Problem 1:

Let $f \in \mathcal{O}(\mathbb{C})$. Then consider $\int\limits_{|z|=2} \frac{f(z)}{z-1} dz$.

Note that $\{z : |z| = 2\}$ is the boundary of the open disc of radius 2, and that 1 is a point in this disc. Thus, Cauchy's formula applies; $f(1) = 1/(2\pi i) \int\limits_{|z|=2} \frac{f(z)}{z-1} dz$, so $\int\limits_{|z|=2} \frac{f(z)}{z-1} dz = 2\pi i f(1)$.

Problem 2:

Let $f \in \mathcal{O}(\mathbb{C})$. Then consider $\int\limits_{|z|=2} \frac{f(z)}{z^2-1} dz$.

Note that $\int\limits_{|z|=2} \frac{f(z)}{z^2-1} dz = \int\limits_{|z|=2} \frac{f(z)}{(z+1)(z-1)} dz$. Now, the function $f/(z+1)$ is holomorphic except at -1, and the function $f/(z-1)$ is holomorphic except at 1. So, we

Problem 3:

If Ω is an open set, f is holomorphic on some open set containing Ω's closure, and $w \notin \Omega$, then $\int\limits_{\partial\Omega} \frac{f(z)}{z-w} dz$ vanishes; $\frac{f(z)}{z-w}$ is a product of two holomorphic functions and is thus holomorphic, so the integral vanishes by the theorem we use to prove Cauchy's Formula.

Problem 4: