Test

1 **Hypothesis**: → ข้าวสารยี่ห้อหนึ่ง บรรจุถุงละ 500g/pack

Null hypothesis: $H_0: \mu = 500$

Feedback: บรรจุภัณฑ์ไม่ถึง 500g ต่อถุง

Alternative hypothesis $H_1: \mu \neq 500$

Hypothesis Testing: → ทดสอบ (ว่า) < ยอมรับ H_0 / H_1 (ปฏิเสธ)

Remark: ถ้าปฏิเสธ → สำหรับปัญหา ยอมรับ H_0 & H_1

2 **ผิดพลาด** (error)

→ ฆาตกร

→ ไม่ใช่ฆาตกร

	ฆาตกร	ไม่ใช่ฆาตกร
ตัดสิน	OK	error I
ไม่ตัดสิน	error II	OK

สิ่งที่จริง (data)

	500g	≠500g
$\mu = 500$	OK	error I
$\mu \neq 500$	error II	OK

③ รูปแบบของ test

$H_0: \mu = 500$ (2-tailed test)

$H_1: \mu \neq 500$

(ถ้ามี "=" ต้องอยู่ใน H_0)

$H_0: \mu \geq 500$ one-tailed test

$H_1: \mu < 500$

④ Significance level → Proba: $\alpha = 5\%$

[25 – 35] 95%

⑤ วิธีตัดสินใจ: เปรียบเทียบคู่
- test statistic (data)
- critical value (table)

Test for mean (μ) .. 5 steps

Step 1 : $H_0: \mu = 500$

$H_1: \mu \neq 500$

<u>Step 2</u> : Significance level $\alpha = 5\% = 0.05$

<u>Step 3</u> : Test statistic → เพื่อทดสอบข้อความ μ

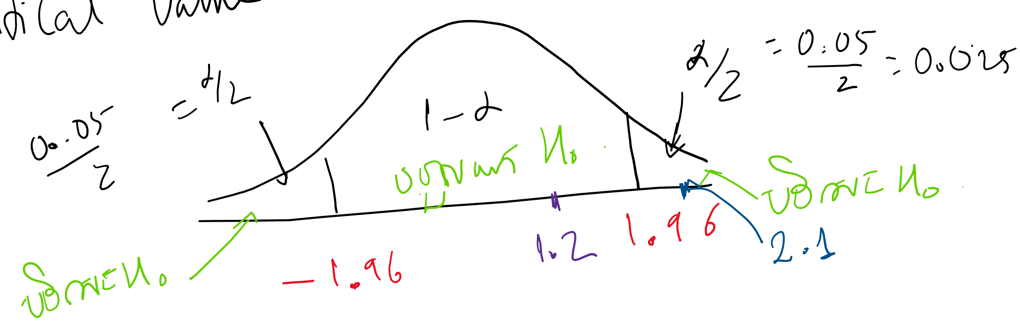
ดังนั้น $\bar{X}$: point estimator $\in \mu$.

$$\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$$

$$z = \frac{\bar{X} - \mu}{\sqrt{\frac{\sigma^2}{n}}} = \boxed{\sqrt{n}\left(\frac{\bar{X}-\mu}{\sigma}\right)} \sim N(0,1)$$

$z = \sqrt{n}\left(\frac{\bar{X}-\mu}{\sigma}\right)$ คือ test statistics.

<u>Step 4</u> : Critical Value



<u>Step 5</u> : เก็บข้อมูลจริง $\underbrace{x_1, x_2, \ldots, x_n}_{\bar{X},\ \sigma,\ \mu = 500} \sim N(\mu, \sigma^2)$.

test statistic $z = \sqrt{n}\left(\frac{\bar{x}-\mu}{\sigma}\right) = 2.1$ not accept H_0

$$z = \sqrt{n}\left(\frac{\bar{x}-\mu}{\sigma}\right) = 1.2 \to \text{accept } H_0$$

1. Jamestown Steel Company manufactures and assembles desks and other office equipment at several plants in western New York State. The weekly production of the Model A325 desk at the Fredonia Plant follows a normal probability distribution with a mean of 200 and a standard deviation of 16. Recently, because of market expansion, new production methods have been introduced and new employees hired. The vice president of manufacturing would like to investigate whether there has been a change in the weekly production of the Model A325 desk. Is the mean number of desks produced at the Fredonia Plant different from 200 at the .01 significance level?

X

$X \sim N(200, 16^2)$.

Step 1:

$H_0: \mu = 200$

$H_1: \mu \neq 200$

Step 2:

$\alpha = 0.01$

step 3

test statistic $z = \sqrt{n}\left(\frac{\bar{X}-\mu}{\sigma}\right)$ (use data).

n = sample size, $\bar{X}$ = sample mean.

$\mu = 200$, $\sigma = 16$

We take a sample from the population (weekly production), compute a test statistic, apply the decision rule, and arrive at a decision to reject H_0 or not to reject H_0. The mean number of desks produced last year (50 weeks because the plant was shut down 2 weeks for vacation) is 203.5. The standard deviation of the population is 16 desks per week. Computing the z value

$x_1, x_2, \ldots, x_{50}$

$\bar{x}$

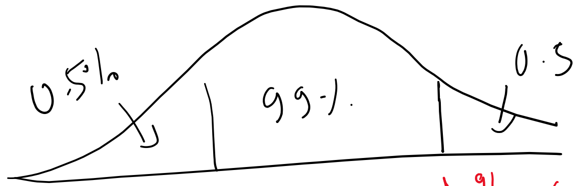
$n = 50$, $\bar{X} = 203.5$

$$z = \sqrt{50}\left(\frac{203.5 - 200}{16}\right) = \frac{\sqrt{50}}{16}(3.5) = 1.218$$

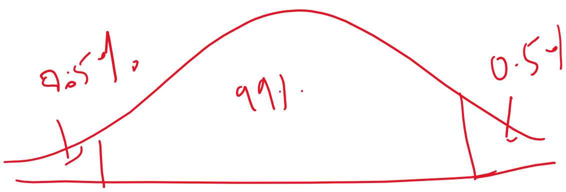
Step 4: $\alpha = 0.05$

(Critical Value)

$\alpha = 10\%$



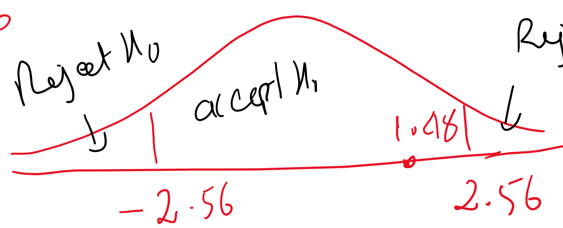
-1.96	$1.96 \leftarrow 5\%$
-1.645	$1.645 \leftarrow 10\%$
-2.56	$2.56 \leftarrow 1\%$



$$\frac{0.5}{100} = 0.005$$

$P(Z > \lambda) = 0.005$ or $P(Z < \lambda) = 0.995$

$\lambda = 2.56$



$-2.56 < Z = 1.48 < 2.56$

$\rightarrow$ We accept H_0

$H_0 : \mu = 200$

$H_1 : \mu \neq 200$

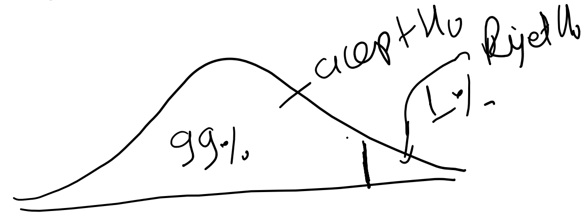
(2-tailed)

$H_0 : \mu \leq 200$

$H_1 : \mu > 200$

(one-tail)

