Post-Lecture Question 6

(a)

$$\left(\frac{\partial P}{\partial V}\right)_T = \frac{2aN^2}{V^3} - \frac{Nk_BT}{(V-Nb)^2} = 0$$
$$\left(\frac{\partial^2 P}{\partial V^2}\right)_T = -\frac{6aN^2}{V^4} + \frac{2Nk_BT}{(V-Nb)^3} = 0$$

$$\begin{cases} 2aN^2(V_c - Nb)^2 = V^3 Nk_B T_c & (1) \\ 3aN^2(V_c - Nb)^3 = V^4 Nk_B T_c & (2) \end{cases}$$

$$\frac{(2)}{(1)} \quad \Rightarrow \quad V_c = \frac{3}{2}(V_c - Nb)$$
$$V_c = 3Nb$$

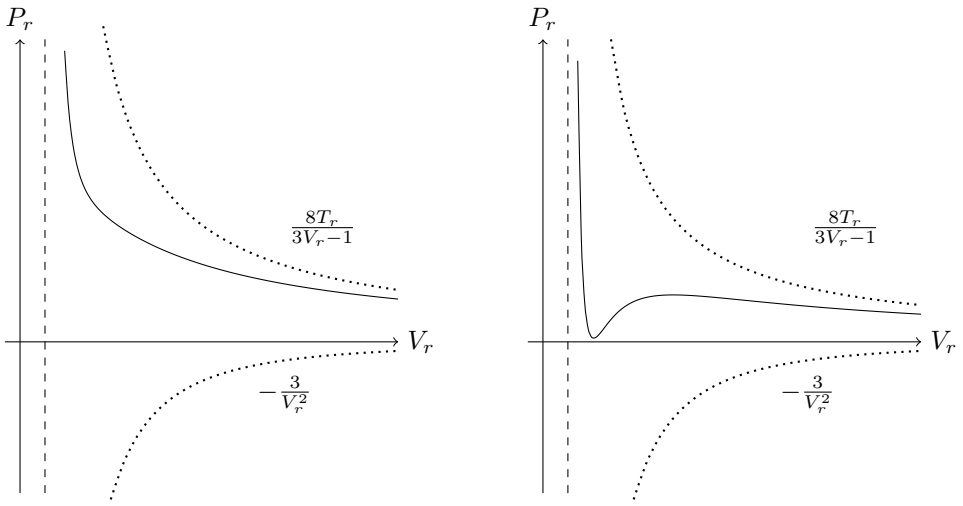
$$(1) \quad \Rightarrow \quad 2aN^2 \cdot (2Nb)^2 = (3Nb)^3 Nk_B T_c$$
$$k_B T_c = \frac{8a}{27b}$$

$$P_c = -\frac{aN^2}{V_c^2} + \frac{Nk_BT_c}{V_c - Nb}$$
$$= \frac{a}{27b^2}$$

(b)

$$P_r = \frac{P}{T_c} = \frac{27b^2}{a}\left[-a\frac{N^2}{V_c^2 V_r^2} + \frac{Nk_BT_rT_c}{(V_rV_c - Nb)}\right]$$
$$= -\frac{3}{V_r^2} + \frac{8T_r}{3V_r - 1}$$

(c)



(a) $T_r > 1$ (b) $T_r < 1$