

# 1. Digital System & Binary Numbers

# 1.1 Digital System

- Digital System
  - Communication, Business transaction, Traffic control, other commercial, industrial, scientific enterprises etc.
  - Discrete elements of information
- Signal
  - Discrete elements of information are represented in a digital system by physical quantities
- Binary Code
  - Discrete elements of information are represented with groups of bits
  - bit : binary + digit

# 1.2 Binary Number

•  $a_5a_4a_3a_2a_1a_0.a_{-1}a_{-2}a_{-3}$

$$= a_n r^n + a_{n-1} r^{n-1} + \dots + a_2 r^2 + a_1 r + a_0 + a_{-1} r^{-1} + a_{-2} r^{-2} + \dots + a_{-m} r^{-m}$$

$$7392 = 7 \times 10^3 + 3 \times 10^2 + 9 \times 10^1 + 2 \times 10^0$$

$$(11010.11)_2 = 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2_0 + 1 \times 2_{-1} + 1 \times 2_{-2}$$
$$= (26.75)_{10}$$

$$2^{10} = 1 \text{ Kilo}$$

$$2^{20} = 1 \text{ Mega}$$

$$2^{30} = 1 \text{ Giga}$$

**Table 1-1**  
*Powers of Two*

n	$2^n$	n	$2^n$	n	$2^n$
0	1	8	256	16	65,536
1	2	9	512	17	131,072
2	4	10	1,024	18	262,144
3	8	11	2,048	19	524,288
4	16	12	4,096	20	1,048,576
5	32	13	8,192	21	2,097,152
6	64	14	16,384	22	4,194,304
7	128	15	32,768	23	8,388,608

# 1.2 Binary Number

Augend	101101	Minuend:	101101	Multiplicand:	1011
Addend	+100111	Subtrahend:	-100111	Multiplier:	*101
sum	1010100	Difference:	000110		1011
					0000
					1011
				Product:	110111

# 1.3 Number Base Conversions

Ex 1-1) Convert decimal 41 to binary.

	Integer Quotient		Remainder	Coefficient	Integer	Remainder
$41/2 =$	20	+	$\frac{1}{2}$	$a_0 = 1$	41	
$20/2 =$	10	+	0	$a_1 = 0$	20	1
$10/2 =$	5	+	0	$a_2 = 0$	10	0
$5/2 =$	2	+	$\frac{1}{2}$	$a_3 = 1$	5	0
$2/2 =$	1	+	0	$a_4 = 0$	2	1
$1/2 =$	0	+	$\frac{1}{2}$	$a_5 = 1$	1	0
					0	1

answer :  $(41)_{10} = (a_5a_4a_3a_2a_1a_0)_2 = (101001)_2$   
 $= 101001$

# 1.3 Number Base Conversions

- Ex 1-2) Convert decimal 153 to octal.

$$\begin{array}{r} 153 \\ 19 \Big| 1 \\ 2 \Big| 3 \\ 0 \Big| 2 \end{array} = (231)_8$$

- Ex 1-3) Convert  $(0.6875)_{10}$  to binary.

	Integer		Fraction	Coefficient
$0.6875 * 2 =$	1	+	0.3750	$a_{-1} = 1$
$0.3750 * 2 =$	0	+	0.7500	$a_{-2} = 0$
$0.7500 * 2 =$	1	+	0.5000	$a_{-3} = 1$
$0.5000 * 2 =$	1	+	0.0000	$a_{-4} = 1$

$$\text{Answer: } (0.6875)_{10} = (0.a_{-1}a_{-2}a_{-3}a_{-4})_2 = (0.1011)_2$$

# 1.4 Octal and Hexadecimal Numbers

**Table 1-2**  
*Numbers with Different Bases*

<b>Decimal (base 10)</b>	<b>Binary (base 2)</b>	<b>Octal (base 8)</b>	<b>Hexadecimal (base 16)</b>
00	0000	00	0
01	0001	01	1
02	0010	02	2
03	0011	03	3
04	0100	04	4
05	0101	05	5
06	0110	06	6
07	0111	07	7
08	1000	10	8
09	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

$$( \underline{10} \underline{110} \underline{001} \underline{101} \underline{011} . \underline{111} \underline{100} \underline{000} \underline{110} )_2 = (26153.7460)_8$$

2    6    1    5    3    7    4    0    6

$$( \underline{10} \underline{1100} \underline{0110} \underline{1011} . \underline{1111} \underline{0010} )_2 = (2C6B.F2)_{16}$$

2    C    6    B    F    2

## 1.5 Complements – Diminished Radix Complement

- $(r-1)$ 's complements of  $N$  is  $(r^n-1)-N$
- $r=10$ ,  $r-1=9$ , 9'complements of  $N$  is  $(10^n-1)-N$   
Ex) the 9's complements of 546700 is  $999999-546700 = 453299$   
the 9's complements of 012398 is  $999999-012398 = 987601$
- For binary number,  $r=2$ ,  $r-1=1$   
1'complements of  $N$  is  $(2^n-1)-N$   
Ex) the 1's complements of 1011000 is 0100111  
the 1's complements of 0101101 is 1010010

## 1.5 Complements – Radix Complement

- The  $r$ 's complements of an  $n$ -digit number  $N$  is  $r^n - N$ ,  $N \neq 0$   
 $0, \quad N=0$
- $r^n - N = [(r^n - 1) - N] + 1$   
→ The  $r$ 's complements is obtained by adding 1 to the  $(r-1)$ 's complements
- Ex) The 10's complements of 012398 is 987602  
The 10's complements of 246700 is 753300  
  
The 2's complements of 1101100 is 0010100  
The 2's complements of 0110111 is 1001001

## 1.5 Complements – Subtraction with Complements

- Ex1-5) using 10's complement, subtract 72532-3250.

$$\begin{array}{r} M = 72532 \\ 10\text{'s complement of } N = +96750 \\ \hline \text{Sum} = 169282 \\ \text{Discard end carry } 10^5 = -100000 \\ \hline \text{Answer} = 69282 \end{array}$$

- Ex1-6) Using 10's complement, subtract 3250-72532.

$$\begin{array}{r} M = 03250 \\ 10\text{'s complement of } N = +27468 \\ \hline \text{There is no end carry} \quad \text{Sum} = 30718 \end{array}$$

Therefore, the answer is  $-(10\text{'s complement of } 30718) = -69282$

## 1.5 Complements – Subtraction with Complements

- Ex1-7)  $X=1010100$ ,  $Y=1000011$ , (a)  $X-Y$ , (b)  $Y-X$

(a)  $X-Y$

$$X = \begin{array}{r} 1010100 \\ \hline \end{array}$$

$$2\text{'s complement of } Y = \begin{array}{r} +0111101 \\ \hline \end{array}$$

$$\text{Sum} = \begin{array}{r} 10010001 \\ \hline \end{array}$$

$$\text{Discard end carry } 2^7 = \begin{array}{r} -10000000 \\ \hline \end{array}$$

$$\text{Answer: } X-Y = \begin{array}{r} 0010001 \\ \hline \end{array}$$

(b)  $Y-X$

$$Y = \begin{array}{r} 1000011 \\ \hline \end{array}$$

$$2\text{'s complement of } X = \begin{array}{r} +0101100 \\ \hline \end{array}$$

$$\text{Sum} = \begin{array}{r} 1101111 \\ \hline \end{array}$$

There is no carry.

The answer is  $Y-X = -(2\text{'s complement of } 1101111) = -0010001$

## 1.5 Complements – Subtraction with Complements

- Ex1-8) Repeat Example 1-7 using 1'complement.

(a)  $X - Y = 1010100 - 1000011$

$$X = \begin{array}{r} 1010100 \\ +0111100 \\ \hline \end{array}$$

$$1\text{'s complement of } Y = \begin{array}{r} 1001000 \\ + \quad \quad \quad 1 \\ \hline \end{array}$$

$$\text{Sum} = \begin{array}{r} 1001000 \\ + \quad \quad \quad 1 \\ \hline \end{array}$$

$$\text{End-around carry} = \begin{array}{r} + \quad \quad \quad 1 \\ \hline \end{array}$$

$$\text{Answer: } X - Y = \begin{array}{r} 0010001 \\ + \quad \quad \quad 1 \\ \hline \end{array}$$

(b)  $Y - X = 1000011 - 1010100$

$$Y = \begin{array}{r} 1000011 \\ +0101011 \\ \hline \end{array}$$

$$1\text{'s complement of } X = \begin{array}{r} 1101110 \\ + \quad \quad \quad 1 \\ \hline \end{array}$$

$$\text{Sum} = \begin{array}{r} 1101110 \\ + \quad \quad \quad 1 \\ \hline \end{array}$$

There is no carry.

The answer is  $Y - X = -(1\text{'s complement of } 1101110) = -0010001$

# 1.6 Signed Binary Numbers

- Ex) The number 9 represented in binary with eight bit

+9 : 00001001

-9 : 10001001 (signed-magnitude representation)

11110110 (signed-1's-complement representation)

11110111 (signed-2's-complement representation)

**Table 1-3**  
*Signed Binary Numbers*

Decimal	Signed-2's complement	Signed-1's complement	Signed magnitude
+7	0111	0111	0111
+6	0110	0110	0110
+5	0101	0101	0101
+4	0100	0100	0100
+3	0011	0011	0011
+2	0010	0010	0010
+1	0001	0001	0001
+0	0000	0000	0000
-0	—	1111	1000
-1	1111	1110	1001
-2	1110	1101	1010
-3	1101	1100	1011
-4	1100	1011	1100
-5	1011	1010	1101
-6	1010	1001	1110
-7	1001	1000	1111
-8	1000	—	—

# 1.6 Signed Binary Numbers

## Arithmetic Addition

- signed-magnitude system follows the rules of ordinary arithmetic.
- signed-complement system requires only addition.

+6	00000110	-6	11111010
+13	00001101	+13	00001101
<hr/>		<hr/>	
+19	00010011	+7	00000111
<hr/>		<hr/>	
+6	00000110	-6	11111010
-13	11110011	-13	11110011
<hr/>		<hr/>	
-7	11111001	-19	11101101

## Arithmetic Subtraction

$$(\pm A) - (+B) = (\pm A) + (-B)$$
$$(\pm A) - (-B) = (\pm A) + (+B)$$

# 1.7 Binary Code-BCD code

- the 4-bit code for one decimal

$$(185)_{10} = (0001\ 1000\ 0101)_{BCD} = (10111001)_2$$

**Table 1-4**  
*Binary Coded Decimal (BCD)*

## ● BCD Addition

4	0100	4	0100	8	1000		0	0000
+5	+0101	+8	+1000	+9	+1001		1	0001
—	—	—	—	—	—	—	2	0010
9	1001	12	1100	17	10001		3	0011
							4	0100
							5	0101
			+0110		+0110		6	0110
			—	—	—	—	7	0111
			10010		10111		8	1000
							9	1001

Decimal symbol	BCD digit
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001

- if the binary sum is greater or equal to 1010, we add 0110 to obtain the correct BCD

# 1.7 Binary Code-Other Decimal Codes

**Table 1-5**  
*Four Different Binary Codes for the Decimal Digits*

<b>Decimal digit</b>	<b>BCD 8421</b>	<b>2421</b>	<b>Excess-3</b>	<b>8 4-2-1</b>
0	0000	0000	0011	0 0 0 0
1	0001	0001	0100	0 1 1 1
2	0010	0010	0101	0 1 1 0
3	0011	0011	0110	0 1 0 1
4	0100	0100	0111	0 1 0 0
5	0101	1011	1000	1 0 1 1
6	0110	1100	1001	1 0 1 0
7	0111	1101	1010	1 0 0 1
8	1000	1110	1011	1 0 0 0
9	1001	1111	1100	1 1 1 1
Unused bit combinations	1010	0101	0000	0 0 0 1
	1011	0110	0001	0 0 1 0
	1100	0111	0010	0 0 1 1
	1101	1000	1101	1 1 0 0
	1110	1001	1110	1 1 0 1
	1111	1010	1111	1 1 1 0

## 1.7 Binary Code-Gray Code

**Table 1-6**  
*Gray Code*

<b>Gray code</b>	<b>Decimal equivalent</b>
0000	0
0001	1
0011	2
0010	3
0110	4
0111	5
0101	6
0100	7
1100	8
1101	9
1111	10
1110	11
1010	12
1011	13
1001	14
1000	15

# 1.7 Binary Code- ASCII Character Code

**Table 1-7**  
*American Standard Code for Information Interchange (ASCII)*

$b_4 b_3 b_2 b_1$	000	001	010	011	100	101	110	111
0000	NUL	DLE	SP	0	@	P	`	p
0001	SOH	DC1	!	1	A	Q	a	q
0010	STX	DC2	"	2	B	R	b	r
0011	ETX	DC3	#	3	C	S	c	s
0100	EOT	DC4	\$	4	D	T	d	t
0101	ENQ	NAK	%	5	E	U	e	u
0110	ACK	SYN	&	6	F	V	f	v
0111	BEL	ETB	'	7	G	W	g	w
1000	BS	CAN	(	8	H	X	h	x
1001	HT	EM	)	9	I	Y	i	y
1010	LF	SUB	*	:	J	Z	j	z
1011	VT	ESC	+	;	K	[	k	{
1100	FF	FS	,	<	L	\	l	
1101	CR	GS	-	=	M	]	m	}
1110	SO	RS	.	>	N	^	n	~
1111	SI	US	/	?	O	-	o	DEL

## 1.7 Binary Code – Error-Detecting Code

- Error-Detecting Code

	With even parity	With odd parity
ASCII A = 1000001	01000001	11000001
ASCII T = 1010100	11010100	01010100

# 1.8 Binary Storage and Registers

- Registers – A register with  $n$  cells can store any discrete quantity of information that contains  $n$  bits.
- Register Transfer

