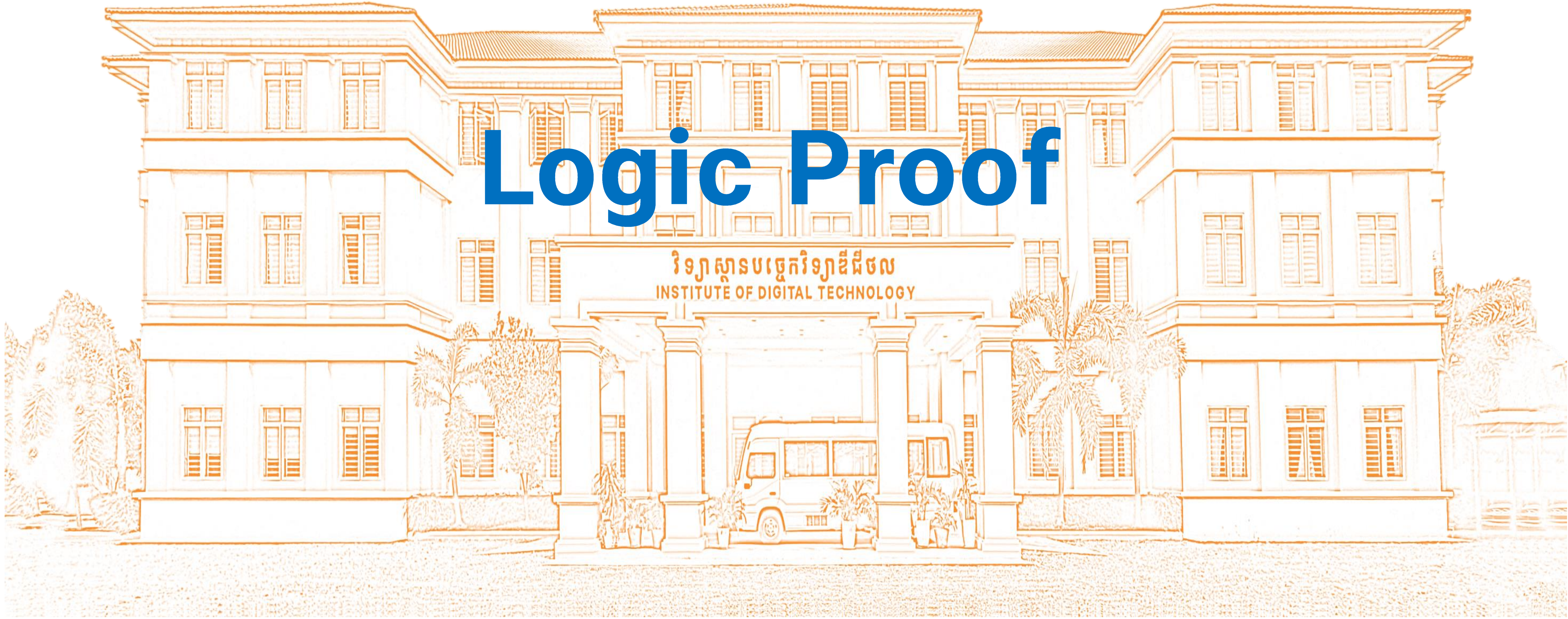
CADT IDT

វិទ្យាស្ថានបច្ចេកវិទ្យាឌីជីថល

Institute of Digital Technology

Department of Foundation Year

Logic Proof



NGETH YOUDARITH

youdarith.ngeth@cadt.edu.kh

(+855) 12 600 012

Prove that $n \in \mathbb{N}$, $1 + (-1)^n(2n - 1)$ is a multiple of 4.

$n \begin{cases} \text{even} \\ \text{odd} \end{cases}$

if n is even and $(-1)^n = 1$

$n = 2k$

but $1 + (-1)^n(2n-1)$ is a multiple of 4

$1 + (-1)^n(2n-1) = 4p$, $p \in \mathbb{N}$

$1 + (-1)^{2k}(2(2k)-1)$

$= 1 + 4k - 1$

$= 4k$ True

if n odd and $(-1)^n = -1$

$n = 2k+1$

$1 + (-1)^n(2n-1) = 1 - (2(2k+1) - 1)$

$= 1 - (4k + 2 - 1)$

$= 1 - 4k - 1$

$1 + (-1)^n(2n-1) = -4k$ True

so $\boxed{1 + (-1)^n(2n-1) \text{ is a multiple of } 4 \text{ for } n \in \mathbb{N}.}$

Prove that if $x \in \mathbb{Q}$ and $y \notin \mathbb{Q}$ then $x + y \notin \mathbb{Q}$

$x \in \mathbb{Q}$: $x = \frac{a}{b}$, $a, b \in \mathbb{Z}$

$y \notin \mathbb{Q}$ $y = c$, $c \in \mathbb{Z}$

P statement

$\Rightarrow p \Rightarrow q$ true

$\Rightarrow \bar{q} \Rightarrow \bar{p}$ true

$q : x + y \notin \mathbb{Q}$ $\bar{p} : y \in \mathbb{Q}$

$\bar{q} : x + y \in \mathbb{Q}$

$x = \frac{a}{b}$

$y = c$

$x + y = \frac{k}{m}$, $k, m \in \mathbb{Z}$

$y = \frac{k}{m} - \frac{a}{b}$

$y = \frac{kb - ma}{mb}$

y is rational number

because $kb - ma$, mb are integer

$\bar{q} \Rightarrow \bar{p}$ is true

so $p \Rightarrow q$ true

if $x \in \mathbb{Q}$ & $y \notin \mathbb{Q}$

then $x + y \notin \mathbb{Q}$

Prove that $n \in \mathbb{N}, 4 \dagger (n^2 + 2)$.

$a | b = \frac{b}{a}$ or $b = ak$ $\quad a \nmid b$, $b \neq ak$

Assume $4 | (n^2+2)$

$n^2 + 2 = 4k$

n even

n odd

if $n = 2m$ $\quad n^2 = 4m^2$

$n^2 + 2 = 4k$ ①.

$n^2 + 2 = (2m)^2 + 2$

$= \underbrace{4m^2}_{4k} + 2$

$= k4 + c$

$n = 2m + 1$

$n^2 + 2 = (2m+1)^2 + 2$

$= 4m^2 + 4m + 1 + 2$

$= 4m^2 + m4 + 3$

$= 4(m^2 + m) + 3$

$= 4k + 3$

so $4 \dagger (n^2+2)$ for $n \in \mathbb{N}$

Prove that if $7x + 9$ is even then x is odd for all $x \in \mathbb{Z}$.

$p \Rightarrow q \iff \bar{q} \Rightarrow \bar{p}$

p: $7x+9$ is even
q: x is odd

$\Rightarrow p \Rightarrow q$

if we want to solve $p \Rightarrow q$ true
we just solve $\bar{q} \Rightarrow \bar{p}$ is true

$\bar{q}$: x is even
$\bar{p}$: $7x+9$ is odd

$x = 2k$

$7x + 9 = 14k + 9$

$7x + 9 = 14k + 8 + 1$
$= 2(7k + 4) + 1$
$= 2m + 1$ odd

Now $\bar{q} \Rightarrow \bar{p}$ is true

Since $p \Rightarrow q$ is also true

$7x+9$ is even then x is odd for all $x \in \mathbb{Z}$

Suppose $x, y \in \mathbb{Z}$ if $5 \nmid xy$, prove that $5 \nmid x$ and $5 \nmid y$

Assume $5 \mid x$

$x = 5k$

$xy = 5ky$

$5 \mid y$

$y = 5k$

$xy = 5kx$

$p : 5 \nmid xy$

$q : 5 \nmid x \ \& \ 5 \nmid y$

$p \Rightarrow q$

$\bar{q} : 5 \mid x \ \& \ 5 \mid y$

$\bar{p} : 5 \mid xy$

$\bar{q} \Rightarrow \bar{p}$

Suppose $a, b \in \mathbb{Z}$ *and* $n \in \mathbb{N}$, if $\underbrace{12a \not\equiv 12b}_{p}$ $(mod\ n)$ prove that $\underbrace{n \dagger 12}_{q}$.

Assume $n | 12$

$\underline{12 = nk}$, $k \in \mathbb{N}$

$12(a-b) = nk(a-b)$

$\underbrace{12a - 12b} = kn(a-b)$

$12a - 12b \equiv 12a \equiv 12b$

$kn(a-b) = [\underbrace{12a \equiv 12b}]$

from $12 = nk$ we get $\underline{12a \equiv 12b}$

mod n

$12a \equiv 12b$

$12a - 12b \equiv kn$

$12(a-b) \equiv kn$

$7 = 2\ (mod\ 5)$

$\bar{q} \Rightarrow \bar{p}$ true

$p \Rightarrow q$ is true

if $n \in \mathbb{N}$, prove that $1 + 3 + 5 + \cdots + (2n - 1) = n^2$.

(Induction proof)

$P(n) = 1 + 3 + 5 + \dots + (2n-1) = n^2$

$P(n) = (2n-1) = n^2$

if $n = 1$: $1 = 1^2$

$n = 2$: $1 + 3 = 2^2$

$n = 3$: $1 + 3 + 5 = 3^2$

$n = 4$: $1 + 3 + 5 + 7 = 4^2$

Assume $n = K$:

$1 + 3 + 5 + \dots + (2K-1) = K^2$

we will solve if $n = K+1$ we get

$P(K+1) = (K+1)^2$

$P(K) = 1 + 3 + 5 + 7 + \dots + (2K-1)$

$P(K+1) = \underbrace{1 + 3 + 5 + 7 + \dots + (2K-1)}_{K^2} + (2K+1)$

$P(K+1) = K^2 + 2K + 1$

$P(K+1) = (K+1)^2$ true

so $1 + 3 + 5 + \dots + (2n-1) = n^2$ for $n \in \mathbb{N}$

$P(K) = 2K-1$

$P(K+1) = 2(K+1) - 1$

$= 2K + 2 - 1$

$= 2K + 1$

if $n \in \mathbb{N}$, prove that $\sum_{i=0}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$

$n=1: 1^2 = \frac{1(1+1)(2.1+1)}{6}$

$1^2 + 2^2 = \frac{2(2+1)(2.2+1)}{6}$

$1^2 + 2^2 + 3^2 = \frac{3(3+1)(2.3+1)}{6}$

$1^2 + 2^2 + 3^2 + \cdots + k^2 = \frac{k(k+1)(2k+1)}{6}$

$\underbrace{1 + 2^2 + 3^2 + \cdots + k^2}_{} + (k+1)^2 = \frac{(k+1)(k+2)(2k+3)}{6}$

$\frac{k(k+1)(2k+1)}{6} + (k+1)^2$