$Z = 1.48$ test statistic



2.1381

2.131

$H_0: \mu \leq 200$

$H_1: \mu > 200$

2. Heinz, a manufacturer of ketchup, uses a particular machine to dispense 16 ounces of its ketchup into containers. From many years of experience with the particular dispensing machine, Heinz knows the amount of product in each container follows a normal distribution with a mean of 16 ounces and a standard deviation of 0.15 ounce. A sample of 50 containers filled last hour revealed the mean amount per container was 16.017 ounces. Does this evidence suggest that the mean amount dispensed is different from 16 ounces? Use the .05 significance level.
   (a) State the null hypothesis and the alternate hypothesis.
   (b) What is the probability of a Type I error?
   (c) Give the formula for the test statistic.
   (d) State the decision rule.
   (e) Determine the value of the test statistic.
   (f) What is your decision regarding the null hypothesis?
   (g) Interpret, in a single sentence, the result of the statistical test.

3. Refer to exercises 2.
   (a) Suppose the next to the last sentence is changed to read: Does this evidence suggest that the mean amount dispensed is more than 16 ounces? State the null hypothesis and the alternate hypothesis under these conditions.
   (b) What is the decision rule under the new conditions stated in part (a)?
   (c) A second sample of 50 filled containers revealed the mean to be 16.040 ounces. What is the value of the test statistic for this sample?
   (d) What is your decision regarding the null hypothesis?
   (e) Interpret, in a single sentence, the result of the statistical test.
   (f) What is the p-value? What is your decision regarding the null hypothesis based on the p-value? Is this the same conclusion reached in part (d)?

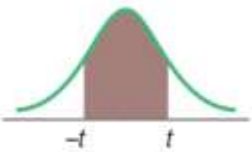
4. The McFarland Insurance Company Claims Department reports the mean cost to process a claim is $60. An industry comparison showed this amount to be larger than most other insurance companies, so the company instituted cost-cutting measures. To evaluate the effect of the cost-cutting measures, the supervisor of the Claims Department selected a random sample of 26 claims processed last month

and recorded the cost to process each claim. The sample information is reported below. At the .01 significance level, is it reasonable to conclude that the mean cost to process a claim is now less than $60?

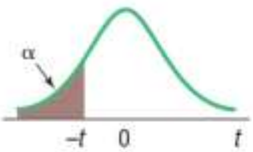
$45	$49	$62	$40	$43	$61
48	53	67	63	78	64
48	54	51	56	63	69
58	51	58	59	56	57
38	76				

5. The mean life of a battery used in a digital clock is 305 days. The lives of the batteries follow the normal distribution. The battery was recently modified to last longer. A sample of 20 of the modified batteries had a mean life of 311 days with a standard deviation of 12 days. Did the modification increase the mean life of the battery?
(a) State the null hypothesis and the alternate hypothesis.
(b) Show the decision rule graphically. Use the .05 significance level.
(c) Compute the value of t. What is your decision regarding the null hypothesis? Briefly summarize your results.

