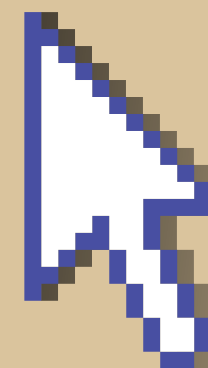


AED 2 - 2025.1

INTERSEÇÃO DE LISTAS ORDENADAS

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TRABALHO 8

Descrição:

Implementar um algoritmo para encontrar eficientemente a interseção entre dois arrays ordenados.

Entrada:

Dois arrays ordenados.

Saída:

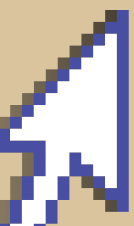
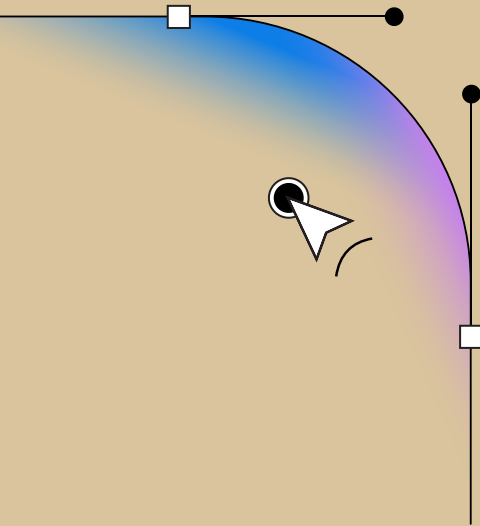
Array com os elementos que aparecem em ambos os arrays.

Exemplo:

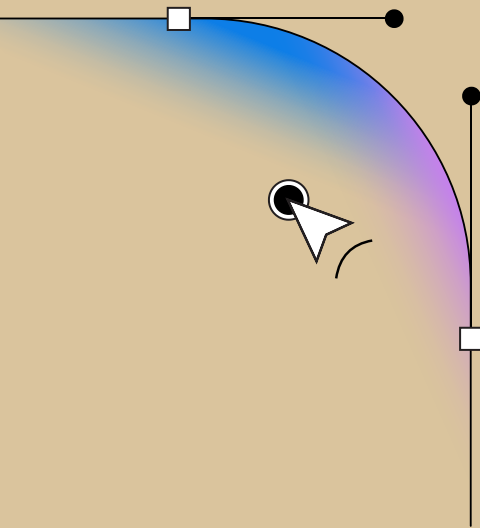
Lista 1 = [1, 3, 4, 5, 7, 9, 11, 11]

Lista 2 = [3, 5, 6, 7, 8, 9, 9, 11]

Saída esperada = [3, 5, 7, 9, 11]

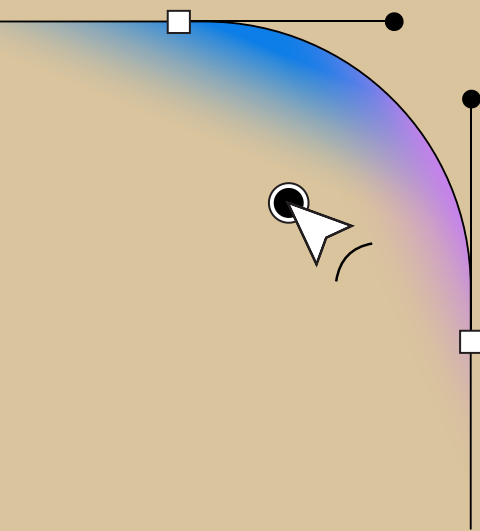


CÓDIGO 1 - COM REPETIÇÃO



```
IntersecaoComRepeticao.py > intersecao
1  def buscaBinaria(L, inicio, fim, numero):
2      if inicio > fim:
3          return []
4      else:
5          meio = (inicio + fim)//2
6          if L[meio] > numero:
7              return buscaBinaria(L, inicio , meio - 1, numero)
8          elif L[meio] < numero:
9              return buscaBinaria(L, meio + 1, fim, numero)
10         else:
11             return [L[meio]]
12
13 def intersecao2(A, inicioA, fimA, B):
14     if inicioA > fimA:
15         return []
16     else:
17         meio = (inicioA + fimA)//2
18         arrayMenor = intersecao2(A, inicioA, meio - 1, B)
19         arrayMaior = intersecao2(A, meio + 1, fimA, B)
20         termoComum = buscaBinaria(B, 0, len(B) - 1, A[meio])
21         return arrayMenor + termoComum + arrayMaior
22
23 def intersecao(A, B):
24     tamA = len(A)
25     tamB = len(B)
26     if tamB >= tamA:
27         return intersecao2(A, 0, tamA - 1, B)
28     else:
29         return intersecao2(B, 0, tamB - 1, A)
```

CÓDIGO 2 - SEM REPETIÇÃO



```
IntersecaoSemRepeticao.py > intersecao2
1  def buscaBinaria(L, inicio, fim, numero):
2      if inicio > fim:
3          return []
4      else:
5          meio = (inicio + fim)//2
6          if L[meio] > numero:
7              return buscaBinaria(L, inicio, meio - 1, numero)
8          elif L[meio] < numero:
9              return buscaBinaria(L, meio + 1, fim, numero)
10         else:
11             return [L[meio]]
12 def intersecao2(A, inicioA, fimA, B):
13     if inicioA > fimA:
14         return []
15     else:
16         meio = (inicioA + fimA)//2
17         arrayMenor = intersecao2(A, inicioA, meio - 1, B)
18         arrayMaior = intersecao2(A, meio + 1, fimA, B)
19         termoComum = buscaBinaria(B, 0, len(B) - 1, A[meio])
20         if len(termoComum) != 0:
21             existeNoMenor = len(arrayMenor) != 0 and termoComum[0] <= arrayMenor[-1]
22             existeNoMaior = len(arrayMaior) != 0 and termoComum[0] >= arrayMaior[0]
23             if existeNoMenor or existeNoMaior:
24                 termoComum = []
25         return arrayMenor + termoComum + arrayMaior
26 def intersecao(A, B):
27     tamA = len(A)
28     tamB = len(B)
29     if tamB >= tamA:
30         return intersecao2(A, 0, tamA - 1, B)
31     else:
32         return intersecao2(B, 0, tamB - 1, A)
```

TEOREMA MESTRE

Sejam:

- n = tamanho de A
- m = tamanho de B
- A é o array que está sendo dividido;
- B é o array onde é feita a busca binária.

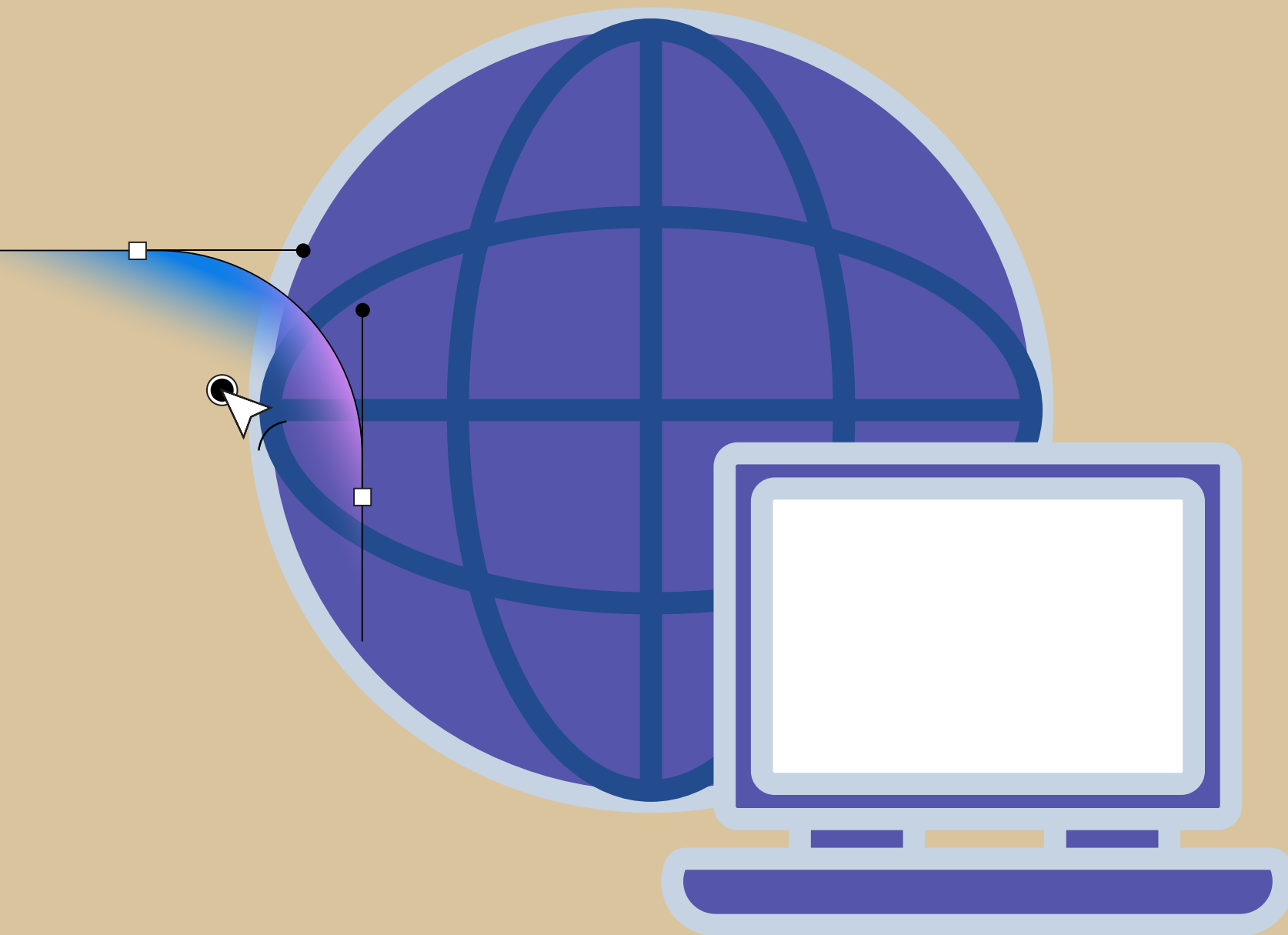
O problema é dividido em duas chamadas recursivas, cada uma com metade de A.

Em cada nível de recursão, se faz 1 busca binária no array B de tamanho m e o custo de uma busca binária é $O(\log m)$

Temos então:

$$T(n) = 2T(n/2) + O(\log m)$$

- $a = 2$ (chamadas recursivas)
- $b = 2$ (dividindo o problema ao meio)
- $f(n) = O(\log m)$ (trabalho fora da recursão)



TEOREMA MESTRE

Comparar $f(n)$ com $n^{\log_b(a)}$

Queremos saber como $f(n) = \log m$ se compara com n .

Se considerarmos $m \geq n$, então:

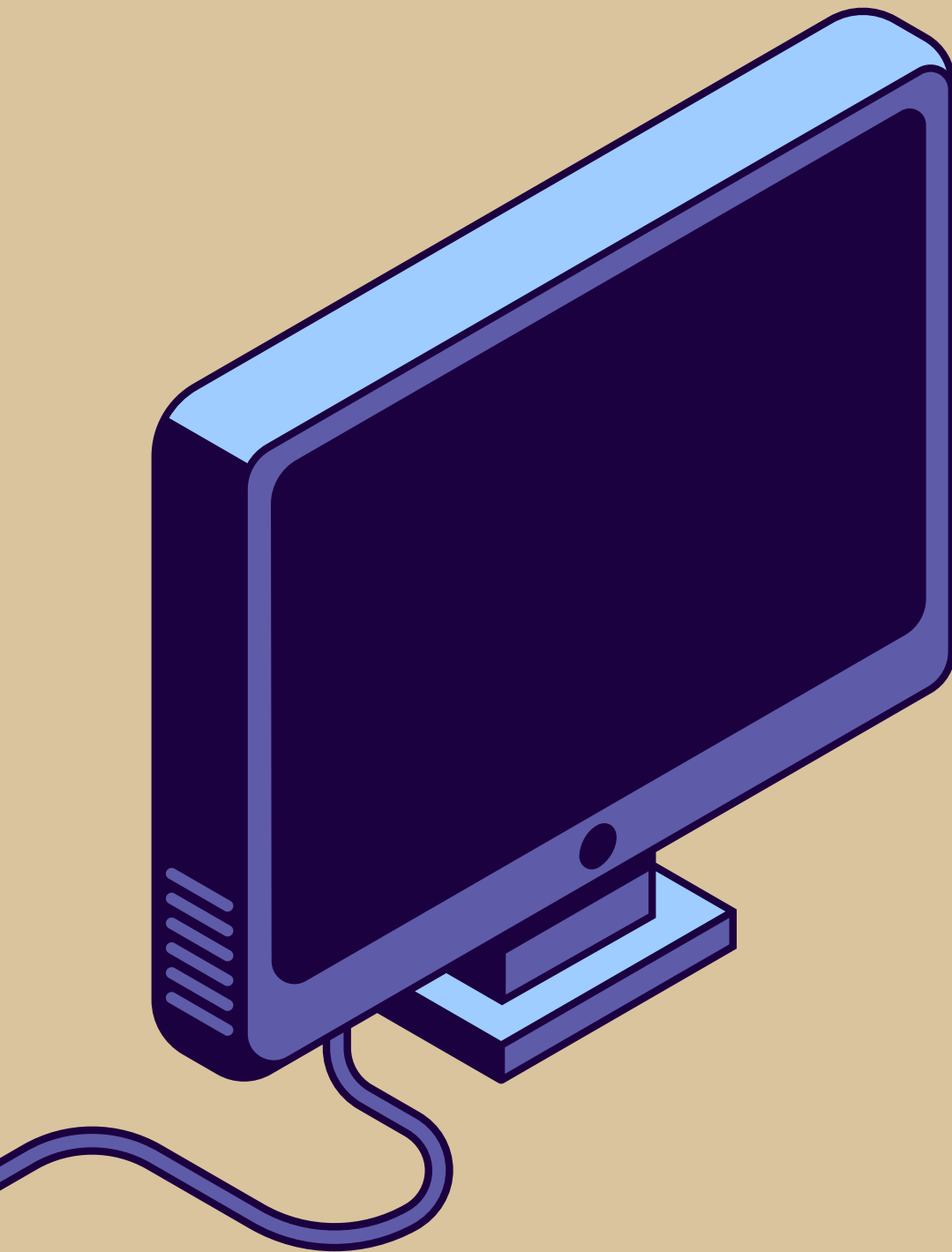
- $\log m = O(\log n)$
- E sabemos que $\log n = o(n)$ (estritamente menor que n)

