Problem 1:

Part a:

The set of points z in the complex plane described by $|z - z_1| = |z - z_2|$ is the set of points equidistant from z_1 and z_2.

That is, it's described by the line going through the midpoint of z_1 and z_2 that is perpendicular to the line going through z_1 and z_2.

Part b:

The set of points z in the complex plane described by $1/z = \overline{z}$ is the set of points with $z\overline{z} = 1$, so that $|z| = \sqrt{z\overline{z}} = 1$.

That is, it's the unit circle centered at the origin.

Part c:

The set of points z in the complex plane described by $\mathrm{Re}(z) = 3$ is the set of points with real component 3.

That is, it's described by a vertical line with x-intercept 3.

Part d:

The set of points z in the complex plane described by $\mathrm{Re}(z) > c$ is the set of points with real component larger than c.

That is, it's described by the right half of a plane cut by the vertical line with x-intercept c (excluding the line itself).

Problem 2:

Let $z, w \in \mathbb{C}$, with $z \neq w$, be vertices of a square, with $z = (x, y)$ and $w = (x', y')$.

Either z and w are the opposite vertices of a square or they are adjacent vertices of a square.

If they are opposite vertices of a square, then there is a uniquely determined square given these vertices. (And this is geometrically clear.)

In this case, we proceed by finding the center of the square, finding a line perpendicular to the line through z and w through the midpoint, and finding points a and b on this line that make a square.

The center of the square is given by $A = (\frac{x+x'}{2}, \frac{y+y'}{2})$; it is the midpoint of two opposite sides.

Now, consider the vector $p = z - w = (x - x', y - y')$, and the vector $q = (y - y', -(x - x'))$. It is readily checked that $p \cdot q = 0$, so that these vectors are perpendicular.

Now, the points given by $a = A + q/2 = (\frac{x+x'+y-y'}{2}, \frac{y+y'-(x-x')}{2})$ and $b = A - q/2 = (\frac{x+x'-(y-y')}{2}, \frac{y+y'+x-x'}{2})$ are the desired points, as a, b, z, w are four points equidistant from A with the property that $(a - b) \cdot (z - w) = 0$ (that is, we can make opposite sides make a right angle with a point). We've already shown the second part: $p = z - w$ and $q = a - b$. Also, z and w are equidistant from A, as A is their midpoint. It is also readily checked that A is a and b's midpoint. Moreover, the length of p and q are the same (by symmetry). So $z = A + p/2$ and $a = A + q/2$ are equidistant from A; all of a, b, w, z are the same distance from A.

If they are adjacent vertices of a square, then there are exactly two different squares given these vertices. (And this is geometrically clear.)

In this case, we proceed by making a line through z perpendicular to the line through z and w, finding points of the appropriate length, and repeating the process with a line through w perpendicular to the line through z and w.

Now, consider the vector $p = z - w = (x - x', y - y')$, and the vector $q = (y - y', -(x - x'))$. It is readily checked that $p \cdot q = 0$, so that these vectors are perpendicular.

The points given by $a = z + q$ and $b = w + q$ together with z and w make a square.

Also, the points given by $a' = z - q$ and $b' = w - q$ together with z and w make a square.

To summarize the above:

If z and w are the opposite corners of the square, then...

If z and w are adjacent corners of the square, then...

Problem 3:

Let $|a| = |b| = 1$, with $a = (x, y)$ and $b = (x', y')$, with $a \neq b$ and $\overline{a}b \neq 1$.

Then we have

$$\begin{aligned}(1-\overline{a}b)\overline{(1-\overline{a}b)} &= (1-xx'+yy', -xy'-x'y)(1-xx'+yy', xy'+x'y)\\ &= ((1-xx'+yy')(1-xx'+yy')-(-xy'-x'y)(xy'+x'y), (-xy'-x'y)(1-xx\end{aligned}$$