Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Lecture with Computer Exercises:
Modelling and Simulating Social Systems with MATLAB

Project Report

Human Bees

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Zurich
May 2014

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1 Abstract

We present an extension to an existing model for the description of work allocation in insect societies as developed by Theraulaz et al. The extensions to the model allow for measuring the performance and welfare of a society, the investigation of external perturbations and population growth. Based on the Theraulaz model, we developed the Productivity-Boredom-Money (PBM) model that is able to describe complementary features of a society. It is more adequate to describe a society of rational workers that work for their own benefit, as it includes money. The PBM model also allows the investigation of work division, and, being more oriented towards a human society, is also able to describe features such as social inequalities.

2 Individual contributions

All team members contributed equally to this project. During the process, A.C.V focused on the PBM model and J.S. focused on the implementation, whereas M.S. focused on the paper.

3 Introduction and Motivations

In the last centuries, division of labor has become one of the most pronounced characteristics of our society. Since Prehistory, human society has become more and more complex, and the advance in technology has made it more difficult for a human being to master all tasks in the society. Today, this fact is illustrated by the astronomic number of existing jobs.

Division of labor is an important aspect of human society, but it has also been shown that even insect societies feature division of labor[1]. It is therefore interesting to investigate the factors leading to this phenomenon, and to examine if a model would also be able to reproduce more complicated characteristics of a society such as social inequality.

Theraulaz et al. have successfully developed a model describing the emergence of division of labor in insect societies[2]. Their model uses few parameters and describes time-dependent expressed through learning and forgetting. It is still quite simple and leaves room for extensions: for instance, only systems exhibiting two tasks have been investigated. The authors did account for the fact that skills are developped in time, but they used fix population sizes, ignored aging, considered an isolated system without external disturbances and did not account for personal preferences.

In this work, we expand the model of Theraulaz et al., who investigated the division of labor in insect societies, based on variable response thresholds. We investigate the effect produced by perturbations on the system such as reinitialization of the abilities of a worker or increasing size of the society. We also present the Productivity-Boredom-Money (PBM) Model, which we derived from the Theraulaz model in order to investigate other characteristics of a society. The PBM model was developed with the intention to describe a society of individualists driven by the search of the own happiness more than that of the society as is the case in the Theraulaz model. This for example allows the introduction of money in the model and makes the model able to give insights into more complicated properties of a system such as social inequality.

4 Description of the Model

4.1 The Theraulaz Model

The model describes the development of a bee hive. The colony consists of M bees, denoted with index i ($i = 1, 2, .., M$). Assume that there are N tasks, denoted by j ($j = 1, 2, .., N$). These tasks need to be performed in order for the colony to survive. They are associated with dynamic stimuli, denoted with s_j. The stimuli s_j, represent a measure of how urgent a specific task j needs to be performed. If a task is not pursued by enough workers of the hive or not in a sufficient rate, the need or the stimulus for the specific task will increase. Now, let θ_{ij} be the treshold for an individuum i with respect to task j. Then the probability for an individuum i to take up task j is given by

$$T_{\theta_{ij}} = \frac{s_j^2}{s_j^2 + \theta_{ij}^2} \tag{1}$$

One can see that for

$$\begin{array}{l} s_j \gg \theta_{ij},\, T_{\theta_{ij}} \to 1; \\ s_j = \theta_{ij},\, T_{\theta_{ij}} = 0.5; \\ s_j \ll \theta_{ij},\, T_{\theta_{ij}} \to 0. \end{array}$$

So the higher the demand (or stimulus) for a task, the more probable it is for a bee to engages in it. Each individual bee i has a specific threshold θ_{ij} towards a given task j. Those bees i, that have a low θ_{ij} towards a task j will perform this task with a higher probability $T_{\theta_{ij}}$. Moreover, the thresholds vary over time as they reflect the ability of a human bee to perform a specific task. In full analogy to humans, a human bee i becomes better as it performs a specific task j during the time fraction x_{ij}("learning"). *Vice versa*, during time fraction $(1 - x_{ij})$, in which it does not perform this task, its ability to perform it, slowly decreases ("forgetting"). Let ϕ be a fixed parameter associated with the learning process of a task and ζ

fixed parameter associated with the forgetting process of a task. Then the temporal change of θ_{ij} is described by

$$\partial\theta_{ij} = [(1 - x_{ij})\phi - x_{ij}\zeta]\Theta(\theta_{ij} - \theta_{min})\Theta(\theta_{max} - \theta_{ij}) \tag{2}$$

where $\Theta(y)$ simply denotes a step-function:

$$\Theta(y) = \begin{cases} 0, & \text{if } y \leq 0 \\ 1, & \text{if } y > 0. \end{cases} \tag{3}$$

Θ is used to maintain θ_{ij} within the chosen boundaries $[\theta_{min}, \theta_{max}]$. Note that ϕ as well as ζ assume the same, constant value for each task. The temporal change of x_{ij} is given by

$$\partial x_{ij} = T_{\theta_{ij}}(s_j)(1 - \sum_{i=1}^{M} x_{ij}) - px_{ij} \tag{4}$$

The first term of the right hand side describes how the fraction of potentially free time $(1 - \sum_{i=1}^{M} x_{ij})$ is actually allocated to task performance j. The second term describes that a human bee stops performing a task and becomes inactive with probability p. p is identical and constant for all tasks and $1/p$ denotes the average time spent on task j. So our model requires a human bee to become inactive before it can continue to pursue a task in the next time interval, if the stimulus is high enough.

Assuming that the demand for each task increases at a fixed rate δ per unit time, the temporal change of s is given by

$$\partial s_j = \delta - \frac{\alpha}{N}\sum_{i=1}^{N} x_{ij} \tag{5}$$

where α denotes a scale factor measuring the efficiency of task performance. Note that α assumes the same, constant value for all tasks.

Moreover, we have introduced a model to measure the welfare of the hive population. The rationale of the model can be explained as follows. The more individuals work the better the hive performs. The output of the conducted work grows and the stimuli for the performed tasks decrease as a consequence. Thus, the welfare of a hive is highest when the sum over all stimuli $\sum_{j=1}^{M} s_j$ is lowest and we can describe the total welfare W by

$$W = exp\left(-\frac{\sum_{j=1}^{M} s_j}{100M}\right). \tag{6}$$

For the evaluation of θ_{ij}, s_j and x_{ij} we have used the Euler-Maruyama method. $z \in \{\theta, s, x\}$ is given by

$$z_k = z_{k-1} + h * dz_{k-1} + \sqrt{h}\ \sigma\Psi(h), \tag{7}$$

where k denotes the step index, h the stepsize, σ^2 the variance and $\Psi(h)$ is a centred gaussian stochastic process. This method is used in mathematics for the approximate numerical solution of stochastic differential equations. It is appropriate for our model as it introduces a degree of stochastic variation accounting for the slightly differing environment of each bee.

4.2 The PBM Model

Introduction

In order to be able to investigate the influence of money and social inequalities, we derived a model from the model proposed by Theraulaz et al. presented above. We name this model the PBM model in order to underline the high importance that Productivity, Boredom and Money play in the model.

Basic principle

The PBM model describes the dynamics of the work allocation of different taks in a society of workers. A worker is not equally skilled for all of the tasks; we define his *productivity* for a task as the amount of work (or amount of produced units) in a given time. The productivity of a worker for a given tasks evolves with time, depending on whether he is actually performing the task or not, which reproduces learning and forgetting. Similarly, the *boredom* of the worker regarding the tasks evolves. For a worker, the boredom is equivalent to earning less money. Each worker is remunerated for his work; as all the tasks are considered to be equally important for the society, they all deliver the same total amount of money, which is distributed to the workers proportionally to their productivity. Every now and then, each worker is given the possibility to quit his task and choose another task, which he will do if he can get a better salary/boredom balance by switching to a new task.

Explanation of the model

The initial value for the productivity P_{ij} of worker i at task j is generated randomly according to the equation

$$P_{ij}(t_0) = \mathrm{rnd}(P_\mu, P_\sigma) \tag{8}$$

where $\mathrm{rnd}(\mu, \sigma)$ means that the number is generated according to a normal distribution of mean μ and standard deviation σ. As a first task, a worker will choose the task where his productivity is highest.

In order to describe the fact that some people intrinsically learn faster than other people and have more room for improvement, each worker i is attributed a task-independent *ability* A_i, which is time-independent and generated randomly according to

$$A_i = \mathrm{rnd}(A_\mu, A_\sigma). \tag{9}$$

The maximal productivity of worker i at task j is directly related to his initial productivity at this task and his ability:

$$P_{ij}^{\max} = A_i \cdot P_{ij}(t_0) \tag{10}$$

The productivity evolves in time as a function of the learning factor λ and of the forgetting factor κ,

$$P_{ij}(t + \Delta t) = \begin{cases} P_{ij}(t) + (P_{ij}^{\max} - P_{ij}(t)) \cdot \lambda \Delta t & \text{if worker } i \text{ is performing task } j \\ P_{ij}(t) - (P_{ij}(t) - P_{ij}(t_0)) \cdot \kappa \Delta t & \text{otherwise} \end{cases} \tag{11}$$

in an attempt to reproduce an exponential-like relaxation to the maximal productivity and the initial productivities. Δt is the time step for the time evolution.

The boredom B_{ij} of worker i at task j is initially zero at time t_0. The maximal boredom $B_{ij}^{\max}$ is generated randomly by the following formula:

$$B_{ij}^{\max} = \mathrm{rnd}(B_\mu, B_\sigma) \tag{12}$$

The evolution of the boredom evolves in a similar fashion as the productivity and depends on the parameters ζ and η for the boredom increase and decrease, respectively:

$$B_{ij}(t + \Delta t) = \begin{cases} B_{ij}(t) + (B_{ij}^{\max} - B_{ij}(t)) \cdot \zeta \Delta t & \text{if worker } i \text{ is performing task } j \\ B_{ij}(t) - B_{ij}(t) \cdot \eta \Delta t & \text{otherwise} \end{cases} \tag{13}$$

For a worker, boredom is equivalent to earning less money than granted by his productivity. The “felt” salary is given by the result of the substraction of the boredom $B_{ij}(t)$ at the current task from the earned money.

The *job offer frequency* p_s is responsible for the occasional possibility given to the worker to change his task: at each time step, a worker has this choice if a random number distributed uniformly between 0 and 1 is smaller than $p_s \cdot \Delta t$.

The results described above show that the PBM model is able to describe basic mechanisms of a society, which can lead to social inequalities. However, it considers each one of the workers as a homo economicus and neglects the interactions with his pairs. It also makes the crude approximation that the money is equivalent to social status, which is not totally correct in real life. The PBM model also considers each one of the tasks to be equally important for the society, while in real life some taks are tremendously more important than other ones.

The PBM model could be extended by the introduction of interaction between the individui, for example by introducing a market, where a worker sells his work to another individuum and not to the society. The application of man-made mechanisms such as social security benefits could also be investigated and their influence on social inequality examined.

5 Implementation

The implementation of the two models follows what has been described in detail in the previous sections. We will thus only briefly describe the main ideas of the implementation of the two presented models.

For the first model, there are three interdependent values which need to be solved at a time for each iteration step: x_{ij}, θ_{ij} and s_j. These variables represent vectors of length $N \cdot M$, $N \cdot M$ and M, respectively, and are calculated according to the differential equations given in Section 4.1. Therefore, we have used Matlabs *reshape* function to combine all three variables into a single $2MN + M$ vector. This form of variable storage allows for parallel solving of the target variables, which was done according to the Euler-Maruyama method.

The PBM simulation is also based on an Euler scheme for the time evolution of the different quantities in the model.

6 Simulation Results and Discussion

6.1 Theraulaz Model

Division of labour

The Theraulaz Model is able to simulate the divison of labour. In Fig.1 on the left we see the development of the thresholds θ_{ij} as a function of time. For each individuum i there is one line with respect to each task j. Here, we have used N=5 bees and M=2 tasks, amounting to 10 depicted lines - one line for each task for each bee. We can see that in the beginning the values of θ_{ij} oscillate and then assume a steady state from approximately $t = 3000$ on. In this steady state, exactly five lines assume the constant maximum value $\theta_{ij} = 1000$ and five lines assume the constant minimum value $\theta_{ij} = 0$. This means that the five bees specialize in exactly one task and keep performing this single task in the steady state.

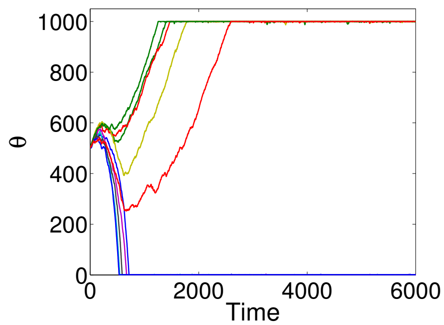

Figure 1: θ as a function of time (M=2, N=5).

This behaviour can be explained as follows. In the beginning, the five bees have no preference for any task as their $\theta = 500$ for all tasks. In other words, there is no specialization yet. First, a bee tries out to perform one of the two tasks and gets inactive with a certain probability over time. It might subsequently decide to continue pursuing its first task or alternatively perform the second one. This decision is influenced by two factors. First, by the choice of the other bees. If all bees perform task 1, the stimulus for this task will be negligible compared to the increasing stimulus for task 2 and it becomes more likely to perform this second task. Second, by the skills the bee has gained or forgotten with respect to a specific task. The more time a bee spends pursuing task 1, the better it gets performing it. In our model, the bee is then more likely to continue pursuing this task. Vice versa is true for a task which is not performed regularly by a bee. Therefore, what we observe is that some bees will exclusively perform task 1, thus $\theta_{j=1} = 0$ and $\theta_{j=2} = 1000$ in the steady state, and others decide to perform task 2, thus $\theta_{j=2} = 0$ and $\theta_{j=1} = 1000$. The model consequently allows for the investigation of the division of labour in societies. Driving force for the division is the specialization by a learning and forgetting process. In Fig.1 on the right we can see x as a function of time.

6.1.1 Measurement of the development and performance of a society

Our model enables the description of the performance of the bee hive. In Fig.2 the thresholds, the corresponding stimuli with respect to task $j = 1, 2$ as well as the corresponding total welfare W of the society is depicted. The behaviour of the thresholds is analogous to what is already described in Fig.1. The corresponding stimuli increase in time, reach a maximum value at approximately time=500 and subsequently decrease to zero. Remarkably, the stimulus of task 2 decreases slower than that of task 1. The development of the welfare curve is closely related to the development of the stimuli. At times the stimuli are high the welfare is low. Thus, the welfare first decreases, goes through a minimum at approximately time=500 and then slowly increases to reach its maximum value of 1. Note that in all graphs all functions reach its steady state value at approximately time=3000.

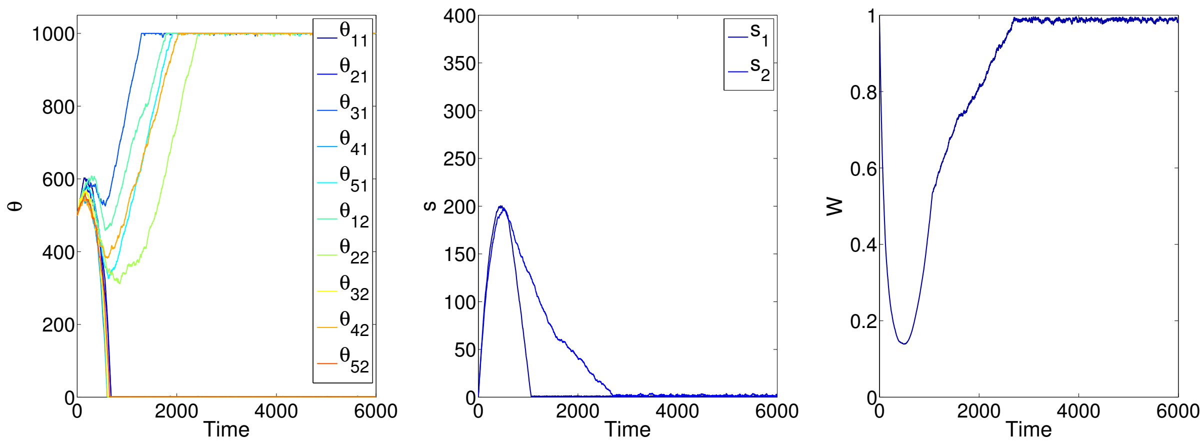

Figure 2: Left: Tresholds θ as a function of time. Middle: Stimuli s_j for task 1 and two as a function of time. Right: Welfare W as a function of time.

The functions can be interpreted as follows. The development of the stimuli can be explained by the fact that it takes time for the bees to reach an equilibrium state – the state where all bees assume exactly one task. Up to this point, the needs of the hive are not sufficiently satisfied. Hence, the stimuli increase. Over time, the bees become more specialized towards a specific task and the stimuli go through a maximum to decrease subsequently. This is an expression that the tasks are performed with a sufficient efficiency. The individual development of the thresholds and stimuli is governed by how fast the bees manage to specialize themselves and satisfy the need for the respective task. In the presented case, for example, one can look at the stimulus and the threshold value which converge last to their equilibrium value - stimulus 2 and threshold θ_{22}. Stimulus 2 decreases slower than stimulus 1, so the need to perform 2 is greater for a longer period of time compared to task 1. This is reflected in the curve of θ_{22}. It remains low as long as stimulus 2 is high and only then converges to 1000. This means that bee 2 engages as long in task 2 as stimulus 2 remains high. Thereafter it becomes inactive with respect to task 2 and is thus the last bee to be fully specialized. The welfare is connected to the sum of the stimuli. The stimuli are high when the hive need that specific tasks need to be performed in order for the hive to survive. Whenever a task is not performed, or to an insufficient extent, the respective stimulus is high. High stimuli thus reflect a poor state. Vice versa, low stimuli show that the hive performs well. Therefore, we have introduced the welfare model which is based on the sum of the stimuli. When the sum of the stimuli is low, indicating a good performance of the hive, the welfare increases. Hence, our model is able to describe how well specific tasks are performed and to measure the total welfare of a population over time.

6.1.2 Perturbations

The Theraulaz Model 1 has been analyzed with respect to an isolated society. It has been used to describe domestic characteristics of a society, such as division of labour and welfare. However, we also want to investigate how the model can be used to describe external perturbations, where the work environment is changed or individuals can get killed (or are otherwise removed). Therefore, we have studied the response of the model with respect to perturbations such as reinitializing either a random bee, or every bee or every bee perform a same task. In the following, we will consider the special case in which the colony consists of five bees and two tasks. Before starting a deep analysis, it is important to know that for this specific setup the bees usually reach an equilibrium state in which two bees work on a task and the three others work on the other task.

Reinitializing one bee Let us initially assume that the model has reached an equilibrium before being subject to a perturbation. We will start with a simple case: reset the associated features x_{ij} and θ_{ij} of a single bee. Here, we need to consider two cases. First, the removed bee was working on the task performed by only a single other bee. Second, the removed bee was performing its task together with two other bees.

In Fig.3 θ_{ij} and x_{ij} of all bees are shown as a function of time. During this time, we have randomly reinitiated a single bee at approximately 2300, 3900 and 7200 iterations. This can be seen at the discontinuities for the corresponding θ_{ij} and x_{ij} graphs. To study case 1, we can look at the perturbation at 3900 iterations, as indicated by an arrow. Here, bee three was reinitialized, thus $x_{i=3,j=1} = 0$ and $\theta_{i=3,j} = 500$. As one can see the x_{ij} and θ_{ij} values for all other bees remain unaffected. The $\theta_{i=3,j}$ and $x_{i=3,j} = 0$ values of bee three are reset but converge to their initital values again. This means that for case one the system is robust in the sense that the same equilibrium is reached as prior to the perturbation. The unaffected bees continue to work and the reinitialized bee returns to its initial work again. In Fig.5 the change in stimuli for the two cases of perturbations is shown. After initial specialization both stimuli converge to a value close to zero and at the time of the perturbation we see a great increase of $s_{j=1}$ after the perturbation. This is due to bee one not performing its initial task efficiently enough. It takes lots of iterations for the stimulus one to reach its its equilibrium again.

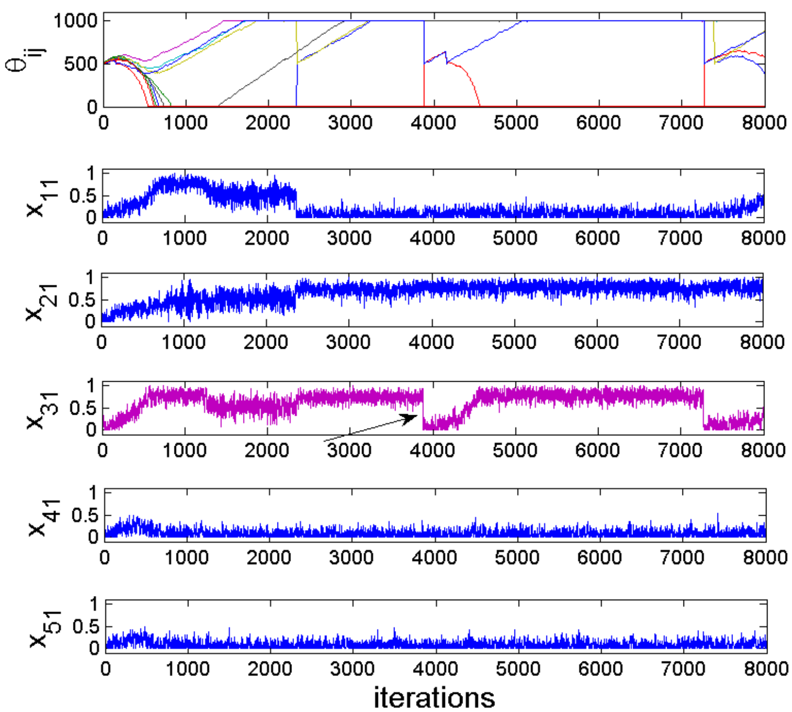

Figure 3: Reinitializing a bee working on a task fulfilled by only one other bee after 3900 iterations.

For the second case, we can look at the reinitialization taking place at 3200 iterations (see arrows in Fig.4). In this case a bee ($i = 1$) was removed, which was working together with two other bees ($i = 2, 3$) on a single task. We observe that $x_{i=2,3,j}$ increase after removing bee one. The corresponding values for bee 4 and 5 remain unaffected, however. Subsequently to the reinitialization, $x_{i=1,j}$ and $\theta_{i=1,j=1,2}$ converge 0 and 1000, respectively. In Fig. 5 we can see that opposed to case one we do not induce a great change in the stimuli (note the zoom of the y-axis). However, we observe a small, though significant increase of both stimuli after the perturbation. Interestingly, after the perturbation the stimuli do not converge to their initial values but remain relatively high. Moreover, the variances of $s_{j=1}$ has changed after the perturbation: Before the shock it was smaller than after the shock. In Fig.6 we have quantified the impact of the perturbance as average over 500 runs: We see that the difference in mean and variance of $s_{j=1}$ increases significantly. The results show that the former partners of bee 1 step up and work more intensively to compensate for the reinitialization of bee 1. Interestingly, bee 1 does not perform any task after the perturbation. The thresholds for task 1 and 2 reach the maximum value so the bee remains inactive and enjoys life on the cost of others. These changes represent a transition from one equilibrium to another. The increase in the mean and variance of $s_{j=1}$ show that the system is more stable (as indicated by a smaller variance) as when only 2 bees are performing a task.

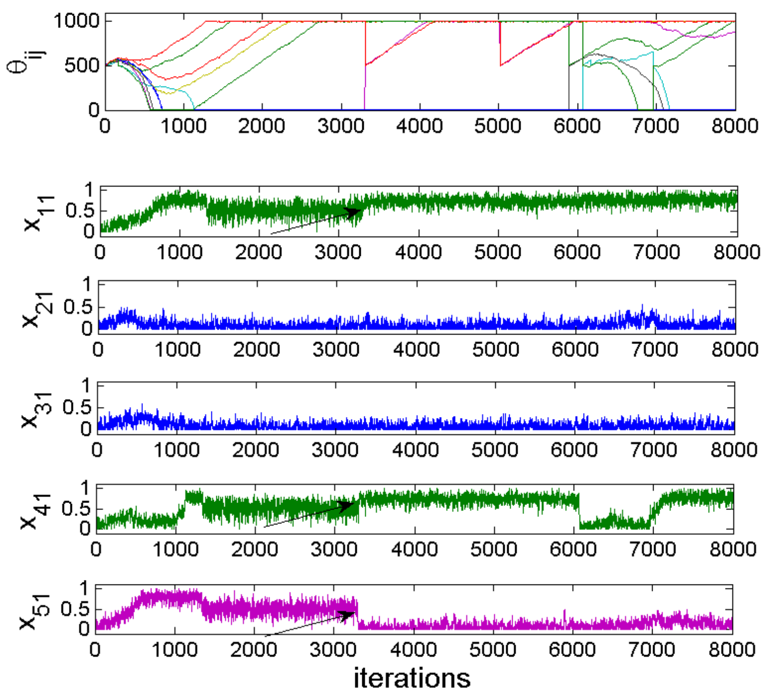

Figure 4: Reinitializing a bee working on a task fulfilled by two other bees after 3250 iterations.

6.1.3 Recovery time

A straightforward measure for the intensity of a perturbation is the time needed to achieve an equilibrium state again. This is achieved when the level of stimuli

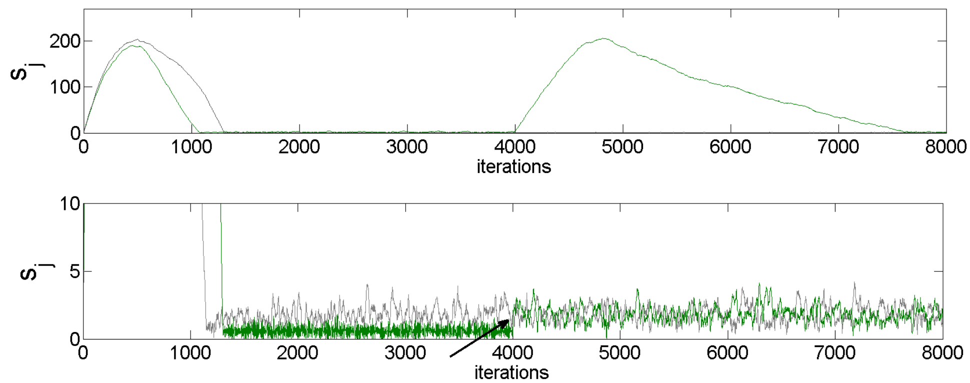

Figure 5: Impact of the perturbation on stimuli

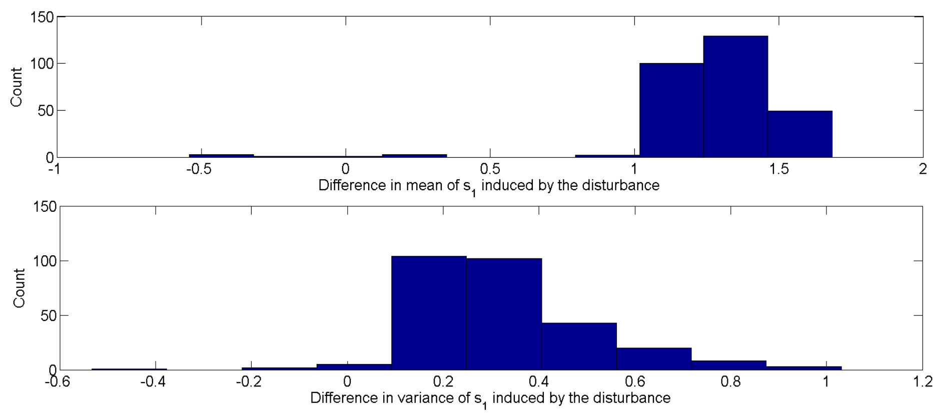

Figure 6: Augmentation of mean and variance of the stimulus after reinitializing a bee working on a task performed by two other bees. The plot represents a superposition of 500 consecutive runs.

assume a low, approximately constant value (we have set the threshold for s to be equal or lower to 5). For this purpose, we have run many simulations to estimate the average recovery time for various perturbations. With perturbations we mean resetting the bees, however, one can easily abstract our model to the real world. Think for example of an accident in a coal mine: All miners could be killed by a single accident and the society needs to train new workers which are skilled enough to work in mines. This scenario can be simulated using our model by resetting individuals working on the same task.

In total four different cases have been explored which are shown in Fig. 7. In

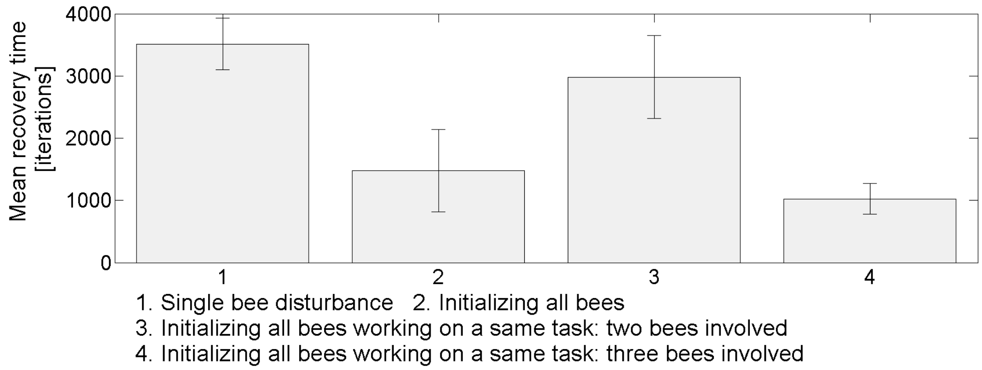

Figure 7: Mean recovery time and associated standard deviation for diverse perturbations. The plot is based on more than 100 runs.

the very left, a single bee is reset. We have considered only cases where this bee was engaged in a task which was performed of two bees in total. In cases where more than two bees were fulfilling a task the recovery was found to be immediate, thus these cases are neglected. We can see that an in average about 3614 iterations were needed to reach equilibrium again. In the second case, we have reinitialized all bees. Here, an average recovery time of 1476 is observed. In the last two cases we have reset all bees responsible for a single task. In case three we have reset 2 out of 2 bees performing a task and in case 4 we have reset 3 out of 3 bees. The corresponding recovery times are 2981 and 10244, respectively. These results are very surprising at first: The recovery time for resetting three bees is the lower than resetting only 2 or only 1 bee. This can probably be explained by a greater increase in the last case: Removing more bees also induces a greater increase in stimulus. This increase consequently leads to a faster adaption of the society. In other words, it reacts faster when the problems are more urgent. This might represent an analogy to human society: Challenges such as climate change illustrate that important tasks do not get tackled quickly if the stimulus is not high enough. Only when the stimulus or the urgency is high and felt by a majority of individuals, then a quick change is done. Coming back to our model we can introduce a rule of thumb: For a given number of tasks (being concerned by the reinitialization) a greater number of bees involved implies a faster recovery and, on the other hand, for a given number of bees, a greater number of tasks implies a slower recovery. Therefore, this method is a good way to quantify the intensity of some perturbation. However this approach is limited to simple cases where recovery can be observed.

