

1. Digital System & Binary Numbers

1.1 Digital System

- Digital System
 - Communication, Business transaction, Traffic control, other commercial, industrial, scientific enterprises etc.
 - Discrete elements of information
- Signal
 - Discrete elements of information are represented in a digital system by physical quantities
- Binary Code
 - Discrete elements of information are represented with groups of bits
 - bit : binary + digit

1.2 Binary Number

$$a_5a_4a_3a_2a_1a_0.a_{-1}a_{-2}a_{-3}$$

$$= a_n r^n + a_{n-1} r^{n-1} + \dots + a_2 r^2 + a_1 r + a_0 + a_{-1} r^{-1} + a_{-2} r^{-2} + \dots + a_{-m} r^{-m}$$

$$7392 = 7 \times 10^3 + 3 \times 10^2 + 9 \times 10^1 + 2 \times 10^0$$

$$(11010.11)_2 = 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 + 1 \times 2_{-1} + 1 \times 2_{-2}$$

$$= (26.75)_{10}$$

$2^{10} = 1\text{Kilo}$
 $2^{20} = 1\text{Mega}$
 $2^{30} = 1\text{Giga}$

Table 1-1
Powers of Two

n	2^n	n	2^n	n	2^n
0	1	8	256	16	65,536
1	2	9	512	17	131,072
2	4	10	1,024	18	262,144
3	8	11	2,048	19	524,288
4	16	12	4,096	20	1,048,576
5	32	13	8,192	21	2,097,152
6	64	14	16,384	22	4,194,304
7	128	15	32,768	23	8,388,608

1.2 Binary Number

Augend	101101	Minuend:	101101	Multiplicand:	1011
Addend	+100111	Subtrahend:	-100111	Multiplier:	*101
sum	<hr/> 1010100	Difference:	<hr/> 000110		<hr/> 1011
					0000
					1011
				Product:	<hr/> 110111

1.3 Number Base Conversions

Ex 1-1) Convert decimal 41 to binary.

	Integer Quotient		Remainder	Coefficient	Integer	Remainder
					41	
$41/2 =$	20	+	$\frac{1}{2}$	$a_0 = 1$	20	1
$20/2 =$	10	+	0	$a_1 = 0$	10	0
$10/2 =$	5	+	0	$a_2 = 0$	5	0
$5/2 =$	2	+	$\frac{1}{2}$	$a_3 = 1$	2	1
$2/2 =$	1	+	0	$a_4 = 0$	1	0
$1/2 =$	0	+	$\frac{1}{2}$	$a_5 = 1$	0	1

Answer
=101001

answer : $(41)_{10} = (a_5 a_4 a_3 a_2 a_1 a_0)_2 = (101001)_2$

1.3 Number Base Conversions

- Ex 1-2) Convert decimal 153 to octal.

$$\begin{array}{r|l} 153 & \\ 19 & 1 \\ 2 & 3 \\ 0 & 2 \end{array} \quad \begin{array}{l} \uparrow \\ \\ \end{array} \quad = (231)_8$$

- Ex 1-3) Convert $(0.6875)_{10}$ to binary.

	Integer		Fraction	Coefficient
$0.6875 \times 2 =$	1	+	0.3750	$a_{-1} = 1$
$0.3750 \times 2 =$	0	+	0.7500	$a_{-2} = 0$
$0.7500 \times 2 =$	1	+	0.5000	$a_{-3} = 1$
$0.5000 \times 2 =$	1	+	0.0000	$a_{-4} = 1$

Answer: $(0.6875)_{10} = (0.a_{-1}a_{-2}a_{-3}a_{-4})_2 = (0.1011)_2$

1.4 Octal and Hexadecimal Numbers

Table 1-2
Numbers with Different Bases

Decimal (base 10)	Binary (base 2)	Octal (base 8)	Hexadecimal (base 16)
00	0000	00	0
01	0001	01	1
02	0010	02	2
03	0011	03	3
04	0100	04	4
05	0101	05	5
06	0110	06	6
07	0111	07	7
08	1000	10	8
09	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

$$\begin{array}{cccccccccc} (& \underline{10} & \underline{110} & \underline{001} & \underline{101} & \underline{011} & . & \underline{111} & \underline{100} & \underline{000} & \underline{110} &)_2 = (26153.7460)_8 \\ & 2 & 6 & 1 & 5 & 3 & & 7 & 4 & 0 & 6 \end{array}$$

$$\begin{array}{cccccc} (& \underline{10} & \underline{1100} & \underline{0110} & \underline{1011} & . & \underline{1111} & \underline{0010} &)_2 = (2C6B.F2)_{16} \\ & 2 & C & 6 & B & & F & 2 \end{array}$$

1.5 Complements – Diminished Radix Complement

- $(r-1)$'s complements of N is $(r^n-1)-N$
- $r=10$, $r-1=9$, 9's complements of N is $(10^n-1)-N$
Ex) the 9's complements of 546700 is $999999-546700 = 453299$
the 9's complements of 012398 is $999999-012398 = 987601$
- For binary number, $r=2$, $r-1=1$
1's complements of N is $(2^n-1)-N$
Ex) the 1's complements of 1011000 is 0100111
the 1's complements of 0101101 is 1010010

1.5 Complements – Radix Complement

- The r 's complements of an n -digit number N is $r^n - N$, $N \neq 0$
 0 , $N = 0$
- $r^n - N = [(r^n - 1) - N] + 1$
→ The r 's complements is obtained by adding 1 to the $(r-1)$'s complements
- Ex) The 10's complements of 012398 is 987602
The 10's complements of 246700 is 753300

The 2's complements of 1101100 is 0010100
The 2's complements of 0110111 is 1001001

1.5 Complements – Subtraction with Complements

- Ex1-5) using 10's complement, subtract 72532-3250.

$$\begin{array}{r} M = \quad \quad 72532 \\ 10\text{'s complement of } N = \quad + 96750 \\ \hline \text{Sum} = \quad \quad 169282 \\ \text{Discard end carry } 10^5 = \quad -100000 \\ \hline \text{Answer} = \quad \quad 69282 \end{array}$$

- Ex1-6) Using 10's complement, subtract 3250-72532.

$$\begin{array}{r} M = \quad \quad 03250 \\ 10\text{'s complement of } N = \quad +27468 \\ \hline \text{Sum} = \quad \quad 30718 \end{array}$$

There is no end carry

Therefore, the answer is $-(10\text{'s complement of } 30718) = -69282$

1.5 Complements – Subtraction with Complements

- Ex1-7) $X=1010100$, $Y=1000011$, (a) $X-Y$, (b) $Y-X$

(a) $X-Y$

$$\begin{array}{r} X = 1010100 \\ 2\text{'s complement of } Y = +0111101 \\ \hline \text{Sum} = 10010001 \\ \text{Discard end carry } 2^7 = -10000000 \\ \hline \text{Answer: } X-Y = 0010001 \end{array}$$

(b) $Y-X$

$$\begin{array}{r} Y = 1000011 \\ 2\text{'s complement of } X = +0101100 \\ \hline \text{Sum} = 1101111 \end{array}$$

There is no carry.

The answer is $Y-X = -(2\text{'s complement of } 1101111) = -0010001$

1.5 Complements – Subtraction with Complements

- Ex1-8) Repeat Example 1-7 using 1's complement.

(a) $X - Y = 1010100 - 1000011$

X =	1010100
1's complement of Y =	+0111100
Sum =	<hr/> 10010000
End-around carry =	+ 1
Answer: X-Y =	<hr/> 0010001

(b) $Y - X = 1000011 - 1010100$

Y =	1000011
1's complement of X =	+0101011
Sum =	<hr/> 1101110

There is no carry.

The answer is $Y - X = -(1's \text{ complement of } 1101110) = -0010001$

1.6 Signed Binary Numbers

- Ex) The number 9 represented in binary with eight bit

+9 : 00001001

-9 : 10001001 (signed-magnitude representation)

11110110 (signed-1's-complement representation)

11110111 (signed-2's-complement representation)

Table 1-3
Signed Binary Numbers

Decimal	Signed-2's complement	Signed-1's complement	Signed magnitude
+7	0111	0111	0111
+6	0110	0110	0110
+5	0101	0101	0101
+4	0100	0100	0100
+3	0011	0011	0011
+2	0010	0010	0010
+1	0001	0001	0001
+0	0000	0000	0000
-0	—	1111	1000
-1	1111	1110	1001
-2	1110	1101	1010
-3	1101	1100	1011
-4	1100	1011	1100
-5	1011	1010	1101
-6	1010	1001	1110
-7	1001	1000	1111
-8	1000	—	—

1.6 Signed Binary Numbers

• Arithmetic Addition

- signed-magnitude system follows the rules of ordinary arithmetic.
- signed-complement system requires only addition.

+6	00000110	-6	11111010
+13	00001101	+13	00001101
<hr/>		<hr/>	
+19	00010011	+7	00000111
+6	00000110	-6	11111010
-13	11110011	-13	11110011
<hr/>		<hr/>	
-7	11111001	-19	11101101

• Arithmetic Subtraction

$$(\pm A) - (+B) = (\pm A) + (-B)$$

$$(\pm A) - (-B) = (\pm A) + (+B)$$

1.7 Binary Code-BCD code

- the 4-bit code for one decimal

$$(185)_{10} = (0001\ 1000\ 0101)_{\text{BCD}} = (10111001)_2$$

• BCD Addition

4	0100	4	0100	8	1000
+5	+0101	+8	+1000	+9	+1001
<hr/>	<hr/>	<hr/>	<hr/>	<hr/>	<hr/>
9	1001	12	1100	17	10001
			+0110		+0110
			<hr/>		<hr/>
			10010		10111

Table 1-4
Binary Coded Decimal (BCD)

Decimal symbol	BCD digit
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001

- if the binary sum is greater or equal to 1010, we add 0110 to obtain the correct BCD

1.7 Binary Code-Other Decimal Codes

Table 1-5
Four Different Binary Codes for the Decimal Digits

Decimal digit	BCD 8421	2421	Excess-3	8 4-2-1
0	0000	0000	0011	0 0 0 0
1	0001	0001	0100	0 1 1 1
2	0010	0010	0101	0 1 1 0
3	0011	0011	0110	0 1 0 1
4	0100	0100	0111	0 1 0 0
5	0101	1011	1000	1 0 1 1
6	0110	1100	1001	1 0 1 0
7	0111	1101	1010	1 0 0 1
8	1000	1110	1011	1 0 0 0
9	1001	1111	1100	1 1 1 1
Unused bit combinations	1010	0101	0000	0 0 0 1
	1011	0110	0001	0 0 1 0
	1100	0111	0010	0 0 1 1
	1101	1000	1101	1 1 0 0
	1110	1001	1110	1 1 0 1
	1111	1010	1111	1 1 1 0

1.7 Binary Code-Gray Code

Table 1-6
Gray Code

Gray code	Decimal equivalent
0000	0
0001	1
0011	2
0010	3
0110	4
0111	5
0101	6
0100	7
1100	8
1101	9
1111	10
1110	11
1010	12
1011	13
1001	14
1000	15

1.7 Binary Code- ASCII Character Code

Table 1-7
American Standard Code for Information Interchange (ASCII)

$b_4b_3b_2b_1$	$b_7b_6b_5$							
	000	001	010	011	100	101	110	111
0000	NUL	DLE	SP	0	@	P	`	p
0001	SOH	DC1	!	1	A	Q	a	q
0010	STX	DC2	"	2	B	R	b	r
0011	ETX	DC3	#	3	C	S	c	s
0100	EOT	DC4	\$	4	D	T	d	t
0101	ENQ	NAK	%	5	E	U	e	u
0110	ACK	SYN	&	6	F	V	f	v
0111	BEL	ETB	'	7	G	W	g	w
1000	BS	CAN	(8	H	X	h	x
1001	HT	EM)	9	I	Y	i	y
1010	LF	SUB	*	:	J	Z	j	z
1011	VT	ESC	+	;	K	[k	{
1100	FF	FS	,	<	L	\	l	
1101	CR	GS	-	=	M]	m	}
1110	SO	RS	.	>	N	^	n	~
1111	SI	US	/	?	O	_	o	DEL

1.7 Binary Code – Error-Detecting Code

- Error-Detecting Code

	With even parity	With odd parity
ASCII A = 1000001	01000001	11000001
ASCII T = 1010100	11010100	01010100

1.8 Binary Storage and Registers

- Registers – A register with n cells can store any discrete quantity of information that contains n bits.
- Register Transfer

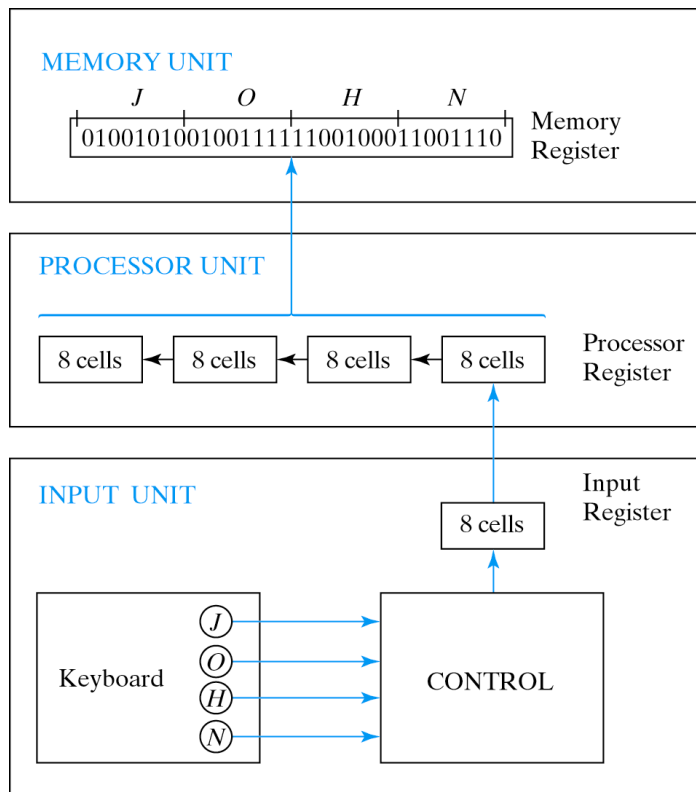


Fig. 1-1 Transfer of information with registers

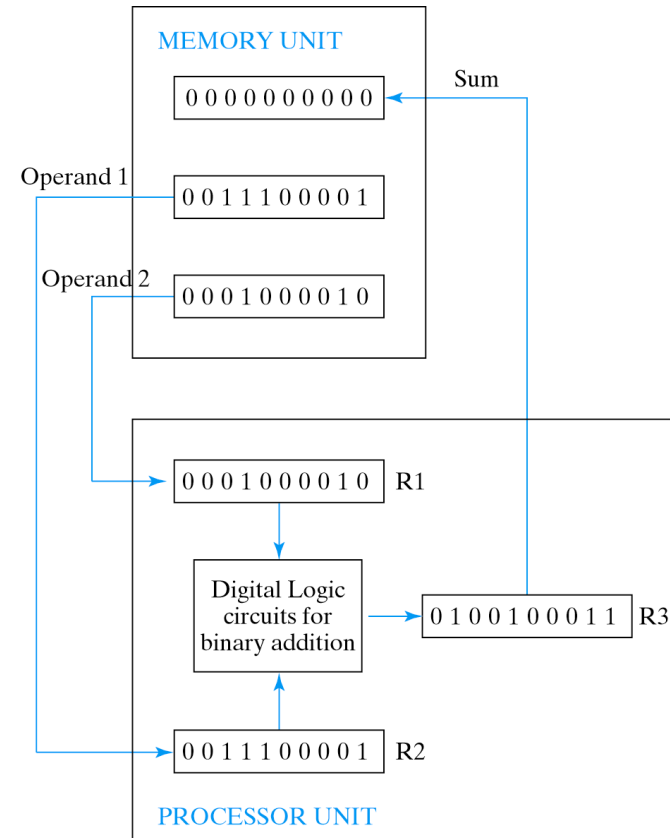


Fig. 1-2 Example of binary information processing

1.9 Binary Logic

Definition of Binary Logic

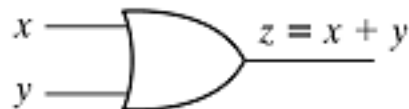
Table 1-8
Truth Tables of Logical Operations

AND			OR			NOT	
x	y	$x \cdot y$	x	y	$x + y$	x	x'
0	0	0	0	0	0	0	1
0	1	0	0	1	1	1	0
1	0	0	1	0	1		
1	1	1	1	1	1		

Logic Gates



(a) Two-input AND gate



(b) Two-input OR gate



(c) NOT gate or inverter

Fig. 1-4 Symbols for digital logic circuits

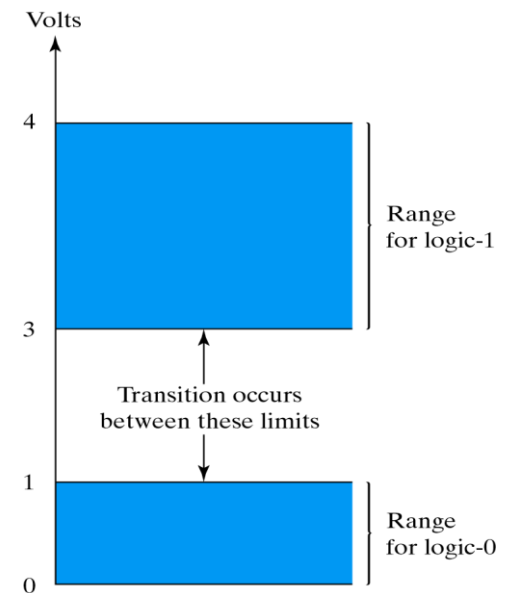


Fig. 1-3 Example of binary signals

1.9 Binary Logic

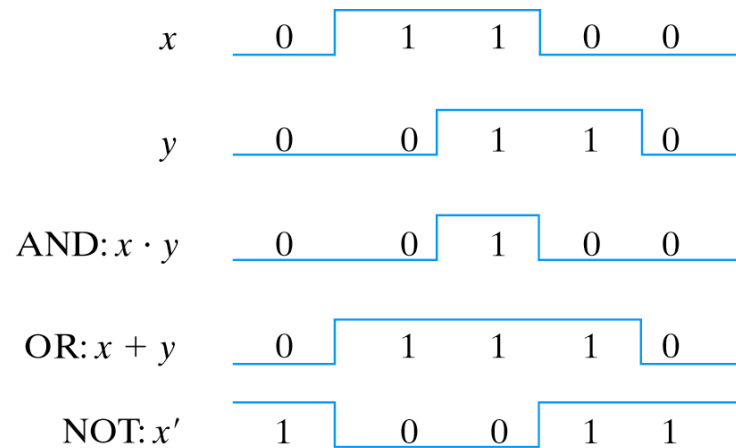


Fig. 1-5 Input-output signals for gates

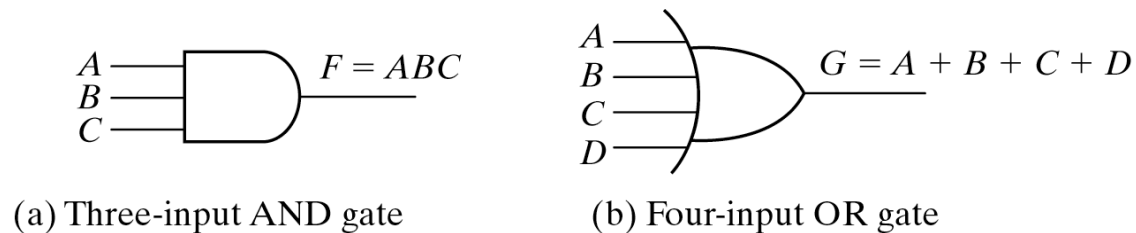


Fig. 1-6 Gates with multiple inputs