Note to grader: the standard notation for "the ball of radius r around the point x in the metric space X is $B_r(x)$. This is terrible; I use the notation $X_r(x)$ to denote "the ball of radius r around the point x in the metric space X, as it is better.

Problem 7a, p111:

Let $f : \mathbb{R} \to \mathbb{R}$ be continuous from the right. Consider $f : \mathbb{R}_\ell \to \mathbb{R}$. Then, by definition, $\forall a \in \mathbb{R} \forall \epsilon > 0 \exists \delta > 0 : |x - a| < \delta$ and $a > x \implies |f(x) - f(a)| < \epsilon$.

Now, let W be open in $\mathbb{R}$. Then for each $a \in W$, there is an interval $I \subset W$ containing a. Consider $f^{-1}(I)$; (This is open, prove it.).

That is, for each $a \in W$, there is a set, U, open in $\mathbb{R}_\ell$ with $a \in U \subset f^{-1}(W)$. Thus, for each $a' \in f^{-1}(W)$, there is a set, U, open in $\mathbb{R}_\ell$ with $a \in U \subset f^{-1}(W)$. Thus, $f^{-1}(W)$ is open.

That is, for all W open in $\mathbb{R}$, $f^{-1}(W)$ is open in $\mathbb{R}_\ell$; so $f : \mathbb{R}_\ell \to \mathbb{R}$ is continuous.

Problem 13, p112:

Problem 2, p118:

Problem 3, p118:

Problem 6, p118:

Problem 7, p118:

Problem 3b, p126:

Problem 4b, p126:

Problem A:

Let X be a metric space, and let A be a countable subset of X with $\overline{A} = X$.

Consider the collection $\mathcal{C} = \{X_r(x) : x \in A, r \in \mathbb{Q}\}$. Then $\mathcal{C}$ is countable; it's a countable union of countable sets.

Next, note that $\bigcup\limits_{C \in \mathcal{C}} C = X$;

Problem B:

Problem C, part i:

Consider $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = x^2$. Then $f(\{1\}) = \{1\}$, so that $f^{-1}(f(\{1\})) = \{-1, 1\}$, so that $f^{-1}(f(\{1\})) \neq \{1\}$.

That is, $f^{-1}(f(A)) = A$ isn't always true.

Problem C, part ii: