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Dynamic Behavior of a SCARA Robot by using N-E Method for a Straight Line and Simulation of Motion by using Solidworks and Verification by Matlab/Simulink

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Design, path planning and dynamic modeling for serial robots by using Solidworks (2013) and Matlab/Simulink (2012)

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ABSTRACT

SCARA (Selective Compliant Assembly Robot Arm) robot of serial architecture is widely used in assembly operations and operations "pick-place", it has been shown that use of robots improves the accuracy of assembly, and saves assembly time and cost as well. The most important condition for the choice of this kind of robot is the dynamic behavior for a given path, no closed solution for the dynamics of this important robot has been reported. This paper presents the study of the kinematics (forward and inverse) by using D-H notation and the dynamics of SCARA robot by using N-E methods. A computer code is developed for trajectory generation by using inverse kinematics, and calculates the variations of the torques of the links for a straight line (path rest to rest) between two positions for operation "pick-place". SCARA robot is constructed to achieve “pick-place» operation using Solid Works software. And verification by Matlab/Simulink. The results of simulations were discussed. An agreement between the two softwares is certainly obtained herein.

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Nomenclature:

A_i^{i-1} D-H transformation matrix for adjacent frames, i and $i-1$

C_i Cosine θ_i

C_{ijk} Cosine θ_{ijk} = cosine {(θ_i + θ_j) + θ_k }

d_i Distance from the origin of the ($i-1$) the coordinate frame to the intersection of the Z_{i-1} axis with the X_i axis along Z_{i-1} *axis*

e_i is the position vector of the COM of link *i* with respect to frame *i*

F_i Input force for i th joint

f_i Force exerted on link i by link $i-1$ at the coordinate frame ($X_{i-1}, Y_{i-1}, Z_{i-1}$) to support link i and the links above it

Γ_i Input torque for i th joint

I_i Inertia matrix of link i about its center of mass with reference to the coordinate system (X_0, Y_0, Z_0)

J_i Inertia matrix of link i about its center of mass referred to its own link coordinate system (X_i, Y_i, Z_i)

l_i The shortest distance between Z_{i-1} and Z_i axes

m_{eff} Effective mass.

m_i Mass of the i th link

n_i Moment exerted on link- i by link $i-1$ at the coordinate frame ($X_{i-1}, Y_{i-1}, Z_{i-1}$)

p_i^* is the displacement from the origin of frame $i-1$ to frame i with respect to frame i

θ_i The joint angle from X_{i-1} axis to the X_i axis about the Z_{i-1} axis (using the right hand rule)

R^{i-1} A 3 × 3 rotation matrix which transforms any vector with reference to coordinate frame (X_i, Y_i, Z_i) to the coordinate system ($X_{i-1}, Y_{i-1}, Z_{i-1}$)

S_i Sine θ_i

S_{ijk} Sine θ_{ijk} = sine {($\theta_i + \theta_j$) + θ_k }

V_i Linear velocity of the coordinate system (X_i, Y_i, Z_i) with respect to base coordinate system (X_0, Y_0, Z_0)

ω_i Angular velocity of the coordinate system (X_i, Y_i, Z_i) with respect to base coordinate system (X_0, Y_0, Z_0)

1. INTRODUCTION

Pick And Place cycle is the time, in seconds, to execute the following motion sequence: Move down one inch, grasp a rated payload; move up one inch; move across twelve inches; move down one inch; ungrasp; move up one inch; and return to start location.

The SCARA Selective Compliant Assembly Robot Arm or Selective Compliant Articulated Robot Arm is widely used for operations “pick-place”. The robot was called Selective Compliance Assembly Robot Arm, SCARA. Its arm was rigid in the Z-axis and pliable in the XY-axes, which allowed it to adapt to holes in the XY-axes.

By virtue of the SCARA's parallel-axis joint layout, the arm is slightly compliant in the X-Y direction but rigid in the ‘Z’ direction, hence the term: Selective Compliant. This is advantageous for many types of assembly operations: pick-place, inserting a round pin in a round hole without binding.

The second attribute of the SCARA is the jointed two-link arm layout similar to our human arms, hence the often-used term, Articulated. This feature allows the arm to extend into confined areas and then retract or “fold up” out of the way. This is advantageous for transferring parts from one cell to another or for loading/ unloading process stations that are enclosed.

The SCARA robots are generally faster and cleaner than comparable Cartesian systems. Their single pedestal mount requires a small footprint and provides an easy, unhindered form of mounting. On the other hand, SCARA's can be more expensive than comparable Cartesian systems and the controlling software requires inverse kinematics for linear interpolated moves. This software typically comes with the SCARA though and is usually transparent to the end-user.

In this work, 4 axes« R-R-P-R » robot systems for operation pick and place will be designed and developed using Solidworks program as shown in figure 1, and modeled by Matlab/Simulink as shown in figure 2.Simulation by using MATLAB/Simulink software will be carried out. The Results of both sofwares will be presented and discussed. In the paper, the equations of kinematics for « R-R-P-R » robot with the robot dynamics for each joint were developed with D-H formulation.

The paper is organized as follows: First, an introduction to SCARA robot, kinematics ispresented in section 2. In section 3, the dynamic behavior.In section 4,the application. Sections 5, 6 and 7, the dynamics simulation, discussion and conclusion respectively and followed by the the references.

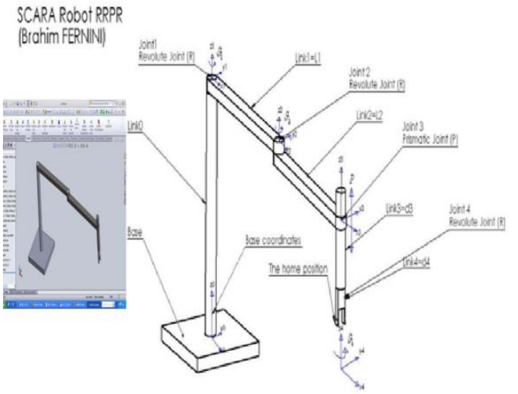

Figure 1. SCARA robot modeled by Solid Works

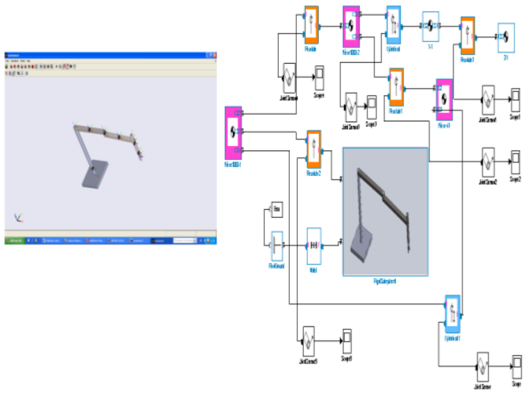

Figure 2. SCARA robot modeled by Matlab/Simulink

Previous Work

The previous work **[1] [2]** studied the dynamic of this robot by using N-L method, but this method is not commonly used for real time control as its need large amount of computation time and space, and the study of the dynamic behavior is done for path created by the joint space, this last does not give the desired trajectories like (straight line, circle,..).

Present Work

The present analysis of this robot is carried out to study the dynamics behavior for a straight line (rest to rest path) by using N-E method. The significance of this study lies in the fact that it gives insight into the dynamic behavior of this robot.

The direct kinematics allows us to find the relationship between the angular displacement and the position of the end-effector, the inverse kinematics allow us to connect between two positions by a straight line (rest to rest path).

SolidWorks and Matlab Simulink softwares are used to model and check the robot motion simulation.

2. ROBOT KINEMATICS

2.1 Direct Kinematics

The Denavit-Hartenberg (D-H) parameters for SCARA robot shown in Fig1are defined in table:

Table 1. D-H parameters of SCARA Robot.

Link	a_i	α_i	d_i	θ_i
1	l_1	0	0	θ_1^*
2	l_2	0	0	θ_2^*
3	0	0	d_3^*	0
4	0	0	d_4^*	θ_4^*

*: joint variables

The expression for the end effector frame relative to the base frame is given by the arm matrix (T_4^0) as:

$T_4^0 = T_1^0 T_2^1 T_3^2 T_4^3$, where:

$$T_1^0 = \begin{bmatrix} c_1 & -s_1 & 0 & l_1c_1 \\ s_1 & c_1 & 0 & l_1s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad T_2^1 = \begin{bmatrix} c_2 & -s_2 & 0 & l_2c_2 \\ s_2 & c_2 & 0 & l_2s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad T_3^2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad T_4^3 = \begin{bmatrix} c_4 & -s_4 & 0 & 0 \\ s_4 & c_4 & 0 & 0 \\ 0 & 0 & 1 & -d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

After multiplication and use of addition matrices, one gets the homogeneous transformation matrix;

$$T_4^0 = \begin{bmatrix} c_{124} & -s_{124} & 0 & l_1c_1 + l_2c_{12} \\ s_{124} & c_{124} & 0 & l_1s_1 + l_2s_{12} \\ 0 & 0 & 1 & -d_3 - d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

2.2 Inverse Kinematics

2.2.1 Inverse Solution for Positions

Desired location of Robot:

$$T_H^R = \begin{pmatrix} n_X & o_X & a_X & p_X \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

The final equation representing the robot is [3]:

$$T_4^0 = T_H^R$$

We get:

$$p_X = l_1c_1 + l_2c_{12}, \; p_y = l_1s_1 + l_2s_{12}.$$

By using Kramer methods we find [4];

Equation of elbow up:

$$\theta_2 = -a\tan\frac{s_2}{c_2}, \qquad \theta_1 = a\tan\frac{+p_xl_2s_2 + p_y(l_1 + l_2c_2)}{p_x(l_1 + l_2c_2) - (p_yl_2s_2)}$$

Equation of elbow down:

$$\theta_2 = +a\tan\frac{s_2}{c_2}, \qquad \theta_1 = a\tan\frac{-p_xl_2s_2 + p_y(l_1 + l_2c_2)}{p_x(l_1 + l_2c_2) + (p_yl_2s_2)}$$

Inverse solution for velocity:

$$\dot{\theta}_1 = \frac{\dot{P}_X c_{12} + \dot{P}_y s_{12}}{l_1s_2} \quad \dot{\theta}_2 = \frac{-\dot{P}_X(l_1c_1 + l_2c_{12}) - \dot{P}_y(l_1s_1 + l_2s_{12})}{l_1l_2s_2}$$

Inverse solution for acceleration:

$$\ddot{\theta}_1 = \frac{(-\dot{P}_X s_{12} + \dot{P}_y c_{12})\dot{\theta}_{12} + (\ddot{P}_X c_{12} + \ddot{P}_y s_{12}) - l_1c_2\dot{\theta}_1\dot{\theta}_2}{l_1s_2} \quad \ddot{\theta}_2 = -\frac{\left[(\ddot{P}_y s_1 - \ddot{P}_x c_1)l_1 + (\ddot{P}_y s_{12} + \ddot{P}_x c_{12})l_2\right]}{l_1l_2s_2}$$

$$-\frac{\left[(\dot{P}_y c_1 - \dot{P}_x s_1)l_1\dot{\theta}_1 + (\dot{P}_X s_{12} + \dot{P}_y c_{12})l_2\dot{\theta}_{12} + l_1l_2c_2\dot{\theta}_2^2\right]}{l_1l_2s_2}$$

3. ROBOT DYNAMICS

We find the dynamics equations of motion of robots by two methods:

Newton-Euler and Lagrange. The Newton-Euler method is more fundamental and finds the dynamic equations to determine the required actuators' force and torque to move the robot, as well as the joint forces. Lagrange method provides only the required differential equations that determine the actuators' force and torque**. [5]**

The N-E method is based on two recursions forward and backward recursive equations. The forward recursive equation is used for the kinematics information such as velocities and accelerations at the center of mass of each link. The backward recursive equation is used for the forces and moments exerted on each link from the end effector to the base of the robot.

The rotation matrices are as follows:

$$R_1^0 = \begin{pmatrix} C_1 & -S_1 & 0 \\ S_1 & C_1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, R_2^1 = \begin{pmatrix} C_2 & -S_2 & 0 \\ S_2 & C_2 & 0 \\ 0 & 0 & 1 \end{pmatrix}, R_4^3 = \begin{pmatrix} C_4 & -S_4 & 0 \\ S_4 & C_4 & 0 \\ 0 & 0 & 1 \end{pmatrix}, R_3^2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$R_2^0 = \begin{pmatrix} C_{12} & -S_{12} & 0 \\ S_{12} & C_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}, R_3^0 = R_2^0, R_4^0 = \begin{pmatrix} C_{124} & -S_{124} & 0 \\ S_{124} & C_{124} & 0 \\ 0 & 0 & 1 \end{pmatrix}, R_0^1 = \begin{pmatrix} C_1 & S_1 & 0 \\ -S_1 & C_1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$R_1^2 = \begin{pmatrix} C_2 & S_2 & 0 \\ -S_2 & C_2 & 0 \\ 0 & 0 & 1 \end{pmatrix}, R_0^2 = \begin{pmatrix} C_{12} & S_{12} & 0 \\ -S_{12} & C_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$R_0^3 = R_0^2, P_1^* = [l_1C_1, l_1S_1, 0]^T, P_2^* = [l_1C_{12}, l_1S_{12}, 0]^T, P_3^* = [0, 0, -d_3]^T$$

$$P_4^* = [0,0,0]^T, \omega_0 = \dot{\omega}_0 = V_0 = 0, \dot{V}_0 = (0,0,g)^T$$

Forward recursive:

$$R_0^1\omega_1 = R_0^1\left(\omega_0 + Z_0\dot{\theta}_1\right) = [0 \quad 0 \quad 1]^T\, \dot{\theta}_1$$

$$R_0^2\omega_2 = R_1^2\left(R_0^1\omega_1 + Z_0\dot{\theta}_2\right) = [0 \quad 0 \quad 1]^T\left(\dot{\theta}_1 + \dot{\theta}_2\right)$$

$$R_0^3\omega_3 = R_2^3\left(R_0^2\omega_2\right) = [0 \quad 0 \quad 1]^T\left(\dot{\theta}_1 + \dot{\theta}_2\right)$$

$$R_0^4\omega_4 = R_3^4\left(R_0^3\omega_3 + Z_0\dot{\theta}_4\right)$$

$$= [0 \quad 0 \quad 1]^T\left(\dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_4\right)$$

$$R_0^1\dot{\omega}_1 = R_0^1\left[\dot{\omega}_0 + Z_0\ddot{\theta}_1 + \omega_0 \times Z_0\theta_1\right]$$

$$= [0 \quad 0 \quad 1]^T\, \ddot{\theta}_1$$

$$R_0^2\dot{\omega}_2 = R_1^2\left[R_0^1\dot{\omega}_1 + Z_0\ddot{\theta}_2 + \left(R_0^1\omega_1\right) \times Z_0\dot{\theta}_2\right]$$

$$= [0 \quad 0 \quad 1]^T\left(\ddot{\theta}_1 + \ddot{\theta}_2\right)$$

$$R_0^3\dot{\omega}_3 = R_2^3\left[R_0^2\dot{\omega}_2\right] = [0 \quad 0 \quad 1]^T\left(\ddot{\theta}_1 + \ddot{\theta}_2\right)$$

$$R_0^4\dot{\omega}_4 = R_3^4\left[R_0^3\dot{\omega}_3 + Z_0\ddot{\theta}_4 + \left(R_0^3\omega_3\right) \times Z_0\dot{\theta}_4\right]$$

$$= [0 \quad 0 \quad 1]^T\left(\ddot{\theta}_1 + \ddot{\theta}_2 + \ddot{\theta}_4\right)$$

$$R_0^1V_1 = \left(R_0^1\dot{\omega}_1\right) \times \left(R_0^1p_1^*\right) + \left(R_0^1\omega_1\right) \times$$

$$\left[\left(R_0^1\omega_1\right) \times \left(R_0^1p_1^*\right)\right] + R_0^1\left(R_0^1\dot{V}_0\right) = \left[-l_1\dot{\theta}_1^2, l_1\ddot{\theta}_1, g\right]^T$$

$$R_0^2V_2 = \left(R_0^2\dot{\omega}_2\right) \times \left(R_0^2p_2^*\right) + \left(R_0^2\omega_2\right)$$

$$\times\left[\left(R_0^2\omega_2\right) \times \left(R_0^2p_2^*\right)\right] + R_1^2\left(R_0^1\dot{V}_1\right)$$

$$= \begin{bmatrix} l_1\ddot{\theta}_1S_2 - l_1\dot{\theta}_1^2C_2 - l_2\left(\dot{\theta}_1 + \dot{\theta}_2\right)^2, \\ l_2\left(\ddot{\theta}_1 + \ddot{\theta}_2\right) + l_1\dot{\theta}_1^2S_2 + l_1\ddot{\theta}_1C_2, g \end{bmatrix}^T$$

$$R_0^3V_3 = R_2^3\left(Z_0\ddot{\theta}_3 + R_0^2\dot{V}_2\right) + \left(R_0^3\dot{\omega}_3\right) \times \left(R_0^3p_3^*\right)$$

$$+\left[2\left(R_0^3\omega_3\right) \times \left(R_2^3Z_0\dot{\theta}_3\right)\right]$$

$$+R_0^3\omega_2 \times \left[\left(R_0^3\omega_3\right) \times \left(R_0^3p_3^*\right)\right]$$

$$= R_2^3\left(R_0^2\dot{V}_2\right) + \left(R_0^3\dot{\omega}_3\right) \times \left(R_0^3p_3^*\right) +$$

$$R_0^3\omega_3 \times \left[\left(R_0^3\omega_3\right) \times \left(R_0^3p_3^*\right)\right]$$

$$= \begin{bmatrix} l_1\ddot{\theta}_1S_2 - l_1\dot{\theta}_1^2C_2 - l_2\left(\dot{\theta}_1 + \dot{\theta}_2\right)^2, \\ l_1\ddot{\theta}_1C_2 + l_1\dot{\theta}_1^2S_2 + l_2\left(\ddot{\theta}_1 + \ddot{\theta}_2\right), g \end{bmatrix}^T$$

$$R_0^4 V_4 = \left(R_0^4 \dot{\omega}_4\right) \times \left(R_0^4 p_4^*\right)$$
$$+\left(R_0^4 \omega_4\right) \times \left[\left(R_0^4 \omega_4\right) \times \left(R_0^4 p_4^*\right)\right]$$
$$+R_3^4\left(R_0^3 \dot{V}_3\right)$$
$$= \begin{bmatrix} l_1\ddot{\theta}_1 S_{24} - l_1\dot{\theta}_1^2 C_{24} - l_2\left\{\left(\dot{\theta}_1+\dot{\theta}_2\right)^2 C_4 - \left(\ddot{\theta}_1+\ddot{\theta}_2\right) S_4\right\}, \\ l_1\ddot{\theta}_1 C_{24} + l_1\dot{\theta}_1^2 S_{24} + l_2\left\{\left(\ddot{\theta}_1+\ddot{\theta}_2\right)^2 S_4 - \left(\ddot{\theta}_1-\ddot{\theta}_2\right) C_4\right\}, g \end{bmatrix}^T$$

The position of center of mass:

$$e_1 = \left[-l_1 C_1 / 2, -l_1 S_1 / 2, 0\right]^T$$
$$e_2 = \left[-l_2 C_{12} / 2, -l_2 S_{12} / 2, 0\right]^T$$
$$e_3 = \left[0, 0, d_3 / 2\right]^T$$
$$e_4 = \left[0, 0, 0\right]^T$$
$$R_0^1 a_1 = \left(R_0^1 \dot{\omega}_1\right) \times \left(R_0^1 e_1\right) + \left(R_0^1 \omega_1\right) \times \left[\left(R_0^1 \omega_1\right) \times \left(R_0^1 e_1\right)\right]$$
$$+\left(R_0^1 \dot{V}_1\right) = R_0^1 \dot{V}_1 = \left[-l_1\dot{\theta}_1^2 / 2, l_1\ddot{\theta}_1 / 2, g\right]^T$$
$$R_0^2 a_2 = \left(R_0^2 \dot{\omega}_2\right) \times \left(R_0^2 e_2\right) + \left(R_0^2 \omega_2\right) \times \left[\left(R_0^2 \omega_2\right) \times \left(R_0^2 e_2\right)\right]$$
$$+\left(R_0^2 \dot{V}_2\right) = R_0^2 \dot{V}_2$$
$$= \begin{bmatrix} l_1\ddot{\theta}_1 S_2 - l_1\dot{\theta}_1^2 C_2 - l_2\left(\dot{\theta}_1+\dot{\theta}_2\right)^2 / 2, \\ l_1\ddot{\theta}_1 C_2 + l_1\dot{\theta}_1^2 S_2 + l_2\left(\ddot{\theta}_1+\ddot{\theta}_2\right) / 2, g \end{bmatrix}^T$$
$$R_0^3 a_3 = \left(R_0^3 \dot{\omega}_3\right) \times \left(R_0^3 e_3\right) + \left(R_0^3 \omega_3\right) \times \left[\left(R_0^3 \omega_3\right) \times \left(R_0^3 e_3\right)\right]$$
$$+\left(R_0^3 \dot{V}_3\right) = R_0^3 \dot{V}_3$$
$$= \begin{bmatrix} l_1\ddot{\theta}_1 S_2 - l_1\dot{\theta}_1^2 C_2 - l_2\left(\dot{\theta}_1+\dot{\theta}_2\right)^2, \\ l_1\ddot{\theta}_1 C_2 + l_1\dot{\theta}_1^2 S_2 + l_2\left(\ddot{\theta}_1+\ddot{\theta}_2\right), g \end{bmatrix}^T$$
$$R_0^4 a_4 = \left(R_0^4 \dot{\omega}_4\right) \times \left(R_0^4 e_4\right)$$
$$+\left(R_0^4 \omega_4\right) \times \left[\left(R_0^4 \omega_4\right) \times \left(R_0^4 e_4\right)\right]$$
$$+\left(R_0^4 \dot{V}_4\right) = R_0^4 \dot{V}_4$$
$$= \begin{bmatrix} l_1\ddot{\theta}_1 S_{24} - l_1\dot{\theta}_1^2 C_{24} - l_2\left\{\left(\dot{\theta}_1+\dot{\theta}_2\right)^2 C_4 - \left(\ddot{\theta}_1+\ddot{\theta}_2\right) S_4\right\}, \\ l_1\ddot{\theta}_1 C_{24} + l_1\dot{\theta}_1^2 S_{24} + l_2\left\{\left(\ddot{\theta}_1+\ddot{\theta}_2\right)^2 S_4 + \left(\ddot{\theta}_1+\ddot{\theta}_2\right) C_4\right\}, g \end{bmatrix}^T$$

Backward recursive:

We have: $f_5 = n_5 = 0$

$$R_0^4 f_4 = R_5^4\left(R_0^5 f_5\right) + m_4 R_0^4 a_4 = m_4 R_0^4 a_4$$
$$= m_4 \begin{bmatrix} l_1\ddot{\theta}_1 S_{24} - l_1\dot{\theta}_1^2 C_{24} - l_2\left\{\left(\dot{\theta}_1+\dot{\theta}_2\right)^2 C_4 - \left(\ddot{\theta}_1+\ddot{\theta}_2\right) S_4\right\}, \\ l_1\ddot{\theta}_1 C_{24} + l_1\dot{\theta}_1^2 S_{24} + l_2\left\{\left(\dot{\theta}_1+\dot{\theta}_2\right)^2 S_4 + \left(\ddot{\theta}_1+\ddot{\theta}_2\right) C_4\right\}, g \end{bmatrix}^T$$

$$R_0^3 f_3 = R_4^3\left(R_0^4 f_4\right) + m_3 R_0^3 a_3$$

$$= \left(m_3 + m_4\right)\begin{bmatrix} l_1\ddot{\theta}_1 S_2 - l_1\dot{\theta}_1^2 C_2 - l_2\left(\dot{\theta}_1 + \dot{\theta}_2\right)^2, \\ l_1\ddot{\theta}_1 C_2 + l_1\dot{\theta}_1^2 S_2 + l_2\left(\ddot{\theta}_1 + \ddot{\theta}_2\right), g \end{bmatrix}^T$$

$$R_0^2 f_2 = R_3^2\left(R_0^3 f_3\right) + m_2 R_0^2 a_2$$

$$= \begin{bmatrix} \left\{ x\left(l_1\ddot{\theta}_1 S_2 - l_1\dot{\theta}_1^2 C_2\right) - y l_2\left(\dot{\theta}_1 + \dot{\theta}_2\right)^2 \right\}, \\ x\left(l_1\ddot{\theta}_1 C_2 + l_1\dot{\theta}_1^2 S_2\right) + y l_2\left(\ddot{\theta}_1 + \ddot{\theta}_2\right), xg \end{bmatrix}^T$$

$$R_0^1 f_1 = R_2^1\left(R_0^2 f_2\right) + m_1 R_0^1 a_1$$

$$= \begin{bmatrix} \begin{Bmatrix} -l_1\dot{\theta}_1^2\left(x + m_1/2\right) - y l_2\left(\dot{\theta}_1 + \dot{\theta}_2\right)^2 C_2 \\ +\left(\ddot{\theta}_1 + \ddot{\theta}_2\right) S_2 \end{Bmatrix}, \\ \begin{Bmatrix} l_1\ddot{\theta}_1\left(x + m_1/2\right) \\ -y l_2\left(\left(\dot{\theta}_1 + \dot{\theta}_2\right)^2 S_2 - \left(\ddot{\theta}_1 + \ddot{\theta}_2\right) C_2\right) \end{Bmatrix}, \left(x + m_1\right) g \end{bmatrix}^T$$

$$x = m_2 + m_3 + m_4$$

$$y = m_2/2 + m_3 + m_4$$

The moments exerted on the links:

$$R_0^4 n_4 = R_5^4\left[R_0^5 n_5 + \left(R_0^5 p_4^*\right)\times\left(R_0^5 f_5\right)\right]$$
$$+\left(R_0^4 p_4^* + R_0^4 e_4\right)\times\left(m_4 R_0^4 a_4\right) + J_4 R_0^4 \dot{\omega}_4$$
$$+\left[\left(R_0^4 \omega_4\right)\times J_4\left(R_0^4 \omega_4\right)\right]$$

$$J_i = R_0^i I_i R_i^0$$

$$i = 1, 2, 3, 4$$

Generally, the mass and the length of link (4) are very small in comparison to other links; the inertia of link (4) is evaluated to be zero.

$$R_0^4 n_4 = 0$$

$$R_0^3 n_3 = R_4^3\left[R_0^4 n_4 + \left(R_0^4 p_3^*\right)\times\left(R_0^4 f_4\right)\right]$$
$$+\left(R_0^3 p_3^* + R_0^3 e_3\right)\times\left(m_3 R_0^3 a_3\right) + J_3 R_0^3 \dot{\omega}_3$$
$$+\left[\left(R_0^3 \omega_3\right)\times J_3\left(R_0^3 \omega_3\right)\right] = \begin{Bmatrix} d_3\left(m_2 + m_4\right) \\ -m_3 l_3/2 \end{Bmatrix}$$
$$\times \begin{bmatrix} l_1\ddot{\theta}_1 C_2 + l_1\dot{\theta}_1^2 S_2 + l_2\left(\ddot{\theta}_1 + \ddot{\theta}_2\right), \\ -l_1\ddot{\theta}_2 S_2 + l_1\dot{\theta}_1^2 C_2 + l_2\left(\dot{\theta}_1 + \dot{\theta}_2\right)^2, 0 \end{bmatrix}^T$$

$$R_0^2 n_2 = R_3^2 \left[R_0^3 n_3 + \left(R_0^3 p_2^* \right) \times \left(R_0^3 f_3 \right) \right]$$
$$+ \left(R_0^2 p_2^* + R_0^2 e_2 \right) \times \left(m_2 R_0^2 a_2 \right) + J_2 R_0^2 \dot{\omega}_2$$
$$+ \left[\left(R_0^2 \omega_2 \right) \times J_2 \left(R_0^2 \omega_2 \right) \right]$$
$$= \begin{bmatrix} \Omega \left\{ l_1 \ddot{\theta}_1 C_2 + l_1 \dot{\theta}_1^2 S_2 + l_2 \left(\ddot{\theta}_1 + \ddot{\theta}_2 \right) \right\} \\ , \Omega \left\{ - l_1 \ddot{\theta}_2 S_2 + l_1 \dot{\theta}_1^2 C_2 + l_2 \left(\dot{\theta}_1 + \dot{\theta}_2 \right)^2 \right\} - l_2 g y , \\ l_2 y \left(l_1 \ddot{\theta}_1 C_2 + l_1 \dot{\theta}_1^2 S_2 \right) + l_2^2 \Lambda \left(\ddot{\theta}_1 + \ddot{\theta}_2 \right) \end{bmatrix}^T$$
$$\Omega = d_2 \left(m_3 + m_4 \right) + d_3 \left(m_3 + m_4 \right) - m_3 l_3 / 2$$
$$\Lambda = m_2 / 3 + m_3 + m_4$$
$$R^1 n_1 = R_0^1 \left[R_0^2 n_2 + \left(R_0^2 p_1^* \right) \times \left(R_0^2 f_2 \right) \right]$$
$$+ \left(R_0^1 p_1^* + R_0^1 e_1 \right) \times \left(m_1 R_0^1 a_1 \right)$$
$$+ J_1 R_0^1 \dot{\omega}_1 + \left[\left(R_0^1 \omega_1 \right) \times J_1 \left(R_0^1 \omega_1 \right) \right]$$
$$= \begin{bmatrix} \Omega \begin{Bmatrix} l_1 \ddot{\theta}_1 + l_2 \left(\ddot{\theta}_1 + \ddot{\theta}_2 \right) C_2 \\ - l_2 \left(\dot{\theta}_1 + \dot{\theta}_2 \right)^2 S_2 \end{Bmatrix} + l_2 g y S_2 , \\ \Omega \left\{ l_1 \dot{\theta}_1^2 + l_2 \left(\ddot{\theta}_1 + \ddot{\theta}_2 \right) S_2 - l_2 \left(\dot{\theta}_1 + \dot{\theta}_2 \right)^2 C_2 \right\} \\ - l_2 g \left(x + y C_2 + m_1 / 2 \right), \\ \begin{Bmatrix} \left(l_1^2 \left(x + m_1 / 3 \right) + l_2^2 \Lambda + 2 l_1 l_2 C_2 y \right) \ddot{\theta}_1 \\ + \left(l_2^2 \Lambda + l_1 l_2 C_2 y \right) \ddot{\theta}_2 - l_1 l_2 S_2 y \dot{\theta}_2 \left(2 \dot{\theta}_1 + \dot{\theta}_2 \right) \end{Bmatrix} \end{bmatrix}$$

The joint torque of link (1):

$$\Gamma_1 = \left[R_0^1 n_1 \right]^T \left(R_0^1 Z_0 \right) = \begin{pmatrix} l_1^2 \left(x + m_1 / 3 \right) \\ + l_2^2 \Lambda + 2 l_1 l_2 C_2 y \end{pmatrix} \ddot{\theta}_1$$
$$+ \left(l_2^2 \Lambda + l_1 l_2 C_2 y \right) \ddot{\theta}_2 - l_1 l_2 S_2 y \dot{\theta}_2 \left(2 \dot{\theta}_1 + \dot{\theta}_2 \right)$$

The joint torque of link (2):

$$\Gamma_2 = \left[R^2 n_2 \right]^T \left(R_1^2 Z_0 \right) = \left(l_1 l_2 y C_2 + l_2^2 \Lambda \right) \ddot{\theta}_1 + l_2^2 \Lambda \ddot{\theta}_2 + l_1 l_2 y S_2 \dot{\theta}_1^2$$

The force exerted on the link (3):

$$F_3 \left[R_0^3 f_3 \right]^T \left(R_2^3 Z_0 \right) = \left(m_3 + m_4 \right) g = m_{eff} g$$

4. APPLICATION:

Consider a rest-to-rest Cartesian path from point (1.5, 1) to point (1.5,-1) on straight line x=1.5 during 1s with $l_1 = l_2 = 1$.A cubic polynomial can satisfy the position and velocity constraints at initial and final points.

$y(0) = y_0 = 1 \; \dot{y}(0) = \dot{y}_0 = 0$

$y(1) = y_f = -1 \; \dot{y}(1) = \dot{y}_f = 0$

The coefficients of the polynomial are:

$a_0 = 1 \; a_1 = 0 \; a_2 = -6 \; a_3 = 4$;

The Cartesian path is :

$y(t) = 1 - 6t^2 + 4t^3 \quad x = 1.5$

The trajectory simulation:

For the trajectory simulation we use elbow down and elbow up.

The figure shows the simulation block to simulate the trajectory by inverse kinematics of SCARA robot.

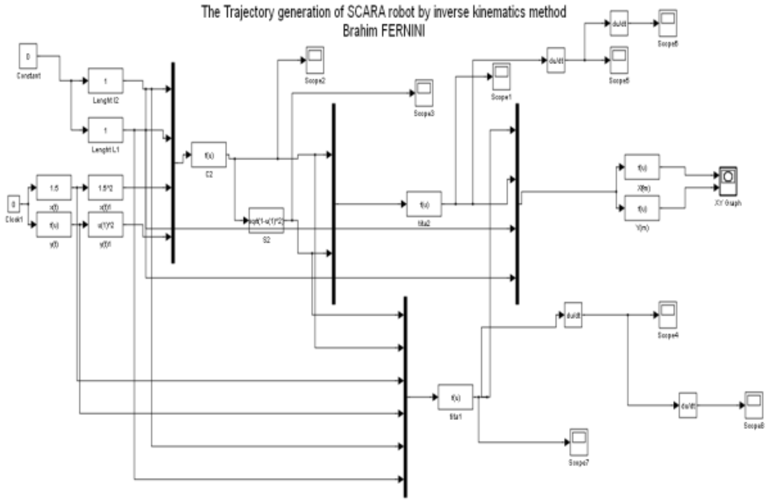

Figure 3. The trajectory generation of a SCARA robot with Matlab/Simulink by using inverse kinematics

Trajectory simulation:
Elbow down

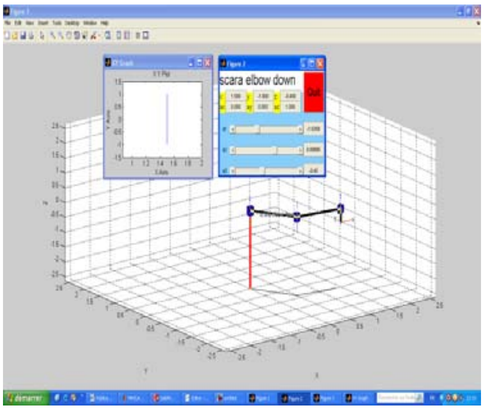

Figure 4. Matlab/Simulink

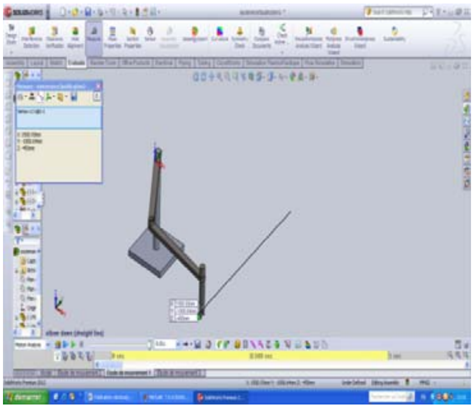

Figure 5. Solidworks

Elbow up

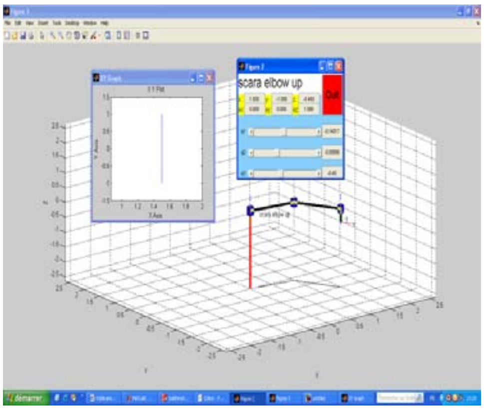

Figure 6. Matlab/Simulink

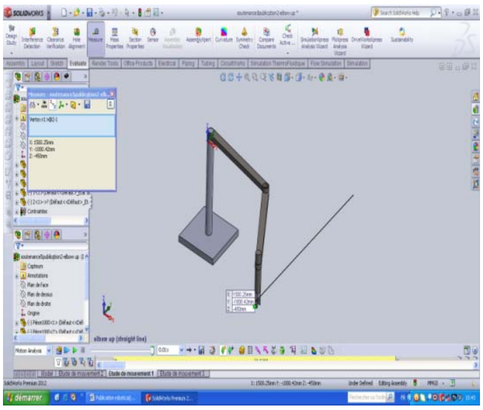

Figure 7. Solidworks

The Trajectory obtained whether by using Solid Works or by MATLAB/Simulink is exactly the same (a straight line), so the position constraint is verified at initial and final points.

The joint velocity of the robot by Matlab/Simlink:

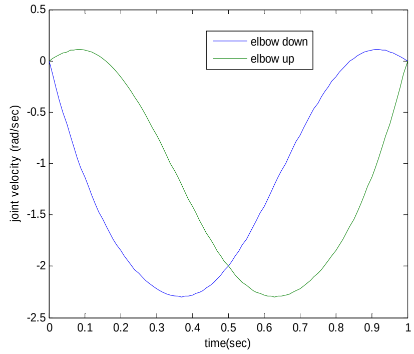

Figure 8. The joint velocity(1)

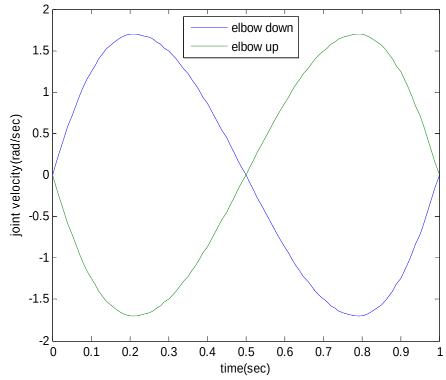

Figure 9. The joint velocity(2)

The angular velocity of the links of robot by Solidworks:
Elbow down

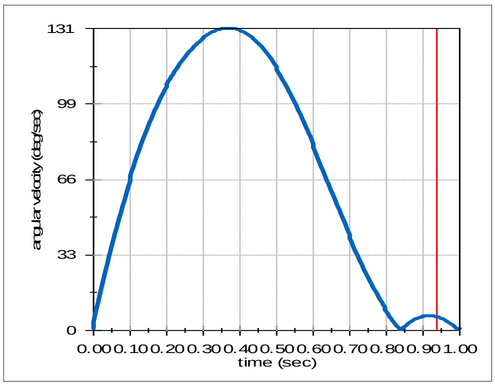

Figure 10. Link (1)

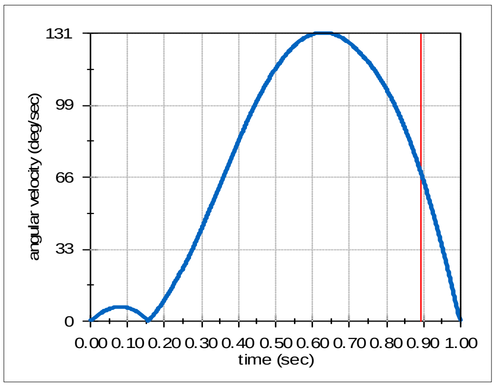

Figure 11. Link (2)

Elbow up:

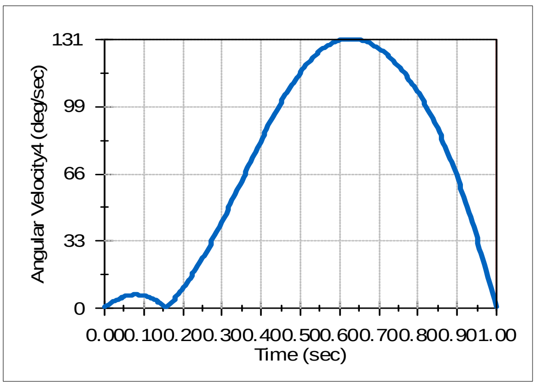

Figure 12. Link (1)

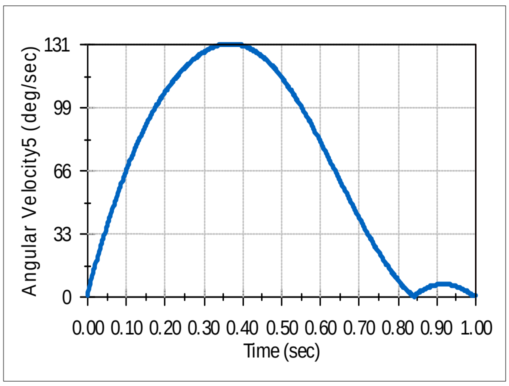

Figure 13. Link (2)

The results obtained by the two softwars Matlab/Simlulink and Solidworks about the joint velocity and the angular velocity show us that the velocity constraint is verified at initial and final points by the two softwares,

The similarity of results of both softwars Solidworks and Matlab/Simulink confirms the reliability of the kinematic model. The SCARA robot achieved a straight line motion between two positions with respect the constraints position and velocity.

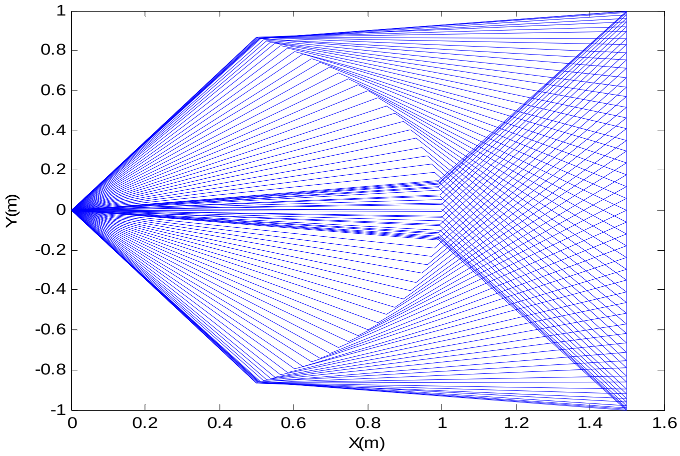

Figure 14. Arbitrary change of the two links of SCARA robot (elbow up and elbow down)

Orientationof thehomogeneous transformation matrix:

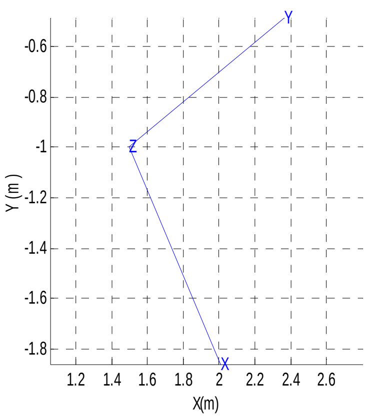

Figure 15. Elbow up

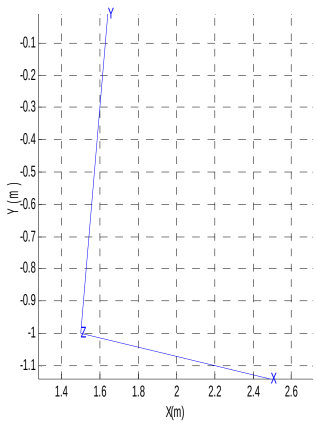

Figure 16. Elbow down

5. DYNAMIC SIMULATION:

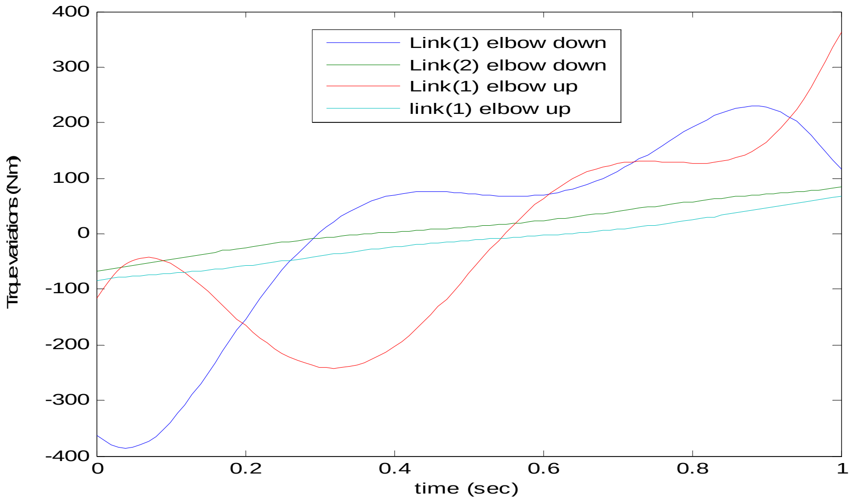

Figure 17. The torque variations.

6. DISCUSSION:

The dynamic equations found by N-E method show that there is no coupling between the link2 and the link3 because the link3 has only motion in vertical direction and there is no torque acting for this link there is only a force to achieve the vertical motion. For these reasons the effective mass can be added to the link1 and the link2 while determining the torques. This fact is clear from the torque equations. And it's found the torques are independents of angular positions and this makes the robot very compliant. Another fact that the joint torques are independent by the lengths of link3 and 4,they are dependent only by their masses as shown the torque equations.

The torques time analysis for SCARA robot is carried out taking the link masses of link 1, link2, effective mass are: 16.92Kg, 16.92Kg, 2Kg respectively .The fig.17 illustrates the variations of torques with time of elbow up and elbow down. It's found the magnitude of the torque of link 1 is higher than the torque requirement for link 2. There is an increasing difference in the two torques as time increased for elbow up and elbow down.

The following table shows the torques value at T=0s and T=1s.

Table 2. The torque values at T=0s and T=1s for elbow up and elbow down

Time(sec)	Elbow up		Elbow down	
	Torque (N.m) Link1	Torque (N.m) (Link2)	Torque (N.m) (Link1)	Torque (N.m) (Link2)
T=0s	-116.0985	-83.5574	-362.0553	-66.9041
T=1s	362.0553	66.9041	116.0985	83.5574

The torques of link1 and link2 of elbow down are symmetric with the torques of link1 and link2 of elbow down respectively, this fact is clear from the results of the table2 and the figure 17.
The results obtained from the table2 and accordingly the figure 17 show that energy consumption is the same for elbow up and elbow down for this operation, in which means elbow up and elbow down swept the same area as shown in figure 14.

7. CONCLUSION

The use of both softwares SolidWorks and Matlab/Simulink permitted to us to qualitatively develop and highlight the relevance of the studied of the kinematic model of SCARA robot.
From the dynamic model by using N-E, it's found that the effective mass can be added to the link1 and the link2 while determining the torques, and there is no coupling between link 3 and link2.Another fact the torques are independents of angular positions and the masses of link3 and link4 and this makes the robot very compliant.

In our case, we can conclude depending from the dynamic analysis by N-E the choice of elbow down or elbow up is the same for this operation “pick-place” with straight line while respecting the constraints, because the final energy balance is the same.

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