

Post-Lecture Question 6

(a)

$$\begin{aligned}\left(\frac{\partial P}{\partial V}\right)_T &= \frac{2aN^2}{V^3} - \frac{Nk_B T}{(V-Nb)^2} = 0 \\ \left(\frac{\partial^2 P}{\partial V^2}\right)_T &= -\frac{6aN^2}{V^4} + \frac{2Nk_B T}{(V-Nb)^3} = 0 \\ \begin{cases} 2aN^2(V_c - Nb)^2 = V^3 Nk_B T_c & (1) \\ 3aN^2(V_c - Nb)^3 = V^4 Nk_B T_c & (2) \end{cases}\end{aligned}$$

$$\begin{aligned}\frac{(2)}{(1)} &\Rightarrow V_c = \frac{3}{2}(V_c - Nb) \\ V_c &= 3Nb\end{aligned}$$

$$\begin{aligned}(1) &\Rightarrow 2aN^2 \cdot (2Nb)^2 = (3Nb)^3 Nk_B T_c \\ k_B T_c &= \frac{8a}{27b}\end{aligned}$$

$$\begin{aligned}P_c &= -\frac{aN^2}{V_c^2} + \frac{Nk_B T_c}{V_c - Nb} \\ &= \frac{a}{27b^2}\end{aligned}$$

(b)

$$\begin{aligned}P_r &= \frac{P}{T_c} = \frac{27b^2}{a} \left[-a \frac{N^2}{V_c^2 V_r^2} + \frac{Nk_B T_r T_c}{(V_r V_c - Nb)} \right] \\ &= -\frac{3}{V_r^2} + \frac{8T_r}{3V_r - 1}\end{aligned}$$

(c)

