Laser Beam Profiling

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1 Introduction

Lasers have been revolutionary in optics, engineering, and medical fields. Many experiments in optics rely on the use of lasers due to their ability to achieve monochromatic, coherent outputs. They can also be focused to a small small spot size, making them ideal for manufacturing precision parts and performing sensitive surgeries. Whatever the application of a given beam, it is useful to know its intensity profile. In this lab, the intensity profile of a laser diode's beam is found using two methods: by obtaining 1-D information using a knife-edge, and 2-D information from photography of the profile. Data obtained from these methods is then used to find the $1/e^2$ beam radii, i.e. the dimensions of the beam profile.

2 Beam Properties

2.1 Laser Modes

To understand the nature of a beam's transverse profile, it is useful to understand how a laser works.

A laser works to amplify light through a gain-feedback system. This is done by the use of a laser cavity. In its most basic form, a laser cavity consists of two aligned mirrors, one with complete reflection, and one with a small amount of transmission, similar in construction to a Fabry-Perot Interferometer. This property of the cavity allows for light waves to interfere with themselves and produce a high intensity output relative to the individual contributions by the gain medium. Suppose a parallel, flat mirror cavity shown in Figure 1.

At cavity lengths of $L = \frac{l\lambda}{2}$ where l is an integer, constructive interference is achieved in the output; i.e. all exiting electromagnetic waves are in phase with one another. The output of this laser cavity is maximum at these lengths. It is said that the laser cavity is at "resonance" and the waveform in the cavity is a "longitudinal mode."

The light also has a "transverse mode" which manifests itself as the intensity profile of the beam in the transverse plane, the plane normal to the light ray.[1] Section 2.2 is dedicated to solving for these modes.

In order to supply power to the laser cavity, a gain medium is set within the cavity, usually a gas, semiconductor, or

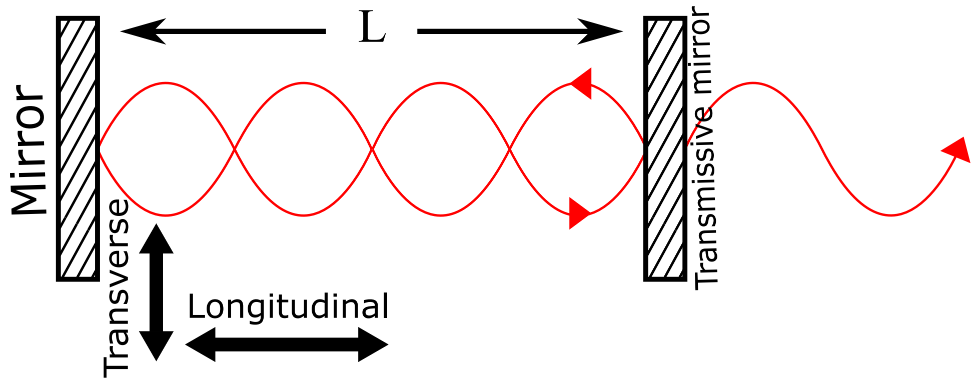

Figure 1: Plane parallel resonator cavity with length L

crystal. This medium serves to interact with light passing through the laser cavity which stimulates emission of new light waves which are in-phase with the old light waves. When more light is added than what is lost in each round trip in the cavity, amplification occurs until a high intensity output is achieved.[2]

2.2 Solving for Transverse Modes

The laser light created by many lasers satisfies certain transverse modes called "Hermite-Gaussian modes." To derive this solution, one must start by recognizing that the electric component (and magnetic component) of light in the cavity acts as a wave, satisfying the 3-Dimensional Wave Equation of light in a vacuum:

$$\nabla^2 \vec{E} - \left(\frac{1}{c}\right)^2 \frac{\partial^2 \vec{E}}{\partial t^2} = 0 \tag{1}$$

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$, $\vec{E}$ is the electric field vector of light, and c is the speed of light.

An ansatz, i.e. an assumption about the solution of the equation, is made about $\vec{E}$. Let $\vec{E}$ have the following general form:

$$\vec{E}\left(\vec{r}, t\right) = \hat{\epsilon} A\left(\vec{r}\right) e^{i(kz-\omega t)} \tag{2}$$

where $\vec{r}$ is the coordinates (x, y, z), $\hat{\epsilon}$ is the polarization vector, and $A\left(\vec{r}\right)$ is a "slowly varying" maximum amplitude of the electric field. Note that $A\left(\vec{r}\right)$ is slowly varying in the sense that A has negligible change in the z-direction on the order of λ. Also note $A\left(\vec{r}\right)$ manifests itself as the transverse electric field profile of the light. $e^{i(kz-\omega t)}$ is the "fast phase carrier" of light in the form of a plane wave where $k = \frac{2\pi}{\lambda}$, and $\omega = 2\pi f$ where f is the frequency of light. By Euler's identity, this value is equivalent to the following:

$$e^{i(kz-\omega t)} = cos(kz - \omega t) + isin(kz - \omega t)$$

While calculations are done with this "complex" $\vec{E}$, it is important remember that physical values of $\vec{E}$ are found by taking the real part, i.e. $Re[\vec{E}]$.

Plug in the ansatz for the electric field into the 3-Dimensional Wave Equation and simplify while seeking to preserve the $A(\vec{r})$ term. Let $\nabla^2 = \nabla_T^2 + \frac{\partial^2}{\partial z^2}$ where $\nabla_T^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$:

$$\nabla_T^2 A + \frac{\partial^2 A}{\partial z^2} + 2ik\frac{\partial A}{\partial z} = 0 \tag{3}$$

To further simplify this differential equation, use the *paraxial approximation*. That is, since $A(\vec{r})$ is slowly varying in the z-direction on the order of λ, light rays travel roughly parallel in the laser cavity. Further, this means that higher order derivatives of $A(\vec{r})$ can be neglected. The consequence of this is the following:

$$\left|\lambda\frac{\partial}{\partial z}\left(\frac{\partial A}{\partial z}\right)\right| \ll \left|4\pi\frac{\partial A}{\partial z}\right|$$

This is equivalent to the following relationship:

$$\left|\frac{\partial^2 A}{\partial z^2}\right| \ll \left|2ik\frac{\partial A}{\partial z}\right|$$

This allows for $\frac{\partial^2 A}{\partial z^2}$ to be neglected, obtaining the following *Paraxial Wave Equation*:[3]

$$\nabla_T^2 A(\vec{r}) + 2ik\frac{\partial A}{\partial z} = 0 \tag{4}$$

By use of mathematical methods not covered here, the Paraxial Wave Equation can be solved to find the transverse modes of a beam. One family of solutions to this differential equation are the "Hermite-Gaussian modes" with rectangular symmetry. (Other modes also solve this equation but focus will be placed on the Hermite-Gaussian modes in this report as they are some of the most common.)

The Hermite-Gaussian modes take on the following form:

$$A(x, y, z) = u_m(x, z)\, u_n(y, z)\, A_0 \tag{5}$$

$$u_m(x, z) = (\text{Phase and z-dependence}) \times H_m\left(\frac{\sqrt{2}x}{w(z)}\right) exp\left(\frac{ikx^2}{2\tilde{q}(z)}\right)$$

where H_m is a Hermitian polynomial with order m, $w(z)$ is the $1/e^2$ radius of the beam at a location z (i.e. when $H_m = 1$ and z is fixed, the distance from the point of maximal amplitude of the beam to where the amplitude drops off to $1/e^2$ of max amplitude), and $\tilde{q}(z) = z - iz_R$ is the complex beam parameter where z_R is the Rayleigh range of the beam.

The following relations can be derived from Equation 5 and are useful:

$$w(z) = w_0\left(1 + \left(\frac{z}{z_R}\right)\right)^{1/2}$$

$$z_R = \frac{\pi w_0^2}{\lambda}$$

where w_0 is the waist of the beam, i.e. where the $1/e^2$ radius of the beam is the smallest.

$u_n(y, z)$ is equivalent to u_m, except one should swap out x for y, and m for n. The resulting plots of electric field amplitude give the Transverse Electro-Magnetic modes, i.e. TEM_{mn} modes:

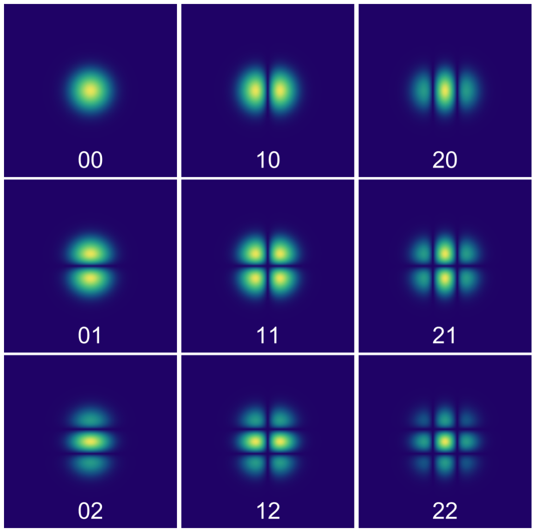

Figure 2: A sample of different Hermite-Gaussian modes which satisfy the paraxial wave equation. Generated using Mathematica v13.

In this lab, the laser diode used has an output with a mode best approximated by the TEM_{00} mode. Since the Hermitian polynomial at the zeroth order has the value $H_0 = 1$, the TEM_{00} profile behaves entirely as a Gaussian radially from the center.

In this lab, the photosensors used actually measure the intensity I of the transverse profile rather than the electric field amplitude A. However, I is related to A by $I \propto A^2$. Since A behaves as a Gaussian in the TEM_{00} case, so does I as any Gaussian squared returns another Gaussian.

3 Imaging Techniques

The aim of this report is to use different imaging techniques to make observations about the transverse profile of a laser beam and find dimensions of the beam profile.

3.1 Knife-Edge Profiling

One method to obtain information about a laser beam's intensity profile in the transverse plane is by a knife-edge scan. By progressively blocking more of a beam's profile with a razor, 1-dimensional information can be found about the beam.

By shining the beam at the photodetector and varying the amount of razor blockage, a function of power that passes the razor versus position of razor can be constructed. This graph is the integral of the 1D intensity profile of the beam. Therefore, the derivative of the plot simply needs to be taken to obtain the intensity profile.

See Section 4.1 for analysis of this data.

3.2 Beam Photography

Another typical way to observe the beam profile of a laser is by the use of a CCD camera to obtain a 2D heatmap of its intensity profile. Since this project was remote and had limited resources, the beam was photographed using an ordinary smartphone camera to obtain 2D information. In order to find as much information as possible, the following steps were followed:

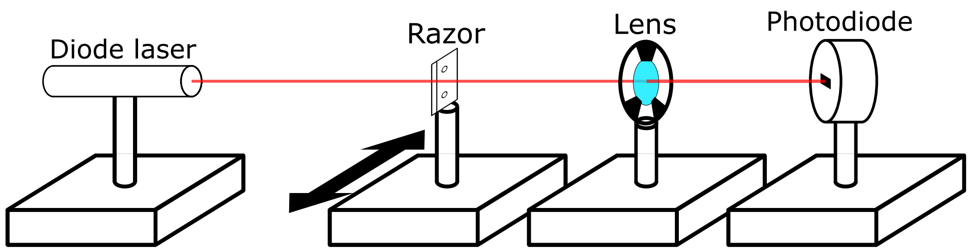

Figure 3: Knife-edge profiling set-up

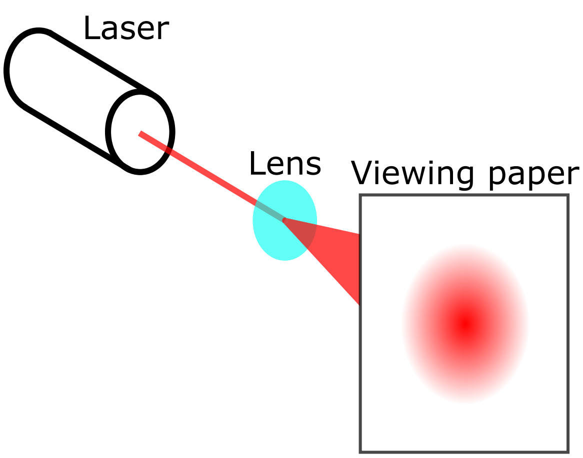

Figure 4: Beam photography set-up

- Magnify the beam with a lens. This allows for the camera to better resolve details of the beam's intensity profile. It also lowers the intensity picked up by the camera and prevents saturation, i.e. the loss of intensity information.

- Shine the beam on a plain, smooth piece of printer paper. This allows for the beam to be projected onto a viewing screen able to be picked up by the camera. Emphasis is placed on using a white, mark-free paper to obtain a clear image of the profile.

- Use manual focus and ISO to obtain the best picture of the beam. A Samsung A50 camera with Adobe Lightroom Pro-Camera was used to take the picture. Since ISO reduces the camera's sensitivity to light, this is another way to avoid saturation and save information about the beam.

- For especially intense beams, a single polarizer or a pair of crossed polarizers can be used to reduce the intensity output of the beam to counteract saturation of the camera.

While this is an accessible way to learn more about a laser's profile, the use of a lens loses information about the scale of the naked beam's profile.

See Section 4.2 for data analysis using beam photography.

4 Data Analysis

4.1 Knife-Edge Measurements

In this lab, the laser beam from a Thorlabs laser diode, Model CPS635, is profiled. A razor mounted to a micrometer is placed in front of the laser beam $7.5 \pm 0.5cm$ from the laser casing. This gives the razor sufficient space to completely block and unblock the beam. On the other side of the razor, a lens with focal length $f = 200mm$ is used to focus the light onto a photodiode. This photodiode produces a current output measured by an ammeter that is proportional to the total power input.

From inspection, the laser beam has an oblong shape. Initially, the laser is positioned in a mount such that the longer axis of the beam profile is positioned parallel to the direction at which the razor would be displaced using a micrometer translation stage. Call this orientation "horizontal." See Figure 5 for data collected in this orientation.

The laser is then mounted such that the shorter axis is positioned parallel to the direction of razor movement. Call this orientation "vertical." See Figure 6 for the collected data.

To obtain some sort of idea about the dimensions of the beam, each 1-D scan can be used to learn about how "wide" the beam is in each direction. Since a Gaussian beam theoretically has no hard cutoff to make this measurement, a standard about what the "beam radius" is must be decided. Recall from Section 2.2 the

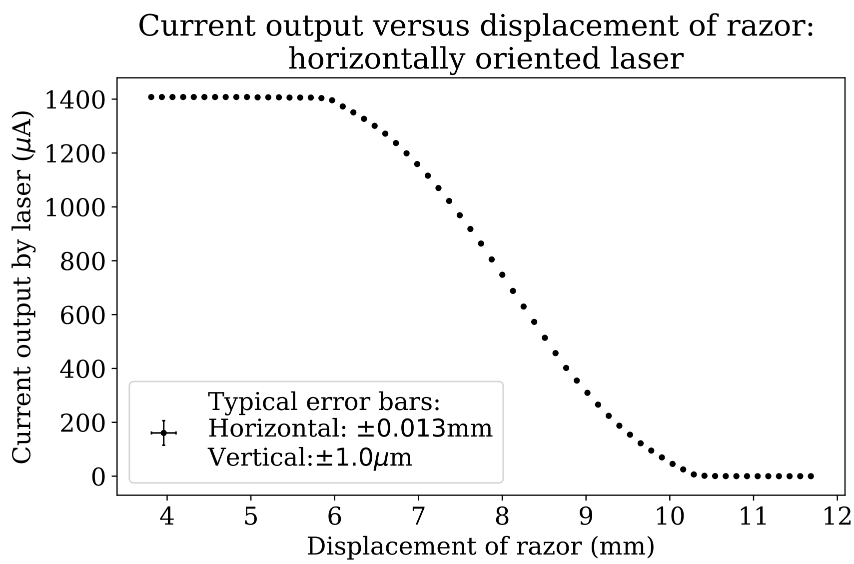

Figure 5: Laser profile aligned horizontally

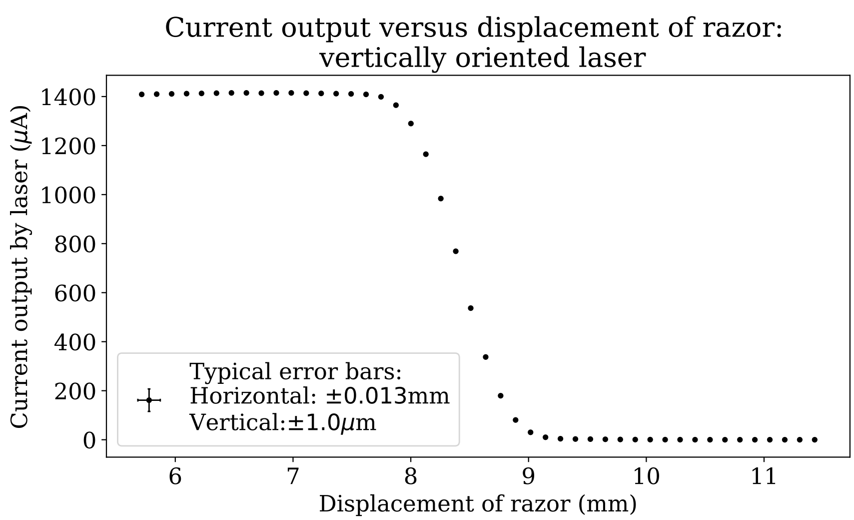

Figure 6: Laser profile aligned vertically

idea of $w(z)$ being the $1/e^2$ radius, or the distance from the maximal point of the Gaussian amplitude profile to where the amplitude drops down to $1/e^2$ of the maximum (approximately 13.5% of the max). This standard is reused, but now the $1/e^2$ pertains to the distance from the maximal intensity of Gaussian profile to where the intensity drops off to $\approx 13.5\%$.

The data from Figures 5 and 6 resemble the *Complementary Error Function* (ERFC), an integral of the Gaussian. By obtaining parameterizations of the ERFC and the Gaussian function, with one parameter being the $1/e^2$ radius, one can fit data which satisfies each relationship to learn what the $1/e^2$ radius is in each case.

The Gaussian intensity profile satisfies the following parameterization:

$$I = I_0 \cdot exp\left(\frac{-2(x-B)^2}{r^2}\right) \qquad (6)$$

where I_0 is the maximum intensity of the beam, B is the centering of the profile, and r is the $1/e^2$ radius of the profile.

The ERFC can be equivalently parameterized in the following way:

$$\mathscr{I} = \mathscr{I}_0 \cdot ERFC\left(\frac{x\sqrt{2}-B}{r}\right) \qquad (7)$$

where B and r represent the same quantities as the Gaussian fit, but $\mathscr{I}$ represents cumulative intensity.

From the ERFC fitting formula, it is possible to directly extract the $1/e^2$ radii from the data. However, further inspection of the data is necessary before fitting to make sure the intensity profile is indeed Gaussian.

The numerical derivative of each plot is determined using the central difference formula:

$$f'(x_i) = \frac{f(x_{i+1}) - f(x_{i-1})}{x_{i+1} - x_{i-1}}$$

4.1.1 Horizontal data

See Figure 7 for the numerical derivative of the horizontal data. From inspection, this plot suggests that the beam profile is not strictly Gaussian. It appears that an aperture is blocking off the edges of the beam. However, this means that the data within the cutoff range can be fit to the Gaussian. On top of this data, a Gaussian fit was created with the points within the cutoff region (red dots are excluded from the fit):

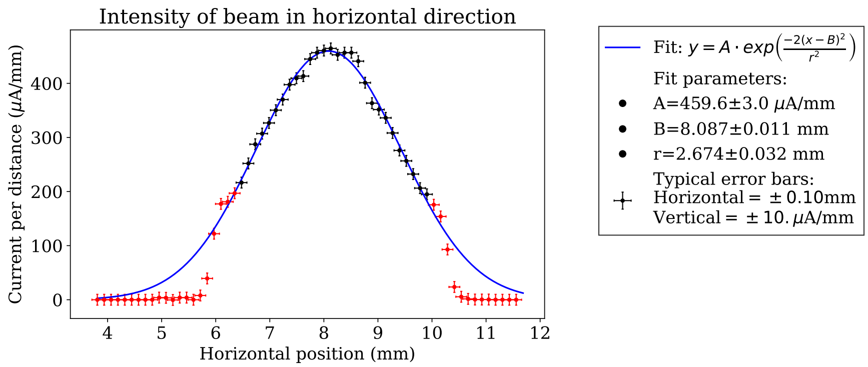

Figure 7: Fitting to numerical derivative of horizontal data; red dots are excluded from the fit. r is the $1/e^2$ radius in the horizontal direction.

Using the fit parameters, had the beam not been cut off, the beam radius along this direction is determined to be $r_{1/e^2} = 2.674 \pm 0.032mm$.

While fitting the Gaussian to the numerical derivative is one way to find the $1/e^2$ radii, the numerical derivative's data points tend to deviate quite a lot from the fit. Alternatively, it is desirable to go back to the ERFC plot and fit the data corresponding to locations of the profile within the cutoff range using the ERFC formula. The following is the result:

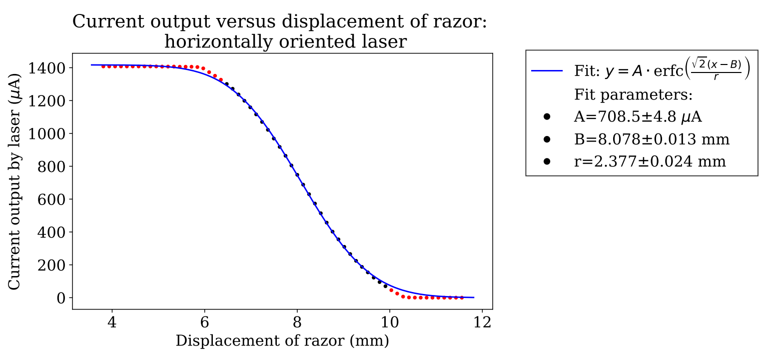

Figure 8: Fitting horizontal data to ERFC; red dots are excluded from the fit. r is the $1/e^2$ radius in the horizontal direction.

This gives the result that had the beam not been cut off, the radius would have been $r_{1/e^2} = 2.377 \pm 0.024mm$.

It is important to note that the transition from the beam to the cut-off region is not a hard boundary. Around any corner, in this case the aperture, diffraction of light occurs which causes an interference pattern. In this case, it likely artificially inflated the radius measurements.

4.1.2 Vertical data

See Figure 9 for the numerical derivative of the vertical data. Unlike the horizontal data, this appears to fit nicely with the Gaussian.

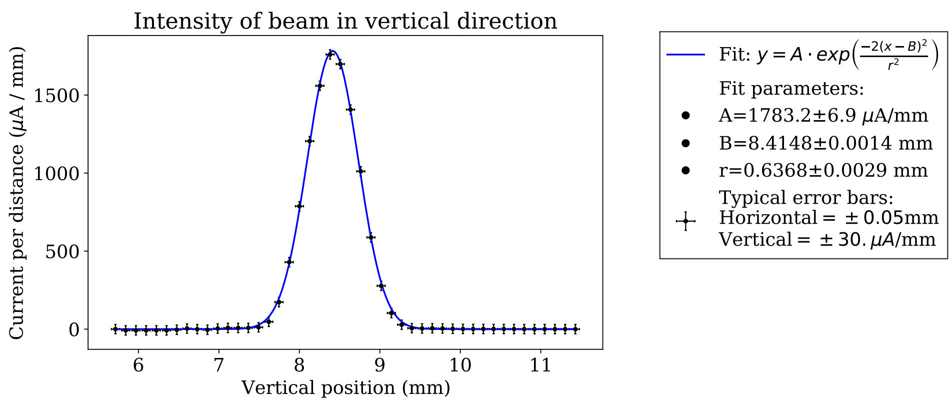

Figure 9: Fitting to numerical derivative to vertical data. r is the $1/e^2$ radius in the vertical direction.

The fitted Gaussian gives the $1/e^2$ radius of the beam, $r_{1/e^2} = 0.6368 \pm 0.0029mm$. The ERFC data is also fitted:

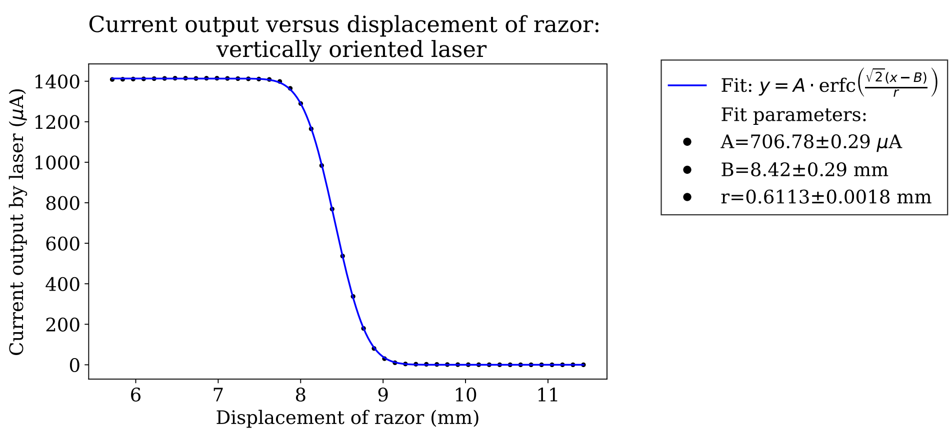

Figure 10: Fitting vertical data to ERFC. r is the $1/e^2$ radius in the vertical direction.

From this, it is found that the radius of the beam is $r_{1/e^2} = 0.6113 \pm 0.0018mm$.

4.2 Camera Measurements

By following the principles outlined in Section 3.2, the following picture of the laser was obtained and turned into a heatmap with colors corresponding to pixel intensity:

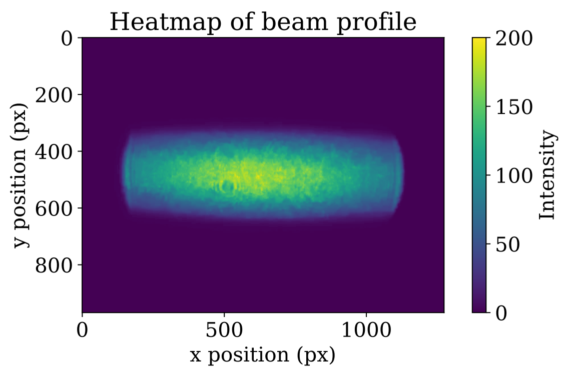

Figure 11: Heatmap of beam profile

Just by inspection, it is obvious that this beam is not Gaussian; there are sudden cutoffs on the ends of the horizontal axis as observed in Section 4.1. It is also oblong, suggesting that the beam has an elliptical output from the diode and is truncated by an aperture as it exits the casing. Further data analysis can be done by doing lineouts on chosen axes. The lineouts can be treated like the 1D scans in Section 4.1. Since information was lost regarding the scale of the beam size, this method cannot be used to determine beam radii. Rather, a ratio between the long and short $1/e^2$ radii can be found and compared to the ratio of radii results from Section 4.1.

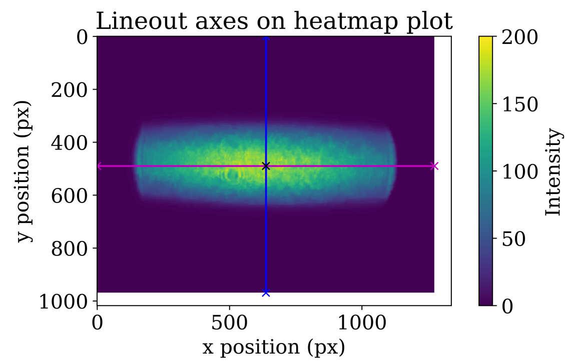

Figure 12: Axes of lineouts of heatmap. Code offered by courtesy of Eric Jones.

Figure 13 was made by fitting a Gaussian to the horizontal lineout data, i.e. the magenta axis. Note that the fit was created excluding the red data points. This follows the same reasoning as Section 4.1 where data within the cutoff region can be fitted to a Gaussian to predict the horizontal $1/e^2$ radius had the aperture not truncated the beam profile.

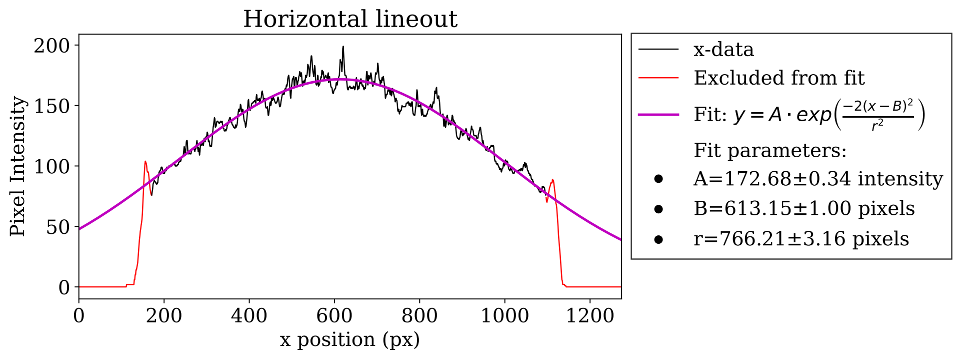

Figure 13: Horizontal lineout plot; note that the horizontal lineout fit is fitted only to the black data within the cutoff range. r is the $1/e^2$ radius in the horizontal direction.

The following plot was made by fitting a Gaussian to the vertical lineout data, i.e. the blue axis:

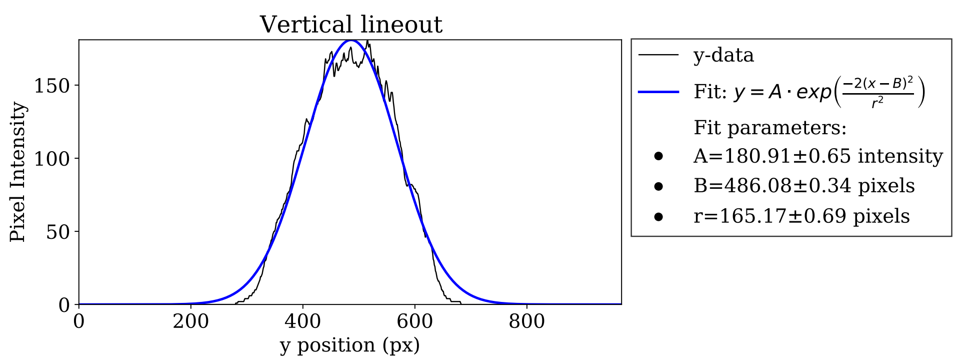

Figure 14: Vertical lineout plot. r is the $1/e^2$ radius in the vertical direction.

From these radii, one can divide the horizontal radius by the vertical radius to obtain a ratio. The following is the ratio from the camera measurements:

$$R_{camera} = \frac{\text{Horizontal radius}}{\text{Vertical radius}} = 4.639 \pm 0.027$$

Compare this to the ratio computed from the razor measurements in Section 4.1 (using ERFC radius parameters):

$$R_{knife} = \frac{\text{Horizontal radius}}{\text{Vertical radius}} = 3.888 \pm 0.041$$

These ratios do not agree with each other within the bounds of error. A likely explanation for this discrepancy is the astigmatism of the beam. Rather than be perfectly collimated, each axis of the beam profile has a different focal length which causes the beam to diverge and converge at different rates depending on the axis. The result is that the ratio of these radii are spatially dependent in the direction of beam propagation.

5 Conclusion

Using a knife-edge measurement, it was observed that the beam was truncated on one axis of its profile, likely due to an aperture. This conclusion was backed up by the photos taken of the magnified beam profile. Regardless, the data from the knife-edge measurements which behaved like the expected Gaussian intensity profile was fitted, thus ignoring points which were in the cutoff regions. For the beam $7.5 \pm 0.5cm$ from the edge of the laser casing, the $1/e^2$ radii of the longer and shorter axes, had the beam not been cut off, were found to be $2.377 \pm 0.024mm$ and $0.6113 \pm 0.0018mm$ respectively (using ERFC radius parameters).

Similar fits were done to the photos of the beam profile. However, since no information about the scale of the beam was obtained, only the ratio R_{camera} of long axis to short axis of the photographed profile could be compared to the knife-edge ratio R_{knife}. The ratios were found to be $R_{camera} = 4.639 \pm 0.027$ and $R_{knife} = 3.888 \pm 0.041$. Unsurprisingly, these ratios differ, likely because of the astigmatism of the beam which causes one axis of the profile to diverge at a different rate from the other.

Notes

[1] G. R. Fowles, *Introduction to Modern Optics*, 2nd ed. (Holt, Rinehart and Winston, Inc., New York, 1989), pp. 272-273.

[2] Ibid, p. 268.

[3] L. C. Andrews, and R. L. Phillips, "Free-Space Propagation of Gaussian-Beam Waves" in *Laser Beam Propagation Through Random Media*, 2nd ed. (SPIE, 2005).