Problem 2:

Let u be as described.

Then there is a $B_r(x)$ (with $x \in \Omega$) such that $B_r(x) \cap \partial\Omega \subset A$ and $B_r(x) \setminus \overline{\Omega} \neq \emptyset$.

Define $v : B_r(x) \cup \Omega \to \mathbb{R}$ by $v(x) = u(x)$ for $x \in \Omega$, else $x = 0$.

Then v is harmonic on $B_r(x) \cup \Omega$: it is clear that v is harmonic on Ω and on $B_r(x) \setminus \overline{\Omega}$. Next, on $\Omega \cap B_r(x)$, we have that v is continuous (as $v(x) \to 0$ as $x \to \Omega \cap B_r(x)$), that v is differentiable (as $\frac{\partial v}{\partial \nu}(x) \to 0$ as $x \to \Omega \cap B_r(x)$), and that v has a continuous second derivative (...reasons). So, $\Delta v = 0$ on $\Omega \cap B_r(x)$, because (limits).

So, we have that v is harmonic on $B_r(x) \cup \Omega$ and that v vanishes on some nonempty open subset of $B_r(x) \cup \Omega$; thus, v vanishes identically.

So u vanishes identically.

Problem 3:

Let G be the Green's function for Ω, a bounded domain.

Part a:

Consider $H(x, y) = G(x, y) - G(y, x)$. Then for all u harmonic on Ω, we have

$$u(x) = \int\limits_{\partial\Omega} u(y)\frac{\partial G(x, y)}{\partial \nu} - G(x, y)\frac{\partial u(y)}{\partial \nu} dS_y$$

Part b:

Part c: