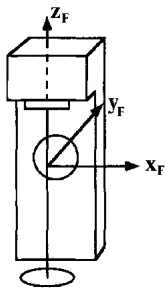


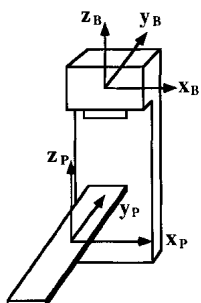
Fixed coordinate system



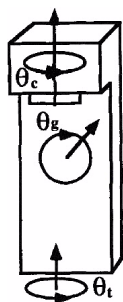
The Fixed coordinate system is a right-handed system centered at isocenter and fixed with respect to the machine.

RVP internal coordinate system

These coordinate systems are Cartesian right-handed and defined according to the conventions set by the IEC-61217 standard and ICRU Report 42. When all device and patient rotation angles are set to zero, all coordinate system Z-axes are vertically upward.

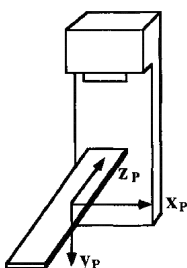


The Beam coordinate system is centered at the radiation source and fixed with respect to the collimator. The Patient coordinate system is centered at table lateral center, table top, and some patient-specific longitudinal landmark. It is fixed with respect to table/patient ensemble.



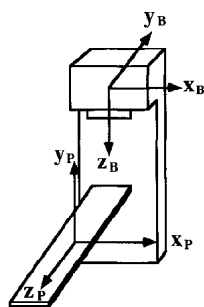
The gantry angle (θ_g) is clockwise around y_F . It is zero when the head is straight up as illustrated. The table angle (θ_t) is counterclockwise around z_F . It is zero when the table longitudinal axis is parallel to the y_F axis and the foot of the table is away from the gantry. The collimator angle (θ_c) is clockwise around z_B . It is zero when the collimator is at center of its excursion.

DICOM patient coordinate system

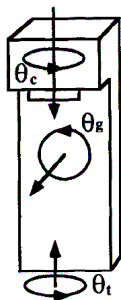


The DICOM Patient Coordinate System is related to an internal reference point in the patient. This reference point determines the origin of the Patient coordinate system and is, in general, set to the center of the central image of the patient CT study. The DICOM Patient Coordinate System is obtained by rotating the IEC Patient Coordinate System by 90 degrees counter-clockwise (in the negative direction) about the x-axis.

Patient to beam (PB) transformation coordinate system



The Beam coordinate system is a left-handed system centered at the radiation source and fixed with respect to the collimator. The Patient coordinate system is a right-handed system centered at table lateral center, table top, and some patient-specific longitudinal landmark. It is fixed with respect to table/patient ensemble.



The three machine rotations are defined as counterclockwise with respect to positive axes of corresponding coordinate systems. The gantry angle (θ_g) is counterclockwise around $-y_F$. It is zero when the head is straight up as illustrated. The table angle (θ_t) is counterclockwise around z_F . It is zero when the table longitudinal axis is parallel to the y_F axis and the foot of the table is away from the gantry. The collimator angle (θ_c) is counterclockwise around $-z_B$. It is zero when the collimator is at center of its excursion.

Coordinate system conversions in RVP

$$\text{DICOM} \rightarrow \text{RVP}: (x_p^{\text{RVP}}, y_p^{\text{RVP}}, z_p^{\text{RVP}}) = (x_p^{\text{DICOM}}, z_p^{\text{DICOM}}, -y_p^{\text{DICOM}})$$

$$\text{RVP} \rightarrow \text{PB}: (x_p^{\text{PB}}, y_p^{\text{PB}}, z_p^{\text{PB}}) = (x_p^{\text{RVP}}, z_p^{\text{RVP}}, -y_p^{\text{RVP}}), (\theta_g^{\text{PB}}, \theta_t^{\text{PB}}, \theta_c^{\text{PB}}) = (-\theta_g^{\text{RVP}}, \theta_t^{\text{RVP}}, \theta_c^{\text{RVP}})$$

$$\text{PB} \rightarrow \text{RVP}: (x_b^{\text{RVP}}, y_b^{\text{RVP}}, z_b^{\text{RVP}}) = (x_b^{\text{RVP}}, y_b^{\text{RVP}}, -z_b^{\text{RVP}})$$

Patient to beam coordinate transformation

Ref: Sherouse, G W. (1991) Coordinate transformation as a primary representation of radiotherapy beam geometry. *Med. Phys.*, 19, 175-179.

$$\mathbf{M}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\mathbf{I}_p[x] & -\mathbf{I}_p[y] & -\mathbf{I}_p[z] & 1 \end{bmatrix}$$

Representing translation along the three Patient system axes. This places the Beam coordinate origin at isocenter in Patient coordinates, \mathbf{I}_p

$$\mathbf{M}_1 = \mathbf{M}_0 \cdot \mathbf{R}_y(-\theta_t)$$

Representing table angle as a counterclockwise rotation of the patient around the +y axis

$$\mathbf{M}_2 = \mathbf{M}_1 \cdot \mathbf{R}_z(\theta_g)$$

Representing gantry angle as a counterclockwise rotation of the coordinate system around the +z axis

$$\mathbf{M}_3 = \mathbf{M}_2 \cdot \mathbf{R}_y(\theta_c)$$

Representing collimator angle as an additional counterclockwise rotation of the coordinate system around its +y axis

$$\mathbf{PB} = \mathbf{M}_3 \cdot \mathbf{T}([0 \quad -SAD \quad 0]) \cdot \mathbf{R}_x\left(-\frac{\pi}{2}\right) \cdot \mathbf{S}([1 \quad 1 \quad -1])$$

Patient to beam transform is formed by translation of the origin from isocenter to the source position, rotation to properly orient the y and z axes, and scaling the z axis by -1 to achieve the left-handed Beam system.

$$\mathbf{BP} = \mathbf{PB}^{-1}$$

Beam to Patient transformed is formed by inverting the Patient to Beam transform

Transformation operators

$$\mathbf{T}(\mathbf{t}) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \mathbf{t}[x] & \mathbf{t}[y] & \mathbf{t}[z] & 1 \end{bmatrix}$$

Translation operator

Note that a translation of the coordinate system is equivalent to a negative translation of the objects in the system.

$$\mathbf{R}_x(\theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\theta) & -\sin(\theta) & 0 \\ 0 & \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rotation operators

θ is positive in the clockwise direction.

$$\mathbf{R}_y(\theta) = \begin{bmatrix} \cos(\theta) & 0 & \sin(\theta) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Note that a counterclockwise rotation of a coordinate system is equivalent to a clockwise rotation of the objects in the coordinate system.

$$\mathbf{R}_z(\theta) = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 & 0 \\ \sin(\theta) & \cos(\theta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{S}(\mathbf{s}) = \begin{bmatrix} \mathbf{s}[x] & 0 & 0 & 0 \\ 0 & \mathbf{s}[y] & 0 & 0 \\ 0 & 0 & \mathbf{s}[z] & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Scale operator

Solution

$$\mathbf{M}_1 = \mathbf{M}_0 \cdot \mathbf{R}_y(-\theta_i) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\mathbf{I}_p[x] & -\mathbf{I}_p[y] & -\mathbf{I}_p[z] & 1 \end{bmatrix} \begin{bmatrix} \cos(\theta_i) & 0 & -\sin(\theta_i) & 0 \\ 0 & 1 & 0 & 0 \\ \sin(\theta_i) & 0 & \cos(\theta_i) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos(\theta_i) & 0 & -\sin(\theta_i) & 0 \\ 0 & 1 & 0 & 0 \\ \sin(\theta_i) & 0 & \cos(\theta_i) & 0 \\ -\mathbf{I}_p[x] \cdot \cos(\theta_i) - \mathbf{I}_p[z] \cdot \sin(\theta_i) & -\mathbf{I}_p[y] & \mathbf{I}_p[x] \cdot \sin(\theta_i) - \mathbf{I}_p[z] \cdot \cos(\theta_i) & 1 \end{bmatrix}$$

$$\mathbf{M}_2 = \mathbf{M}_1 \cdot \mathbf{R}_z(\theta_g) = \mathbf{M}_1 \cdot \begin{bmatrix} \cos(\theta_g) & -\sin(\theta_g) & 0 & 0 \\ \sin(\theta_g) & \cos(\theta_g) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos(\theta_g) \cdot \cos(\theta_i) & -\sin(\theta_g) \cdot \cos(\theta_i) & -\sin(\theta_i) & 0 \\ \sin(\theta_g) & \cos(\theta_g) & 0 & 0 \\ \cos(\theta_g) \cdot \sin(\theta_i) & -\sin(\theta_g) \cdot \sin(\theta_i) & \cos(\theta_i) & 0 \\ -\mathbf{I}_p[x] \cdot \cos(\theta_g) \cdot \cos(\theta_i) - \mathbf{I}_p[y] \cdot \sin(\theta_g) \cdot \cos(\theta_i) - \mathbf{I}_p[z] \cdot \cos(\theta_g) \cdot \sin(\theta_i) & \mathbf{I}_p[x] \cdot \sin(\theta_g) \cdot \cos(\theta_i) - \mathbf{I}_p[y] \cdot \cos(\theta_g) + \mathbf{I}_p[z] \cdot \sin(\theta_g) \cdot \sin(\theta_i) & \mathbf{I}_p[x] \cdot \sin(\theta_i) - \mathbf{I}_p[z] \cdot \cos(\theta_i) & 1 \end{bmatrix}$$

$$\mathbf{M}_3 = \mathbf{M}_2 \cdot \mathbf{R}_y(\theta_c) = \mathbf{M}_2 \cdot \begin{bmatrix} \cos(\theta_c) & 0 & \sin(\theta_c) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(\theta_c) & 0 & \cos(\theta_c) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \left\{ \cos(\theta_c) \cdot \cos(\theta_g) \cdot \cos(\theta_i) + \sin(\theta_g) \cdot \sin(\theta_i) \right\} & -\sin(\theta_g) \cdot \cos(\theta_i) & \left\{ \sin(\theta_c) \cdot \cos(\theta_g) \cdot \cos(\theta_i) - \cos(\theta_c) \cdot \sin(\theta_i) \right\} & 0 \\ \cos(\theta_c) \cdot \sin(\theta_g) & \cos(\theta_g) & \sin(\theta_c) \cdot \sin(\theta_g) & 0 \\ \left\{ \cos(\theta_c) \cdot \cos(\theta_g) \cdot \sin(\theta_i) - \sin(\theta_c) \cdot \cos(\theta_i) \right\} & -\sin(\theta_g) \cdot \sin(\theta_i) & \left\{ \cos(\theta_c) \cdot \cos(\theta_g) + \sin(\theta_c) \cdot \cos(\theta_g) \cdot \sin(\theta_i) \right\} & 0 \\ \left\{ \mathbf{I}_p[x] \cdot (-\cos(\theta_g) \cdot \cos(\theta_c) \cdot \cos(\theta_i) - \sin(\theta_c) \cdot \sin(\theta_i)) - \mathbf{I}_p[y] \cdot \cos(\theta_c) \cdot \sin(\theta_g) + \mathbf{I}_p[z] \cdot (\sin(\theta_c) \cdot \cos(\theta_i) - \cos(\theta_c) \cdot \cos(\theta_g) \cdot \sin(\theta_i)) \right\} & \left\{ \mathbf{I}_p[x] \cdot \sin(\theta_g) \cdot \cos(\theta_i) - \mathbf{I}_p[y] \cdot \cos(\theta_c) - \mathbf{I}_p[z] \cdot \sin(\theta_g) \cdot \sin(\theta_i) \right\} & \left\{ \mathbf{I}_p[x] \cdot (\cos(\theta_c) \cdot \cos(\theta_i) - \sin(\theta_c) \cdot \cos(\theta_g) \cdot \cos(\theta_i)) - \mathbf{I}_p[y] \cdot \sin(\theta_c) \cdot \sin(\theta_g) + \mathbf{I}_p[z] \cdot (-\cos(\theta_c) \cdot \cos(\theta_i) - \sin(\theta_c) \cdot \cos(\theta_g) \cdot \sin(\theta_i)) \right\} & 1 \end{bmatrix}$$

$$\mathbf{PB} = \mathbf{M}_3 \cdot \mathbf{T}(0 \quad -SAD \quad 0) \cdot \mathbf{R}_x\left(-\frac{\pi}{2}\right) \mathbf{S}(1 \quad 1 \quad -1) =$$

$$= \mathbf{M}_3 \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -SAD & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\left(\frac{\pi}{2}\right) & \sin\left(\frac{\pi}{2}\right) \\ 0 & -\sin\left(\frac{\pi}{2}\right) & \cos\left(\frac{\pi}{2}\right) \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

$$= \begin{bmatrix} \left\{ \cos(\theta_c) \cdot \cos(\theta_g) \cdot \cos(\theta_i) + \sin(\theta_g) \cdot \sin(\theta_i) \right\} & -\sin(\theta_g) \cdot \cos(\theta_i) & \left\{ \sin(\theta_c) \cdot \cos(\theta_g) \cdot \cos(\theta_i) - \cos(\theta_c) \cdot \sin(\theta_i) \right\} & 0 \\ \cos(\theta_c) \cdot \sin(\theta_g) & \cos(\theta_g) & \sin(\theta_c) \cdot \sin(\theta_g) & 0 \\ \left\{ \cos(\theta_c) \cdot \cos(\theta_g) \cdot \sin(\theta_i) - \sin(\theta_c) \cdot \cos(\theta_i) \right\} & -\sin(\theta_g) \cdot \sin(\theta_i) & \left\{ \cos(\theta_c) \cdot \cos(\theta_g) + \sin(\theta_c) \cdot \cos(\theta_g) \cdot \sin(\theta_i) \right\} & 0 \\ \left\{ \mathbf{I}_p[x] \cdot (-\cos(\theta_c) \cdot \cos(\theta_g) \cdot \cos(\theta_i) - \sin(\theta_c) \cdot \sin(\theta_i)) - \mathbf{I}_p[y] \cdot \cos(\theta_c) \cdot \sin(\theta_g) + \mathbf{I}_p[z] \cdot (\sin(\theta_c) \cdot \cos(\theta_i) - \cos(\theta_c) \cdot \cos(\theta_g) \cdot \sin(\theta_i)) \right\} & \left\{ \mathbf{I}_p[x] \cdot \sin(\theta_g) \cdot \cos(\theta_i) - \mathbf{I}_p[y] \cdot \cos(\theta_c) - \mathbf{I}_p[z] \cdot \sin(\theta_g) \cdot \sin(\theta_i) \right\} & \left\{ \mathbf{I}_p[x] \cdot (\cos(\theta_c) \cdot \cos(\theta_i) - \sin(\theta_c) \cdot \cos(\theta_g) \cdot \cos(\theta_i)) - \mathbf{I}_p[y] \cdot \sin(\theta_c) \cdot \sin(\theta_g) + \mathbf{I}_p[z] \cdot (-\cos(\theta_c) \cdot \cos(\theta_i) - \sin(\theta_c) \cdot \cos(\theta_g) \cdot \sin(\theta_i)) \right\} & 1 \end{bmatrix}$$

$$\mathbf{C}_B = \begin{bmatrix} \left[\begin{aligned} &(\mathbf{C}_p[x] - \mathbf{I}_p[x]) \cdot (\cos(\theta_c) \cdot \cos(\theta_g) \cdot \cos(\theta_t) + \sin(\theta_c) \cdot \sin(\theta_t)) + \\ &+ (\mathbf{C}_p[y] - \mathbf{I}_p[y]) \cdot \cos(\theta_c) \cdot \sin(\theta_g) + \\ &+ (\mathbf{C}_p[z] - \mathbf{I}_p[z]) \cdot (\cos(\theta_c) \cdot \cos(\theta_g) \cdot \sin(\theta_t) - \sin(\theta_c) \cdot \cos(\theta_t)) \end{aligned} \right] \\ \\ \left[\begin{aligned} &(\mathbf{C}_p[x] - \mathbf{I}_p[x]) \cdot (\cos(\theta_c) \cdot \sin(\theta_t) \cdot \sin\left(\frac{\pi}{2}\right) - \sin(\theta_c) \cdot \cos(\theta_g) \cdot \cos(\theta_t) \cdot \sin\left(\frac{\pi}{2}\right) - \sin(\theta_g) \cdot \cos(\theta_t) \cdot \cos\left(\frac{\pi}{2}\right)) + \\ &+ (\mathbf{C}_p[y] - \mathbf{I}_p[y]) \cdot (\cos(\theta_g) \cdot \cos\left(\frac{\pi}{2}\right) - \sin(\theta_c) \cdot \sin(\theta_g) \cdot \sin\left(\frac{\pi}{2}\right)) + \\ &+ (\mathbf{C}_p[z] - \mathbf{I}_p[z]) \cdot (-\cos(\theta_c) \cdot \cos(\theta_t) \cdot \sin\left(\frac{\pi}{2}\right) - \sin(\theta_c) \cdot \cos(\theta_g) \cdot \sin(\theta_t) \cdot \sin\left(\frac{\pi}{2}\right) - \sin(\theta_g) \cdot \sin(\theta_t) \cdot \cos\left(\frac{\pi}{2}\right)) - \\ &- SAD \cdot \cos\left(\frac{\pi}{2}\right) \end{aligned} \right] \\ \\ \left[\begin{aligned} &(\mathbf{C}_p[x] - \mathbf{I}_p[x]) \cdot (\cos(\theta_c) \cdot \sin(\theta_t) \cdot \cos\left(\frac{\pi}{2}\right) - (\sin(\theta_c) \cdot \cos(\theta_g) \cdot \cos\left(\frac{\pi}{2}\right) - \sin(\theta_g) \cdot \sin\left(\frac{\pi}{2}\right)) \cdot \cos(\theta_t)) - \\ &- (\mathbf{C}_p[y] - \mathbf{I}_p[y]) \cdot (\sin(\theta_c) \cdot \sin(\theta_g) \cdot \cos\left(\frac{\pi}{2}\right) + \cos(\theta_g) \cdot \sin\left(\frac{\pi}{2}\right)) - \\ &- (\mathbf{C}_p[z] - \mathbf{I}_p[z]) \cdot (\cos(\theta_c) \cdot \cos(\theta_t) \cdot \cos\left(\frac{\pi}{2}\right) + (\sin(\theta_c) \cdot \cos(\theta_g) \cdot \cos\left(\frac{\pi}{2}\right) - \sin(\theta_g) \cdot \sin\left(\frac{\pi}{2}\right)) \cdot \sin(\theta_t)) + \\ &+ SAD \cdot \sin\left(\frac{\pi}{2}\right) \end{aligned} \right] \end{bmatrix}$$