Problem 1:

Consider $\text{Ind}(\partial D(a, r), a)$. Define $\theta : [0, 1] \to \mathbb{R}$ by $x \to 2\pi x$. Note that θ satisfies $\gamma(t) - a = |\gamma(t) - a| e^{i\theta(t)}$, where $\gamma(t) = re^{2\pi t} + a$From the definitions, we have

$$\begin{aligned}\text{Ind}(\partial D(a, r), a) &= \frac{1}{2\pi}(\theta(1) - \theta(0)) \\ &= \frac{1}{2\pi}(\theta(1) - \theta(0))\end{aligned}$$

Problem 2:

Problem 3:

Problem 4:

Problem 5:

Problem 6:

Problem 7:

Problem 8:

Problem 9:

Consider f, holomorphic on some disk, Ω, centered at z. Consider $g(w) = \frac{f(w)}{w-z}$; then we have that $\int\limits_{\partial Om} g(w)dw = 2\pi i \text{Res}_z g$. (Note that g's only singularity is at z.) Moreover, note that $g(w) = \frac{\sum\limits_{n=0}^{\infty} a_n(w-z)^n}{w-z}$. Thus, by the residue theorem, $\int\limits_{\partial\Omega} \frac{f(w)}{w-z}dw = f(z)$.

Problem 10:

Problem 11:

Problem 12:

Problem 13:

Problem 14:

Problem 15:

Problem 16:

Problem 17:

Problem 18:

Problem 19:

Problem 20:

Problem 21:

Problem 22:

Problem 23: