whether such coordination produces 'predictable' patterns in the series of aggregate outcomes. In fact, such a good level of coordination could be produced by the players' use of repeated choice patterns that could reflect themselves into regularities at the aggregate level.

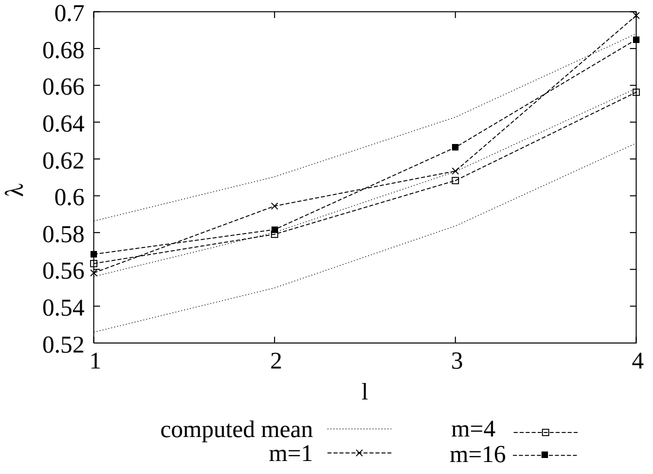

Fig. 1. Average informational efficiency for $m = 1, 4, 16$ computed with different values of l in the aggregate information case.

4.2. Informational efficiency

In order to investigate the issue, we introduce a measure of *informational efficiency*, which describes the amount of 'informational content' of the history of interaction. In fact, the presence of 'structure' in the time series would indicate the presence of some 'slack' in the players' coordination patterns that could in principle be exploited.

Note that the degree of informational efficiency of the resulting 'market' is an aggregate measure of closeness to the mixed strategy equilibrium solution. In fact, mixed strategy equilibrium behavior implies that the series of winning sides be a sequence of unrelated draws.

We introduce a measure of *informational efficiency* Λ defined as

$$\Lambda(l) = \sum_{h \in I_l} \max\{F(0|h), F(1|h)\} , \tag{3}$$

where I_l is the set of all the binary strings of length l and $N(i|h)$ with $i \in \{0, 1\}$ is the number of times i has been the winning side when h was the preceding history of length l (i.e. the string of the previous l winning sides). From its definition, $\Lambda(l)$ represents the average payoff of the (ex-post defined) best strategy of a hypothetical player who has access to the last l rounds of the history of play. It follows that $\Lambda(l)$ is a non-decreasing function of l.

The results for the informational efficiency are shown in Figs. 1 and 2 for the aggregate and full information treatments, respectively. We calculated the values of Λ for different values of the history length l. The dotted lines represent the 2 st. dev. band relative to the usual 'benchmark' case. Interestingly, as the figures show, irrespectively of the information and memory amounts, the observed informational efficiency departs

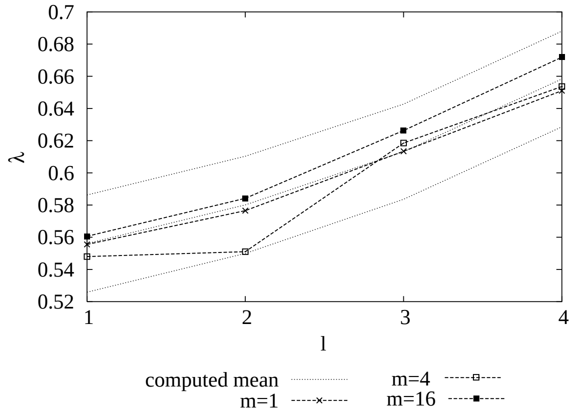

Fig. 2. Average informational efficiency for $m = 1, 4, 16$ computed with different values of l in the full information case.

from the ‘benchmark’ values for less than 1 st. dev., suggesting a substantial lack of structure in the outcome time series.

4.3. Analysis of payoffs distribution

The last aggregate analysis concerns the total payoff distribution over the players population in all treatments. The natural benchmark to which we compare the observed distribution is again the “random players” case where all the players involved choose randomly with equal probabilities between the two sides.

If N is the number of players the winning side can be of size $0, \ldots, (N-1)/2$ and the probability for any given player of belonging to a winning side of size k is k/n. Then the probability to be a winner can be computed as

$$p_{\mathrm{w}} = \sum_{h=0}^{(N-1)/2} 2 \binom{N}{h} \frac{h}{N} , \tag{4}$$

where the factor 2 accounts for the symmetry of the game, and the probability of winning k matches out of M is given by a binomial distribution

$$p(k; M, p_{\mathrm{w}}) = \binom{M}{k} p_{\mathrm{w}}^{k} (1 - p_{\mathrm{w}})^{M-k} . \tag{5}$$

In our case $N = 5$ and since we are concerned with the total payoff in each treatment, $M = 100$. Table 5 shows the mean and average payoff for all treatments and for the subpopulation obtained considering only treatments with a given m. The distributions so obtained are then compared with the theoretical distribution defined in Eq. (5) by using the Kolmogorov–Smirnov statistics. As can be seen, the null hypothesis of identity between the observed distributions and the theoretical prediction based on

Table 5
Payoff distribution analysis

	Aggregate Info.			Full Info.		
	Mean	St. dev.	K.S. test	Mean	St. dev.	K.S. test
All	32.6	6.67	0.039	33.15	6.41	0.008
$m = 1$	31.7	5.46	0.56	31.9	5.22	0.56
$m = 4$	33.7	9.13	0.016	32.75	4.89	0.11
$m = 16$	32.6	4.77	0.16	34.8	8.48	0.03
Theoretical	31.25	4.63				

purely random players can never be disproved with high significance, even if the variance of the observed payoffs seems larger especially in the $m = 4$ case.

5. Conclusion and outlooks

The main research question of this work was to investigate, first, the extent to which groups of human players are able to coordinate efficiently in a minority game, and, second, if and to what extent the amount of information available to them makes a difference in terms of efficiency. Our main results indicate that the degree of both allocative and informational efficiency achieved by our groups are remarkably high in all treatments when compared to the theoretical benchmark represented by the case of symmetric players following the mixed strategy equilibrium. The lack of a significant difference between treatments suggests that players only need minimal information to coordinate efficiently, and disclosing more information does not necessarily lead to improved aggregate performance.

An additional treatment whereby groups' compositions vary from round to round (random assignment) would allow to measure the extent to which the high level of coordination observed rests upon some forms of "self-organization", implying players learning over time to adapt to one another.

In addition, further research might involve a sound confrontation with experimental data from similar coordination games (e.g., [4–6] and references therein), tests of learning models and further simulations studies in order to identify the behavioral and institutional conditions that favor the emergence of spontaneous, decentralized coordination in this large class of interactive settings.

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