Let $g \in C^1(\mathbb{R})$.

Problem 1:

Consider $u_t - 2u_x = x$ for $x \in \mathbb{R}$, $t > 0$, with $u(x, 0) = g(x)$.

Fix $x \in \mathbb{R}, t > 0$. Define $z(s) = u(x + 2s, t + s)$. Then $\frac{d}{ds}z(s) = 2u_x(x + 2s, t + s) + u_t(x + 2s, t + s) = x + 2s$.

Also, $u(x, t) - g(x + 2t) = u(x, t) - u(x + 2t, 0) = z(0) - z(-t) = \int_{-t}^{0} \frac{d}{ds}z(s)ds = \int_{0}^{t}(x + 2(s - t))ds$.

Therefore, $u(x, t) = g(x+2t)+\int_{0}^{t}(x+2(s-t))ds = g(x+2t)+xt+t^2-2t^2 = g(x + 2t) + xt - t^2$.

Problem 2:

Consider $u_t - 2u_x = u$ for $x \in \mathbb{R}$, $t > 0$, with $u(x, 0) = g(x)$.

Fix $x \in \mathbb{R}, t > 0$. Define $z(s) = u(x + 2s, t + s)$. Then $\frac{d}{ds}z(s) = 2u_x(x + 2s, t + s) + u_t(x + 2s, t + s) = u(x + 2s, t + s) = z(s)$. That is, $z(s) = \frac{d}{ds}z(s)$, so $z(s) = Ce^s$ for some $C \in \mathbb{R}$ (this is elementary differential equations). Because $z(-t) = u(x - 2t, 0) = g(x - 2t)$, we determine that $z(s) = g(x - 2t)e^te^s$.

Also, $u(x, t) - g(x - 2t) = u(x, t) - u(x + 2t, 0) = z(0) - z(-t) = g(x - 2t)e^t - g(x - 2t)$.

So $u(x, t) = g(x - 2t)e^t$.

Problem 3:

Consider $u_t - 2u_x = u^2$ for $x \in \mathbb{R}$, $t > 0$, with $u(x, 0) = g(x)$.

Fix $x \in \mathbb{R}, t > 0$. Define $z(s) = u(x + 2s, t + s)$. Then $\frac{d}{ds}z(s) = 2u_x(x + 2s, t+s)+u_t(x+2s, t+s) = (u(x+2s, t+s))^2 = z(s)^2$. That is, $z(s)^2 = \frac{d}{ds}z(s)$, so $z(s) = \frac{-1}{s+C}$ for some $C \in \mathbb{R}$ (this is elementary differential equations). Because $z(-t) = u(x - 2t, 0) = g(x - 2t)$, we determine that $C = \frac{-1}{g(x-2t)} - t$ so that $z(s) = \frac{-1}{s-t+\frac{-1}{g(x-2t)}}$.

Also, $u(x, t)-g(x-2t) = z(0)-z(-t)u(x, t)-u(x+2t, 0) = z(0)-z(-t) = \frac{-1}{-t+\frac{-1}{g(x-2t)}} - \frac{-1}{-2t+\frac{-1}{g(x-2t)}}$.

By simplifying, $u(x, t) = g(x - 2t) + \frac{-1}{-t+\frac{-1}{g(x-2t)}} - \frac{-1}{-2t+\frac{-1}{g(x-2t)}} = g(x - 2t) +$

$\frac{t}{2t^2+3\frac{t}{g(x-2t)}+\frac{1}{g(x-2t)^2}} = g(x-2t) + \frac{tg(x-2t)^2}{2t^2g(x-2t)^2+3tg(x-2t)+1}$.