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	Jeffrey Hui	
Netid:		
	ihui8	

CS 441 - HW2: PCA and Linear Models

Complete the sections below. You do not need to fill out the checklist.

Total Points Available	[]/160
1. PCA on MNIST	
a. Display 10 principal component vectors	[]/5
b. Display scatterplot	[]/5
c. Plot cumulative explained variance	[]/5
d. Compression and 1-NN experiment	[]/15
2. MNIST Classification with Linear Models	
 a. LLR / SVM error vs training size 	[]/20
b. Error visualization	[]/10
c. Parameter selection experiments	[]/15
3. Temperature Regression	
 a. Linear regression test 	[]/10
 Feature selection results 	[]/15
4. Stretch Goals	
 a. PR and ROC curves 	[]/10
b. Visualize weights	[]/10
c. Other embeddings	[]/15
d. One city is all you need	[]/15
e. SVM with RBF kernel	[]/10

1. PCA on MNIST

a. Display 10 principal component vectors















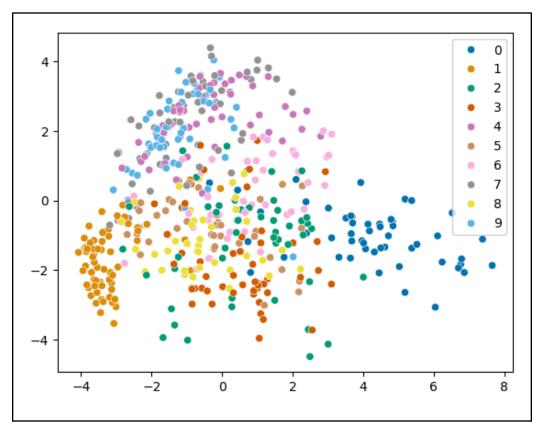




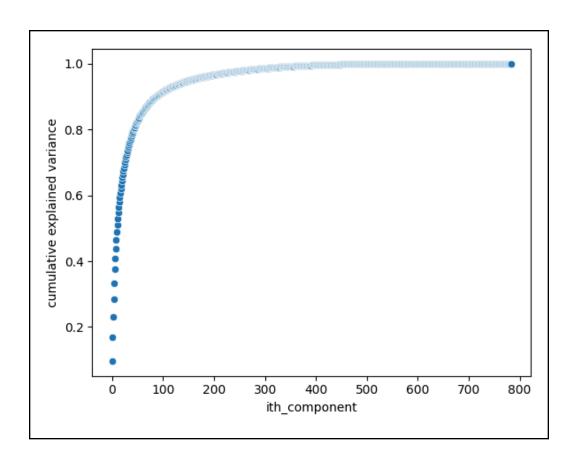


b. Display scatterplot

Scatterplot $x_{train[:500]}$ for the first two PCA dimensions. Show a different color for each label.



c. Plot cumulative explained variance



d. Compression and 1-NN experiment

Number of components selected

	Total Time (s)	Test Error (%)	Dimensions
Brute Force (PCA)	1.28	2.68	87
Brute Force	5.49	3.09	784

2. MNIST Classification with Linear Models

a. LLR / SVM error vs training size

Test error (%)

# training samples	LLR	SVM
100	32.5%	32.4%
1,000	13.64	16.11

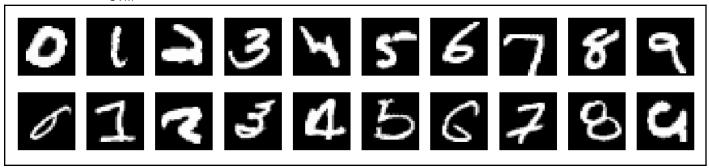
10,000	9.5	11.12
60,000	7.39	8.17

b. Error visualization

LLR



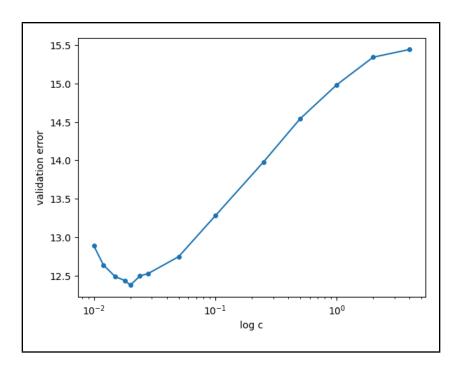
SVM



c. Parameter selection experiments

	Logistic Regression
Best C value	0.02
Validation error (%)	12.38
Test error (%)	13.57

Plot C value vs validation error for values tested



3. Temperature Regression

a. Linear regression test

Test RMSE

	Linear regression
Original features	2.161
Normalized features	2.163

Why might normalizing features in this way not be as helpful as it is for KNN?

Linear regression is not sensitive to the scale of the features as much as KNN. Additionally, if the relationship between the features and the target variable is non-linear, normalizing the data might not necessarily improve performance and could even lead to an increase in RMSE.

b. Feature selection results

Feature Rank	Feature number	City	Day
1	334	Chicago	-1
2	347	Minneapolis	-1
3	405	Grand Rapids	-1
4	336	Kansas City	-1
5	361	Cleveland	-1
6	307	Omaha	-2
7	367	Indianapolis	-1
8	264	Minneapolis	-2
9	9	Boston	-5
10	236	Springfield	-3

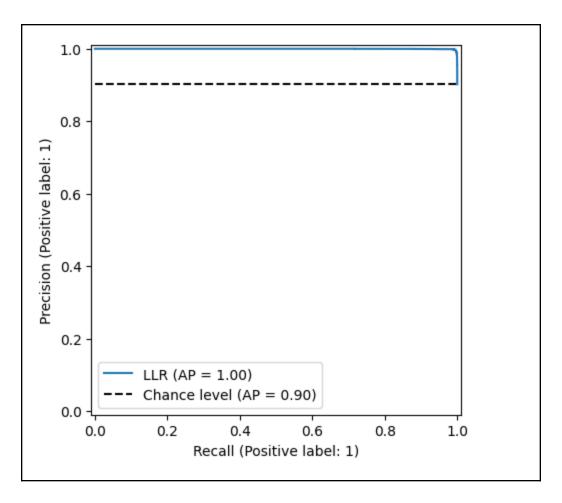
Test error using only the 10 most important features for regression

	Linear Regression
RMS Error	2.058

4. Stretch Goals

a. PR and ROC curves

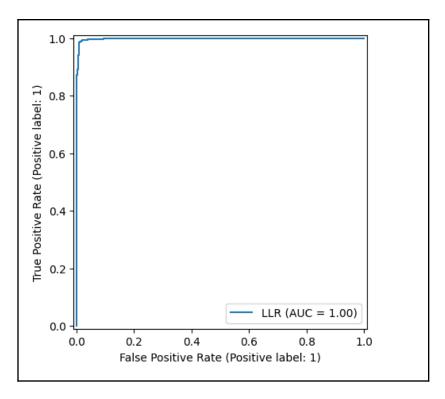
PR plot



Average Precision

1.0

ROC plot



Area under the curve (AUC)

1.0

b. Visualize weights

LLR - L2





















LLR - L1













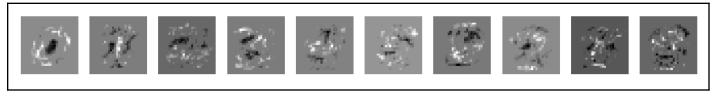




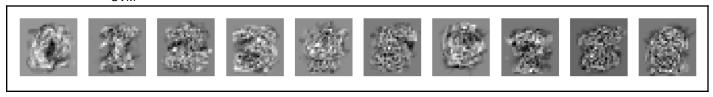




LLR - elastic



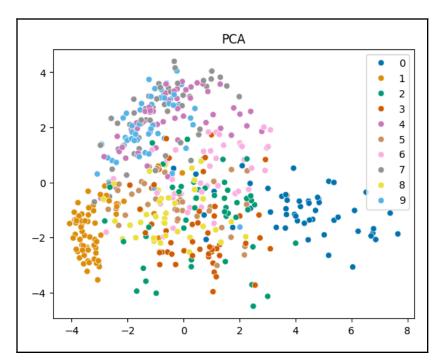
SVM



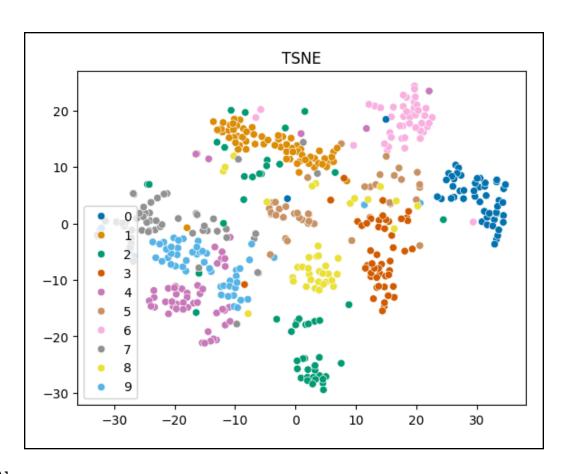
c. Other embeddings

Display 2+ plots for TSNE, MDA, and/or LDA, and copy PCA plot from 1b here.

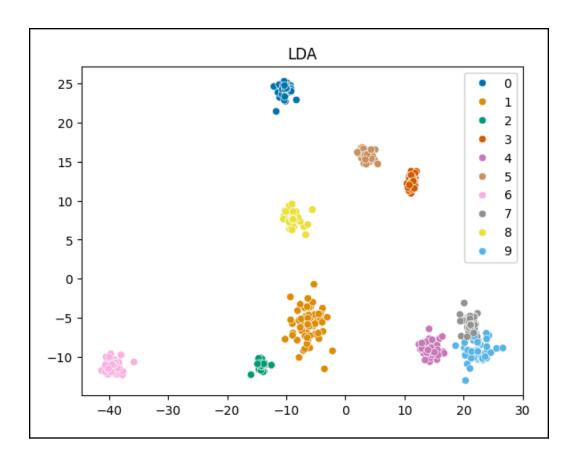
PCA



[TSNE]



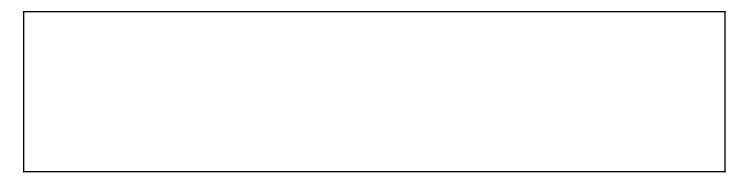
[LDA]



d. One city is all you need

City	
Test error using features only fror	n that city

Explain your process (in words):



e. Compare linear SVM and SVM with RBF kernel

Test accuracy (%)

# training samples	SVM-Linear	SVM-RBF
100	67.5%	65.59%
1,000	86.36%	90.83%
10,000	90.5%	95.94%
60,000	92.61%	97.92%

Acknowledgments / Attribution

List any outside sources for code or ideas or "None".

None

cs441-sp24-hw2-starter

February 20, 2024

0.1 CS441: Applied ML - HW 2

0.1.1 Parts 1-2: MNIST

Include all the code for generating MNIST results below

```
[80]: # initialization code
      import numpy as np
      from keras.datasets import mnist
      %matplotlib inline
      from matplotlib import pyplot as plt
      from scipy import stats
      from sklearn.linear model import LogisticRegression
      import cache_magic
      def load mnist():
        111
        Loads, reshapes, and normalizes the data
        (x_train, y_train), (x_test, y_test) = mnist.load_data() # loads MNIST data
        x_train = np.reshape(x_train, (len(x_train), 28*28)) # reformat to 768-d_
       \rightarrowvectors
        x_test = np.reshape(x_test, (len(x_test), 28*28))
        maxval = x_train.max()
        x_train = x_train/maxval # normalize values to range from 0 to 1
        x test = x test/maxval
        return (x_train, y_train), (x_test, y_test)
      def display_mnist(x, subplot_rows=1, subplot_cols=1):
        Displays one or more examples in a row or a grid
        if subplot_rows>1 or subplot_cols>1:
          fig, ax = plt.subplots(subplot_rows, subplot_cols, figsize=(15,15))
          for i in np.arange(len(x)):
            ax[i].imshow(np.reshape(x[i], (28,28)), cmap='gray')
            ax[i].axis('off')
            plt.imshow(np.reshape(x, (28,28)), cmap='gray')
```

```
plt.axis('off')
plt.show()
```

0.1.2 Part 1: PCA and Data Compression

```
[81]: from sklearn.decomposition import PCA
  import matplotlib.pyplot as plt

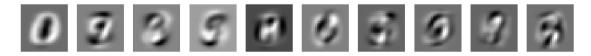
  (x_train, y_train), (x_test, y_test) = load_mnist()

# Compute the first 10 principal components using x_train

%cache pca1a = PCA(n_components=10).fit(x_train)

print(pca1a.components_.shape)
# Display First 10 Components
display_mnist(pca1a.components_, subplot_rows=1, subplot_cols=10)
```

Loading cached value for variable 'pca1a'. Time since caching: 8 days, 1:38:03.816843 (10, 784)

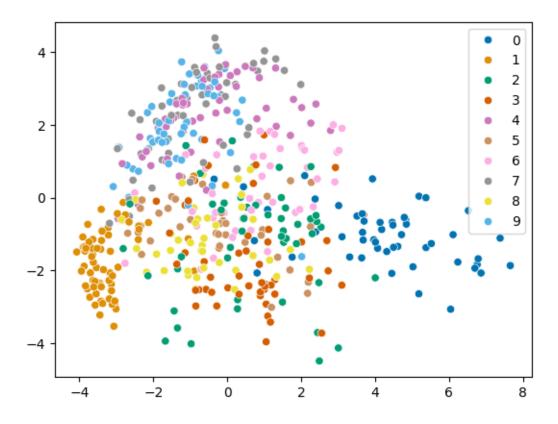


```
[82]: # Scatter plot of first two PCA dimensions
import seaborn as sns

# use pca.transform
%cache pca1b = PCA(n_components=2).fit(x_train)
x = pca1b.transform(x_train[:500])
ind = np.arange(500)
sns.scatterplot(x=x[ind,0],y=x[ind,1], hue=y_train[ind], palette="colorblind")

Loading cached value for variable 'pca1b'. Time since caching: 8 days,
1:40:06.955217
```

[82]: <Axes: >

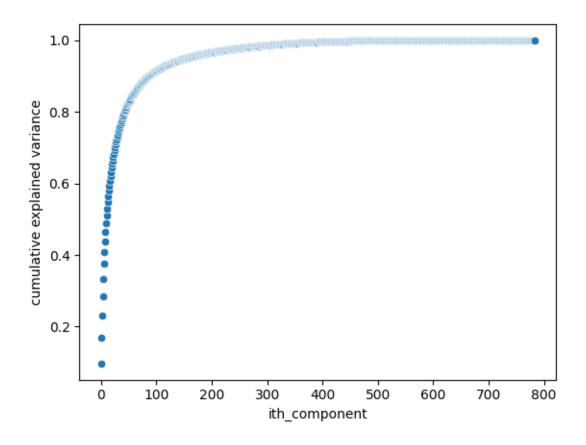


```
[83]: # Plot cumulative explained variance ratio
    # cumsum and pca.explained_variance_ratio_ will be useful

%cache pca1c = PCA().fit(x_train)
    cumul = np.cumsum(pca1c.explained_variance_ratio_)
    sns.scatterplot(
        data = {
          "ith_component": np.arange(0, len(cumul), 1),
          "cumulative explained variance": cumul
        },
        x = "ith_component",
        y = "cumulative explained variance"
)
```

Loading cached value for variable 'pca1c'. Time since caching: 8 days, 1:37:55.480961

[83]: <Axes: xlabel='ith_component', ylabel='cumulative explained variance'>



```
[84]: # Select number of dimensions that explains 90% of variance, according to your
       ⇔plot above
      # !apt install libomp-dev > /dev/null 2>&1
      # !pip install faiss-cpu > /dev/null 2>&1
      import faiss
      import time
      # plus 1 because of 0 indexing
      M = np.argwhere(cumul >= 0.9)[0].item() + 1
      print(f"M = {M}")
      # compress training data and test data
      %cache pcald = PCA(n_components=M).fit(x_train)
      compressed_train, compressed_test = pcald.transform(x_train), pcald.
       ⇔transform(x_test)
      def create_and_add(dim, data):
          idx = faiss.IndexFlatL2(dim)
          idx.add(data)
          return idx
      # Get time and error when using original features with brute force 1-NN
```

```
%cache original_index = create_and_add(x_train.shape[1], x_train)
      t1 = time.time()
      dist, prediction = original_index.search(x_test, 1)
      t2 = time.time()
      timing = t2 - t1
      # prediction = y_train[prediction]
      # error = prediction[prediction != y_test.reshape(-1, 1)].sum() / _ _ _ _
      \hookrightarrow len(prediction) * 100
      error = np.mean(y_train[prediction.flatten()] != y_test) * 100
      print(f"original error = {error}% | time = {timing}")
      # print(np.mean(dist))
      # Get time and error when using compressed features with brute force 1-NN
      %cache comp_index = create_and_add(compressed_train.shape[1], compressed_train)
      t1 = time.time()
      dist, prediction = comp_index.search(compressed_test, 1)
      t2 = time.time()
      timing = t2 - t1
      # prediction = y_train[prediction]
      error = np.mean(y_train[prediction.flatten()] != y_test) * 100
      print(f"compressed error = {error}% | time = {timing}")
      # print(np.mean(dist))
     M = 87
     Loading cached value for variable 'pca1d'. Time since caching: 8 days,
     1:37:52.756670
     Loading cached value for variable 'original index'. Time since caching: 8 days,
     1:37:50.756619
     original error = 3.09\% | time = 5.727076530456543
     Loading cached value for variable 'comp_index'. Time since caching: 8 days,
     1:37:50.332103
     compressed error = 2.68% | time = 1.317983627319336
[85]: %cache
     <IPython.core.display.HTML object>
     0.1.3 Part 2: MNIST Classification with Linear Models
```

```
[86]: from sklearn.linear_model import LogisticRegression from sklearn import svm from sklearn.metrics import accuracy_score
```

LLR/SVM vs training size

```
[87]: # LLR sizes = [100, 1000, 10000, 60000]
```

Loading cached value for variable 'llr_estimators'. Time since caching: 8 days, 1:37:11.062065 [0.32499999999999, 0.13639999999999, 0.09499999999999, 0.07389999999997]

Loading cached value for variable 'svm_estimators'. Time since caching: 7 days, 23:17:33.032995 [0.3235, 0.1611000000000000, 0.111199999999997, 0.0817]

Error visualization

```
[89]: scores = llr_estimators[-1].predict_proba(x_test)
      llrhigh, llrlow = [], []
      for cls in range(10):
          # get samples with true label of class
          cls_sample = x_test[y_test == cls]
          # get their scores
          cls_sample_scores = scores[y_test == cls][:, cls]
          # show
          llrhigh.append(cls_sample[np.argmax(cls_sample_scores)])
          llrlow.append(cls_sample[np.argmin(cls_sample_scores)])
      print("LLR high and low confidence")
      display_mnist(llrhigh, subplot_rows=1, subplot_cols=10)
      display_mnist(llrlow, subplot_rows=1, subplot_cols=10)
      scores = svm_estimators[-1].decision_function(x_test)
      svmhigh, svmlow = [], []
      for cls in range(10):
          # get samples with true label of class
          cls_sample = x_test[y_test == cls]
          # get their scores
```

```
cls_sample_scores = scores[y_test == cls][:, cls]
# show
svmhigh.append(cls_sample[np.argmax(cls_sample_scores)])
svmlow.append(cls_sample[np.argmin(cls_sample_scores)])

print("SVM high and low confidence")
display_mnist(svmhigh, subplot_rows=1, subplot_cols=10)
display_mnist(svmlow, subplot_rows=1, subplot_cols=10)
```

LLR high and low confidence



SVM high and low confidence



Parameter selection

```
[90]: # Try multiple C parameters, select one that minimizes validation error
# Often, you need to try a few values and see those results to determine what_
other values to try

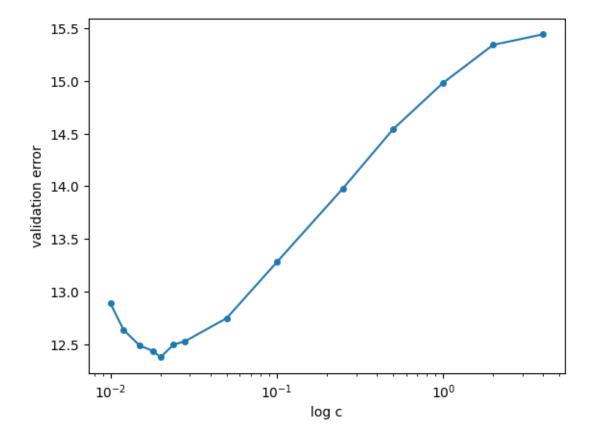
get_error = lambda pred, y: (pred != y).sum() / len(y) * 100

C = [0.01, 0.012, 0.015, 0.018, 0.02, 0.024, 0.028, 0.05, 0.1, 0.25, 0.5, 1, 2, 4]

x_train2c, x_val2c = x_train[:1000], x_train[50000:]
```

```
y_train2c, y_val2c = y_train[:1000], y_train[50000:]
val_err = []
for c in C:
    model = svm.LinearSVC(max_iter=10000, dual='auto', C=c)
    model.fit(x_train2c, y_train2c)
    val_err.append(get_error(model.predict(x_val2c), y_val2c))
plt.semilogx(C, val_err, marker='o', markersize=4)
plt.xlabel('log c')
plt.ylabel('validation error')
```

[90]: Text(0, 0.5, 'validation error')



```
[91]: # Get test result for selected parameter
%cache best_svm_model = svm.LinearSVC(max_iter=10000, dual='auto', C=0.02).

fit(x_train2c, y_train2c)
print(f"Validation error: {get_error(best_svm_model.predict(x_val2c),__
y_val2c)}")
print(f"Test error: {get_error(best_svm_model.predict(x_test), y_test)}")
```

Loading cached value for variable 'best_svm_model'. Time since caching: 7 days, 15:46:20.024406

0.2 Part 3: Temperature Regression

```
[92]: import numpy as np
      # from google.colab import drive
      %matplotlib inline
      from matplotlib import pyplot as plt
      from sklearn.linear_model import Ridge
      from sklearn.linear_model import Lasso
      # load data (modify to match your data directory or comment)
      def load temp data():
       datadir = "./"
       T = np.load(datadir + 'temperature_data.npz')
       x_train, y_train, x_val, y_val, x_test, y_test, dates_train, dates_val,_
       dates_test, feature_to_city, feature_to_day = \
       T['x_train'], T['y_train'], T['x_val'], T['y_val'], T['x_test'], T['y_test'],
       →T['dates_train'], T['dates_val'], T['dates_test'], T['feature_to_city'], □

¬T['feature_to_day']
       return (x_train, y_train, x_val, y_val, x_test, y_test, dates_train,_
       ⇒dates_val, dates_test, feature_to_city, feature_to_day)
      # plot one data point for listed cities and target date
      def plot_temps(x, y, cities, feature to_city, feature to_day, target_date):
       nc = len(cities)
       ndays = 5
        xplot = np.array([-5, -4, -3, -2, -1])
        yplot = np.zeros((nc,ndays))
       for f in np.arange(len(x)):
          for c in np.arange(nc):
            if cities[c] == feature to city[f]:
              yplot[feature_to_day[f]+ndays,c] = x[f]
       plt.plot(xplot,yplot)
       plt.legend(cities)
       plt.plot(0, y, 'b*', markersize=10)
       plt.title('Predict Temp for Cleveland on ' + target_date)
       plt.xlabel('Day')
       plt.ylabel('Avg Temp (C)')
       plt.show()
```

```
[93]: # load data
(x_train, y_train, x_val, y_val, x_test, y_test, dates_train, dates_val,u_dates_test, feature_to_city, feature_to_day) = load_temp_data()
''' Data format:
```

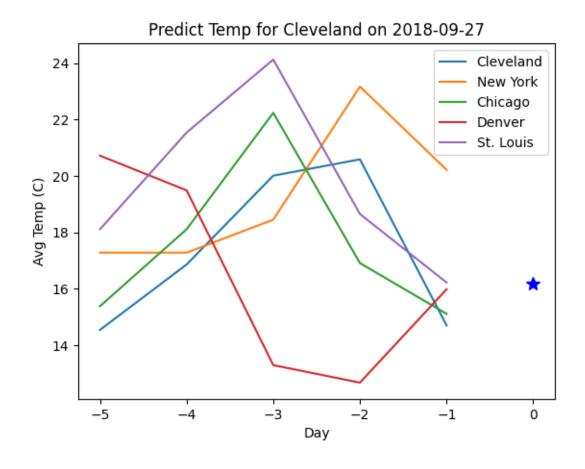
```
x\_train, y\_train: features and target value for each training sample \sqcup
 \hookrightarrow (used to fit model)
      x_val, y_val: features and target value for each validation sample (used
 \hookrightarrowto select hyperparameters, such as regularization and K)
      x_{test}, y_{test}: features and target value for each test sample (used to_\sqcup
 ⇔evaluate final performance)
      dates_xxx: date of the target value for the corresponding sample
      feature_to_city: maps from a feature number to the city
      feature_to_day: maps from a feature number to a day relative to the_
 \hookrightarrow target value, e.g. -2 means two days before
      Note: 361 is the temperature of Cleveland on the previous day
111
f = 361
print('Feature {}: city = {}, day= {}'.format(f,feature_to_city[f],__

→feature_to_day[f]))
baseline_rmse = np.sqrt(np.mean((y_val[1:]-y_val[:-1])**2)) # root mean squared_
\rightarrow error
print('Baseline - prediction using previous day: RMSE={}'.format(baseline_rmse))
# plot first two x/y for val
plot_temps(x_val[0], y_val[0], ['Cleveland', 'New York', 'Chicago', 'Denver', _

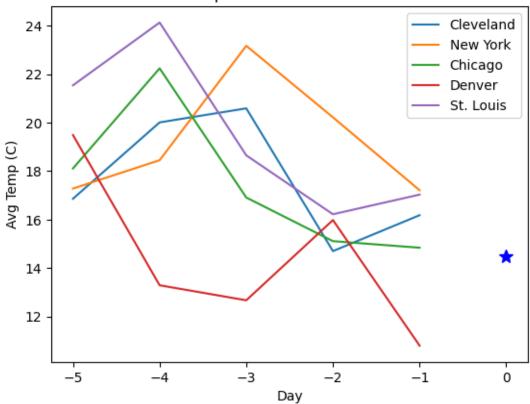
¬'St. Louis'], feature_to_city, feature_to_day, dates_val[0])

plot_temps(x_val[1], y_val[1], ['Cleveland', 'New York', 'Chicago', 'Denver', _
```

Feature 361: city = Cleveland, day= -1
Baseline - prediction using previous day: RMSE=3.460601246750482







Linear regression test

```
[95]: F = 361
    from sklearn.metrics import root_mean_squared_error
    # linear regression (use Ridge)
    # original features
    %cache original_ridge = Ridge().fit(x_train, y_train)
```

```
print(f"original features rmse: {np.sqrt(np.mean((original_ridge.
       →predict(x_test) - y_test) ** 2))}")
      # normalized features
      (x_train_norm, y_train_norm), (x_test_norm, y_test_norm) = ___
       ⊸normalize_features(x_train, y_train, F), normalize_features(x_test, y_test, u_
      %cache normalized_ridge = Ridge().fit(x_train_norm, y_train_norm)
      print(f"normalized features rmse: {np.sqrt(np.mean((normalized ridge.
       →predict(x_test_norm) - y_test_norm) ** 2))}")
     Loading cached value for variable 'original_ridge'. Time since caching: 6 days,
     0:30:16.487390
     original features rmse: 2.160860526080926
     Loading cached value for variable 'normalized ridge'. Time since caching: 6
     days, 0:22:20.693291
     normalized features rmse: 2.1630698027573665
     Feature selection
[96]: # feature analysis (select important features using Lasso)
      lasso = Lasso().fit(x_train, y_train)
      lasso_indices = [(i, v) for i, v in enumerate(np.abs(lasso.coef_)) if v > 0.001]
      lasso_indices.sort(key = lambda x: x[1], reverse=True)
      top10 indices = [x[0] for x in lasso indices[:10]]
      print(f"Top 10: {top10_indices}")
      print(f"Top 10 cities: {[feature_to_city[x] for x in top10_indices]}")
      print(f"Top 10 days: {[feature_to_day[x] for x in top10_indices]}")
      # predict using best features
      selected_indices = [x[0] for x in lasso_indices]
      root_mean_squared_error(
          Ridge().fit(
              np.take(x_train, selected_indices, axis=1), y_train)
              .predict(np.take(x_test, selected_indices, axis=1))
          , y_test
      )
     Top 10: [334, 347, 405, 366, 361, 307, 367, 264, 9, 236]
     Top 10 cities: ['Chicago', 'Minneapolis', 'Grand Rapids', 'Kansas City',
     'Cleveland', 'Omaha', 'Indianapolis', 'Minneapolis', 'Boston', 'Springfield']
     Top 10 days: [-1, -1, -1, -1, -1, -2, -1, -2, -5, -3]
[96]: 2.0579879892446384
[97]: %cache
```

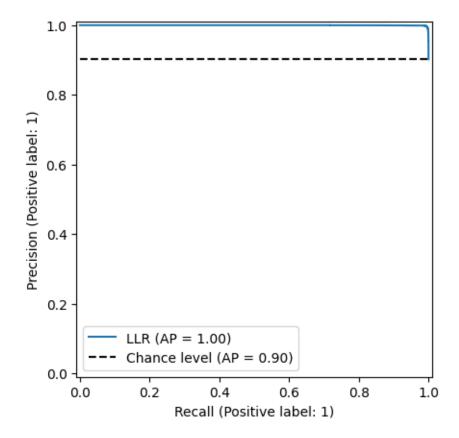
<IPython.core.display.HTML object>

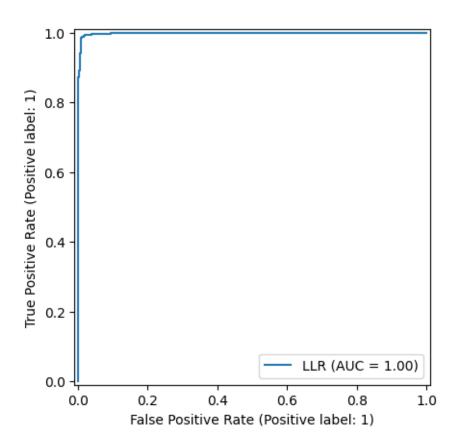
0.3 Part 4: Stretch Goals

Include all your code used for any stretch goals in this section. Add headings where appropriate.

a) PR and ROC curves

[98]: <sklearn.metrics._plot.roc_curve.RocCurveDisplay at 0x1da02c62b90>



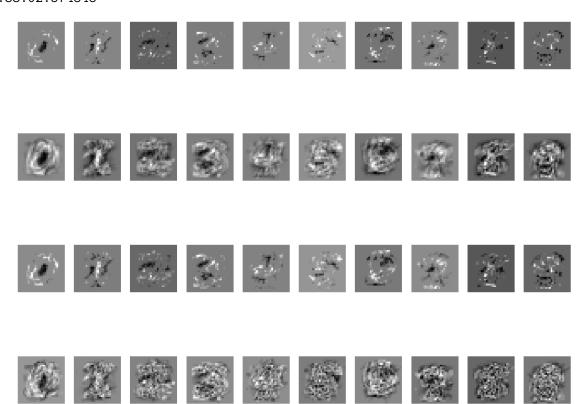


b) Visualize weights

Loading cached value for variable 'p4bllrl1'. Time since caching: 5 days, 18:34:20.956202

Loading cached value for variable 'p4bllrl2'. Time since caching: 5 days, 18:34:17.953454

Loading cached value for variable 'p4bllrel'. Time since caching: 5 days, 18:33:02.574848

