yacoda

Let's do some coding...

7. Optimal control

Optimal control problem (OCP)

$$\underset{x(.),u(.)}{minimize} \quad J(x(.),u(.))$$ → Objective functional

$$s.t. \quad \dot{x}(t)=f(x(t),u(t)) \quad \forall t \in [0,T]$$ → Dynamic constraints

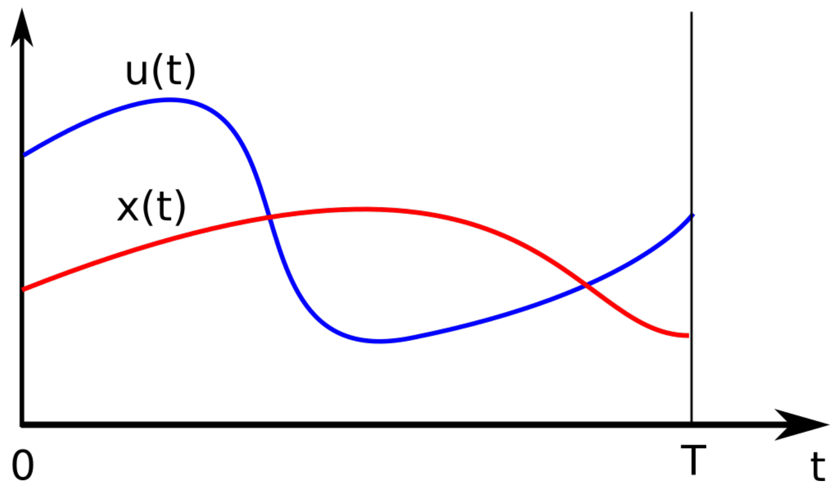
$$h(x(t),u(t)) \leq 0 \quad \forall t \in [0,T]$$ → Path constraints

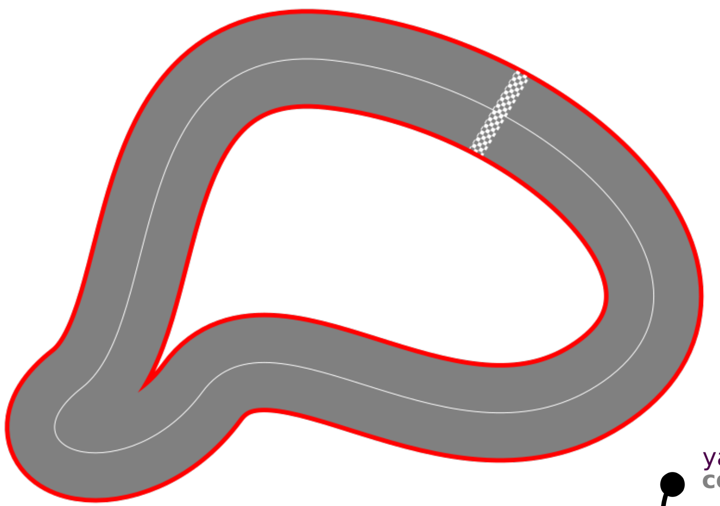
$$B(x(0),x(T))=0$$ → Boundary (initial+terminal) constraints

Trajectories

$x(.) \quad \mathbb{R} \rightarrow \mathbb{R}^n$

$u(.) \quad \mathbb{R} \rightarrow \mathbb{R}^m$





Infinite-dimensional

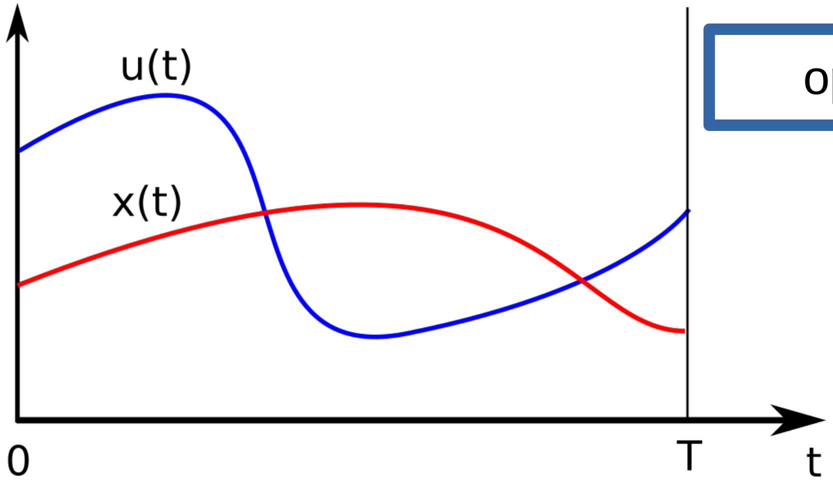
$x(.), u(.)$

Discretize!

Trajectories

$x(.) \quad \mathbb{R} \rightarrow \mathbb{R}^n$

$u(.) \quad \mathbb{R} \rightarrow \mathbb{R}^m$



Direct methods	Indirect methods	Dynamic programming
discretize (time)	optimize	discretize (time and space)
↓	↓	↓
optimize	discretize (time)	optimize

First discretize, then optimize

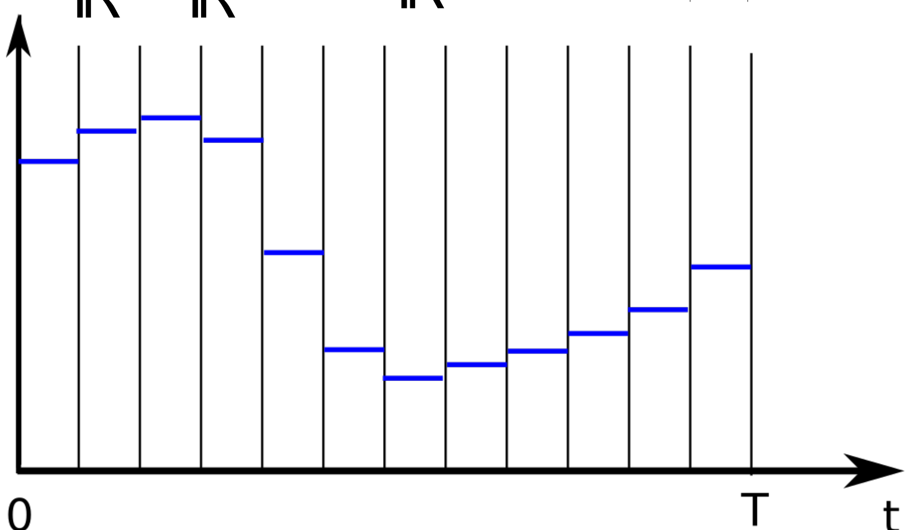
$u(.)\quad \mathbb{R} \rightarrow \mathbb{R}^m$

Other options: polynomial, spline, fourier series...

