CADT IDT វិទ្យាស្ថានបច្ចេកវិទ្យាឌីជីថល Institute of Digital Technology

Assignment of Matrices and Determinants

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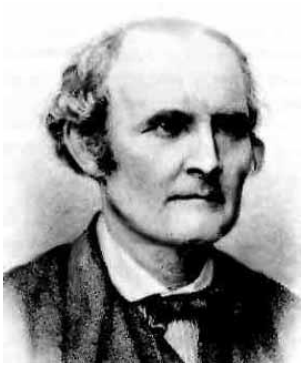
Matrices

A matrix is a rectangular array of elements arranged into rows and columns . If a matrix has m rows and n columns , then the **order** of the matrix is denoted by m x n (read as m by n) . A general matrix of m rows and n columns is

James Sylvester

$$A = [\ a_{ij}\ ]_{m \times n} = \begin{bmatrix} a_{11} & a_{12}\cdots & \cdots a_{1j}\cdots & a_{1n} \\ a_{21} & a_{22}\cdots & \cdots a_{2j}\cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{i1} & a_{i2}\cdots & \cdots a_{ij}\cdots & a_{in} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2}\cdots & \cdots a_{mj}\cdots & a_{mn} \end{bmatrix}_{mxn}$$



Arthur Cayley

$$\text{eg :- } A = [\ a_{ij}\ ]_{3 \times 3} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

1. In the matrix $\begin{bmatrix} 2 & 5 & 19 & 7 \\ \sqrt{2} & 3 & -5 & 11 \\ 6 & -4 & 5 & 2 \end{bmatrix}$

   (a) write the order of the matrix
   (b) The number of elements
   (c) Write the elements a_{12} , a_{31} , a_{33} , a_{42}

2. If a matrix has 15 elements , what are the possible orders it have ?

3. If a matrix has 24elements ,list out all possible orders of that matrix? What, if it has 17 elements ?

4. Construct a 3 x 2 matrix A such that A = $\left|a_{ij}\right|$where a_{ij} = 2i + j

5. Construct a 3 x 3 matrix A such that A = $\left|a_{ij}\right|$where a_{ij} = i - j

6. Construct a 3 X 4 matrix A=[a_{ij}] where $a_{ij} = \dfrac{(i-j)^2}{2}$

7. Construct a 2 x 2 matrix ,$a_{ij} = i^2 + j^2 + 7$

8. Construct a 4 x 3 matrix B such that $b_{ij} = \frac{(i+2j)^2}{2}$

9. Construct a 2 x 3 matrix B = [b_{ij}] whose elements b_{ij} are given by $b_{ij} = \frac{|-3i+2j|}{5}$

10. Construct a 3 x 2 matrix A such that A = $\lfloor a_{ij} \rfloor$ where $a_{ij} = \begin{cases} i+j, i \geq j \\ i-j, i<j \end{cases}$

11. Construct a 3 x 3 matrix A where A = [a_{ij}] where $a_{ij} = 4i - 2j$, $i \neq j$ and $a_{ij} = 0$ when $i = j$.

12. Write a 2 x 2 matrix whose general term is given by $a_{ij} = \left[\frac{i}{j}\right]$ whete [x] denote greatest integer function

13. Construct a 4 x 3 matrix A = [a_{ij}], where $a_{ij} = 2i + \frac{i}{j}$

14. Construct a 2 x 2 matrix A = [a_{ij}], where $a_{ij} = \begin{cases} i-j, i \geq j \\ i+j, i<j \end{cases}$

Different types of Matrices.

(a) **Row matrix** :- A matrix having only one row is called row matrix . Its order is 1 x n

eg :- $[\,1 \;\; 2 \;\; 3 \;\; 4\,]_{1 \times 4}$

(b) **Column matrix** :- A matrix having only one column is column matrix . Its order is n x 1

eg :- $\begin{bmatrix} 1 \\ 3 \\ -4 \\ 5 \end{bmatrix}_{4x1}$

(c) **Zero matrix** :- If all elements of a matrix are zeros , it is called zero matrix or null Matrix denoted by O . eg :- $O = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

(d) **Square matrix** :- If a matrix has same number of rows and columns,it is called square matrix ,order is n x n .

eg :- $A = \begin{bmatrix} 3 & -1 & 1 \\ 0 & 11 & 2 \\ -6 & 78 & 9 \end{bmatrix}_{3x3}$, $B = \begin{bmatrix} 9 & 0 \\ 5 & 2 \end{bmatrix}_{2x2}$

In the matrix A the elements 3 , 11 , 9 are called the principal diagonal elements

(e) **Diagonal matrix** :- A square matrix is said to be diagonal if all the elements except the principal diagonal elements are zero

$$\text{eg :- } A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}_{3x3} = \text{diag}(1, 2, 3)$$

(f) **Scalar matrix** :- A diagonal matrix is said to be scalar matrix if all the principal diagonal elements are same. i.e $a_{ij} = k$ for $i = j$, $a_{ij} = 0$ for $i \neq j$

$$\text{eg :- } A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}_{3x3}$$

(g) **Unit (identity) matrix** :- A scalar matrix is said to be identical if all the principal diagonal elements are unity and is denoted by I_n . i.e $a_{ij} = \begin{cases} 0, i \neq j \\ 1, i = j \end{cases}$

$$\text{eg :- } I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3x3}$$

(h) **Triangular matrices** . They are of two types

(1) **Upper triangular matrix** :- A square matrix is called an upper triangular matrix , if all its elements below the principal diagonal elements are zero .

i.e $[a_{ij}]_{mxn}$ is an upper triangular if $m = n$ and $a_{ij} = 0$ for $i > j$

$$\text{eg:-} \begin{bmatrix} 4 & 3 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 5 \end{bmatrix}_{3x3}$$

(2) **Lower triangular matrix** :- A square matrix is called a lower triangular matrix , if all its elements above the principal diagonal elements are zero .

i.e $[a_{ij}]_{mxn}$ is a lower triangular if $m = n$ and $a_{ij} = 0$ for $i < j$

$$\text{eg:-} \begin{bmatrix} 9 & 0 & 0 \\ 12 & -1 & 0 \\ 5 & 9 & 4 \end{bmatrix}_{3x3}$$

(i) **Comparable Matrices** :- Two matrices are comparable if they are of same order

🗝 **Algebra of matrices.**

(a) Two matrices A and B are equal, written as A = B, iff they are of the same order and their corresponding entries are equal.

15. Find x , y and z if $\begin{bmatrix} 3 & 2 & 1 \\ x & y & 5 \\ 1 & -1 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 1 \\ -1 & 0 & 5 \\ 1 & -1 & z \end{bmatrix}$

16 Find x and y if $\begin{bmatrix} 2x-1 \\ 5 \end{bmatrix} = \begin{bmatrix} 3 \\ x+y \end{bmatrix}$

17. Given a matrix A = $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & 2 & 0 \\ 3 & 4 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ equal ? Give reasons?

18. If $\begin{bmatrix} x+3 & z+4 & 2y-7 \\ -6 & a-1 & 0 \\ b-3 & -21 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 6 & 3y-2 \\ -6 & -3 & 2c+2 \\ 2b+4 & -21 & 0 \end{bmatrix}$, find a , b , c , x , y and z ?

19. If $\begin{bmatrix} 3a+2b & a+3b \\ 5c-d & 3c+4d \end{bmatrix} = \begin{bmatrix} 6 & 7 \\ 8 & 11 \end{bmatrix}$ find a , b , c , d ?

20. Find a and b if A = B where A = $\begin{bmatrix} a+4 & 3b \\ 8 & -6 \end{bmatrix}$ and B = $\begin{bmatrix} 2a+2 & b^2+2 \\ 8 & b^2-10 \end{bmatrix}$

(b) **Addition of matrices**

If A and B are two matrices of same order then A + B is a matrix of same order whose element are obtained by adding corresponding elements of A and B

Properties :- (1) Matrix addition is commutative
(2) Matrix addition is associative
(3)If A , B , C are matrices of same order , then A + B = A + C implies A = C [cancellation law]

(c) If A is a matrix then kA is a matrix obtained by multiplying all elements of A by k

21. Find A + B and A – B if A = $\begin{bmatrix} 5 & 2 & -1 \\ 6 & 4 & -2 \end{bmatrix}$ and B = $\begin{bmatrix} 2 & 8 & 3 \\ -4 & 5 & 9 \end{bmatrix}$

22. If A = $\begin{bmatrix} 2 & -1 \\ 3 & 1 \end{bmatrix}$, B = $\begin{bmatrix} 1 & 4 \\ 7 & 2 \end{bmatrix}$, Find 2A + 3B and 3A – 2B

23. Find x and y if $2\begin{bmatrix}1 & 3\\ 0 & x\end{bmatrix}+\begin{bmatrix}y & 0\\ 1 & 2\end{bmatrix}=\begin{bmatrix}5 & 6\\ 1 & 8\end{bmatrix}$

24. If A = $\begin{bmatrix}2 & -2\\ 4 & 2\\ -5 & 1\end{bmatrix}$ and B = $\begin{bmatrix}8 & 0\\ 4 & -2\\ 3 & 6\end{bmatrix}$, find X such that 2A + 3X = 5 B

25. A = $\begin{bmatrix}-1 & 0 & 2\\ 3 & 1 & 4\end{bmatrix}$, B = $\begin{bmatrix}0 & -2 & 5\\ 1 & 3 & -2\end{bmatrix}$, C = $\begin{bmatrix}5 & 2 & -1\\ 6 & 4 & -2\end{bmatrix}$ compute 2A – 3B + 4C

26. Solve the matrix equation $\begin{bmatrix}x^2\\ y^2\end{bmatrix} - 3\begin{bmatrix}x\\ 2y\end{bmatrix} = \begin{bmatrix}-2\\ 9\end{bmatrix}$

27. Find X and Y if X + Y = $\begin{bmatrix}7 & 0\\ 2 & 5\end{bmatrix}$ and X - Y = $\begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix}$

29. Two farmers Anil and Rajesh cultivate only three varities of rice namely Basmati , Permal and Naura. The sale in rupees of these varities by both farmers on the month of may and june are given below by the following matrices A and B

May sales

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$$A = \begin{array}{c} \\ \\ \end{array}\begin{array}{cc} \begin{array}{ccc} \textit{Basmati} & \textit{Permal} & \textit{Naura} \end{array} & \\ \begin{bmatrix}10000 & 20000 & 30000\\ 50000 & 30000 & 10000\end{bmatrix} & \begin{array}{l} \textit{Anil}\\ \textit{Rajesh}\end{array} \end{array}$$

June sales

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$$A = \begin{array}{cc} \begin{array}{ccc} \textit{Basmati} & \textit{Permal} & \textit{Naura} \end{array} & \\ \begin{bmatrix}5000 & 10000 & 6000\\ 20000 & 10000 & 10000\end{bmatrix} & \begin{array}{l} \textit{Anil}\\ \textit{Rajesh}\end{array} \end{array}$$

Find what were the combined sales in may and june for each farmer in each variety
Find change in sales from may to june

30. Find x , y , z if $\begin{pmatrix}x+y+z\\ x+z\\ y+z\end{pmatrix}=\begin{pmatrix}9\\ 5\\ 7\end{pmatrix}$, x = 2 , y = 4 , z = 3

31. Find X and Y if (a) X + Y = $\begin{bmatrix}7 & 0\\ 2 & 5\end{bmatrix}$ and X – Y = $\begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix}$

(b) 2X + 3Y = $\begin{bmatrix}2 & 3\\ 4 & 0\end{bmatrix}$ and 3X + 2Y = $\begin{bmatrix}2 & -2\\ -1 & 5\end{bmatrix}$

(d) Multiplication of matrices

If $A = [a_{ij}]_{m\times n}$ and $B = [b_{jk}]_{n\times p}$ then $AB = [c_{ik}]_{m\times p}$ where $c_{ik} = \sum_{j=1}^{n} a_{ij}b_{jk}$, $i = 1,2,3,\ldots,m$ and $k = 1,2,3,\ldots p$.

If A is a m x n matrix and B is a n x p matrix , then AB is a m x p matrix .

{ ***Two matrices can be subjected for multiplication , only if the number of columns of the first matrix and the number of row of the second matrix are same*** }

Points to note

(a) If A is a square matrix, then $A^m = A^{m-1}A = AA^{m-1}$, for all $m \in N$.
i.e $A^2 = A\,.A$ & $A^3 = A^2\,.\,A$

(b) If $f(x) = a_0 + a_1 x + a_2x_2 + \ldots\ldots + a_mx^m$ is any polynomial with real (or complex) Coefficients then
$f(A) = a_0I + a_1 A + a_2A^2 + \ldots\ldots + a_m A^m$, A is any square matrix.

(c) If A and B are two matrices such that AB exists , then BA may or may not exist.

(d) Matrix multiplication is not commutative in general

(e) Matrix multiplication is associative i.e (AB) C = A (BC)

(f) matrix multiplication is distributive over matrix addition A(B+C)= AB + AC

(g) **The product of two matrices can be a null matrix while neither of them is the null matrix .**

For example :- If $A = \begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix}$ **and** $B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, **then** $AB = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

32. If $A = \begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$ find AB and BA

33. If $A = \begin{bmatrix} 3 & 2 \\ -1 & 0 \\ -1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 5 & 6 \\ 0 & 1 & 2 \end{bmatrix}$ Find AB and BA

34. If $A = \begin{bmatrix} a & 0 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$ find a so that $A^2 = B$

35. Evaluate $\begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$

36. If $A_\theta = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ Show that $A_\theta \, . \, A_\phi = A_{\theta+\phi}$

37. Evaluate $[\ 1\ 2\ 3\ ] \begin{bmatrix} 1 & 0 & 2 \\ 2 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$

38. If $f(x) = 3x^2 - 9x + 7$, find f(A)?

39. Find x if $\begin{bmatrix} x & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x \\ 3 \end{bmatrix} = 0$

40. Given an example of twonon zero matrices A and B such that AB = 0 ?

41. If $A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$, Find the value of $A^2 - 5A - 14I$?

42. If $M = \begin{bmatrix} 1 & -3 & 2 \\ 4 & 7 & 8 \\ 8 & -3 & -1 \end{bmatrix}$ Evaluate $M^2 - 5M + 22I$?

43. Let $A = \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}$, show that $I + A = (I - A) \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

44. If A is a m x n matrix such that AB and BA are both defined , then write the order of matrix B ?

45. For what values of x , $\begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = 0$

46. Find the number of all possible matrices of order 3 x 3 with each entry 0 or 1 ?

47. What is the condition for m and n for which a matrix $A = [a_{ij}]_{m \times n}$ is a square matrix

48. If A is of order 2 x 3 and B is of order 3 x 2 , then write the order of AB ?

49. Let $f(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$, find f(x) . f(-x)

50. Find f (A) , where $f(x) = x^2-5x+7$ for $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

51. If $f(x) = 2x^2 - 3x$, find f(A) if $A = \begin{bmatrix} -2 & 1 \\ 0 & 3 \end{bmatrix}$

52 If $A = \begin{bmatrix} \cos^2 x & \cos x \sin x \\ \cos x \sin x & \sin^2 x \end{bmatrix}$, $B = \begin{bmatrix} \cos^2 y & \cos y \sin y \\ \cos y \sin y & \sin^2 y \end{bmatrix}$, show that AB is a zero matrix if x and y differ by an odd multiple of $\pi/2$

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53. If $A = \begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix}$, verify that $A^2 = A$

54. If A is a square matrix such that $A^2 = A$, prove that $(I + A)^3 - 7A = A$

55. If $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & -1 \end{bmatrix}$, Evaluate $A^3 - 23A - 40I$

56. If $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$, $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then find k such that $A^2 = kA - 2I$

57. Find the matrix A such that $\begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix} A = \begin{bmatrix} 3 & 3 & 5 \\ 1 & 0 & 1 \end{bmatrix}$

58. If $A = \begin{bmatrix} a & b \\ 0 & 1 \end{bmatrix}$, prove by P.M.I. that $A^n = \begin{bmatrix} a^n & \dfrac{b(a^n-1)}{(a-1)} \\ 0 & 1 \end{bmatrix}$

59. If $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ prove by P.M.I. that $A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$, where n is any positive integer

60. If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$, prove using induction that $A^m = \begin{bmatrix} 3^{m-1} & 3^{m-1} & 3^{m-1} \\ 3^{m-1} & 3^{m-1} & 3^{m-1} \\ 3^{m-1} & 3^{m-1} & 3^{m-1} \end{bmatrix}$

61. If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$, using P.M.I prove that $A^n = \begin{bmatrix} \cos n\alpha & -\sin n\alpha \\ \sin n\alpha & \cos n\alpha \end{bmatrix}$

62. There are three families. Family A consists of 2 men ,3 women and 1 child . Family B consists of 2 men , 1 women and 3 children . Family C has 4 men , 2 women and 6 children . Daily income of men and women are Rs. 65 and 42 respectively and children have no income . Calculate the daily income of each family .

63. If A = $\begin{bmatrix} 0 & -\tan\alpha/2 \\ \tan\alpha/2 & 0 \end{bmatrix}$ and I is an identity matrix of order 2 ,

P.T $I + A = (I - A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

64. Asok , Ramesh and Ravi are friends . Ashok need 5 note books , 3 pen and 2 pencils . Ramesh needs 7 note books , 5 pen and a pencil . Ravi needs 4 note books , 2 pen and 5 pencils . They require the prices in three shops A , B and C . In shops Rs.12 for a note book , Rs.8 for a pen and Rs.2 for a pencil . In shop B , the respective process are Rs.11 , Rs.9 and Rs.3 . In shop C , it is Rs. 15 , Rs.6 and Rs.4 respectively . Express the above information into two matrices P and Q . Find PQ and thus prepare an estimate

65. A trust fund has Rs.30,000 that must be invested in two different types of bonds. The first bond pays 5% interest per year and the second bond pays 7% per year . Using matrix multiplication determine how to divide Rs.30,000 among the two bonds if the trust fund must obtaon an annual total interest of Rs.1800

Transpose of a matrix

If A is a matrix , then a matrix obtained by interchanging rows and columns of A is called the transpose of a matrix and is denoted by A ' . Thus if A is a m x n matrix , then A ' is a n x m matrix

eg:- $If\ A = \begin{bmatrix} 5 & -2 \\ 3 & 4 \\ 9 & 1 \end{bmatrix},\ then\ A^T = \begin{bmatrix} 5 & 3 & 9 \\ -2 & 4 & 1 \end{bmatrix}$

Properties

1. (A ') ' = A 2. (A + B)' = A' + B'
3. (AB)' = B' A'. 4. (k A)' = k A'.

Symmetric and Skew-Symmetric Matrices.

A square matrix A is said to be **symmetric** if **A ' = A**(i.e$a_{ij} = a_{ji}$) and is said to be **skew – symmetric** if A ' = - A (i.e$a_{ij} = -a_{ji}$)

All principal diagonal elements of a skew - symmetric matrix are zeroes.

Note:- (1) **Every square matrix A can be uniquely expressed as the sum of two matrices of which one is symmetric and the other is skew- symmetric**

i.e A = P + Q where

P $= \frac{1}{2}(A + A')$ **a symmetric matrix ,**

Q $= \frac{1}{2}(A - A')$ **a skew-symmetric matrix.**

(2) If A and B are symmetric matrices , then AB is symmetric only when AB = BA or iff A and B commute

(3) For a symmetric matrix , all its positive integral powers are symmetric and for a skew symmetric matrix , all even integral powers are symmetric and all odd integral powers are skew symmetric

(4) If A and B are symmetric matrices of same order , then AB + BA is symmetric Where as AB – BA is skew symmetric matrix

66. If A’ = $\begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix}$ and B = $\begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$, then find (A + 2B)’

67. If A = $\begin{pmatrix} 2 & 1 & 3 \\ 4 & 1 & 0 \end{pmatrix}$ and B = $\begin{pmatrix} 1 & -1 \\ 0 & 3 \\ 5 & 8 \end{pmatrix}$ Verify that $(AB)^T = B^TA^T$

68. Show that every square matrices can be expressed as the sum of two matrices , of which one is symmetric and other is skew-symmetric?

69. Express $\begin{bmatrix} 4 & 2 & -1 \\ 3 & 5 & 7 \\ 1 & 2 & -1 \end{bmatrix}$ as the sum of two matrices of which one is symmetric and other skew symmetric

70. Express $\begin{bmatrix} 1 & 0 & -9 \\ -6 & 2 & 7 \\ 3 & -5 & 6 \end{bmatrix}$ as the sum of two matrices of whichs one is symmetric and other is skew symmetric .

71. If A and B are symmetric matrices of same order , prove that AB + BA is symmetric and AB - BA is a skew-symmetric matrix?

72. Prove that the principal diagonal elements of a skew symmetric matrix are all zero

73. For the matrix A , prove that A + A^T is a symmetric matrix

74. For the matrix A , prove that A - A^T is a skew-symmetric matrix

75. If B is a skew symmetric matrix , prove that (ABA^T) is a skew symmetric matrix ?

76. If A is a 3 x 4 matrix and B is a matrix such that A’B and BA’ are both defined , find the order of B ?

77. Express $\begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrices

78. If A = [1 2 3] , write AA^T

79. Let A = $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ then find AA^T.

80. Name a matrix which is both symmetric as well as skew symmetric

81. If the matrix A = $\begin{bmatrix} 0 & a & -3 \\ 2 & 0 & -1 \\ b & 1 & 0 \end{bmatrix}$ is skew symmetric , find a and b

Determinants

Associated with every square matrix A there defines a real number called determinant of A ,denoted as $|A|$ or det(A) and is defined as follows :

(a) If A = [a_{11}] , then det A = $|A|$ = a_{11}.

(b) If A = $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$, *then* det A = $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$ = $a_{11}a_{22} - a_{21}a_{12}$.

(c) If A = $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$, then det A = $\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

$= a_{11}a_{22}a_{33} - a_{11}a_{32}a_{23} - a_{12}a_{21}a_{33} + a_{12}a_{31}a_{23} + a_{13}a_{21}a_{32} - a_{13}a_{31}a_{22}$.

A square matrix is said to be singular if det A = 0 , otherwise non-singular

If A is a n x n matrix and λ is a constant , then

(a) $|\lambda A| = \lambda^n|A|$ (b) $|\lambda AB| = \lambda^n|A||B|$

1. Evaluate the determinant of following matrices

a) $\begin{bmatrix} 3 & -5 \\ -8 & 9 \end{bmatrix}$ b) $\begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix}$ c) $\begin{bmatrix} 0 & \sin\alpha & -\cos\alpha \\ -\sin\alpha & 0 & \sin\beta \\ -\cos\alpha & -\sin\beta & 0 \end{bmatrix}$ d) $\begin{bmatrix} 1 & 2 & 5 \\ 2 & 3 & 1 \\ -1 & 1 & 1 \end{bmatrix}$

2. For what value of x , the matrix $\begin{bmatrix} 5-x & x+1 \\ 2 & 4 \end{bmatrix}$ is singular ?

3. If $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix}$, prove that $|3A| = 27|A|$

4. Find the integral value of x if $\begin{vmatrix} x^2 & x & 1 \\ 0 & 2 & 1 \\ 3 & 1 & 4 \end{vmatrix} = 28$

5.

6.

7. Evaluate the determinant $\begin{vmatrix} \log_3 512 & \log_4 3 \\ \log_3 8 & \log_4 9 \end{vmatrix}$

8. What positive value of x that makes the pair of determinants equal $\begin{vmatrix} 2x & 3 \\ 5 & x \end{vmatrix}, \begin{vmatrix} 16 & 3 \\ 5 & 2 \end{vmatrix}$

9. If A is a 3 x 3 matrix and if $|A| = 4$ and $|B| = 5$, find $|2AB|$

10. For any 2 x 2 matrix A, $|A| = 5$, find $|kA|$?

Minors and Cofactors.

Let $A = [a_{ij}]$ be a square matrix of order n and A_{ij} be the square matrix obtained from A by deleting the ith row and jth column, then**M_{ij} = minor of a_{ij} = det A_{ij} and A_{ij} = cofactor of a_{ij}= $(-1)^{i+j}$ det A_{ij} = $(-1)^{i+j}M_{ij}$.**

Adjoint of a matrix

Let A be a square matrix , then the transpose of the cofactor matrix is called Adjoint of the matrix and is denoted by **Adj A**.

Properties

1 . $|adj\ A| = |A|^{n-1}$

2. adj AB = (adj B) (adj A)

3. adj A^T = (adj A $)^T$

4. adj (adj A) = $|A|^{n-2}$ A

If A is a n x n matrix and λ is a constant , then

(a) A . (adj A) = $|A| I_n$(b) $|A| |adj\ A| = |A|^n$

11. If A is a square matrix of order 3 such that $|adj\ A| = 100$, find $|A|$

12. What is the value of $|3I_3|$ where I_3 is the identity matrix of order 3 ?

13. A square matrix A is of order 3 , and has $|A| = 5$, find $|A \text{ adj. } A|$

14. Find the minors and cofactors for A = $\begin{bmatrix} 1 & -3 & 2 \\ 4 & 7 & 8 \\ 8 & -3 & -1 \end{bmatrix}$

15. A is a square matrix of order 3 and $|A| = 7$, write the value of $|\text{adj} . A|$

16. Write the adjoint of the following matrix $\begin{pmatrix} 2 & -1 \\ 4 & 3 \end{pmatrix}$

17. If A = $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, verify that A (adj. A) = (adj . A) = $|A| I_2$.

18. Find the adjoint of A if A = $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & -3 \\ -1 & 2 & 3 \end{bmatrix}$

Inverse of a square matrix

Let A be a non singular square matrix of order n , then A is said to be invertible , if there exists a square matrix B such that $AB = I_n = BA$, then B is called the **inverse** of A and is denoted by A^{-1}.

Properties

1 . $A^{-1} = \dfrac{adjA}{|A|}$
2. $(A^{-1})^{-1} = A$
3. If A and B are invertible matrices of same order , then AB is invertible of same order and $(AB)^{-1} = B^{-1} A^{-1}$
4. If A and B are invertible square matrices of same order then adj (AB) = (adj B) (adj A)
5. For singular matrices , inverse doesn't exists .
6. The inverse of a matrix , if it exists is unique
7. $(A^{-1})^T = (A^T)^{-1}$
8. Let A , B , C are square matrices of same order and if AB = AC , then B = C

Elementary row operations

The elementary row operations on a matrix are defined as follows .

(a) The interchange of any two rows

(b) The multiplication of every element of a row by a non-zero constant

(c) The addition to the elements of a row , the products of the corresponding elements of any other row by any non-zero constant .

Method to find the inverse of a matrix by elementary row transformations :-

Let A be a square matrix

Step I : Write A = IA

Step II : Apply a sequence of operations on L.H.S. and prefactor of product IA till we get I = BA

Step III : The matrix B is the required inverse of the matrix A

Note :- If after applying one or more elementary operations , we get all zeros in one row , then A^{-1} does not exists

19. Find the inverse of the matrix $\begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}$.

20. Find the inverse of the matrix $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$.

21. Find the inverse of $\begin{bmatrix} a & b \\ c & \dfrac{1+bc}{a} \end{bmatrix}$

22. Prove that $(AB)^{-1} = B^{-1} . A^{-1}$, where A = $\begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}$ and B = $\begin{bmatrix} 6 & 7 \\ 8 & 9 \end{bmatrix}$

23. If A = $\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, show that $A^2 – 5A + 7I = 0$ and find A^{-1}

24. Show that A = $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ satisfies the equation $A^3 – 6A^2 + 9A – 4I = 0$. Hence find A^{-1}

25. Show that A = $\begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$ satisfies the equation $x^2 – 3x – 7 = 0$, thus find A^{-1} ?

26. If A and B are invertible matrices of order n , then show that AB is also invertible and $(AB)^{-1} = B^{-1}A^{-1}$

27. If A = $\begin{bmatrix} 3 & 1 \\ 4 & 0 \end{bmatrix}$ and B = $\begin{bmatrix} 4 & 0 \\ 2 & 5 \end{bmatrix}$, verify that $(AB)^{-1} = B^{-1}A^{-1}$.

28. Find the inverse of the matrix A = $\begin{bmatrix} a & b \\ c & \dfrac{1+bc}{a} \end{bmatrix}$ and show that $aA^{-1} = (a^2+ bc + 1)I_2 –aA$.

29. If A = $\begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$, show that $A'A^{-1} = \begin{bmatrix} \cos 2x & -\sin 2x \\ \sin 2x & \cos 2x \end{bmatrix}$

30. If A is a square matrix such that $A^3 = I$. Prove that A is non – singular and prove that $A^{-1} = A^2$.

31. Obtain the inverse of the following matrix using elementary operations:

(a) $\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$ (c) $\begin{bmatrix} 3 & 2 \\ 10 & 7 \end{bmatrix}$ (d) $\begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix}$

(e) $\begin{bmatrix} 2 & 3 & 1 \\ 2 & 4 & 1 \\ 3 & 7 & 2 \end{bmatrix}$ (f) $\begin{bmatrix} 1 & 1 & 2 \\ 3 & 1 & 1 \\ 2 & 3 & 1 \end{bmatrix}$ (g) $\begin{bmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$

Applications of determinants

(1)Area of the triangle

If $A(x_1,y_1)$, $B(x_2,y_2)$, $C(x_3,y_3)$ be three vertices of a triangle then the area of thetriangle ABC is ½ $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

Note :- If the three points are collinear , then the area is zero

32. Find the area of a triangle with vertices (5 , 4) , (-2, 4) and (2 , -6)
33. Prove that the points (a, b + c) , (b, c + a) , (c , a + b) are collinear
34. Using determinants , find the equation of the line joining the points (1 , 2)&(3,6)
35. Find the value of x , if area of triangle is 35 sq.units with vertices (x , 4) , (2 , -6) and (5 , 4)

(2) Solution of linear equations (*Cramer's rule*)

Consider 3 linear equations in 3 unknowns :-

$$a_1 x_1 + a_2 x_2 + a_3 x_3 = d_1$$
$$b_1 x_1 + b_2 x_2 + b_3 x_3 = d_2$$
$$c_1 x_1 + c_2 x_2 + c_3 x_3 = d_3$$

Let Δ be the determinant of the coefficients and Δ_1 be the determinant obtained by replacing the first column of Δ by the columns matrix(i.e d_1,d_2,d_3) . Similarly we define Δ_2 ,Δ_3

Case – I

If $\Delta \neq 0$, then the system is said to be **consistent** and has **unique solution** given by

$x_1 = \frac{\Delta_1}{\Delta}$,$x_2 = \frac{\Delta_2}{\Delta}$, $x_3 = \frac{\Delta_3}{\Delta}$

Case – II

If $\Delta = 0$ and each Δ_1 ,Δ_2 , $\Delta_3 = 0$, then we say the system is **consistent** and has **infinite** solutions

Case – III

If $\Delta = 0$ and any of $\Delta_1, \Delta_2, \Delta_3 \neq 0$, then we say the system is **inconsistent** and has **no** solutions

<u>System of linear equations :-</u>(***Matrix method***/ solution by inverse of a matrix)

Consider a system of linear equations

$$a_{11} x_1 + a_{12} x_2 + \dots\dots\dots + a_{1n} x_n = b_1$$
$$a_{21} x_1 + a_{22} x_2 + \dots\dots\dots + a_{2n} x_n = b_1$$
$$\dots\dots \quad \dots\dots \quad \dots\dots \quad \dots\dots \quad \dots..$$
$$\dots\dots \quad \dots\dots \quad \dots\dots \quad \dots\dots \quad \dots..$$
$$a_{n1} x_1 + a_{n2} x_2 + \dots\dots\dots + a_{nn} x_n = b_n$$

These equations can be expressed as AX = B where

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots\dots & a_{1n} \\ a_{21} & a_{22} & \dots\dots & a_{2n} \\ \dots\dots & \dots\dots & \dots\dots & \dots\dots \\ \dots\dots & \dots\dots & \dots\dots & \dots\dots \\ a_{n1} & a_{n2} & \dots\dots & a_{nn} \end{bmatrix}, \; X = \begin{bmatrix} x_1 \\ x_2 \\ : \\ : \\ x_n \end{bmatrix}, \text{ and } B = \begin{bmatrix} b_1 \\ b_2 \\ : \\ : \\ b_n \end{bmatrix}$$ **So that $X = A^{-1} B$**

<u>Checking the consistency</u>

<u>Case – I</u>

If det A $\neq 0$,then the system is said to be **<u>consistent</u>** and has **<u>unique solution</u>** given by $X = A^{-1} B$

<u>Case – II</u>

If det A = 0 and adjA . B = 0 , then we say the system is **<u>consistent</u>** and has **<u>infinite solutions</u>**

<u>Case – III</u>

If det A = 0 and adj A . B $\neq 0$, then we say the system is **<u>inconsistent</u>** and has **<u>no solutions</u>**

36. Using matrices , solve the following system of equations :

$$x + y + z = 6 , \; x + 2z = 7 , \; 3x + y + z = 12$$

37. Using matrices , solve the system of equations

$$x + 2y - 3z = -4$$
$$2x + 3y + 2z = 2$$

$3x - 3y - 4z = 11$

38. Solve by using matrix method $x - y + 2z = 7$, $3x + 4y - 5z = -5$, $2x - y + 3z = 12$

39. If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1} and then solve the system of equations

$2x - 3y + 5z = 11$, $3x + 2y - 4z = -5$, $x + y - 2z = -3$

40. Find the product $A = \begin{bmatrix} 2 & 3 & 4 \\ 5 & 4 & -6 \\ 3 & -2 & -2 \end{bmatrix}$, $B = \begin{bmatrix} 20 & 2 & 34 \\ 8 & 16 & -32 \\ 22 & -13 & 7 \end{bmatrix}$. Use it to solve the system of equations $\frac{2}{x}+\frac{3}{y}+\frac{4}{z}=-3$, $\frac{5}{x}+\frac{4}{y}-\frac{6}{z}=4$, $\frac{3}{x}-\frac{2}{y}-\frac{2}{z}=6$

41. Use the product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}, \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ to solve the system of equations $x - 2y + 2z = 1$, $2y - 3z = 1$, $3x - 2y + 4z = 2$

42. Find the matrix P satisfying the matrix equation $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} P \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$

43. The cost of 4 kg onion , 3 kg wheat and 2 kg rice is Rs. 60 . The cost of 2 kg onion , 4 kg wheat and 6 kg rice is Rs. 90 . The cost of 6 kg onion , 2 kg wheat and 3 kg rice is Rs. 70 . Find cost of each item per kg by matrix method ?

44. Using matrix method , solve the following system of equations

(a) $x + 2y + z = 11$
$x + 3z = 11$
$2x - 3y = 1$.

(b) $2x + 3y + 5z = 15$
$3x + 5y + 2z = 12$
$5x + 2y + 3z = 13$

(c) $\frac{2}{x}+\frac{3}{y}+\frac{10}{z}=4$, $\frac{4}{x}-\frac{6}{y}+\frac{5}{z}=1$, $\frac{6}{x}+\frac{9}{y}-\frac{20}{z}=2$

45. Find A^{-1}, if $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & 4 \end{bmatrix}$ and hence solve the equations

$x + 2y - 3z = -4$
$2x + 3y + 2z = 2$
$3x - 3y - 4z = 11$

46. If $A = \begin{pmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{pmatrix}$ are two square matrices , verify that $AB = BA = 6I_3$ and hence solve $x - y = 3$, $2x + 3y + 4z = 17$, $y + 2z = 7$

47. Find A^{-1}, where $A = \begin{bmatrix} -1 & 2 & 5 \\ 2 & -3 & 1 \\ -1 & 1 & 1 \end{bmatrix}$. Hence solve the equations :-

$-x + 2y + 5z = 2$; $2x - 3y + z = 15$; $-x + y + z = -3$

🗝 **Properties of Determinants.**

(a) det (I_n) = 1, where I_n is identity matrix of order n.

(b) Let A be a square matrix of order n then the sum of products of elements of a row (orcolumn) with their corresponding cofactors. = det A .

(c) If A and B are square matrices of same order , then det (AB) = det A det B

(d) The value of a determinant remain unaltered if its rows and columns are interchanged i.e det A = det A'

(e) If two rows (or columns) are interchanged , the value of the determinant changes its sign

(f) If two rows (or columns) of a determinant are identical , then the value of the determinant is zero

(g)If each element of a row (or column) is multiplied by the same number , then the value of the determinant is also multiplied by that number

(h) If A is a square matrix of order n and α is any scalar , then det (αn) =α^n det A

(i) If each element in any row (or column)of a determinant consists of two terms , then the determinant can be expressed as the sum of two determinants

48. Without expanding , prove that $\begin{vmatrix} x & a & x+a \\ y & b & y+b \\ z & c & z+c \end{vmatrix} = 0$

49. Using properties of determinants , prove that $\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix} =0$

31. Prove that $\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3$

32. Prove that $\begin{vmatrix} a+b+nc & na-a & nb-b \\ nc-c & b+c+na & nb-b \\ nc-c & na-a & c+a+nb \end{vmatrix} = n(a+b+c)^3$.

33. Prove that $\begin{vmatrix} -a^2 & ab & ac \\ ab & -b^2 & bc \\ ac & bc & -c^2 \end{vmatrix} = 4a^2b^2c^2$

34. Prove that $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$

35. Prove that $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix}$ = (a-b)(b-c)(c-a)(a+b+c)

36. If x , y , z are all different and , $\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$ Show that xyz = -1

37. Prove that $\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$

38. Using properties of determinants , prove that $\begin{vmatrix} ab & a+b & 1 \\ bc & b+c & 1 \\ ca & c+a & 1 \end{vmatrix} = (a-b)(b-c)(c-a)$

39. Prove by using properties of determinants , $\begin{vmatrix} b^2+c^2 & ab & ac \\ ba & c^2+a^2 & bc \\ ac & bc & a^2+b^2 \end{vmatrix} = 4a^2b^2c^2$

40. If [.] denote the greatest integer function and $-1 \le x < 0$, $0 \le y < 1$, $1 \le z < 2$, then find the value of the determinant $\begin{vmatrix} [x]+1 & [y] & [z] \\ [x] & [y]+1 & [z] \\ [x] & [y] & [z]+1 \end{vmatrix}$

41. Evaluate the determinant $\Delta = \begin{vmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{vmatrix}$. Also prove that $2 \le \Delta \le 4$

42. Prove that $\begin{vmatrix} b+c & c+a & a+b \\ q+r & r+p & p+q \\ y+z & z+x & x+y \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$

43. Show that $\begin{vmatrix} x+y & x & x \\ 5x+4y & 4x & 2x \\ 10x+8y & 8x & 3x \end{vmatrix} = x^3$.

44. Show that $\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1 + a^2 + b^2)^3$.

45. Prove that $\begin{vmatrix} a & a^2 & bc \\ b & b^2 & ac \\ c & c^2 & ab \end{vmatrix} = (a - b) (b - c) (c - a) (ab + bc + ca)$

46. Prove that $\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2$

47. Show that $\begin{vmatrix} a & b-c & c+a \\ a+c & b & c-a \\ a-b & b+a & c \end{vmatrix} = (a + b + c) (a^2 + b^2 + c^2)$

48. Prove that $\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ yz & xz & xy \end{vmatrix} = (y - z) (z - x) (x - y) (yz + zx + xy)$

49. If a , b , c are in A.P. prove that $\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix} = 0$

50. Show that $\begin{vmatrix} a & a+b & a+b+c \\ 2a & 3a+2b & 4a+3b+2c \\ 3a & 6a+3b & 10a+6b+3c \end{vmatrix} = a^3$

51. Prove that $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$

h52. Prove that $\begin{vmatrix} a^2 & 2ab & b^2 \\ b^2 & a^2 & 2ab \\ 2ab & b^2 & a^2 \end{vmatrix} = \left(a^3+b^3\right)^2$

53 Using properties of determinants , prove that $\begin{vmatrix} -bc & b^2+bc & c^2+bc \\ a^2+ac & -ac & c^2+ac \\ a^2+ab & b^2+ab & -ab \end{vmatrix} = (ab+bc+ca)^3$

54. If a , b , c are the pth , qth , rth terms of a G.P. , prove that $\begin{vmatrix} \log a & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix} = 0$

55. Using properties of determinants prove that $\begin{vmatrix} \sin\alpha & \cos\alpha & \cos(\alpha+\delta) \\ \sin\beta & \cos\beta & \cos(\beta+\delta) \\ \sin\gamma & \cos\gamma & \cos(\gamma+\delta) \end{vmatrix} = 0$

56. Prove without expanding that

(a) $\begin{vmatrix} b+c & q+r & y+z \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix} = 2\begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix}$ h(b) $\begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ca & cb & -c^2 \end{vmatrix}$ = 4 $a^2b^2c^2$

(c) $\begin{vmatrix} x & x^2 & yz \\ y & y^2 & xz \\ z & z^2 & xy \end{vmatrix}$ = (x – y)(y – z)(z – x)(xy + yz + xz)

h57. Prove that $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$

h58. Prove that $\begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix} = (5x+4)(4-x)^2$

h59. Prove that $\begin{vmatrix} 3a & -a+b & -a+c \\ -b+a & 3b & -b+c \\ -c+a & -c+b & 3c \end{vmatrix}$ = 3(a + b + c)(ab + bc + ca)

60. If a , b , c are in A.P. then prove that the value of $\begin{vmatrix} x+2 & x+3 & x+2a \\ x+3 & x+4 & x+2b \\ x+4 & x+5 & x+2c \end{vmatrix} = 0$

61. Prove that the determinant $\begin{vmatrix} x & \sin t & \cos t \\ -\sin t & -x & 1 \\ \cos t & 1 & x \end{vmatrix}$ is independent of t

62. If $\begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix} X = \begin{bmatrix} -4 & 8 & 4 \\ -1 & 2 & 1 \\ -3 & 6 & -3 \end{bmatrix}$, find X

63. If $\begin{bmatrix} 2 & 5 \\ -3 & 7 \end{bmatrix}$ x A = $\begin{bmatrix} 17 & -1 \\ 47 & -13 \end{bmatrix}$, then (1) Find the 2 x 2 matrix A (2) Find A^2

(3) show that $A^2 + 5A - 6I = 0$, where I is the identity matrix of order 2 (4) What is A^{-1} ?

64. Match the following (A is a non singular matrix)

A	B
i. A. (adj . A)	a. A^{-1}
ii. A^2 = I , then A^{-1} =	b.I
iii. A is orthogonal matrix if AA^T =	c. $\lvert A \rvert$ I
iv. $\dfrac{adj\,A}{\lvert A \rvert}$	d. A

HIGHER ORDER THINKING SKILLS

65. Prove that the inverse of every square matrix , if it exists is unique ?

66. If A is a square matrix such that A^2 = A , then using mathematical induction , prove that $(I + A)^n = I + (2^n-1)A$

67. If A is a symmetric as well as skew symmetric matrix , prove that A is a null matrix

68. Prove that the determinant $\begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$ is independent of θ

69. If the system of equations ax + y + z = 0 , x + by + z = 0 , x + y + cz = 0 where a , b , c ≠ 1 has a non trivial solution , prove that $\frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}=1$

70. If a ≠ p , b ≠ q , c ≠r and $\begin{vmatrix} p & b & c \\ a & q & c \\ a & b & r \end{vmatrix} = 0$, then find the value $\frac{p}{p-a}+\frac{q}{q-b}+\frac{r}{r-c}$

71. Show that $\begin{vmatrix} a & b-c & c+b \\ a+c & b & c-a \\ a-b & b+a & c \end{vmatrix} = (a+b+c)(a^2+b^2+c^2)$

72. Show that $\begin{vmatrix} -2a & a+b & a+c \\ b+a & -2b & b+c \\ c+a & c+b & -2c \end{vmatrix} = 4(b+c)(c+a)(a+b)$

73. For a fixed number n , if $\begin{vmatrix} n! & (n+1)! & (n+2)! \\ (n+1)! & (n+2)! & (n+3)! \\ (n+2)! & (n+3)! & (n+4)! \end{vmatrix}$,show that $\frac{D}{(n!)^3}-4$ is divisible by n

74. If $a_1 , a_2 , \ldots , a_n , \ldots$ are in G.P., find the value of $\begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix}$

75. Prove that $\begin{vmatrix} xC_1 & xC_2 & xC_3 \\ yC_1 & yC_2 & yC_3 \\ zC_1 & zC_2 & zC_3 \end{vmatrix} = \frac{xyz}{12}(x-y)(y-z)(z-x)$

OBJECTIVE QUESTIONS

1. If A and B are two matrices of order 3 x m and 3 x n respectively and m = n , then order of 5A – 2B is

a) m x 3 b) 3 x 3 c) m x n d) 3 x n

2. If A = $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ then A^2 is

a) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$ c) $\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$ d) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

3. If A = $\begin{bmatrix} 2 & m & -3 \\ 0 & 2 & 5 \\ 1 & 1 & 3 \end{bmatrix}$ then A^{-1} exists if

a)2 b) ≠ 2 c) ≠ -2 d)none of these

4. If A is a matrix of order 3 x 3 , then the number of minors of A are

a)3 b)9 c)27 d)81

5. $\begin{bmatrix} 2x+y & 4x \\ 5x-7 & 4x \end{bmatrix} = \begin{bmatrix} -7 & 7y-13 \\ y & x+6 \end{bmatrix}$ then the value of x + y is

a) 4 b)5 c)6 d)7

6. There are two values of a which makes the determinant $\begin{vmatrix} 1 & -2 & 5 \\ 2 & a & -1 \\ 0 & 4 & 2a \end{vmatrix} = 86$, then the sum of these values is

a)4 b)5 c)-4 d)9

7. ……………matrix is both symmetric and skew symmetric

8. If for matrix A , $|A| = 5$, find $|4A|$ where A is a matrix of order 2 x 2

9. Show by an example of two matrices A ≠0 , B ≠ 0 but AB = 0

10. The matrix $\begin{bmatrix} 0 & -5 & 8 \\ 5 & 0 & 12 \\ -8 & -12 & 0 \end{bmatrix}$ is a

a) diagonal matrix b) symmetric matrix c) skew symmetric matrix d)scalar

11. The matrix $\begin{bmatrix} 5 & 10 & 3 \\ -2 & -4 & 6 \\ -1 & -2 & b \end{bmatrix}$ is a singular matrix , then b is

a) -3 b)3 c)0 d)any real number

12. A is a matrix of order 3x 3 such that $|A| = 2$ then the value of $|\text{adj adj } A|$ is

a) – 16 b) 16 c)0 d) 2

13. A matrix which is nota square matrix is called ……… matrix

14. If A and B are skew symmetric matrices then AB – BA is a …….

15. If A is a square matrix , then $A + A^T$ is a …… matrix and $A - A^T$ is a ……… matrix

16. Given a 2 x 2 matrix , A=[a_{ij}] where $a_{ij} = \dfrac{(i-2j)^2}{3}$ then a_{21} is

a)0 b) 1/3 c)3 d)2/3

17. If A is a symmetric matrix then A^3 is a ……matrix

18. If for matrix A , $|A| = 5$ then find $|4A|$ where A is of order 2 x 2

19. The adjoint of the matrix A = $\begin{bmatrix} p & q \\ r & s \end{bmatrix}$ is

a) $\begin{bmatrix} s & -q \\ -r & p \end{bmatrix}$ b) $\begin{bmatrix} s & q \\ r & -p \end{bmatrix}$ c) $\begin{bmatrix} 0 & 0 \\ 0 & q \end{bmatrix}$ d) none of these

20. The value of $\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$ is

a) 1 b) 0 c) a + b d) a – b

21. If A = $\begin{bmatrix} \cos\vartheta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, then for any natural number n , the value of det (A ‘) is …

22. If a matrix has 5 elements , write all possible orders it can have

23. If $\begin{bmatrix} 3x-2y & 5 \\ x & -2 \end{bmatrix} = \begin{bmatrix} 3 & 5 \\ -3 & -2 \end{bmatrix}$, find the value of y

24. If A is a square matrix such that $A^2 = I$, then $(A - I)^3 + (A + I)^3 - 7A$ is

a) A b) I – A c) I + A d) 3A

25. Sum of two skew symmetric matrix is always ……..matrix

26. Write the value of the determinant $\begin{vmatrix} 102 & 18 & 36 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix}$

27. If A = $\begin{bmatrix} 2 & m & -3 \\ 0 & 2 & 5 \\ 1 & 1 & 3 \end{bmatrix}$ and A^{-1} exists , then find the value of m

28. If A is a 3 x 3 invertible matrix then what will be value of k if $|A^{-1}| = |A|^k$

29. If A is a matrix of order m x n and B is a matrix such that AB' and BA' are both defined , then the order of B is

a) n x m b) n x n c) n x m d) m x n

30. For any matrices , we have

a) AB = BA b) AB ≠ BA C) AB = 0 d) none of these

31. The sum of product of elements of any row with the cofactors of corresponding elements is equal to ……

32. Write a square matrix of order two , which is both symmetric and skew symmetric

33. The matrix P = $\begin{bmatrix} 0 & 0 & 4 \\ 0 & 4 & 0 \\ 4 & 0 & 0 \end{bmatrix}$ is a

a) square matrix b) Diagonal matrix c) Unit matrix d) none

34. The trace of the matrix $\begin{bmatrix} 2 & 7 & 4 \\ 0 & -3 & 2 \\ 4 & 7 & 9 \end{bmatrix}$ is ….

35. Father of matrix theory is ……

36. The negative of a matrix is obtained by multiplying it by ……

37. If A and B are square matrices of same order , then (AB)' = ….

38. f(x) = $\begin{bmatrix} 0 & x-a & x-b \\ x+a & 0 & x-c \\ x+b & x+c & 0 \end{bmatrix}$ then

a) f(a) =0 (b) f(b) = 0 c) f(0) = 0 d) f(1) = 0