Problem 11:

Let Ω be a domain with a C^2 boundary $\partial\Omega$.

Pick a point, x, on $\partial\Omega$. Without loss of generality, say that $x = 0$.

Then on some neighborhood, U, of 0, $\partial\Omega$ is the graph of a C^2 function, γ, upon reorienting/relabeling coordinate axes. Without loss of generality, we can require that $\nabla_u \gamma = 0$ for all $u = (u_1, u_2 \dots u_{n-1}, 0)$ and that $\{x \in \mathbb{R}^n : x_i = 0 \text{ if } i \neq n, 0 < x_n < \epsilon\}$ lies outside of Ω for some $\epsilon > 0$. (This is geometrically clear.)

Now, γ has a continuous second derivative where $\gamma(x') = 0$ (without loss of generality, we can assume that $x' = 0$); that is, there is a $\delta > 0$ with γ having a continuous second derivative on $B(0, \delta)$. So, γ has a bounded second derivative on $B(0, \delta)$; say that M is a bound for the second derivative on $B(0, \delta)$.

Fix $u = (u_1, u_2 \dots u_{n-1}, 0)$, with $\|u\| = 1$. Consider $\gamma|_{x:x=cu}$. Then

Problem 12: