Problem 1:

Find a representation for the linear functionals on ℓ^p, where ℓ^p consists of sequences $\langle x_n \rangle$ of real numbers such that

$$(\sum |x_n|^p)^{1/p} < \infty$$

Problem 2:

Let $f \in L^p$, and let $T_\Delta(f)$ denote the Δ-approximant of f. Prove that

$$\|T_\Delta(f)\|_p \leq \|f\|_p$$

Problem 3:

Prove that ℓ^p, $1 \leq p < \infty$, and L^∞ are complete.

Problem 4:

Let ℓ^∞ denote the set of all bounded sequences of real numbers. Set $\|(x_n)\|_\infty = \sup |x_n|$. Prove that this is a norm, and ℓ^∞ is a Banach Space.

Problem 5:

Prove the Minkowski inequality for $0 < p < 1$.

Problem 6:

Young's inequality states that if $a, b \geq 0$, $1 < p < \infty$, and $1/p + 1/q = 1$, then

$$ab \leq a^p/p + b^q/q$$

Prove the Holder inequality using this.

Problem 7:
I cannot pick up your dry cleaning.
I don't have a car, as I am too poor to afford one.