Gain Scheduled LPV H_∞ Control Based on LMI Approach for a Robotic Manipulator

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A new approach to the design of a gain scheduled linear parameter-varying (LPV) H_∞ controller, which places the closed-loop poles in the region that satisfies the specified dynamic response, for an n-joint rigid robotic manipulator, is presented. The nonlinear time-varying robotic manipulator is modeled to be a LPV system with a convex polytopic structure with the use of the LPV convex decomposition technique in a filter introduced. State feedback controllers, which satisfy the H_∞ performance and the closed-loop pole-placement requirements, for each vertex of the convex polyhedron parameter space, are designed with the use of the linear matrix inequality (LMI) approach. Based on these designed feedback controllers for each vertex, a LPV controller with a smaller on-line computation load and a convex polytopic structure is synthesized. Simulation and experiment results verify that the robotic manipulator with the LPV controller always has a good dynamic performance along with the variations of the joint positions.

1. INTRODUCTION

Gain scheduling control is an engineering design approach applied widely to deal with nonlinear time-varying systems. In many practical control applications, for example, in aircraft control and process control,[1,2] it has been proved to be very effective. Its principle is to design the local controller first and then use interpolation to obtain the global controller. Its intrinsic character is to design controllers for parameter-dependent or nonlinear time-varying systems by using linear control design methods.[3] Recently, due to the development of robust control, the gain scheduling control has been widely studied.[4] Es-

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pecially, the gain scheduling control based on the linear parameter-varying (LPV) approach has found its applications in practical engineering designs.[5] Though the dynamics of a robotic manipulator is of a nonlinear nature, a LPV model with the joint positions and their changing rates as its varying parameters can be obtained by some transformation.

Up to now, the gain scheduling control for a robotic LPV system has not been explored in detail theoretically. The heuristic method frequently used is to divide the parameter space (the joint position space) into many small regions, in each of which the robotic manipulator is regarded as a linear time-invariant (LTI) system and LTI controllers are designed for each of the fixed parameters to achieve a synthetical controller with the use of interpolation or other techniques. The main deficiency of this method lies in the nonconsideration of the time-varying nature of the LPV system such that no system robustness and satisfactory performance is guaranteed for all possible trajectories of the parameters. In fact, the aforementioned gain scheduled controller only guarantees system stability when the parameters are in a slow variation.[6] In order to overcome this deficiency, the measured values of the parameters are introduced in the controller design in this paper so that the designed controllers are changing together with the variation of the robotic dynamics so as to achieve system stability and high performance for all parameter trajectories. Usually, there are two methods in parameter controller synthesis. One is based on the small gain theory and is good for a LPV plant, which relies on parameters in a linear fractional transformation (LFT) form.[4,7] One of the shortcomings of the LFT description is that the parameters are allowed to be complex numbers. Thus, when the known parameters are real numbers, conservatism is introduced. An improvement for this method is to utilize the concept of quadratic H_∞ performance, which is to find a Lyapunov function to guarantee all H_∞ performance for all possible trajectories of the LPV plant. In this method, parameters are regarded as real numbers. They come into the state space matrix of the LPV plant by an affine form. In fact, the essence of this method is the ability of the quadratic H_∞ performance to deal with real parameters. Thus, the other method in parameter controller synthesis is the LPV synthesis with quadratic H_∞ performance for those plants, whose state space matrix affinely relies on time-varying parameters. With certain assumptions, such kind of synthesis problems can be simplified to finding the solution of a group of linear matrix inequalities (LMIs).

There is dynamic uncertainty and external disturbance in robotic motion, for example, the coupling among joints, friction, the noise of sensors and actuators, and so on. Their existence certainly debases the performance and influences the stability of the robotic manipulator. Therefore, our object is to design a controller with disturbance attenuation, robust stability, and closed-loop response satisfying some requirements. These objects all can be expressed by the H_∞ performance and settled with H_∞-synthesis techniques.

A new approach to the design of a gain scheduled LPV H_∞ controller, which combines the gain scheduling theory with the H_∞ theory and uses the LPV synthesis, for an n-joint rigid robotic manipulator, is presented in this paper. This approach first makes sure of the range of the manipulator joints' motion, including those of the joint positions and joint velocities. The robotic manipulator is then modeled to be a LPV system with a convex polytopic structure with the use of the LPV convex decomposition technique in an introduced filter. State feedback controllers, which satisfy the H_∞ performance and the closed-loop pole-placement requirements, for each vertex of the convex polyhedron parameter space, are designed with the use of the LMI approach. Based on these designed feedback controllers for each vertex, a LPV controller with a smaller on-line computation load and a convex polytopic structure is synthesized.

Along with the application of the LMI approach in control theory, the H_∞ control problem and the pole-placement problem can be formulated as a convex optimization problem involving LMIs.[8] Moreover, these LMIs can be solved by efficient interior-point optimization algorithms. Therefore, the multiobject controller, which not only satisfies the H_∞ performance, but also has a good dynamic performance with pole placement, can be designed by solving a set of LMIs.[7] In this paper, the LMIs, which satisfy the H_∞ performance and the pole-placement requirements, are solved by the MATLAB LMI Control Toolbox.[9]

2. THE MODEL OF THE ROBOTIC MANIPULATOR AND ITS LPV EXPRESSION

The dynamic equation of n-joint rigid robotic manipulators is as follows:

$$M(q)\ddot{q}+C(q,\dot{q})\dot{q}+g(q)=\tau, \tag{1}$$

where $q\in R^n$ is the joint position vector, $M(q)$

$\in R^{n\times n}$ is the inertial matrix, $C(q,\dot{q})\dot{q}\in R^n$ is the centrifugal and the Coriolis term, $g(q)\in R^n$ is the gravity term, and $\tau\in R^n$ is the control torque.

Let $x_1=q$, $x_2=\dot{q}$, and $\hat{x} = (x_1\ x_2)^T$. Equation (1) can be expressed in the state space as follows:

$$\dot{\hat{x}}=F(\hat{x})+G(\hat{x})\tau, \tag{2}$$

where

$$F(\hat{x})=\begin{bmatrix} [0_{n\times n} \quad I_{n\times n}]\hat{x} \\ -M^{-1}(q)[C(\hat{x})+g(q)] \end{bmatrix}, \quad G(\hat{x})=\begin{bmatrix} 0_{n\times 1} \\ M^{-1}(q) \end{bmatrix}.$$

In real motion, the joint angular velocity $\dot{q}$ is always bounded. Equation (2) can always be approximately expressed as

$$\dot{\hat{x}}=A(q)\hat{x}+B(q)\hat{u}, \tag{3}$$

where $A(q)$ and $B(q)$ are the functions of the joint position vector of the manipulator q, and $\hat{u}=\tau$. Equation (3) is the LPV expression of the robotic dynamics. It can be seen that the coefficient matrices are the functions of the joint positions.

This paper presents the design of a state feedback LPV controller, whose gains vary along with the variations of the joint positions. This controller satisfies the specified H_∞ performance and guarantees that the closed-loop poles are placed in the required region.

3. THE DESIGN OF THE LPV CONTROLLER

3.1. Related Definitions and Theorems

Definition 1. *(the LMI region):*[7] *Suppose D is a subset of a complex plane. If there exists a symmetric matrix $\alpha=[\alpha_{kl}]\in R^{m\times m}$ and a matrix $\beta=[\beta_{kl}]\in R^{m\times m}$ such that $D=\{z\in C: f_D(z)<0\}$, where $f_D(z):=\alpha+z\beta+\bar{z}\beta^T=[\alpha_{kl}+\beta_{kl}z+\beta_{lk}\bar{z}]_{1\leq k,l\leq m}$ is the characteristic function of D and takes value in the $m\times m$ Hermitian matrix space, then D is called a LMI region. In the above definition, $M=[\mu_{kl}]_{1\leq k,l\leq m}$ expresses that M is a $m\times m$ matrix with a general term μ_{kl}.*

Definition 2. *(quadratic D stability):*[7] *Suppose that for the LPV $\dot{x}=A(p)x$ with respect to p, when p is a fixed value, its pole location in the LMI region D can be described in the following:*

$$M_D[A(p),X]=[\alpha_{kl}X+\beta_{kl}A(p)X + \beta_{lk}XA(p)^T]_{1\leq k,l\leq m},$$

where X is a positive definite matrix, and $M_D[A(p),X]$ and $f_D(z)$ can be related by the following substitution, $[X,A(p)X,XA(p)^T]\leftrightarrow(1,z,\bar{z})$. Then, the matrix $A(p)$ is quadratic D stable if and only if there exists a symmetric positive definite matrix X such that $M_D[A(p),X]<0$ for all admissible values of the parameter p.

Definition 3. *(quadratic H_∞ performance):*[2] *The linear parameter-varying system with respect to p,*

$$\dot{x}=A(p)x+B(p)u, \tag{4}$$

$$y=C(p)x+D(p)u, \tag{5}$$

has quadratic H_∞ performance γ if and only if there exists a positive definite matrix $X>0$ such that

$$B^o_{[A(p),B(p),C(p),D(p)]}(X,\gamma) := \begin{bmatrix} A^T(p)X+XA(p) & XB(p) & C^T(p) \\ B^T(p)X & -\gamma I & D^T(p) \\ C(p) & D(p) & -\gamma I \end{bmatrix}<0$$

for all admissible values of the parameter p.

Theorem 1. *(vertex property): For the polytopic linear parameter-varying plant (4) and (5), where*

$$\begin{bmatrix} A(p) & B(p) \\ C(p) & D(p) \end{bmatrix}\in P:=Co\left\{\begin{bmatrix} A_i & B_i \\ C_i & D_i \end{bmatrix}, \quad i=1,\ldots,r\right\},$$

the following three items are equivalent:

(**1**) *The system is quadratic D-stable with quadratic H_∞ performance γ.*

(**2**) *There exists a positive definite matrix $X>0$ such that, for all $\left[\begin{smallmatrix} A(p) & B(p) \\ C(p) & D(p) \end{smallmatrix}\right]\in P$,*

$$M_D(A(p),X)<0,$$

$$B^o_{[A(p),B(p),C(p),D(p)]}(X,\gamma)<0.$$

(3) *There exists a positive definite matrix $X>0$, which satisfies the following LMIs:*

$$M_{\mathrm{D}}(A_i, X)<0,$$

$$B^{o}_{[A_i, B_i, C_i, D_i]}(X,\gamma)<0, \quad i=1,2,\ldots,r.$$

Proof. According to Definitions 2 and 3, it is evident that (1) and (2) are equivalent. Note that $\left[\begin{smallmatrix} A(p) & B(p) \\ C(p) & D(p)\end{smallmatrix}\right] = \Sigma_{i=1}^{r}\alpha_i\left[\begin{smallmatrix} A_i & B_i \\ C_i & D_i\end{smallmatrix}\right]$, where $\alpha_i \geq 0$, $\Sigma_{i=1}^{r}\alpha_i=1$. The polytopic feature of the above linear parameter-varying system leads to the equivalence of (2) and (3) directly. ■

3.2. The Controller Design

Suppose that the range of the manipulator joints' positions are $q_{i\,\min} \leq q_i \leq q_{i\,\max}$, $i=1,2,\ldots,n$ and therefore the motion range has $N=2^n$ vertices, i.e., $\{(q_{1\,\max}) OR (q_{1\,\min}), (q_{2\,\max}) OR (q_{2\,\min}), \ldots, (q_{n\,\max}) OR (q_{n\,\min})\}$, denoted as ϑ_{mi} $(i=1,2,\ldots,N)$. Thus, the N vertices form a convex polyhedron. With the use of the convex decomposition technique,[5] the LPV expression of the robotic dynamics (3) can be expressed by the N vertices in the convex polyhedron:

$$\dot{\hat{x}}=\left[\sum_{i=1}^{N}\rho_i(q)A(\vartheta_{mi})\right]\hat{x}+\left[\sum_{i=1}^{N}\rho_i(q)B(\vartheta_{mi})\right]u, \quad (6)$$

where

$$\sum_{i=1}^{N}\rho_i(q)=1, \quad \rho_i(q)>0.$$

Reference 2 states that it is difficult to design a gain scheduled controller with the use of the polytopic technique for the LPV system with a structure as in Eq. (6). A first-order filter is then introduced in this paper to solve the problem. Thus, a new control input u is defined and the state equation of the filter is

$$\dot{x}_u=A_u x_u+B_u u,$$

$$\hat{u}=C_u x_u, \quad (7)$$

where the coefficient matrices $A_u \in R^{n\times n}$, $B_u \in R^{n\times n}$, and $C_u \in R^{n\times n}$ are the design parameters of the filter. In the filter design, the bandwidth of the filter is managed to be broader than the expected bandwidth of the system. We define $x=[\hat{x}\ x_u]^T$. By combining Eqs. (6) and (7), the augmented system can be expressed as

$$\dot{x}=\bar{A}x+\bar{B}u, \quad (8)$$

where
$$\tilde{A}=\begin{bmatrix}\sum_{i=1}^{N}\rho_i(q)A(\vartheta_{mi}) & \sum_{i=1}^{N}\rho_i(q)B(\vartheta_{mi})C_u \\ 0 & A_u\end{bmatrix} =\sum_{i=1}^{N}\rho_i(q)A_i, \quad \tilde{B}=\begin{bmatrix}0\\ B_u\end{bmatrix},$$

$$A_i=\begin{bmatrix}A(\vartheta_{mi}) & B(\vartheta_{mi})C_u \\ 0 & A_u\end{bmatrix}.$$

It is seen that after the filter is introduced, the $\tilde{B}$ matrix in the augmented plant (8) becomes a constant matrix. Since there are model errors in modeling the robotic manipulator (e.g., the unmodeled part in the high-frequency range), transformation errors in the LPV transforming of the robotic model, and dynamic uncertainties and external disturbances (e.g., the coupling among joints, frictions, noise in sensors, and executors, etc.), a disturbance term $w(t) \in R^{n\times 1}$ is set in this paper to represent an equivalent disturbance for all the aforementioned factors. The performance index $Z(t) \in R^{k\times 1}$ represents the disturbance repression performance for disturbance $w(t)$. With the consideration of the disturbance and the performance index, Eq. (8) can be expanded to be a polytopic linear parameter-varying system:

$$\dot{x}(t)=\left[\sum_{i=1}^{N}\rho_i(q)A_i\right]x(t)+B_1 w(t)+B_2 u(t), \quad (9)$$

$$Z(t)=C_1 x(t)+D_{11}w(t)+D_{12}u(t), \quad (10)$$

where

$$B_2=\tilde{B}, \quad B_1=\begin{bmatrix}0_{n\times n}\\ I_n\\ 0_{n\times n}\end{bmatrix} \text{ in Eq. (9).}$$

The coefficient matrixes C_1, D_{11}, and D_{12} of the performance index $Z(t)$ in Eq. (10) satisfy $C_1^T C_1 = P$, $D_{11} = 0_{k\times n}$, and $D_{12}^T D_{12} = Q$. Since it is guaranteed that the closed-loop system possesses H_∞ performance γ for any disturbance $w \in L_2(0,+\infty)$, i.e., $1/\gamma \int_0^\infty (x^T P x + u^T Q u)dt < \gamma \int_0^\infty w^T w dt$, where $P>0$ and $Q>0$ are weighted matrices.

For the LPV system expressed by Eqs. (9) and (10), we only need to design the state feedback gain K $(i=1,2,\ldots,N)$ with respect to each vertex ϑ_{mi} $(i=1,2,\ldots,N)$, respectively. At any position q in the polytopic system, a LPV controller with a convex polytopic structure can be synthesized with the use of the K_is:

$$K(q) = \sum_{i=1}^{N} \rho_i(q) K_i. \tag{11}$$

The following theorem proves the feasibility of this design.

Theorem 2. *In the design of the state feedback controller (11) for a system expressed by Eqs. (9) and (10), if there exists a positive definite matrix $X_{cl}>0$ such that K_i satisfies $M_D(A_i+K_i, X_{cl})<0$, $B^o_{[A_i+K_i, B_1, C_1+D_{12}K_i, D_{11}]}(X_{cl},\gamma)<0$, $i=1,2,\ldots,N$, the designed polytopic controller (11) guarantees that the closed-loop system is quadratic D-stable with a quadratic H_∞ performance γ between $w(t)$ and $z(t)$ for all admissible values of the varying parameter q.*

Proof. Let the controller (11) be substituted into the system (9) and (10). The closed-loop system is

$$\dot{x}_{cl} = \left\{ \sum_{i=1}^{N} \rho_i(q)(A_i + B_2 K_i) \right\} x_{cl} + B_1 w,$$

$$z = \left\{ \sum_{i=1}^{N} \rho_i(q)(C_1 + D_{12} K_i) \right\} x_{cl} + D_{11} w. \tag{12}$$

It is seen that the closed system has a polytopic structure. According to Theorem 1, for a linear parameter-varying system (12), as long as all the vertices satisfy $M_D(A_i+K_i, X_{cl})<0$, $B^o_{[A_i+K_i, B_1, C_1+D_{12}K_i, D_{11}]}(X_{cl},\gamma)<0$, $i=1,2,\ldots,N$, the polytopic controller (11) guarantees that the closed-loop system is quadratic D-stable with a quadratic H_∞ performance γ between $w(t)$ and $z(t)$ for all admissible values of the varying parameter q. ∎

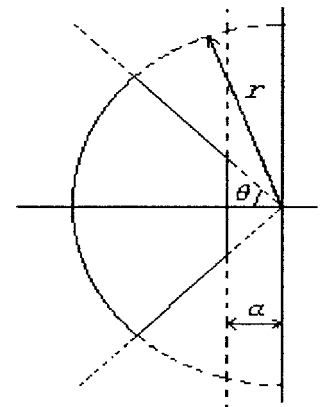

Figure 1. The system pole-placement region $S(\alpha,r,\theta)$.

Theorem 2 shows that the key to the design of a polytopic controller is to find a positive definite matrix $X_{cl}>0$ and the vertex controller K_is. The LMI technique is used in this paper to find them. For the polytopic parameter-varying system (9) and (10), the LTI systems at the vertices are

$$\left.\begin{aligned} \dot{x} &= A_i x + B_1 w + B_2 u, \\ z &= C_1 x + D_{11} w + D_{12} u \end{aligned}\right\} \quad i=1,2,\ldots,N. \tag{13}$$

$T_{wzi}(s)$ denotes the closed-loop transfer function between $w(t)$ and $Z(t)$, and its closed-loop realization is supposed to be $(A_{cli}, B_{cli}, C_{cli}, D_{cli})$.

To obtain satisfied dynamic performance, the closed-loop poles are required to be placed in the region $S(\alpha,r,\theta)$ in Figure 1.

Confining the closed-loop poles in this region ensures a minimum decay rate α, a minimum damping ratio $\zeta = \cos\theta$, and a maximum undamped natural frequency $\omega_d = r\sin\theta$. These, in turn, bound the maximum overshoot, the frequency of oscillation, the decay time, the rise time, and the settling time. As is seen from Definition 1, region $S(\alpha,r,\theta)$ is a LMI region.

Theorem 3. *Given $\gamma>0$, there exists a positive definite symmetric matrix X and $L_i = K_i X$, which satisfy the following LMIs:*

$$A_i X + X A_i^T + B_2 L_i + L_i^T B_2^T < (-2\alpha) X, \tag{14}$$

$$\begin{bmatrix} -rX & A_i X + B_2 L_i \\ X A_i^T + L_i^T B_2^T & -rX \end{bmatrix} < 0, \tag{15}$$

$$\begin{bmatrix} I_{11i} & I_{12i} \\ I_{21i} & I_{22i} \end{bmatrix} < 0, \tag{16}$$

$$\begin{bmatrix} A_i X + X A_i^T + B_2 L_i + L_i^T B_2^T & B_1 & X C_1^T + L_i^T D_{12}^T \\ B_1^T & -\gamma & D_{11}^T \\ C_1 X + D_{12} L_i & D_{11} & -\gamma \end{bmatrix} < 0, \tag{17}$$

where

$$I_{11i} = \sin\theta \cdot A_i X + X(\sin\theta \cdot A_i)^T + (\sin\theta \cdot B_2) L_i + L_i^T (\sin\theta \cdot B_2)^T,$$

$$I_{21i} = \cos\theta \cdot A_i X - \cos\theta \cdot X A_i^T + \cos\theta \cdot B_2 L_i - \cos\theta \cdot L_i^T B_2^T,$$

$$I_{12i} = I_{21i}^T,$$

$$I_{22i} = I_{11i},$$

$$i = 1,2,\ldots,N.$$

Suppose (X^*, L^*) *is one feasible solution of the above LMIs. Then the state* X^* *and the feedback gain* $K_i^* = L_i^*(X^*)^{-1}$ *are the positive definite matrix* $X_{cl} > 0$ *and the vertex controller, respectively, which satisfy Theorem 2.*

Proof. For the pole-placement region $S(\alpha, r, \theta)$ as shown in Figure 1, according to the relationship between $M_D(A, X)$ and $f_D(z)$, the following LMIs, which satisfy the pole-placement requirements, can be obtained from Definition 2 and Theorem 1: there exists $X_D > 0$ such that

$$A_{cli} X_D + X_D A_{cli}^T + 2\alpha X_D < 0, \tag{18}$$

$$\begin{bmatrix} -r X_D & A_{cli} X_D \\ X_D A_{cli}^T & -r X_D \end{bmatrix} < 0, \tag{19}$$

$$\begin{bmatrix} \sin\theta(A_{cli} X_D + X_D A_{cli}^T), & \cos\theta(A_{cli} X_D - X_D A_{cli}^T), \\ \cos\theta(X_D A_{cli}^T - A_{cli} X_D), & \sin\theta(A_{cli} X_D + X_D A_{cli}^T) \end{bmatrix} < 0. \tag{20}$$

Also, from Definition 3 and Theorem 1, the LMI, which guarantees the H_∞ performance to be $\|T_{wzi}\|_\infty < \gamma$, is: there exists $X_\infty > 0$ such that

$$\begin{bmatrix} A_{cli} X_\infty + X_\infty A_{cli}^T & B_{cli} & X_\infty C_{cli}^T \\ B_{cli}^T & -\gamma I & D_{cli}^T \\ C_{cli} X_\infty & D_{cli} & -\gamma I \end{bmatrix} < 0. \tag{21}$$

From Eq. (13), $A_{cli} = A_i + B_2 K_i$, $B_{cli} = B_1$, $C_{cli} = C_1 + D_{12} K_i$, and $D_{cli} = D_{11}$. Suppose $X = X_D = X_\infty > 0$ and $L_i = K_i X$. With the substitution of the above into Eqs. (18)–(21), it is easy to obtain Eqs. (14)–(17). ■

The above inequality constraints can be easily solved with some LMI optimization software, such as MATLAB LMI Control Toolbox.[9] It is worth noticing that all the feedback gains for the vertexes are obtained off line. Real-time calculation is only for Eq. (11). Therefore, in practical control, the controller designed in this paper has a small on-line computation load and thus is easy to be realized. See Figure 2 for the overall control scheme.

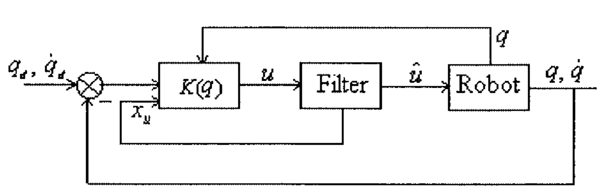

Figure 2. The block diagram of a closed-loop control.

4. SIMULATION STUDIES

Simulation studies are carried out on a self-designed direct drive (DD) two-joint planar robotic manipulator. Its dynamics equation can be expressed as[10]

$$\begin{bmatrix} a & b\cos(q_2 - q_1) \\ b\cos(q_2 - q_1) & c \end{bmatrix} \cdot \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} -b\dot{q}_2^2 \sin(q_2 - q_1) \\ b\dot{q}_1^2 \sin(q_2 - q_1) \end{bmatrix} = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}, \tag{22}$$

where

$a = 5.6794\ \text{kg m}^2, \quad b = 1.4730\ \text{kg m}^2,$ and

$c = 1.7985\ \text{kg m}^2.$

In real robotic motion, the joint angular velocities $\dot{q}_1$, $\dot{q}_2$ are always bounded. Suppose $|\dot{q}_1| < \nu_1$ and $|\dot{q}_2| < \nu_2$. Let the state vector $\hat{x} = \lfloor q_1\ q_2\ \dot{q}_1\ \dot{q}_2 \rfloor^T$ and the control vector $\hat{u} = \lfloor \tau_1\ \tau_2 \rfloor^T$ and, thus, Eq. (22) can be approximately expressed in a LPV form as shown in Eq. (3):

$$\dot{\hat{x}} = A(q)\hat{x} + B(q)\hat{u}, \tag{23}$$

where

$$A(q) = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -m_{12} b \nu_1 \sin(q_2 - q_1) & m_{11} b \nu_2 \sin(q_2 - q_1) \\ 0 & 0 & -m_{22} b \nu_1 \sin(q_2 - q_1) & m_{12} b \nu_2 \sin(q_2 - q_1) \end{bmatrix},$$

$$B(q) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix},$$

$$m_{11} = c / [ac - b^2 \cos^2 (q_2 - q_1)],$$

$$m_{12} = m_{21} = -b \cos (q_2 - q_1) / [ac - b^2 \cos^2 (q_2 - q_1)],$$

$$m_{22} = a / [ac - b^2 \cos^2 (q_2 - q_1)].$$

Suppose the motion range of the DD two-joint planar robotic manipulator is $q_{1\,\min} \leq q_1 \leq q_{1\,\max}$ and $q_{2\,\min} \leq q_2 \leq q_{2\,\max}$. Thus, the four vertices $\vartheta_{m1} = (q_{1\,\min}, q_{2\,\min})$, $\vartheta_{m2} = (q_{1\,\max}, q_{2\,\min})$, $\vartheta_{m3} = (q_{1\,\max}, q_{2\,\max})$, and $\vartheta_{m4} = (q_{1\,\min}, q_{2\,\max})$ from this motion range form a convex tetrahedron. Due to Eq. (6), the LPV robotic expression (23) can be convex decomposed into Eq. (24) by using the four vertices in the convex tetrahedron:

$$\dot{\hat{x}} = \left[\sum_{i=1}^{4} \rho_i(q) A(\vartheta_{mi}) \right] \hat{x} + \left[\sum_{i=1}^{4} \rho_i(q) B(\vartheta_{mi}) \right] \hat{u}, \tag{24}$$

where

$$\rho_1(q) = (q_{2\,\max} - q_2)(q_{1\,\max} - q_1) / (q_{2\,\max} - q_{2\,\min}) \times (q_{1\,\max} - q_{1\,\min}),$$

$$\rho_2(q) = (q_{2\,\max} - q_2)(q_1 - q_{1\,\min}) / (q_{2\,\max} - q_{2\,\min}) \times (q_{1\,\max} - q_{1\,\min}),$$

$$\rho_3(q) = (q_2 - q_{2\,\min})(q_1 - q_{1\,\min}) / (q_{2\,\max} - q_{2\,\min}) \times (q_{1\,\max} - q_{1\,\min}),$$

$$\rho_4(q) = (q_2 - q_{2\,\min})(q_{1\,\max} - q_1) / (q_{2\,\max} - q_{2\,\min}) \times (q_{1\,\max} - q_{1\,\min}),$$

$$\sum_{i=1}^{4} \rho_i(q) = 1, \quad \rho_i(q) > 0.$$

By choosing a filter with $A_u = \text{diag}\{-h, -h\}$, $B_u = \text{diag}\{d_1, d_2\}$, and $C_u = \text{diag}\{l, l\}$, the coefficient matrices $\tilde{A}$ and $\tilde{B}$ in the augmented system (8) are obtained. In real simulations, the parameters of the filter are chosen as $h = 20$, $d_1 = 1$, and $d_2 = 1$, and the performance weighting matrices P and Q are chosen as $P = \text{diag}\{2.25, 9, 1, 16\}$ and $Q = \text{diag}\{1.44, 1.44\}$. Then the coefficient matrices in the performance index (10) are

$$C_1 = \begin{bmatrix} 1.5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$D_{11} = 0_{6\times 2}, \quad D_{12} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1.2 & 0 \\ 0 & 1.2 \end{bmatrix}.$$

In our simulation, let $q_{1\,\min} = 0$, $q_{1\,\max} = \pi$, $q_{2\,\min} = 0$, and $q_{2\,\max} = \pi$. The system closed-loop pole placement is required to be in the region of $\alpha = 1$, $r = 3$, and $\theta = \pi/4$. By using the MATLAB LMI Control Toolbox,

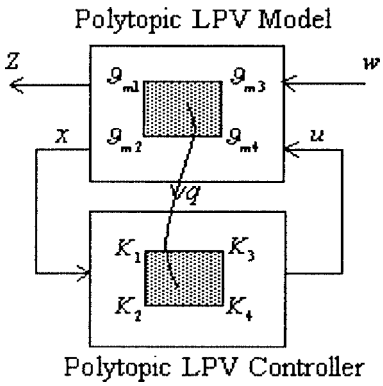

Figure 3. The polytopic LPV controller design principle.

optimized $\gamma = 0.0109$ can be obtained from Theorem 3. The state feedback gain matrices for the four vertices are, respectively,

$$K_1 = \begin{bmatrix} 25.2205 & 6.5411 & 20.4850 & 5.3130 & 9.1816 & -9.0301 \\ 6.5411 & 7.9866 & 5.3130 & 6.4870 & -3.6809 & 11.5475 \end{bmatrix},$$

$$K_2 = \begin{bmatrix} 25.2205 & -6.5411 & 20.4850 & -5.3130 & 9.1816 & 9.0301 \\ -6.5411 & 7.9866 & -5.3130 & 6.4870 & 3.6809 & 11.5475 \end{bmatrix},$$

$$K_3 = \begin{bmatrix} 25.2205 & 6.5411 & 20.4850 & 5.3130 & 9.1816 & -9.0301 \\ 6.5411 & 7.9866 & 5.3130 & 6.4870 & -3.6809 & 11.5475 \end{bmatrix},$$

$$K_4 = \begin{bmatrix} 25.2205 & -6.5411 & 20.4850 & -5.3130 & 9.1816 & 9.0301 \\ -6.5411 & 7.9866 & -5.3130 & 6.4870 & 3.6809 & 11.5475 \end{bmatrix}.$$

The closed-loop poles corresponding to the four vertexes are all

$$\{-1.8035 \pm 1.0980i, -1.8035 \pm 1.0980i,$$
$$-2.0736 \pm 1.2977i\}.$$

It is seen that the designed controller not only satisfies the pole-placement requirements, but also has a good H_∞ performance due to a small optimized γ. Thus, the controller satisfies the design requirements. After the state feedback gain matrices for the four vertexes are obtained, the LPV controller can be obtained from Eq. (11) (see Figure 3 for the controller design principle),

$$K(q) = \rho_1(q)K_1 + \rho_2(q)K_2 + \rho_3(q)K_3 + \rho_4(q)K_4, \tag{25}$$

where $\Sigma_{i=1}^{4}\rho_i(q) = 1$, $\rho_i(q) > 0$.

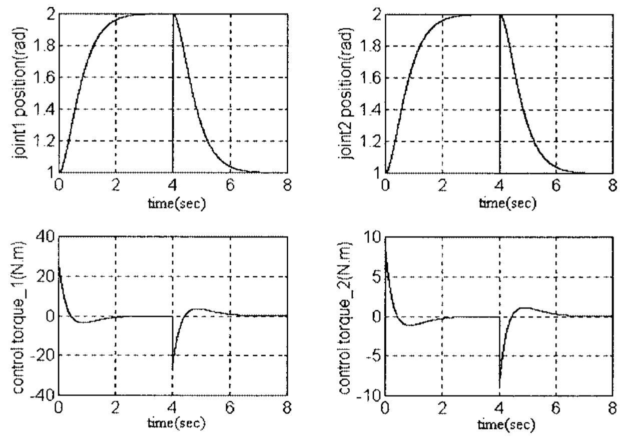

Figure 4. Simulation result.

In our simulation, the reference input is a square wave. The position response curves and the control torque curves resulted from the LPV controller (25) are demonstrated in Figure 4. It is seen that under the functioning of the LPV controller with a convex polytopic structure, the response of the system has no overshoot as well as a small rise time. It also has a strong tracking ability for a fast varying reference input.

5. EXPERIMENTS

The proposed gain scheduled LPV H_∞ controller in Section 4 is applied to our self-designed direct drive (DD) two-joint planar robotic manipulator for physical experiments. The inputs for joint 1 and joint 2 are a square wave with a frequency of 0.125 Hz and a sinusoidal wave, i.e., $\theta_{2d} = 0.5 + 0.5\sin(0.25\pi t)$, respectively. The joint position response and the control torque curves are shown in Figure 5. Also, a comparison between the proposed controller and the single H_∞ LTI controller is made and results are shown in Figure 5.

It is seen from Figure 5(a) that under the functioning of the gain scheduled LPV H_∞ controller, the robotic manipulator has no overshooting, a smooth motion, and a small rise time in its step response. Also, the system has a strong switching ability between different points. And it is evident that the step response features under the functioning of a single H_∞ LTI controller are much worse. This is due to the pole placement requirements proposed in this paper to adjust the transient response of the robotic manipulator. Figure 5(b) demonstrates that though there is a sudden increase of the load on joint 2 as a result of the po-

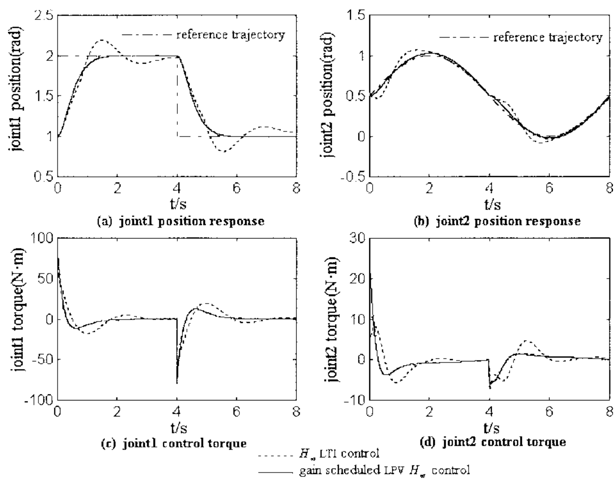

Figure 5. Experiment result. (a) Joint 1 position response, (b) joint 2 position response, (c) joint 1 control torque, and (d) joint 2 control torque.

sition switching of joint 1, the manipulator catches up the reference input right away under the functioning of the gain scheduled LPV H_∞ controller. While under the functioning of the single H_∞ LTI controller, the manipulator deviates from the reference trajectory a lot and it takes a fairly long time for the manipulator to catch up to the reference input. This proves that the proposed controller has good robustness. The spikes in Figure 5(c) correspond to the boundaries of the square-wave input, while those for joint 2 shown in Figure 5(d) are due to the large impulse on joint 2 as the result of the sudden position switching of joint 1.

6. CONCLUSIONS

A new approach to the design of a gain scheduled LPV robust H_∞ controller, which places the closed-loop poles in the region that satisfies the dynamic response, is presented. This approach combines the gain scheduled theory with the H_∞ theory and uses the LPV synthesis technique. The robotic manipulator is modeled to be a LPV system with a convex polytopic structure with the use of the LPV convex decomposition technique in an introduced filter. State feedback controllers, which satisfy the H_∞ performance and the closed-loop pole-placement requirements for each vertex of the convex polyhedron parameter space, are designed with the use of the LMI approach. Based on these designed feedback controllers for each vertex, a LPV controller with a smaller on-line computation load and a convex polytopic structure is synthesized. The designed controller has small on-line computation load and is easy to be realized. Simulation results verify that the robotic manipulator with the LPV controller always has a good dynamic performance along with the variations of the joint positions. A control performance comparison between the proposed controller and the single H_∞ LTI controller is made by experiments. The results demonstrate the advantage of the proposed gain scheduled LPV H_∞ controller.

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