yacoda

Let's do some coding...

6. Fitting

Hands-on CasADi course on optimal control

Joris Gillis

Least-squares

- Simplest form of fitting

$$\underset{x}{\text{minimize}} \quad \frac{1}{2}\|h(x)-\hat{h}\|_2^2$$

Unknown parameters

$\mathbb{R}^n$

Model prediction

$\mathbb{R}^n \rightarrow \mathbb{R}^m$

Measurements

$\mathbb{R}^m$

- Write as:

$$\underset{x}{\text{minimize}} \quad \frac{1}{2}\|F(x)\|_2^2 \qquad F: \mathbb{R}^n \rightarrow \mathbb{R}^m$$

Least-squares

- Simplest form of fitting

$$\underset{x}{\text{minimize}} \quad \frac{1}{2}\|h(x)-\hat{h}\|_2^2$$

Unknown parameters $\mathbb{R}^n$

Model prediction $\mathbb{R}^n \to \mathbb{R}^m$

Measurements $\mathbb{R}^m$

- Write as:

$$\underset{x}{\text{minimize}} \quad \frac{1}{2} F^T F \qquad\qquad F: \mathbb{R}^n \to \mathbb{R}^m$$

Rootfinding: $\Delta x = -\left(\nabla^2 f\right)^{-1} \nabla f$

$$\nabla f = J^T F \qquad\qquad \nabla^2 f = \sum_{i=1}^{m} \frac{\partial^2 F_i}{\partial x^2}^T F_i + J^T J$$

$\mathbb{R}^n$ $\qquad\qquad$ $\mathbb{R}^{n \times n}$

$$\nabla^2 f \qquad \Delta x = -\left(\nabla^2 f\right)^{-1} \nabla f$$

Least-squares

- Exact Hessian $$\sum_{i=1}^{m} \frac{\partial^2 F_i}{\partial x^2}^T F_i + J^T J$$ (CasADi)
- Gauss-Newton Hessian $$J^T J$$
- Steepest-Descent $$\alpha I$$
- Levenberg $$J^T J + \alpha I$$
- Levenberg-Marquardt $$J^T J + \alpha\, diag\left(J^T J\right)$$

$\nabla^2 f$ $\Delta x = -\left(\nabla^2 f\right)^{-1} \nabla f$

Least-squares

- Exact Hessian

$$\sum_{i=1}^{m} \frac{\partial^2 F_i}{\partial x^2}^T F_i + J^T J$$

(CasADi)

$$H + \beta I$$

Note: solver requires $\nabla^2 f \succcurlyeq 0$

E.g. choose β to be $-min(eig(H))$

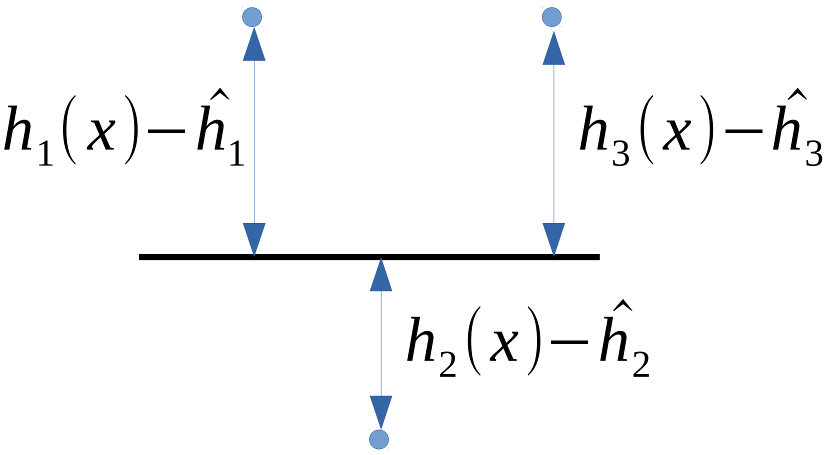
Alternative: eigen-decomposition of H

Linear least-squares

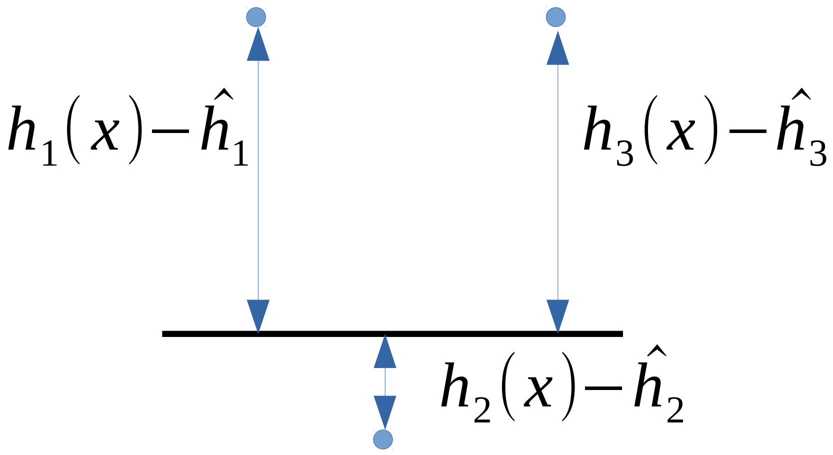
- Exact Hessian $J^T J$ (CasADi)

Satisfied: $\nabla^2 f \succcurlyeq 0$

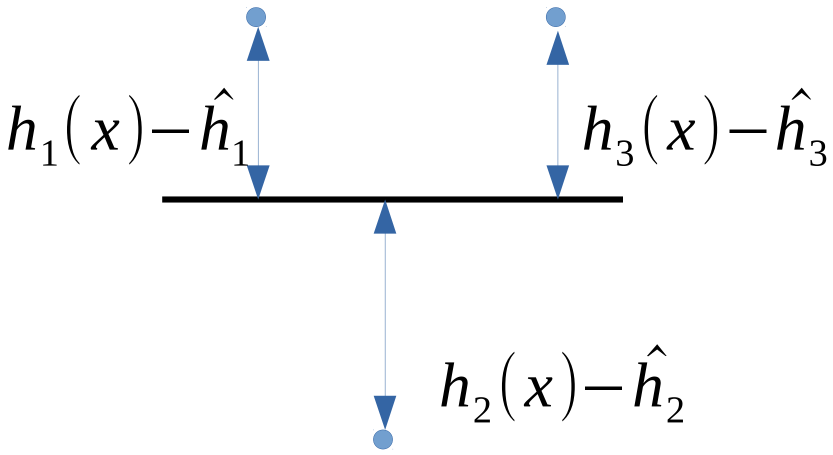
L2 versus L1



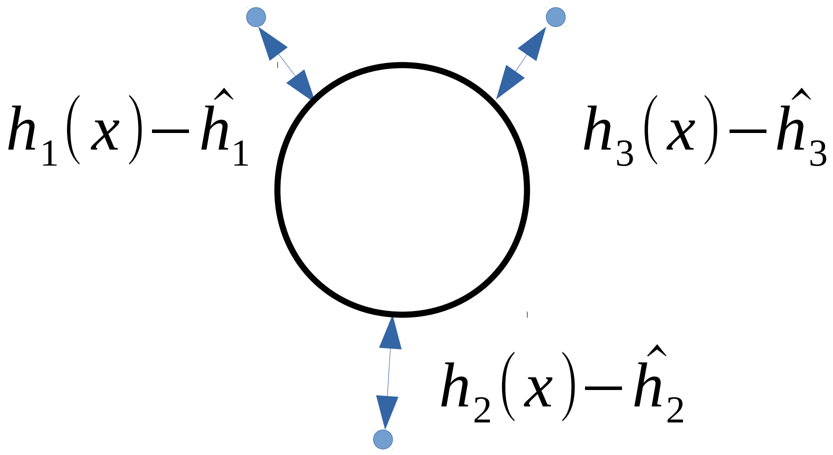
L2 versus L1



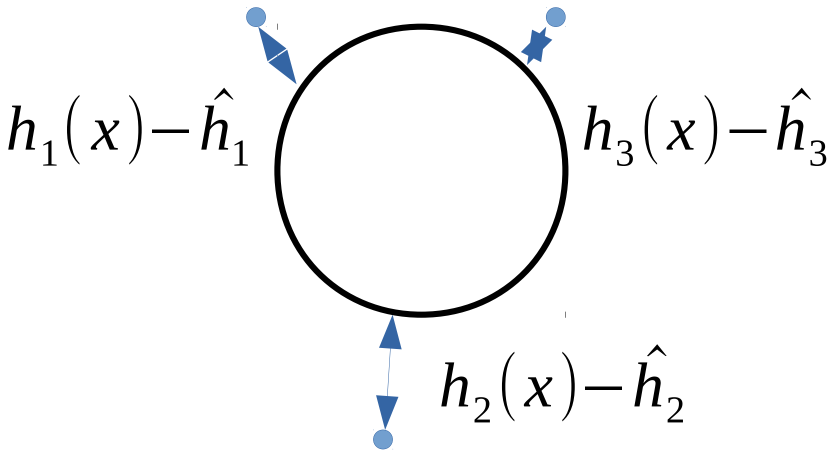
L2 versus L1



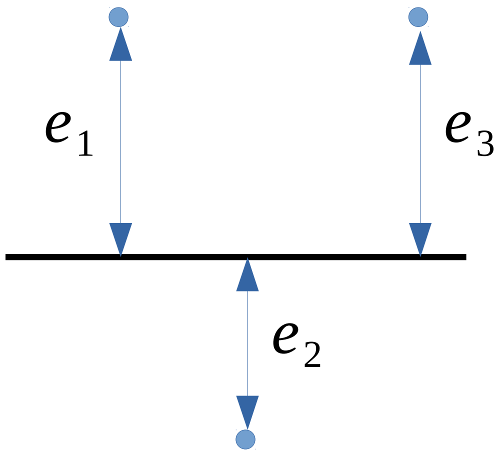
L2 versus L1



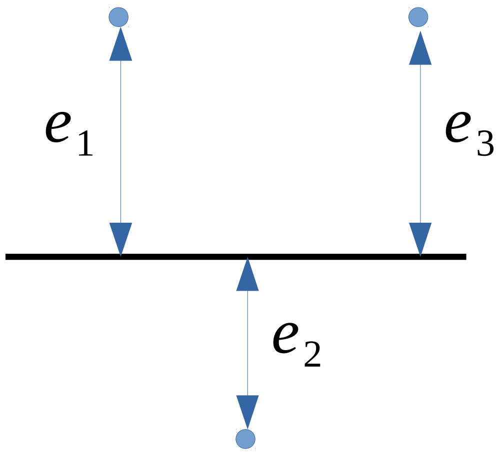
L2 versus L1



L2 versus L1



L2 versus L1



$$\underset{x}{\text{minimize}} \quad \|e\|_2$$

$$\sum_i e_i^2$$

$$\underset{x}{\text{minimize}} \quad \|e\|_1$$

$$\sum_i |e_i|$$

Big deviations are...

...penalized extra hard

...penalized proportionally

L2 versus L1

$$\underset{x}{\text{minimize}} \sum_i |e_i|$$

$$\Big\downarrow \text{ Smooth reformulation}$$

$$\underset{x,L}{\text{minimize}} \sum_i L_i$$

$$s.t. \quad -L_i \leq e_i \leq L_i$$

yacoda

Let's do some coding...

6. Fitting