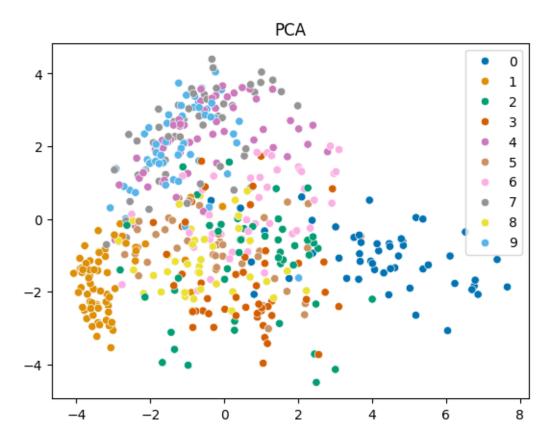


c) Other embeddings

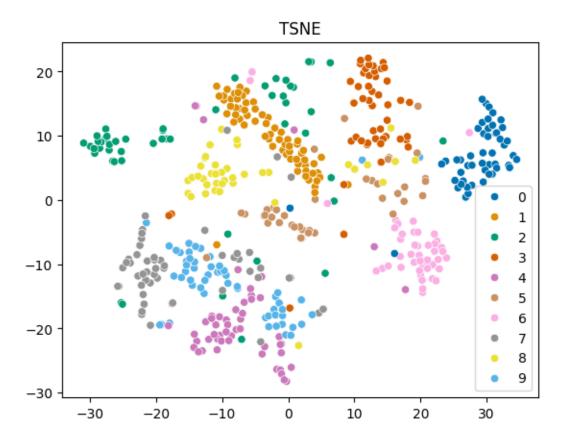
```
[100]: x = pca1b.transform(x_train[:500])
ind = np.arange(500)
sns.scatterplot(x=x[ind,0],y=x[ind,1], hue=y_train[ind], palette="colorblind").

set(title='PCA')
```

[100]: [Text(0.5, 1.0, 'PCA')]

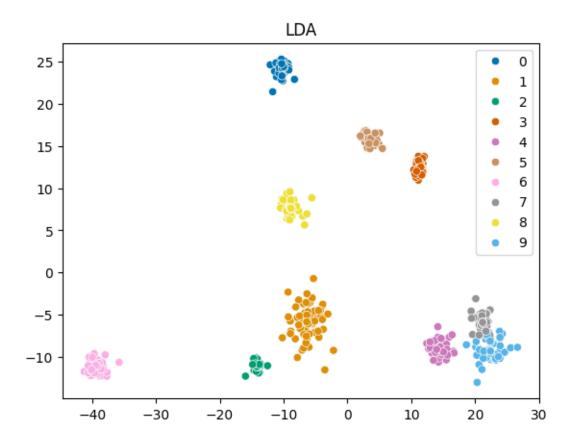


[101]: [Text(0.5, 1.0, 'TSNE')]



```
[102]: from sklearn.discriminant_analysis import LinearDiscriminantAnalysis p4clda = LinearDiscriminantAnalysis(n_components=2).fit(x_train[:500], y_train[: $\infty 500])  
x = p4clda.transform(x_train[:500])  
sns.scatterplot(x=x[ind,0],y=x[ind,1], hue=y_train[ind], palette="colorblind").  
$\infty set(title='LDA')$
```

[102]: [Text(0.5, 1.0, 'LDA')]



e) SVM with RBF Kernel

```
[0.675, 0.8636, 0.905, 0.9261]
Loading cached value for variable 'svmrbf_estimators'. Time since caching: 5 days, 17:37:32.071795
Loading cached value for variable 'svmrbfscores'. Time since caching: 5 days, 17:30:58.110247
[0.6559, 0.9083, 0.9594, 0.9792]
```

```
[104]: %cache
```

<IPython.core.display.HTML object>

```
[105]: | # from https://gist.github.com/jonathanagustin/b67b97ef12c53a8dec27b343dca4abba
      # install can take a minute
      # import os
       # # @title Convert Notebook to PDF. Save Notebook to given directory
       # NOTEBOOKS_DIR = "/content/drive/My Drive/CS441/24SP/hw2" # @param {type:
      # NOTEBOOK NAME = "CS441 SP24 HW2 Solution.ipynb" # @param {type:"string"}
      #__
        ⇔#----
      # from google.colab import drive
       # drive.mount("/content/drive/", force_remount=True)
      # NOTEBOOK_PATH = f"{NOTEBOOKS_DIR}/{NOTEBOOK_NAME}"
       # assert os.path.exists(NOTEBOOK_PATH), f"NOTEBOOK_NOT FOUND: {NOTEBOOK_PATH}"
       # !apt install -y texlive-xetex texlive-fonts-recommended texlive-plain-generic_
       →> /dev/null 2>&1
      # !jupyter nbconvert "$NOTEBOOK_PATH" --to pdf > /dev/null 2>&1
      # NOTEBOOK PDF = NOTEBOOK PATH.rsplit('.', 1)[0] + '.pdf'
       # assert os.path.exists(NOTEBOOK_PDF), f"ERROR MAKING PDF: {NOTEBOOK_PDF}"
       # print(f"PDF CREATED: {NOTEBOOK_PDF}")
```