

Post-Lecture Question 4

(a)

$$\begin{aligned} dE &= T dS - p dV + \mu dN \\ \Rightarrow dS &= \frac{1}{T} dE + \frac{P}{T} dV - \frac{\mu}{T} dN \end{aligned}$$

$$\begin{aligned} dS &= dS_\alpha + dS_\beta \\ &= \frac{1}{T_\alpha} dE_\alpha + \frac{P_\alpha}{T_\alpha} dV_\alpha - \frac{\mu_{\alpha 1}}{T_\alpha} dN_{\alpha 1} - \frac{\mu_{\alpha 2}}{T_\alpha} dN_{\alpha 2} \\ &\quad + \frac{1}{T_\beta} dE_\beta + \frac{P_\beta}{T_\beta} dV_\beta - \frac{\mu_{\beta 1}}{T_\beta} dN_{\beta 1} - \frac{\mu_{\beta 2}}{T_\beta} dN_{\beta 2} \\ &= 0 \end{aligned}$$

Have

$$\begin{aligned} dE_\alpha &= -dE_\beta \Rightarrow \left(\frac{1}{T_\alpha} - \frac{1}{T_\beta} \right) dE_\alpha = 0 \\ dE_\alpha &= -dE_\beta \Rightarrow \left(\frac{1}{T_\alpha} - \frac{1}{T_\beta} \right) dE_\alpha = 0 \\ dN_{\alpha 1} &= -dN_{\beta 1} \Rightarrow \frac{1}{T} (\mu_{\beta 1} - \mu_{\alpha 1}) dN_{\beta 1} = 0 \\ dN_{\alpha 2} &= -dN_{\beta 2} \Rightarrow \frac{1}{T} (\mu_{\beta 2} - \mu_{\alpha 2}) dN_{\beta 2} = 0 \end{aligned}$$

$$\Rightarrow \begin{cases} T_\alpha = T_\beta = T \\ P_\alpha = P_\beta = P \\ \mu_{\beta 1} = \mu_{\alpha 1} = \mu_1 \\ \mu_{\beta 2} = \mu_{\alpha 2} = \mu_2 \end{cases}$$

It is unnecessary that $\mu_1 = \mu_2$.

(b)

$$\begin{cases} T_\alpha = T_\beta = T_{\text{bath}} \\ P_\alpha = P_\beta = P_{\text{bath}} \\ \mu_{\beta 1} = \mu_{\alpha 1} \\ \mu_{\beta 2} = \mu_{\alpha 2} \end{cases}$$

(c)

$$dG = -S dT - V dP + \mu dN$$

$$\begin{aligned} \Delta G_1 &= \frac{1}{2} \left(\frac{\partial^2 G_1}{\partial N_{\alpha 1}^2} \right)_{N_2, T, P} (\Delta N_{\alpha 1})^2 + \frac{1}{2} \left(\frac{\partial^2 G_1}{\partial N_{\beta 1}^2} \right)_{N_2, T, P} (\Delta N_{\beta 1})^2 \\ &= \left(\frac{\partial^2 G_1}{\partial N_1^2} \right)_{N_2, T, P} (\Delta N_1)^2 \\ &= \left(\frac{\partial \mu_1}{\partial N_1} \right)_{N_2, T, P} (\Delta N_1)^2 \geq 0 \\ &\Rightarrow \left(\frac{\partial \mu_1}{\partial N_1} \right)_{N_2, T, P} \geq 0 \end{aligned}$$