MAST 90014 - Optimisation for Industry
Group Project 2025

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1 Introduction

Distribution planning during promotional periods represents one of the most challenging problems in retail supply chain management. When demand can surge to 10 times normal levels within a single day, traditional inventory and transportation strategies often fail, leading to either costly stock-outs or excessive inventory holding costs. This project examines how mixed-integer programming can optimise distribution decisions for electronics retailers during Black Friday, one of the year's most intense promotional period.

Black Friday has evolved from a single-day sales event into a week-long phenomenon that tests the limits of retail supply chains. For electronics retailers like JB Hi-Fi, Harvey Norman, and Best Buy, success during this period can determine annual profitability. The challenge extends beyond simply having enough inventory-retailers must position the right products at the right locations while managing constrained transportation capacity and escalating logistics costs.

Our study is motivated by three key industry trends:

- The increasing concentration of sales during promotional periods means retailers cannot afford distribution failures during Black Friday.
- The rise of omnichannel retail has created customer expectations for product availability that penalise stock-outs more severely than ever before.
- The growing cost of logistics capacity during peak periods forces retailers to make strategic trade-offs between service levels and operational efficiency.

We model a small-scale distribution network comprising 5 warehouses serving 5 retail stores over a 7-day Black Friday period. The network must distribute three representative products: high-value smartphones experiencing 100-fold day-on-day demand increases, headphones with concentrated Black Friday purchasing, and earphones facing obsolescence risks and heavy discounting. These products capture the diversity of challenges retailers face, from managing extreme demand spikes to balancing inventory risks for promotional items.

The optimisation problem addresses two fundamental decisions: how to manage truck fleet capacity (comparing flexible daily rentals against economical weekly contracts) and how to route products through the network over time. With truck availability limited to 10 vehicles per day and each truck constrained to 10,500 volume units, the model must balance transportation efficiency against inventory costs while ensuring product availability during demand surges.

Following this introduction, Section 2 formally defines the distribution optimisation problem, explaining key assumptions and simplifications.

Section 3 reviews relevant literature in supply chain optimisation, inventory management, and promotional period logistics.

Section 4 details our data collection methodology and parameter specifications.

Section 5 presents the mathematical model formulation.

Section 6 describes our solution strategies for improving computational performance.

Section 7 analyses results from both the case study.

Section 8 defines and analyses the results from the computation cases study.

Finally, Section 9 provides conclusions and managerial recommendations for implementing these optimisation approaches in practice.

2 Problem Definition

We address the distribution planning problem faced by electronics retailers during Black Friday promotional periods. A retailer operating multiple warehouses must efficiently distribute products to retail stores over a one-week period encompassing Black Friday, when demand can spike to 10 times normal levels between two subsequent days.

2.1 Problem Context

The retailer manages a network of 5 warehouses serving 5 retail stores, distributing three representative categories of consumer electronics: high-value smartphones, mid-range headphones, and heavily discounted earphones. We chose to limit the scope of our case study to three representative products as we wanted to hand-craft data for a more realistic, tailored problem. Each product category exhibits distinct demand patterns during Black Friday week, with:

- Smartphones experiencing almost no demand prior to Black Friday as consumers await heavy discounts followed by a sustained 10x demand over the weekend,
- Headphones showing a moderate increase in demand for Black Friday (4x), and
- Earphones showing a 100x in daily demand between Thursday and Black Friday.

The full demand profiles for these products can be seen in Table 2.

The central challenge is determining optimal truck fleet management and product distribution strategies that minimise total operational costs while meeting customer demand. This requires coordinating decisions across multiple time periods, balancing immediate needs against future demand spikes and the resulting costs from these decisions.

2.2 Key Decisions

The retailer must make two interconnected sets of decisions:

- **Fleet Management**: Whether to commit to a fixed truck fleet for the entire week (weekly rental) or maintain flexibility to adjust fleet size daily (daily rental). Daily rentals offer operational flexibility at a 50% cost premium, allowing the retailer to scale capacity for Black Friday's demand spike.
- **Distribution Planning**: How many units of each product to ship from each warehouse to each retailer in each time period. These decisions must account for truck capacity constraints, varying transportation costs between locations, and the trade-off between holding inventory and risking stockouts.

2.3 Problem Characteristics

Several factors make this problem particularly challenging:

- **Extreme Demand Volatility**: Demand for earphones increases from 10 units on Thursday to 1,000 units on Black Friday - a 100-fold increase. Headphones show even more extreme behavior with zero demand immediately before Black Friday as customers wait for promotions.
- **Capacity Constraints**: The retailer faces a hard constraint of 10 trucks per day due to driver availability. With each truck having a set capacity and products occupying different space, efficient packing becomes critical during peak periods.
- **Heterogeneity of Goods**: Retailers and transportation companies must deal with incredibly varied goods, with significantly different characteristics in terms of shape, size, fragility, value and obsolescence. These characteristics heavily impact transportation considerations and the various costs faced by the retailer.
- **Cost Trade-offs**: The retailer must balance four competing cost components:
  - Transportation costs varying by distance and by good, capturing the overall cost of transporting each good and their unique characteristics applicable to transportation, like fragility.
  - Truck rental costs.
  - Inventory holding costs, including obsolescence risk for discounted items, expressed as a percentage of the retail price of the good.
  - Shortage costs, reflecting lost profit margins and calculated as a percentage of the retail price.

2.4 Simplifications and Assumptions

To make the problem tractable while maintaining practical relevance, we adopt several simplifications:

- **Deterministic Demand**: We assume demand is known with certainty based on historical Black Friday patterns. While real demand contains uncertainty, the promotional nature of Black Friday creates relatively predictable patterns. Furthermore, adding uncertainty to demand forecasts would overcomplicate the problem and detract from the results and learnings of the study being performed.

- **No Inter-Retailer Shipments**: Products cannot be transferred between retail stores. This reflects common practice where inter-store transfers are logistically complex and time-consuming during peak periods.
- **Unlimited Warehouse Supply**: Warehouses have sufficient inventory to meet all distribution requests. This assumes proper upstream planning has positioned inventory at warehouses before Black Friday week.
- **Single Transportation Mode**: All shipments use trucks with identical capacity. While retailers might use multiple transportation modes, trucks represent the dominant mode for regional distribution.

2.5 Problem Variants

We analyze four variants of the base problem, creating a 2×2 matrix of scenarios:

- **Truck Rental Strategy**:
  - Weekly rental: Lower daily cost but requires committing to fixed fleet size
  - Daily rental: Higher cost but allows dynamic fleet adjustment
- **Truck Market Conditions**:
  - High truck costs: Representing peak season rates when logistics capacity is scarce
  - Low truck costs: Representing normal market conditions

These variants allow us to examine how optimal distribution strategies change under different operational constraints and market conditions, providing robust insights for retail decision-makers. This also allows for companies to adapt to prevailing market conditions and assess the cost/benefit of a flexible fleet during highly-volatile periods.

The data for these variants is available in Section 4, *Data*.

2.6 Relevance and Applications

While framed around Black Friday, this problem represents a broader class of distribution challenges during promotional periods. Similar patterns occur during:

- Product launches with anticipated demand spikes
- Seasonal sales events (back-to-school, holiday shopping)
- Flash sales and limited-time promotions in e-commerce
- Emergency response requiring rapid inventory deployment

The insights from our analysis apply to any situation where retailers must balance distribution efficiency against service levels under extreme demand volatility and capacity constraints.

3 Literature Review

The optimisation of supply chain distribution systems has been extensively studied in operations research, with mixed-integer programming (MIP) emerging as a dominant methodology for addressing complex logistics challenges. This literature review examines relevant research in supply chain optimisation, inventory management, and retail distribution planning, with particular attention to multi-period models and promotional period management.

3.1 Mixed-Integer Programming in Supply Chain Management

Mixed-integer programming has proven to be a powerful tool for optimising complex supply chain networks. Mixed Integer Linear Programming (MILP) has emerged as a powerful tool for optimising complex supply chain networks [9], enabling companies to model production planning, network design, and transportation logistics while achieving significant cost reductions. The discrete nature of many supply chain decisions - such as the number of trucks to hire or the selection of warehouse locations - makes MIP particularly suitable for these applications.

IBM Systems and Technology Group uses operations research models and methods extensively for solving large-scale supply chain optimisation (SCO) problems for planning its extended enterprise semiconductor supply chain [4], demonstrating the scalability of MIP approaches to industrial applications. However, the computational complexity of large-scale MIP models has led researchers to develop specialised solution techniques. Pure optimisation methods are computationally infeasible, and fast heuristic methods alone generate poor results [4], necessitating hybrid approaches that combine exact methods with heuristics.

3.2 Multi-Period Inventory Management

The management of inventory across multiple time periods presents unique challenges that have been addressed through various modeling approaches. Qiu, R., Sun, M., & Lim, Y. F. (2017) consider a finite-horizon single-product periodic-review inventory management problem with demand distribution uncertainty [14], highlighting the importance of robust optimisation approaches when dealing with uncertain demand patterns.

Periodic review policies, particularly the (s, S) policy, have received significant attention in the literature. The (s, S) policy is a well-studied strategy [1], summarised as when an item drops below some reserve amount s, a purchase order is placed to replenish the item back to a standard level S. In another paper, periodic review of the (s, S) policy is used to optimise inventories from an integrated perspective of inventory management across the supply chain over time [7]. Under this variation, optimisation of the parameters s, S themselves over time as a sort of meta-optimisation problem over the planning horizon [7], providing a practical framework for dynamic inventory replenishment decisions.

The coordinated management of multiple products adds another layer of complexity. A multi-item multi-period inventory control model is developed for known-deterministic variable demands under limited available budget [6], addressing the resource allocation challenges that arise when managing diverse product portfolios. This is particularly relevant for retail environments where interaction effects across complementary products plays an important role in characterising the optimal inventory policy [8].

3.3 Promotional Period Supply Chain Management

While the academic literature on Black Friday-specific optimisation is limited, industry reports highlight the unique challenges of promotional period supply chain management. 85% of retailers are at least somewhat concerned about inventory shortages during BFCM (Black Friday and Cyber Monday) [2], emphasizing the critical importance of effective distribution planning during these peak periods.

The demand volatility characteristic of promotional periods creates particular challenges. In order to meet the targets being set by Black Friday's continued growth, supply chains are required to deliver high quantities of stock to specific locations with short deadlines [11]. This aligns with our modeling approach, which incorporates demand scaling factors ranging from 0.2x to 10.0x normal levels throughout the Black Friday week.

Recent industry trends show that '*Early sales events like Black Friday and Cyber Monday have conditioned shoppers to start planning and purchasing well in advance*' [10], extending the planning horizon and requiring more sophisticated multi-period optimisation models. This evolution in consumer behavior reinforces the need for flexible distribution strategies that can adapt to changing demand patterns over extended promotional periods.

3.4 Integration of Distribution and Inventory Decisions

The integration of transportation and inventory decisions represents a critical advancement in supply chain optimisation. When consumers shop online and pick up at store their orders, stores are typically visited by a truck that supplies the collection points and by a transport that replenishes the inventory of the store [12], highlighting the need for coordinated logistics planning.

Recent research has emphasised the importance of considering multiple cost components simultaneously. This paper attempts to integrate both forward and reverse logistics to design a general closed loop supply chain (CLSC) network consisting of manufacturing plant, distribution center and customer market [5], though our focus remains on forward logistics given the nature of Black Friday sales.

3.5 Solution Approaches and Computational Considerations

The computational complexity of multi-period, multi-product distribution problems has led to various solution approaches. Since these models are very difficult to solve, they require exploiting their properties and developing special solution techniques to reduce the computational effort [13]. Decomposition methods, including Lagrangean relaxation and branch-and-bound algorithms, have proven effective for large-scale problems.

The book by Sawik, T. (2011), '*Scheduling in Supply Chains Using Mixed Integer Programming*' [3], provides comprehensive coverage of MIP applications to supply chain scheduling, emphasising the importance of preprocessing and modern MIP software capabilities in solving practical-scale problems. This aligns with our use of Gurobi, a state-of-the-art MIP solver, for our optimisation model.

3.6 Research Gap and Contribution

While extensive literature exists on general supply chain optimisation and inventory management, there is limited research specifically addressing the unique characteristics of Black Friday distribution challenges using real-world data. Most studies either focus on theoretical models with synthetic data or examine steady-state operations rather than promotional periods with extreme demand volatility.

Our research addresses this gap by developing a MIP model that explicitly incorporates Black Friday-specific characteristics: dramatic demand fluctuations, product-specific obsolescence risks (particularly for heavily discounted items), and the trade-off between flexible daily truck rentals versus cost-effective weekly contracts. By grounding our analysis in actual product specifications and industry-standard cost structures, we provide a practical framework that retailers can directly apply to their Black Friday distribution planning.

4 Data

4.1 Product Selection

We selected three products representing typical Black Friday electronics categories:

Table 1: Product specifications

Product	Retail Price	Shortage Cost	Holding Cost	Units/Truck
Samsung Galaxy S25 Ultra	\$2,200 [19]	\$1,100	\$44/day	350 [20]
Audio-Technica ATH-R50x	\$400 [21]	\$200	\$8/day	60 [22]
Samsung Galaxy Buds FE	\$200 [23]	\$50	\$10/day	1,500 [24]

Product dimensions were sourced from manufacturer specifications. Truck capacity allocations assume these products represent 1% of total cargo, yielding the units per truck shown above.

4.2 Cost Parameters

- **Holding costs** are based on 2% daily rate for Black Friday inventory (2.5x normal rate):
  - Standard calculation: based on reported industry standards [15] that carry costs are typically 15% - 30% of the value of a company's inventory. Assuming this is an annual figure, this gives us around a 0.8% of the item's retail price per day.
  - Black Friday multiplier accounts for warehouse premiums and time sensitivity.
  - Earphones have elevated holding cost (\$10 vs \$4 standard) due to obsolescence risk.
- **Shortage costs** reflect lost profit margins:

  - Standard calculation: based on reported industry standards [16] that 50% - 70% are considered good margins, and that the shortage cost is close to the opportunity cost of a lost sale (ie. the margin)
  - Phones and headphones: 50% margin (industry standard for electronics)
  - Earphones: 25% margin due to heavy discounting

- **Truck rental costs** from commercial providers:
  - Weekly: \$1,100 for 7 days (50m^3 truck)
  - Daily: \$100-\$200 depending on demand
  - Daily premium: 50% surcharge over weekly rate

4.3 Demand Profiles

We model identical demand patterns across all five retailers to isolate the effects of transportation costs and truck scheduling decisions. The demand profiles reflect typical Black Friday consumer behavior:

Table 2: Daily demand by product (units per retailer)

Product	Mon	Tue	Wed	Thu	Fri (BF)	Sat	Sun
Earphones (Good 0)	100	50	30	10	1,000	600	400
Headphones (Good 1)	50	20	0	0	200	10	20
Phones (Good 2)	10	5	3	1	100	80	80

Key demand characteristics:

- **Earphones**: Steady decline from Monday to Thursday (100→10 units), massive spike on Black Friday (1,000 units), gradual decline over weekend
- **Headphones**: No demand Wednesday-Thursday as consumers wait for deals, concentrated spike on Black Friday (200 units)
- **Phones**: Minimal pre-Black Friday demand, sustained weekend demand (80 units/day) as consumers purchase accessories

4.4 Network Structure

- 5 warehouses representing distribution centers
- 5 retailers representing metropolitan locations
- Transportation costs varying by distance (range: \$0.40-\$2.60 per unit)
- Maximum 10 trucks daily (driver availability constraint)

4.5 Computational Scale Study

Beyond analyzing the specific Black Friday case, we investigate how the problem characteristics and solution approaches scale to larger supply chain networks. This scale study serves two critical purposes:

- **Practical Relevance**: Real-world retailers often operate dozens of warehouses serving hundreds of stores with thousands of SKUs. Understanding how our optimisation approach performs at different scales ensures the methodology remains viable for enterprise-level implementation.
- **Computational Insights**: As problem size increases, the number of decision variables grows exponentially (warehouses $\times$ retailers $\times$ products $\times$ periods). The scale study reveals whether solution times increase linearly, polynomially, or exponentially, informing decisions about model granularity and decomposition strategies.

Further information on the Computational Case Study, including scales studied and the methods for creating synthetic data is available in section 8.

5 Model

The following is the specification of the model for weekly truck rentals. To see the modifications required to support daily truck rental, see Section 5.3 *Modifications for Daily Truck Rental.* To simplify the model, we introduce two dummy decision variables *carried_retailer_stock* and *short_retailer_stock* to:

- make calculation of holding and shortage costs easier,
- to enforce supply-demand balance in each period.

5.1 Parameters

- W: set of warehouses, $W = \{1, \ldots, n_w\}, n_w \in \mathbb{N}$.
- G: set of distinct goods, $G = \{1, \ldots, n_g\}, n_g \in \mathbb{N}$.
- R: set of retailers, $R = \{1, \ldots, n_r\}, n_r \in \mathbb{N}$.
- P: set of periods (number of days), $P = \{1, \ldots, n_p\}, n_p \in \mathbb{N}$.
- K_T: storage capacity of each truck, $K_T \in \mathbb{N}$.
- R_T: daily rental cost of each truck, $R_T \in \mathbb{N}$.
- M_T: maximum number of trucks that can be rented, $M_T \in \mathbb{N}$.
- K_g: size of good g, $K_g \in \mathbb{N}$.
- H_g: holding cost of good g, $H_g \geq 0$.
- S_g: shortage cost of good g, $S_g \geq 0$.
- $T_{w,r,g}$: transportation cost of good g from warehouse w to retailer r, $T_{w,r,g} \geq 0$.
- $D_{r,g,p}$: unit demand of good g at retailer r in period p, $D_{r,g,p} \geq 0$.

5.2 Variables and Model

- r: total number of trucks rented for the week, $r \in \mathbb{N}$.
- $a_{w,p}$: number of trucks allocated to warehouse w in period p, $a_{w,p} \in \mathbb{N}$.
- $t_{w,r,g,p}$: amount of good g transported from warehouse w to retailer r in period p, $t_{w,r,g,p} \in \mathbb{N}$.
- $h_{r,g,p}$: dummy variable to track the amount of good g held at retailer r at start of period p (ie. left from previous day), $h_{r,g,p} \in \mathbb{N}$.
- $s_{r,g,p}$: dummy variable to track the amount of good g short of demand at retailer r in period p, $s_{r,g,p} \in \mathbb{N}$.

