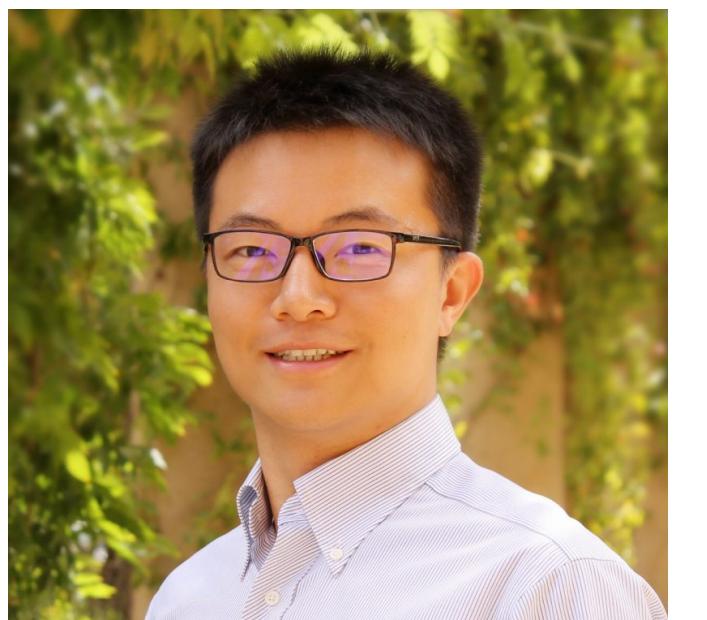


EfficientML.ai Lecture 03: Pruning and Sparsity

Part I



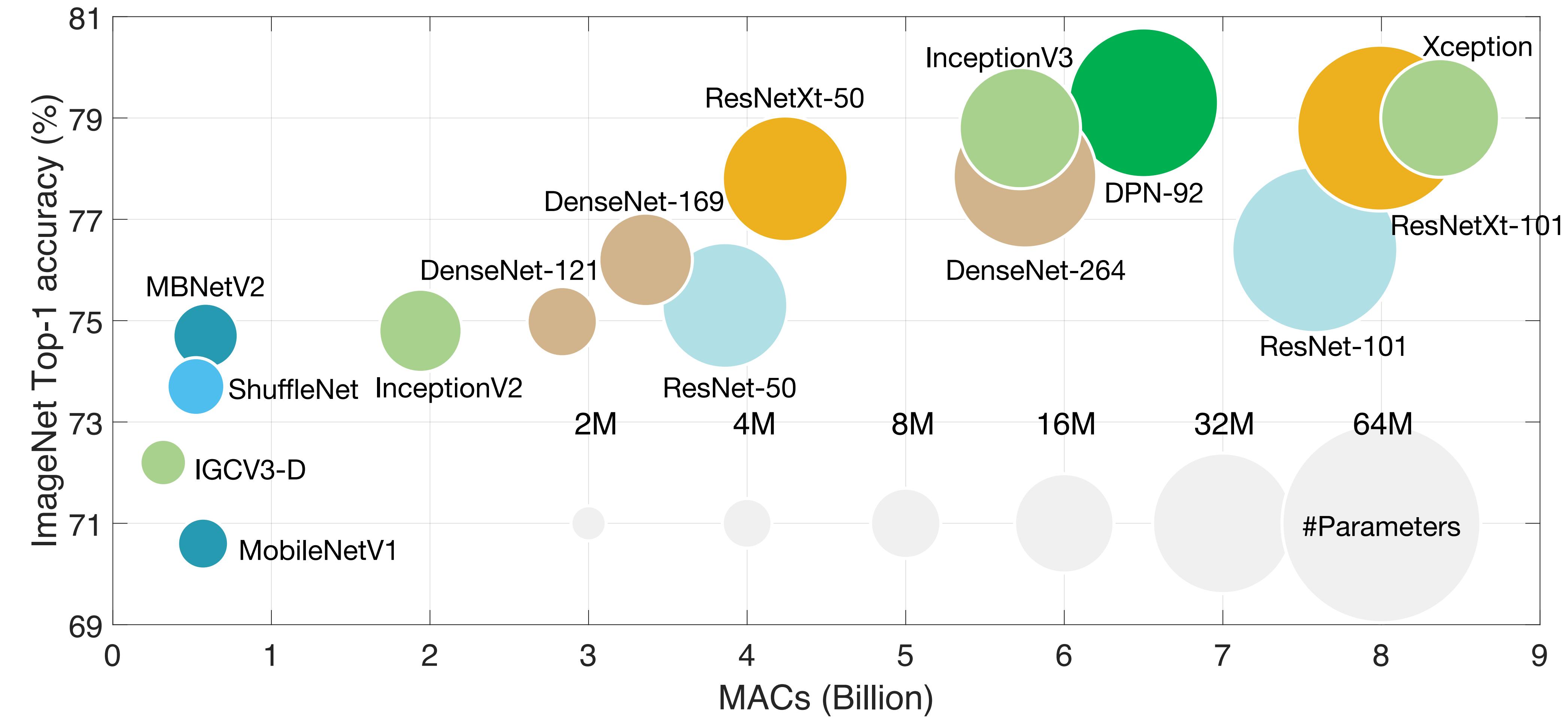
Song Han

Associate Professor, MIT
Distinguished Scientist, NVIDIA

 @SongHan/MIT



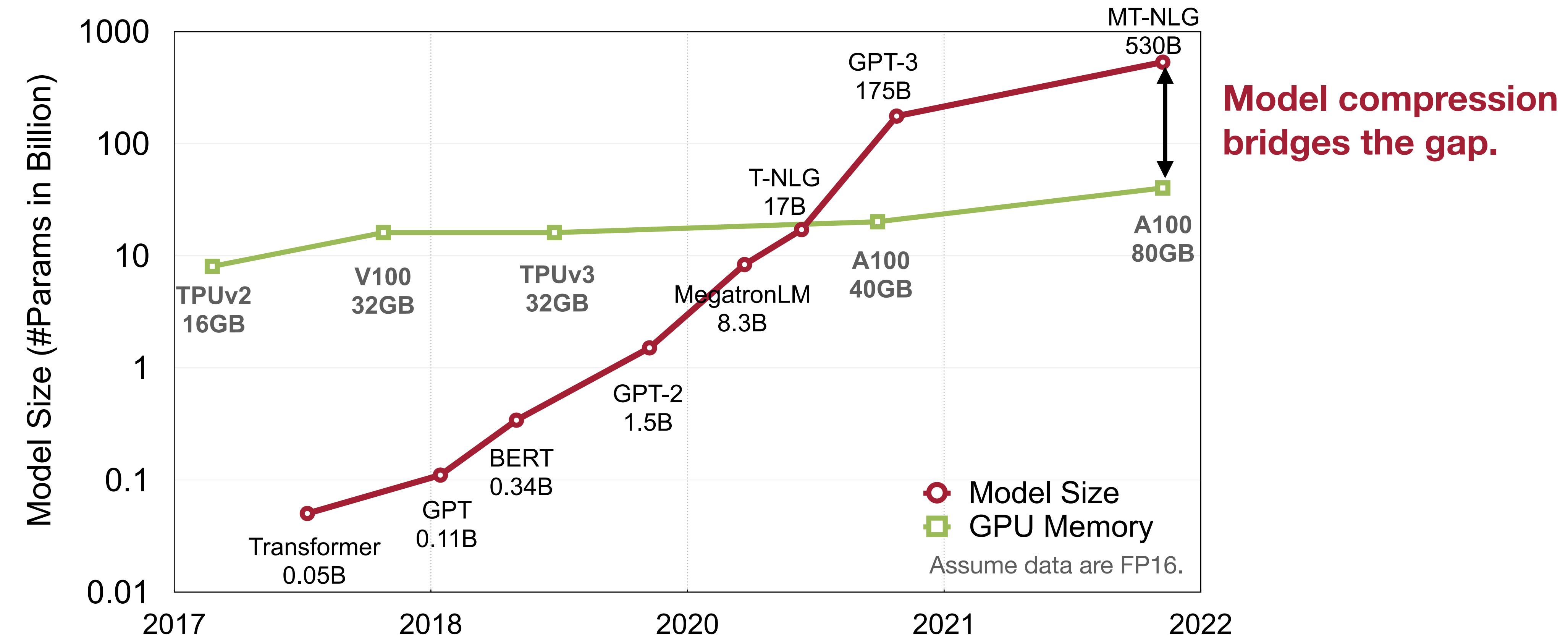
Today's AI is too BIG!



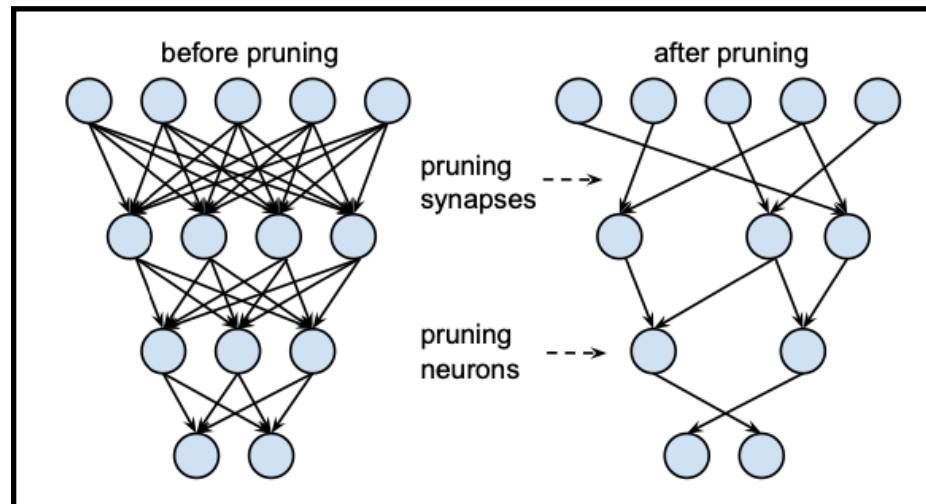
Model Compression and Hardware Acceleration for Neural Networks: A Comprehensive Survey [Deng et al., IEEE 2020]

Efficient Deep Learning Techniques are Essential

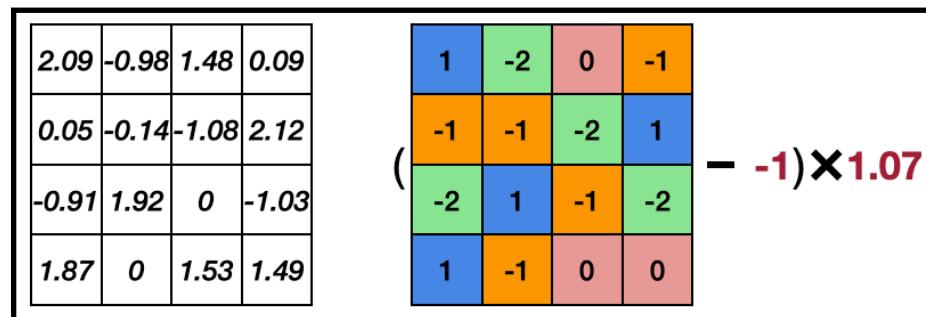
Bridges the Gap between the Supply and Demand of Computation



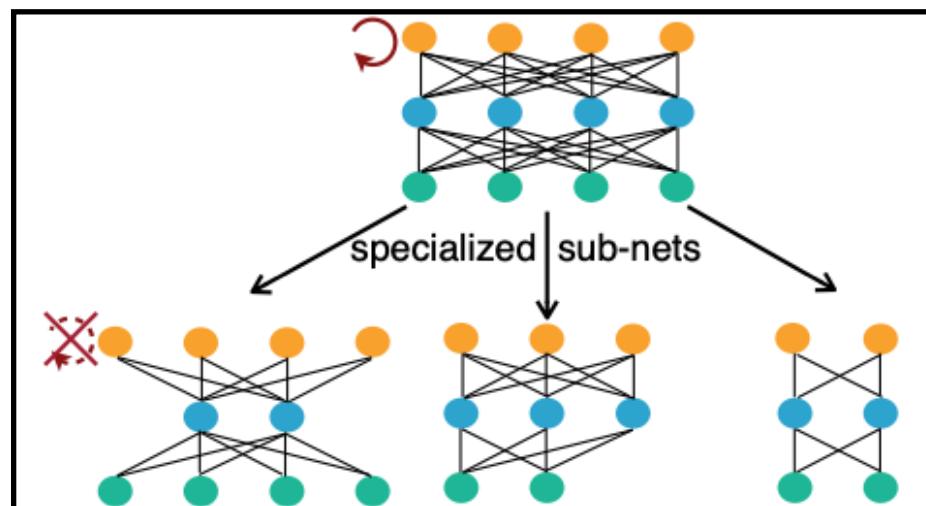
Part 1 of This Course: Efficient Inference



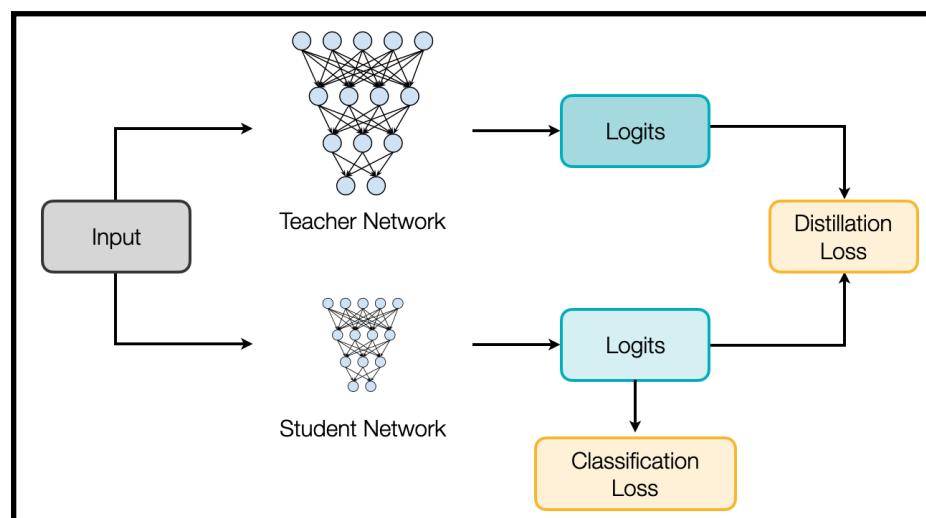
Pruning



Quantization



Neural Architecture Search



Knowledge Distillation

Chapter I: Efficient Inference

Sep 12	Lecture 3: Pruning and Sparsity (Part I) [Slides] [Video] [Video (Live)]	
Sep 17	Lecture 4: Pruning and Sparsity (Part II) [Slides] [Video] [Video (Live)]	Lab 1 out
Sep 19	Lecture 5: Quantization (Part I) [Slides] [Video] [Video (Live)]	Lab 0 due
Sep 24	Lecture 6: Quantization (Part II) [Slides] [Video] [Video (Live)]	
Sep 26	Lecture 7: Neural Architecture Search (Part I) [Slides] [Video] [Video (Live)]	Lab 1 due, Lab 2 out
Oct 1	Lecture 8: Neural Architecture Search (Part II) [Slides] [Video] [Video (Live)]	
Oct 3	Lecture 9: Knowledge Distillation [Slides] [Video] [Video (Live)]	
Oct 8	Lecture 10: MCUNet: TinyML on Microcontrollers [Slides] [Video] [Video (Live)]	Lab 2 due, Lab 3 out
Oct 10	Lecture 11: TinyEngine and Parallel Processing [Slides] [Video] [Video (Live)]	

MLPerf (the Olympic Game for AI Computing)

Pruning on Large Language Models

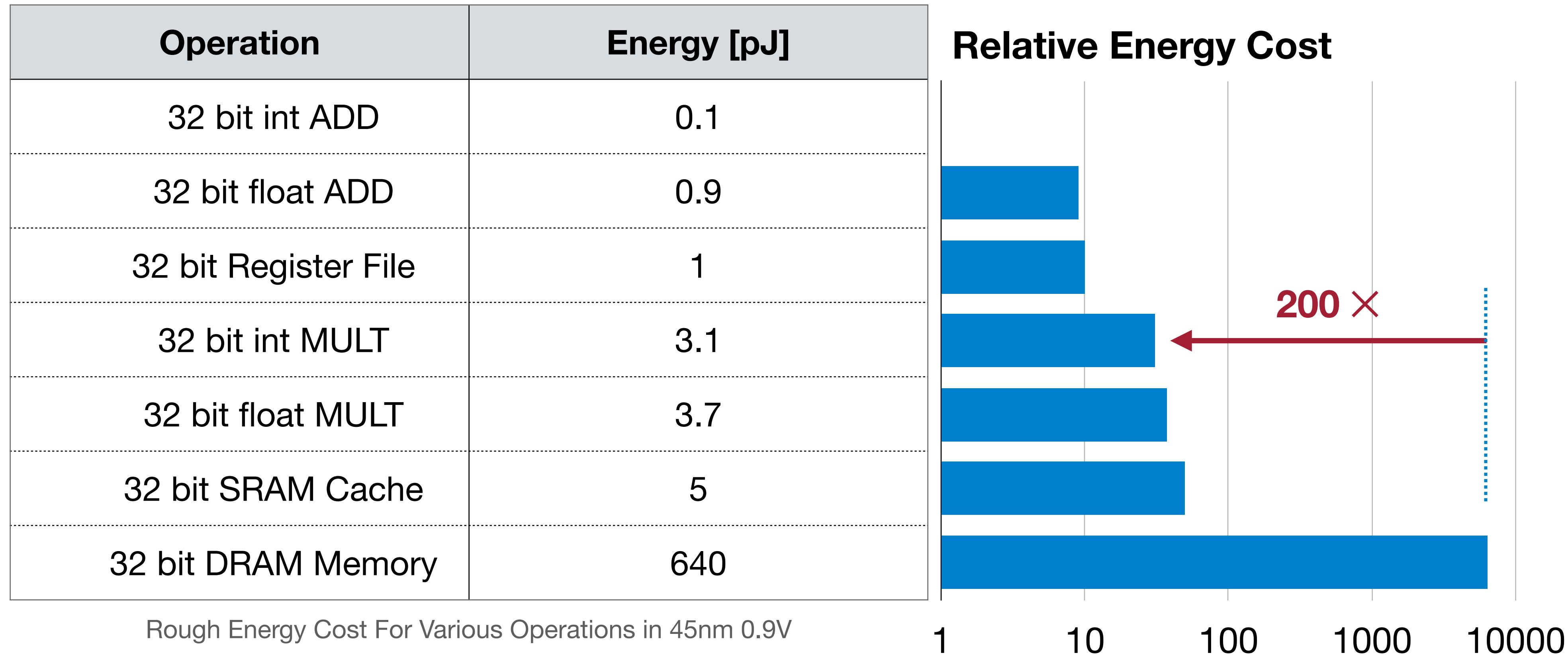
- The open division submission on Llama 2 70B: 2.5x speedup while maintaining 99% accuracy.
 - Depth pruning: 80 layers -> 32 layers
 - Width pruning: 28,762 intermediate dimensions -> 14,336 intermediate dimensions

	Closed Division	Open Division	Speedup
Offline samples/sec	4488	11189	2.5x

Llama 2 70B performance metrics for both closed division and open division.
Measured on a single NVIDIA H200 GPU.

Memory is Expensive

Data Movement → More Memory Reference → More Energy



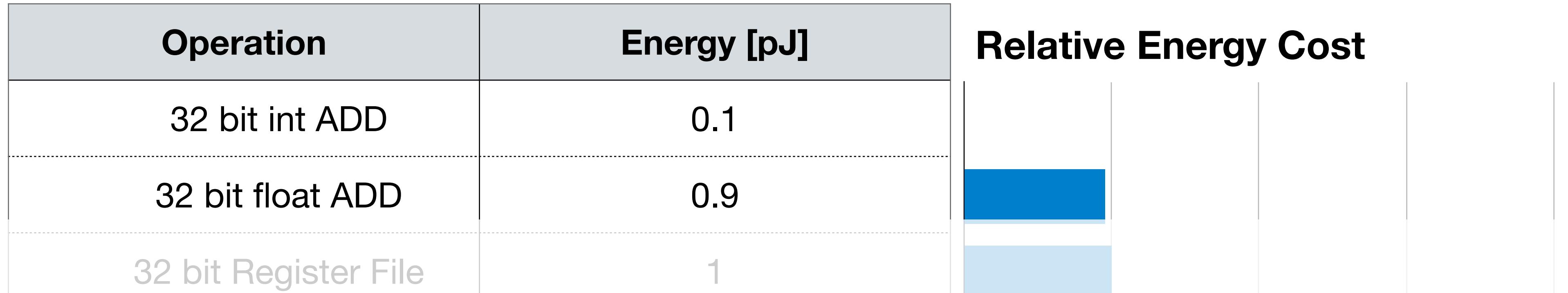
1  = 200 ×+

This image is in the public domain

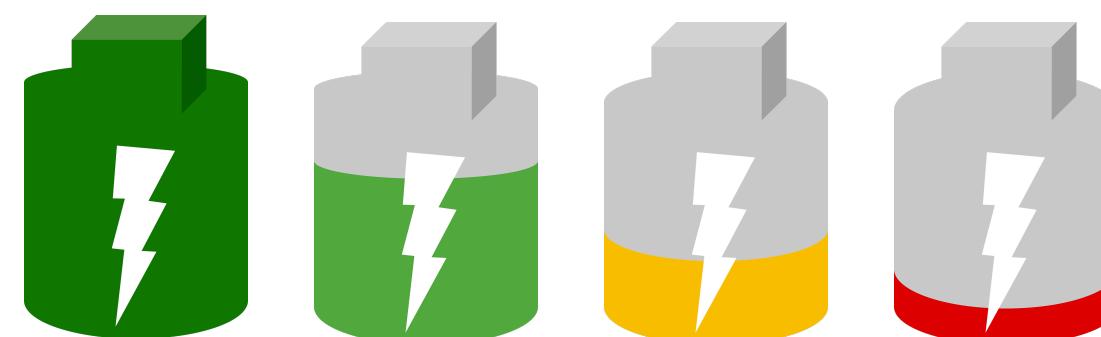
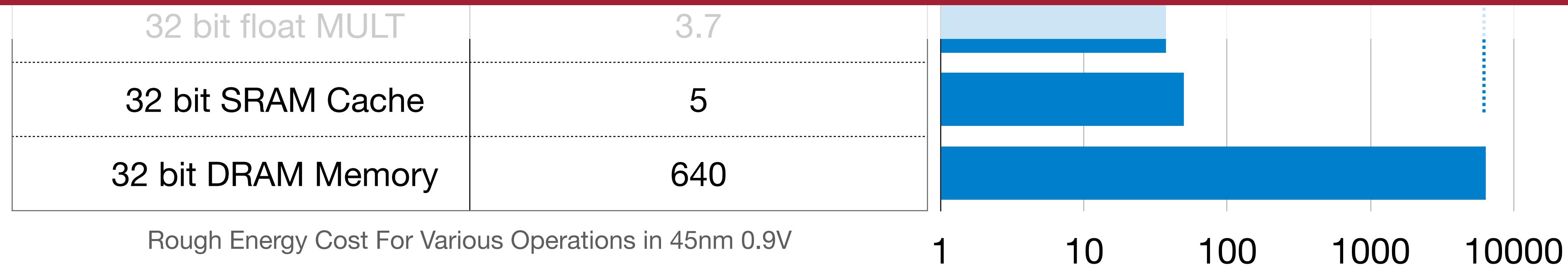
Computing's Energy Problem (and What We Can Do About it) [Horowitz, M., IEEE ISSCC 2014]

Memory is Expensive

Data Movement → More Memory Reference → More Energy



How should we make deep learning more efficient?

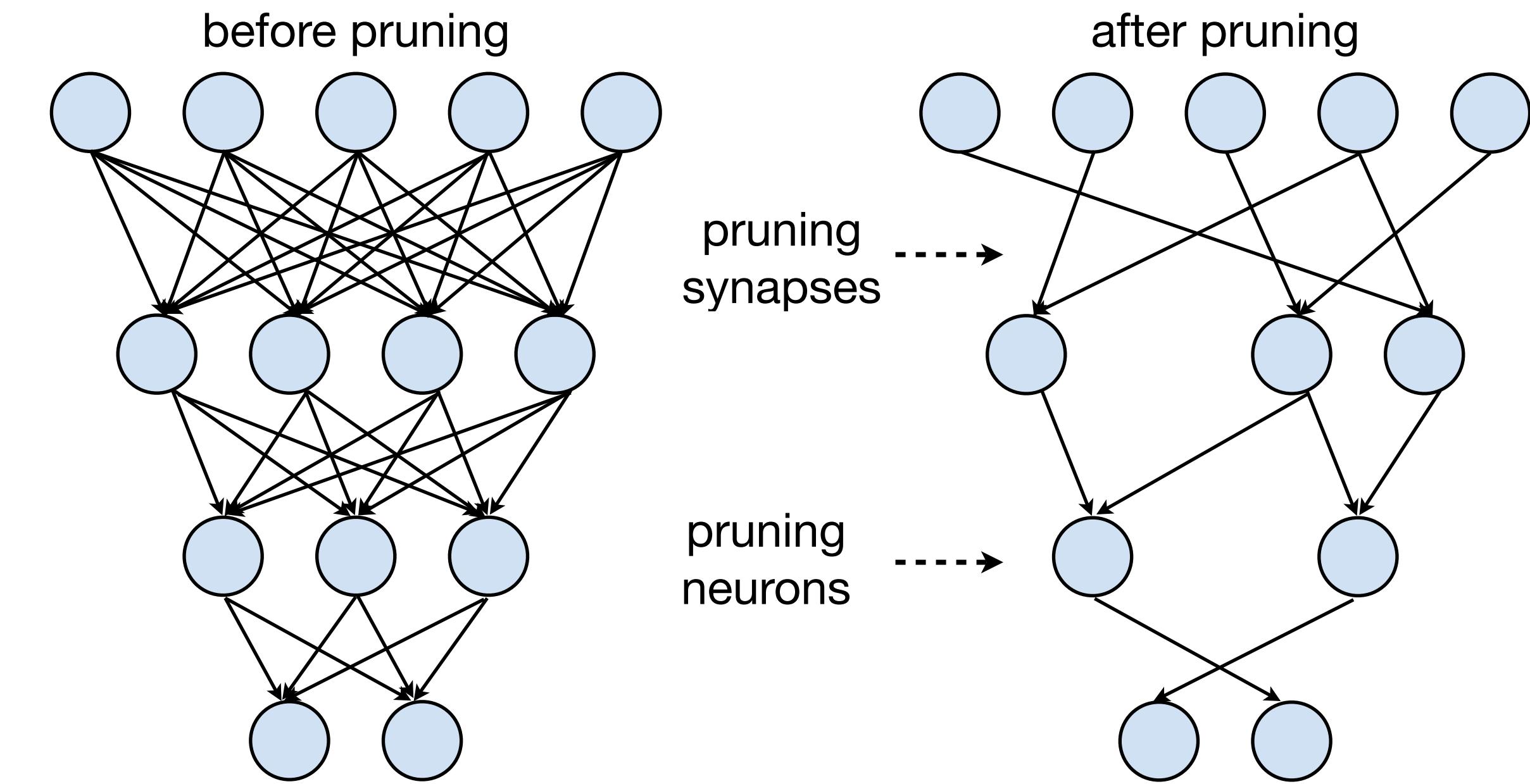


Battery images are in the public domain
[Image 1](#), [Image 2](#), [Image 2](#), [Image 4](#)

Computing's Energy Problem (and What We Can Do About it) [Horowitz, M., IEEE ISSCC 2014]

Neural Network Pruning

- **Introduction to Pruning**
 - What is pruning?
 - How should we formulate pruning?
- **Determine the Pruning Granularity**
 - In what pattern should we prune the neural network?
- **Determine the Pruning Criterion**
 - What synapses/neurons should we prune?
- **Determine the Pruning Ratio**
 - What should target sparsity be for each layer?
- **Fine-tune/Train Pruned Neural Network**
 - How should we improve performance of pruned models?



Learning Both Weights and Connections for Efficient Neural Network [Han et al., NeurIPS 2015]

Neural Network Pruning

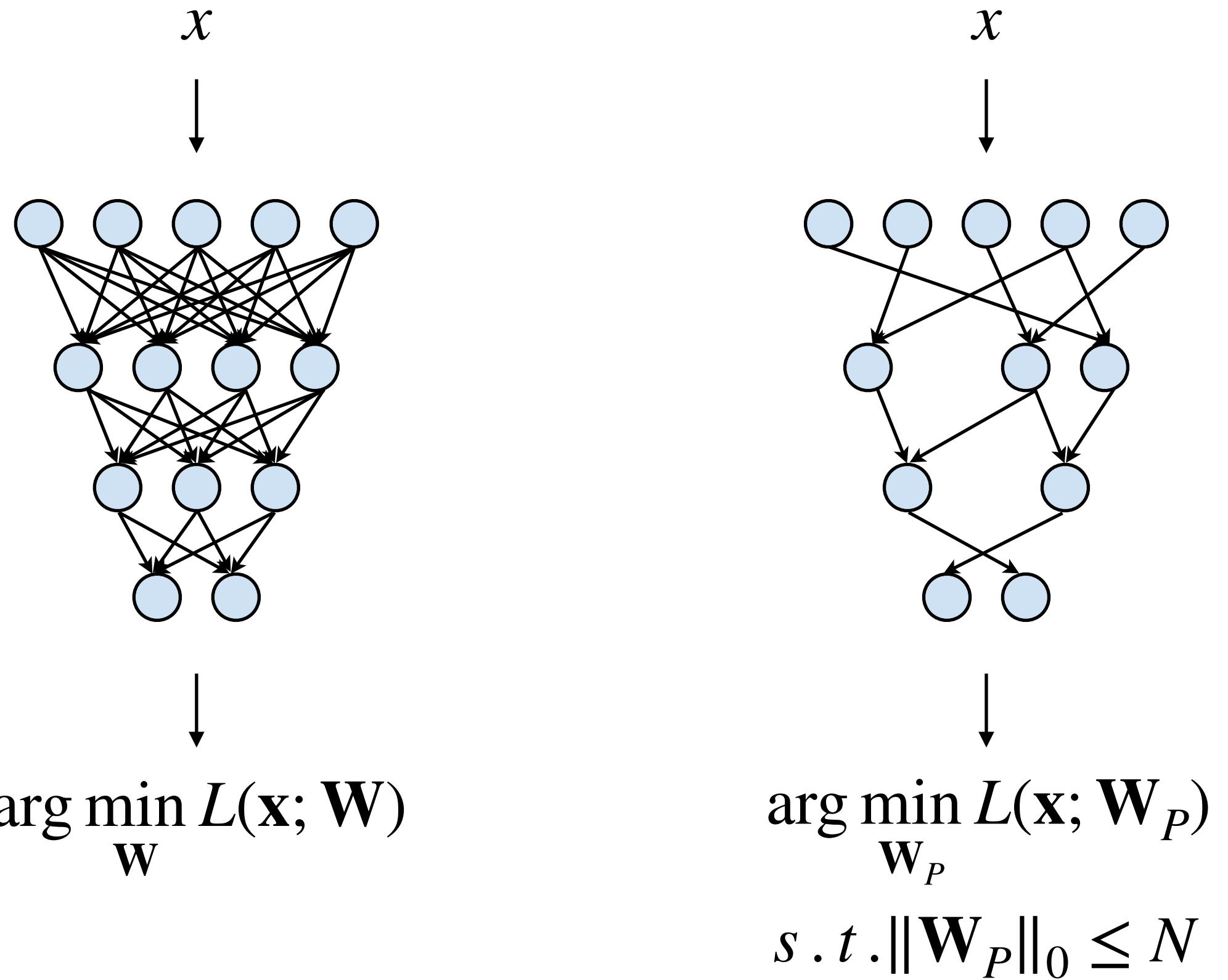
- In general, we could formulate the pruning as follows:

$$\arg \min_{\mathbf{W}_P} L(\mathbf{x}; \mathbf{W}_P)$$

subject to

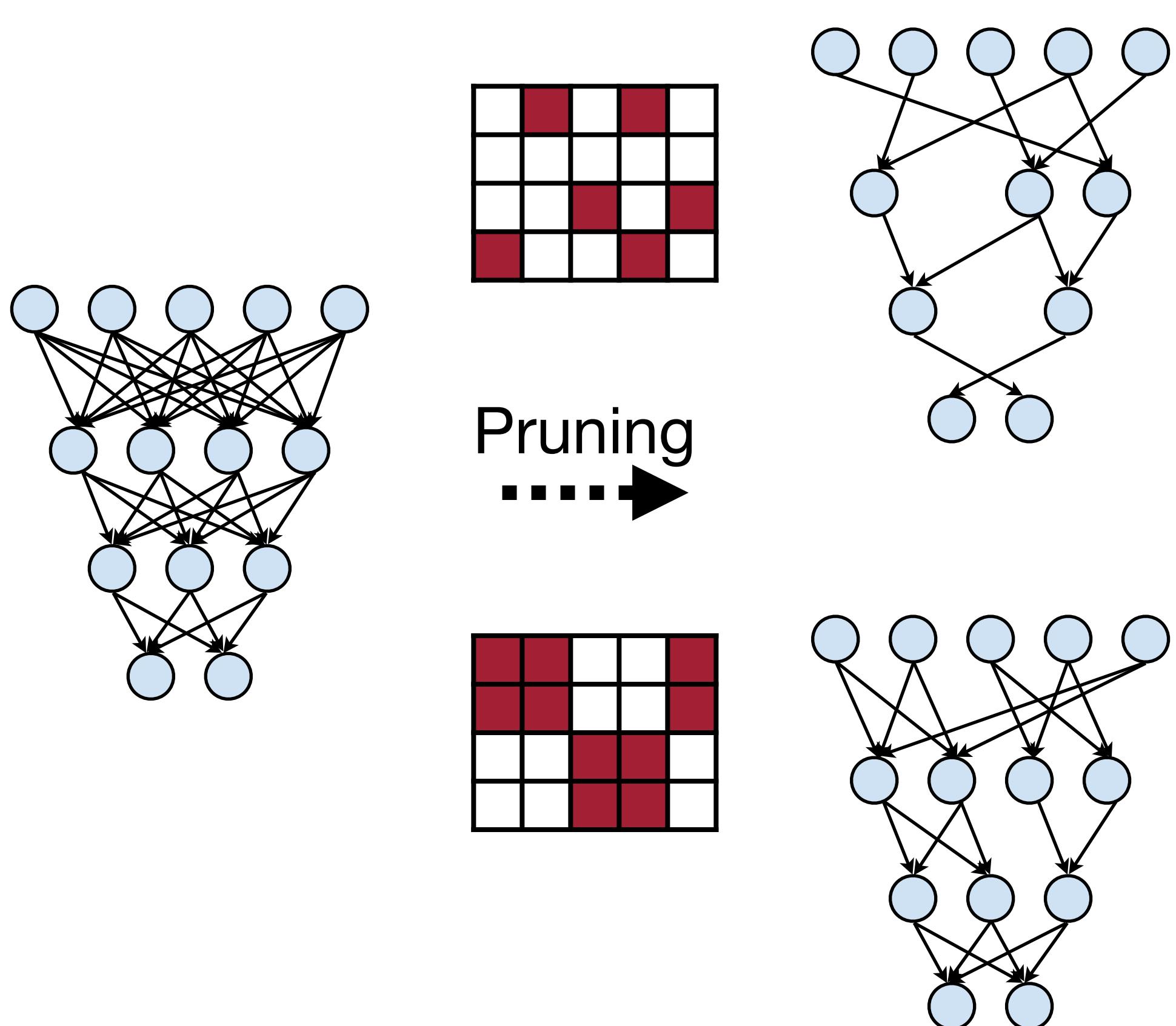
$$\|\mathbf{W}_p\|_0 < N$$

- L represents the objective function for neural network training;
- \mathbf{x} is input, \mathbf{W} is original weights, \mathbf{W}_P is pruned weights;
- $\|\mathbf{W}_p\|_0$ calculates the #nonzeros in W_P , and N is the target #nonzeros.



Neural Network Pruning

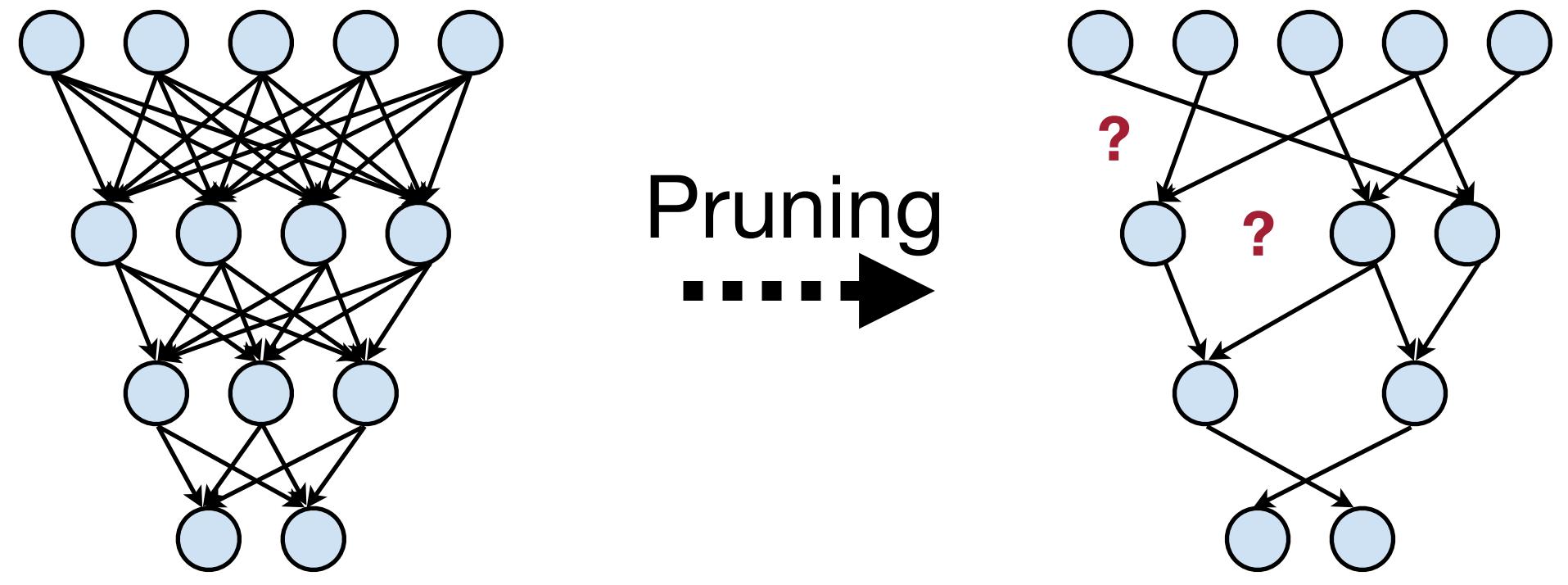
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Learning Both Weights and Connections for Efficient Neural Network [Han et al., NeurIPS 2015]

Neural Network Pruning

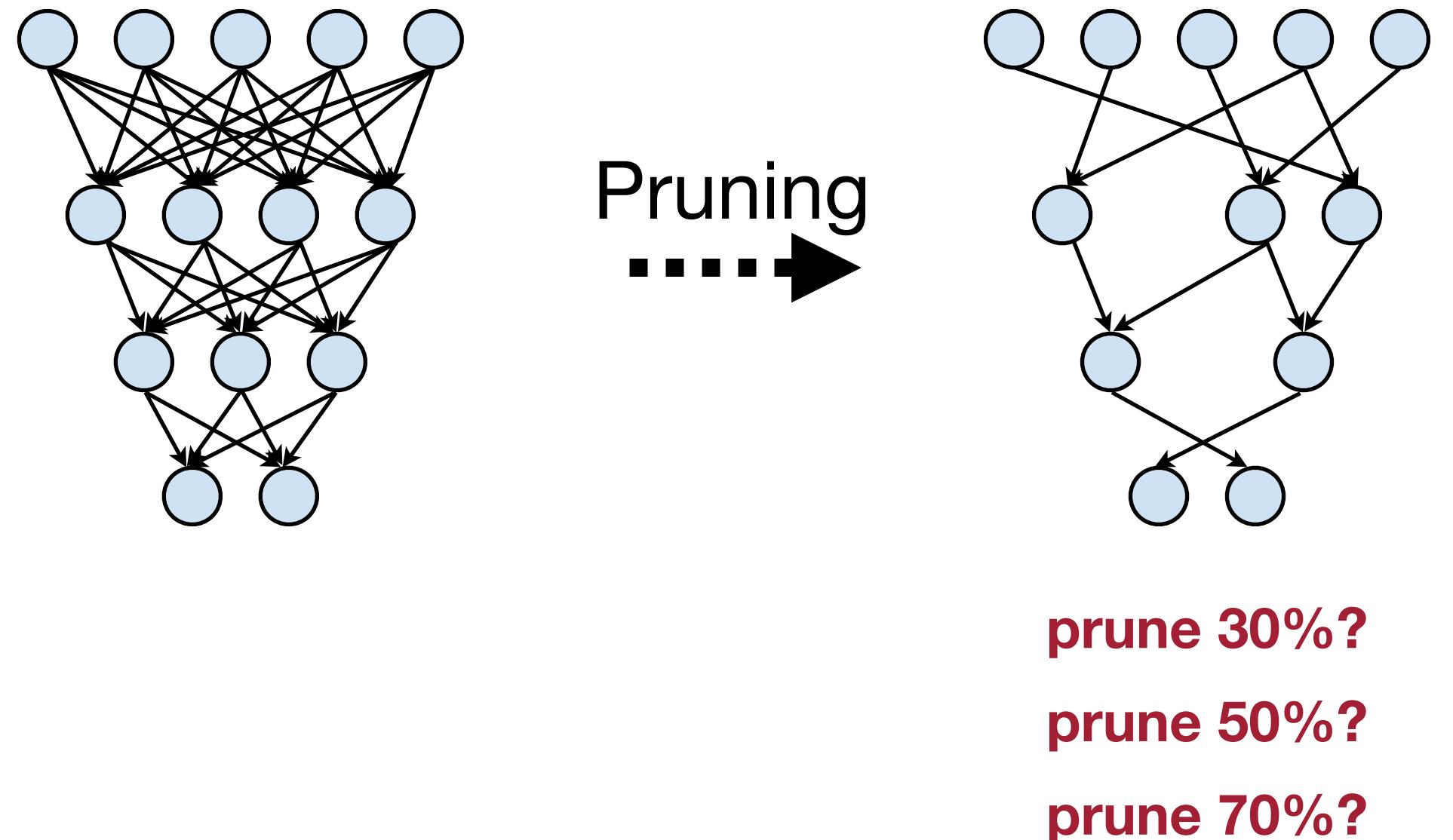
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which synapses?
which neurons?

Neural Network Pruning

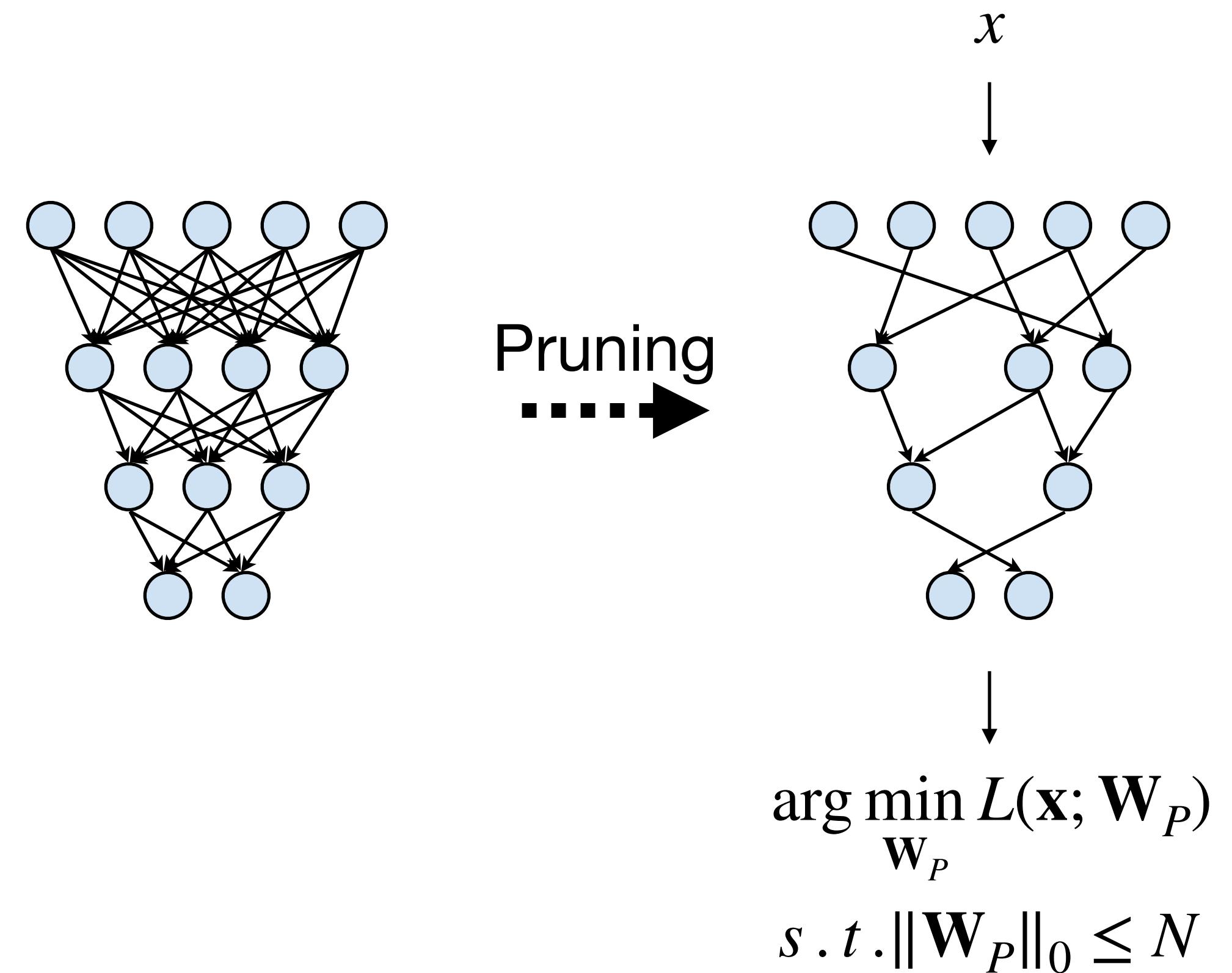
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Learning Both Weights and Connections for Efficient Neural Network [Han et al., NeurIPS 2015]

Neural Network Pruning

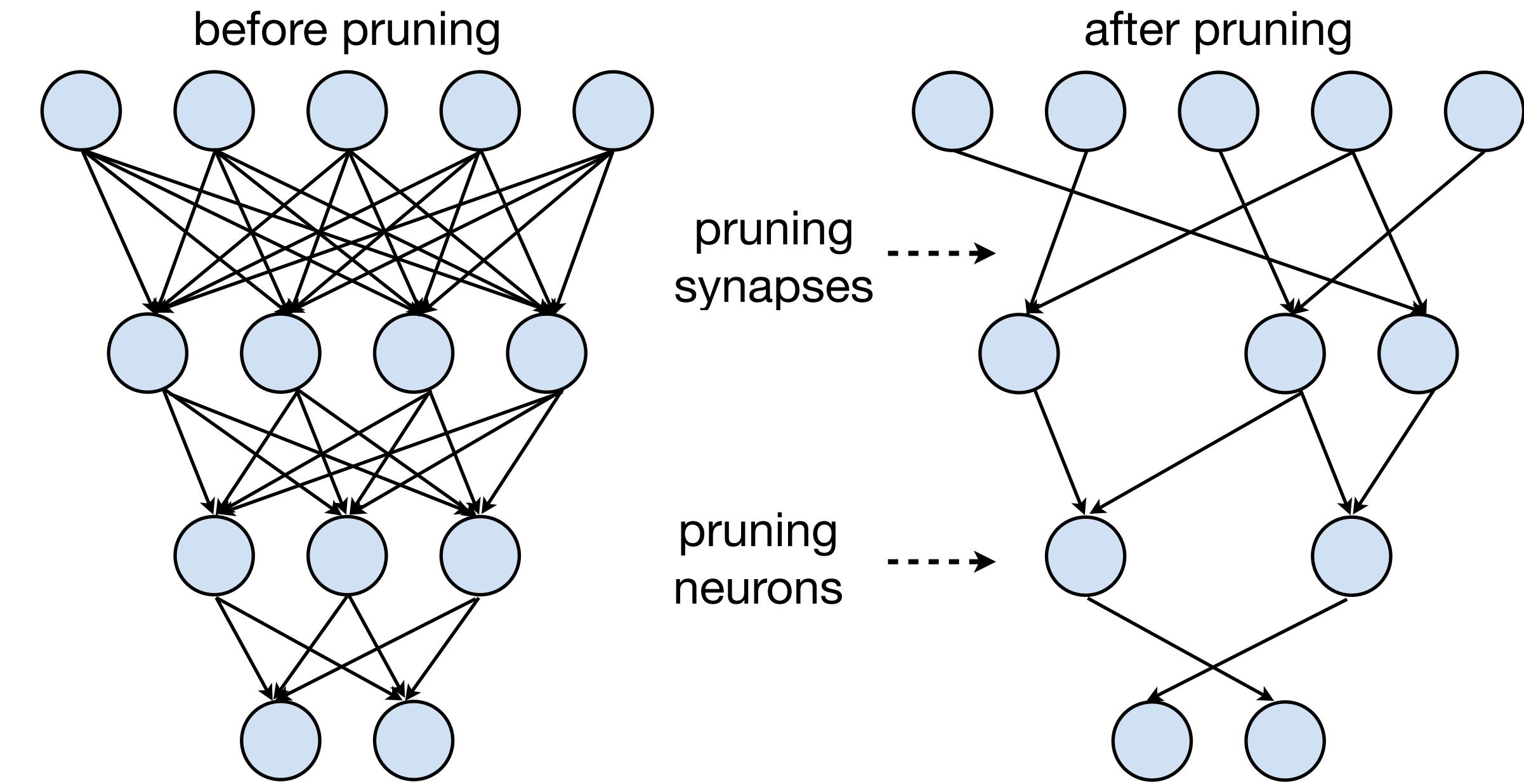
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Learning Both Weights and Connections for Efficient Neural Network [Han et al., NeurIPS 2015]

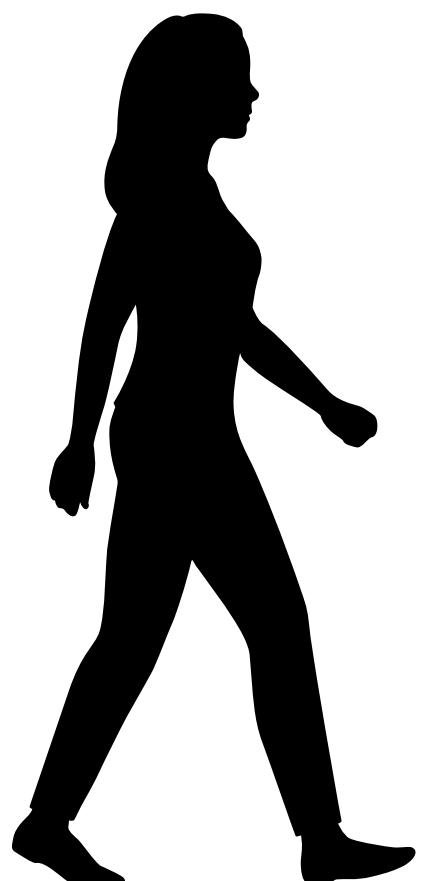
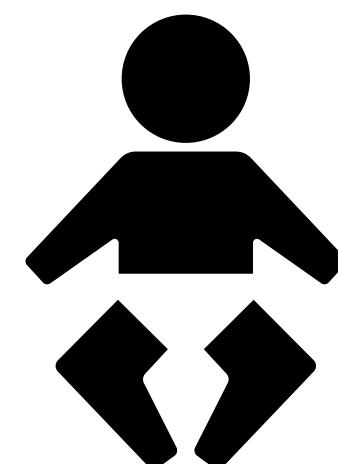
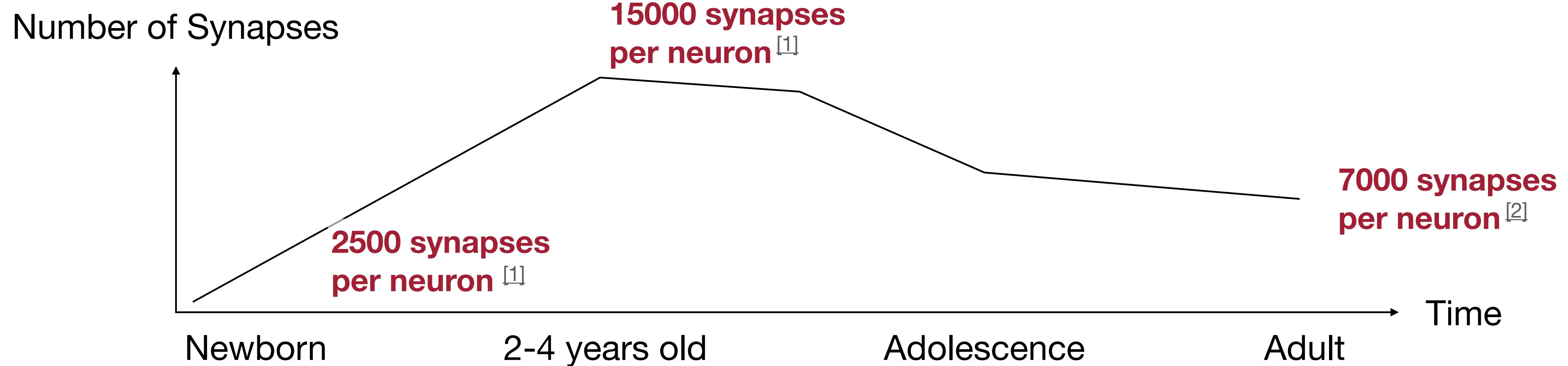
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Learning Both Weights and Connections for Efficient Neural Network [Han et al., NeurIPS 2015]

Pruning Happens in Human Brain

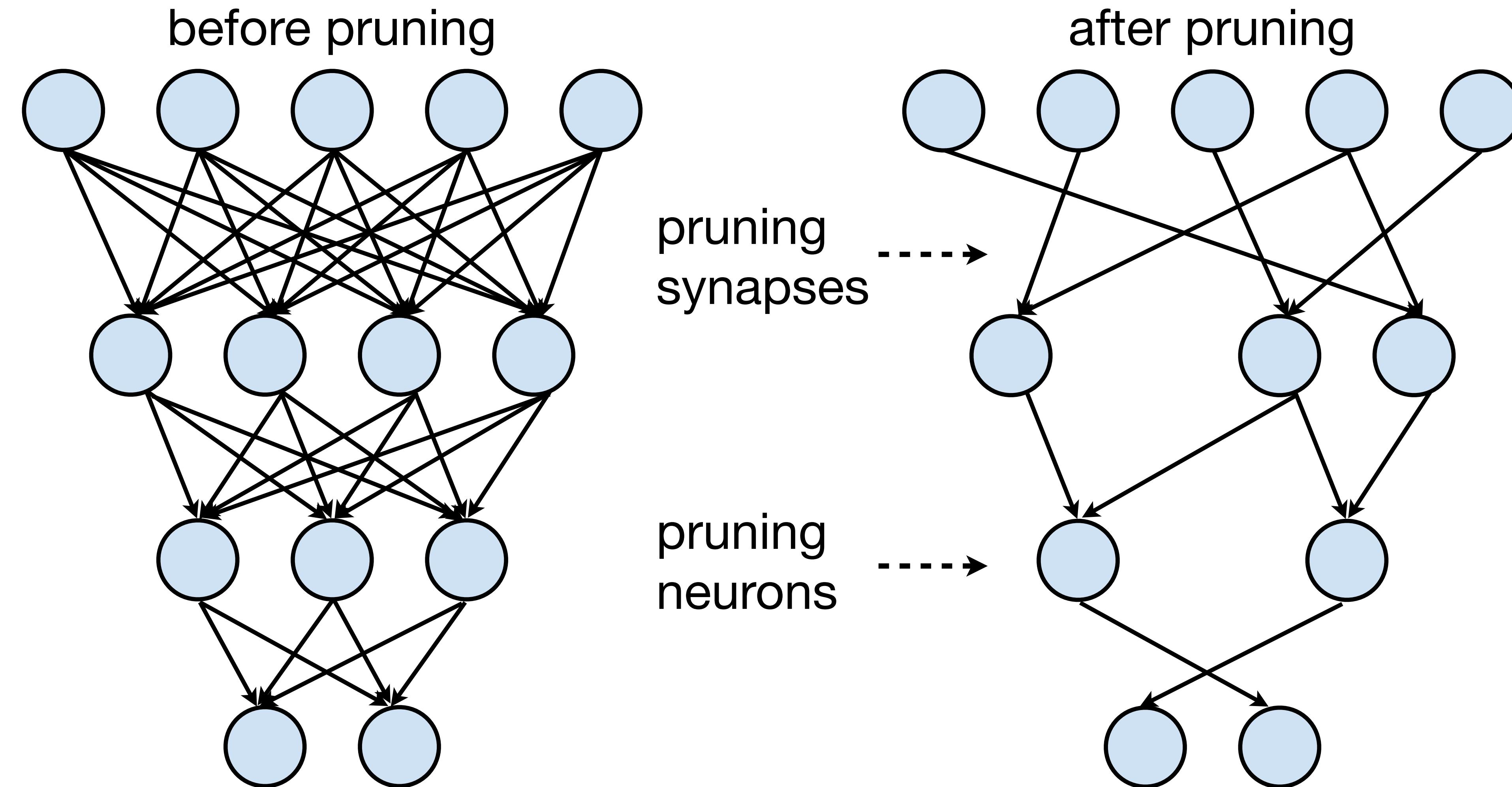


Do We Have Brain to Spare? [Drachman DA, Neurology 2004]
Peter Huttenlocher (1931–2013) [Walsh, C. A., Nature 2013]

Data Source: 1, 2
Slide Inspiration: Alila Medical Media

Neural Network Pruning

Make neural network smaller by removing synapses and neurons

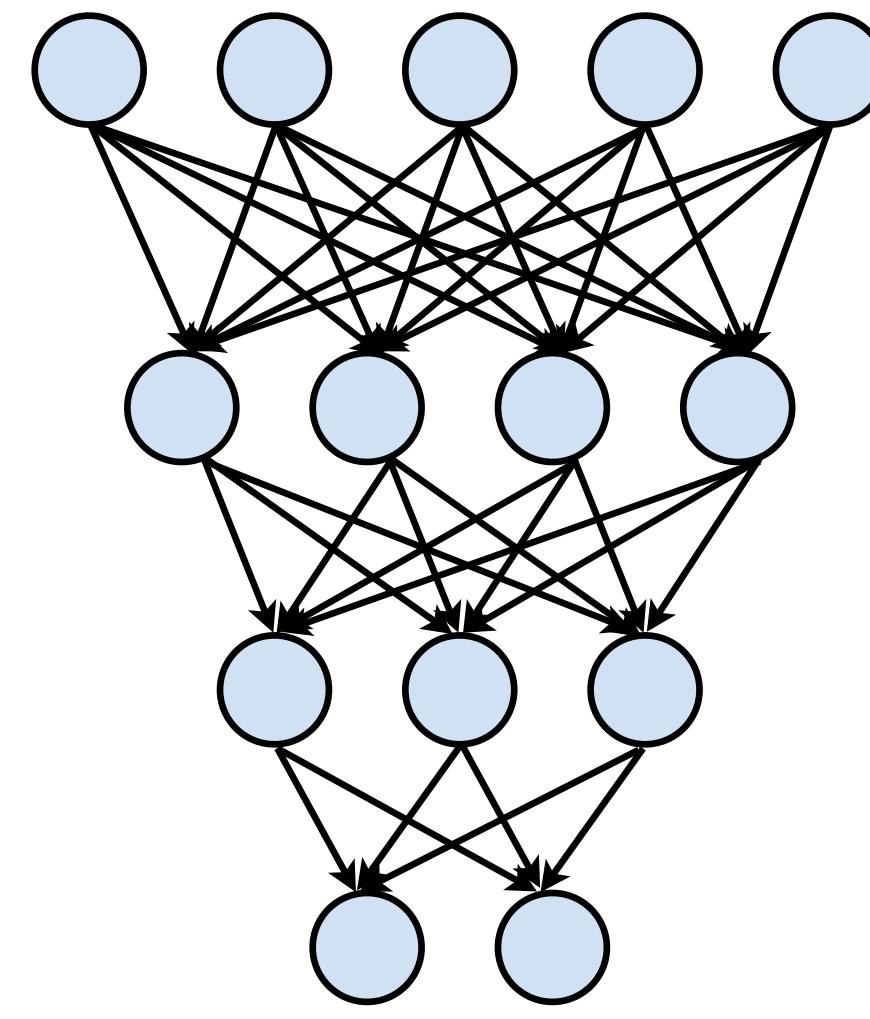


Optimal Brain Damage [LeCun et al., NeurIPS 1989]

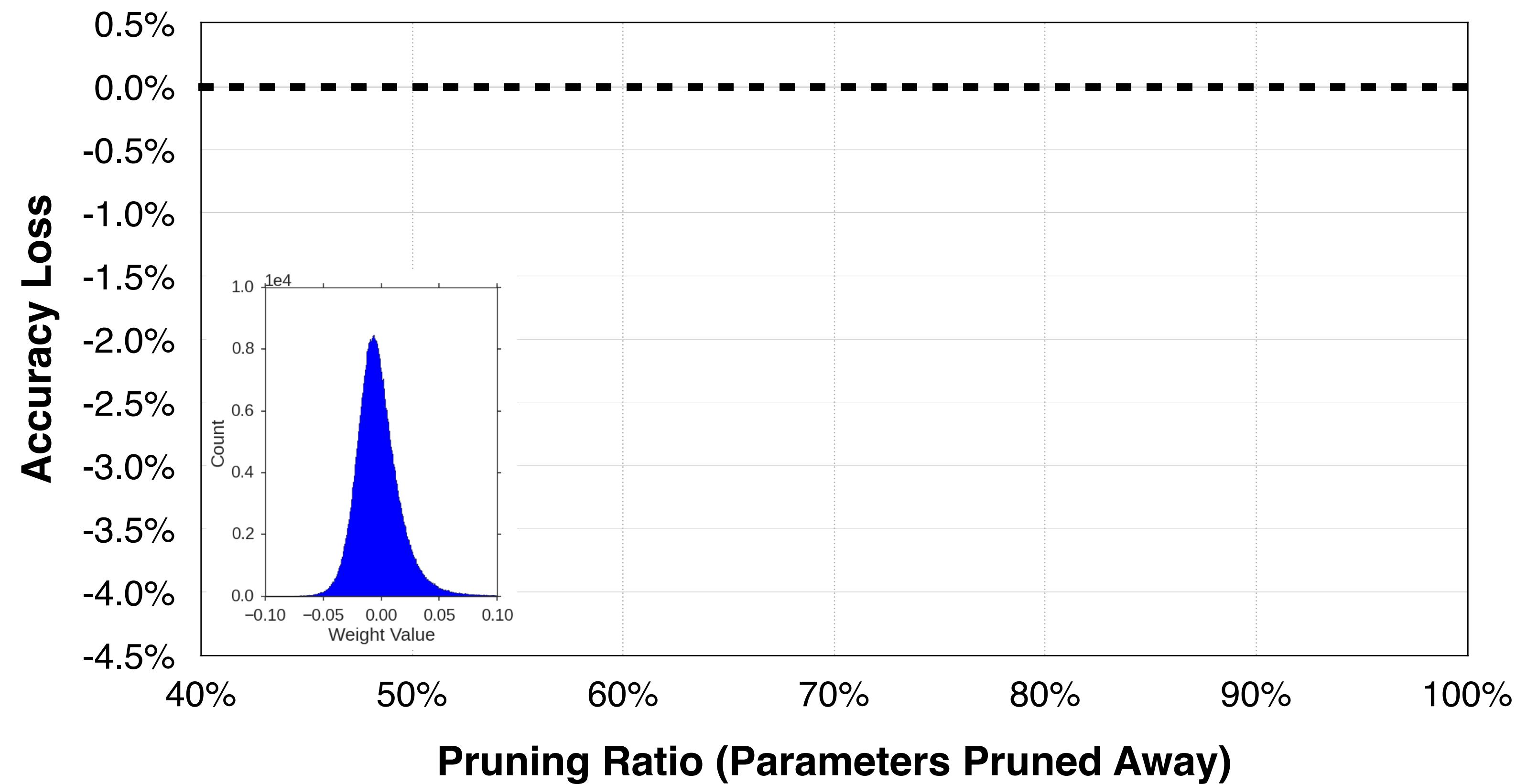
Learning Both Weights and Connections for Efficient Neural Network [Han et al., NeurIPS 2015]

Neural Network Pruning

Make neural network smaller by removing synapses and neurons



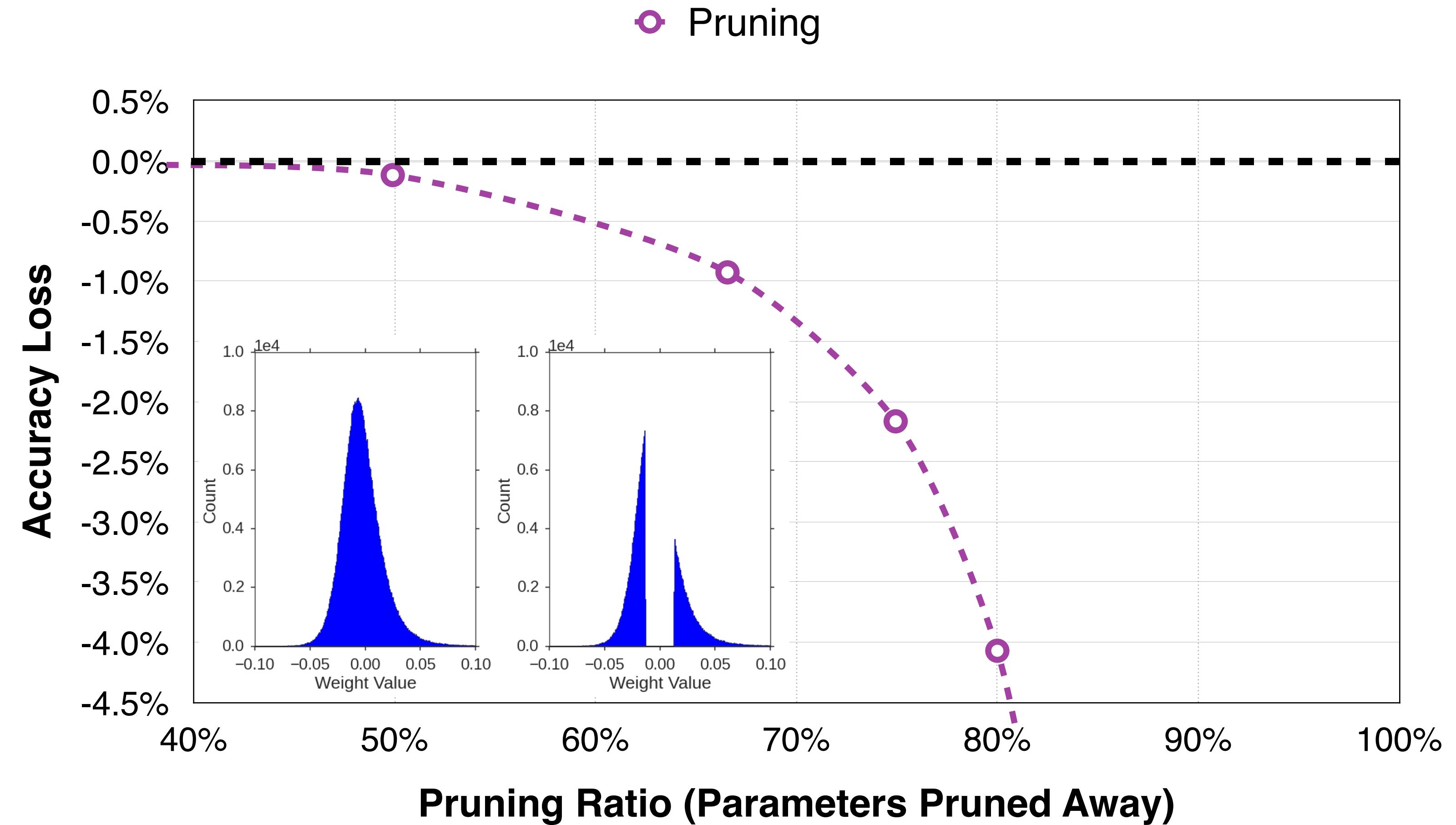
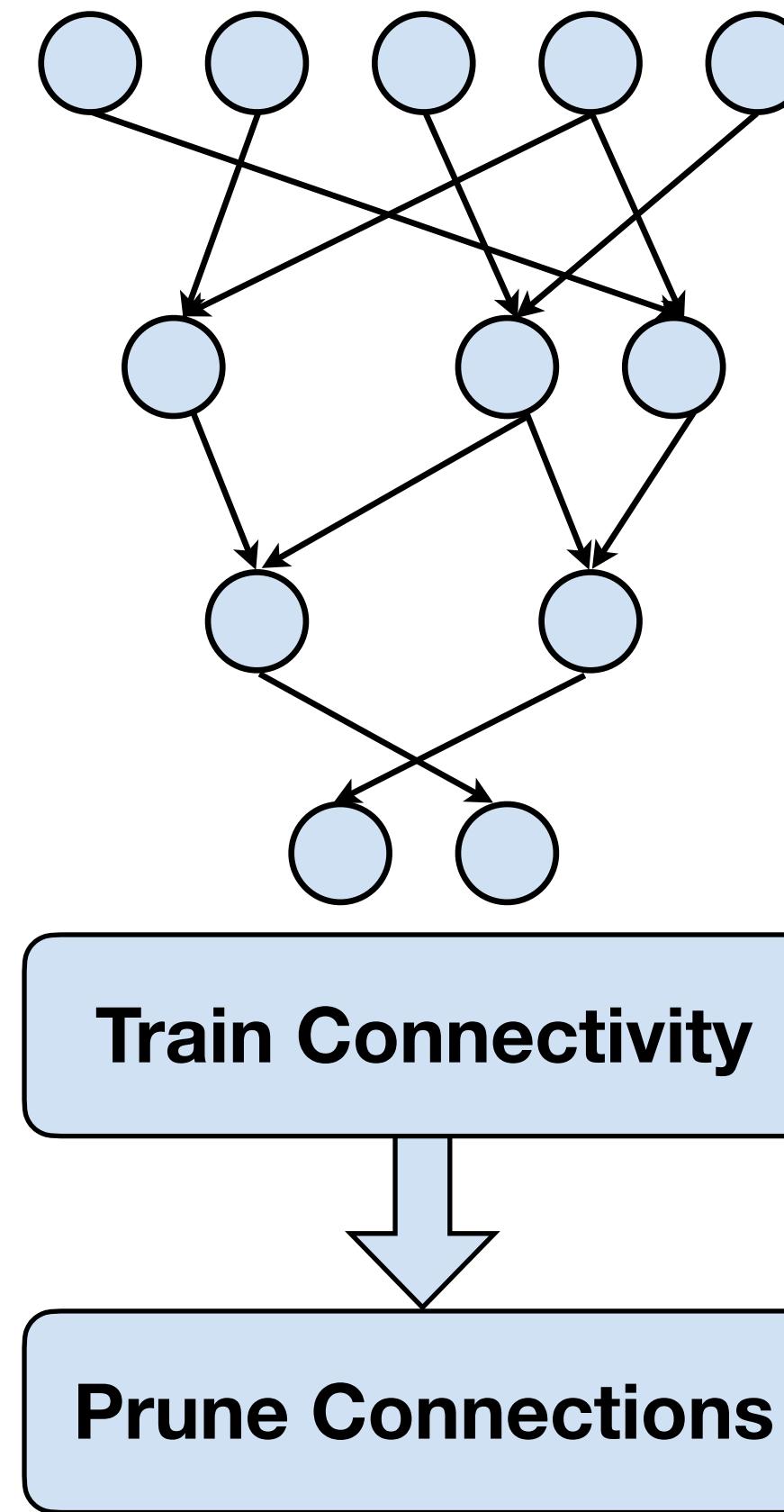
Train Connectivity



Learning Both Weights and Connections for Efficient Neural Network [Han et al., NeurIPS 2015]

Neural Network Pruning

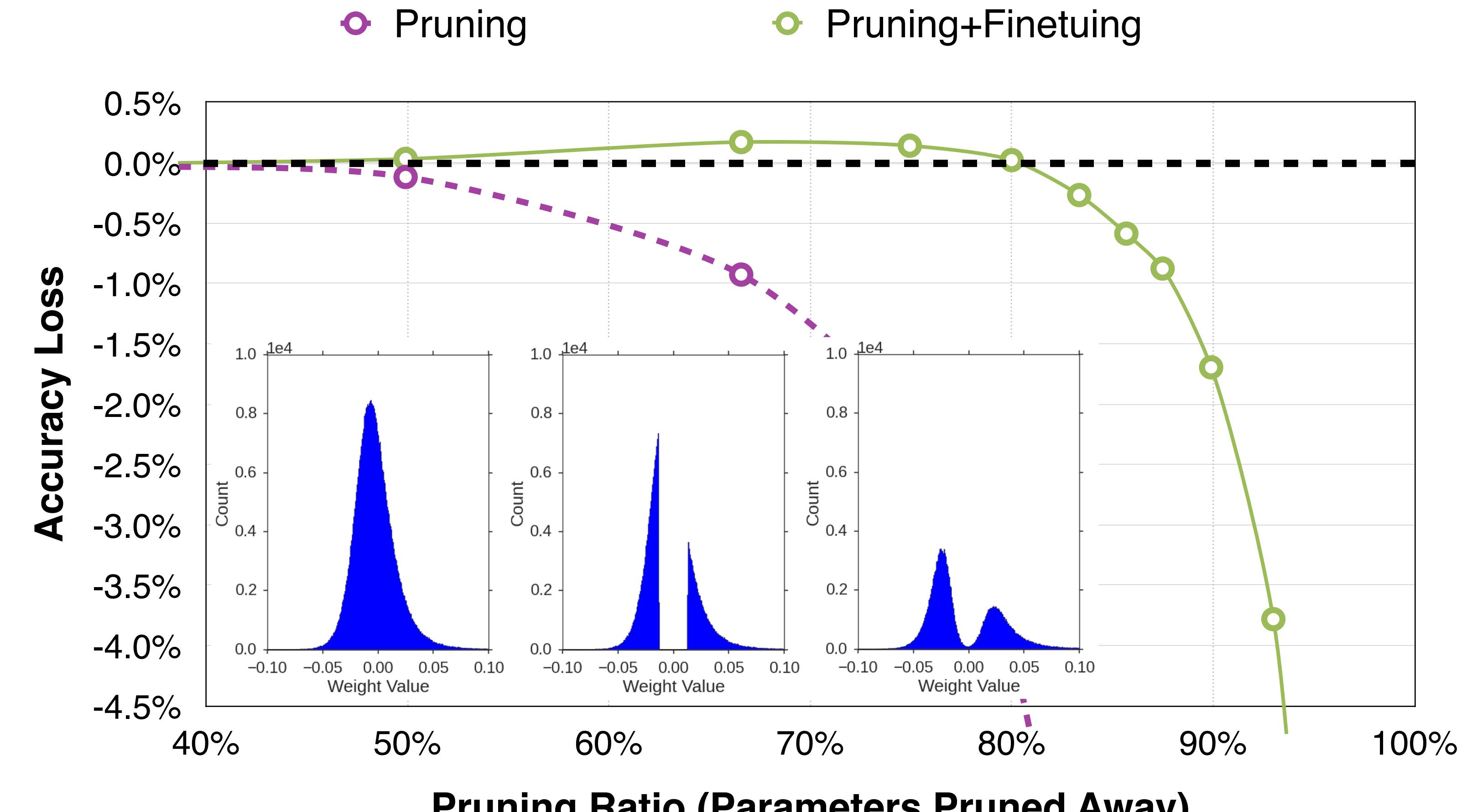
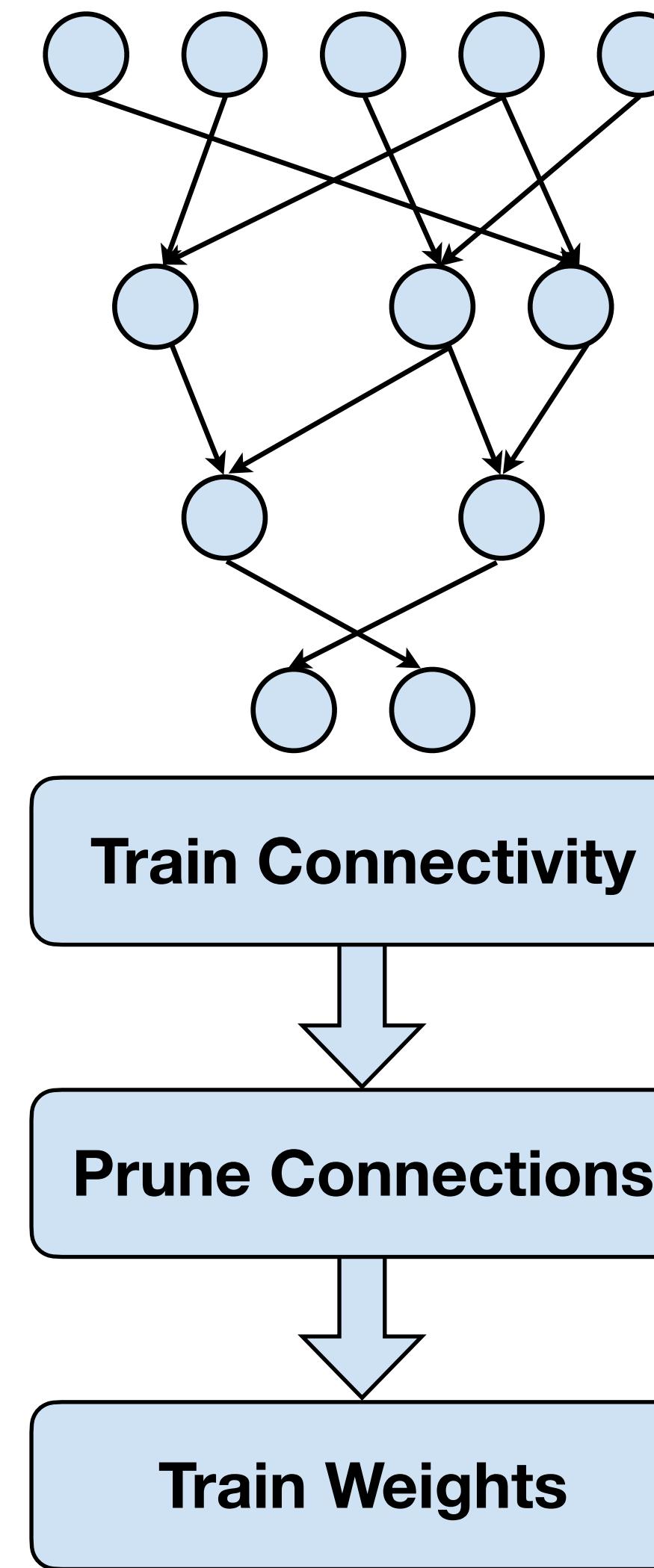
Make neural network smaller by removing synapses and neurons



Learning Both Weights and Connections for Efficient Neural Network [Han et al., NeurIPS 2015]

Neural Network Pruning

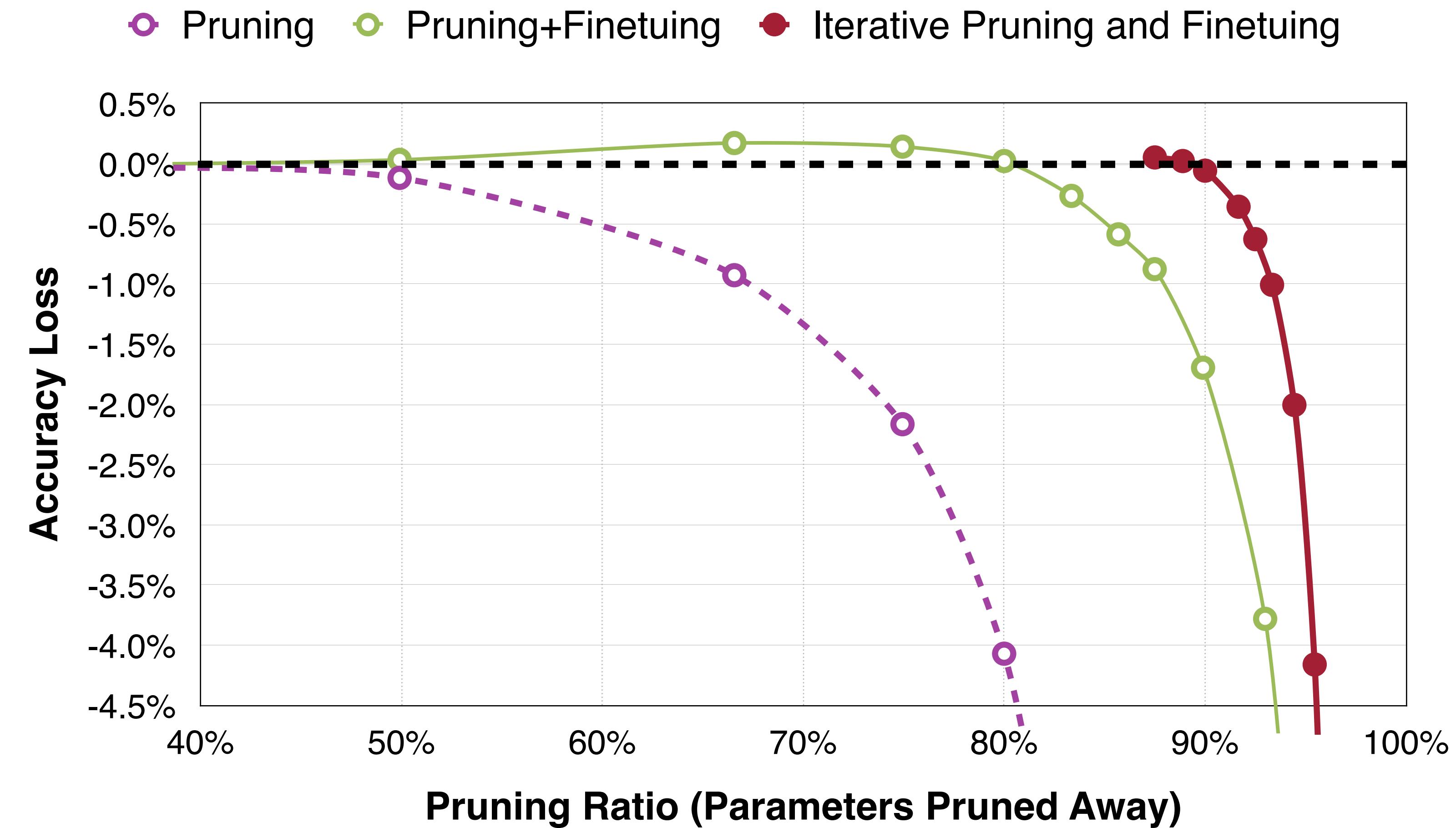
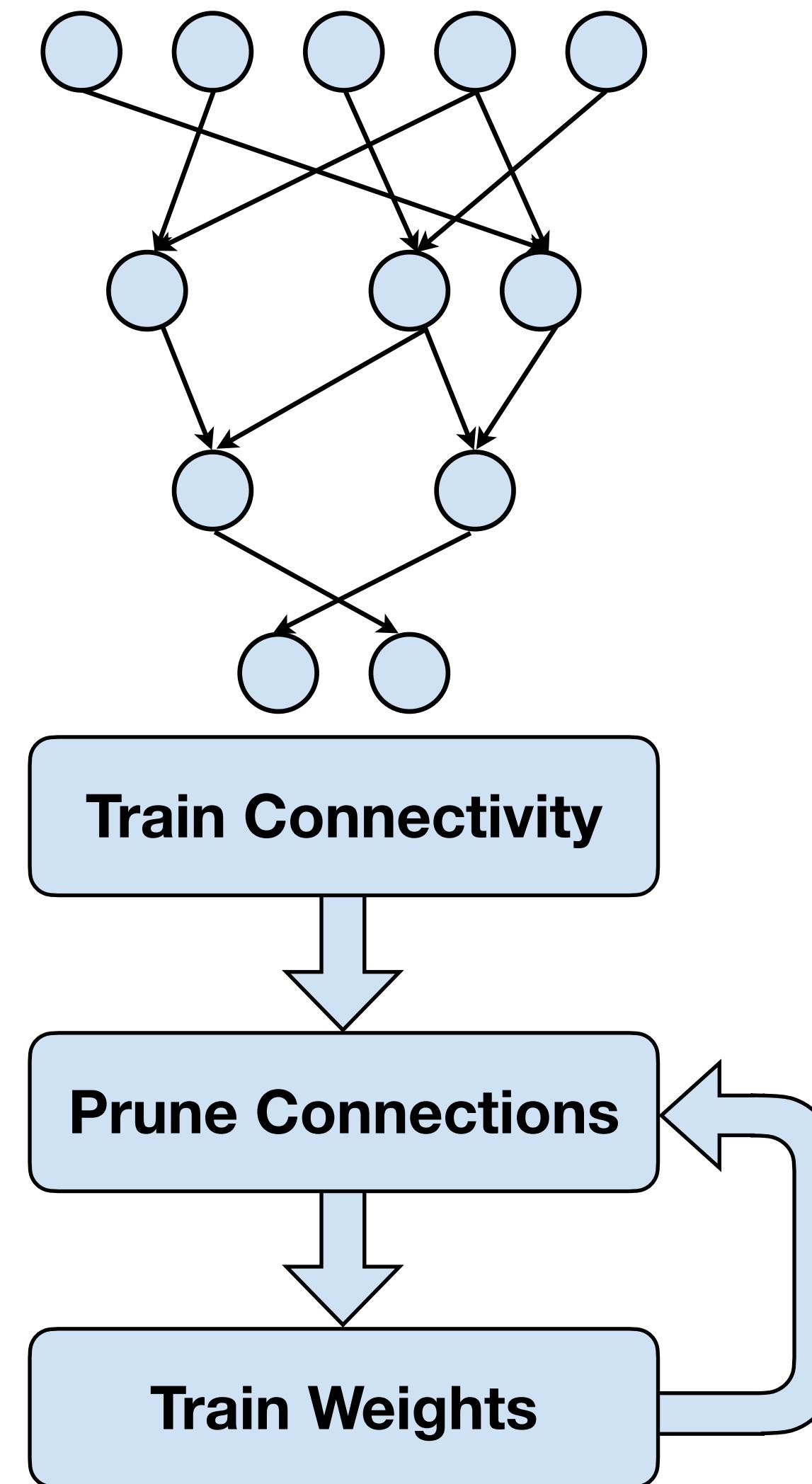
Make neural network smaller by removing synapses and neurons



Learning Both Weights and Connections for Efficient Neural Network [Han et al., NeurIPS 2015]

Neural Network Pruning

Make neural network smaller by removing synapses and neurons



Learning Both Weights and Connections for Efficient Neural Network [Han et al., NeurIPS 2015]

Neural Network Pruning

Make neural network smaller by removing synapses and neurons

Neural Network	#Parameters			MACs
	Before Pruning	After Pruning	Reduction	Reduction
AlexNet	61 M	6.7 M	9 ×	3 ×
VGG-16	138 M	10.3 M	12 ×	5 ×
GoogleNet	7 M	2.0 M	3.5 ×	5 ×
ResNet50	26 M	7.47 M	3.4 ×	6.3 ×
SqueezeNet	1 M	0.38 M	3.2 ×	3.5 ×

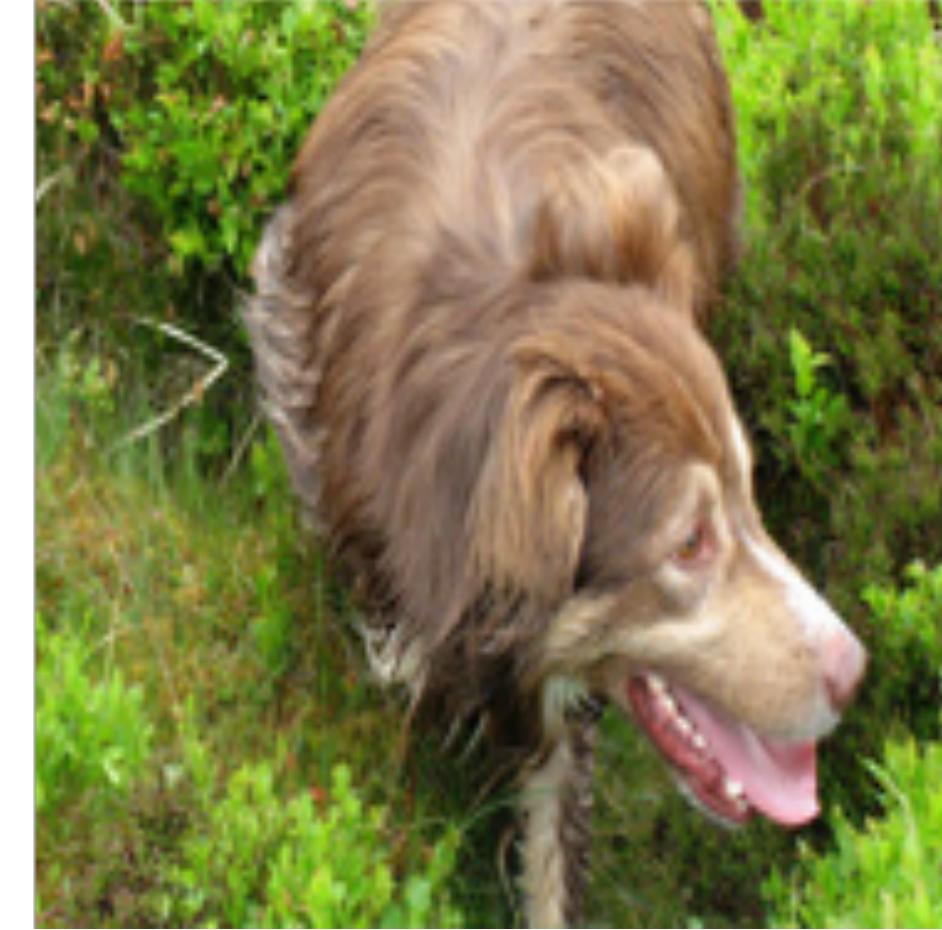
Neural Network Pruning

Pruning the NeuralTalk LSTM does not hurt image caption quality.



Baseline: a basketball player in a white uniform is playing with a **ball**.

Pruned 90%: a basketball player in a white uniform is playing with a **basketball**.



Baseline: a brown dog is running through a grassy field.

Pruned 90%: a brown dog is running through a grassy **area**.



Baseline: a man **is riding a surfboard on a wave**.

Pruned 90%: a man **in a wetsuit is riding a wave on a beach**.

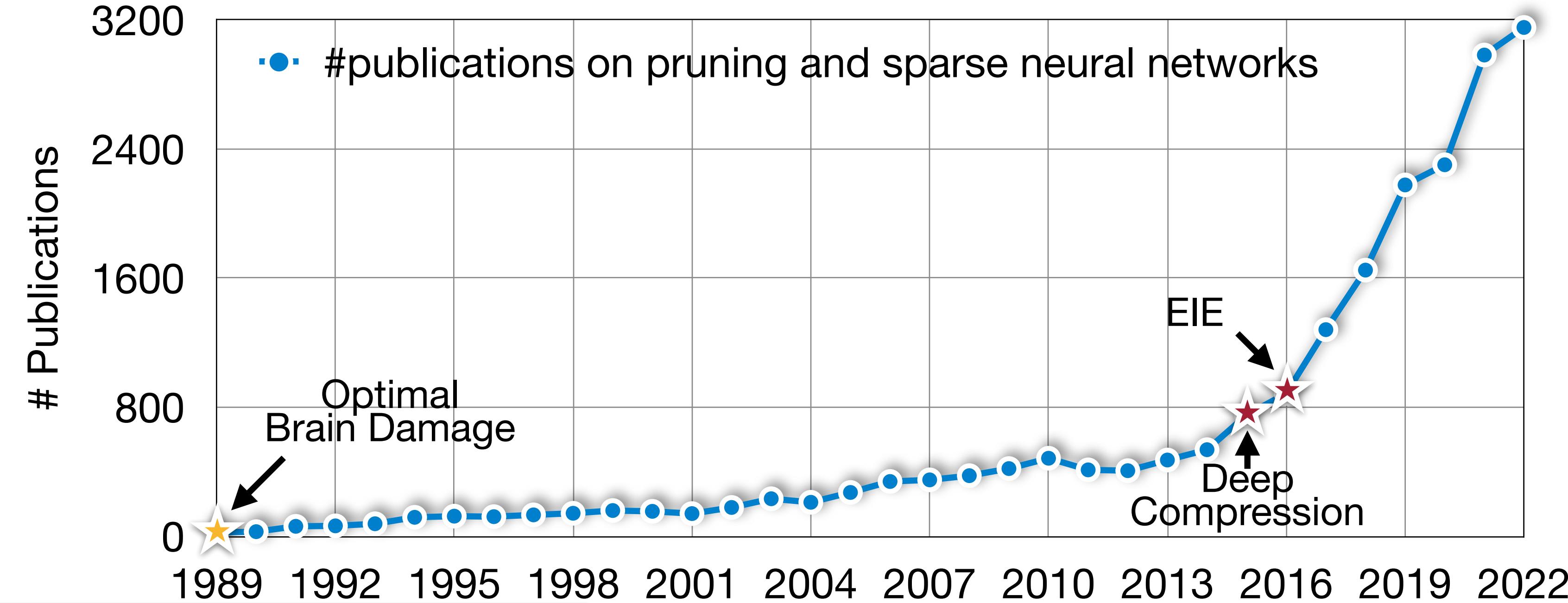


Baseline: a soccer player in red is running in the field.

Pruned 95%: a man **in a red shirt and black and white black shirt** is running through a field.

Neural Network Pruning

Make neural network smaller by removing synapses and neurons



598 Le Cun, Denker and Solla

Optimal Brain Damage

Yann Le Cun, John S. Denker and Sara A. Solla
AT&T Bell Laboratories, Holmdel, N. J. 07733

Learning both Weights and Connections for Efficient Neural Networks

Song Han
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Jeff Pool
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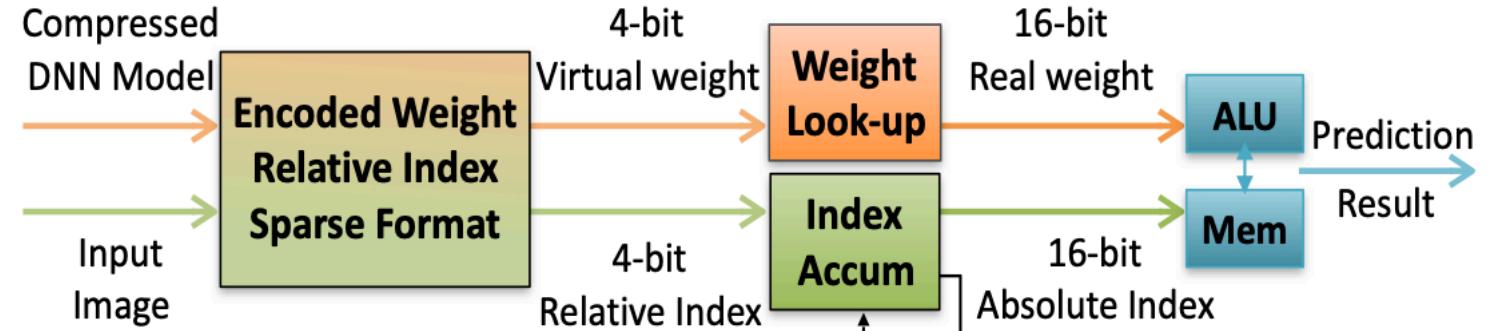
John Tran
NVIDIA
johntran@nvidia.com

William J. Dally
Stanford University
NVIDIA
dally@stanford.edu

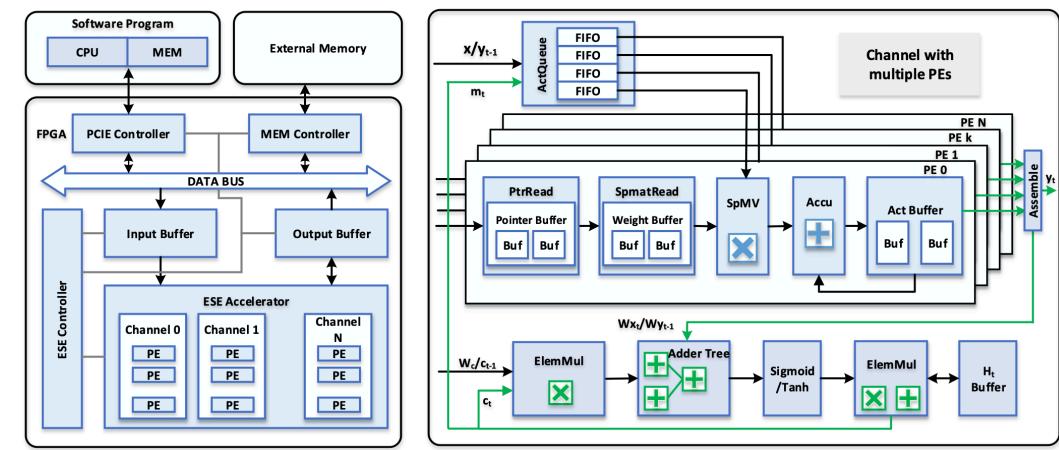
Source: <https://github.com/mit-han-lab/pruning-sparsity-publications>

Pruning in the Industry

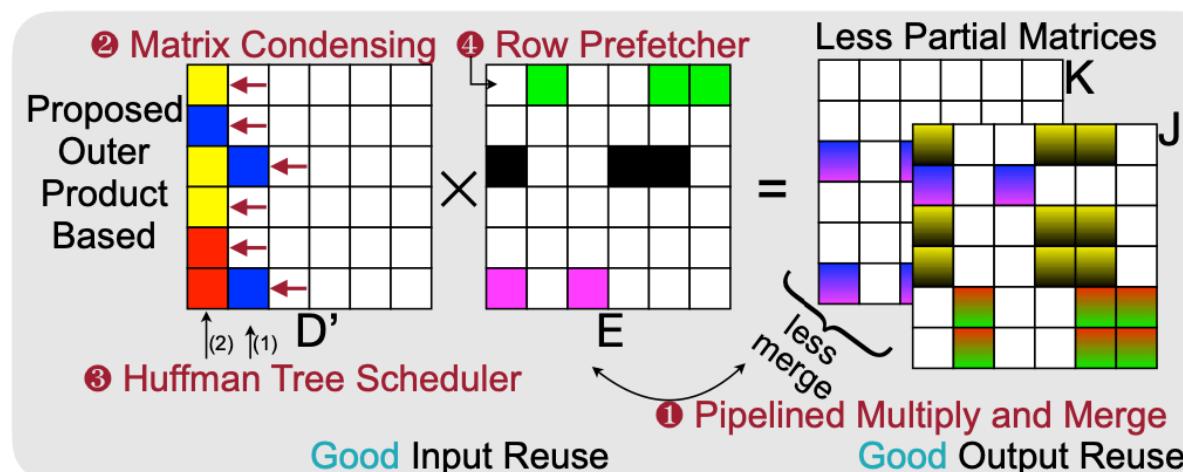
Hardware support for sparsity



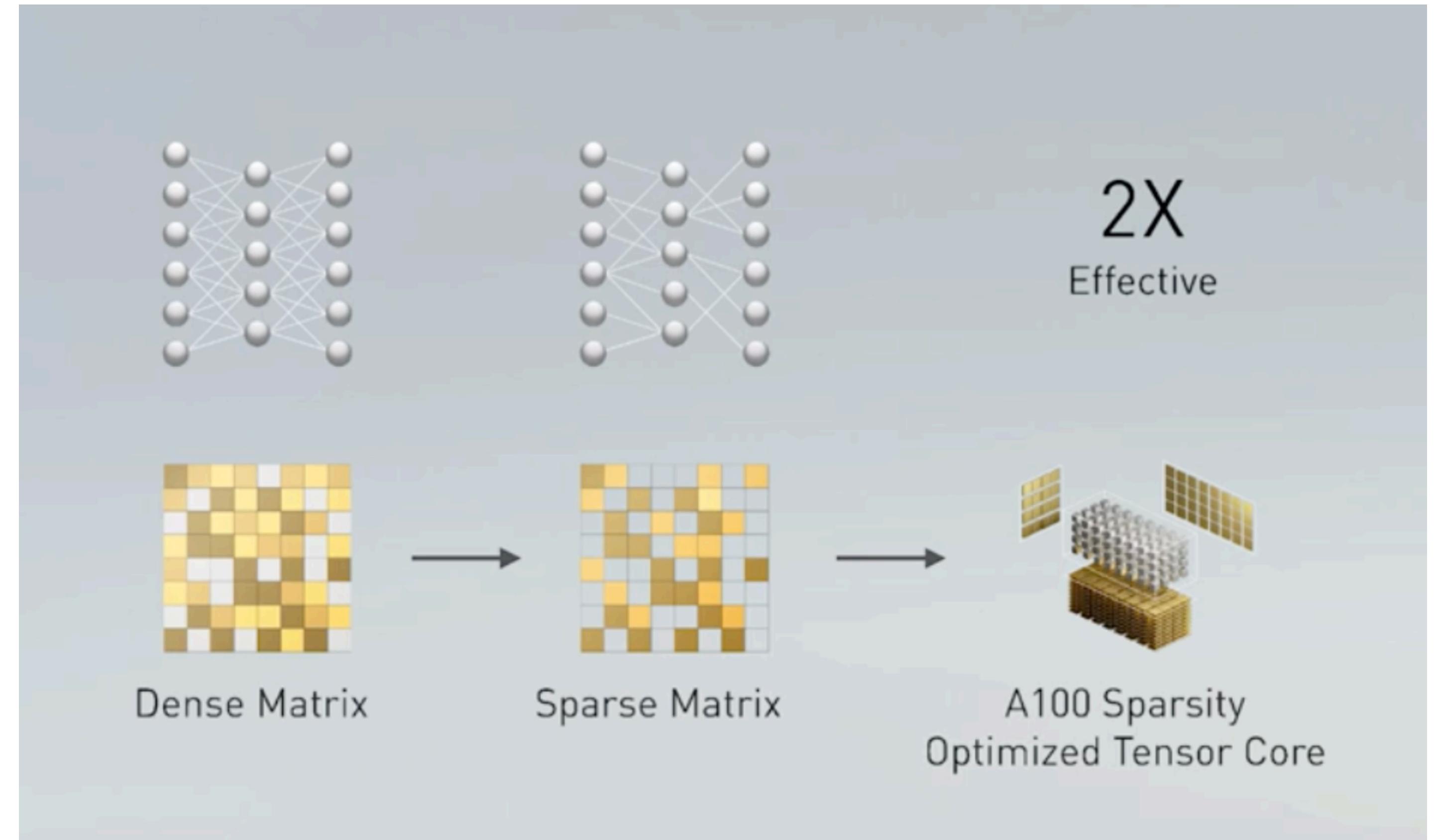
EIE [Han et al., ISCA 2016]



ESE [Han et al., FPGA 2017]



SpArch [Zhang et al., HPCA 2020]
SpAtten [Wang et al., HPCA 2021]



2:4 sparsity in A100 GPU
2X peak performance, 1.5X measured BERT speedup

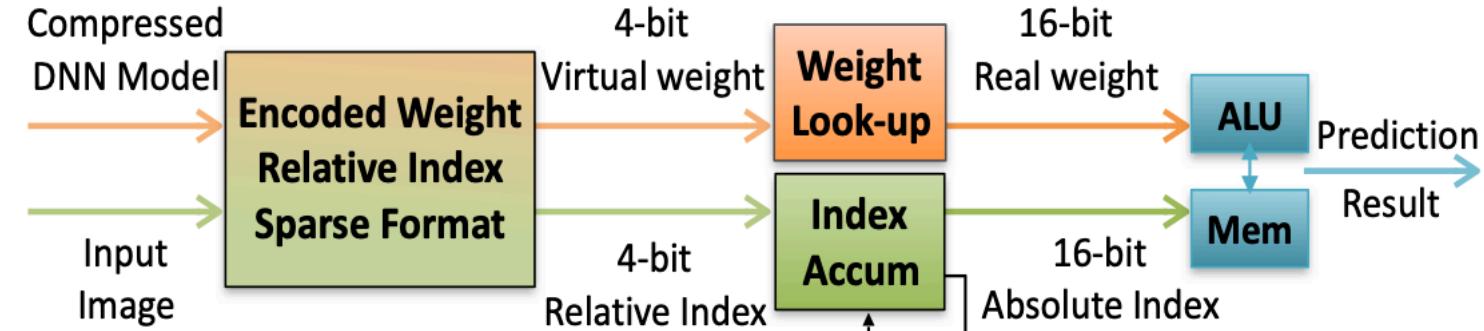
Pruning in the Industry

Hardware support for sparsity

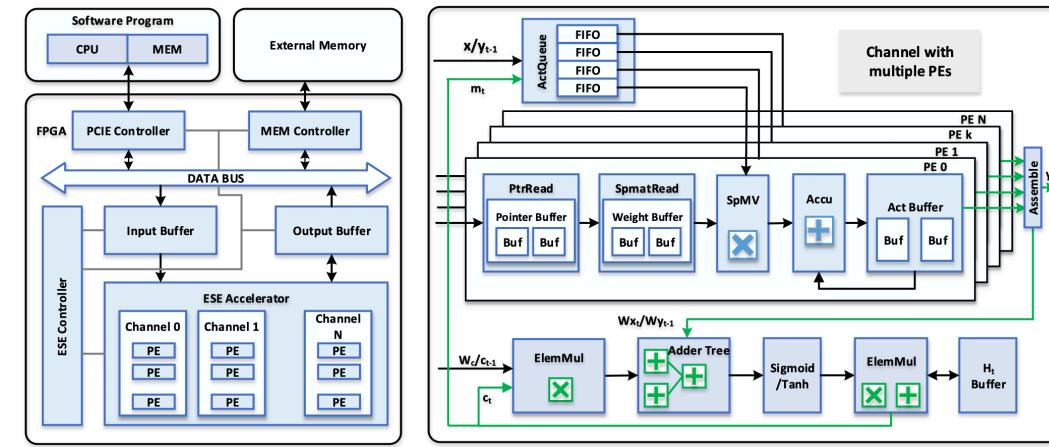
AMD
XILINX



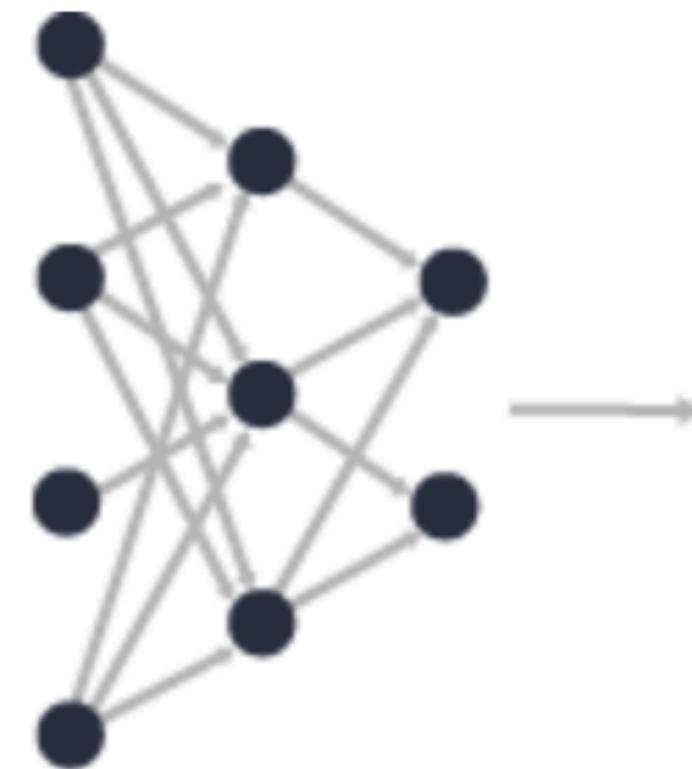
XILINX
VITIS™



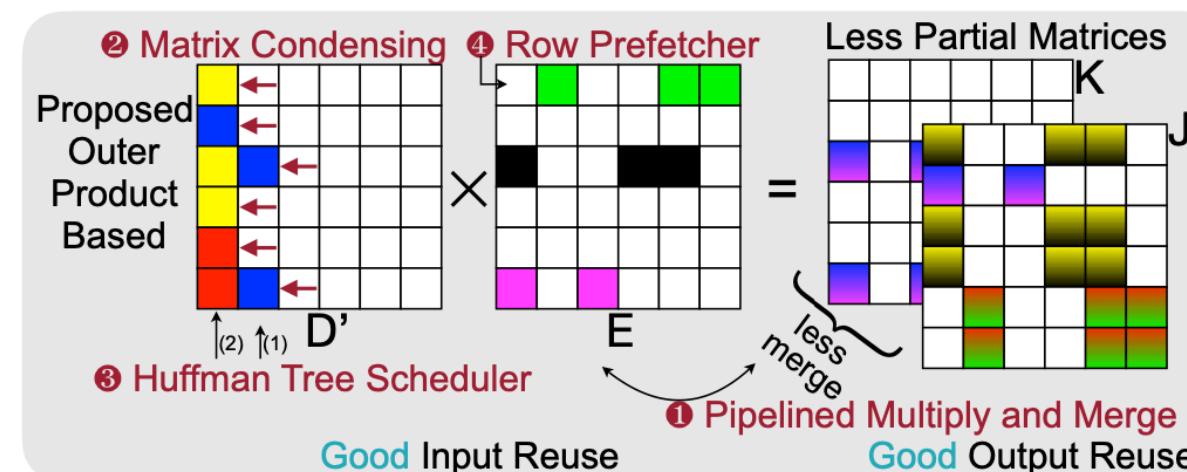
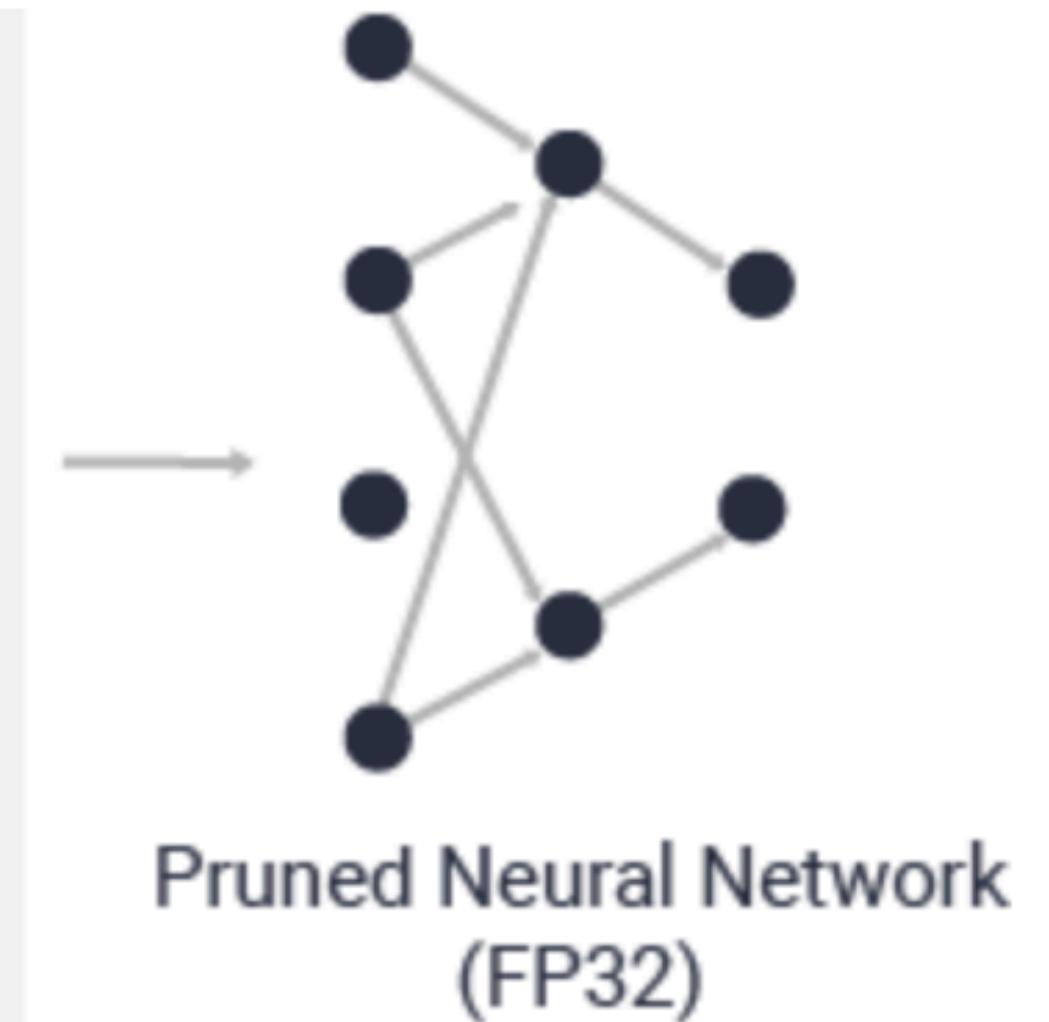
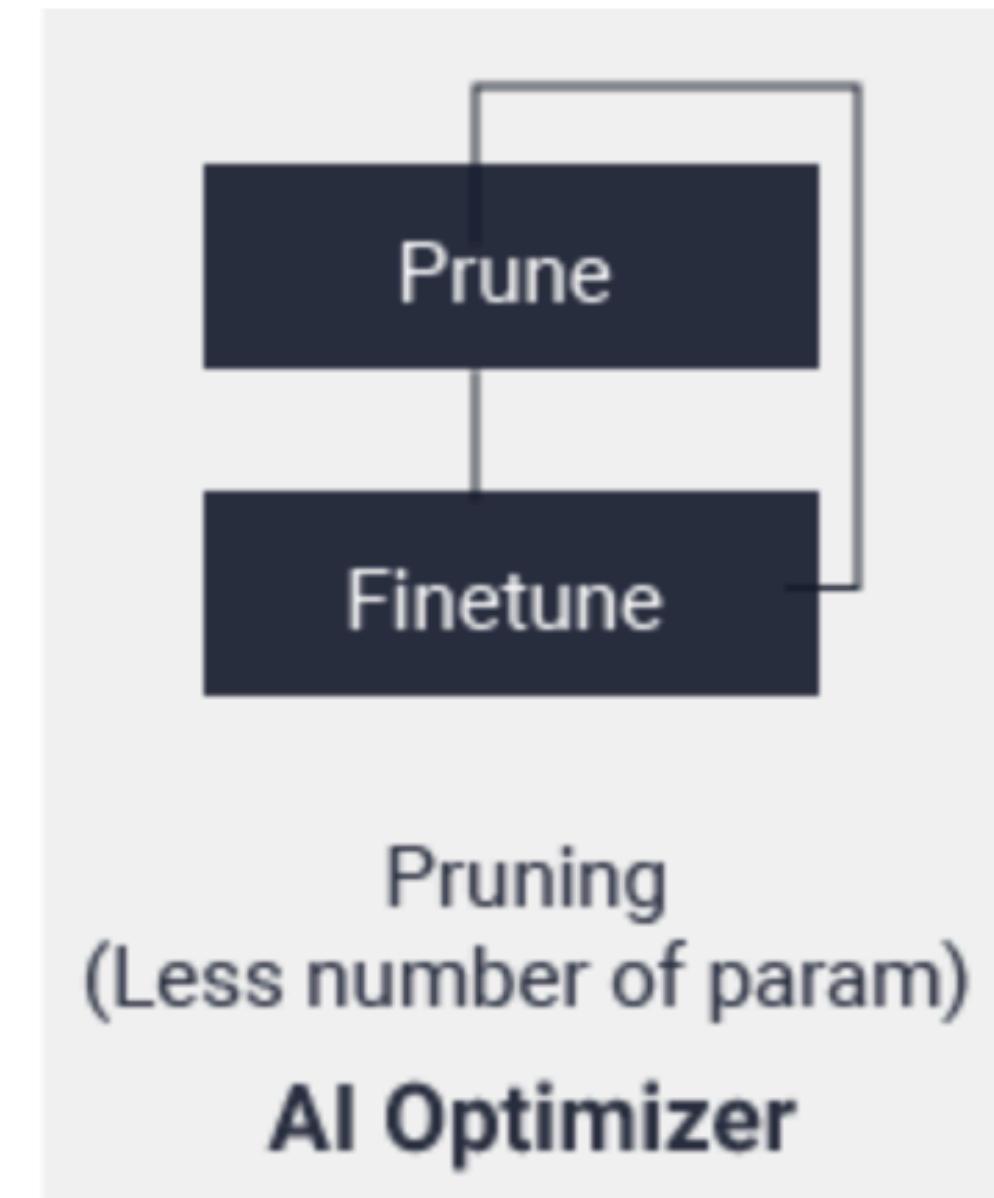
EIE [Han et al., ISCA 2016]



ESE [Han et al., FPGA 2017]



Dense Neural Network
(FP32)

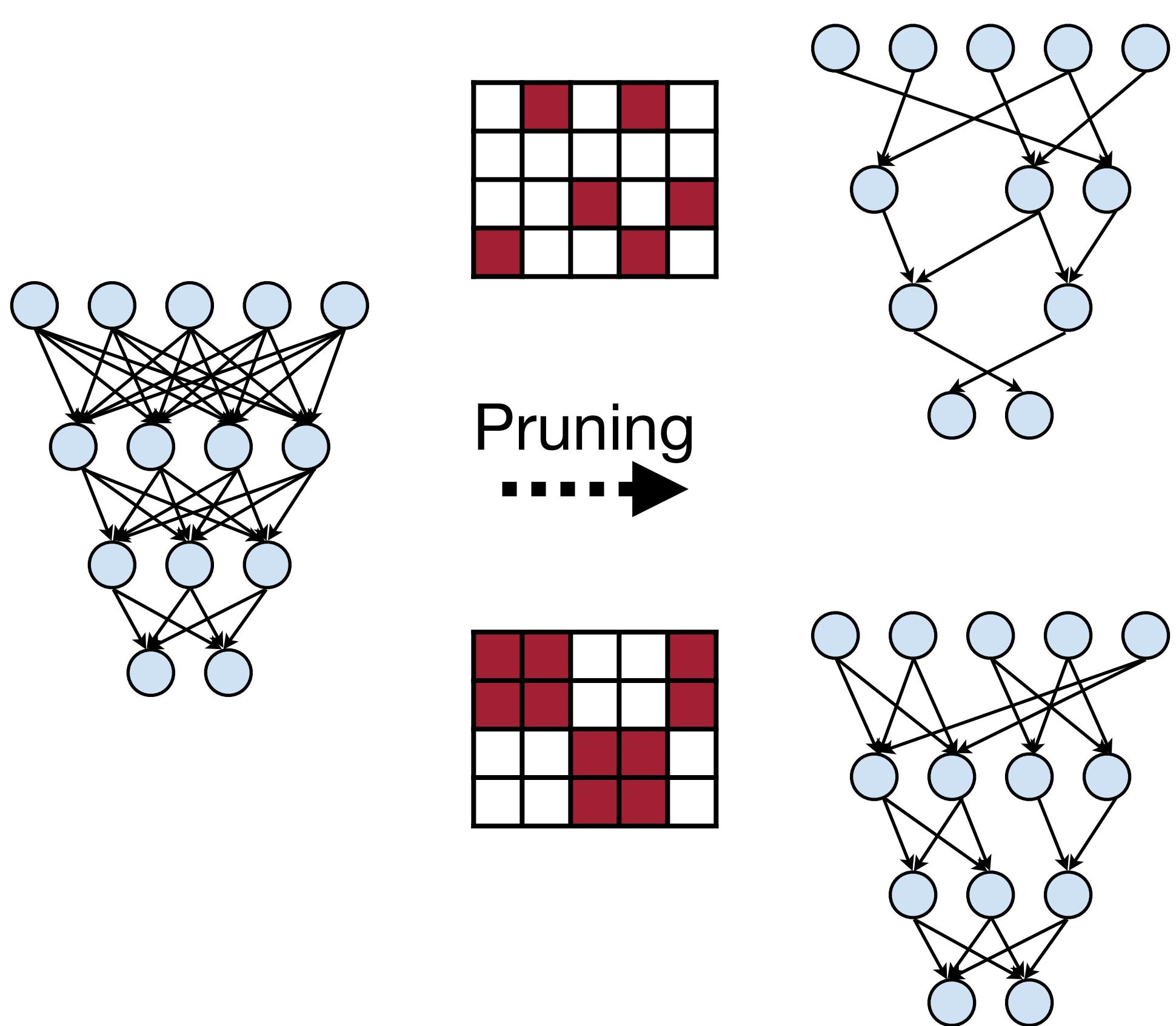


SpArch [Zhang et al., HPCA 2020]
SpAtten [Wang et al., HPCA 2021]

Reduce model complexity by 5x to 50x with minimal accuracy impact

Neural Network Pruning

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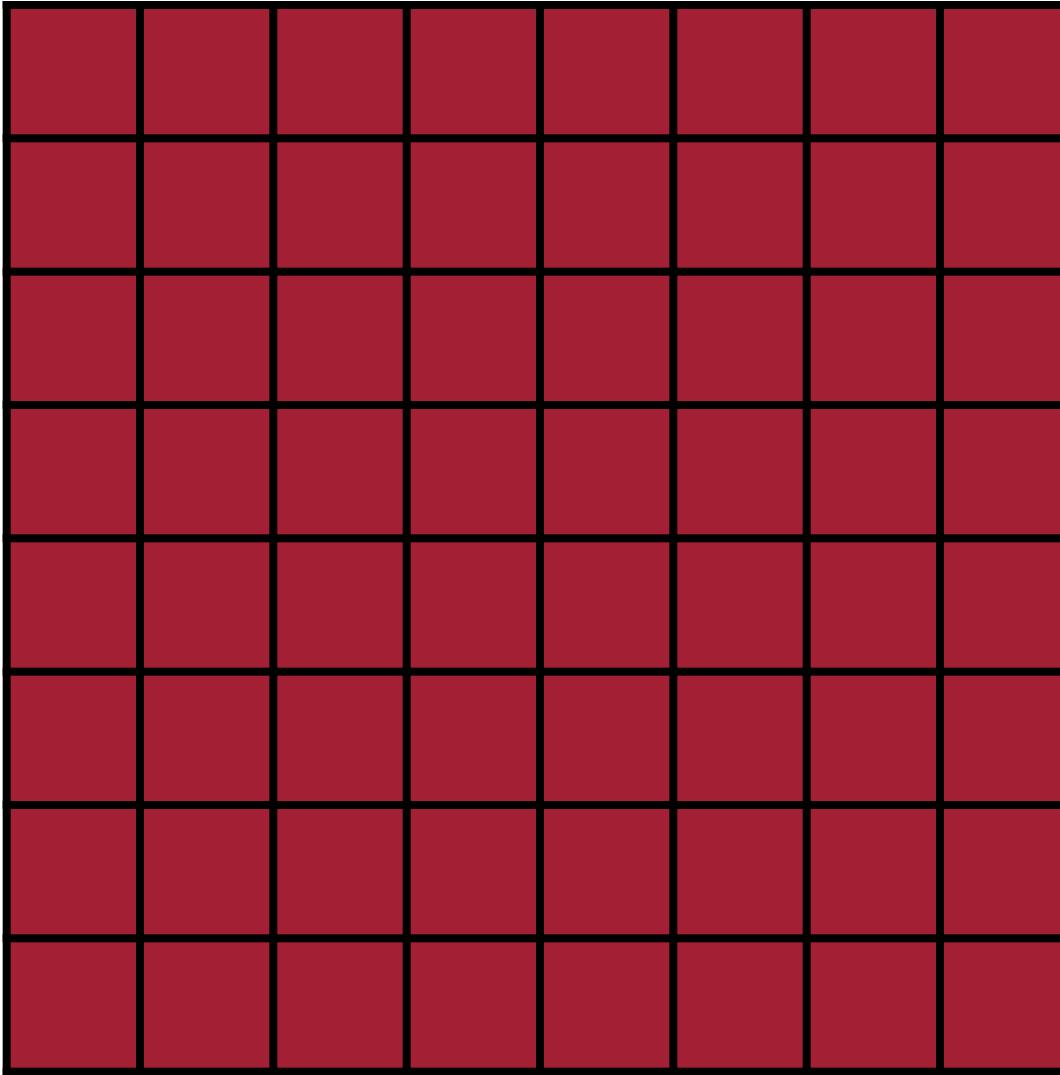
Learning Both Weights and Connections for Efficient Neural Network [Han et al., NeurIPS 2015]

Section 2: Pruning Granularity

Pruning can be performed at different granularities, from structured to non-structured.

Pruning at Different Granularities

A simple example of 2D weight matrix



Pruning at Different Granularities

A simple example of 2D weight matrix

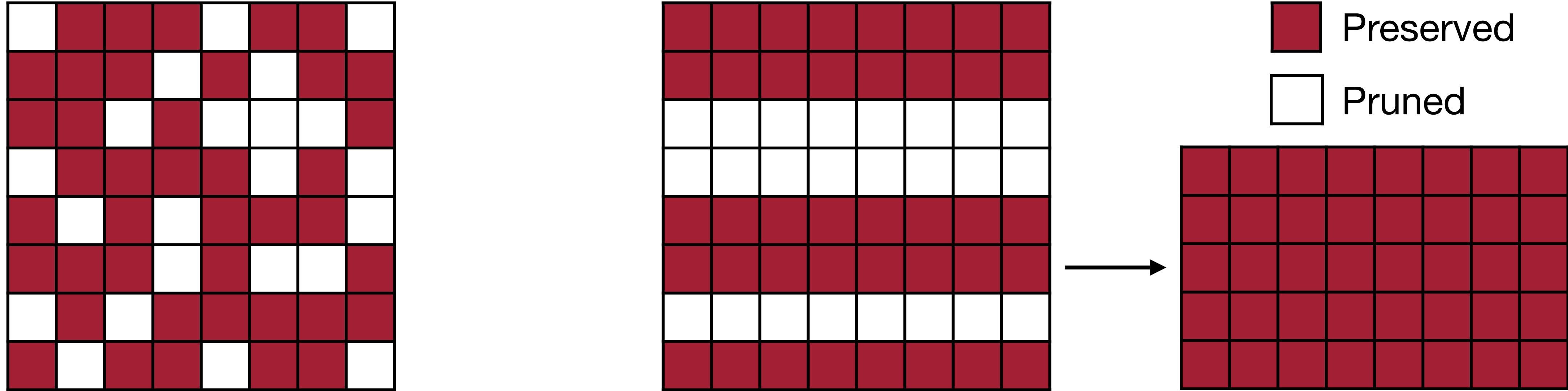


Fine-grained/Unstructured

- More flexible pruning index choice
- Hard to accelerate (irregular)

Pruning at Different Granularities

A simple example of 2D weight matrix



Fine-grained/Unstructured

- More flexible pruning index choice
- Hard to accelerate (irregular)

Coarse-grained/Structured

- Less flexible pruning index choice (a subset of the fine-grained case)
- Easy to accelerate (just a smaller matrix!)

