Problem 1:

Consider $f : \mathbb{C} \to \mathbb{C}$ given by $z \mapsto z^2 \sin(1/z)$.

The "proof" that f is differentiable at 0 relies on this line:

$$\left|\frac{f(h) - f(0)}{h - 0}\right| = \left|\frac{h^2 \sin(1/h)}{h}\right| = |h \sin(1/h)| \leq |h|$$

The last step relies on the fact that $|\sin(x)| \leq 1$, which is not true in $\mathbb{C}$; this is where the "proof" fails.

Problem 2:

Part i:

Consider $[b, a]$. Then for all continuous functions f, $\int\limits_{[b,a]} f(z)dz = \int\limits_0^1 f(\gamma(t))\gamma'(t)dt = -\int\limits_1^0 f(\gamma(t))\gamma'(t)dt = -\int\limits_{[a,b]} f(z)dz$, where $\gamma : [0, 1] \to \mathbb{C}$ is given by $t \mapsto tb + (1 - t)a$. That is, $[b, a] = \dot{-}[a, b]$.

Part ii:

Let $\gamma_1 : [0, 1] \to \mathbb{C}$ be a smooth curve in the plane. We can define γ_2 so that $\gamma_2 = \dot{-}\gamma_1$ by choosing $\gamma_2(t) = \gamma_1(1 - t)$.

Problem 3:

Let $p \in [a, b]$. Then there is a $T \in [0, 1]$ such that $p = aT + (1 - T)b$. Now, for all continuous functions f,:

$$\begin{aligned}
\int\limits_{[a,b]} f(z)dz &= \int\limits_0^1 f(\gamma(t))\gamma'(t)dt \\
&= \int\limits_0^T f(\gamma(t))\gamma'(t)dt + \int\limits_T^1 f(\gamma(t))\gamma'(t)dt \\
&= \int\limits_{[a,p]} f(z)dz + \int\limits_{[p,b]} f(z)dz
\end{aligned}$$

That is, $[a, b] = [a, p]\dot{+}[p, b]$.

Problem 4:

Let T be a triangle, with vertices a, b, c. Then

$$\begin{aligned}
\partial T &= [a,b]\dot{+}[b,c]\dot{+}[c,a] \\
&= [a,p]\dot{+}[p,b]\dot{+}[b,q]\dot{+}[q,c]\dot{+}[c,r]\dot{+}[r,a] \\
&= [a,p]\dot{+}[p,b]\dot{+}[b,q]\dot{+}[q,c]\dot{+}[c,r]\dot{+}[r,a]\dot{+}[p,q]\dot{-}[p,q]\dot{+}[q,r]\dot{-}[q,r]\dot{+}[r,p]\dot{-}[r,p] \\
&= [a,p]\dot{+}[p,b]\dot{+}[b,q]\dot{+}[q,c]\dot{+}[c,r]\dot{+}[r,a]\dot{+}[p,q]\dot{+}[q,p]\dot{+}[q,r]\dot{+}[r,q]\dot{+}[r,p]\dot{+}[p,r] \\
&= [a,p]\dot{+}[p,r]\dot{+}[r,a]\dot{+}[b,q]\dot{+}[q,p]\dot{+}[p,b]\dot{+}[q,c]\dot{+}[c,r]\dot{+}[r,q]\dot{+}[p,q]\dot{+}[q,r]\dot{+}[r,p] \\
&= \partial T_1\dot{+}\partial T_2\dot{+}\partial T_3\dot{+}\partial T_4
\end{aligned}$$

The above isn't very illuminating. You really should be looking at the picture while doing this proof.

Problem 5:

Problem 6: