CADT IDT វិទ្យាស្ថានបច្ចេកវិទ្យាឌីជីថល Institute of Digital Technology

Summarize of 1’s , 2’s , 9’s and 10’s complement

Lecture: Ngeth Youdarith

Email: youdarith.ngeth@cadt.edu.kh

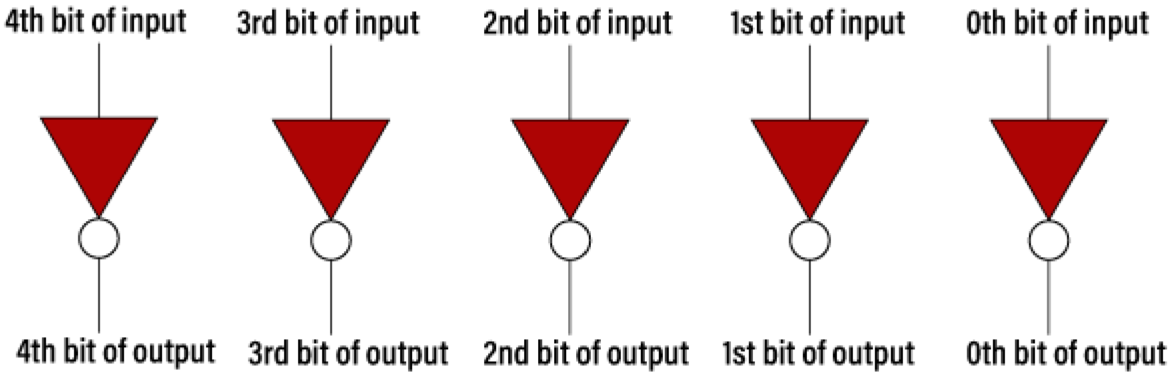
1's complement

In number representation techniques, the binary number system is the most used representation technique in digital electronics. The complement is used for representing the negative decimal number in binary form. Different types of complement are possible of the binary number, but 1's and 2's complements are mostly used for binary numbers. We can find the 1's complement of the binary number by simply inverting the given number. For example, 1's complement of binary number 1011001 is 0100110. We can find the 2's complement of the binary number by changing each bit(0 to 1 and 1 to 0) and adding 1 to the least significant bit. For example, 2's complement of binary number 1011001 is (0100110)+1=0100111.

For finding 1's complement of the binary number, we can implement the logic circuit also by using NOT gate. We use NOT gate for each bit of the binary number. So, if we want to implement the logic circuit for 5-bit 1's complement, five NOT gates will be used.

Binary Number	1's Complement
0000	1111
0001	1110
0010	1101
0011	1100
0100	1011
0101	1010
0110	1001
0111	1000
1000	0111
1001	0110
1010	0101

1011	0100
1100	0011
1101	0010
1110	0001
1111	0000



Example 1: 11010.1101

For finding 1's complement of the given number, change all 0's to 1 and all 1's to 0. So the 1's complement of the number 11010.1101 comes out **00101.0010**.

Example 2: 100110.1001

For finding 1's complement of the given number, change all 0's to 1 and all 1's to 0. So, the 1's complement of the number 100110.1001 comes out **011001.0110**.

1's Complement Table

Use of 1's complement

1's complement plays an important role in representing the signed binary numbers. The main use of 1's complement is to represent a signed binary number. Apart from this, it is also used to perform various arithmetic operations such as addition and subtraction.

In signed binary number representation, we can represent both positive and negative numbers. For representing the positive numbers, there is nothing to do. But for representing negative numbers, we have to use 1's complement technique. For representing the negative number, we first have to represent it with a positive sign, and then we find the 1's complement of it.

Let's take an example of a positive and negative number and see how these numbers are represented.

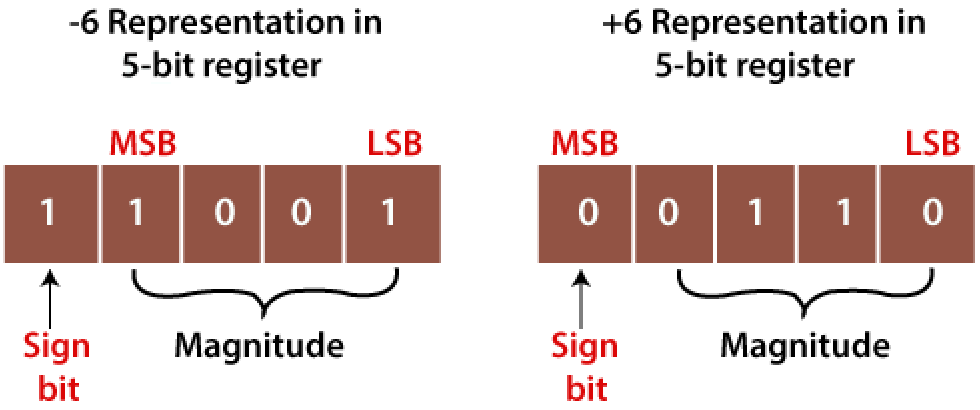
Example 1: +6 and -6

The number +6 is represented as same as the binary number. For representing both numbers, we will take the 5-bit register.

So the +6 is represented in the 5-bit register as 0 0110.

The -6 is represented in the 5-bit register in the following way:

1. +6=0 0110
2. Find the 1's complement of the number 0 0110, i.e., 1 1001. Here, MSB denotes that a number is a negative number.



Here, MSB refers to Most Significant Bit, and LSB denotes the Least Significant Bit.

Example 2: +120 and -120

The number +120 is represented as same as the binary number. For representing both numbers, take the 8-bit register.

So the +120 is represented in the 8-bit register as 0 1111000.

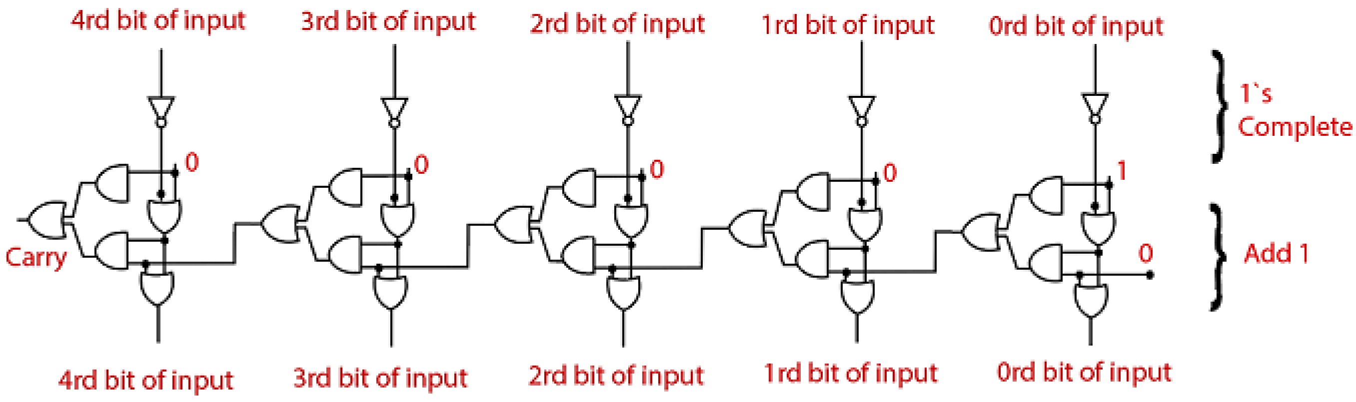
The -120 is represented in the 8-bit register in the following way:

1. +120=0 1111000
2. Now, find the 1's complement of the number 0 1111000, i.e., 1 0000111. Here, the MSB denotes the number is the negative number.

2's complement

Just like 1's complement, 2's complement is also used to represent the signed binary numbers. For finding 2's complement of the binary number, we will first find the 1's complement of the binary number and then add 1 to the least significant bit of it.

For example, if we want to calculate the 2's complement of the number 1011001, then firstly, we find the 1's complement of the number that is 0100110 and add 1 to the LSB. So, by adding 1 to the LSB, the number will be (0100110)+1=0100111. We can also create the logic circuit using OR, AND, and NOT gates. The logic circuit for finding 2's complement of the 5-bit binary number is as follows:



Example 1: 110100

For finding 2's complement of the given number, change all 0's to 1 and all 1's to 0. So the 1's complement of the number 110100 is 001011. Now add 1 to the LSB of this number, i.e., (001011)+1=001100.

Backward Skip 10sPlay VideoForward Skip 10s

Example 2: 100110

For finding 1's complement of the given number, change all 0's to 1 and all 1's to 0. So, the 1's complement of the number 100110 is 011001. Now add one the LSB of this number, i.e., (011001)+1=011010.

2's Complement Table

Binary Number	1's Complement	2's complement
0000	1111	0000
0001	1110	1111
0010	1101	1110
0011	1100	1101
0100	1011	1100
0101	1010	1011
0110	1001	1010
0111	1000	1001
1000	0111	1000

1001	0110	0111
1010	0101	0110
1011	0100	0101
1100	0011	0100
1101	0010	0011
1110	0001	0010
1111	0000	0001

Use of 2's complement

2's complement is used for representing signed numbers and performing arithmetic operations such as subtraction, addition, etc. The positive number is simply represented as a magnitude form. So there is nothing to do for representing positive numbers. But if we represent the negative number, then we have to choose either 1's complement or 2's complement technique. The 1's complement is an ambiguous technique, and 2's complement is an unambiguous technique. Let's see an example to understand how we can calculate the 2's complement in signed binary number representation.

Example 1: +6 and -6

The number +6 is represented as same as the binary number. For representing both numbers, take the 5-bit register.

So the +6 is represented in the 5-bit register as 0 0110.

The -6 is represented in the 5-bit register in the following way:

1. +6=0 0110
2. Now, find the 1's complement of the number 0 0110, i.e. 1 1001.
3. Now, add 1 to its LSB. When we add 1 to the LSB of 11001, the newly generated number comes out 11010. Here, the sign bit is one which means the number is the negative number.

Example 2: +120 and -120

The number +120 is represented as same as the binary number. For representing both numbers, take the 8-bit register.

So the +120 is represented in the 8-bit register as 0 1111000.

The -120 is represented in the 8-bit register in the following way:

1. +120=0 1111000
2. Now, find the 1's complement of the number 0 1111000, i.e. 1 0000111. Here, the MSB denotes the number is the negative number.
3. Now, add 1 to its LSB. When we add 1 to the LSB of 1 0000111, the newly generated number comes out 1 0001000. Here, the sign bit is one, which means the number is the negative number.

9's and 10's Complement

If the number is binary, then we use 1's complement and 2's complement. But in case, when the number is a decimal number, we will use the 9's and 10's complement. The 10's complement is obtained from the 9's complement of the number, and we can also find the 9's and 10's complement using the r's and (r-1)'s complement formula.

9's Complement

The 9's complement is used to find the subtraction of the decimal numbers. The 9's complement of a number is calculated by subtracting each digit of the number by 9. For example, suppose we have a number 1423, and we want to find the 9's complement of the number. For this, we subtract each digit of the number 1423 by 9. So, the 9's complement of the number 1423 is 9999-1423= 8576.

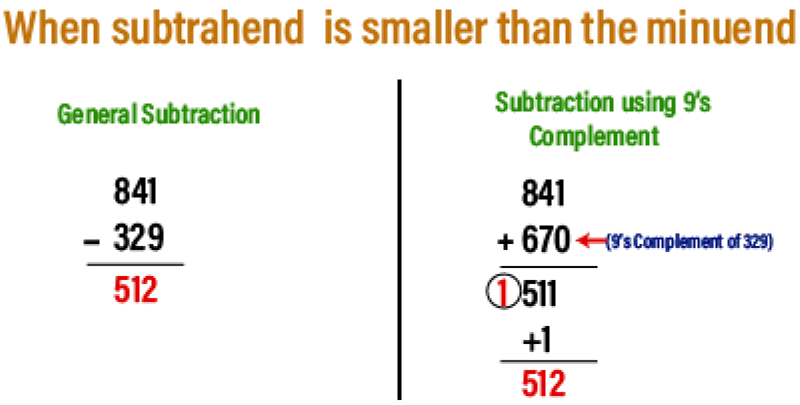
Subtraction using 9's complement

With the help of the 9's complement, the process of subtraction is done in a much easier way. Generally, we subtract the subtrahend from the minuend, but in a case when we perform subtraction using 9's complement, there is no need to do the same.

For subtracting two numbers using 9's complement, we first have to find the 9's complement of the subtrahend and then we will add this complement value with the minuend. There are two possible cases when we subtract the numbers using 9's complement.

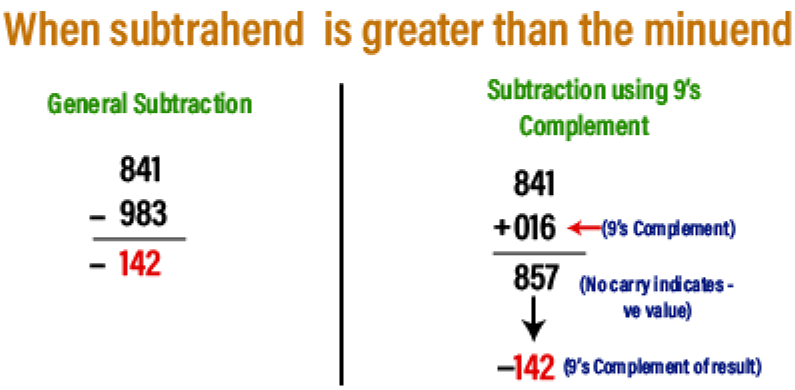
Case 1: When the subtrahend is smaller than the minuend.

For subtracting the smaller number from the larger number using 9's complement, we will find the 9's complement of the subtrahend, and then we will add this complement value with the minuend. By adding both these values, the result will come in the formation of carry. At last, we will add this carry to the result obtained previously.



Case 2: When the subtrahend is greater than the minuend.

In this case, when we add the complement value and the minuend, the result will not come in the formation of carry. This indicates that the number is negative, and for finding the final result, we need to find the 9's complement of the result.



10's Complement

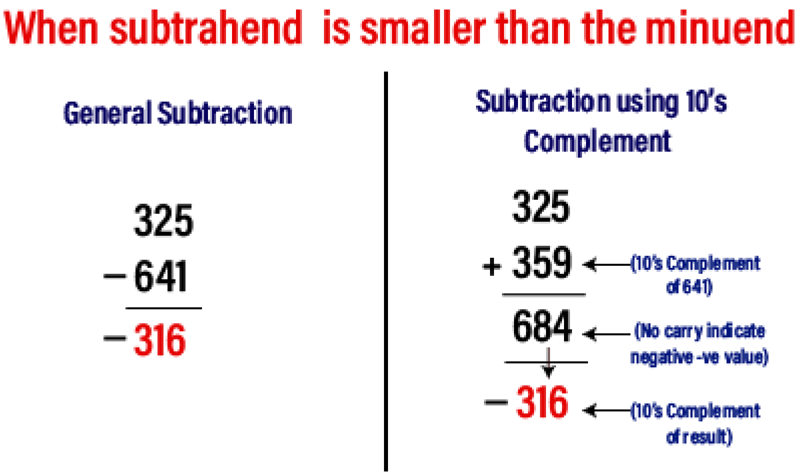
The 10's complement is also used to find the subtraction of the decimal numbers. The 10's complement of a number is calculated by subtracting each digit by 9 and then adding 1 to the result. Simply, by adding 1 to its 9's complement we can get its 10's complement value. For example, suppose we have a number 1423, and we want to find the 10's complement of the number. For this, we find the 9's complement of the number 1423 that is 9999-1423= 8576, and now we will add 1 to the result. So the 10's complement of the number 1423 is 8576+1=8577.

Subtraction using 10's complement

For subtracting two numbers using 10's complement, we first have to find the 10's complement of the subtrahend, and then we will add this complement value with the minuend. There are two possible cases when we subtract the numbers using 10's complement.

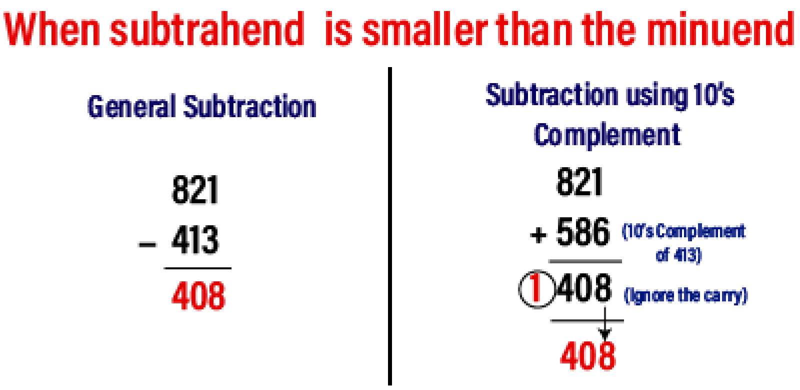
Case 1: When the subtrahend is smaller than the minuend.

For subtracting the smaller number from the larger number using 10's complement, we will find the 10's complement of the subtrahend and then we will add this complement value with the minuend. By adding both these values, the result will come in the formation of carry. We ignore this carry and the remaining digits will be the answer.



Case 2: When the subtrahend is greater than the minuend.

In this case, when we add the complement value and the minuend, the result will not come in the formation of carry. This indicates that the number is negative and for finding the final result, we need to find the 10's complement of the result obtained by adding complement value of subtrahend and minuend.



Radix and Diminished Radix complement

The mostly used complements are 1's, 2's, 9's, and 10's complement. Apart from these complements, there are many more complements from which mostly peoples are not familiar. For finding the subtraction of the number base system, the complements are used. If **r** is the base of the number system, then there are two types of complements that are possible, i.e., r's and (r-1)'s. We can find the r's complement, and (r-1)'s complement of the number, here r is the radix. The r's complement is also known as **Radix complement** (r-1)'s complement, is known as **Diminished Radix complement**.

If the base of the number is 2, then we can find 1's and 2's complement of the number. Similarly, if the number is the octal number, then we can find 7's and 8's complement of the number.

There is the following formula for finding the r's and (r-1)'s complement:

r' s= complement=$(r^n)_{10}$-N
(r-1)' s complement={$(r^n)_{10}$-1}-N

In the above formulas,

- The n is the number of digits in the number.
- The N is the given number.
- The r is the radix or base of the number.

Advantages of r's complement

These are the following advantages of using r's complement:

- In r's complement, we can further use existing addition circuitry means there is no special circuitry.
- There is no need to determine whether the minuend and subtrahend are larger or not because the result has the right sign automatically.
- The negative zeros are eliminated by r's complement.

Let's take some examples to understand how we can calculate the r's and (r-1)'s complement of binary, decimal, octal, and hexadecimal numbers.

Example 1: $(1011000)_2$

This number has a base of 2, which means it is a binary number. So, for the binary numbers, the value of r is 2, and r-1 is 2-1=1. So, we can calculate the 1's and 2's complement of the number.

1's complement of the number 1011000 is calculated as:

$=\{(2^7)_{10}-1\}-(1011000)_2$
$=\{(128)_{10}-1\}-(1011000)_2$
$=\{(127)_{10}\}-(1011000)_2$
$=1111111_2-1011000_2$
$=0100111$

2's complement of the number 1011000 is calculated as:

$=(2^7)_{10}-(1011000)_2$
$=(128)_{10}-(1011000)_2$

$=10000000_2-1011000_2$
$=0101000_2$

Example 2: $(155)_{10}$

This number has a base of 10, which means it is a decimal number. So, for the decimal numbers, the value of r is 10, and r-1 is 10-1=9. So, we can calculate the 10's and 9's complement of the number.

9's complement of the number 155 is calculated as:

$=\{(10^3)_{10}-1\}-(155)_{10}$
$=(1000-1)-155$
$=999-155$
$=(844)_{10}$

10's complement of the number 1011000 is calculated as:

$=(10^3)_{10}-(155_{10}$
$=1000-155$
$=(845)_{10}$

Example 3: $(172)_8$

This number has a base of 8, which means it is an octal number. So, for the octal numbers, the value of r is 8, and r-1 is 8-1=7. So, we can calculate the 8's and 7's complement of the number.

7's complement of the number 172 is calculated as:

$=\{(8^3)_{10}-1\}-(172)_8$
$=((512)_{10}-1)-(132)_8$
$=(511)_{10}-(122)_{10}$
$=(389)_{10}$
$=(605)_8$

8's complement of the number 172 is calculated as:

$=(8^3)_{10}-(172)_8$
$=(512)_{10}-172_8$
$=512_{10}-122_{10}$
$=390_{10}$
$=606_8$

Example 4: $(F9)_{16}$

This number has a base of 16, which means it is a hexadecimal number. So, for the hexadecimal numbers, the value of r is 16, and r-1 is 16-1=15. So, we can calculate the 16's and 15's complement of the number.

15's complement of the number F9 is calculated as:

$\{(16^2)_{10}-1\}-(F9)_{16}$
$(256-1)_{10}-F9_{16}$
$255_{10}-249_{10}$
$(6)_{10}$
$(6)_{16}$

16's complement of the number F9 is calculated as:

$\{(16^2)_{10}\}-(F9)_{16}$
$256_{10}-249_{10}$
$(7)_{10}$
$(7)_{16}$

Addition and Subtraction using 1's complement

In our previous section, we learned about different complements such as 1's complement, 2's complement, 9's complement, and 10's complement, etc.. In this section, we will learn to perform the arithmetic operations such as addition and subtraction using 1's complement. We can perform addition and subtraction using 1's, 2's, 9's, and 10's complement.

Addition using 1's complement

There are three different cases possible when we add two binary numbers which are as follows:

Case 1: Addition of the positive number with a negative number when the positive number has a greater magnitude.

Initially, calculate the 1's complement of the given negative number. Sum up with the given positive number. If we get the end-around carry 1, it gets added to the LSB.

Example: 1101 and -1001

1. First, find the 1's complement of the negative number 1001. So, for finding 1's complement, change all 0 to 1 and all 1 to 0. The 1's complement of the number 1001 is 0110.
2. Now, add both the numbers, i.e., 1101 and 0110;
   1101+0110=1 0011
3. By adding both numbers, we get the end-around carry 1. We add this end around carry to the LSB of 0011.
   0011+1=0100

Case 2: Adding a positive value with a negative value in case the negative number has a higher magnitude.

Initially, calculate the 1's complement of the negative value. Sum it with a positive number. In this case, we did not get the end-around carry. So, take the 1's complement of the result to get the final result.

Note: The resultant is a negative value.

Example: 1101 and -1110

1. First find the 1's complement of the negative number 1110. So, for finding 1's complement, we change all 0 to 1, and all 1 to 0. 1's complement of the number 1110 is 0001.
2. Now, add both the numbers, i.e., 1101 and 0001;
   1101+0001= 1110
3. Now, find the 1's complement of the result 1110 that is the final result. So, the 1's complement of the result 1110 is 0001, and we add a negative sign before the number so that we can identify that it is a negative number.

Case 3: Addition of two negative numbers

In this case, first find the 1's complement of both the negative numbers, and then we add both these complement numbers. In this case, we always get the end-around carry, which get added to the LSB, and for getting the final result, we take the 1's complement of the result.

Note: The resultant is a negative value.

Example: -1101 and -1110 in five-bit register

1. Firstly find the 1's complement of the negative numbers 01101 and 01110. So, for finding 1's complement, we change all 0 to 1, and all 1 to 0. 1's complement of the number 01110 is 10001, and 01101 is 10010.
2. Now, we add both the complement numbers, i.e., 10001 and 10010;
   10001+10010= 1 00011
3. By adding both numbers, we get the end-around carry 1. We add this end-around carry to the LSB of 00011.
   00011+1=00100
4. Now, find the 1's complement of the result 00100 that is the final answer. So, the 1's complement of the result 00100 is 110111, and add a negative sign before the number so that we can identify that it is a negative number.

Subtraction using 1's complement

These are the following steps to subtract two binary numbers using 1's complement

- In the first step, find the 1's complement of the subtrahend.
- Next, add the complement number with the minuend.
- If got a carry, add the carry to its LSB. Else take 1's complement of the result which will be negative

Note: The subtrahend value always get subtracted from minuend.

Example 1: 10101 - 00111

We take 1's complement of subtrahend 00111, which comes out 11000. Now, sum them. So,

10101+11000 =1 01101.

In the above result, we get the carry bit 1, so add this to the LSB of a given result, i.e., 01101+1=01110, which is the answer.

Example 2: 10101 - 10111

We take 1's complement of subtrahend 10111, which comes out 01000. Now, add both of the numbers. So,

10101+01000 =11101.

In the above result, we didn't get the carry bit. So calculate the 1's complement of the result, i.e., 00010, which is the negative number and the final answer.

Addition and Subtraction using 2's complement

In our previous section, we learned how we could perform arithmetic operations such as addition and subtraction using 1's complement. In this section, we will learn to perform these operations using 2's complement.

Addition using 2's complement

There are three different cases possible when we add two binary numbers using 2's complement, which is as follows:

Case 1: Addition of the positive number with a negative number when the positive number has a greater magnitude.

Initially find the 2's complement of the given negative number. Sum up with the given positive number. If we get the end-around carry 1 then the number will be a positive number and the carry bit will be discarded and remaining bits are the final result.

Example: 1101 and -1001

1. First, find the 2's complement of the negative number 1001. So, for finding 2's complement, change all 0 to 1 and all 1 to 0 or find the 1's complement of the number 1001. The 1's complement of the number 1001 is 0110, and add 1 to the LSB of the result 0110. So the 2's complement of number 1001 is 0110+1=0111
2. Add both the numbers, i.e., 1101 and 0111;
   1101+0111=1 0100
3. By adding both numbers, we get the end-around carry 1. We discard the end-around carry. So, the addition of both numbers is 0100.

Case 2: Adding of the positive value with a negative value when the negative number has a higher magnitude.

Initially, add a positive value with the 2's complement value of the negative number. Here, no end-around carry is found. So, we take the 2's complement of the result to get the final result.

> Note: The resultant is a negative value.

Example: 1101 and -1110

1. First, find the 2's complement of the negative number 1110. So, for finding 2's complement, add 1 to the LSB of its 1's complement value 0001.
   0001+1=0010
2. Add both the numbers, i.e., 1101 and 0010;
   1101+0010= 1111
3. Find the 2's complement of the result 1110 that is the final result. So, the 2's complement of the result 1110 is 0001, and add a negative sign before the number so that we can identify that it is a negative number.

Case 3: Addition of two negative numbers

In this case, first, find the 2's complement of both the negative numbers, and then we will add both these complement numbers. In this case, we will always get the end-around carry, which will be added to the LSB, and forgetting the final result, we will take the2's complement of the result.

> Note: The resultant is a negative value.

Example: -1101 and -1110 in five-bit register

1. Firstly find the 2's complement of the negative numbers 01101 and 01110. So, for finding 2's complement, we add 1 to the LSB of the 1's complement of these numbers. 2's complement of the number 01110 is 10010, and 01101 is 10011.
2. We add both the complement numbers, i.e., 10001 and 10010;
   10010+10011= 1 00101
3. By adding both numbers, we get the end-around carry 1. This carry is discarded and the final result is the 2.s complement of the result 00101. So, the 2's complement of the result 00101 is 11011, and we add a negative sign before the number so that we can identify that it is a negative number.

Subtraction using 2's complement

These are the following steps to subtract two binary numbers using 2's complement

- In the first step, find the 2's complement of the subtrahend.
- Add the complement number with the minuend.

- If we get the carry by adding both the numbers, then we discard this carry and the result is positive else take 2's complement of the result which will be negative.

Example 1: 10101 - 00111

We take 2's complement of subtrahend 00111, which is 11001. Now, sum them. So,

10101+11001 =1 01110.

In the above result, we get the carry bit 1. So we discard this carry bit and remaining is the final result and a positive number.

Example 2: 10101 - 10111

We take 2's complement of subtrahend 10111, which comes out 01001. Now, we add both of the numbers. So,

10101+01001 =11110.

In the above result, we didn't get the carry bit. So calculate the 2's complement of the result, i.e., 00010. It is the negative number and the final answer.