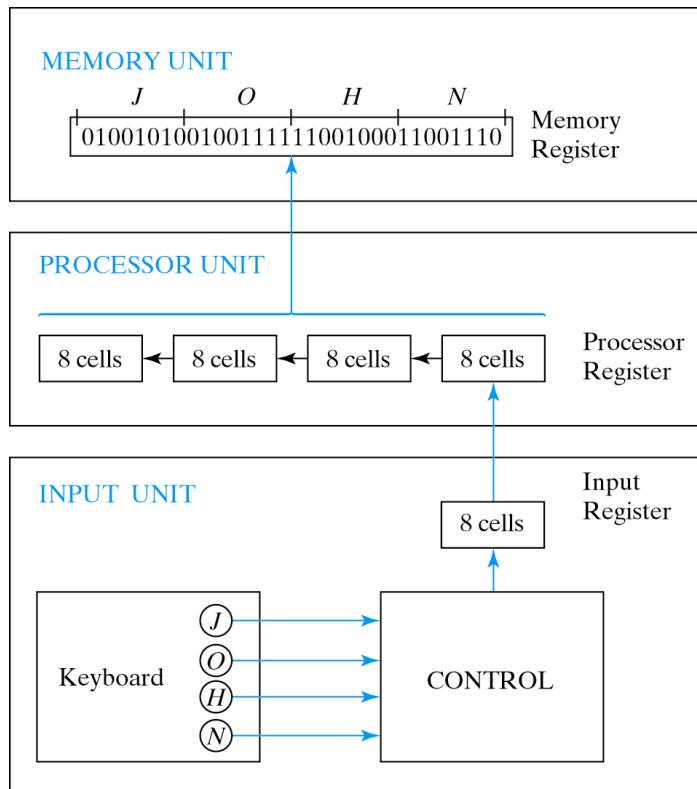


Fig. 1-1 Transfer of information with registers

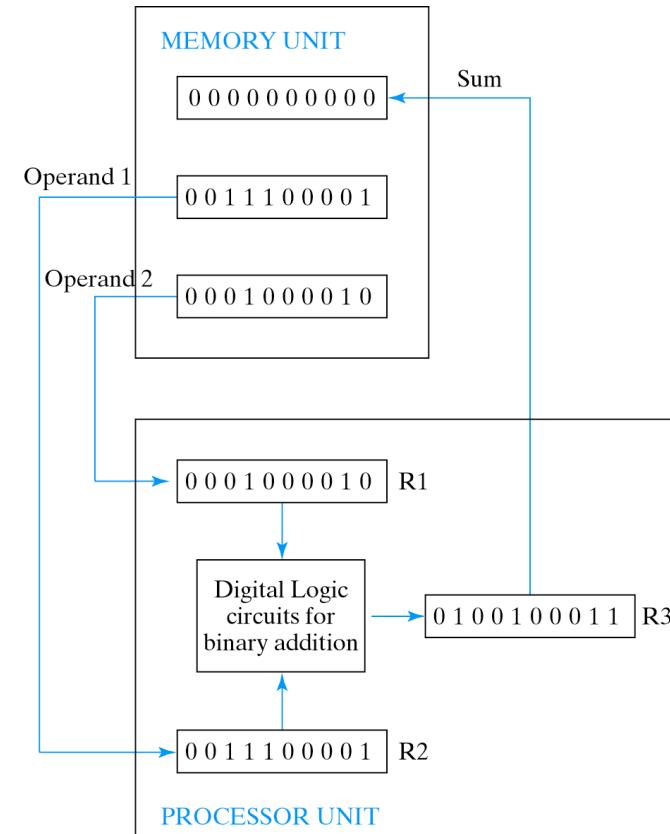


Fig. 1-2 Example of binary information processing

# 1.9 Binary Logic

## Definition of Binary Logic

Table 1-8

Truth Tables of Logical Operations

AND		OR		NOT	
$x$	$y$	$x \cdot y$	$x$	$y$	$x + y$
0	0	0	0	0	0
0	1	0	0	1	1
1	0	0	1	0	1
1	1	1	1	1	1

## Logic Gates



(a) Two-input AND gate



(b) Two-input OR gate



(c) NOT gate or inverter

Fig. 1-4 Symbols for digital logic circuits

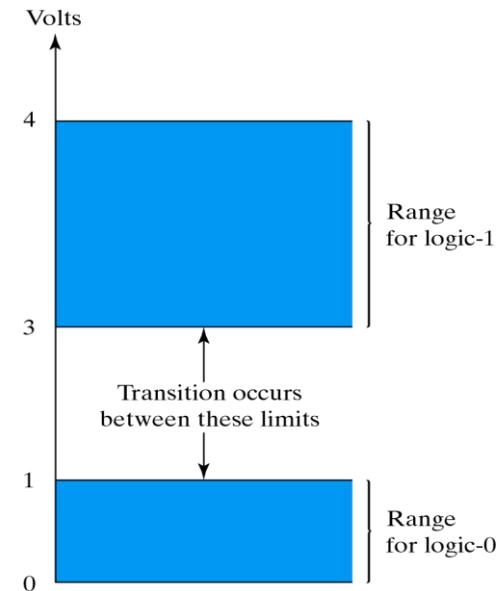


Fig. 1-3 Example of binary signals

# 1.9 Binary Logic

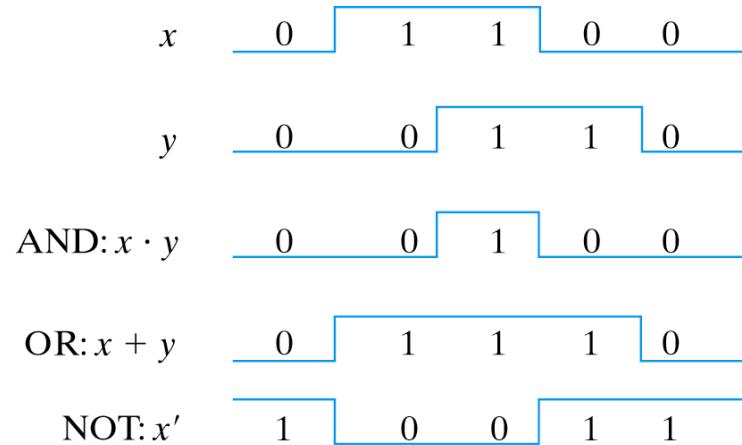
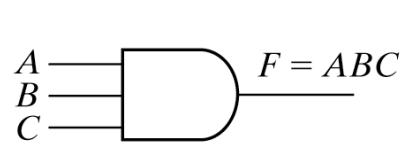
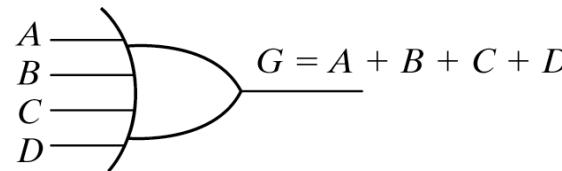


Fig. 1-5 Input-output signals for gates



(a) Three-input AND gate



(b) Four-input OR gate

Fig. 1-6 Gates with multiple inputs