Knife Edge measurement of Gaussian Beam

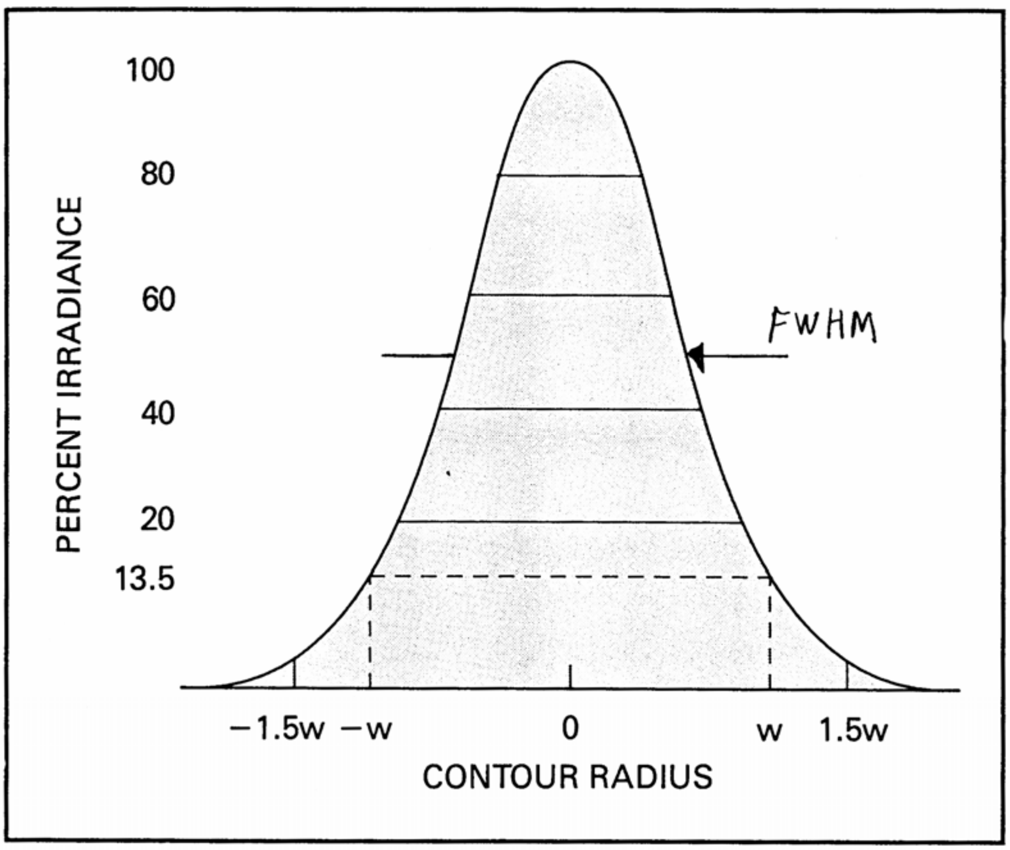
- Consider a Gaussian shaped beam

$$I(r) = I_0 \exp\left(\frac{-2r^2}{w^2}\right) = \frac{2P}{\pi w^2} \exp\left(\frac{-2r^2}{w^2}\right)$$

Where P = total power in the beam

w = $1/e^2$ beam radius at point w(z)

- This is in cylindrical coordinates
- r is the radius of the central area



GAUSSIAN IRRADIANCE PROFILE for TEM_{00} mode, showing definitions of beam radius w.

Knife Edge and Gaussian

- Straight knife edge cutting into a Gaussian shaped beam
- Measure the total power seen when knife move in x direction
- Must convert to Cartesian coordinates & integrate
- Assume -∞ is when the knife fully below the beam

$$I(x) = \frac{2P}{\pi w^2} \int_{-\infty}^{x} \exp\left(\frac{-x^2}{w^2} \right) dx \int_{-\infty}^{\infty} \exp\left(\frac{-y^2}{w^2} \right) dy$$

Where P is the total power of the beam

I(x) is the intensity measured at position x

- In x direction the beam is cut: Integrate from x to -∞
- In y direction get full beam: integrate from -∞ to +∞
- To solve this use the error function or integral of the normal

$$erf(x) = \frac{2}{\sqrt{\pi}} \int_{0}^{x} e^{-s^2} ds$$

- Two ways of fitting this:
- Fit the power measured: that is the integral
- Fit the derivative

Fit the power measured

- That is the integral
- x_0 = centre of beam
- Then the power measured is given by for x >x_0

$$I(x)=\frac{P}{2}erf\left(\frac{(x-x_0)}{w}\right)$$

For x<x_0

$$I(x)=\frac{P}{2}\left[1+erf\left(\frac{(-x+x_0)}{w}\right)\right]$$

- Must also assume some background light level B
- In Excel use the Normdist (Normal distribution function)
- This is slightly different from erf function
- Fit with the excel function of the following formula

I(x)=P*normdist(-x,-x_0,w/1.414,1)+B

Where x is the position (starting with x below x_0)
    x_0 is the fitted centre point of the beam
    w = 1/e^2 point you fit
    1 is to make it the integration of the normal distribution
    B is the background or offset level

- Set up a spreadsheet with initial estimates of each parameter
- Have columns with x, I(x_n), I(x_n)-I(x_{n-1}), fitted P, $(\text{fit-}I(x))^2$, error
- Set a column to sum the error^2 (sum of the squares)
- Plot the difference (gives an estimate of w and x_0)
- P estimate is max power, B is background level
- Then use solver under tools to fit with minimizing sum of squares
- Use sum of squares as fit, others as variables
- Useful to plot the errors against position (called residuals)
- Should be on both sides of plots
- See plots next page

He-Ne Glenn

first round: (this is used to roughly profile the laser beam and decide on the resolution for the z-axis needed)

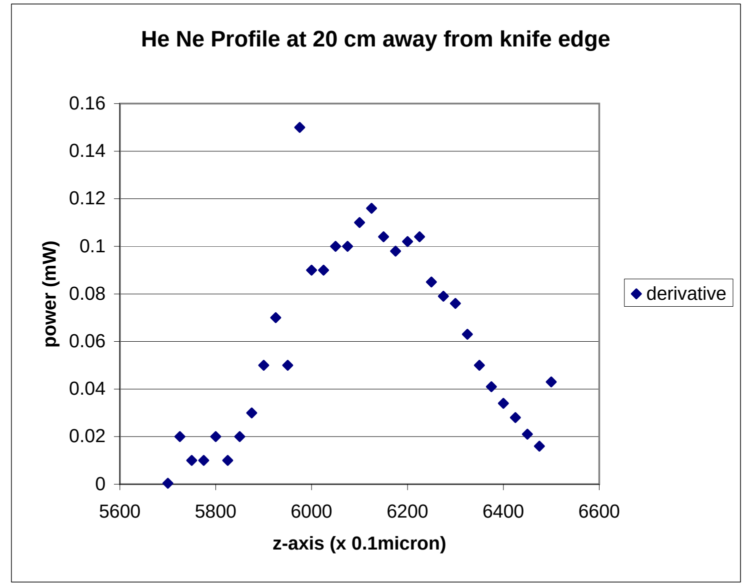
z-axis	power
5700	2
5775	1.98
5850	1.96
5925	1.87
6000	1.72
6075	1.39
6150	1.08
6225	0.76
6300	0.467
6375	0.25
6450	0.114
6525	0.046

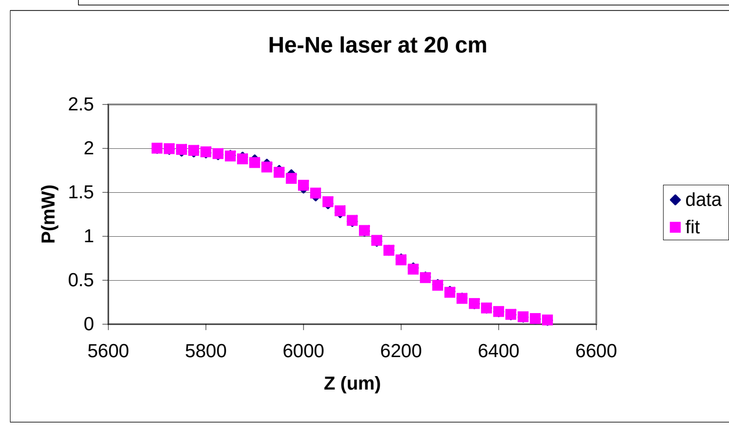
without laser: 0.05 mW

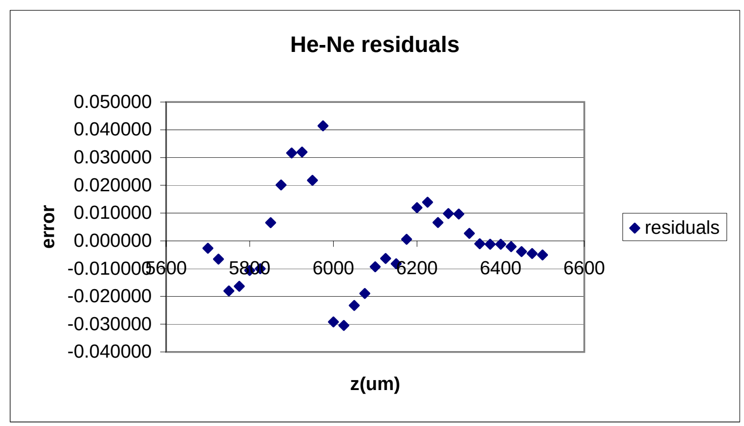
can see that the increments is too much
need to have higher resolution so need to decrease increment size

			Z0	W/1.414	I0	P	erf version	W
second round:			6136.773	175.2052	0.009834	2.005552		247.7402
z-axis(um)	power	deriviative	P	error^2	fit error			
5700	2	0.0004	2.002681	7.19E-06	-0.002681		2.002694	
5725	1.99	0.02	1.996572	4.32E-05	-0.006572		1.99659	
5750	1.97	0.01	1.988034	0.000325	-0.018034		1.988057	
5775	1.96	0.01	1.976341	0.000267	-0.016341		1.97637	
5800	1.95	0.02	1.960649	0.000113	-0.010649		1.960685	
5825	1.93	0.01	1.940015	0.0001	-0.010015		1.940059	
5850	1.92	0.02	1.913427	4.32E-05	0.006573		1.913479	
5875	1.9	0.03	1.879859	0.000406	0.020141		1.879918	
5900	1.87	0.05	1.838329	0.001003	0.031671		1.838394	
5925	1.82	0.07	1.787983	0.001025	0.032017		1.788053	
5950	1.75	0.05	1.728177	0.000476	0.021823		1.72825	
5975	1.7	0.15	1.658564	0.001717	0.041436		1.658637	
6000	1.55	0.09	1.579166	0.000851	-0.029166		1.579235	
6025	1.46	0.09	1.490428	0.000926	-0.030428		1.490491	
6050	1.37	0.1	1.393249	0.000541	-0.023249		1.393302	
6075	1.27	0.1	1.288967	0.00036	-0.018967		1.289007	
6100	1.17	0.11	1.179313	8.67E-05	-0.009313		1.179338	
6125	1.06	0.116	1.066331	4.01E-05	-0.006331		1.066339	
6150	0.944	0.104	0.952263	6.83E-05	-0.008263		0.927347	
6175	0.84	0.098	0.839415	3.42E-07	0.000585		0.839389	
6200	0.742	0.102	0.73002	0.000144	0.011980		0.729979	
6225	0.64	0.104	0.626107	0.000193	0.013893		0.626053	
6250	0.536	0.085	0.529386	4.37E-05	0.006614		0.529323	
6275	0.451	0.079	0.441172	9.66E-05	0.009828		0.441102	
6300	0.372	0.076	0.362335	9.34E-05	0.009665		0.362262	
6325	0.296	0.063	0.293296	7.31E-06	0.002704		0.293223	
6350	0.233	0.05	0.234054	1.11E-06	-0.001054		0.233984	
6375	0.183	0.041	0.18424	1.54E-06	-0.001240		0.184175	
6400	0.142	0.034	0.143199	1.44E-06	-0.001199		0.14314	
6425	0.108	0.028	0.110064	4.26E-06	-0.002064		0.110013	
6450	0.08	0.021	0.083852	1.48E-05	-0.003852		0.083808	
6475	0.059	0.016	0.063532	2.05E-05	-0.004532		0.063496	
6500	0.043	0.043	0.048098	2.6E-05	-0.005098		0.048069	
				0.009046				

without laser: 0.05 mW
using the same cover as that for trial 1







Fit the difference of Power Measured

- That is the derivative
- x_0 = centre of beam
- Take a derivative of the measurements
- Best if take a simple derivative

$$\frac{dI_0}{dx} = \frac{[I(x_1) - I(x_0)]}{[x_1 - x_0]}$$

- Then the plot is Gaussian with the formula:

$$\frac{dI(x)}{dx} = \frac{P}{w\sqrt{\pi}} \exp\left(- \frac{[x - x_0]^2}{w^2} \right)$$

- Note need to be careful with the derivatives units you use