6.2 Dynamical Population within the Theraulaz Model

The assumption that the colony size does not evolve in time is an important simplification of the reality. Therefore, we have attempted to extend the previous model to a more dynamical one.

6.2.1 Model Extension

The crucial extension is that N is now depending on time. This is a fundamental change in the model and it raises many new implementation problems. First, the size of the vector that is processed by the ordinary differential equation solver changes frequently; to solve this problem a more dynamical solver is required. Second, the number of bees is one of the dominant factors influencing the computational intensity of the simulation; N should therefore reach some sort of dynamical equilibrium or stay within a given range. And finally, it is more complicated to keep track of the full history of the variables.

About the extension, on one hand, a probability of birth of a bee was introduced which dependent linearly on the welfare of the colony. This appears to be a natural choice, since the welfare was defined as the ability of the society to fulfill all its needs. On the other hand, a probability of death was also introduced which is inversely proportional to the welfare. This stochastic approach allows to get a more realistic model, since birth can occur even in difficult moments. Those are the main changes, the other ones are mainly technical and will not be presented in this report.

6.2.2 First results

Before considering the result of the extend model, we will have a look at some simple cases

First, we can observe in Fig. 8 that killing a bee that used to work on a task performed by only one other bee, implies a sudden increase in stimulus for the concerned task, which reaches a maximum after approximately 6700 iterations. We can also notice that at the same time abrupt changes in the level of x_{21} and x_{22} happen and that the level of x_{32} and x_{42} slightly increase. Those changes show that, on one side, under the increasing pressure induced by the stimulus, a bee switches its specialization. On the other side, the two bees that did not make the effort to switch task, need to work harder in order to compensate the loss induced by the change in specialization.

Second, we can observe in Fig. 9 that introducing two new bees simultaneously increases the stimuli. The new introduced θ tend to go first to the maximal value of 1000, before that two of them suddently decrease to zero. This abrupt decrease in θ coincides exactly with a kick in x_{61}. The initial low level of x_{61} tells us that the

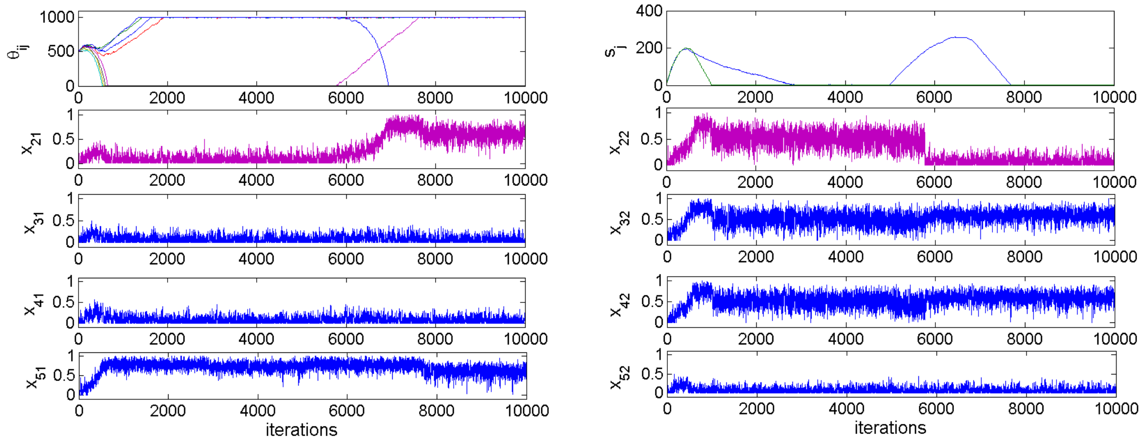

Figure 8: Reorganization of the colony after the death of a bee after 5000 iterations

two new bees decide first not to work at all. However, the transition in the θ and s shows that at a certain time the stimulus is so high that those bees are pressured to work and after a few iterations an equilibrium is reached.

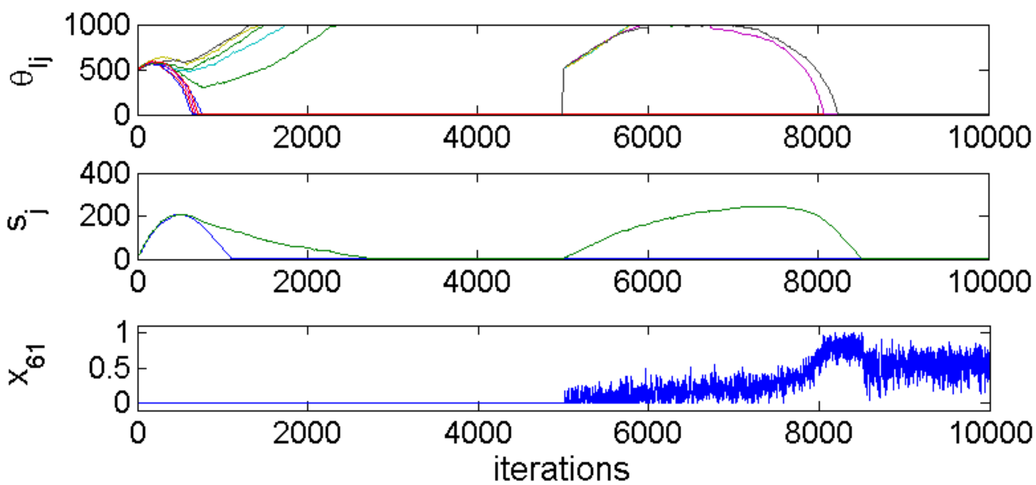

Figure 9: Introduction of two new bees in the society after 5000 iterations

We can notice with those two examples, that birth or death of a bee often disturbed the state the society. However, in our example those effects are amplified by the small size of the society and it can be observed in Fig. 10 that larger colonies are less perturbed by the incorporation a few numbers of new bees.

6.2.3 Main result

After a few runs of our new dynamical model, we can start to make some observations. The dynamics of the θ_{ij} are hard to capture, because of the complexity of the process. However, simple observations can be made by considering the s_j and the $N(t)$. First, the number of bees increases drastically at the beginning of the simulation before reaching an oscillatory state. Second, cycles can be observed for the welfare and for the stimuli. The reason behind this cycling effects is that when the welfare is high many new bees are introduced. Such bees have medium thresholds and low x_{ij} - to many of those is going to increase the instability of the colony and by extension the stimuli. In opposition, if the amount of welfare is moderate the number of bees introduced is lower and then the above mentioned perturbation does not occur.

It is important to notice that the cycles in stimuli have been observed for most of the runs, however the nice welfare cycles in Fig. 10 can only be observe if the phase difference between the two stimuli are not to important.

6.2.4 Comments

Deeper analysis could be useful to have a more precise idea of the dynamics behind this model. For example, it would be interesting to determine the conditions either under which the colony disappears or under which a dynamical equilibrium can be found. Furthermore, it would be interesting to test the reactions of the model to a higher number of tasks. In conclusion, this dynamical evolution of the colony size opens a wide range of new investigation possibilities and this extension is a key step towards a more precise and general model.

6.3 PBM Model

The PBM model is quite robust and a broad range of parameters deliver sensible results. Below we describe the influence of the more important parameters on time-dependant quantities such as allocated tasks, productivities, boredoms and salaries.

