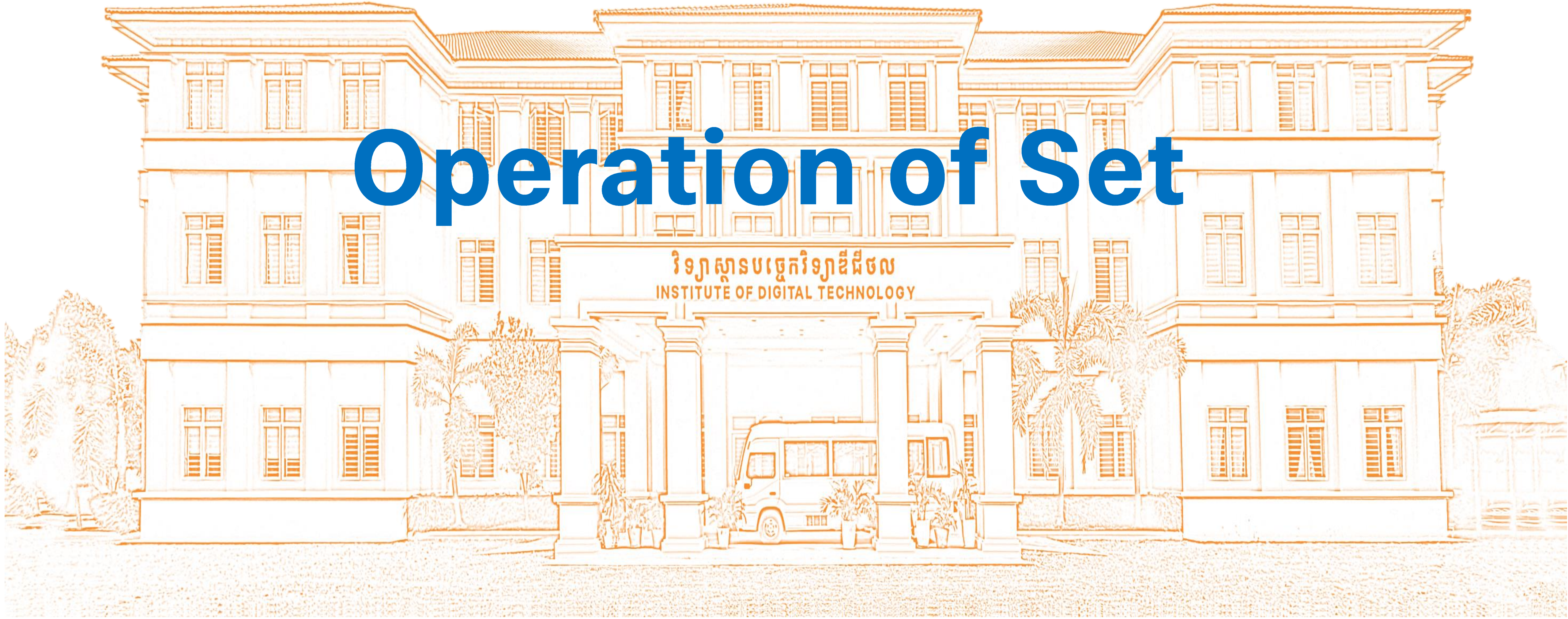
CADT IDT វិទ្យាស្ថានបច្ចេកវិទ្យាឌីជីថល
Institute of Digital Technology

Department of Foundation Year



Operation of Set

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Set Operation

There are five operation on set

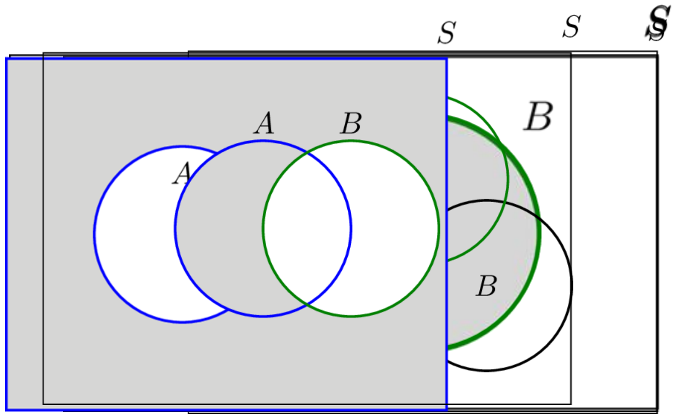
1.Union Set $\Rightarrow A \cup B = \{x \mid x \in A \, or \, x \in B\}$

2.Intersection Set $\Rightarrow A \cap B = \{x \mid x \in A \, and \, x \in B\}$

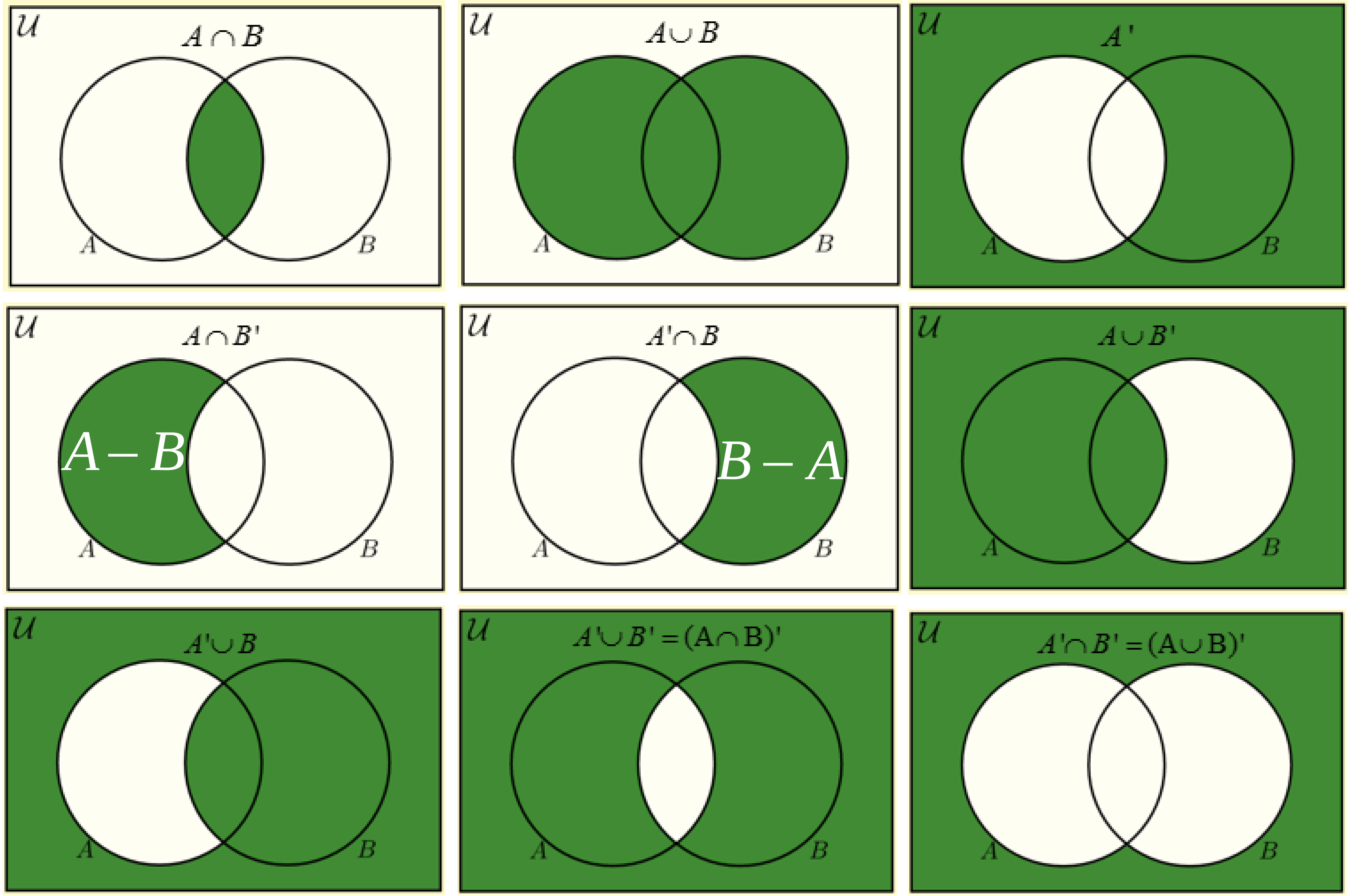
3.The complement of Set $\Rightarrow \bar{A} = \{x \mid x \in U \text{ and } x \notin A\}$

4.The Difference of Set $\Rightarrow A - B = \{x \mid x \in A \, and \, x \notin B\}$, $A \Delta B = \{x \mid x \in A, x \notin B \text{ or } x \in B, x \notin A\}$

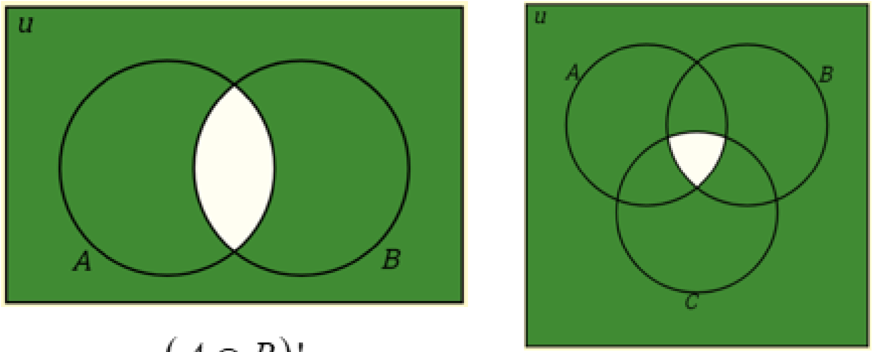
5.Cartesian Product of sets $\Rightarrow A \times B = \{(x, y) \mid x \in A \text{ and } y \in B\}$, $A \times B \neq B \times A$