$$u_1, u_2, \dots, u_N$$

$\mathbb{R}^m \quad \mathbb{R}^m \quad \mathbb{R}^m$

Control interval

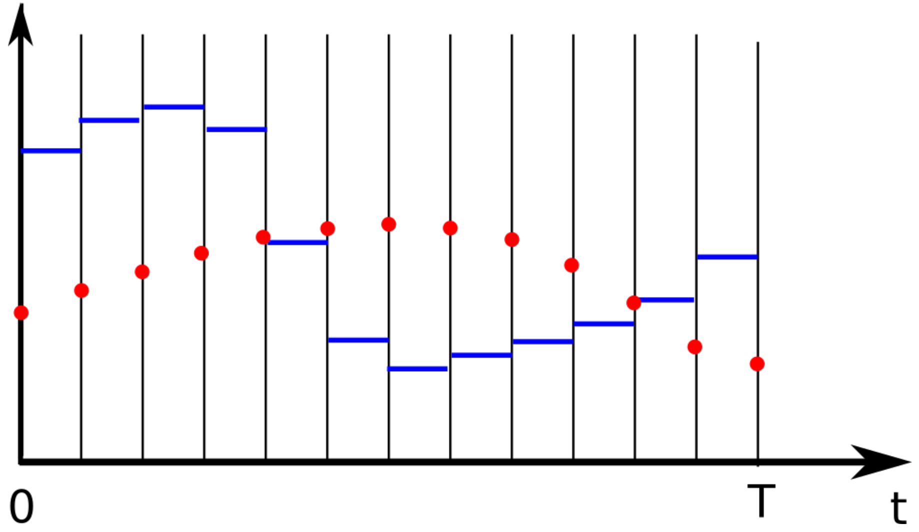


$$t_1, t_2, \dots, t_{N+1}$$

First discretize, then optimize

$u(.)$ $\downarrow$ $u_1, u_2, \ldots, u_N$

$x(.)$ $\downarrow$ $x_1, x_2, \ldots, x_{N+1}$



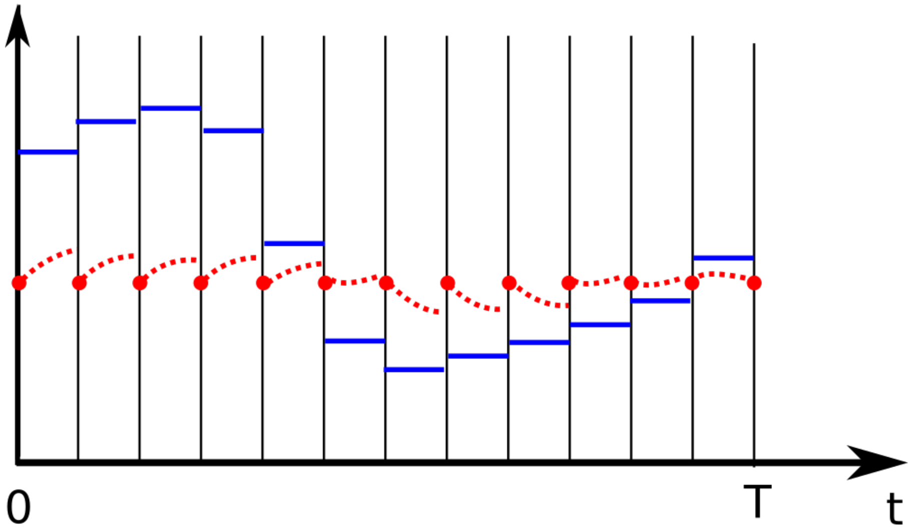
First discretize, then optimize

$u(.)$ → $u_1, u_2, \ldots, u_N$

$x(.)$ → $x_1, x_2, \ldots, x_{N+1}$

x_k, u_k, f → Discretized dynamics

$$F(x_k, u_k)$$



Multiple shooting

$$\underset{x(.),u(.)}{minimize} \; J(x(.),u(.))$$ → Objective functional

$$s.t. \;\; \dot{x}(t)=f(x(t),u(t)) \;\; \forall t\in[0,T]$$ → Dynamic constraints

$$h(x(t),u(t))\leq 0 \;\; \forall t\in[0,T]$$ → Path constraints

$$B(x(0),x(T))=0$$ → Boundary (initial+terminal)constraints

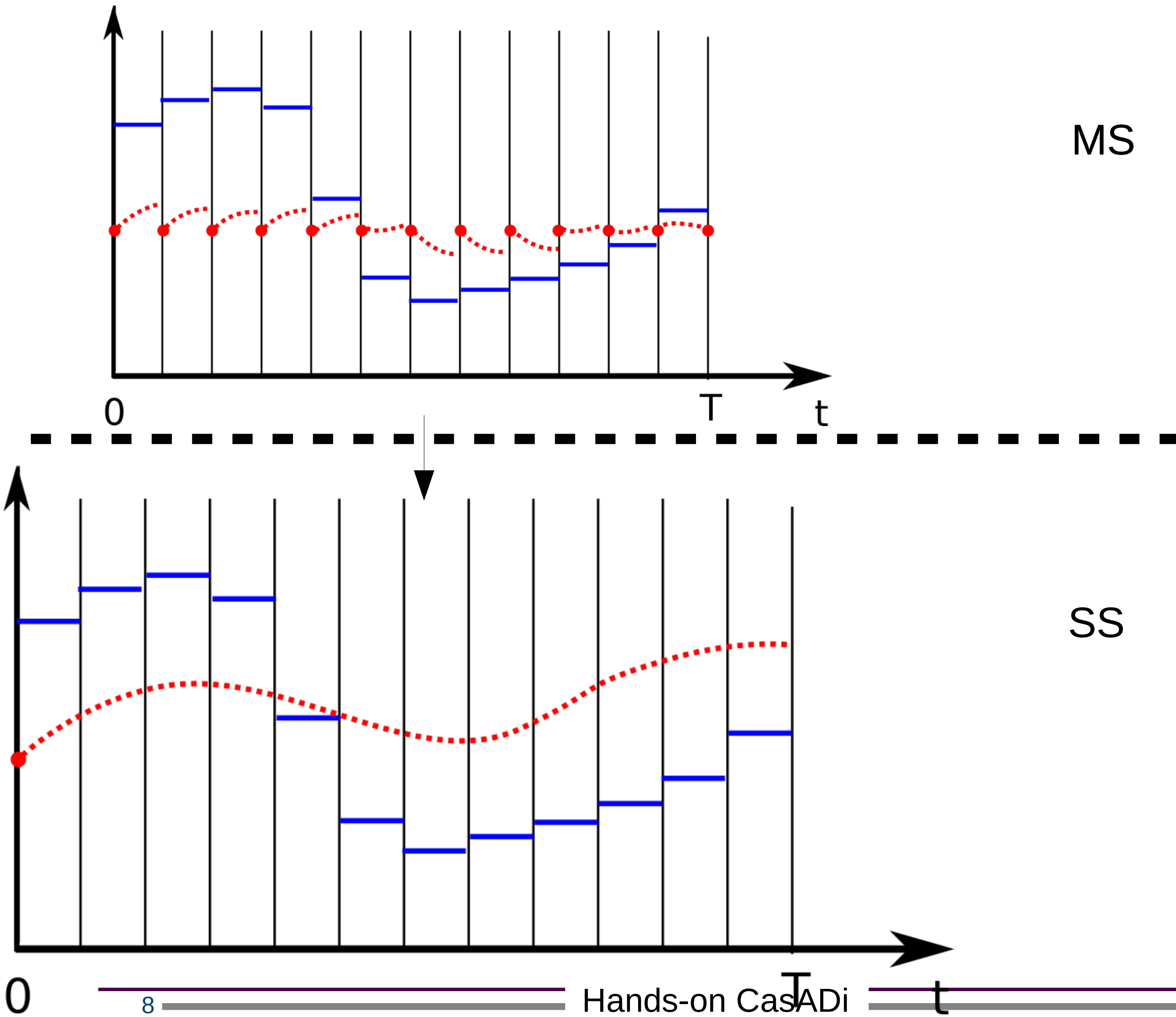
$$\underset{x_\bullet,u_\bullet}{minimize} \; \tilde{J}(x_\bullet,u_\bullet)$$ → Objective function

$$s.t. \;\; x_{k+1}=F(x_k,u_k) \;\; k=1\ldots N$$ → Dynamic constraints

$$h(x_k,u_k)\leq 0 \;\; k=1..N$$ → Path constraints

$$B(x_1,x_{N+1})=0$$ → Boundary (initial+terminal)constraints

Single shooting



Single shooting

MS

$$\tilde{J}(x_\bullet, u_\bullet) \longleftrightarrow \tilde{J}(x_1, x_2, x_3, \ldots, u_\bullet)$$

$$\tilde{J}(x_1, x_2, x_3, \ldots, u_\bullet)$$

SS

$$\tilde{J}(x_1, F(x_1, u_1), F(F(x_1, u_1), u_2), \ldots, u_\bullet)$$

$$\underset{x_1, u_\bullet}{minimize} \quad \hat{J}(x_1, u_\bullet)$$

$$x_k \equiv \begin{cases} F(x_{1,} u_k) & if\ k=2 \\ F(x_{k-1}, u_{k-1}) & else \end{cases}$$

$$s.t. \quad h(x_k, u_k) \leq 0 \quad k = 1 \ldots N$$

$$B(x_1, x_{N+1}) = 0$$

7. Optimal control