Então temos o Caso 1 do teorema:

$$T(n) = \Theta(n^{\log_b(a)}) = \Theta(n)$$

E o tempo total fica:

$$T(n) = \Theta(n \cdot \log m)$$



TEMPOS COM REPETIÇÃO

10 números:

```
Interseção COM repetição 10 elementos
Array 1: [1, 3, 4, 5, 7, 9, 11, 11, 12, 15]
Array 2: [1, 2, 2, 5, 6, 10, 10, 11, 12, 20]
Resultado: [1, 5, 11, 11, 12]
Tempo: 0.000000 segundos
```

100 números:

```

Recebe: [0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15,
, 96, 100, 104, 105, 108, 110, 112, 115,
96, 200, 204, 208, 212, 216, 220, 224, 2
, 328, 332, 336, 340, 344, 348, 352, 356
Tempo: 0.001999 segundos

```

1.000 números:

```
69, 7144, 7144, 7381, 7519
69, 9378, 9434, 9654, 9747
Tempo: 0.003140 segundos
```

10.000 números:

```
Interseção COM repetição 10.000 elemen
Resultado: Squeezed text (474 lines).
Tempo: 0.092202 segundos
```

100.000 números:

```
Resultado: Squeezed text (3636 lines).
Tempo: 0.632804 segundos
```

1.000.000 números:

```
Interseção COM repetição 1.000.000 e
Resultado: Squeezed text (36355 lines).
Tempo: 4.311645 segundos
```

TEMPOS SEM REPETIÇÃO

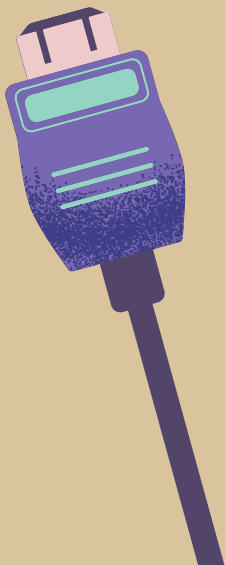
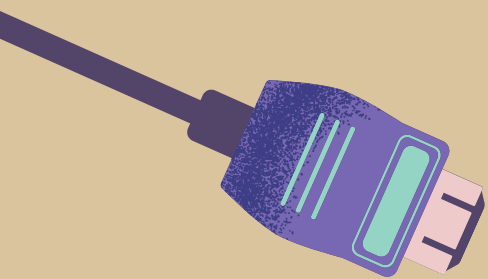
10 números:

```
Interseção SEM repetição 100 elementos  
Resultado: [0, 4, 5, 8, 10, 12, 15, 16, 20, 24, 25, 28, 30, 32, 36, 38, 40, 44, 48, 52, 56, 60, 64, 68, 72, 76, 80, 84, 88, 92, 96, 100, 104, 105, 108, 110, 112, 115, 116, 120, 124, 128, 132, 136, 140, 144, 148, 152, 156, 160, 164, 168, 172, 176, 180, 184, 188, 192, 196, 200, 204, 208, 212, 216, 220, 224, 228, 232, 236, 240, 244, 248, 252, 256, 260, 264, 268, 272, 276, 280, 284, 288, 292, 296, 300, 304, 308, 312, 316, 320, 324, 328, 332, 336, 340, 344, 348, 352, 356, 360, 364, 368, 372, 376, 380, 384, 388, 392, 396, 400, 404, 408, 412, 416, 420, 424, 428, 432, 436, 440, 444, 448, 452, 456, 460, 464, 468, 472, 476, 480, 484, 488, 492, 496, 500, 504, 508, 512, 516, 520, 524, 528, 532, 536, 540, 544, 548, 552, 556, 560, 564, 568, 572, 576, 580, 584, 588, 592, 596, 600, 604, 608, 612, 616, 620, 624, 628, 632, 636, 640, 644, 648, 652, 656, 660, 664, 668, 672, 676, 680, 684, 688, 692, 696, 700, 704, 708, 712, 716, 720, 724, 728, 732, 736, 740, 744, 748, 752, 756, 760, 764, 768, 772, 776, 780, 784, 788, 792, 796, 800, 804, 808, 812, 816, 820, 824, 828, 832, 836, 840, 844, 848, 852, 856, 860, 864, 868, 872, 876, 880, 884, 888, 892, 896, 900, 904, 908, 912, 916, 920, 924, 928, 932, 936, 940, 944, 948, 952, 956, 960, 964, 968, 972, 976, 980, 984, 988, 992, 996, 1000]  
Tempo: 0.001020 segundos
```

100 números:

```
44, 7381, 7519, 7866, 7889, 8012, 8035, 8058, 8081, 8104, 8127, 8150, 8173, 8196, 8219, 8242, 8265, 8288, 8311, 8334, 8357, 8380, 8403, 8426, 8449, 8472, 8495, 8518, 8541, 8564, 8587, 8610, 8633, 8656, 8679, 8702, 8725, 8748, 8771, 8794, 8817, 8840, 8863, 8886, 8909, 8932, 8955, 8978, 9001, 9024, 9047, 9070, 9093, 9116, 9139, 9162, 9185, 9208, 9231, 9254, 9277, 9300, 9323, 9346, 9369, 9392, 9415, 9438, 9461, 9484, 9507, 9530, 9553, 9576, 9599, 9622, 9645, 9668, 9691, 9714, 9737, 9760, 9783, 9806, 9829, 9852, 9875, 9898, 9921, 9944, 9967, 9990, 10013, 10036, 10059, 10082, 10105, 10128, 10151, 10174, 10197, 10220, 10243, 10266, 10289, 10312, 10335, 10358, 10381, 10404, 10427, 10450, 10473, 10496, 10519, 10542, 10565, 10588, 10611, 10634, 10657, 10680, 10703, 10726, 10749, 10772, 10795, 10818, 10841, 10864, 10887, 10910, 10933, 10956, 10979, 11002, 11025, 11048, 11071, 11094, 11117, 11140, 11163, 11186, 11209, 11232, 11255, 11278, 11301, 11324, 11347, 11370, 11393, 11416, 11439, 11462, 11485, 11508, 11531, 11554, 11577, 11600, 11623, 11646, 11669, 11692, 11715, 11738, 11761, 11784, 11807, 11830, 11853, 11876, 11899, 11922, 11945, 11968, 11991, 12014, 12037, 12060, 12083, 12106, 12129, 12152, 12175, 12198, 12221, 12244, 12267, 12290, 12313, 12336, 12359, 12382, 12405, 12428, 12451, 12474, 12497, 12520, 12543, 12566, 12589, 12612, 12635, 12658, 12681, 12704, 12727, 12750, 12773, 12796, 12819, 12842, 12865, 12888, 12911, 12934, 12957, 12980, 13003, 13026, 13049, 13072, 13095, 13118, 13141, 13164, 13187, 13210, 13233, 13256, 13279, 13302, 13325, 13348, 13371, 13394, 13417, 13440, 13463, 13486, 13509, 13532, 13555, 13578, 13601, 13624, 13647, 13670, 13693, 13716, 13739, 13762, 13785, 13808, 13831, 13854, 13877, 13900, 13923, 13946, 13969, 13992, 14015, 14038, 14061, 14084, 14107, 14130, 14153, 14176, 14199, 14222, 14245, 14268, 14291, 14314, 14337, 14360, 14383, 14406, 14429, 14452, 14475, 14498, 14521, 14544, 14567, 14590, 14613, 14636, 14659, 14682, 14705, 14728, 14751, 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COMPARANDO



	Com Repetição	Sem Repetição
10 números	0.000000	0.000000
100 números	0.001999	0.001020
1.000 números	0.003140	0.004020
10.000 números	0.092202	0.047047
100.000 números	0.632804	0.436656
1.000.000 números	4.311645	6.154403

OBRIGADO!