Venn Diagram



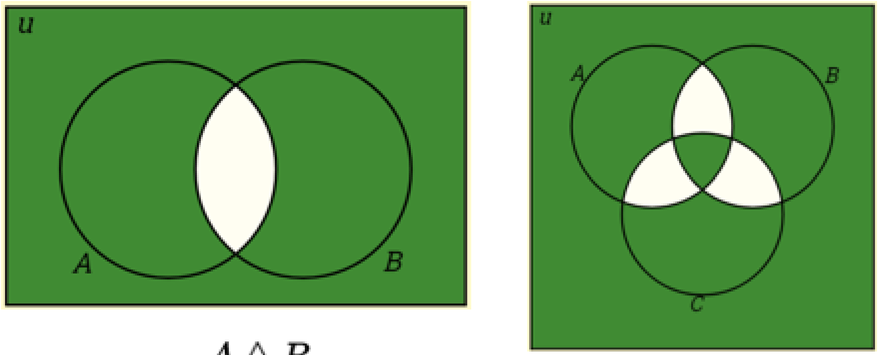
Complement of Intersection of Sets



$(A \cap B)'$

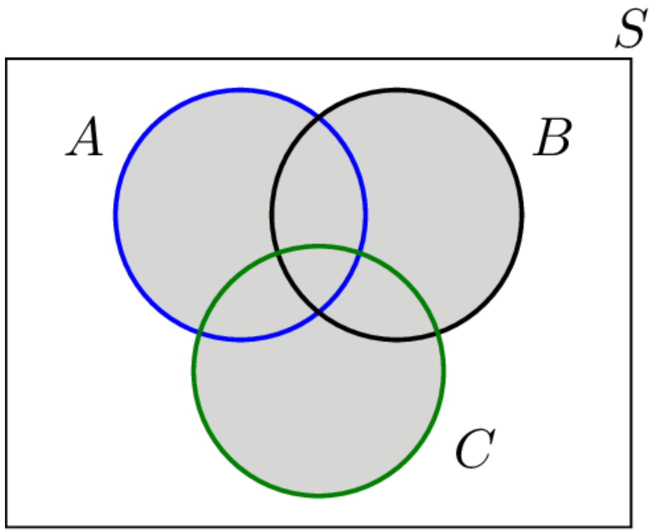
$(A \cap B \cap C)'$

Symmetric Difference of Sets

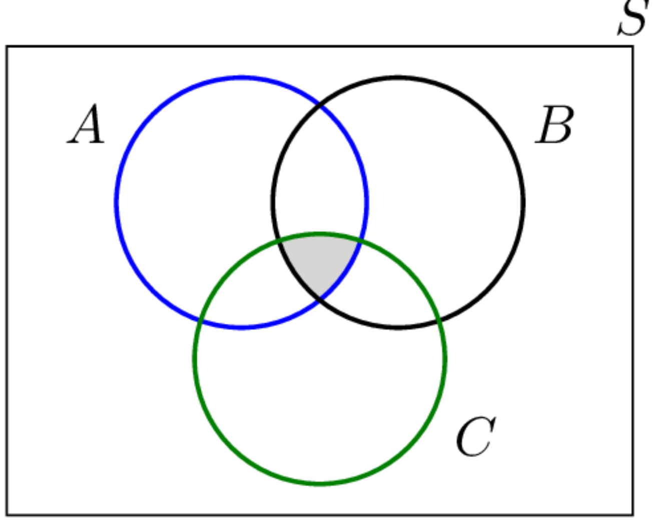


$A \triangle B$

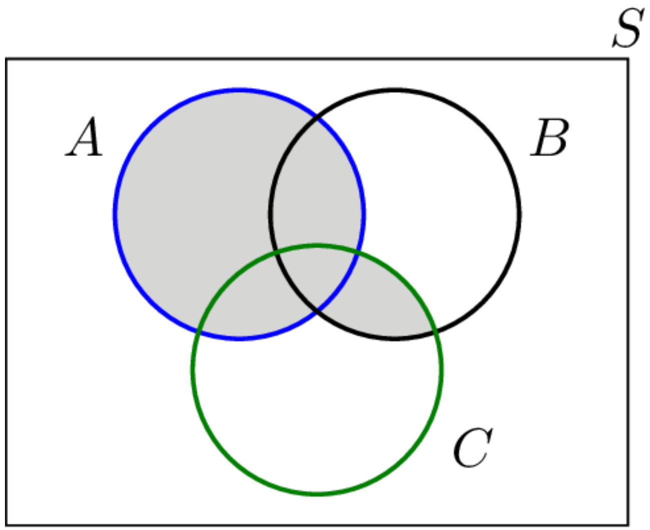
$A \triangle B \triangle C$



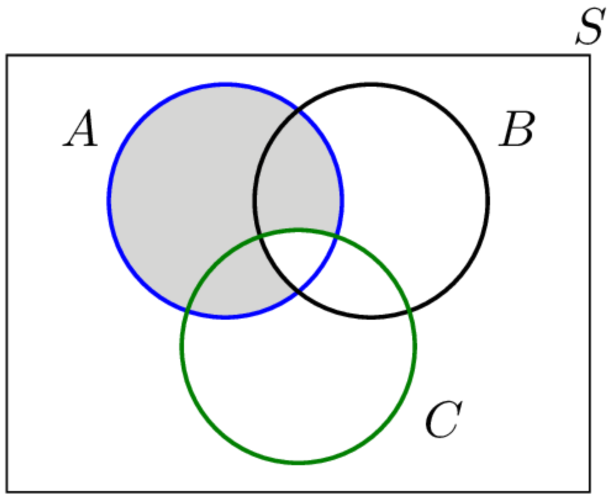
$A \cup B \cup C$



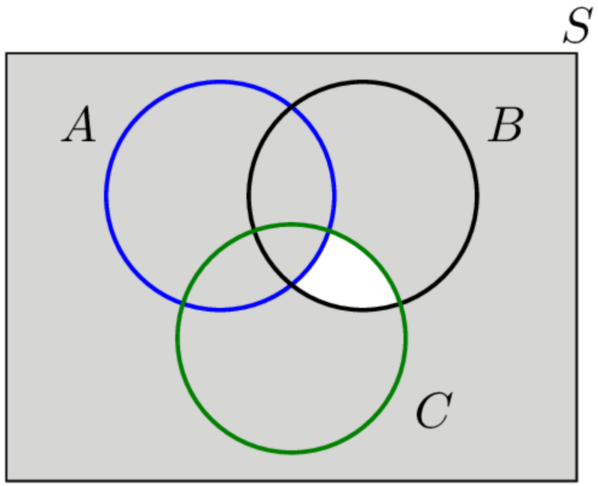
$A \cap B \cap C$



$A \cup (B \cap C)$



$A - (B \cap C)$



$A \cup (B \cap C)^C$

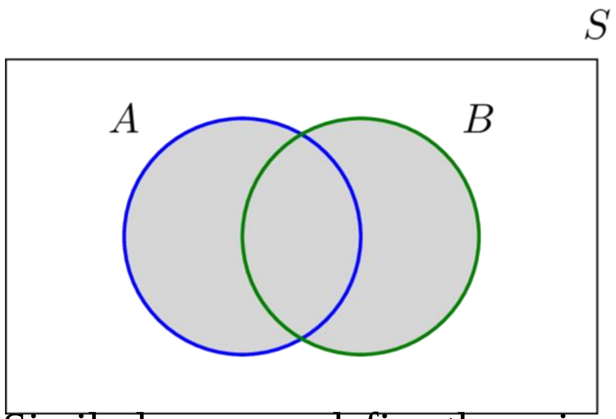
Set Operation

1.Union Set

The union of two sets is a set containing all elements that are in A or in B (possibly both).

For example, $\{1,2,3,4\} \cup \{2,3,5\} = \{1,2,3,4,5\}$. Thus, we can write $x \in (A \cup B)$ if and only if $(x \in A)$ or $(x \in B)$.

Note that : $A \cup B = B \cup A$. In Figure below, the union of sets A and B is shown by the shaded area in the Venn diagram.



Similarly we can define the union of three or more sets. In particular, $if\ A_1, A_2, \cdots, A_n$ are n sets, their union $A_1 \cup A_2 \cup . . \cup A_n$ is a set containing all elements that are in at least one of the sets.

We can write this union more compactly by $\bigcup_{i=1}^{n} A_i$.

For example, if $A_1 = \{a, b, c\}, A_2 = \{c, h\}, A_3 = \{a, d\}, then\ \cup_i A_i = A_1 \cup A_2 \cup A_3 = \{a, b, c, d, h\}$.

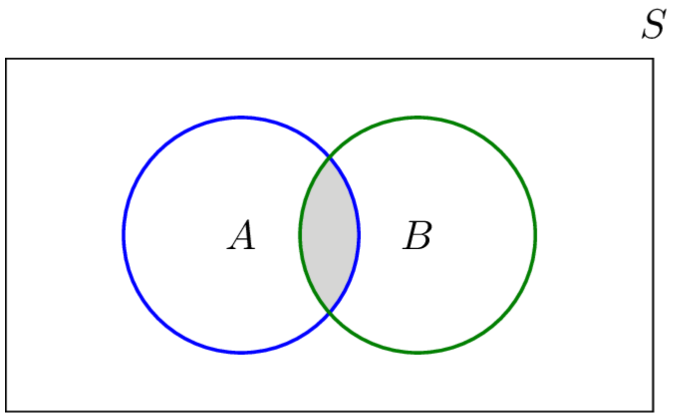
We can similarly define the union of infinitely many sets $A_1 \cup A_2 \cup A_3 \cup \cdots$

Set Operation

2.Intersection Set

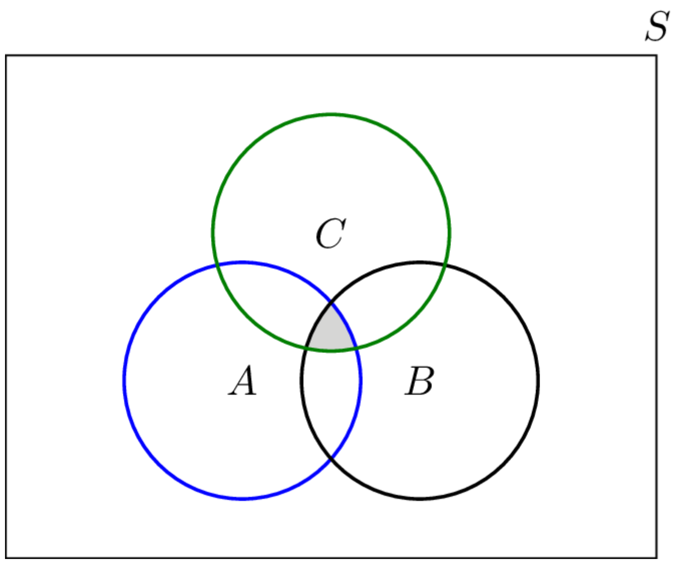
The intersection of two sets A and B, denoted by $A \cap B$, consists of all elements that are both in A and B.

For example, $\{1,2,3,4\} \cap \{2,3,5\} = \{2,3\}$. In Figure, the intersection of sets A and B is shown by the shaded area using a Venn diagram.



More generally, for sets $A_1, A_2, A_3, \cdots,$ their intersection $\bigcap_i A_i$ is defined as the set consisting of the elements that are in all $A_i{}'s$.

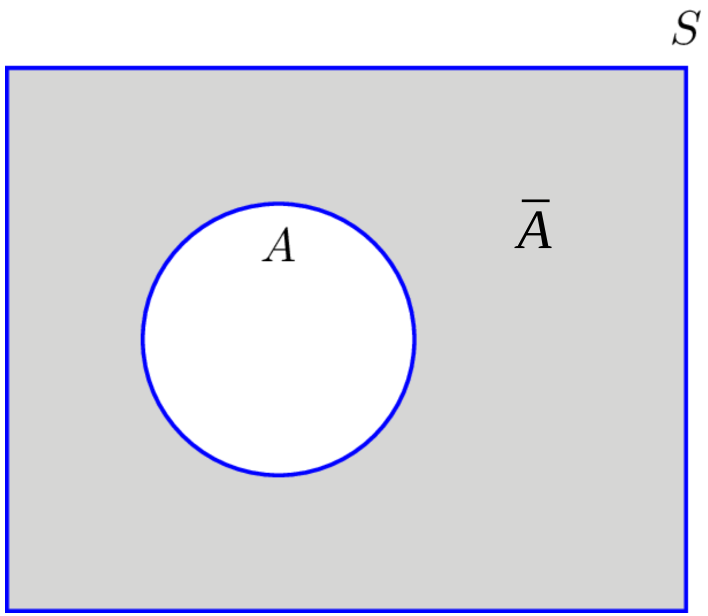
Figure shows the intersection of three sets.



Set Operation

3.The Complement of Set

The complement of a set A, denoted by A^c or $\bar{A}$, is the set of all elements that are in the universal set S but are not in A. In Figure, $\bar{A}$ is shown by the shaded area using a Venn diagram.

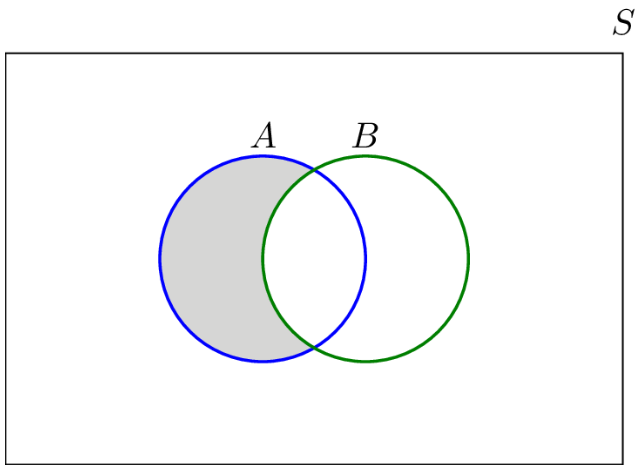


Set Operation

4.The Difference of Set

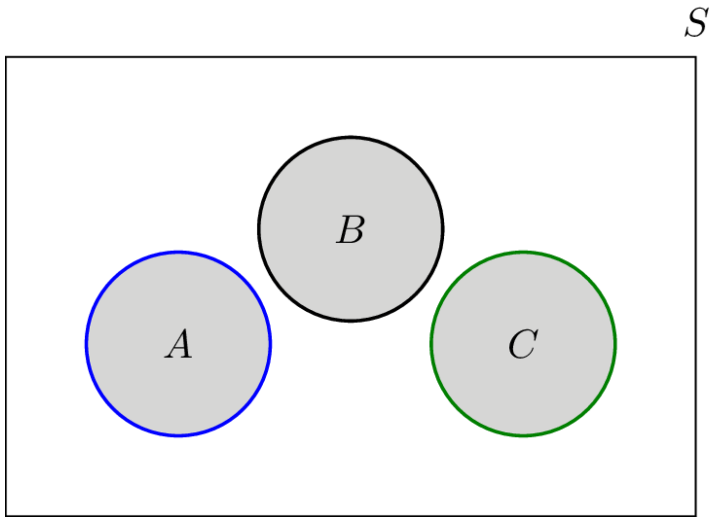
The difference (subtraction) is defined as follows. The set $A - B$ consists of elements that are in A but not in B.

For example $if\ A = \{1,2,3\}\ and\ B = \{3,5\}, then\ A - B = \{1,2\}$. In Figure , $A - B$ is shown by the shaded area using a Venn diagram. Note that $A - B = A \cap \bar{B}$.



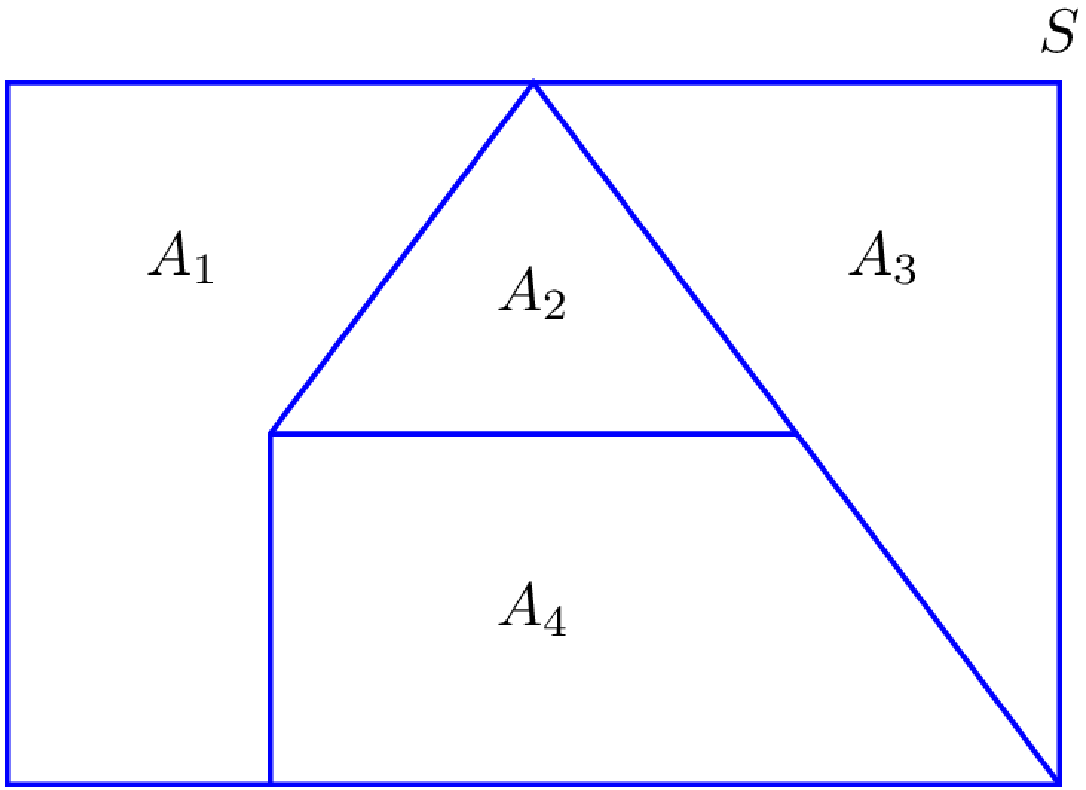
Set Operation

Two sets A and B are mutually exclusive or disjoint if they do not have any shared elements; i.e., their intersection is the empty set, $A \cap B = \emptyset$. More generally, several sets are called disjoint if they are pairwise disjoint, i.e., no two of them share a common elements. Figure shows three disjoint sets.



Set Operation

If the earth's surface is our sample space, we might want to partition it to the different continents. Similarly, a country can be partitioned to different provinces. In general, a collection of nonempty sets $A_1, A_2, \cdots$ is a partition of a set A if they are disjoint and their union is A. In Figure, the sets A_1, A_2, A_3 *and* A_4 form a partition of the universal set S.



Set Properties

❖ Union Property

1. $A \cup A = A$
2. $A \cup \bar{A} = U$
3. $A \cup \emptyset = A$
4. $A \cup B = B \cup A$
5. $A \cup (B \cup C) = (A \cup B) \cup C$
6. $A \subset A \cup B \ \wedge B \subset A \cup B$
7. $A \cup B \subset B \ \Leftrightarrow A \subset B$

Intersection Property

$A \cap A = A$

$A \cap \bar{A} = \emptyset$

$A \cap \emptyset = \emptyset$

$A \cap B = B \cap A$

$A \cap (B \cap C) = (A \cap B) \cap C$

$A \subset A \cap B = A \quad , \ B \subset A \cap B = B$

$A \cap B \subset B \ \Leftrightarrow B \subset A$

Number Element Properties

- $n(A \cap U) = n(A)$
- $n(A \cap \emptyset) = 0$
- $n(A \cap B) \leq n(A)$
- $n(A \cap \bar{A}) = 0$
- $n(U) = n(A) + n(\bar{A})$
- $n(A \cup B) = n(A) + n(B) - n(A \cap B) \ \ , \ A\ joint\ B$
- $n(A \cup B) = n(A) + n(B) \ \ , A\ disjoint\ B$
- $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$
- $n(A \times B) = n(A) \times n(B)$
- $n(\prod_{i=1}^{n} A_i) = n(A_1) \times n(A_2) \times n(A_3) \times \cdots \times n(A_n)$

Set Theory

Theorem : De Morgan's law

For any sets $A_1, A_2, \cdots, A_n$, we have

$$\overline{(A_1 \cup A_2 \cup A_3 \cup \cdots A_n)} = \overline{A_1} \cap \overline{A_2} \cap \overline{A_3} \cdots \cap \overline{A_n}$$

$$\overline{(A_1 \cap A_2 \cap A_3 \cap \cdots A_n)} = \overline{A_1} \cup \overline{A_2} \cup \overline{A_3} \cdots \cup \overline{A_n}$$

Theorem : Distributive law

For any sets $A, B, and\ C$ we have

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

Example

If the universal set is given by $S = \{1,2,3,4,5,6\}, and\ A = \{1,2\}, B = \{2,4,5\}, C = \{1,5,6\}$ are three sets, find the following sets:

a. $A \cup B$

b. $A \cap B$

c. $\bar{A}$

d. $\bar{B}$

e. Check De Morgan's law by finding $\overline{(A \cup B)}$ and $\bar{A} \cap \bar{B}$.

f. Check the distributive law by finding $A \cap (B \cup C)$ and $(A \cap B) \cup (A \cap C)$.

answer for example

$a.\ A \cup B = \{1,2,4,5\}.$

$b.\ A \cap B = \{2\}.$

$c.\ \bar{A} = \{3,4,5,6\}$ ($\bar{A}$consists of elements that are in S but not in A).

$d.\ \bar{B} = \{1,3,6\}.$

e. We have

$\overline{(A \cup B)} = \overline{\{1,2,4,5\}} = \{3,6\},$

which is the same as

$\bar{A} \cap \bar{B} = \{3,4,5,6\} \cap \{1,3,6\} = \{3,6\}.$

f. We have

$A \cap (B \cup C) = \{1,2\} \cap \{1,2,4,5,6\} = \{1,2\},$

which is the same as

$(A \cap B) \cup (A \cap C) = \{2\} \cup \{1\} = \{1,2\}.$

answer for example

A Cartesian product of two sets A and B, written as $A \times B$, is the set containing ordered pairs from A and B. That is, if $C = A \times B$, then each element of C is of the form (x,y), where $x \in A$ and $y \in B$:

$A \times B = \{(x,y) | x \in A \text{ and } y \in B\}$.

For example, if $A = \{1,2,3\}$ and $B = \{H,T\}$, then

$A \times B = \{(1,H),(1,T),(2,H),(2,T),(3,H),(3,T)\}$.

Note that here the pairs are ordered, so for example, $(1,H) \neq (H,1)$. Thus $A \times B$ is not the same as $B \times A$.

Solve by Venn Diagram

Example 1:

150 college freshmen were interviewed.

85 were registered for a math class

70 were registered for a soft skill class

50 were registered for both math and soft skill

How many signed up only for a math class?

