Problem 1 (Problem 2 in book):

Let u be such that $\Delta u = 0$ and let v be such that $v(x) = u(Ox)$ for some orthogonal $n \times n$ matrix O.

Then we have:

$$\begin{aligned}\Delta v(x) &= \Delta u(x) \\ &= \sum_{i=1}^{n} u_{x_i x_i}(Ox) \\ &= \sum_{i=1}^{n} u_{x_i x_i}(x) \\ &= 0\end{aligned}$$

Problem 2 (Problem 3 in book):

Let u be such that

$$\begin{cases} -\Delta u = f & \text{in } B(0,r) \\ u = g & \text{on } \partial B(0,r) \end{cases}$$

with dimension $n \geq 3$.

Then

Thus,

$$u(0) = \fint_{\partial B(0,r)} g dS + \int_{B(0,r)} \left(\frac{1}{|x|^{n-2}} - \frac{1}{r^{n-2}} \right) f dx$$

Problem 3 (Only homework on page):

Let $u(x)$ be a C^2 solution to

$$\Delta u(x) = |x|^2 \text{ on } \mathbb{R}^n$$

Set $m(r) = \fint_{\partial B(0,r)} u(y) dS(y)$.