Problem 3:

Part 1)

Euler method with step size: 0.05, final state:

X = -6236.847 , Y = -7305.782, Z = 1527.51

RK4 method with step size: 0.05, final state:

X = -6211.819, Y = -7289.724, Z = 1527.319

Part 2)

State Change Description	Time	Distance to Waypoint
Waypoint1 to Waypoint2	47.1948	200.0 ft
Waypoint2 to Waypoint3	86.8533	200.0 ft
Waypoint3 to Waypoint4	103.3512	200.0 ft

Part 3)

Step Size	Runtime	Final State	Estimate Error
0.05	1.2450	X = -4623.59458894 Y = 13356.8299059 Z = 1500.03308433	5.6392
0.025	2.4512	X = -4629.20865723 Y = 13356.29788648 Z = 1500.03306853	1.0505
0.0125	4.8419	X = -4630.25220973 Y = 13356.17651859 Z = 1500.03306315	0.4299
0.00625	9.7134	X = -4630.68111271 Y = 13356.14607464 Z = 1500.03306188	-

The step size 0.0125 is the one in which the error estimate measured in the simulations is below 1.0 ft.

State Changes Report for Different Time Steps:

Time Step	State Change Description	Time	Distance to Waypoint
0.05	Waypoint1 to Waypoint2	47.1948	200.0
0.05	Waypoint2 to Waypoint3	86.8533	200.0
0.05	Waypoint3 to Waypoint4	103.3512	200.0
0.025	Waypoint1 to Waypoint2	47.1951	200.0
0.025	Waypoint2 to Waypoint3	86.8537	200.0
0.025	Waypoint3 to Waypoint4	103.3515	200.0
0.0125	Waypoint1 to Waypoint2	47.1946	200.0
0.0125	Waypoint2 to Waypoint3	86.8532	200.0
0.0125	Waypoint3 to Waypoint4	103.3511	200.0
0.00625	Waypoint1 to Waypoint2	47.1945	200.0
0.00625	Waypoint2 to Waypoint3	86.8531	200.0
0.00625	Waypoint3 to Waypoint4	103.3510	200.0

Part 4)

Simulation Runtime	Final State
1.01	X = -4630.669 Y= 13356.119 Z = 1500.033

State Change Description	Time	Distance To Waypoint
Waypoint1 to Waypoint2	47.1934	200.0
Waypoint2 to Waypoint3	86.8522	200.0
Waypoint3 to Waypoint4	103.3501	200.0

The simulation run time with classic RK4 with the error estimate measured in the simulations below 1.0 ft is 4.8419. However, the RK45 simulation run time is 1.01 and is somehow $\frac{1}{4}$ the classic RK4.

The main reason is that RK45 uses dynamic step size which allows RK45 to sometimes use larger step sizes. This will lead to lower simulation run time for RK45 than the classic RK4.