Let $g \in C^1(\mathbb{R})$.

Problem 1:

Consider $u_t - 2u_x = x$ for $x \in \mathbb{R}$, $t > 0$, with $u(x,0) = g(x)$.

Fix $x \in \mathbb{R}, t > 0$. Define $z(s) = u(x+2s, t+s)$. Then $\frac{d}{ds}z(s) = 2u_x(x+2s, t+s) + u_t(x+2s, t+s) = x + 2s$.

Also, $u(x,t) - g(x+2t) = u(x,t) - u(x+2t, 0) = z(0) - z(-t) = \int\limits_{-t}^{0} \frac{d}{ds}z(s)ds = \int\limits_{0}^{t}(x + 2(s-t))ds$.

Therefore, $u(x,t) = g(x+2t) + \int\limits_{0}^{t}(x+2(s-t))ds = g(x+2t) + xt + t^2 - 2t^2 = g(x+2t) + xt - t^2$.

Problem 2:

Consider $u_t - 2u_x = u$ for $x \in \mathbb{R}$, $t > 0$, with $u(x,0) = g(x)$.

Fix $x \in \mathbb{R}, t > 0$. Define $z(s) = u(x+2s, t+s)$. Then $\frac{d}{ds}z(s) = 2u_x(x+2s, t+s) + u_t(x+2s, t+s) = u(x+2s, t+s) = z(s)$. That is, $z(s) = \frac{d}{ds}z(s)$, so $z(s) = Ce^s$ for some $C \in \mathbb{R}$ (this is elementary differential equations). Because $z(-t) = u(x-2t, 0) = g(x-2t)$, we determine that $z(s) = g(x-2t)e^t e^s$.

Also, $u(x,t) - g(x-2t) = u(x,t) - u(x+2t, 0) = z(0) - z(-t) = g(x-2t)e^t - g(x-2t)$.

So $u(x,t) = g(x-2t)e^t$.

Problem 3:

Consider $u_t - 2u_x = u^2$ for $x \in \mathbb{R}$, $t > 0$, with $u(x,0) = g(x)$.

Fix $x \in \mathbb{R}, t > 0$. Define $z(s) = u(x+2s, t+s)$. Then $\frac{d}{ds}z(s) = 2u_x(x+2s, t+s) + u_t(x+2s, t+s) = (u(x+2s, t+s))^2 = z(s)^2$. That is, $z(s)^2 = \frac{d}{ds}z(s)$, so $z(s) = \frac{-1}{s+C}$ for some $C \in \mathbb{R}$ (this is elementary differential equations). Because $z(-t) = u(x-2t, 0) = g(x-2t)$, we determine that $C = \frac{-1}{g(x-2t)} - t$ so that $z(s) = \frac{-1}{s - t + \frac{-1}{g(x-2t)}}$.

Also, $u(x,t) - g(x-2t) = z(0) - z(-t) u(x,t) - u(x+2t, 0) = z(0) - z(-t) = stuff$