Parameters for a standard simulation could be the following: $N = 7$, $M = 3$, $\lambda = 0.01$, $\kappa = 0.003$, $\zeta = 0.001$, $\eta = 0.0003$, $p_s = 0.003$, $\Delta t = 1$, $A_\mu = 3$, $A_\sigma = 0.7$, $P_\mu = 2$, $P_\sigma = 0.7$, $B_\mu = 0.5$, $B_\sigma = 0.15$. Figure 11 shows the time evolution of the tasks performed by the different workers. It allows to see the dynamics of work allocation. Figures 12, 14 and 13 allow to understand the motivation for choosing another task. Figure 12 shows the evolution of the productivity at the current tasks

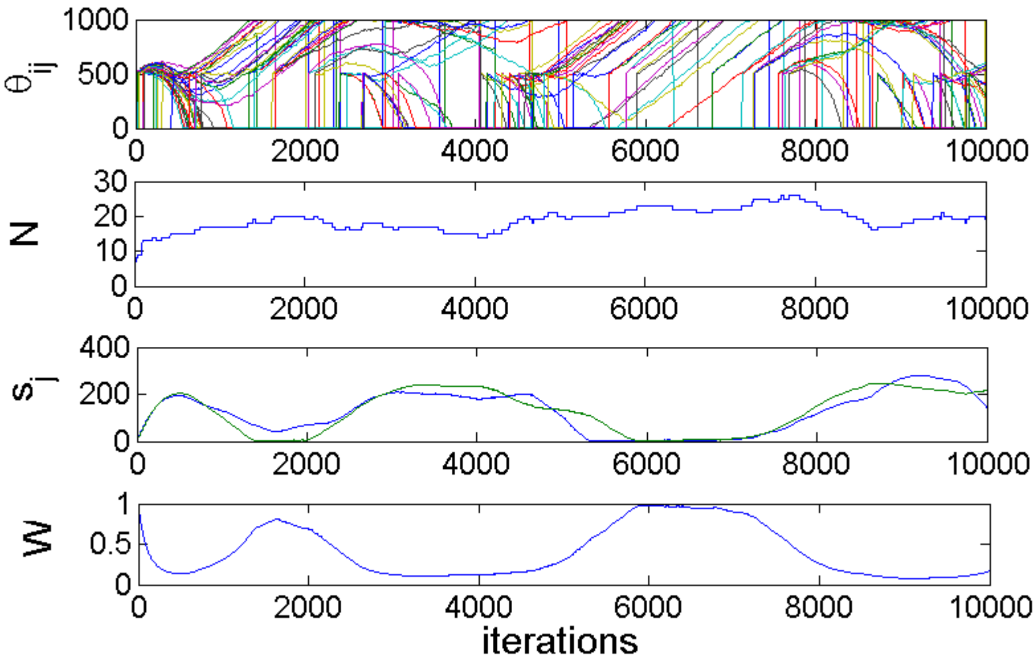

Figure 10: Run of the new dynamical model

and explains why workers performing the same task do not earn the same amount of money, which can be seen in Figure 14. Figure 13 displays the evolution of the boredom. It illustrates that a too high boredom can induce a change of task even if the new task is less paid than the previous one. A general observation is that people working on tasks at which their maximam boredom is high tend to change the task rapidly because of the rapid increase of the boredom. Figure 15 shows the total amount of money earned so far by each of the workers. It features a pronounced social inequality, which is caused by two main factors. Firstly, the inherent characteristics of the workers make some of them much more productive, thence earning more money. The second factor has its origin in the society, more precisely in the production of other individuals; it could be illustrated by the fact that a not particularly skilled individual will be remunerated a lot if he is the only one able to do his job, while two very skilled individuals at the same task will earn much less.

The randomness used in generation of the productivities P_{ij} and of the abilities A_i allow the inspection of social inequality. Both quantities have a similar influence,

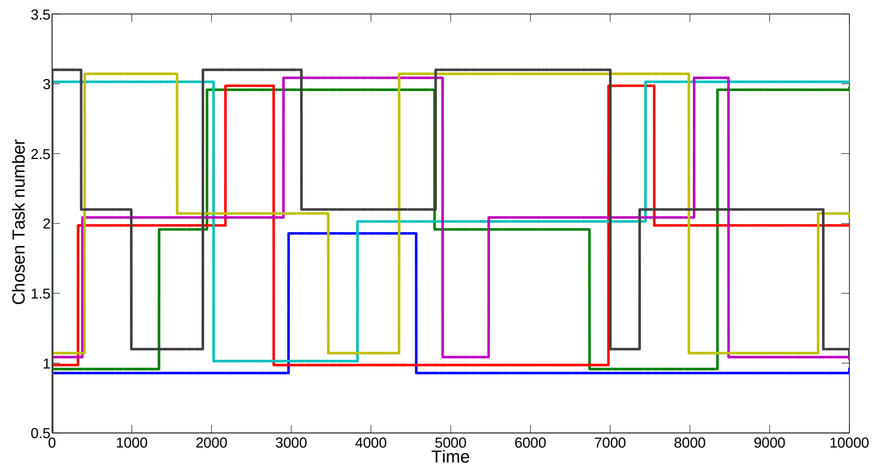

Figure 11: Current tasks of the workers as a function of time. Each worker is represented by a different color. The curves of the different individui have a small vertical shift so that all the lines are visible.

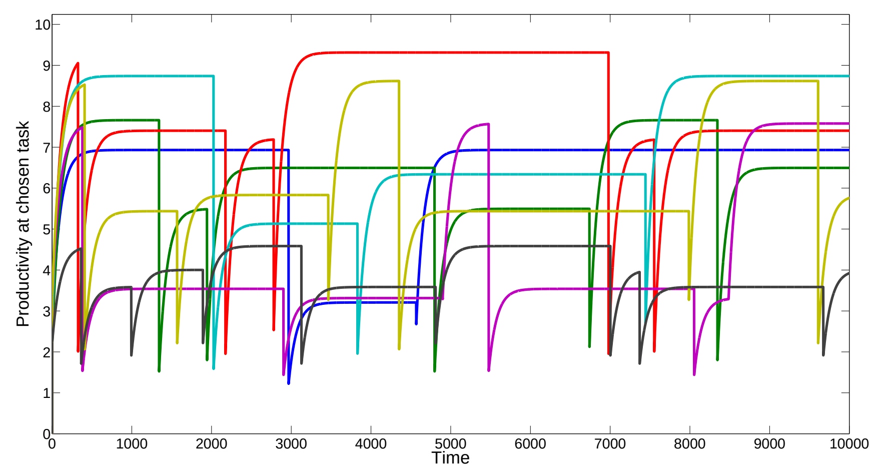

Figure 12: Productivity of the workers at the tasks they are currently performing. The incontinuities mark a change in the task and correspond to what is shown in Figure 11.

with the difference that the differences in P_{ij} are both task- and worker-specific, while the differences in A_i are worker-specific. Setting the standard deviation of the distributions to zero would result in a model with much less social inequality,

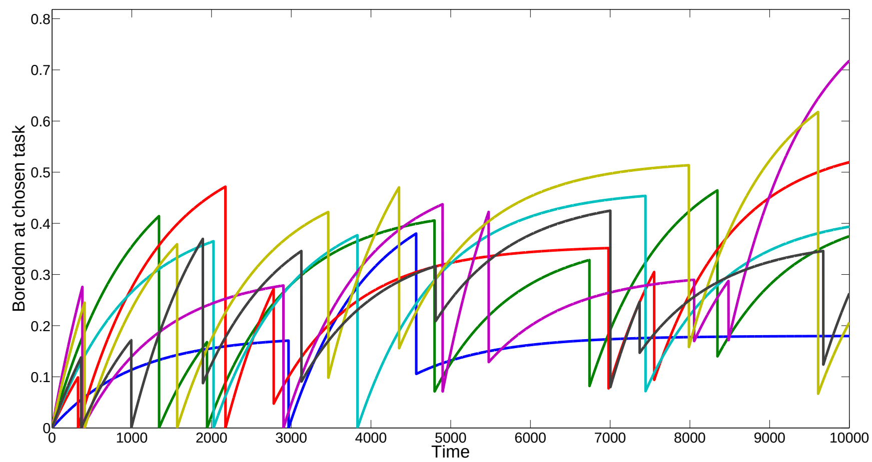

Figure 13: Boredom of the workers at the tasks they are currently performing. It can be seen that the maximal boredom is not achieved in most of the cases, since the boredom becomes too high.

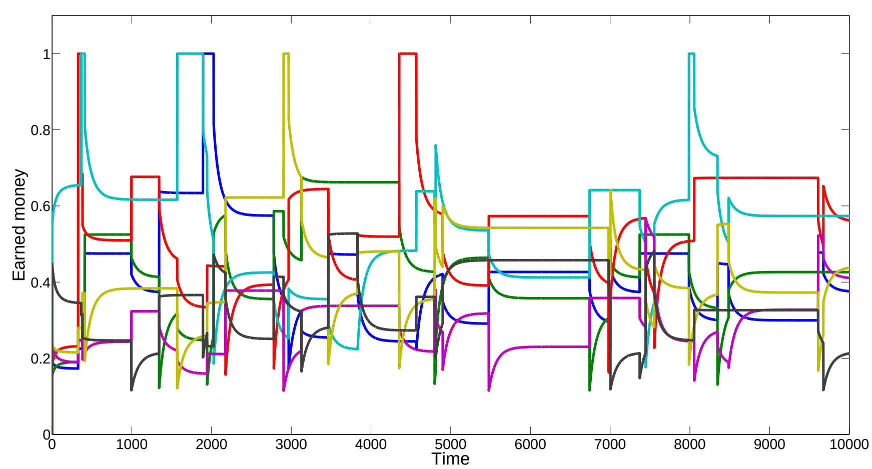

Figure 14: Salary of the workers as a function of time. A salary of 1 means that a worker is the only one to perform his current task and therefore gets all the money granted to the task.

which is not the scope of the present model. The larger the standard deviation, the more pronounced the social inequalities will be. Figures 16 and 17 illustrate the

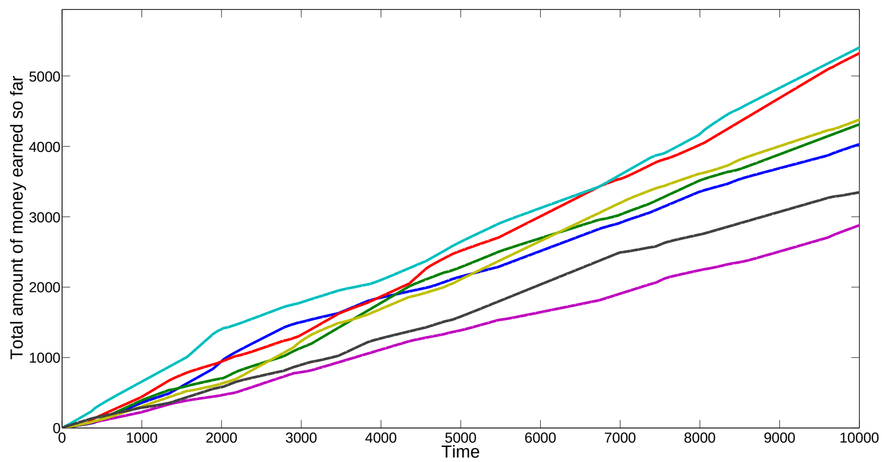

Figure 15: Total money earned by each of the workers. It illustrates how the social inequalities are steadily increasing, and the hierarchy of the society stays the same over during the simulation.

influence of the standard deviation used for the generation of the abilities and the initial productivities on social inequality.

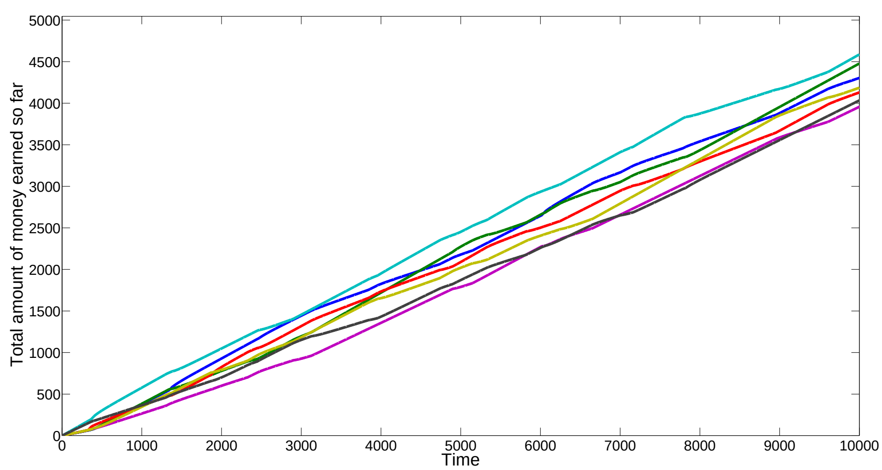

Figure 16: Total money earned by each of the workers. But for $P_\sigma = 0.2$ and $A_\sigma = 0.2$, the same parameters as in Figure 15 were used. The social inequalities remain narrow and do not increase a lot with time.

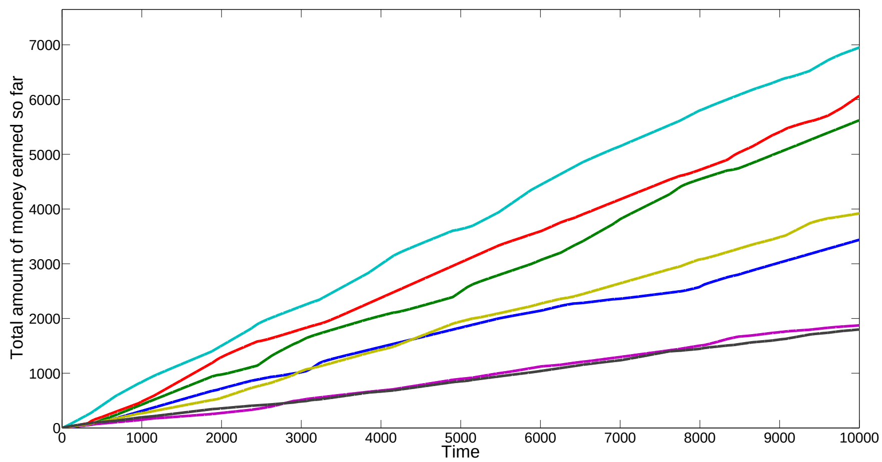

Figure 17: Total money earned by each of the workers. But for $P_\sigma = 1.0$ and $A_\sigma = 1.4$, the same parameters as in Figure 15 were used. The social gap increases a lot and some workers earn several times the salary of other individui.

The boredom can be seen as the main reason for choosing a new task. Figure 18 was obtained by using $\zeta = 0$ instead of $\zeta = 0.001$ as above. It displays a prompt work specialization, illustrated by the fact that after a short equilibration period, there are no more task changes due to the absence of boredom. Figure 19 and 20, on the other hand, were obtained with $\zeta = 0.01$ and $B_\mu = 1.5$. They show the more frequent task changes and the higher average boredom. It is to be noted that the high sensitivity to boredom in this case does not diminish the social inequalities, which can be seen in Figure 21.

The randomness in the generation of $B_{ij}^{\max}$ allows the differenciated sensibility to boredom, which, as mentioned above, has an influence on the rate at which an individual will change tasks.

Another cause for a change in the task is the evolution of the market, meaning that a worker is more likely to leave his current task if a new individuum just joined the new task, thus lowering the average salary.

The parameter p_s becomes important when task changes become frequent and a smaller value for p_s will result in less frequent task changes. However, changing p_s above a specific threshold will have a negligible influence, as the additional time a worker has to wait after a new task has become more profitable will change only little.

