Model predictive control of batch processes based on two-dimensional integration frame

Article *in* Nonlinear Analysis Hybrid Systems · May 2018

DOI: 10.1016/j.nahs.2017.11.002

Han Chao
Shanghai University

li Jia
National Education Examinations Authority

Nonlinear Analysis: Hybrid Systems 28 (2018) 75–86

Contents lists available at ScienceDirect

Nonlinear Analysis: Hybrid Systems

journal homepage: www.elsevier.com/locate/nahs

Model predictive control of batch processes based on two-dimensional integration frame

Chao Han [a], Li Jia [a,*], Daogang Peng [b]

[a] Department of Automation, College of Mechatronics Engineering and Automation, Shanghai University, Shanghai 200072, China
[b] College of Automation Engineering, Shanghai University of Electric Power, Shanghai 200090, China

ARTICLE INFO

Article history:
Received 2 February 2017
Accepted 6 November 2017

Keywords:
Batch process
Integrated model predictive control
Model identification
Dynamic R-parameter
Iterative learning control

ABSTRACT

A novel integrated model predictive control (MPC) strategy for batch processes is proposed in this paper. Both batch-axis and time-axis information are integrated into a two-dimensional control frame. The control law is obtained through the solution of a MPC optimization with time-varying prediction horizon, which leads to superior tracking performance and robustness against disturbance and uncertainty. Moreover, both model identification and dynamic R-parameter are employed to compensate the model-plant mismatch and make zero-error tracking possible. Next, the convergence analysis and tracking performance of the proposed integrated model predictive learning control system are described and proved strictly. Lastly, the effectiveness of the proposed method is verified by an example.

1. Introduction

Batch processes provide the flexibility required for multipurpose facilities and have been widely applied to the manufacture of low-volume, high-value and products, such as specialty chemicals, pharmaceuticals, food and consumer products [1]. However, most traditional control techniques are not suitable for batch processes for its strong nonlinearity, inherent dynamic nature and discontinuous operations. Therefore, the optimization and control of batch processes remains to be challenging in modern industrial control.

Batch processes have the characteristic of repetition, and thus iterative learning control (ILC) can be used in the optimization control of batch processes [2–4]. After its initial development for industrial robot [5], ILC has been increasingly practiced for batch processes with repetitive natures to realize perfect tracking and control optimization [6,7]. Xiong and Zhang presented a batch-to-batch iterative optimal control method based on recurrent neural network models to solve the model prediction errors problem [8]. Lee et al. proposed the optimal iterative learning algorithm based on linear time-varying models for the temperature control of batch processes [9,10]. In above mentioned results, only the batch-to-batch control is taken into account and it is difficult to guarantee the performance of the batch process when the real-time uncertainties and disturbances exist. Therefore, an integrated control system is required for the maximization benefit of batch processes, in which the information of time-axis and batch-axis are processed synchronously. Rogers first employed two-dimensional (2D) theory to solve the above-mentioned problem [11]. Li et al. presented an ILC strategy for 2D time-invariant linear repetitive systems with fixed time delays [12]. Chin et al. proposed a two-stage iterative learning control technique by using the real-time feedback information to modify the ILC parameters for independent disturbance rejection [13]. For piecewise affine batch processes, Liu et al. proposed a 2D closed-loop ILC method for robust tracking of the set-point profile against

* Corresponding author.
E-mail address: jiali@staff.shu.edu.cn (L. Jia).

https://doi.org/10.1016/j.nahs.2017.11.002
1751-570X/

uncertainties and disturbances [14]. Liu et al. also proposed robust PI based set-point learning control for batch process with time-varying uncertainties and load disturbances [15]. Chen et al. combined the model predictive control (MPC) with the ILC based on 2-D theory for linear batch processes [16]. To guarantee robust convergence along both time and batch directions, a 2D ILC scheme which integrates feedback control with feedforward control was developed by Gao et al. for robust tracking of desired trajectory [17]. For multi-phase batch processes, Wang et al. proposed a iterative learning model predictive control scheme to reject the uncertain disturbances in time-axis [18]. Duran et al. presented a iterative learning modeling method and used it to control batch fermentation process [19]. Based on data-driven model, predictive quality control strategy for batch process was proposed by Aumi [20]. Lu et al. proposed natural gradient method based model-free optimization to control the quality of batch process [21]. Recently, Wang et al. presented an average dwell time based optimal ILC for multi-phase batch processes [22]. Su et al. adopted Just-In-Time-Learning modeling and based on that, an extended prediction self-adaptive control was proposed [23]. B. Corbett proposed a subspace identification methods for data-driven modeling and quality control of batch processes [24].

However, most 2D controllers proposed in the references assume that the batch processes are described by linear differential equations or can be locally linearized for the feasibility of proof and analysis. How to extend the traditional 2D controllers to a more general 2D control frame for batch process with strong nonlinearity remains to be a problem.

Inspired by MPC with time-varying prediction horizon and iterative learning control (ILC), a novel 2D control frame for batch process with strong nonlinearity is proposed in this paper, combining batch-axis discrete information and time-axis continuous information through the cost function of time-varying MPC strategy. The new integrated control system not only created a feedback controller in time-axis with iterative learning convergence in batch-axis, but also improved the accuracy of the model through online parameter modification. Furthermore, the convergence and tracking performance of such nonlinear 2D control system are given rigorous description and proof for the first time. Lastly, simulation examples illustrated the effectiveness of the strategy.

The paper is structured as follows. Section 2 gives the description of batch processes discussed in this paper. Section 3 proposes the integrated MPC system with model identification and convergence and tracking performance analysis is presented in Section 4, followed by the simulation example given in Section 5. In the end, the conclusion is given in Section 6.

2. System description

A batch process is a reaction process repeatedly conducting a given task over a limited duration of time. The discussed nonlinear batch process in this paper can be described by the following discrete-time state-space representation.

$$\begin{cases} x_k\left(t+1\right) = f\left(x_k\left(t\right), u_k\left(t\right), t\right) \\ y_k\left(t\right) = g\left(x_k\left(t\right), t\right) \\ x_k(0) = x_0, t = 1, 2, \ldots, T;\ k = 1, 2, \ldots \end{cases} \tag{1}$$

where t and k denote time step and cycle index, respectively. $x_k\left(t\right) \in R^n$, $u_k\left(t\right) \in R^m$ and $y_k\left(t\right) \in R^l$ are, respectively, the state, the control input and the batch process output at time t in kth cycle, and x_0 is the initial state of each cycle. $f\left(\cdot,\cdot,\cdot\right) : R^n \times R^m \times R^+ \to R^n$ and $g\left(\cdot,\cdot\right) : R^n \times R^+ \to R^l$ represent the dynamic characteristics of system, T is the duration of each batch.

3. Integrated MPC strategy with model identification

The formulation of the proposed integrated MPC strategy with model identification is depicted in Fig. 1, where $u_k(t|t)$ and $y_k\left(t+1\right)$ are the real time control signal and corresponding output within the kth batch. P_t is the time-varying prediction horizon of the integrated MPC controller. Based on the information of previous batch input and real-time feedback, the MPC optimizer can calculate an input sequence $\mathbf{U}_k(t|t)$ by solving an optimization problem online. The control signal $u_k(t|t)$, which is the first component of $\mathbf{U}_k(t|t)$, is sent to the process. Then the $\mathbf{U}_k(t+1|t+1)$ is recalculated as the previous instant with shrinking prediction horizon. The step is implemented repeatedly until the end of current batch. At next batch, this whole procedure is repeated to let the output trajectory asymptotically converge towards the reference trajectory while disturbances and uncertainties are compensated in real time.

3.1. Model identification with online updated parameter algorithm

Neuro-fuzzy model (NFM) [25] is employed to identify the proposed batch process in this paper. The NFM is described by the function $\hat{\mathbf{Y}}_k = \Phi\left(\mathbf{U}_k\right)\mathbf{W}_k$, where $\mathbf{W}_k = \left[w_1\left(k\right), w_2\left(k\right), \ldots, w_N\left(k\right)\right]^T$ are model adjustable parameters, and $\Phi(\mathbf{U}_k)$ is a matrix decided by $\mathbf{U}_k$. It is evident that $\mathbf{U}_k$ is the variable of function $\Phi(\cdot)$. More specificity, $\hat{\mathbf{Y}}_k = \Phi\left(\mathbf{U}_k\right)\mathbf{W}_k$ can be written as

$$\begin{aligned} \hat{\mathbf{Y}}_k &= \sum_{i=1}^{N} \hat{\alpha}_i \cdot f_i\left(\mathbf{U}_k\right) \\ &= \left(f_1\left(\mathbf{U}_k\right), f_2\left(\mathbf{U}_k\right), \ldots, f_N\left(\mathbf{U}_k\right)\right) \cdot \left(\hat{\alpha}_1, \hat{\alpha}_2, \ldots, \hat{\alpha}_N\right)^T \\ &= \Phi(\mathbf{U}_k) \cdot \mathbf{W}_k \end{aligned}$$

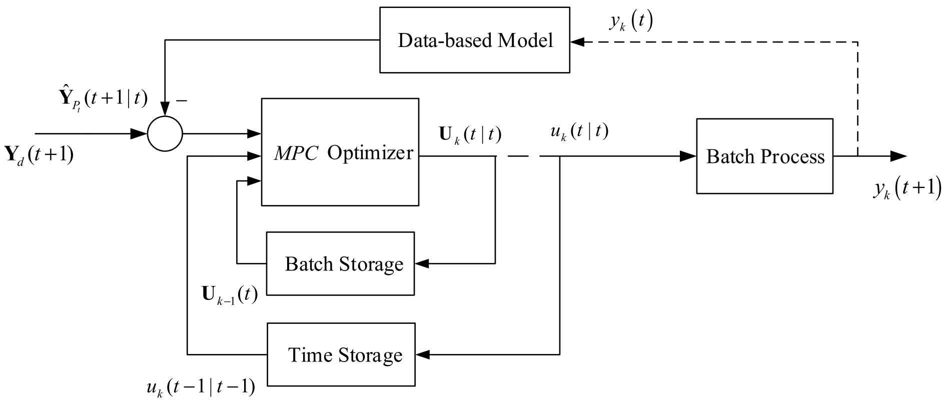


Fig. 1. Integrated model predictive control system.

And the function $\Phi(\mathbf{U}_k)$ is

$$\Phi\left(\mathbf{U}_k\right)=\frac{\left[\mu_1\left(V\left(\mathbf{U}_k\right)\right),\mu_2\left(V\left(\mathbf{U}_k\right)\right),\ldots,\mu_N\left(V\left(\mathbf{U}_k\right)\right)\right]}{\sum_{j=1}^{N}\mu_j\left(V\left(\mathbf{U}_k\right)\right)}$$

where $V\left(\mathbf{U}_k\right)=\left[\mathbf{Y}_k,\mathbf{U}_k\right]=\left[v_1,v_2,\ldots,v_M\right]$, v_i is the ith input variable of the NFM ($i=1,2,\ldots,M$), $\mu_j\left(V\left(\mathbf{U}_k\right)\right)=\exp\left(-\sum_{i=1}^{M}\frac{(v_i-c_{ji})^2}{\sigma_j^2}\right)$, where c_{ji} and σ_j are, respectively, the central value and width of the Gaussian membership function.

The model-plant mismatch is termed as model prediction error defined by

$$\hat{e}_k(t)=y_k(t)-\hat{y}_k(t) \tag{2}$$

And the tracking error is defined as

$$e_k(t)=y_d(t)-y_k(t) \tag{3}$$

The parameters of NFM are modified along time-axis by the following objective function to compensate the model prediction error.

$$\min J\left(\mathbf{W}_k(t)\right)=\left\|\Phi\left(\bar{\mathbf{U}}_k(t)\right)\mathbf{W}_k(t)-\bar{\mathbf{Y}}_k(t+1)\right\|_{\mathbf{P}_1}^2+\left\|\mathbf{W}_k(t)-\mathbf{W}_{k-1}(t)\right\|_{\mathbf{P}_2}^2 \tag{4}$$

where

$$\bar{\mathbf{U}}_k(t)=\begin{bmatrix}u_k(0)\\u_k(1)\\\vdots\\u_k(t)\end{bmatrix}_{(t+1)\times 1},\bar{\mathbf{Y}}_k(t+1)=\begin{bmatrix}y_k(1)\\y_k(2)\\\vdots\\y_k(t+1)\end{bmatrix}_{(t+1)\times 1},$$

$$\mathbf{P}_1=p_1\cdot\mathbf{I}_{t+1},\mathbf{P}_2=p_2\cdot\mathbf{I}_N,t=0,1,\ldots,T-1,$$

where $p_2=\beta k^2$, p_1 and β are both positive numbers.

The optimal $\mathbf{W}_k(t)$ can be calculated by setting $\nabla J\left(\mathbf{W}_k(t)\right)=0$

$$\mathbf{W}_k(t)=\left(p_1\cdot\Phi^T\left(\bar{\mathbf{U}}_k(t)\right)\Phi\left(\bar{\mathbf{U}}_k(t)\right)+p_2\cdot\mathbf{I}_N\right)^{-1}\cdot\left(p_1\cdot\Phi^T\left(\bar{\mathbf{U}}_k(t)\right)\bar{\mathbf{Y}}_k(t)+p_2\cdot\mathbf{W}_{k-1}(t)\right) \tag{5}$$

Then we can update online the parameters of model according to Eq. (5) to compensate model-plant mismatch.

Assumption. Assume that the matrices $\mathbf{W}_k(t)$ and $\Phi\left(\bar{\mathbf{U}}_k(t)\right)$ are both bounded.

According to Eq. (5), we have

$$\left(p_1\cdot\Phi^T\left(\bar{\mathbf{U}}_k(t)\right)\Phi\left(\bar{\mathbf{U}}_k(t)\right)+p_2\cdot\mathbf{I}_N\right)\mathbf{W}_k(t)-p_2\cdot\mathbf{W}_{k-1}(t)=p_1\cdot\Phi^T\left(\bar{\mathbf{U}}_k(t)\right)\bar{\mathbf{Y}}_k(t) \tag{6}$$

From Assumption and Eq. (6), we obtain that $\Phi^T\left(\bar{\mathbf{U}}_k(t)\right)\bar{\mathbf{Y}}_k(t)$ is bounded.

According to Eq. (5), we also have

$$\begin{aligned}\Delta\mathbf{W}_k(t) &= \mathbf{W}_k(t) - \mathbf{W}_{k-1}(t) \\ &= \frac{p_1}{\beta k^2}\left(\frac{p_1}{\beta k^2}\cdot \Phi^T\left(\bar{\mathbf{U}}_k(t)\right)\Phi\left(\bar{\mathbf{U}}_k(t)\right)+\mathbf{I}_N\right)^{-1}\cdot \Phi^T\left(\bar{\mathbf{U}}_k(t)\right)\left(\bar{\mathbf{Y}}_k - \Phi\left(\bar{\mathbf{U}}_k(t)\right)\mathbf{W}_{k-1}(t)\right)\end{aligned} \tag{7}$$

Next according to Assumption, it is easy to know that the matrix $p_1\left(\frac{p_1}{\beta k^2}\cdot \Phi^T\left(\bar{\mathbf{U}}_k(t)\right)\Phi\left(\bar{\mathbf{U}}_k(t)\right)+\mathbf{I}_N\right)^{-1}\Phi^T\left(\bar{\mathbf{U}}_k(t)\right) \cdot \left(\bar{\mathbf{Y}}_k - \Phi\left(\bar{\mathbf{U}}_k(t)\right)\mathbf{W}_{k-1}(t)\right)$ is bounded, and then we can obtain $\lim_{k\to\infty}\Delta\mathbf{W}_k = 0$.

Although the parameters of the model are convergent, the unpredictable uncertainties and disturbances actually cannot be eliminated completely. Thus, without loss of generality, we assume that the model prediction errors converge to a very small region along batch axes, namely $\hat{e}_k(t) \in \Theta_{\hat{e}}$ as $k \to \infty$, where $\Theta_{\hat{e}} = \left\{\hat{\mathbf{E}}_k \,\middle|\, \left\|\hat{\mathbf{E}}_k\right\|_{\mathbf{Q}} \le M_{\hat{e}}\right\}$ and $\hat{\mathbf{E}}_k = \left[\hat{e}_k(2), \hat{e}_k(3), \ldots, \hat{e}_k(L+1)\right]'$.

Remark 1. It should be noted that the proposed neuro-fuzzy model is updated online by using the model parameter updation formula derived by minimizing the function containing model-plant mismatch, which guarantees the robustness and convergence of the neuro-fuzzy model. In general, most of the identification/modeling methods are computed offline, so it is hard to ensure the online modeling precision. Even though some identification/modeling is online, the real-time computation amount is always huge. The neuro-fuzzy model with online updated parameter algorithm proposed in this paper overcomes these drawbacks mentioned above. The method builds an offline neuro-fuzzy model as initial model firstly, and then update it through an equation without much calculation storage and time consumption.

3.2. Integrated MPC system

For the kth batch, the integrated MPC strategy with time-varying prediction horizon applied along time axis can be given by the following quadratic cost function:

$$\min J\left(\mathbf{U}_k(t|t), k, t\right) = \left\|\mathbf{Y}_d(t+1) - \hat{\mathbf{Y}}_{P_t}(t+1|t)\right\|_{\mathbf{Q}}^2 + \left\|\mathbf{U}_k(t|t) - \mathbf{U}_{k-1}(t)\right\|_{\mathbf{R}_k}^2 + \left\|\Delta\mathbf{U}_k(t|t)\right\|_{\bar{\mathbf{R}}}^2 \tag{8}$$

Subject to

$$\begin{aligned}&\hat{\mathbf{Y}}_{P_t}(t+1|t) = \hat{\mathbf{Y}}_{P_t}\left(\mathbf{U}_k(t|t)\right) \\ &\mathbf{U}^{low}(t) \le \mathbf{U}_k(t|t) \le \mathbf{U}^{up}(t)\end{aligned} \tag{9}$$

where

$$P_t = T - t,\ t = 0, 1, \ldots, T-1,$$

$$\mathbf{Y}_d(t+1) = \begin{bmatrix} y_d(t+1) \\ y_d(t+2) \\ \vdots \\ y_d(T)\end{bmatrix}_{P_t\times 1}, \hat{\mathbf{Y}}_{P_t}(t+1|t) = \begin{bmatrix} \hat{y}(t+1|t) \\ \hat{y}(t+2|t) \\ \vdots \\ \hat{y}(T|t)\end{bmatrix}_{P_t\times 1},$$

$$\mathbf{U}_k(t|t) = \begin{bmatrix} u_k(t|t) \\ u_k(t+1|t) \\ \vdots \\ u_k(T-1|t)\end{bmatrix}_{P_t\times 1}, \mathbf{U}_{k-1}(t) = \begin{bmatrix} u_{k-1}(t) \\ u_{k-1}(t+1) \\ \vdots \\ u_{k-1}(T-1)\end{bmatrix}_{P_t\times 1},$$

$$\mathbf{U}^{low}(t) = \begin{bmatrix} u^{low}(t) \\ u^{low}(t+1) \\ \vdots \\ u^{low}(T-1)\end{bmatrix}_{P_t\times 1}, \mathbf{U}^{up}(t) = \begin{bmatrix} u^{up}(t) \\ u^{up}(t+1) \\ \vdots \\ u^{up}(T-1)\end{bmatrix}_{P_t\times 1}, \Delta\mathbf{U}_k(t|t) = \begin{bmatrix} u_k(t|t) - u_k(t-1|t-1) \\ u_k(t+1|t) - u_k(t|t) \\ \vdots \\ u_k(T-1|t) - u_k(T-2|t)\end{bmatrix}_{P_t\times 1}$$

$\mathbf{U}^{low}(t)$ and $\mathbf{U}^{up}(t)$ are the lower and upper bounds of the time-varying prediction input sequence. $\mathbf{Q}$, $\mathbf{R}_k$ and $\bar{\mathbf{R}}$ are weighting matrices defined as $\mathbf{Q} = q \times \mathbf{I}_{P_t}$, $\mathbf{R} = r_k \times \mathbf{I}_{P_t}$ and $\bar{\mathbf{R}} = \bar{r} \times \mathbf{I}_{P_t}$, where q, r and $\bar{r}$ are all positive numbers.

It should be noted that above MPC prediction horizon P_t is time-varying which ranges from current instant t to the final instant of a batch and the control horizon is the same length as prediction horizon.

The constrained optimization problem can be solved by classical optimization algorithm or heuristic optimization algorithm such as sequential quadratic programming (SQP) [26], particle swarm optimization (PSO) [27], and genetic algorithm (GA) [28].

Remark 2. Since a 2D control frame of batch process is presented in this paper, the word "Integrated" means that the proposed strategy systematically combines batch-axis information with time axis information into a two-dimensional frame through the constrained optimization problem of the time-varying horizon MPC system. It also implies the integration of ILC and MPC algorithm idea. The increment of control along batch axis in the cost function of integrated MPC strategy embodies the ILC idea which guarantees the convergence of the proposed algorithm while the remaining two terms in the cost function make the system to have good robustness against real-time disturbances and uncertainties. This method is a real-time feedback control based on former output information and meanwhile the product qualities asymptotically converge towards the reference trajectory. That is why we refer the proposed strategy as an integrated MPC strategy.

In summary, the following steps describe the algorithm of integrated MPC strategy.

Algorithm.

Step1 Identify data-based model based on historical batch operation date points. Let $k = 1$ and initialize $\mathbf{U}_k$, $\mathbf{Q}$, $\mathbf{R}_k$ and $\bar{\mathbf{R}}$, Set $t = 1$.

Step2 At the tth instant of the k-th batch, update the parameters $\mathbf{W}_k(t)$ of the data-based model according to Eq. (5). Solve the optimization problem Eq. (8) to achieve $u_k(t|t)$ as the actual control signal, and then measure the corresponding output $y_k(t+1)$.

Step3 If $t < T$, set $t = t + 1$ and go back to Step 2, else set $k = k + 1$, $t = 1$ and go to Step 2.

4. Control performance analysis

4.1. convergence analysis

Theorem 1. *Consider a batch process described by Eq. (1), which is controlled by proposed integrated MPC scheme. With the strategy of model identification in Eq. (5), the control sequence $\mathbf{U}_k$ of integrated MPC policy will converge to a constant sequence along batch cycle, whose increment corresponds to zero, namely $\Delta\mathbf{U}_k = \mathbf{U}_{k+1} - \mathbf{U}_k \to \mathbf{0}$ as $k \to \infty$.*

Proof. In the domain of batch-axis, we can rewrite Eq. (8)

$$\begin{aligned} &J\left(\mathbf{U}_k(t|t), k, t\right) \\ &= J\left(u_k(t|t), k, t\right) + J\left(\left[\mathbf{0}_{(P_t-1)\times 1}\mathbf{I}_{(P_t-1)\times(P_t-1)}\right]\mathbf{U}_k(t|t), k, t\right) \\ &= \left\|y_d(t+1) - \hat{y}(t+1|t)\right\|_{\mathbf{Q}}^2 + \|u_k(t|t) - u_{k-1}(t)\|_{\mathbf{R}_k}^2 + \|\Delta u_k(t|t)\|_{\bar{\mathbf{R}}}^2 + J\left(\left[\mathbf{0}_{(P_t-1)\times 1}\mathbf{I}_{(P_t-1)\times(P_t-1)}\right]\mathbf{U}_k(t|t), k, t\right) \end{aligned} \tag{10}$$

At the $t+1$th instant of the k-th batch, the P_{t+1} dimensional solution to the integrated MPC optimization problem in Eq. (8) $\mathbf{U}_k(t+1|t+1)$ is the optimal solution of the $t+1$th instant, namely $J(\mathbf{U}_\mathrm{k}, k, t+1)$ is minimal when $\mathbf{U}_\mathrm{k} = \mathbf{U}_k(t+1|t+1)$

Thus, we have

$$J\left(\mathbf{U}_k\left(t+1|t+1\right), k, t+1\right) \le J\left(\left[\mathbf{0}_{(P_t-1)\times 1}\mathbf{I}_{(P_t-1)\times(P_t-1)}\right]\mathbf{U}_k(t|t), k, t\right) \tag{11}$$

According to Eqs. (10) and (11), we get

$$\begin{aligned} &J\left(\mathbf{U}_k\left(t+1|t+1\right), k, t+1\right) + \left\|y_d(t+1) - \hat{y}(t+1|t)\right\|_{\mathbf{Q}}^2 + \|u_k(t|t) - u_{k-1}(t)\|_{\mathbf{R}_k}^2 + \|\Delta u_k(t|t)\|_{\bar{\mathbf{R}}}^2 \\ &\quad \le J\left(\mathbf{U}_k(t|t), k, t\right) \quad t = 0, 1, \ldots, T-1 \end{aligned} \tag{12}$$

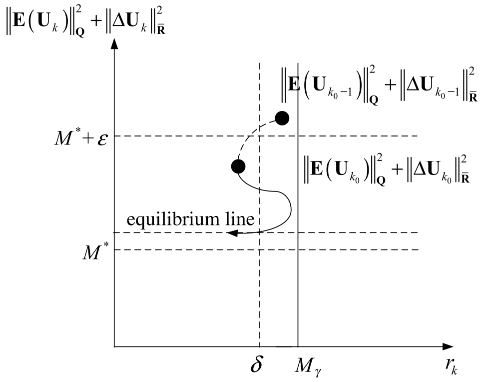
Specially, at the initial instant $t = 0$ of the k-th batch, the T dimensional solution to the integrated MPC optimization problem in Eq. (8) $\mathbf{U}_k(0|0)$ is the optimal solution of the initial instant. Clearly, the following inequality holds.

$$J\left(\mathbf{U}_k(0|0), k, 0\right) \le J(\mathbf{U}_{k-1}, k, 0) = \left\|\mathbf{Y}_d - \hat{\mathbf{Y}}\left(\mathbf{U}_{k-1}\right)\right\|_{\mathbf{Q}}^2 + \|\Delta\mathbf{U}_{k-1}\|_{\bar{\mathbf{R}}}^2 \tag{13}$$

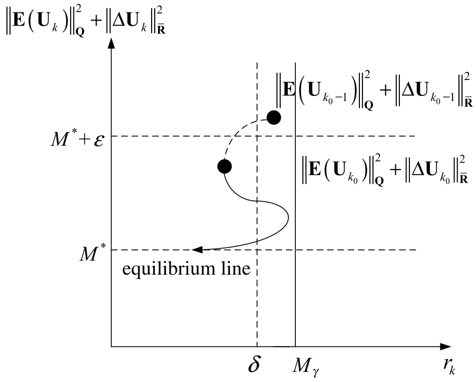
where $\mathbf{U}_{k-1}$ is the T dimensional optimal control sequence of the $k-1$th batch, consisting of the first elements of the optimal solutions of each instant.

From Eqs. (12), (13), after T times iteration, we have the following final form of inequality:

$$\left\|\mathbf{Y}_d - \hat{\mathbf{Y}}\left(\mathbf{U}_k\right)\right\|_{\mathbf{Q}}^2 + \|\mathbf{U}_k - \mathbf{U}_{k-1}\|_{\mathbf{R}_k}^2 + \|\Delta\mathbf{U}_k\|_{\bar{\mathbf{R}}}^2 \le \left\|\mathbf{Y}_d - \hat{\mathbf{Y}}\left(\mathbf{U}_{k-1}\right)\right\|_{\mathbf{Q}}^2 + \|\Delta\mathbf{U}_{k-1}\|_{\bar{\mathbf{R}}}^2 \tag{14}$$



(a) Bounded-tracking.



(b) Zero-tracking.

Fig. 2. The bounded-tracking and zero-tracking system in the condition of perfect model.

Furthermore, we get

$$0 \le \left\|\mathbf{Y}_d - \hat{\mathbf{Y}}(\mathbf{U}_k)\right\|_{\mathbf{Q}}^2 + \|\Delta\mathbf{U}_k\|_{\bar{\mathbf{R}}}^2 \le \left\|\mathbf{Y}_d - \hat{\mathbf{Y}}(\mathbf{U}_{k-1})\right\|_{\mathbf{Q}}^2 + \|\Delta\mathbf{U}_{k-1}\|_{\bar{\mathbf{R}}}^2 \tag{15}$$

$\left\|\mathbf{Y}_d - \hat{\mathbf{Y}}(\mathbf{U}_k)\right\|_{\mathbf{Q}}^2 + \|\Delta\mathbf{U}_k\|_{\bar{\mathbf{R}}}^2$ monotonically decreases as k increases and has lower bound. Therefore, the limit of $\left\|\mathbf{Y}_d - \hat{\mathbf{Y}}(\mathbf{U}_k)\right\|_{\mathbf{Q}}^2 + \|\Delta\mathbf{U}_k\|_{\bar{\mathbf{R}}}^2$ exists, namely

$$\lim_{k\to\infty} \left\|\mathbf{Y}_d - \hat{\mathbf{Y}}(\mathbf{U}_k)\right\|_{\mathbf{Q}}^2 + \|\Delta\mathbf{U}_k\|_{\bar{\mathbf{R}}}^2 = \lim_{k\to\infty} \left\|\mathbf{Y}_d - \hat{\mathbf{Y}}(\mathbf{U}_{k-1})\right\|_{\mathbf{Q}}^2 + \|\Delta\mathbf{U}_{k-1}\|_{\bar{\mathbf{R}}}^2 \tag{16}$$

From Eq. (14), we obtain

$$\lim_{k\to\infty} \|\mathbf{U}_k - \mathbf{U}_{k-1}\|_{\mathbf{R}_k}^2 \le \left[\lim_{k\to\infty}\left(\left\|\mathbf{Y}_d - \hat{\mathbf{Y}}(\mathbf{U}_k)\right\|_{\mathbf{Q}}^2 + \|\Delta\mathbf{U}_k\|_{\bar{\mathbf{R}}}^2\right) - \lim_{k\to\infty}\left(\left\|\mathbf{Y}_d - \hat{\mathbf{Y}}(\mathbf{U}_{k-1})\right\|_{\mathbf{Q}}^2 + \|\Delta\mathbf{U}_{k-1}\|_{\bar{\mathbf{R}}}^2\right)\right] \tag{17}$$

Eqs. (16), (17) show $\lim_{k\to\infty}\|\mathbf{U}_k - \mathbf{U}_{k-1}\|_{\mathbf{R}_k}^2 \le 0$. That is to say, the proposed integrated MPC policy converges to zero along batch cycle, namely $\lim_{k\to\infty}\|\mathbf{U}_k - \mathbf{U}_{k-1}\|_{\mathbf{R}_k}^2 = 0$. □

4.2. Tracking performance analysis

Inspired by the definition of Lyapunov stability, bounded-tracking and zero-tracking are defined in the integrated control system. Based on our previous work [25], we present the theoretical proof of tracking performance and choose a specific function for parameter R in the end on the basis of the theoretical analysis.

Definition 1 (*Bounded-tracking*). If there exists a $\delta = \delta(\varepsilon) > 0$ for any $\varepsilon > 0$ at k_0th batch such that $r_{k_0} < \delta$, then the inequality $\left|\|\mathbf{E}(\mathbf{U}_k)\|_{\mathbf{Q}}^2 + \|\Delta\mathbf{U}_k\|_{\bar{\mathbf{R}}}^2 - M^*\right| < \varepsilon$ holds for every $k \ge k_0$, where k_0 is a certain batch index and M^* is positive.

Definition 2 (*Zero-tracking*). If it is bounded-tracking and there exists a $\delta > 0$ at k_0th batch such that $r_{k_0} < \delta$, then the equality $\lim_{k\to\infty}\left|\|\mathbf{E}(\mathbf{U}_k)\|_{\mathbf{Q}}^2 + \|\Delta\mathbf{U}_k\|_{\bar{\mathbf{R}}}^2 - M^*\right| = 0$ holds, where k_0 is a certain batch index and M^* is positive.

Here Fig. 2 is introduced to help interpreting the definitions above. Fig. 2(a) represents the system of bounded-tracking, and shows that if $r_{k_0} < \delta$ in the k_0th batch, then $\left|\|\mathbf{E}(\mathbf{U}_k)\|_{\mathbf{Q}}^2 + \|\Delta\mathbf{U}_k\|_{\bar{\mathbf{R}}}^2 - M^*\right| < \varepsilon$ is satisfied for each $k \ge k_0$, where $0 \le r_k \le M_r$ for each batch index k. But for bounded-tracking system, $\|\mathbf{E}(\mathbf{U}_k)\|_{\mathbf{Q}}^2 + \|\Delta\mathbf{U}_k\|_{\bar{\mathbf{R}}}^2$ may converge to local optimum solution if $r_k \ne 0$ at every batch. Fig. 2(b) represents the system of zero-tracking, which shows that if $r_{k_0} < \delta$ in the k_0th batch, then $\|\mathbf{E}(\mathbf{U}_k)\|_{\mathbf{Q}}^2 + \|\Delta\mathbf{U}_k\|_{\bar{\mathbf{R}}}^2$ will converge to optimal solution M^* whatever value r_k is after batch k_0. That is to say, $\left(r_k, \|\mathbf{E}(\mathbf{U}_k)\|_{\mathbf{Q}}^2 + \|\Delta\mathbf{U}_k\|_{\bar{\mathbf{R}}}^2\right)$ will converge to the equilibrium line $\|\mathbf{E}(\mathbf{U}_k)\|_{\mathbf{Q}}^2 + \|\Delta\mathbf{U}_k\|_{\bar{\mathbf{R}}}^2 = M^*$ where $0 \le r_k \le M_r$.

Remark 3. The trajectories in Fig. 2 only represent the trend of movement. The straight line parts of the trajectories do not mean the faster convergence rate compared with non-straight ones. The rate of convergence mainly depends on the density of curves, and smaller density results in faster convergence rate.

Lemma. For arbitrary initial control sequence $\mathbf{U}_{k_0}$ at k_0th batch and $\varepsilon > 0$, there exists $\delta = \delta(\varepsilon) > 0$ and $r_{k_0+1} < \delta$ such that the optimal control sequence $\mathbf{U}_{k_0+1}$ at k_0+1th batch satisfies $\mathbf{U}_{k_0+1} \in \Theta_1$, where $\Theta_1 = \left\{\mathbf{U}_{k_0+1} \middle| \left| \left\|\mathbf{E}\left(\mathbf{U}_{k_0+1}\right)\right\|^2_{\mathbf{Q}} + \left\|\Delta\mathbf{U}_{k_0+1}\right\|^2_{\bar{\mathbf{R}}} - M^*\right| < \varepsilon\right\}$.

Theorem 2. *For an arbitrary initial control profile, the tracking error sequence $\mathbf{E}(\mathbf{U}_k)$ over a batch period can finally be bounded by a small region as the batch number k tend to be infinite, namely $\mathbf{E}(\mathbf{U}_k) \in \Theta_e$ as $k \to \infty$, where $\Theta_e = \left\{e\left(\mathbf{U}_k, t_f\right) \middle| \left\|e\left(\mathbf{U}_k, t_f\right)\right\|_{\mathbf{Q}} \le M_{\hat{e}} + \sqrt{\varepsilon}\right\}$.*

Proof. The perfect model assumption is first assumed, and then with the strategy of model identification, the result is extended to general cases, namely model-plant mismatch.

(1) Perfect model

Under this condition, Eq. (8) can be simplified as

$$\begin{aligned} &\min J\left(\mathbf{U}_k(t|t), k, t\right) \\ &= \left\|\mathbf{E}\left(\mathbf{U}_k(t|t)\right) + \hat{\mathbf{E}}\left(\mathbf{U}_k(t|t)\right)\right\|^2_{\mathbf{Q}} + \left\|\mathbf{U}_k(t|t) - \mathbf{U}_{k-1}(t)\right\|^2_{\mathbf{R}_k} + \left\|\Delta\mathbf{U}_k(t|t)\right\|^2_{\bar{\mathbf{R}}} \\ &= \left\|\mathbf{E}\left(\mathbf{U}_k(t|t)\right)\right\|^2_{\mathbf{Q}} + \left\|\mathbf{U}_k(t|t) - \mathbf{U}_{k-1}(t)\right\|^2_{\mathbf{R}_k} + \left\|\Delta\mathbf{U}_k(t|t)\right\|^2_{\bar{\mathbf{R}}} \end{aligned} \tag{18}$$

Based on the former convergence analysis, we construct the following function:

$$f\left(\mathbf{U}_k, k\right) = \left\|\mathbf{E}\left(\mathbf{U}_k\right)\right\|^2_{\mathbf{Q}} + \left\|\mathbf{U}_k - \mathbf{U}_{k-1}\right\|^2_{\mathbf{R}_k} + \left\|\Delta\mathbf{U}_k\right\|^2_{\bar{\mathbf{R}}} \tag{19}$$

For the function described by Eq. (18), set an open set $\Theta_{\mathbf{U}^*} = \left\{\mathbf{U} \middle| \left|\left\|\mathbf{E}\left(\mathbf{U}_k\right)\right\|^2_{\mathbf{Q}} + \left\|\Delta\mathbf{U}_k\right\|^2_{\bar{\mathbf{R}}} - M^*\right| < \varepsilon\right\}$, in which there is no other extreme solutions except for the global optimum solution.

According to **Lemma**, we know that for any ε, there exists a $\delta > 0$. If $r_{k_0+1} < \delta$, then for any initial solution $\mathbf{U}_{k_0}$ we have $\left|\left\|\mathbf{E}\left(\mathbf{U}_{k_0+1}\right)\right\|^2_{\mathbf{Q}} + \left\|\Delta\mathbf{U}_{k_0+1}\right\|^2_{\bar{\mathbf{R}}} - M^*\right| < \varepsilon$, namely $\mathbf{U}_{k_0+1} \in \Theta_{\mathbf{U}^*}$.

Furthermore, the following inequality holds for any batch $k > k_0$

$$\left|\left\|\mathbf{E}\left(\mathbf{U}_k\right)\right\|^2_{\mathbf{Q}} + \left\|\Delta\mathbf{U}_k\right\|^2_{\bar{\mathbf{R}}} - M^*\right| \le \left|\left\|\mathbf{E}\left(\mathbf{U}_{k_0+1}\right)\right\|^2_{\mathbf{Q}} + \left\|\Delta\mathbf{U}_{k_0+1}\right\|^2_{\bar{\mathbf{R}}} - M^*\right| < \varepsilon \tag{20}$$

Therefore, $\mathbf{U}_k \in \Theta_{\mathbf{U}^*}$ for any $k > k_0$. That is to say, for any $\mathbf{U}_{k_0}$ at batch k_0, the optimization problem is bounded-tracking. By setting $\nabla f\left(\mathbf{U}_k\right) = 0$, we have

$$\frac{\partial f\left(\mathbf{U}_k, k\right)}{\partial\mathbf{U}_k} = \frac{\partial\left(\left\|\mathbf{E}\left(\mathbf{U}_k\right)\right\|^2_{\mathbf{Q}} + \left\|\Delta\mathbf{U}_k\right\|^2_{\bar{\mathbf{R}}}\right)}{\partial\mathbf{U}_k} + 2r_k \cdot \left(\mathbf{U}_k - \mathbf{U}_{k-1}\right) = 0 \tag{21}$$

Because of $\lim_{k\to\infty}\left\|\mathbf{U}_k - \mathbf{U}_{k-1}\right\|^2_{\mathbf{R}} = 0$, we have $\lim_{k\to\infty} r_k \cdot \left(\mathbf{U}_k - \mathbf{U}_{k-1}\right) = 0$. Thus, the result of $\lim_{k\to\infty} \frac{\partial\left(\left\|\mathbf{E}(\mathbf{U}_k)\right\|^2_{\mathbf{Q}} + \left\|\Delta\mathbf{U}_k\right\|^2_{\bar{\mathbf{R}}}\right)}{\partial\mathbf{U}_k} = 0$ holds. In other words, $\left\|\mathbf{E}\left(\mathbf{U}_k\right)\right\|^2_{\mathbf{Q}} + \left\|\Delta\mathbf{U}_k\right\|^2_{\bar{\mathbf{R}}}$ converges to extreme solution as k tends to be infinite. Because the condition $\mathbf{U}_k \in \Theta_{\mathbf{U}^*}$ for any $k > k_0$ holds and there are no other extreme solution except for the global optimum solution in $\Theta_{\mathbf{U}^*}$, we make a conclusion that $\left\|\mathbf{E}\left(\mathbf{U}_k\right)\right\|^2_{\mathbf{Q}} + \left\|\Delta\mathbf{U}_k\right\|^2_{\bar{\mathbf{R}}}$ converges to global optimal solution, namely

$$\lim_{k\to\infty} \left\|\mathbf{E}\left(\mathbf{U}_k\right)\right\|^2_{\mathbf{Q}} + \left\|\Delta\mathbf{U}_k\right\|^2_{\bar{\mathbf{R}}} - M^* = 0 \tag{22}$$

Therefore, if there exists only one extreme solution in the open set $\Theta_{\mathbf{U}^*}$, then the system is zero-tracking for arbitrary initial control profile.

(2) Model-plant mismatch

According to Eq. (8), we have

$$\begin{aligned} &\min J\left(\mathbf{U}_k(t|t), k, t\right) \\ &= \left\|\mathbf{E}\left(\mathbf{U}_k(t|t)\right) + \hat{\mathbf{E}}\left(\mathbf{U}_k(t|t)\right)\right\|^2_{\mathbf{Q}} + \left\|\mathbf{U}_k(t|t) - \mathbf{U}_{k-1}(t)\right\|^2_{\mathbf{R}_k} + \left\|\Delta\mathbf{U}_k(t|t)\right\|^2_{\bar{\mathbf{R}}} \end{aligned} \tag{23}$$

According to the results in (1), we conclude that $\left\|\mathbf{E}\left(\mathbf{U}_k(t)\right) + \hat{\mathbf{E}}\left(\mathbf{U}_k(t)\right)\right\|^2_{\mathbf{Q}} + \left\|\Delta\mathbf{U}_k(t)\right\|^2_{\bar{\mathbf{R}}}$ is bounded above by a small constant ε when $k > k_0$ by adjusting r_k, namely

$$\left\|\mathbf{E}\left(\mathbf{U}_k(t)\right) + \hat{\mathbf{E}}\left(\mathbf{U}_k(t)\right)\right\|^2_{\mathbf{Q}} \le \left\|\mathbf{E}\left(\mathbf{U}_k(t)\right) + \hat{\mathbf{E}}\left(\mathbf{U}_k(t)\right)\right\|^2_{\mathbf{Q}} + \left\|\Delta\mathbf{U}_k(t)\right\|^2_{\bar{\mathbf{R}}} < \varepsilon, \quad k > k_0 \tag{24}$$

then

$$\left\|\mathbf{E}\left(\mathbf{U}_k(t)\right) + \hat{\mathbf{E}}\left(\mathbf{U}_k(t)\right)\right\|_{\mathbf{Q}} < \sqrt{\varepsilon} \tag{25}$$

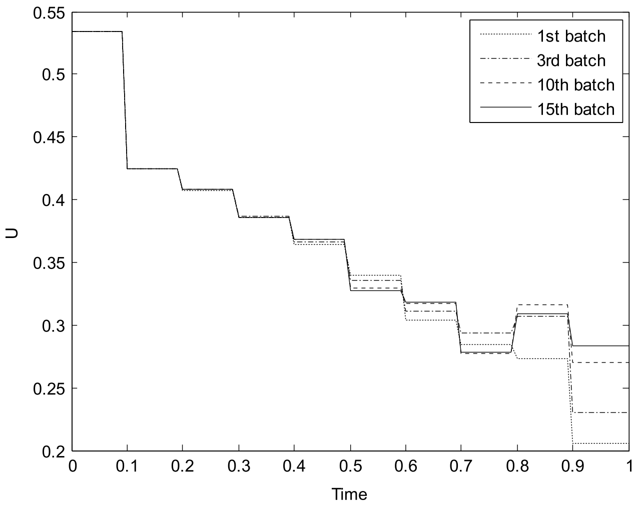


Fig. 3. Control trajectories at 1st, 3rd, 10th, 15th batches.

According to Eq. (25), we can get

$$\left\|\mathbf{E}\left(\mathbf{U}_k(t)\right)\right\|_{\mathbf{Q}} - \left\|\hat{\mathbf{E}}\left(\mathbf{U}_k(t)\right)\right\|_{\mathbf{Q}} \le \left\|\mathbf{E}\left(\mathbf{U}_k(t)\right) + \hat{\mathbf{E}}\left(\mathbf{U}_k(t)\right)\right\|_{\mathbf{Q}}^2 < \sqrt{\varepsilon} \tag{26}$$

namely

$$\left\|\mathbf{E}\left(\mathbf{U}_k(t)\right)\right\|_{\mathbf{Q}} < \sqrt{\varepsilon} + \left\|\hat{\mathbf{E}}\left(\mathbf{U}_k(t)\right)\right\|_{\mathbf{Q}} \tag{27}$$

Because $\sqrt{\varepsilon}$ can be relatively small, $\left\|\mathbf{E}\left(\mathbf{U}_k(t)\right)\right\|_{\mathbf{Q}}$ mainly depends on the model error $\hat{\mathbf{E}}\left(\mathbf{U}_k(t)\right)$. Therefore, tracking error can converge to a very small domain depending on the upper bound of model error, that is to say, $\mathbf{E}\left(\mathbf{U}_k\right) \in \Theta_e$ as $k \to \infty$. □

Remark 4. Because it is impossible to match model with actual plant totally, large r_k at the very beginning would lead the quality falling into suboptimal solution (namely the optimal solution while model-plant mismatch). Therefore, r_k can be designed as the monotone decreasing function about the final tracking error. Thus r_k would increase gradually with the tracking precision increasing, then the system output would run-to-run tend to desire quality. In this paper, the following specific function is chosen for r_k

$$r_k = \frac{\tau_1}{1 + \tau_2 \cdot e^{\tau_3 \cdot \left|y_d(t_f) - y_k(t_f)\right|}} \tag{28}$$

where τ_1, τ_2 and τ_3 are all penalty parameters used to adjust the weight r_k from batch to batch. Eq. (28) indicates that r_k would monotonically increase while $\left|y_d(t_f) - y_k(t_f)\right|$ decreasing to restrain the variation of control signals. Thus if the final system output is close enough to the desired output, i.e., $r_k = \frac{\tau_1}{1+\tau_2}$ after taking into account of $y_k(t_f) = y_d(t_f)$, the change of control input would be very little. Otherwise the value of r_k would be small, which causes control signals change sharply to obtain better approximation to the reference trajectories.

5. Example

To verify the effectiveness and applicability of the proposed strategy, the control system is simulated to control the product quality of a typical nonlinear batch reactor, in which an irreversible exothermic reaction $A \xrightarrow{k_1} B \xrightarrow{k_2} C$ happens [29,30]. This reaction processes are described by the inherent first-order dynamic equations

$$\begin{aligned} \dot{x}_1 &= -k_1 \exp(-E_1/T)x_1^2 \\ \dot{x}_2 &= k_1 \exp(-E_1/T)x_1^2 - k_2 \exp(-E_2/T)x_2 \end{aligned} \tag{29}$$

where x_1 and x_2, respectively, denote the reactant concentration of A and B, and T is the reaction temperature. The values of parameter k_1, k_2, E_1 and E_2 are given in Table 1.

The reaction temperature is normalized through $u = (T - T_{\min})/(T_{\max} - T_{\min})$, where $T_{\min}$ and $T_{\max}$ are 298(k) and 398(k), respectively. u is the control signal confined within $[0,\ 1]$, and $x_2(t)$ is the corresponding output. The control objective is to

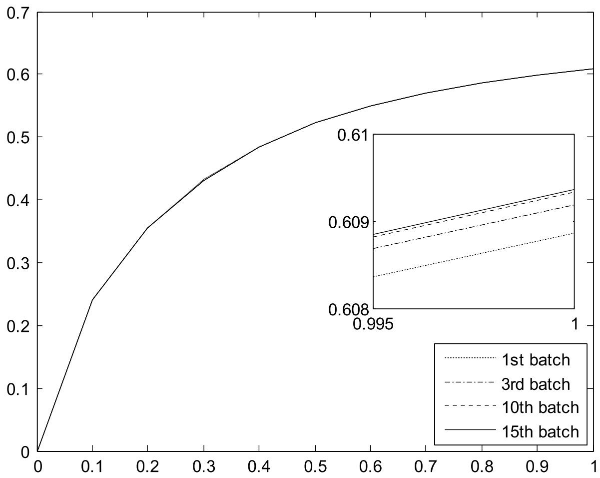


Fig. 4. Output trajectories at 1st, 3rd, 10th, 15th batches.

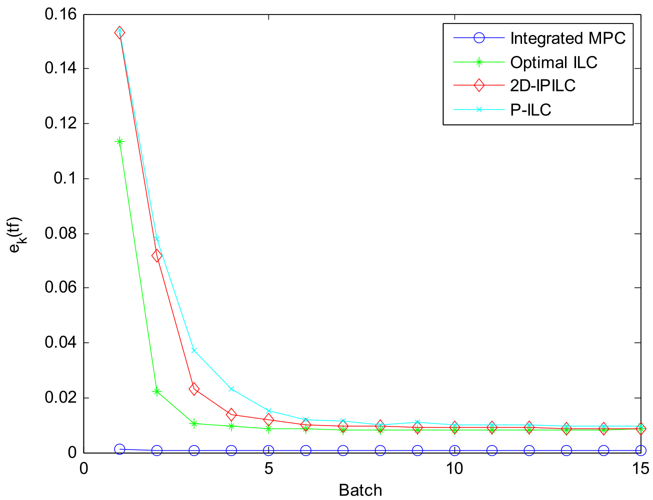


Fig. 5. Curves of tracking error based on four controller systems.

Table 1
Parameter values for the batch reactor.

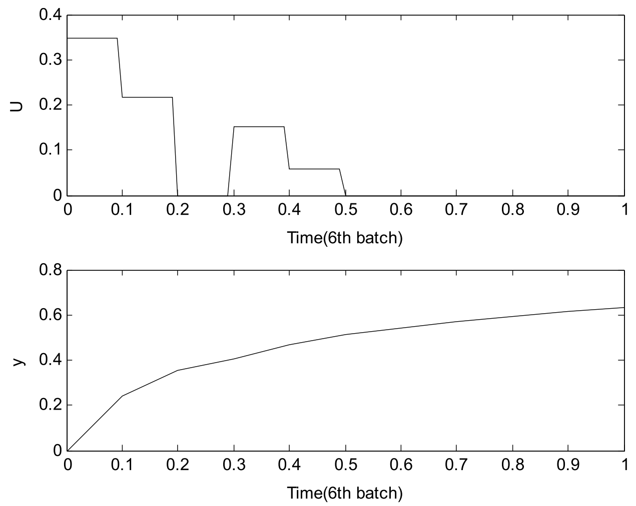
Parameter	Value
k_1	4.0×10^3
k_2	6.2×10^5
E_1	2.5×10^3
E_2	5.0×10^3

approximate the end-time output to an ideal value by adjusting the control signal u from batch to batch. The reaction time t in each batch satisfies $0 \leq t \leq t_f$. The initial operating conditions are as follows: $x_1(0) = 1, x_2(0) = 0$.

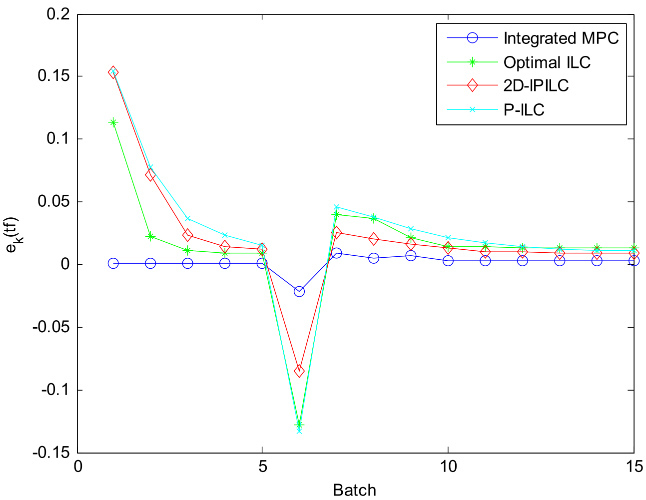
In this part, the neuro-fuzzy model (NFM) is employed. A set of independent random signals with uniform distribution between $[0,\ 1]$ are used to simulate the input data. The parameters of NFM can be trained through the input–output data set. The input form of NFM is chosen as follows:

$$V = \left[x_{2,k}(t-1), x_{2,k}(t), u_k(t-1), u_k(t)\right] \tag{30}$$

where $x_{2,k}(t)$ represents the product concentration at tth time instant of the kth batch. Object is to drive the process output $y = x_2$ to its set-point specified at the batch end, i.e. $y_d(t_f) = 0.61$.



(a) 6th batch control and output trajectories.



(b) Curves of tracking error based on four controller systems.

Fig. 6. Trajectories of controller systems under uncertainties.

R is considered to be a dynamic parameter as follows:

$$\mathbf{R} = r_k \cdot \mathbf{I}_T, r_k = \frac{\tau_1}{1 + \tau_2 \cdot e^{\tau_3 \cdot |y_d(t_f) - y_k(t_f)|}} \tag{31}$$

In the simulation, the values of τ_1, τ_2 and τ_3 are chosen by trial and error. The following parameters are adopted to initialize the simulation system: $\mathbf{U}_0 = 0, \quad \tau_1 = 20, \tau_2 = 1, \quad \tau_3 = 1 \times 10^3, \quad q = 1 \times 10^4, \bar{r} = 0.1$.

Comparison was made between our method and three other typical control algorithms of batch process published in recent years, namely optimal iterative learning control (ILC) [25], 2D Roesser model based integrated predictive iterative learning control (2D-IPILC) [16] and proportion type iterative learning control (P-ILC) [16]. The trajectories of proposed controller systems are, respectively, shown in Figs. 3–4. Fig. 5 shows the final output error curves of these four methods and the final output error values can be seen from Table 2. It is clear that the proposed control strategy has faster convergence rate and yields a more accurate final output than that obtained by other three controller systems.

Furthermore, to test the robustness of this strategy, a scenario in which a kinetic parameter is changed, is considered in the simulation. This case is implemented by changing parameters E_1 and E_2 at the 6th batch to simulate uncertainties.

$$\begin{aligned} E_1 &= 0.95E_{10} \\ E_2 &= 1.1E_{20} \end{aligned} \tag{32}$$

where E_{10} and E_{20} are the nominal values of parameters E_1 and E_2 as shown in Table 1.

Table 2
Final output error value based on two controller systems.

Methods	1st batch	3rd batch	10th batch	15th batch
Integrated MPC	1.2×10^{-3}	8×10^{-4}	7×10^{-4}	6×10^{-4}
Optimal ILC	1.1×10^{-1}	1.1×10^{-2}	8.5×10^{-3}	8.3×10^{-3}
2D-IPILC	1.5×10^{-1}	2.3×10^{-2}	9.1×10^{-3}	8.8×10^{-3}
P-ILC	1.5×10^{-1}	3.7×10^{-2}	9.9×10^{-3}	9.5×10^{-3}

Table 3
Sixth batch final output error value based on four controller systems under uncertainties.

Uncertainties	Integrated MPC	Optimal ILC [25]	2D-IPILC [16]	P-ILC [16]
$E_1 = 0.95E_{10}$ $E_2 = 1.1E_{20}$	-2.2×10^{-2}	-1.28×10^{-1}	-8.5×10^{-2}	-1.3×10^{-1}

As seen from Fig. 6, the proposed control strategy has the best robustness and can maintain good performance despite the existence of disturbances and uncertainties (see Table 3).

6. Conclusion

In this article, a novel integrated model predictive control strategy with model identification was presented, which not only combined real-time feedback with batch convergence into one frame, but also improved the accuracy of the model through online identification. Moreover, strict proof of the convergence and tracking performance analysis of the proposed integrated MPC system are given to verify that perfect tracking can be attained despite the existence of model errors. Lastly, the proposed control strategy was applied to a simulation example and the results demonstrate that the control system has good tracking performance and robustness.

Acknowledgments

This work is supported by National Natural Science Foundation of China (61374044, 61773251), and Shanghai Science Technology Commission (15510722100, 16111106300, 17511109400).

Appendix

Nomenclature

k	batch index
t_f	batch length
T	sampling number
P_t	prediction horizon length at the instant t
$y_d(t)$	reference output at the instant t
$u_k(t+n\|t)$	predicted control value at the instant $t+n$ calculated at instant t of kth batch
$u_k(t)$	actual control signal at the instant t of kth batch
$\hat{y}_k(t+n\|t)$	predicted output value at the instant $t+n$ calculated at instant t of kth batch
$y_k(t)$	actual output variable at the instant t of kth batch
$\hat{e}_k(t)$	model prediction error
$e_k(t)$	output tracking error
$\mathbf{U}_k(t\|t)$	predicted control sequence from instant t to $T-1$
$\hat{\mathbf{Y}}_{P_t}(t+1\|t)$	predicted output sequence from instant $t+1$ to T
$\hat{\mathbf{Y}}_k$	predicted output sequence of the kth batch
$\mathbf{E}(\cdot)$	output tracking error sequence
$\hat{\mathbf{E}}(\cdot)$	model prediction error sequence
$\mathbf{Q}$	weighting matrix for output tracking error
$\mathbf{R}_k$	weighting matrix for control increment along batch axis
$\bar{\mathbf{R}}$	weighting matrix for control increment along time axis
ε	small positive constant

References

[1] D. Bonvin, Optimal operation of batch reactors: a personal view, J. Process Control 8 (1998) 355–368.
[2] Z.H. Xiong, J. Zhang, X. Wang, Y.M. Xu, Integrated tracking control strategy for batch processes using a batch-wise linear time-varying perturbation model, Control Theory Appl. 1 (1) (2007) 178–188.

[3] N. Lu, F. Gao, Stage-based process analysis and quality prediction for batch processes, Ind. Eng. Chem. Res. 44 (10) (2005) 3547–3555.
[4] J.H. Lee, F.K.S. Lee, Iterative learning control applied to batch processes: an overview, Control Eng. Pract. 15 (10) (2007) 1306–1318.
[5] S. Arimoto, S. Kawamura, F. Miyazaki, Bettering operation of robots by learning, J. Robot. Syst. 1 (2) (1984) 123–140.
[6] K.S. Lee, I.S. Chin, H.J. Lee, J.H. Lee, Model predictive control technique combined with iterative learning control for batch processes, AIChE J. 45 (1999) 2175–2187.
[7] C.J. Chien, J.S. Liu, P2 type iterative learning controller for robust output tracking of nonlinear time varying systems, Internat. J. Control 64 (2) (1996) 319–334.
[8] Z. Xiong, J. Zhang, A batch-to-batch iterative optimal control strategy based on recurrent neural network models, J. Process Control 15 (2005) 11–21.
[9] K.S. Lee, J.H. Lee, Convergence of constrained model based predictive control for batch processes, IEEE Trans. Automat. Control 45 (10) (2000) 1928–1932.
[10] J.H. Lee, K.S. Lee, W.C. Kim, Model based iterative learning control with a quadratic criterion for time varying linear systems, Automatica 36 (5) (2000) 641–657.
[11] E. Rogers, D.H. Owens, Stability Analysis for Linear Repetitive Processes, Springer-Verlag, Berlin, Heidelberg, 1992.
[12] X.D. Li, T.W.S. Chow, J.K.L. Ho, 2-D system theory based iterative learning control for linear continuous systems with time delays, IEEE Trans. Circuits Syst. 52 (7) (2005) 1421–1430.
[13] I. Chin, S.J. Qin, K.S. Lee, M. Cho, A two-stage iterative learning control technique combined with real-time feedback for independent disturbance rejection, Automatica 40 (2004) 1913–1922.
[14] T. Liu, Y. Wang, A synthetic approach for robust constrained iterative learning control of piecewise affine batch processes, Automatica 48 (2012) 2762–2775.
[15] T. Liu, C. Shao, X.Z. Wang, Robust PI based set-point learning control for batch processes subject to time-varying uncertainties and load disturbance, in: IFAC Proceedings Volumes, 2014, pp. 1272–1277.
[16] C. Chen, Z.H. Xiong, Y. Zhong, Design and analysis of integrated predictive iterative learning control for batch process based on two-dimensional system theory, Chin. J. Chem. Eng. 22 (2014) 762–768.
[17] T. Liu, F. Gao, Robust two-dimensional iterative learning control for batch processes with state delay and time-varying uncertainties, Chem. Eng. Sci. 65 (2010) 6134–6144.
[18] Y.Q. Wang, D.H. Zhou, F. Gao, Iterative learning model predictive control for multi-phase batch processes, J. Process Control 18 (2008) 543–557.
[19] C.A. Duran-Villalobos, B. Lennox, Iterative learning modelling and control of batch fermentation processes, in: IFAC Proceedings Volumes, 2013, pp. 511–516.
[20] S. Aumi, B. Corbett, T.C. Pringle, P. Mhaskar, Data-driven model predictive quality control of batch processes, AIChE J. 59 (8) (2013) 2852–2861.
[21] N.Y. Lu, F. Zhao, J.H. Lu, R.Y. Qi, Quality control of batch process using natural gradient based model-free optimization, in: IFAC Proceedings Volumes, 2014, pp. 8335–8340.
[22] L.M. Wang, X. He, D.H. Zhou, Average dwell time-based optimal iterative learning control for multi-phase batch processes, J. Process Control 40 (2016) 1–12.
[23] Q.L. Su, M.W. Hermanto, R.D. Braatz, M.S. Chiu, Just-in-time- learning based extended prediction self-adaptive control for batch processes, J. Process Control 43 (2016) 1–9.
[24] B. Corbett, P. Mhaskar, Subspace identification for data-driven modeling and quality control of batch processes, AIChE J. 62 (5) (2016) 1581–1601.
[25] L. Jia, J.P. Shi, M.S. Chiu, Integrated neuro-fuzzy model and dynamic r-parameter based quadratic criterion-iterative learning control for batch process control technique, Neurocomputing 98 (2012) 24–33.
[26] Z.B. Teng, S.Y. Zhang, A successive quadratic programming algorithm with global and superlinear convergence properties, J. UEST China 30 (1) (2001) 103–106.
[27] L. Jia, D.S. Cheng, Particle swarm optimization algorithm based iterative learning algorithm for batch processes, Control Eng. China 18 (3) (2011) 341–344.
[28] J.H. Mou, S.M. Su, A hybrid genetic algorithm for constrained optimization, Comput. Simul. 26 (8) (2009) 184–187.
[29] W.H. Ray, Advanced Process Control, McGraw-Hil, New York, 1981.
[30] Z.H. Xiong, J. Zhang, Product quality trajectory tracking in batch processes using iterative learning control based on time-varying perturbation models, Am. Chem. Soc. 42 (25) (2003) 6802–6814.