Let

$$C_t = \sum_{w \in W} \sum_{r \in R} \sum_{g \in G} \sum_{p \in P} T_{w,r,g} t_{w,r,g,p} \qquad \forall w \in W, r \in R, g \in G, p \in P \quad \text{(Total transport costs)} \tag{1}$$

$$C_s = \sum_{r \in R} \sum_{g \in G} \sum_{p \in P} S_g s_{r,g,p} \qquad \forall r \in R, g \in G, p \in P \quad \text{(Total shortage costs)} \tag{2}$$

$$C_h = \sum_{r \in R} \sum_{g \in G} \sum_{p \in P} H_g h_{r,g,p} \qquad \forall r \in R, g \in G, p \in \{P \cup \{n_p + 1\}\} \quad \text{(Total holding costs)} \tag{3}$$

$$C_r = \sum_{w \in W} \sum_{p \in P} R_T a_{w,p} \qquad \forall w \in W, p \in P \quad \text{(Total truck rental costs)} \tag{4}$$

Then our model is

$$\min \quad C_t + C_s + C_h + C_r \qquad \text{(Minimise total cost of system)}, \tag{5}$$

$$\text{s.t.} \quad \sum_{w \in W} a_{w,p} = r, \qquad \forall p \in P, \quad \text{(Daily truck allocation = hired)}, \tag{6}$$

$$\sum_{g \in G} K_g \sum_{r \in R} t_{w,r,g,p} \leq a_{w,p} * K_T, \qquad \forall w \in W, p \in P, \quad \text{(Total sent less than capacity)}, \tag{7}$$

$$h_{r,g,1} = 0, \qquad \forall r \in R, g \in G, \quad \text{(Retailers start with no stock)}, \tag{8}$$

$$s_{r,g,p} \geq D_{r,g,p} - h_{r,g,p} - \sum_{w \in W} t_{w,r,g,p}, \qquad \forall r \in R, g \in G, p \in P \quad \text{(Shortage defn)}, \tag{9}$$

$$h_{r,g,(p+1)} = \sum_{w \in W} t_{w,r,g,p} - D_{r,g,p} + h_{r,g,p} + s_{r,g,p}, \quad \forall r \in R, g \in G, p \in P, \quad \text{(Held goods defn)}, \tag{10}$$

$$r \geq 0 \qquad \text{(Scope of variables)}, \tag{11}$$

$$a_{w,p} \geq 0 \qquad \forall w \in W, p \in P, \quad \text{(Scope of variables)}, \tag{12}$$

$$t_{w,r,g,p} \geq 0 \qquad \forall w \in W, r \in R, g \in G, p \in P \quad \text{(Scope of variables)}, \tag{13}$$

$$h_{r,g,p} \geq 0 \qquad \forall r \in R, g \in G, p \in P \quad \text{(Scope of variables)}, \tag{14}$$

$$s_{r,g,p} \geq 0 \qquad \forall r \in R, g \in G, p \in P \quad \text{(Scope of variables)}. \tag{15}$$

The objective function minimises the total cost of the system.

The first constraint ensures all trucks allocated to warehouses within a period equals the total amount of trucks hired (ie. no unused trucks).

The second constraint ensures the total size of goods sent from each warehouse does not exceed the total capacity of all trucks assigned to that warehouse.

The third constraint specifies that each retailer starts with no stock on hand.

The fourth constraint ensures that the shortage of a particular good for a particular store in a particular period is at least equal to the unmet demand. Since any additional units of shortage will contribute positively to the cost, combining this constraint with the objective function gives equality (ie. that shortage is equal to unmet demand).

The fifth constraint ensures the total units of a good left at a particular retailer in a particular period is equal to the amount held initially, plus the amount sent to the store, minus the demand of the day plus the shortage. Equivalently, this constraint says that if shortage is zero, the change in held goods is equal to supply minus demand, and if shortage is positive then by constraint 4 the amount of held goods must be zero.

5.3 Modifications for Daily Truck Rental

For daily truck rental, we make two small modifications. We must modify the variable r to now allow for daily variations, and we must modify the first constraint to take into account this variability. The variable r becomes r_p:

- r_p: total number of trucks rented for each period p, $r_p \geq 0$.

And our first constraint becomes

$$\sum_{w \in W} a_{w,p} = r_p, \quad \forall p \in P, \quad \text{(Daily truck allocation = hired)} \tag{16}$$

Note that we used separate instance data for the daily truck rental scenarios (*daily_trucks_high_cost*, *daily_trucks_low_cost*), and so the 50% daily rental premium is already captured in a change to R_T.

6 Solution strategy

As described in the above section, we propose an IP model to optimise the rental and scheduling plan. Such a formulation is usually solved exactly using the branch-and-bound algorithm. Therefore, in this project, we solve the proposed formulation using the Gurobi solver, which solves IP or MIP model using an improved branch-and-cut algorithm. Compared with the standard branch-and-bound algorithm, this solver integrates some heuristics policy and valid cuts into the branch-and-bound algorithm, significantly accelerating the solving process.

However, this solver cannot handle our formulation when the size is large. Specifically, we observe that the solver needs more than 100 seconds to solve the size of 5 warehouses, 30 types of goods, 15 retailers and 5 periods using our synthetic data. To mitigate this issue, some techniques proposed in this section are applied to accelerate the solving process. Based on appropriate experiments, we observe that these tricks are useful in some scales, helping the algorithm converge more quickly at certain problem sizes.

The challenges mainly include:

- **Large Branch-and-bound Tree**: The proposed formulation is an integer programming problem, requiring more Gomory cuts to tighten the relaxation bound. As a result, the simplex method needs to be executed significantly more often than in binary programming, leading to a longer runtime.

- **Weak relaxation bound**: We note that the best bound provided by Gurobi is difficult to improve, which reduces the efficiency of the solver.

To mitigate these issues, we propose two strategies in this section - branch priority and cut search - which accelerate convergence.

6.1 Branching strategy

The branch priority is significant to the solving efficiency. The branch-and-bound problem is far larger, especially when variables are integer, compared with binary case. To mitigate this issue, we require the solver to branch significant variables with higher priority.

In our case, the priority of the number of trucks rented is assigned with the highest priority, followed by the scheduling variables, and then the shortage calculation. The above process is implemented by setting the parameter ***BranchPriority*** for each of the variables in Gurobi.

With this technique, the Gomory cuts derived by branching the truck number are relatively stronger than ones derived by branching other variables, tightening the relaxation bound.

6.2 Active cuts search

We note that the best bound of the problem provided by the solver is hard to be improved, while the best incumbent is relatively easy to derive with the help of heuristics integrated into the solver. Based on this observation, we attempt to add more valid cuts into the formulation to tighten the relaxation bound. To implement this, we set the parameter ***Cuts*** to **3** in Gurobi to encourage the solver to find more valid cuts.

Most of the cuts derived by this step are different from Gomory cuts, which are usually generated by exploring the structure of the problem.

7 Results and analysis

7.1 Results

Tables 1 to 6 presents the optimized total cost, daily cost and average truck utilization in each scenario.

Scenario	Number of Trucks (Each period)	Truck Hire Cost	Transport- -ation Cost	Shortage Cost	Holding Cost	Total Cost
High Rent, Daily	5,2,2,10,10, 4,4	11100.0	11654.7	0.0	6464.0	29218.7
Low Rent, Daily	5,2,2,10,10, 4,5	5700.0	11621.2	0.0	6200.0	23521.2
High Rent, Weekly	10	14000.0	11518.8	0.0	6200.0	31718.8
Low Rent, Weekly	10	7000.0	11517.9	0.0	6200.0	24717.9

Table 3: Minimized cost from the model

Day	Truck Hire Cost	Transport- -ation Cost	Shortage Cost	Holding Cost	Total Cost
Day 1	1500.0	708.0	0.0	0.0	2208.0
Day 2	600.0	355.4	0.0	0.0	955.4
Day 3	3000.0	235.9	0.0	0.0	3235.9
Day 4	3000.0	680.1	0.0	776.0	4456.1
Day 5	1200.0	4754.4	0.0	5552.0	11506.4
Day 6	1200.0	2827.0	0.0	64.0	4091.0
Day 7	1200.0	2093.9	0.0	72.0	3365.9

Table 4: Minimized daily cost, when trucks are rented daily and expensive

Day	Truck Hire Cost	Transport- -ation Cost	Shortage Cost	Holding Cost	Total Cost
Day 1	750.0	708.0	0.0	0.0	1458.0
Day 2	300.0	355.4	0.0	0.0	655.4
Day 3	300.0	237.9	0.0	0.0	537.9
Day 4	1500.0	733.7	0.0	712.0	2949.7
Day 5	1500.0	4687.5	0.0	5488.0	11675.5
Day 6	600.0	2830.9	0.0	0.0	3430.9
Day 7	750.0	2067.8	0.0	0.0	2817.8

Table 5: Minimized daily cost, when trucks are rented daily and less expensive

7.2 Analysis

As the result shows, when the cost of renting trucks is high and trucks are scheduled on a daily basis, the model tries its best to meet all the demands (and succeeds) to avoid the massive shortage cost. Truck utilization for each warehouse at each period is close to 1 (See table 6), which is well expected for the condition where trucks are expensively rented daily, as wasting the capacity of trucks is very unprofitable. Holding cost is relatively low comparing to the truck renting cost and transportation cost, as trucks can be rented contingently and surplus in retailers can usually be avoided.

When the truck renting cost is low, it would be even more profitable to rent trucks contingently, and truck utilization can be more relaxed, as the ratio of profit gained by reducing the number of trucks rented to reducing carried goods is relatively lower compared to when trucks are expensive to rent (thus the model should favor more on reducing holding cost). This is reflected in the behaviour of the model as expected, where at period 6 (the seventh day), one more truck is rented comparing to the case when renting cost is high. This has enabled the warehouses to send less products in the period before (while still meeting all demands, thus in surplus), resulting a less total holding cost.

When trucks are rented on a weekly basis, the model still decides to minimize shortage cost as much as possible, even when trucks are expensive to hire. The main difference to the case where trucks are rented daily, is that during the periods when demands are low, there would be idle trucks. Therefore, truck utilization can be relaxed (see table 6), and the shipments can be made generally when demanded to reduce holding cost, and there is more freedom to decide on transportation schedule to reduce transportation cost. These effects are reflected on the results, that holding cost and transportation cost are

Day	Truck Hire Cost	Transport--ation Cost	Shortage Cost	Holding Cost	Total Cost
Day 1	2000.0	704.0	0.0	0.0	2704.0
Day 2	2000.0	325.5	0.0	0.0	2325.5
Day 3	2000.0	228.0	0.0	0.0	2228.0
Day 4	2000.0	616.2	0.0	712.0	3328.2
Day 5	2000.0	4802.1	0.0	5488.0	12290.1
Day 6	2000.0	2785.0	0.0	0.0	4785.0
Day 7	2000.0	2058.0	0.0	0.0	4058.0

Table 6: Minimized daily cost, when trucks are rented weekly and expensive

Day	Truck Hire Cost	Transport--ation Cost	Shortage Cost	Holding Cost	Total Cost
Day 1	1000.0	704.0	0.0	0.0	1704.0
Day 2	1000.0	325.5	0.0	0.0	1325.5
Day 3	1000.0	225.0	0.0	0.0	1225.0
Day 4	1000.0	668.9	0.0	712.0	2380.9
Day 5	1000.0	4751.5	0.0	5488.0	11239.5
Day 6	1000.0	2785.0	0.0	0.0	3785.0
Day 7	1000.0	2058.0	0.0	0.0	3058.0

Table 7: Minimized daily cost, when trucks are rented weekly and less expensive

lower than in the case where trucks are rented daily, and to a slight extent offsets the increased truck renting cost due to the change in renting schedule.

When the renting cost of trucks is low, it is interesting to see that the final cost only changes due to halved truck renting cost. This indicates that the decision for transportation and supplying the retailers are already optimal to ensure that the shortage cost is minimal. In fact, holding costs only presents on day 4 and 5 (holding cost is induced from the surplus on previous day) in order to meet the peak demand on Friday.

8 Computational scale study

In this section, we compare the efficiency between our method and the benchmark (using Gurobi directly without any modifications). The maximum runtime of Gurobi is limited to 100 seconds to meet the efficiency requirements of modern business operations. The time spent on constructing the model isn't included since one can change the cost vector and constraint matrix of the existing model efficiently in practice, so we focus on the runtime of the model. Computational experiments are performed on a computer equipped with an AMD Ryzen 7 6800H processor and 16 GB of RAM. All algorithms are written in Python version 3.9.2 and Gurobi 10.0.1.

8.1 Synthetic data

Our collected dataset isn't enough to support the computational evaluation of a large-scale instance, we thus extend it with some modifications. While some parameters are

Scenario	Average Truck Utilization
High Rent, Daily	0.98
Low Rent, Daily	0.95
High Rent, Weekly	0.54
Low Rent, Weekly	0.54

Table 8: Average truck utilization comparison

used directly, other parameters are generated using the random number API provided by Numpy. We denote the uniform distribution with lower bound b and upper bound a as $U(a, b)$. Note that a scaling factor is applied to simulate the Black Friday effect, similar to the last section.

Table 9: Parameters setting

Parameter	Value
good sizes	$U(1, 20)$
holding costs	$U(20, 100)$
transportation costs	$U(1, 4)$
penalty costs	$U(1, 1.2)\times$ holding costs
basic demands	$U(40, 100)$

In addition, we adjust the maximum number of trucks using the following expression to ensure that the optimisation problem is feasible but challenging:

$$maximum\ number\ of\ trucks = \left\lceil 1.2 \times \frac{total\ volume}{number\ of\ periods \times truck\ capacity} \right\rceil .$$

8.2 Results

In practice, the number of customer nodes and SKUs (goods) is typically larger than the number of warehouses and periods. Therefore, we primarily focus on varying the number of customer nodes and SKUs to evaluate scalability, while making only slight adjustments to the number of warehouses and periods.

Each instance size is run 10 times to mitigate the randomness introduced by solver behavior. The computational results are summarized in Table 10.

We observe that all instances can be solved to near optimality within 100 seconds. However, our method performs better than the benchmark method in larger instances in solving efficiency, while similarly in small cases. It is noteworthy that we set the parameter of **MIPGap** of the solver as 0.005, leading to the fact that the Gap value converges to 0.00 to prevent from tedious exploration of branch-and-bound tree. Based on this observation, we suggest that one can set a small target **MIPGap** for the solver to derive a near optimal value (possibly optimal in most cases) within a relatively short duration.

Our strategies outperform the benchmark on certain medium-scale instances, such as $(20, 60, 50, 5)$, while the benchmark is faster on others. However, the differences aren't

Table 10: Performance comparison

Scale	**Improved** Time (s)	Gap (%)	**Benchmark** Time (s)	Gap (%)
(5, 25, 15, 5)	0.38	0.00	0.38	0.00
(5, 25, 15, 10)	**0.67**	0.00	0.78	0.00
(5, 30, 15, 5)	0.25	0.00	0.19	0.00
(5, 35, 15, 5)	**0.4**	0.00	0.55	0.00
(10, 25, 25, 5)	0.53	0.00	0.43	0.00
(20, 60, 50, 5)	**2.8**	0.00	3.99	0.00
(20, 100, 60, 5)	6.93	0.00	5.81	0.00
(20, 120, 80, 5)	10.03	0.00	9.56	0.00

obvious. In most cases, an optimal solution can be obtained within seconds, demonstrating the effectiveness of our model. While both of them cannot dominate the other, in future work, we will attempt to explore more stable acceleration techniques.

9 Conclusion and recommendations

We propose an optimization model to help stakeholders determine the optimal plan of truck rental and scheduling plans to fulfill demands in different scenarios. We focus on minimizing the total costs including rental, holding and penalty costs, while some practical constraints are considered. With the support of an advanced solver, this IP model can be solved to optimality within 15 seconds under different instance sizes, fully meeting the efficiency requirements of modern business operations.

Furthermore, we attempt to propose some useful techniques to improve the solving efficiency. Synthetic data are made to test the performance of the improved method and a benchmark provided by standard Gurobi. Based on our numerical results, we notice that encourage the solver to branch significant variables with higher priority and search for valid cuts aggressively are beneficial to accelerate the convergence for medium scales. However, its superiority isn't obvious under other scales. In the future, we will explore other stable strategies to contribute to this field.

Some managerial insights are derived through numerical experiments supported by real datasets:

- When truck rental is expensive and a daily pattern is applied, the model focuses on avoiding shortage costs by fully utilizing the rented trucks.
- When truck rental is cheap and a daily pattern is applied, the model seeks a more flexible transport plan to minimize holding costs once most demands are met.
- When truck rental is expensive and a weekly pattern is applied, some idle trucks are tolerated, while a more flexible delivery plan is preferred.
- When truck rental is cheap and a weekly pattern is applied, more trucks can be rented, allowing a more flexible delivery plan, which leads to a near-optimal rental and delivery strategy.

In summary, our model adapts its decisions to different cost structures, demonstrating its effectiveness in making strategic choices across varying environments.

In the future, this model can be improved by considering the following effects:

- Several emerging innovations, such as parcel lockers and drones, have been developed to improve last-mile delivery. Exploring the integration of these technologies into the model could yield valuable contributions.
- The surge in demand during Black Friday is difficult to predict. In this report, a scaling factor was applied. However, developing a more robust model in the future could improve order fulfillment.

10 Individual contributions

- **Yitian Wang**
  - Came up with the initial proposal that was submitted.
  - Synthetic data creation.
  - Solution strategy design and computational experiments design.
  - Wrote Computational Case Study portion of the notebook.
  - Wrote Computation Case Study portion of the report, Section 8.
- **Dean Soste**
  - Wrote Case Study notebook including data loading and model formulation.
  - Conducted research into trucks and goods to hand-craft realistic demand patterns, truck storage capacities, holding and shortage costs.
  - Co-ordinated group and facilitated meetings.
  - Suggested the pivot towards a realistic business scenario (Black Friday) and increasing the complexity of the model toward a multi-period model.
  - Wrote the Introduction (Section 1), Problem Definition (Section 2), Literature Review (Section 3) and Data (Section 4) sections of the report.
- **Tony Yue**
  - Created the original dataset defined from the proposal.
  - Created the presentation slides.
- **Zhouyi Cheng**
  - Wrote the Results and Analysis section of the report (Section 7)
- **Abdullah Alsuwat**
  - Wrote the Conclusion and Recommendations section of the report (Section 9)
- **Sahar Bahalgardi**
  - Wrote the Model section of the report (Section 5)

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