Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Lecture with Computer Exercises:
Modelling and Simulating Social Systems with MATLAB

Project Report

Human Bees

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Agreement for free-download

We hereby agree to make our source code for this project freely available for download from the web pages of the SOMS chair. Furthermore, we assure that all source code is written by ourselves and is not violating any copyright restrictions.

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4 Description of the Model

The model describes the development of a bee hive. The colony consists of M bees, denoted with index i ($i = 1, 2, .., M$). Assume that there are N tasks, denoted by j ($j = 1, 2, .., N$). These tasks need to be performed in order for the colony to survive. They are associated with dynamic stimuli, denoted with s_j. The stimuli s_j, represent a measure of how urgent a specific task j needs to be performed. If a task is not pursued by enough workers of the hive or not in a sufficient rate, the need or the stimulus for the specific task will increase. Now, let θ_{ij} be the treshold for an individuum i with respect to task j. Then the probability for an individuum i to take up task j is given by

$$T_{\theta_{ij}} = \frac{s_j^2}{s_j^2 + \theta_{ij}^2} \tag{1}$$

One can see that for

$$\begin{aligned} &s_j \gg \theta_{ij},\, T_{\theta_{ij}} \to 1; \\ &s_j = \theta_{ij},\, T_{\theta_{ij}} = 0.5; \\ &s_j \ll \theta_{ij},\, T_{\theta_{ij}} \to 0. \end{aligned}$$

So the higher the demand (or stimulus) for a task, the more probable it is for a bee to engages in it. Each individual bee i has a specific threshold θ_{ij} towards a given task j. Those bees i, that have a low θ_{ij} towards a task j will perform this task with a higher probability $T_{\theta_{ij}}$. Moreover, the thresholds vary over time as they reflect the ability of a human bee to perform a specific task. In full analogy to humans, a human bee i becomes better as it performs a specific task j during the time fraction x_{ij}("learning"). *Vice versa*, during time fraction $(1 - x_{ij})$, in which it does not perform this task, its ability to perform it, slowly decreases ("forgetting"). Let ϕ be a fixed parameter associated with the learning process of a task and ζ fixed parameter associated with the forgetting process of a task. Then the temporal change of θ_{ij} is described by

$$\partial\theta_{ij} = [(1 - x_{ij})\phi - x_{ij}\zeta]\Theta(\theta_{ij} - \theta_{min})\Theta(\theta_{max} - \theta_{ij}) \tag{2}$$

where $\Theta(y)$ simply denotes a step-function:

$$\Theta(y) = \begin{cases} 0, & \text{if } y \leq 0 \\ 1, & \text{if } y > 0. \end{cases} \tag{3}$$

Θ is used to maintain θ_{ij} within the chosen boundaries $[\theta_{min}, \theta_{max}]$. Note that ϕ as well as ζ assume the same, constant value for each task. The temporal change of x_{ij} is given by

$$\partial x_{ij} = T_{\theta_{ij}}(s_j)(1 - \sum_{i=1}^{M} x_{ij}) - p x_{ij} \tag{4}$$

The first term of the right hand side describes how the fraction of potentially free time $(1 - \sum_{i=1}^{M} x_{ij})$ is actually allocated to task performance j. The second term describes that a human bee stops performing a task and becomes inactive with probability p. p is identical and constant for all tasks and $1/p$ denotes the average time spent on task j. So our model requires a human bee to become inactive before it can continue to pursue a task in the next time interval, if the stimulus is high enough.

Assuming that the demand for each task increases at a fixed rate δ per unit time, the temporal change of s is given by

$$\partial s_j = \delta - \frac{\alpha}{N} \sum_{i=1}^{N} x_{ij} \tag{5}$$

where α denotes a scale factor measuring the efficiency of task performance. Note that α assumes the same, constant value for all tasks.

Moreover, we have introduced a model to measure the welfare of the hive population. The rationale of the model can be explained as follows. The more individuals work the better the hive performs. The output of the conducted work grows and the stimuli for the performed tasks decrease as a consequence. Thus, the welfare of a hive is highest when the sum over all stimuli $\sum_{j=1}^{M} s_j$ is lowest and we can describe the total welfare W by

$$W = exp\left(- \frac{\sum_{j=1}^{M} s_j}{100M} \right). \tag{6}$$

For the evaluation of θ_{ij}, s_j and x_{ij} we have used the Euler-Maruyama method. $z \in \{\theta, s, x\}$ is given by

$$z_k = z_{k-1} + h * dz_{k-1} + \sqrt{h}\ \sigma \Psi(h) \tag{7}$$

, where k denotes the step index, h the stepsize, σ^2 the variance and $\Psi(h)$ is a centred gaussian stochastic process. This method is used in mathematics for the approximate numerical solution of stochastic differential equations. It is appropriate for our model as it introduces a degree of stochastic variation accounting for the slightly differing environment of each bee.

5 Implementation

6 Simulation Results and Discussion

Division of labour

Our model is able to simulate the divison of labour. In Fig.1 on the left we see the development of the thresholds θ_{ij} as a function of time. For each individuum i there is one line with respect to each task j. Here, we have used N=5 bees and M=2 tasks, amounting to 10 depicted lines - one line for each task for each bee. We can see that in the beginning the values of θ_{ij} oscillate and then assume a steady state from approximately $t = 3000$ on. In this steady state, exactly five lines assume the constant maximum value $\theta_{ij} = 1000$ and five lines assume the constant minimum value $\theta_{ij} = 0$. This means that the five bees specialize in exactly one task and keep performing this single task in the steady state.

