Problem 1:
Let $f_n \to f$ in measure.

Problem 2:
Let f be continuous on $[a, b]$, with one of its derivates everywhere non-negative on (a, b).

If this derivate is D_+, then for all $x \in [a, b]$, $\liminf\limits_{h \to 0^+} \frac{f(x+h)-f(x)}{h} \geq 0$.

If this derivate is D_-, then for all $x \in [a, b]$, $\liminf\limits_{h \to 0^+} \frac{f(x)-f(x+h)}{h} \geq 0$.

If this derivate is D^+, then for all $x \in [a, b]$, $\limsup\limits_{h \to 0^+} \frac{f(x+h)-f(x)}{h} \geq 0$.

If this derivate is D^-, then for all $x \in [a, b]$, $\limsup\limits_{h \to 0^+} \frac{f(x)-f(x+h)}{h} \geq 0$.

So in all cases, f is nondecreasing.

Problem 3:
Suppose that $f_n(x) \to f(x)$ at each $x \in [a, b]$.

Problem 4:
Suppose that $f \in BV([a, b])$. Then f' exists, by a theorem in class.

Problem 5:
Let g be an absolutely continuous monotone function on $[0, 1]$, and E be a set of measure 0.

We know that g is the antiderivative of some function, f. That is, $g(x) = \int\limits_0^x f(t)dt + g(0)$. for some f.

Problem 6:

Let f be a nonnegative measurable function on $[0,1]$.

We know that ln is a concave function on $[0,1]$ (if this is not clear, it's the inverse of a convex function).

So $-\ln$ is a convex function on $[0,1]$.

So Jensen's inequality applies:

$$-\ln \int f \leq -\int \ln f$$
$$\ln \int f \geq \int \ln f$$

This satisfies the problem.