How many signed up only for a soft skill class?

How many signed up for math or soft skill?

How many signed up for neither math nor soft skill?

Solve by Venn Diagram

Example 2:

100 were students interviewed

28 took Algorithm

31 took Soft Skill

42 took Maths

9 took Algorithm and Soft Skill

10 took Algorithm and Maths

6 took Soft Skill and Maths

4 took all three subjects

How many students took none of the three subjects?

How many students took Algorithm, but not Soft Skill or Maths?

How many students took Soft Skill and Algorithm but not Maths?

Solve by Venn Diagram

Example 3 :

110 college freshmen were surveyed

25 took algorithm

45 took Soft Skill

45 took mathematics

10 took Algorithm and mathematics

8 took Soft Skill and mathematics

6 took Algorithm and Soft Skill

5 took all three

a. How many students took Soft Skill, but neither Algorithm nor mathematics?

b. How many students took Soft Skill, Algorithm or mathematics?

c. How many students did not take any of the three subjects?

Applications of Sets

A study was made of 200 students to determine what cartoons they watch.

- 22 students don't watch these cartoons.
- 73 students watch only Tiny Toons.
- 136 students watch Tiny Toons.
- 14 students watch only Animaniacs and Pinky & the Brain.
- 31 students watch only Tiny Toons and Pinky & the Brain.
- 63 students watch Animanics.
- 135 students do not watch Pinky & the Brain.

How many watched only Pinky & the Brain?

Music: http://www.bensound.com

Exercises of set operation

Find the union, intersection and the difference (A - B) of the following pairs of sets.

(a) A = The set of all letters of the word FEAST B = The set of all letters of the word TASTE

(b) $A = \{x : x \in W, 0 < x \leq 7\}$ $B = \{x : x \in W, 4 < x < 9\}$

(c) $A = \{x \mid x \in N, x \text{ is a factor of } 12\}$ $B = \{x \mid x \in N, x \text{ is a multiple of } 2, x < 12\}$

(d) A = The set of all even numbers less than 12 B = The set of all odd numbers less than 11

(e) $A = \{x : x \in I, -2 < x < 2\}$ $B = \{x : x \in I, -1 < x < 4\}$

(f) $A = \{a, l, m, n, p\}$ $B = \{q, r, l, a, s, n\}$

Exercises of set operation

Let $X = \{2, 4, 5, 6\}$ $Y = \{3, 4, 7, 8\}$ $Z = \{5, 6, 7, 8\}$, find

(a) $(X - Y) \cup (Y - X)$

(b) $(X - Y) \cap (Y - X)$

(c) $(Y - Z) \cup (Z - Y)$

(d) $(Y - Z) \cap (Z - Y)$

Exercises of set operation

Let $\xi = \{1, 2, 3, 4, 5, 6, 7\}$ and $A = \{1, 2, 3, 4, 5\}$ $B = \{2, 5, 7\}$ show that

(a) $(A \cup B)' = A' \cap B'$

(b) $(A \cap B)' = A' \cup B'$

(c) $(A \cap B) = B \cap A$

(d) $(A \cup B) = B \cup A$

Let $P = \{a, b, c, d\}$ $Q = \{b, d, f\}$ $R = \{a, c, e\}$ verify that

(a) $(P \cup Q) \cup R = P \cup (Q \cup R)$

(b) $(P \cap Q) \cap R = P \cap (Q \cap R)$

Exercises of set operation

1. Let A and B be two finite sets such that $n(A) = 20$, $n(B) = 28$ and $n(A \cup B) = 36$, find $n(A \cap B)$.

2. If $n(A - B) = 18$, $n(A \cup B) = 70$ and $n(A \cap B) = 25$, then find $n(B)$.

3. In a group of 60 people, 27 like cold drinks and 42 like hot drinks and each person likes at least one of the two drinks. How many like both coffee and tea?

4. There are 35 students in art class and 57 students in dance class. Find the number of students who are either in art class or in dance class.

- When two classes meet at different hours and 12 students are enrolled in both activities.
- When two classes meet at the same hour.

Exercises of set operation

5. In a group of 100 persons, 72 people can speak English and 43 can speak French. How many can speak English only? How many can speak French only and how many can speak both English and French?

6. In a competition, a school awarded medals in different categories. 36 medals in dance, 12 medals in dramatics and 18 medals in music. If these medals went to a total of 45 persons and only 4 persons got medals in all the three categories, how many received medals in exactly two of these categories?

7. Each student in a class of 40 plays at least one indoor game chess, carrom and scrabble. 18 play chess, 20 play scrabble and 27 play carrom. 7 play chess and scrabble, 12 play scrabble and carrom and 4 play chess, carrom and scrabble. Find the number of students who play (i) chess and carrom. (ii) chess, carrom but not scrabble.