Problem 1: Let

$$A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

a) Find A's characteristic polynomial, real eigenvalues, and bases for their associated eigenspaces.

$\lambda^2 + 1 = 0$, no real eigenvalues.

b) What is the row-reduced echelon form of A? For rref(A), find the characteristic polynomial, real eigenvalues, and bases for their associated eigenspaces?

$(\lambda - 1)^2$, eigenvalue is 1, any basis for $\mathbb{R}^2$ works.

Problem 2: Define:

a) Inner Product: An inner product on V is a function (written $(\cdot,\cdot)$) from V^2 to $\mathbb{R}$ such that for all $u, v, w \in V$, $c \in \mathbb{R}$:

1) $(u, u) \geq 0$, $(u, u) = 0$ if and only if $u = 0_V$

2) $(v, u) = (u, v)$

3) $(u + v, w) = (u, w) + (v, w)$

4) $(cu, v) = c(u, v)$

b) Orthonormal: A set of vectors, S, in V is orthonormal if each pair of vectors is orthogonal and $\|u\| = 1$ for all $u \in S$.

c) Eigenvalue: We say that λ is an eigenvalue of a matrix (or linear transformation) if $Ax = \lambda x$ (or $T(x) = \lambda x$ for some vector x.

d) Eigenvector: The eigenvectors associated with an eigenvalue, la, are the vectors x satisfying $Ax = \lambda x$ (or $T(x) = \lambda x$).

Anything close to "the definition in the book" gets points.

Problem 3: Must a matrix with real entries have any real eigenvalues?

No. See problem 1.