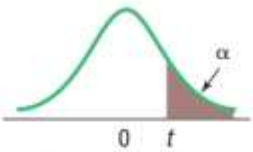
Student's t Distribution



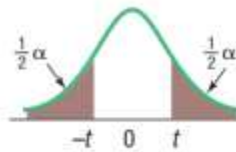
Confidence interval



Left-tailed test



Right-tailed test



Two-tailed test

df (degrees of freedom)	Confidence Intervals, c					
	80%	90%	95%	98%	99%	99.9%
	Level of Significance for One-Tailed Test, α					
	0.10	0.05	0.025	0.01	0.005	0.0005
	Level of Significance for Two-Tailed Test, α					
	0.20	0.10	0.05	0.02	0.01	0.001
1	3.078	6.314	12.706	31.821	63.657	636.619
2	1.886	2.920	4.303	6.965	9.925	31.599
3	1.638	2.353	3.182	4.541	5.841	12.924
4	1.533	2.132	2.776	3.747	4.604	8.610
5	1.476	2.015	2.571	3.365	4.032	6.869
6	1.440	1.943	2.447	3.143	3.707	5.959
7	1.415	1.895	2.365	2.998	3.499	5.408
8	1.397	1.860	2.306	2.896	3.355	5.041
9	1.383	1.833	2.262	2.821	3.250	4.781
10	1.372	1.812	2.228	2.764	3.169	4.587
11	1.363	1.796	2.201	2.718	3.106	4.437
12	1.356	1.782	2.179	2.681	3.055	4.318
13	1.350	1.771	2.160	2.650	3.012	4.221
14	1.345	1.761	2.145	2.624	2.977	4.140
15	1.341	1.753	2.131	2.602	2.947	4.073
16	1.337	1.746	2.120	2.583	2.921	4.015
17	1.333	1.740	2.110	2.567	2.898	3.965
18	1.330	1.734	2.101	2.552	2.878	3.922
19	1.328	1.729	2.093	2.539	2.861	3.883
20	1.325	1.725	2.086	2.528	2.845	3.850
21	1.323	1.721	2.080	2.518	2.831	3.819
22	1.321	1.717	2.074	2.508	2.819	3.792
23	1.319	1.714	2.069	2.500	2.807	3.768
24	1.318	1.711	2.064	2.492	2.797	3.745
25	1.316	1.708	2.060	2.485	2.787	3.725
26	1.315	1.706	2.056	2.479	2.779	3.707
27	1.314	1.703	2.052	2.473	2.771	3.690
28	1.313	1.701	2.048	2.467	2.763	3.674
29	1.311	1.699	2.045	2.462	2.756	3.659
30	1.310	1.697	2.042	2.457	2.750	3.646
31	1.309	1.696	2.040	2.453	2.744	3.633
32	1.309	1.694	2.037	2.449	2.738	3.622
33	1.308	1.692	2.035	2.445	2.733	3.611
34	1.307	1.691	2.032	2.441	2.728	3.601
35	1.306	1.690	2.030	2.438	2.724	3.591
36	1.306	1.688	2.028	2.434	2.719	3.582
37	1.305	1.687	2.026	2.431	2.715	3.574
38	1.304	1.686	2.024	2.429	2.712	3.566
39	1.304	1.685	2.023	2.426	2.708	3.558
40	1.303	1.684	2.021	2.423	2.704	3.551
41	1.303	1.683	2.020	2.421	2.701	3.544
42	1.302	1.682	2.018	2.418	2.698	3.538
43	1.302	1.681	2.017	2.416	2.695	3.532
44	1.301	1.680	2.015	2.414	2.692	3.526
45	1.301	1.679	2.014	2.412	2.690	3.520
46	1.300	1.679	2.013	2.410	2.687	3.515
47	1.300	1.678	2.012	2.408	2.685	3.510
48	1.299	1.677	2.011	2.407	2.682	3.505
49	1.299	1.677	2.010	2.405	2.680	3.500
50	1.299	1.676	2.009	2.403	2.678	3.496
51	1.298	1.675	2.008	2.402	2.676	3.492
52	1.298	1.675	2.007	2.400	2.674	3.488
53	1.298	1.674	2.006	2.399	2.672	3.484
54	1.297	1.674	2.005	2.397	2.670	3.480
55	1.297	1.673	2.004	2.396	2.668	3.476
56	1.297	1.673	2.003	2.395	2.667	3.473
57	1.297	1.672	2.002	2.394	2.665	3.470
58	1.296	1.672	2.002	2.392	2.663	3.466
59	1.296	1.671	2.001	2.391	2.662	3.463
60	1.296	1.671	2.000	2.390	2.660	3.460
61	1.296	1.670	2.000	2.389	2.659	3.457
62	1.295	1.670	1.999	2.388	2.657	3.454
63	1.295	1.669	1.998	2.387	2.656	3.452
64	1.295	1.669	1.998	2.386	2.655	3.449
65	1.295	1.669	1.997	2.385	2.654	3.447
66	1.295	1.668	1.997	2.384	2.652	3.444
67	1.294	1.668	1.996	2.383	2.651	3.442
68	1.294	1.668	1.995	2.382	2.650	3.439
69	1.294	1.667	1.995	2.382	2.649	3.437
70	1.294	1.667	1.994	2.381	2.648	3.435

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