

I. Find LU-factorization for the following matrix

1.  $\begin{bmatrix} 2 & 4 & 2 \\ 1 & 1 & 2 \\ -1 & 0 & 2 \end{bmatrix}$

2.  $\begin{bmatrix} 2 & 4 & 2 \\ 1 & -1 & 3 \\ -1 & 7 & -7 \end{bmatrix}$

3.  $\begin{bmatrix} 1 & 1 & -1 & 3 \\ 2 & 4 & -6 & 5 \\ -1 & -2 & 3 & 0 \\ 2 & 5 & -10 & 1 \end{bmatrix}$

II. Show that  $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 1 & 3 & 4 \end{bmatrix}$  does not have an LU decomposition.

III. Which, if any, of these matrices have LU decomposition.

a.  $A = \begin{bmatrix} 3 & 2 \\ 0 & 1 \end{bmatrix}$       b.  $A = \begin{bmatrix} 0 & 1 \\ 3 & 2 \end{bmatrix}$       c.  $A = \begin{bmatrix} 1 & -3 & 7 \\ -2 & 6 & 1 \\ 0 & 3 & -2 \end{bmatrix}$

IV. Reorder the rows of  $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 1 & 3 & 4 \end{bmatrix}$  so that the reordered matrix has LU decomposition.

V. Reorder the rows of  $A = \begin{bmatrix} 1 & -3 & 7 \\ -2 & 6 & 1 \\ 0 & 3 & -2 \end{bmatrix}$  so that the reordered matrix has LU decomposition.

VI. The ministry of Health had survey people in one village (using health service) and they got the data as

In 1985 227 people use health service

In 1995 249 people use health service

In 2005 281 people use health service

a. write the quadratic equation which through of the points above.

b. How many people using healthy service in 2025.

VII. Calculation  $\det(A)$ ,  $\text{adj}(A)$ ,  $A^{-1}$ ,  $\det(A^{-1})$ ,  $\det(5A)$  of matrix  $A = \begin{bmatrix} 2 & 0 & 1 \\ -3 & 3 & -1 \\ -3 & 1 & -1 \end{bmatrix}$

VIII. Solve the system of linear equation by LU Decomposition  $\begin{cases} x_1 + 2x_2 + 3x_3 = 1 \\ 2x_1 + 3x_2 + 4x_3 = 3 \\ x_1 + 2x_2 + x_3 = 3 \end{cases}$

IX. Consider  $A = \begin{bmatrix} 1 & 6 & 2 \\ 2 & 12 & 5 \\ -1 & -3 & -1 \end{bmatrix}$

a. Show that A doesn't have an LU decomposition.

b. Re-order the rows of A and find an LU decomposition of the new matrix.

c. Hence solve

$$x_1 + 6x_2 + 2x_3 = 9$$

$$2x_1 + 12x_2 + 5x_3 = -4$$

$$-x_1 - 3x_2 - x_3 = 17$$

X. Find Cholesky Decomposition of Matrices below

$$1. A = \begin{bmatrix} 16 & 20 & -12 \\ 20 & 29 & -5 \\ -12 & -5 & 35 \end{bmatrix}$$

$$2. A = \begin{bmatrix} 4 & 4 & 8 \\ 4 & 13 & 2 \\ 8 & 2 & 56 \end{bmatrix}$$

$$3. A = \begin{bmatrix} 9 & 3 & 6 \\ 3 & 2 & 0 \\ 6 & 0 & 12 \end{bmatrix}$$

XI. Find Eigenvalues and Eigenvectors of Matrices

$$1. A = \begin{bmatrix} 2 & 3 \\ 4 & 10 \end{bmatrix}$$

$$2. A = \begin{bmatrix} 1 & 1 & 1 \\ -1 & -3 & -3 \\ 2 & 4 & 4 \end{bmatrix}$$

$$3. A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

$$4. A = \begin{bmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{bmatrix}$$