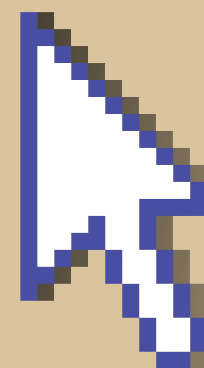


AED 2 - 2025.1

# INTERSECÇÃO DE LISTAS ORDENADAS

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# TRABALHO 8

## Descrição:

Implementar um algoritmo para encontrar eficientemente a interseção entre dois arrays ordenados.

## Entrada:

Dois arrays ordenados.

## Saída:

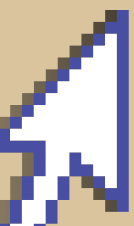
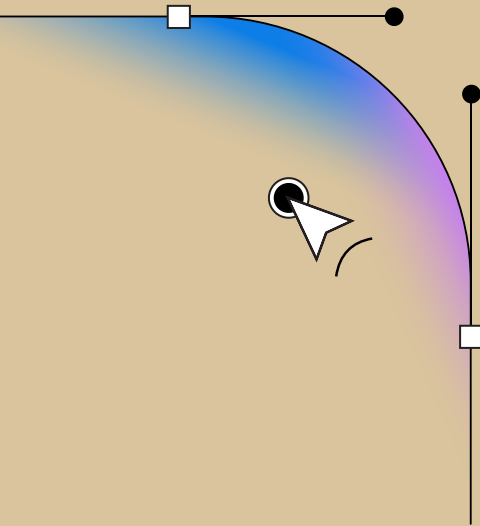
Array com os elementos que aparecem em ambos os arrays.

## Exemplo:

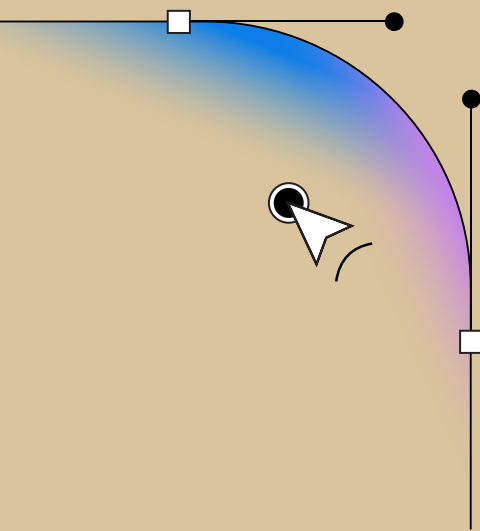
Lista 1 = [1, 3, 4, 5, 7, 9, 11, 11]

Lista 2 = [3, 5, 6, 7, 8, 9, 9, 11]

Saída esperada = [3, 5, 7, 9, 11]

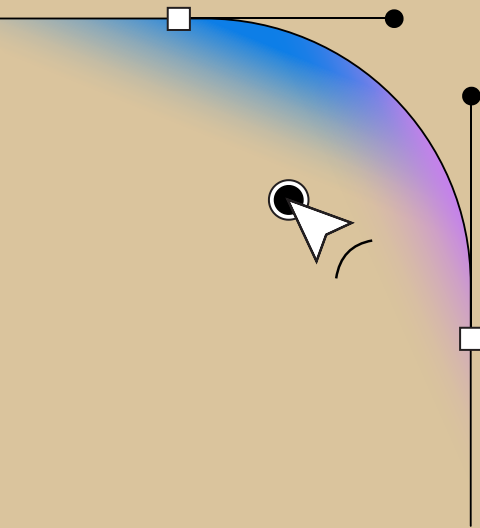


# CÓDIGO 1 - COM REPETIÇÃO



```
IntersecaoComRepeticao.py > intersecao
1  def busca_bin(L, inicio, fim, numero):
2      if inicio > fim:
3          return []
4      else:
5          meio = (inicio+fim)//2
6          if L[meio] > numero:
7              return busca_bin(L, inicio , meio-1, numero)
8          elif L[meio] < numero:
9              return busca_bin(L, meio+1, fim, numero)
10         else:
11             return [L[meio]]
12
13 def intersecao2(A, inicioA, fimA, B, inicioB, fimB):
14     if inicioA > fimA or inicioB > fimB:
15         return []
16     else:
17         meio = (inicioA+fimA)//2
18         arrayMenor = intersecao2(A, inicioA, meio - 1, B, inicioB, fimB)
19         arrayMaior = intersecao2(A, meio+1, fimA, B, inicioB, fimB)
20         termoComum = busca_bin(B, 0, len(B)-1, A[meio])
21         return arrayMenor + termoComum + arrayMaior
22
23 def intersecao(A, B):
24     tamA = len(A)
25     tamB = len(B)
26     if tamB >= tamA:
27         return intersecao2(A, 0, tamA - 1, B, 0, tamB -1)
28     else:
29         return intersecao2(B, 0, tamB - 1, A, 0, tamA -1)
30
```

# CÓDIGO 2 - SEM REPETIÇÃO



```
IntersecaoSemRepeticao.py > intersecao2
1 def busca_bin(L, inicio, fim, numero):
2     if inicio > fim:
3         return []
4     else:
5         meio = (inicio+fim)//2
6         if L[meio] > numero:
7             return busca_bin(L, inicio, meio-1, numero)
8         elif L[meio] < numero:
9             return busca_bin(L, meio+1, fim, numero)
10        else:
11            return [L[meio]]
12 def intersecao2(A, inicioA, fimA, B, inicioB, fimB):
13     if inicioA > fimA or inicioB > fimB:
14         return []
15     else:
16         meio = (inicioA+fimA)//2
17         arrayMenor = intersecao2(A, inicioA, meio - 1, B, inicioB, fimB)
18         arrayMaior = intersecao2(A, meio+1, fimA, B, inicioB, fimB)
19         termoComum = busca_bin(B, 0, len(B)-1, A[meio])
20         if len(termoComum) != 0:
21             existeNoMenor = len(arrayMenor) != 0 and termoComum[0] <= arrayMenor[-1]
22             existeNoMaior = len(arrayMaior) != 0 and termoComum[0] >= arrayMaior[0]
23             if existeNoMenor or existeNoMaior:
24                 termoComum = []
25         return arrayMenor + termoComum + arrayMaior
26 def intersecao(A, B):
27     tamA = len(A)
28     tamB = len(B)
29     if tamB >= tamA:
30         return intersecao2(A, 0, tamA - 1, B, 0, tamB -1)
31     else:
32         return intersecao2(B, 0, tamB - 1, A, 0, tamA -1)
```

# TEOREMA MESTRE

Sejam:

- $n$  = tamanho de A
- $m$  = tamanho de B
- A é o array que está sendo dividido;
- B é o array onde é feita a busca binária.

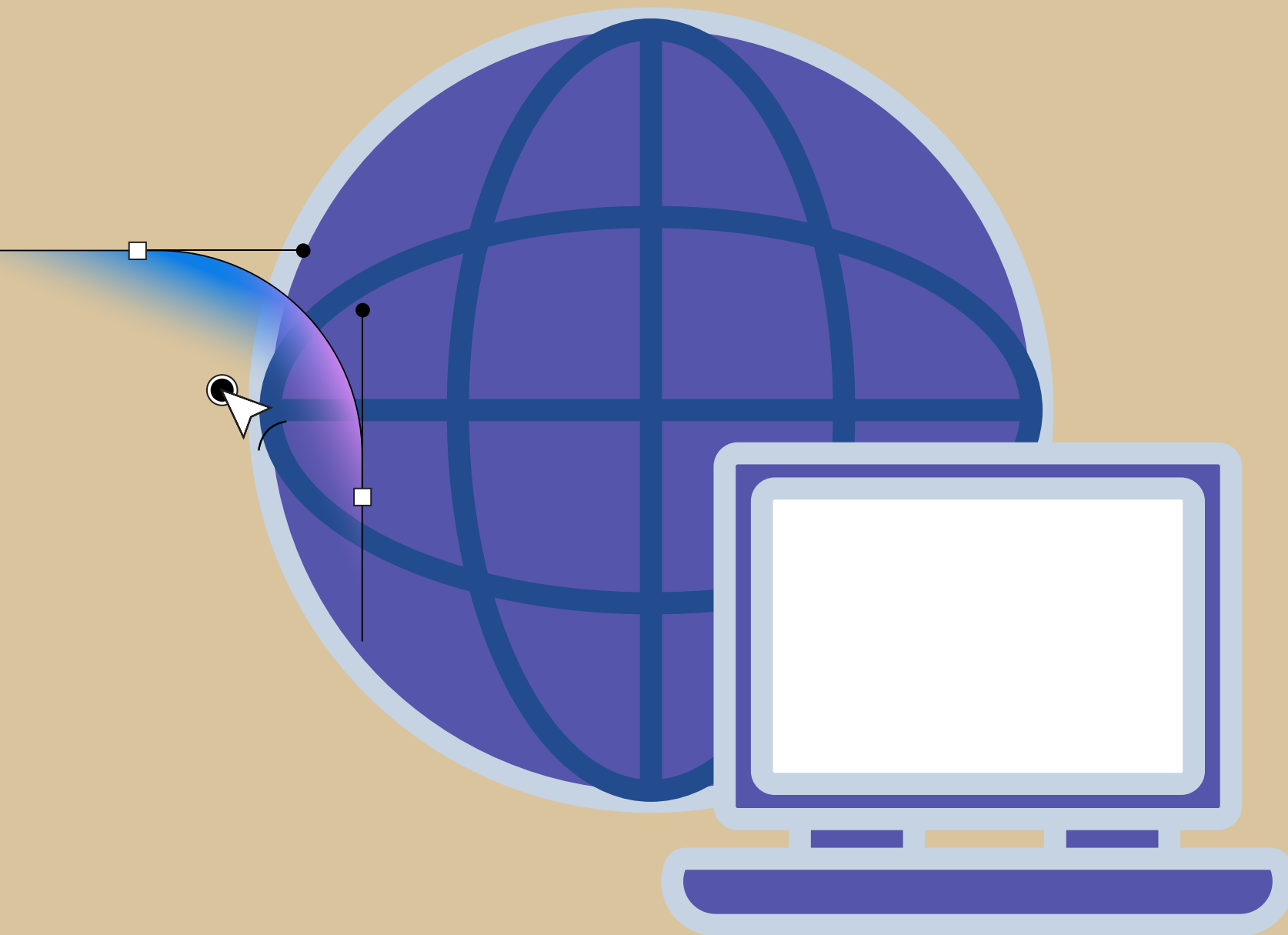
O problema é dividido em duas chamadas recursivas, cada uma com metade de A.

Em cada nível de recursão, se faz 1 busca binária no array B de tamanho  $m$  e o custo de uma busca binária é  $O(\log m)$

Temos então:

$$T(n) = 2T(n/2) + O(\log m)$$

- $a = 2$  (chamadas recursivas)
- $b = 2$  (dividindo o problema ao meio)
- $f(n) = O(\log m)$  (trabalho fora da recursão)



# TEOREMA MESTRE

Comparar  $f(n)$  com  $n^{\log_b(a)}$

Queremos saber como  $f(n) = \log m$  se compara com  $n$ .

Se considerarmos  $m \geq n$ , então:

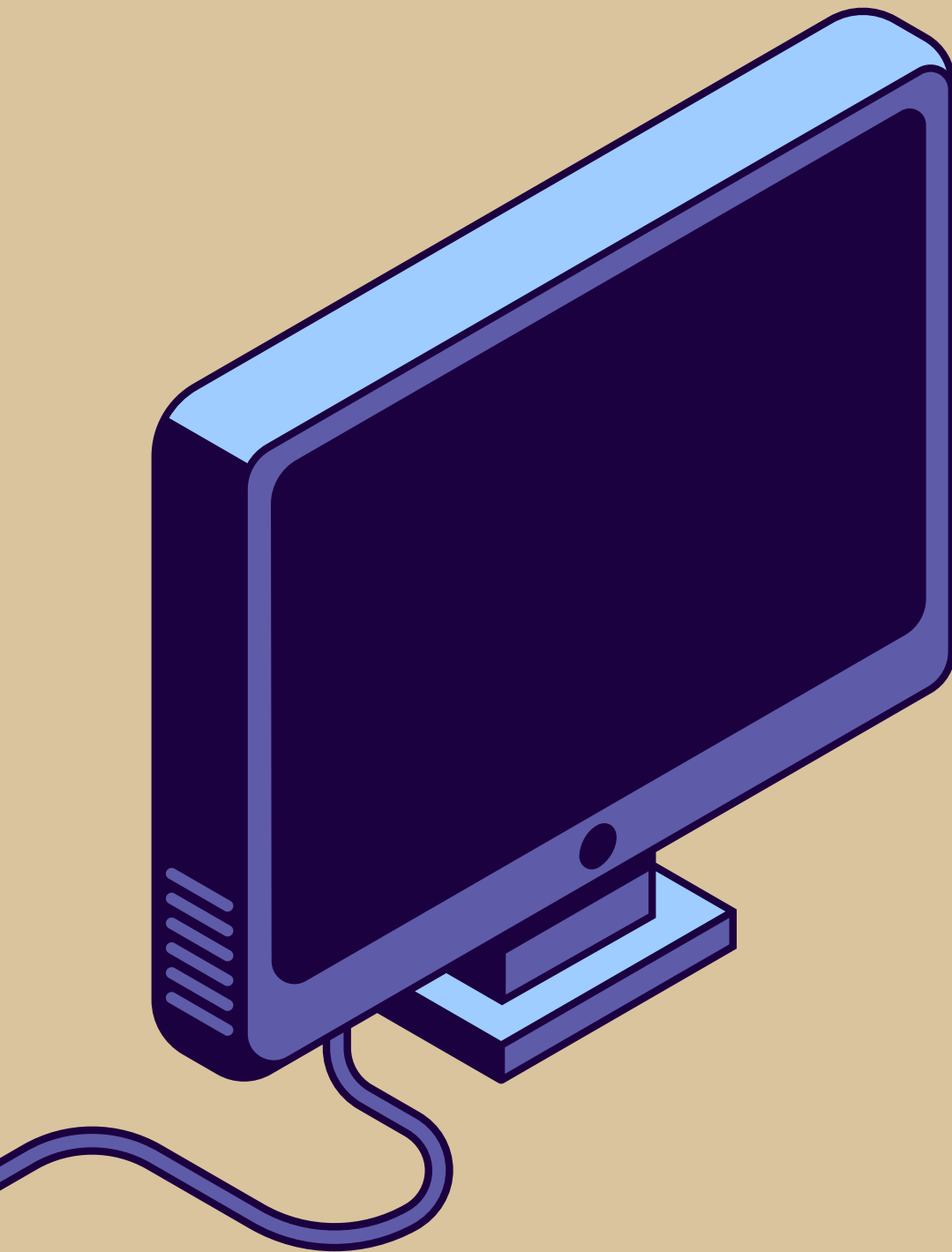
- $\log m = O(\log n)$
- E sabemos que  $\log n = o(n)$  (estritamente menor que  $n$ )

Então temos o Caso 1 do teorema:

$$T(n) = \Theta(n^{\log_b(a)}) = \Theta(n)$$

E o tempo total fica:

$$T(n) = \Theta(n \cdot \log m)$$



# TEMPOS COM REPETIÇÃO

# 10 números:

```
Interseção COM repetição 10 elementos
Array 1: [1, 3, 4, 5, 7, 9, 11, 11, 12, 15]
Array 2: [1, 2, 2, 5, 6, 10, 10, 11, 12, 20]
Resultado: [1, 5, 11, 11, 12]
Tempo: 0.000000 segundos
```

# 100 números:

```

Recebe: [0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15,
, 96, 100, 104, 105, 108, 110, 112, 115,
96, 200, 204, 208, 212, 216, 220, 224, 2
, 328, 332, 336, 340, 344, 348, 352, 356
Tempo: 0.001999 segundos

```

# 1.000 números:

```
69, 7144, 7144, 7381, 7519
69, 9378, 9434, 9654, 9747
Tempo: 0.003140 segundos
```

## 10.000 números:

```
Interseção COM repetição 10.000 elemen
Resultado: Squeezed text (474 lines).
Tempo: 0.092202 segundos
```

# 100.000 números:

```
Resultado: Squeezed text (3636 lines).
Tempo: 0.632804 segundos
```

# 1.000.000 números:

```
Interseção COM repetição 1.000.000 e
Resultado: Squeezed text (36355 lines).
Tempo: 4.311645 segundos
```

# TEMPOS SEM REPETIÇÃO

10 números:

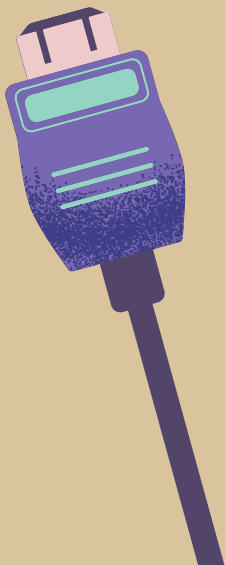
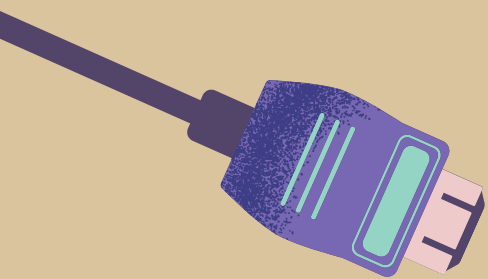
```
Interseção SEM repetição 100 elementos  
Resultado: [0, 4, 5, 8, 10, 12, 15, 16, 20, 24, 25, 28, 30, 32, 36, 38, 40, 44, 48, 52, 56, 60, 64, 68, 72, 76, 80, 84, 88, 92, 96, 100, 104, 105, 108, 110, 112, 115, 116, 120, 124, 128, 132, 136, 140, 144, 148, 152, 156, 160, 164, 168, 172, 176, 180, 184, 188, 192, 196, 200, 204, 208, 212, 216, 220, 224, 228, 232, 236, 240, 244, 248, 252, 256, 260, 264, 268, 272, 276, 280, 284, 288, 292, 296, 300, 304, 308, 312, 316, 320, 324, 328, 332, 336, 340, 344, 348, 352, 356, 360, 364, 368, 372, 376, 380, 384, 388, 392, 396, 400, 404, 408, 412, 416, 420, 424, 428, 432, 436, 440, 444, 448, 452, 456, 460, 464, 468, 472, 476, 480, 484, 488, 492, 496, 500, 504, 508, 512, 516, 520, 524, 528, 532, 536, 540, 544, 548, 552, 556, 560, 564, 568, 572, 576, 580, 584, 588, 592, 596, 600, 604, 608, 612, 616, 620, 624, 628, 632, 636, 640, 644, 648, 652, 656, 660, 664, 668, 672, 676, 680, 684, 688, 692, 696, 700, 704, 708, 712, 716, 720, 724, 728, 732, 736, 740, 744, 748, 752, 756, 760, 764, 768, 772, 776, 780, 784, 788, 792, 796, 800, 804, 808, 812, 816, 820, 824, 828, 832, 836, 840, 844, 848, 852, 856, 860, 864, 868, 872, 876, 880, 884, 888, 892, 896, 900, 904, 908, 912, 916, 920, 924, 928, 932, 936, 940, 944, 948, 952, 956, 960, 964, 968, 972, 976, 980, 984, 988, 992, 996, 1000]  
Tempo: 0.001020 segundos
```

100 números:

```
44, 7381, 7519, 7866, 7889, 8012, 8035, 8058, 8081, 8104, 8127, 8150, 8173, 8196, 8219, 8242, 8265, 8288, 8311, 8334, 8357, 8380, 8403, 8426, 8449, 8472, 8495, 8518, 8541, 8564, 8587, 8610, 8633, 8656, 8679, 8702, 8725, 8748, 8771, 8794, 8817, 8840, 8863, 8886, 8909, 8932, 8955, 8978, 9001, 9024, 9047, 9070, 9093, 9116, 9139, 9162, 9185, 9208, 9231, 9254, 9277, 9300, 9323, 9346, 9369, 9392, 9415, 9438, 9461, 9484, 9507, 9530, 9553, 9576, 9599, 9622, 9645, 9668, 9691, 9714, 9737, 9760, 9783, 9806, 9829, 9852, 9875, 9898, 9921, 9944, 9967, 9990, 10013, 10036, 10059, 10082, 10105, 10128, 10151, 10174, 10197, 10220, 10243, 10266, 10289, 10312, 10335, 10358, 10381, 10404, 10427, 10450, 10473, 10496, 10519, 10542, 10565, 10588, 10611, 10634, 10657, 10680, 10703, 10726, 10749, 10772, 10795, 10818, 10841, 10864, 10887, 10910, 10933, 10956, 10979, 11002, 11025, 11048, 11071, 11094, 11117, 11140, 11163, 11186, 11209, 11232, 11255, 11278, 11301, 11324, 11347, 11370, 11393, 11416, 11439, 11462, 11485, 11508, 11531, 11554, 11577, 11600, 11623, 11646, 11669, 11692, 11715, 11738, 11761, 11784, 11807, 11830, 11853, 11876, 11899, 11922, 11945, 11968, 11991, 12014, 12037, 12060, 12083, 12106, 12129, 12152, 12175, 12198, 12221, 12244, 12267, 12290, 12313, 12336, 12359, 12382, 12405, 12428, 12451, 12474, 12497, 12520, 12543, 12566, 12589, 12612, 12635, 12658, 12681, 12704, 12727, 12750, 12773, 12796, 12819, 12842, 12865, 12888, 12911, 12934, 12957, 12980, 13003, 13026, 13049, 13072, 13095, 13118, 13141, 13164, 13187, 13210, 13233, 13256, 13279, 13302, 13325, 13348, 13371, 13394, 13417, 13440, 13463, 13486, 13509, 13532, 13555, 13578, 13601, 13624, 13647, 13670, 13693, 13716, 13739, 13762, 13785, 13808, 13831, 13854, 13877, 13900, 13923, 13946, 13969, 13992, 14015, 14038, 14061, 14084, 14107, 14130, 14153, 14176, 14199, 14222, 14245, 14268, 14291, 14314, 14337, 14360, 14383, 14406, 14429, 14452, 14475, 14498, 14521, 14544, 14567, 14590, 14613, 14636, 14659, 14682, 14705, 14728, 14751, 14774, 14797, 14820, 14843, 14866, 14889, 14912, 14935, 14958, 14981, 15004, 15027, 15050, 15073, 15096, 15119, 15142, 15165, 15188, 15211, 15234, 15257, 15280, 15303, 15326, 15349, 15372, 15395, 15418, 15441, 15464, 15487, 15510, 15533, 15556, 15579, 15602, 15625, 15648, 15671, 15694, 15717, 15740, 15763, 15786, 15809, 15832, 15855, 15878, 15901, 15924, 15947, 15970, 15993, 16016, 16039, 16062, 16085, 16108, 16131, 16154, 16177, 16200, 16223, 16246, 16269, 16292, 16315, 16338, 16361, 16384, 16407, 16430, 16453, 16476, 16499, 16522, 16545, 16568, 16591, 16614, 16637, 16660, 16683, 16706, 16729, 16752, 16775, 16798, 16821, 16844, 16867, 16890, 16913, 16936, 16959, 16982, 17005, 17028, 17051, 17074, 17097, 17120, 17143, 17166, 17189, 17212, 17235, 17258, 17281, 17304, 17327, 17350, 17373, 17396, 17419, 17442, 17465, 17488, 17511, 17534, 17557, 17580, 17603, 17626, 17649, 17672, 17695, 17718, 17741, 17764, 17787, 17810, 17833, 17856, 17879, 17902, 17925, 17948, 17971, 17994, 18017, 18040, 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24641, 24664, 24687, 24710, 24733, 24756, 24779, 24802, 24825, 24848, 24871, 24894, 24917, 24940, 24963, 24986, 25009, 25032, 25055, 25078, 25101, 25124, 25147, 25170, 25193, 25216, 25239, 25262, 25285, 25308, 25331, 25354, 25377, 25400, 25423, 25446, 25469, 25492, 25515, 25538, 25561, 25584, 25607, 25630, 25653, 25676, 25699, 25722, 25745, 25768, 25791, 25814, 25837, 25860, 25883, 25906, 25929, 25952, 25975, 25998, 26021, 26044, 26067, 26090, 26113, 26136, 26159, 26182, 26205, 26228, 26251, 26274, 26297, 26320, 26343, 26366, 26389, 26412, 26435, 26458, 26481, 26504, 26527, 26550, 26573, 26596, 26619, 26642, 26665, 26688, 26711, 26734, 26757, 26780, 26803, 26826, 26849, 26872, 26895, 26918, 26941, 26964, 26987, 27010, 27033, 27056, 27079, 27102, 27125, 27148, 27171, 27194, 27217, 27240, 27263, 27286, 27309, 27332, 27355, 27378, 27401, 27424, 27447, 27470, 27493, 27516, 27539, 27562, 27585, 27608, 27631, 27654, 27677, 27700, 27723, 27746, 27769, 27792, 27815, 27838, 27861, 27884, 27907, 27930, 27953, 27976, 27999, 28022, 28045, 28068, 28091, 28114, 28137, 28160, 28183, 28206, 28229, 28252, 28275, 28298, 28321, 28344, 28367, 28390, 28413, 28436, 28459, 28482, 28505, 28528, 28551, 28574, 28597, 28620, 28643, 28666, 28689, 28712, 28735, 28758, 28781, 28804, 28827, 28850, 28873, 28896, 28919, 28942, 28965, 28988, 29011, 29034, 29057, 29080, 29103, 29126, 29149, 29172, 29195, 29218, 29241, 29264, 29287, 29310, 29333, 29356, 29379, 29402, 29425, 29448, 29471, 29494, 29517, 29540, 29563, 29586, 29609, 29632, 29655, 29678, 29701, 29724, 29747, 29770, 29793, 29816, 29839, 29862, 29885, 29908, 29931, 29954, 29977, 30000, 30023, 30046, 30069, 30092, 30115, 30138, 30161, 30184, 30207, 30230, 30253, 30276, 30299, 30322, 30345, 30368, 30391, 30414, 30437, 30460, 30483, 30506, 30529, 30552, 30575, 30598, 30621, 30644, 30667, 30690, 30713, 30736, 30759, 30782, 30805, 30828, 30851, 30874, 30897, 30920, 30943, 30966, 30989, 31012, 31035, 31058, 31081, 31104, 31127, 31150, 31173, 31196, 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# COMPARANDO



	Com Repetição	Sem Repetição
10 números	0.000000	0.000000
100 números	0.001999	0.001020
1.000 números	0.003140	0.004020
10.000 números	0.092202	0.047047
100.000 números	0.632804	0.436656
1.000.000 números	4.311645	6.154403

**OBRIGADA!**