In the results described above, the PBM model has been shown to be able to de-

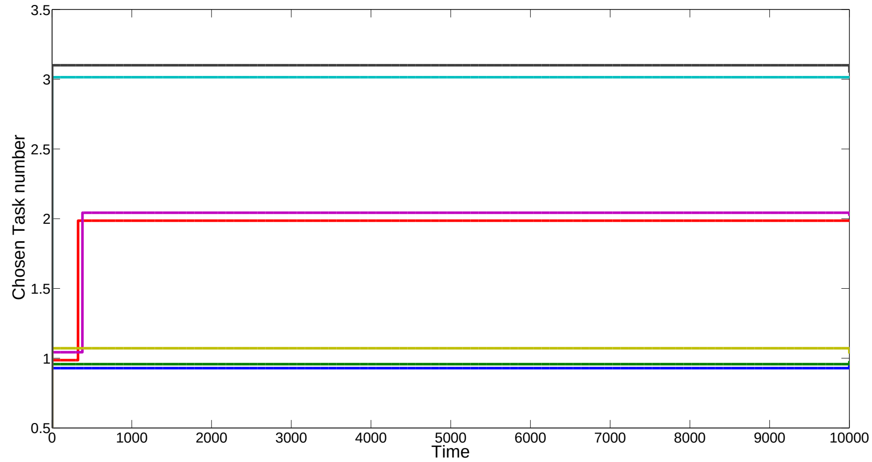

Figure 18: Chosen task number as a function of time. The same parameters as in Figure 11 were used but for $\zeta = 0$, which implies that the workers do not experience any boredom at all. It shows an early change of task for two of the workers, after which the system does not change anymore.

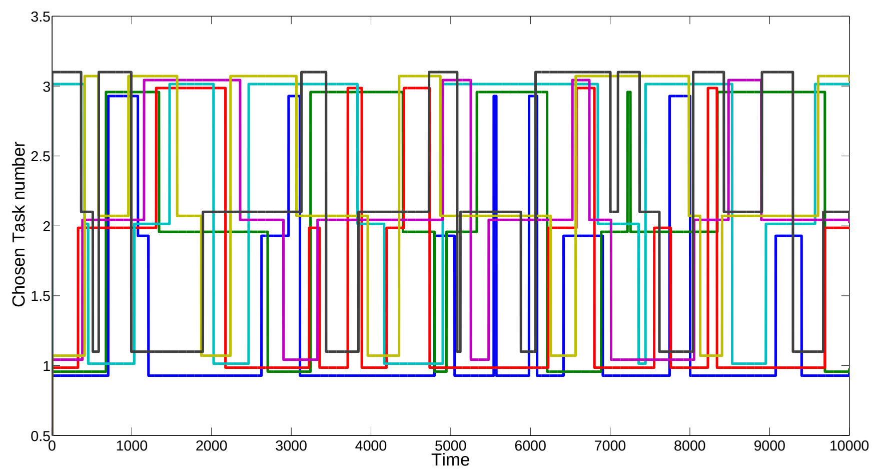

Figure 19: Chosen task number with $\zeta = 0.01$ and $B_\mu = 1.5$. All the other parameters are the same ones as used for Figure 11. The very high rate of task changes is mainly due to the high average boredom, which is displayed in Figure 20.

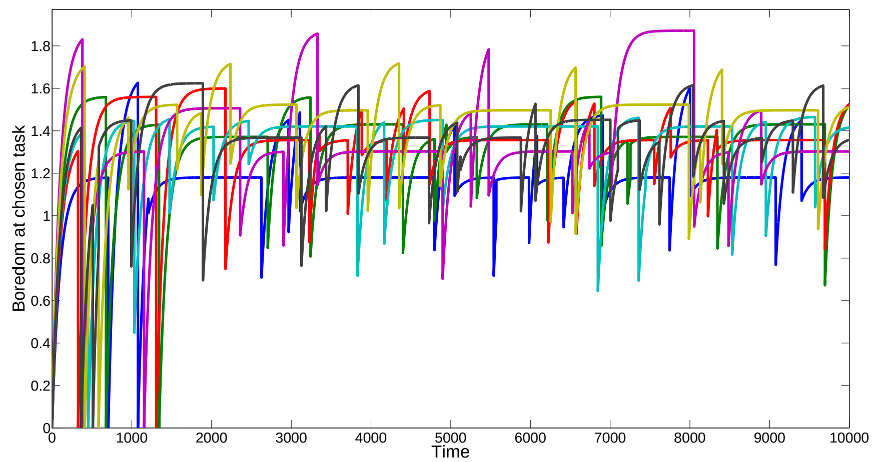

Figure 20: Boredom experienced by the workers at their current task for high maximal boredoms and a high fatigability.

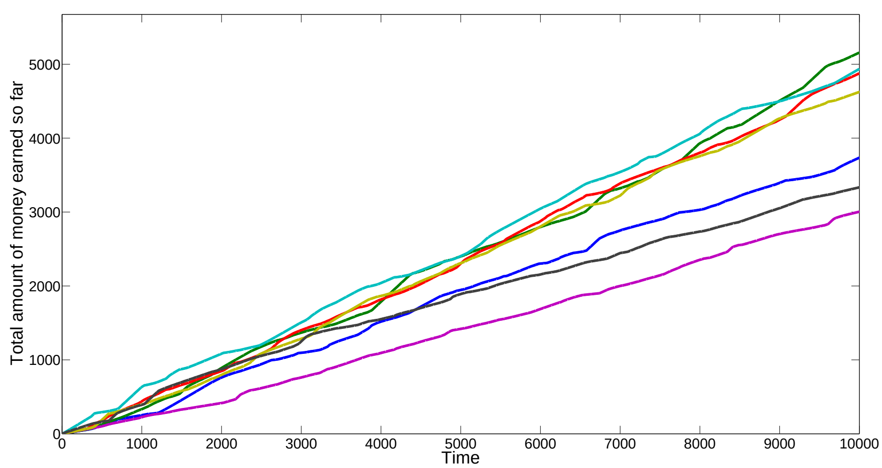

Figure 21: Total amount of money earned by the workers in the case where the fatigability and the maximal boredoms are high. Despite more fluctuations in comparison with Figure 15, the social inequalities are still pronounced.

scribe a society of individualists in a more adapted way than the Theraulaz model. As the Theraulaz model, it is able to investigate work specialization, but the examination of a society of individualists also allows to observe more complicated phenomena such as social inequalities.

The model makes some crude approximations, such as considering money to be equivalent to social status, and it neglects altruism by describing rational workers, each one of them acting as a *homo economicus*.

This model could be extended in order to include interaction between the individui, for example by letting each one of them sell his work to another peer instead of selling it to the society as is the case so far. This could allow some insights into the market phenomenon. Another extension could be the investigation of man-made mechanisms such as social security benefits and their influence on social inequalities.

7 Summary and Outlook

The extended Theralaux model was shown to be useful in a variety of circumstances. It is very important to understand that our discussions might have focused on bees, but can be extended to other colonies of insects and even higher organisms living in societies as well. We have shown that we are able to measure the welfare of a society and that we can model the different needs and urgencies towards specific goods and services achieved by tasks. The dynamics with a simple growth model could be implemented too. Additionally, we have tested the model with respect to a variety of perturbations. These tests allow for interpretations of external effects on a society, such as war, accidents and other incidents that remove workers from a given system. We propose that it lies in the simplicity of our model that makes it applicable to narrowly defined and simple questions of societies for a wide range of different animals.

In order to better address the describtion of societies of higher organisms, especially humans, we also developed an alternative model, the PBM model. The basic premises of this model make it a model useful to investigate the individualist side of a society of workers. Whereas in the Theraulaz model, the workers work for the benefit of the society by working on different tasks simultaneously and doing what the society needs most (i.e. the tasks having large stimuli), the workers in the PBM model are incarnations of the *homo economicus*, who tries to maximize the own benefit. Through its flexibility, the PBM model allows the investigation of a large range of aspects of a society of individualists and is able to simulate the origin of social inequalities. It is therefore rather oriented to the description of a human society than the Theraulaz model. The PBM lacks man-made mechanisms such as social security benefits or altruism, but is able to give an insight into basic mechanisms of a society.

8 References

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