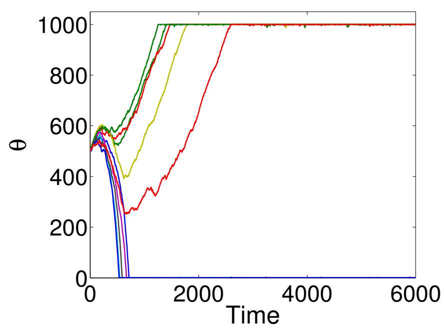

Figure 1: θ as a function of time.

This behaviour can be explained as follows. In the beginning, the five bees have no preference for any task as their $\theta = 500$ for all tasks. In other words, there is no specialization yet. First, a bee tries out to perform one of the two tasks and gets inactive with a certain probability over time. It might subsequently decide to continue pursuing its first task or alternatively perform the second one. This decision is influenced by two factors. First, by the choice of the other bees. If all bees perform task 1, the stimulus for this task will be negligible compared to the increasing stimulus for task 2 and it becomes more likely to perform this second task. Second, by the skills the bee has gained or forgotten with respect to a specific task. The more time a bee spends pursuing task 1, the better it gets performing it. In our model, the bee is then more likely to continue pursuing this task. Vice versa is true for a task which is not performed regularly by a bee. Therefore, what we observe is that some bees will exclusively perform task 1, thus $\theta_{j=1} = 0$ and $\theta_{j=2} = 1000$ in the steady state, and others decide to perform task 2, thus $\theta_{j=2} = 0$ and $\theta_{j=1} = 1000$. The model consequently allows for the investigation of the division of labour in societies. Driving force for the division is the specialization by a learning and forgetting process. In Fig.1 on the right we can see x as a function of time.

Measurement of the development and performance of a society

Our model enables the description of the performance of the bee hive. In Fig.2 the thresholds, the corresponding stimuli with respect to task $j = 1, 2$ as well as the corresponding total welfare W of the society is depicted. The behaviour of the thresholds is analogous to what is already described in Fig.1. The corresponding stimuli increase in time, reach a maximum value at approximately time=500 and subsequently decrease to zero. Remarkably, the stimulus of task 2 decreases slower than that of task 1. The development of the welfare curve is closely related to the development of the stimuli. At times the stimuli are high the welfare is low. Thus, the welfare first decreases, goes through a minimum at approximately time=500 and then slowly increases to reach its maximum value of 1. Note that in all graphs all functions reach its steady state value at approximately time=3000.

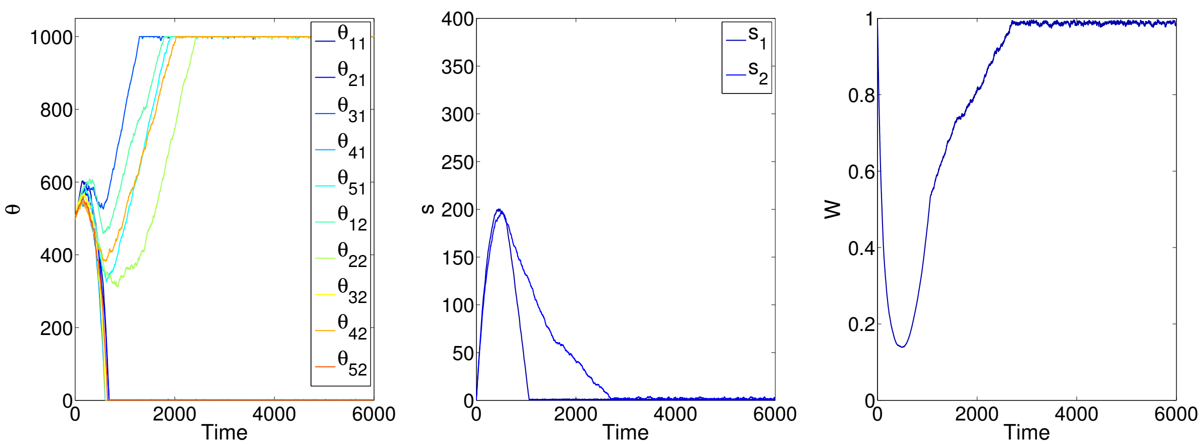

Figure 2: Left: Tresholds θ as a function of time. Middle: Stimuli s_j for task 1 and two as a function of time. Right: Welfare W as a function of time.

The functions can be interpreted as follows. The development of the stimuli can be explained by the fact that it takes time for the bees to reach an equilibrium state – the state where all bees assume exactly one task. Up to this point, the needs of the hive are not sufficiently satisfied. Hence, the stimuli increase. Over time, the bees become more specialized towards a specific task and the stimuli go through a maximum to decrease subsequently. This is an expression that the tasks are performed with a sufficient efficiency. The individual development of the thresholds and stimuli is governed by how fast the bees manage to specialize themselves and satisfy the need for the respective task. In the presented case, for example, one can look at the stimulus and the threshold value which converge last to their equilibrium value - stimulus 2 and threshold θ_{22}. Stimulus 2 decreases slower than stimulus 1, so the need to perform 2 is greater for a longer period of time compared to task 1. This is reflected in the curve of θ_{22}. It remains low as long as stimulus 2 is high and only then converges to 1000. This means that bee 2 engages as long in task 2 as stimulus 2 remains high. Thereafter it becomes inactive with respect to task 2 and is thus the last bee to be fully specialized. The welfare is connected to the sum of the stimuli. The stimuli are high when the hive need that specific tasks need to be performed in order for the hive to survive. Whenever a task is not performed, or to an insufficient extent, the respective stimulus is high. High stimuli thus reflect a poor state. Vice versa, low stimuli show that the hive performs well. Therefore, we have introduced the welfare model which is based on the sum of the stimuli. When the sum of the stimuli is low, indicating a good performance of the hive, the welfare increases. Hence, our model is able to describe how well specific tasks are performed and to measure the total welfare of a population over time.

7 Summary and Outlook

8 References