Problem 1:

(Prove: The real 2x2 matrix blah represents a complex-linear map iff a=d, c=-b)

Let the real matrix $A =$ represent a complex-linear map. Then

Now, let the real matrix A have that $a = d$ and $c = -b$. Write $A =$.

Let $z = x + iy \in \mathbb{C}$, and define the vector $\vec{z} =$ Then:

$$\begin{aligned} A\vec{z} &= stuff \\ &= work \\ &= z \end{aligned}$$

Problem 2:

Consider $\int\limits_{|z|=R} \overline{z}^n dz$. Define $\alpha : [0, 2\pi] \to \mathbb{C}$ by $\alpha(t) = R(\cos(t) + i\sin(t))$.

If $n \neq 1$, we have

$$\begin{aligned} \int\limits_{|z|=R} \overline{z}^n dz &= \int\limits_{\alpha} \overline{z}^n dz \\ &= \int\limits_0^{2\pi} [R(\cos(t) - i\sin(t))]^n [R(-\sin(t) + i\cos(t))] dt \\ &= R^{n+1} \int\limits_0^{2\pi} \frac{-\sin(t) + i\cos(t)}{(\cos(t) + i\sin(t))^n} dt \\ &= R^{n+1} \frac{1}{n-1} [((\cos(2\pi) + i\sin(2\pi)))^{1-n} - (\cos(0) + i\sin(0))^{1-n} \\ &= R^{n+1} \frac{1}{n-1} [1 - 1] \\ &= 0 \end{aligned}$$

(Here, we're using freely the fact that $\overline{z} = 1/z$ if $|z| = 1$, and we gloss over the u-substitution with $u = \cos(t) + i\sin(t)$.)

And if $n = 1$, we have

Problem 3: