Problem 2:

Let u be as described, with u smooth (as was said we could do in class).

Then there is a $B_r(x)$ (with $x \in \Omega$) such that $B_r(x) \cap \partial\Omega \subset A$ and $B_r(x) \setminus \overline{\Omega} \neq \emptyset$.

Define $v : B_r(x) \cup \Omega \to \mathbb{R}$ by $v(x) = u(x)$ for $x \in \Omega$, else $x = 0$. First, v is continuous on $B_r(x) \cup \Omega$ (it is clear except for $\partial\Omega$, and it works on $\partial\Omega$ because $u = 0$ on $\partial\Omega \cap B_r(x)$.) Next, v has a continuous first derivative on $B_r(x) \cup \Omega$ (it is clear except for $\partial\Omega$, and it works on $\partial\Omega$ because $\frac{\partial u}{\partial \nu} \to 0$ in Ω and $\frac{\partial u}{\partial \nu} = 0$ outside of Ω).

Then v is harmonic on $B_r(x) \cup \Omega$: it is clear that v is harmonic on Ω and on $B_r(x) \setminus \overline{\Omega}$.

So, we have that v is harmonic on $B_r(x) \cup \Omega$ and that v vanishes on some nonempty open subset of $B_r(x) \cup \Omega$; thus, v vanishes identically.

So u vanishes identically.

Problem 3:

Let G be the Green's function for Ω, a bounded domain.

Part a:

Consider $H(x, y) = G(x, y) - G(y, x)$. Then for all u harmonic on Ω, we have

$$u(x) = \int\limits_{\partial\Omega} u(y)\frac{\partial G(x, y)}{\partial \nu} - G(x, y)\frac{\partial u(y)}{\partial \nu} dS_y$$

Part b:

Part c: