Problem 1:

Consider $\prod_{n=1}^{\infty} \cos(\frac{z}{2^n})$. Note that if $z = 2^k(m\pi/2)$ for any $k \in \mathbb{N}, m \in \mathbb{Z}$, the product is trivially 0.

To evaluate this, we apply a trick that relies on this converging; we should determine that this actually converges. So, consider that $\sum_{0}^{\infty} |1 - \cos(z/2^n)| \leq$

Now, note that for each $N \in \mathbb{N}$, $\sin(\frac{z}{2^N}) \prod_{n=1}^{N} \cos(\frac{z}{2^n}) = \frac{1}{2^N}\sin(z)$, by applying the same sort of telescoping trick we used in class.

So $\prod_{n=1}^{N} \cos(\frac{z}{2^n}) = \frac{\sin(z)}{2^N \sin(\frac{z}{2^N})}$. By taking limits as $N \to \infty$, we get that $\prod_{n=1}^{\infty} \cos(\frac{z}{2^n}) = \lim_{N\to\infty} \frac{-\ln(2)2^{-N}\sin(z)}{-\ln(2)2^{-N}\cos(z2^{-N})} = \sin(z)$.

Problem 2: