**Problem 1:**

Let $R$ be a UFD and $P$ be a prime ideal containing no proper prime ideals other than (0).

Let $\mathcal{P}$ be a minimal generating set for $P$ (that is, with $a \nmid b$ for all $a, b \in \mathcal{P}$). Then each $a \in \mathcal{P}$ has a prime factorization. We know that $\gcd(\mathcal{P})$ exists; we show that it is in $P$.

First, let $a, b \in \mathcal{P}$. Then there are $c, d$ such that $c \gcd(a, b) = a$ and $d \gcd(a, b) = b$. Now, because $P$ is prime, that means that either $\gcd(a, b) \in P$ or both $c$ and $d$ are. If both $c$ and $d$ are in $P$, then

But naturally, every element of $\mathcal{P}$ is a multiple of $\gcd(\mathcal{P})$. So $P \subset (\gcd(\mathcal{P}))$.

So $P$ is principal if $P$ is a prime ideal containing no proper prime ideals other than (0).

**Problem 2:**

**Problem 3:**

**Problem 4:**

**Problem 5:**