Pruning at Different Granularities

The case of convolutional layers

- The weights of convolutional layers have 4 dimensions $[c_o, c_i, k_h, k_w]$:
 - c_i : input channels (or channels)
 - c_o : output channels (or filters)
 - k_h : kernel size height
 - k_w : kernel size width
- The 4 dimensions give us more choices to select pruning granularities

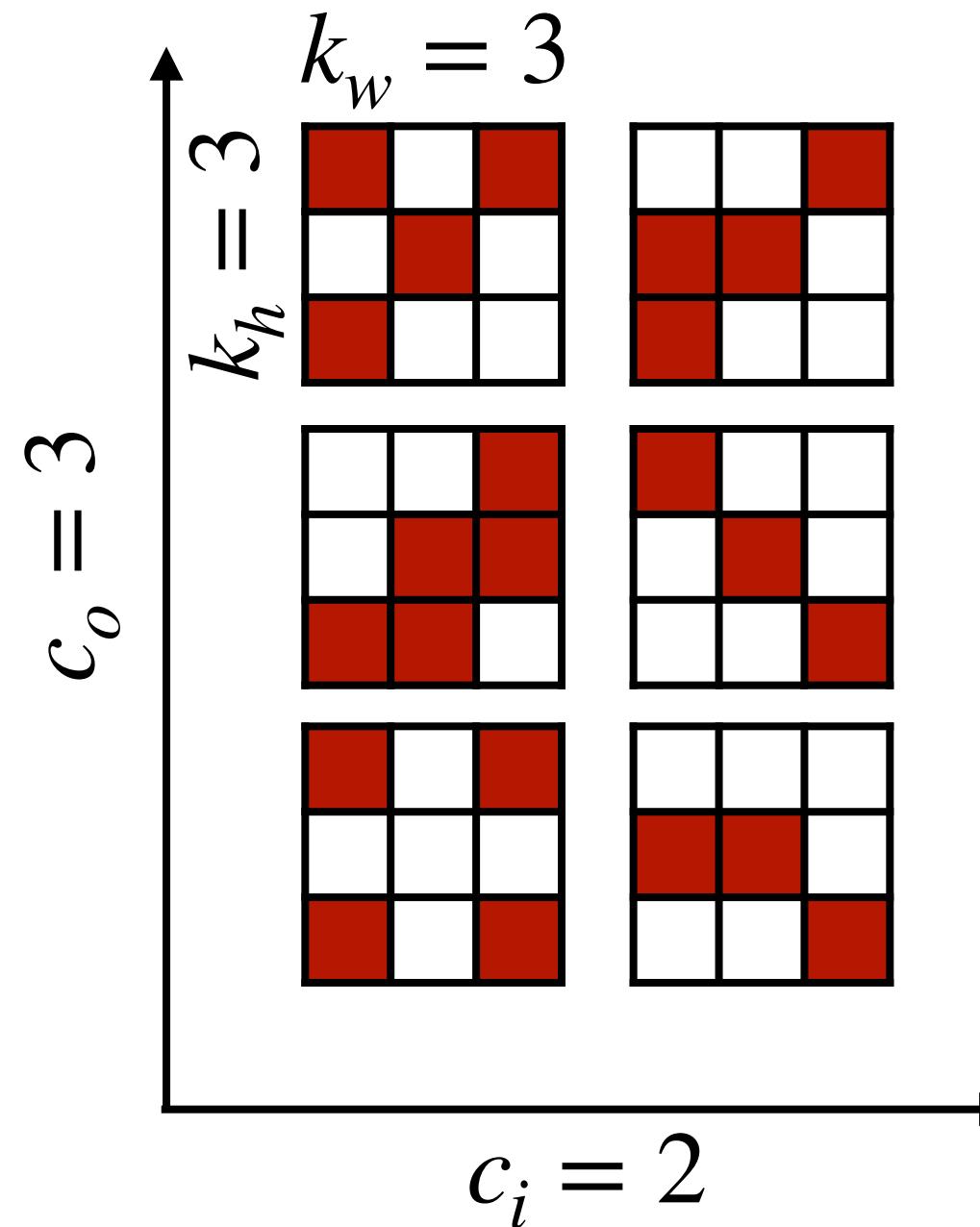
Pruning at Different Granularities

The case of convolutional layers

- Some of the commonly used pruning granularities

■ Preserved
□ Pruned

Notations

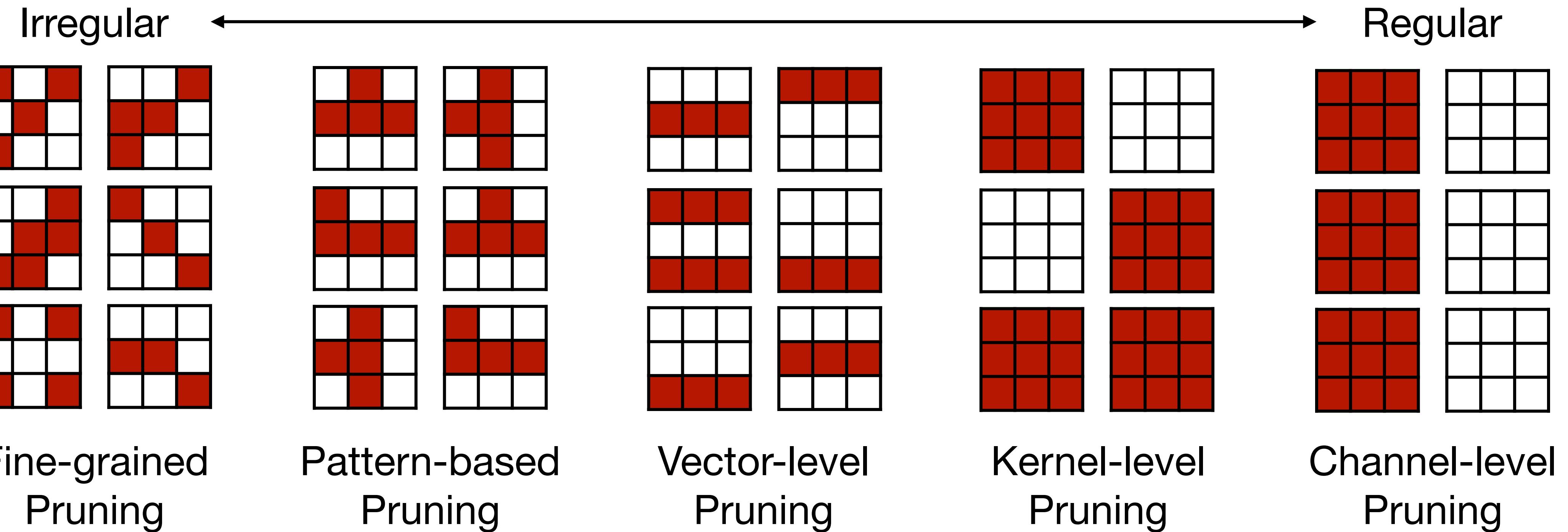
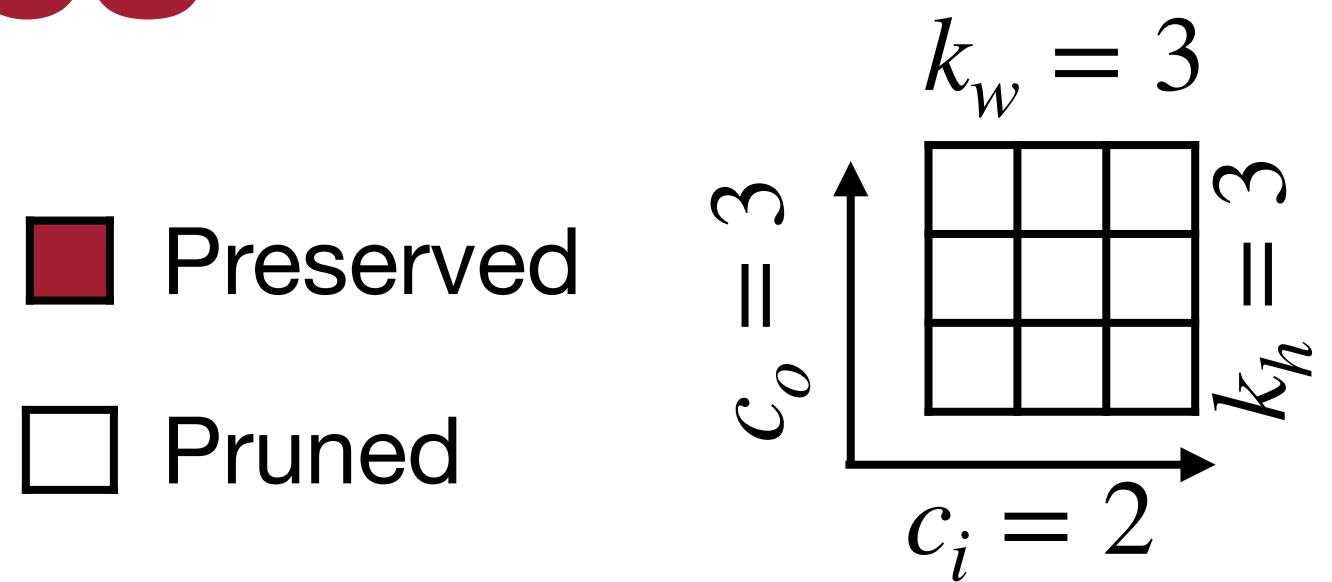


Exploring the granularity of sparsity in convolutional neural networks [Mao et al., CVPR-W]

Pruning at Different Granularities

The case of convolutional layers

- Some of the commonly used pruning granularities



like Tetris :)

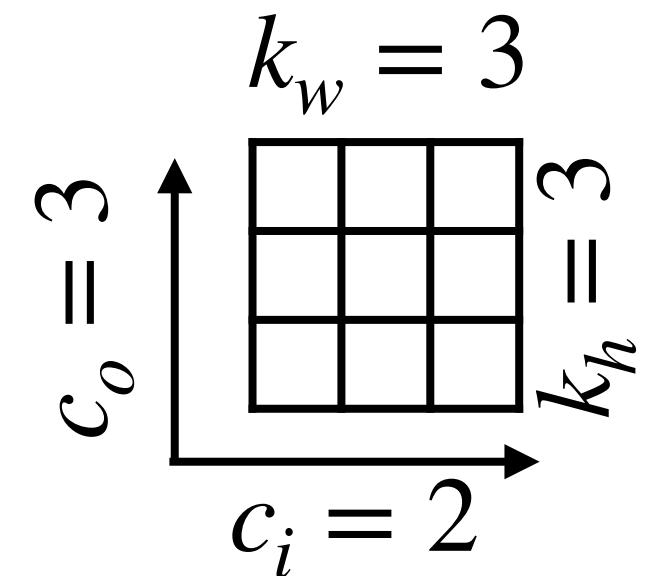
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Pruning at Different Granularities

The case of convolutional layers

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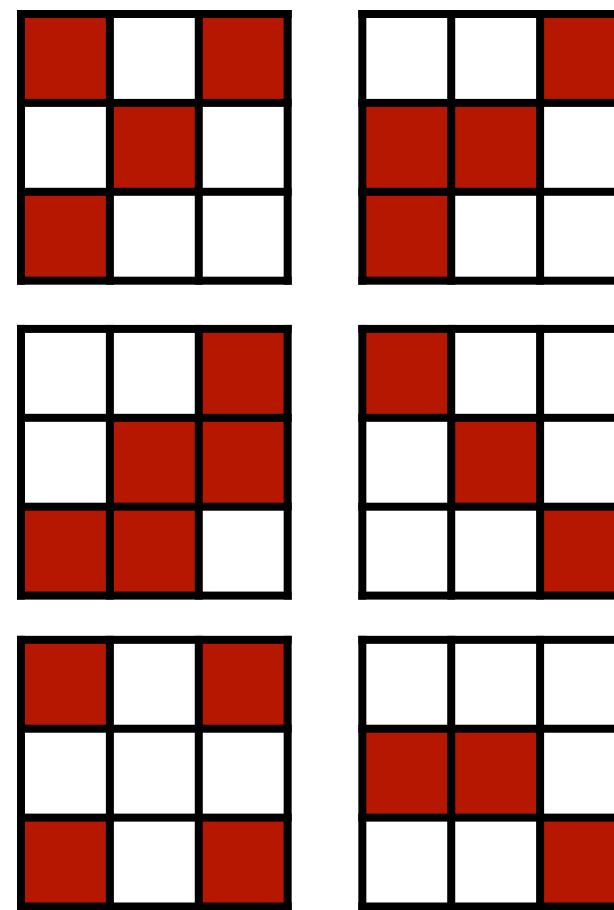
■ Preserved
□ Pruned



Pros? Cons?

Irregular

Regular



Fine-grained
Pruning

Pattern-based
Pruning

Vector-level
Pruning

Kernel-level
Pruning

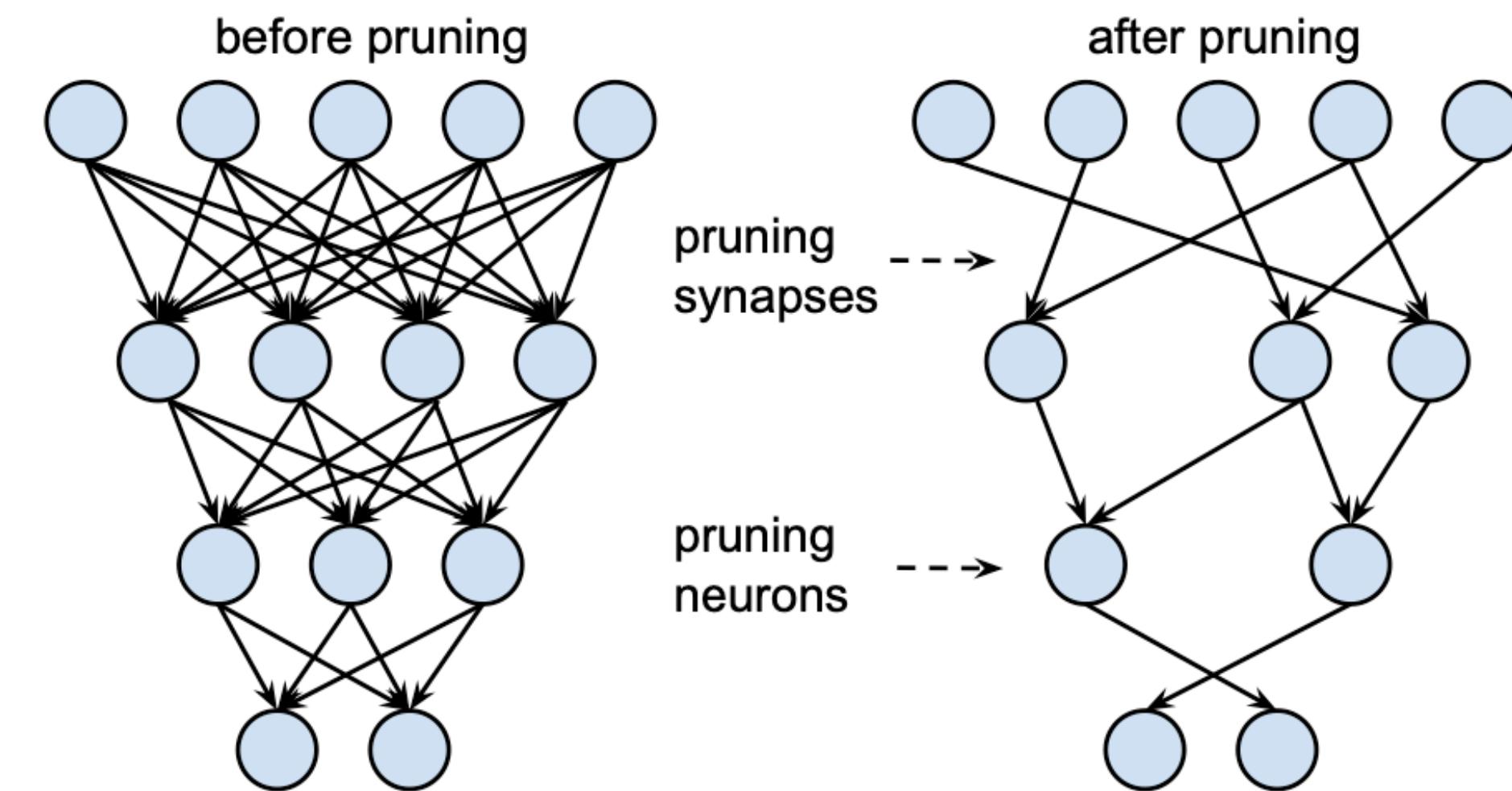
Channel-level
Pruning

Exploring the granularity of sparsity in convolutional neural networks [Mao et al., CVPR-W]

Pruning at Different Granularities

Let's look into some cases

- **Fine-grained Pruning** (the case we show before)
 - Flexible pruning indices



Learning Both Weights and Connections for Efficient Neural Network [Han et al., NeurIPS 2015]

Pruning at Different Granularities

Let's look into some cases

- **Fine-grained Pruning** (the case we show before)
 - Flexible pruning indices
 - Usually larger compression ratio since we can flexibly find “redundant” weights (we will later discuss how we find them)

Neural Network	#Parameters		
	Before Pruning	After Pruning	Reduction
AlexNet	61 M	6.7 M	9 ×
VGG-16	138 M	10.3 M	12 ×
GoogleNet	7 M	2.0 M	3.5 ×
ResNet50	26 M	7.47 M	3.4 ×

Pruning at Different Granularities

Let's look into some cases

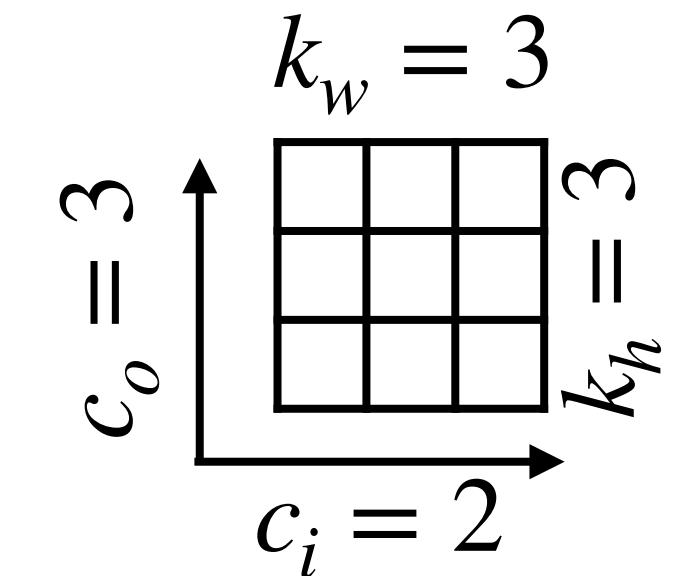
- **Fine-grained Pruning** (the case we show before)
 - Flexible pruning indices
 - Usually larger compression ratio since we can flexibly find “redundant” weights (we will later discuss how we find them)
 - Can deliver speed up on some custom hardware (e.g., EIE) but not GPU (easily)

Pruning at Different Granularities

The case of convolutional layers

- Some of the commonly used pruning granularities

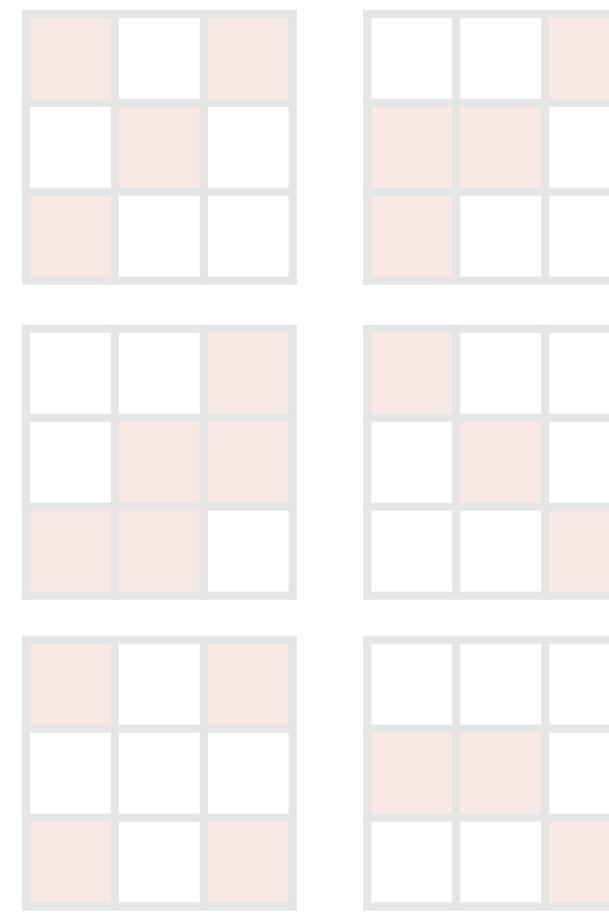
■ Preserved
□ Pruned



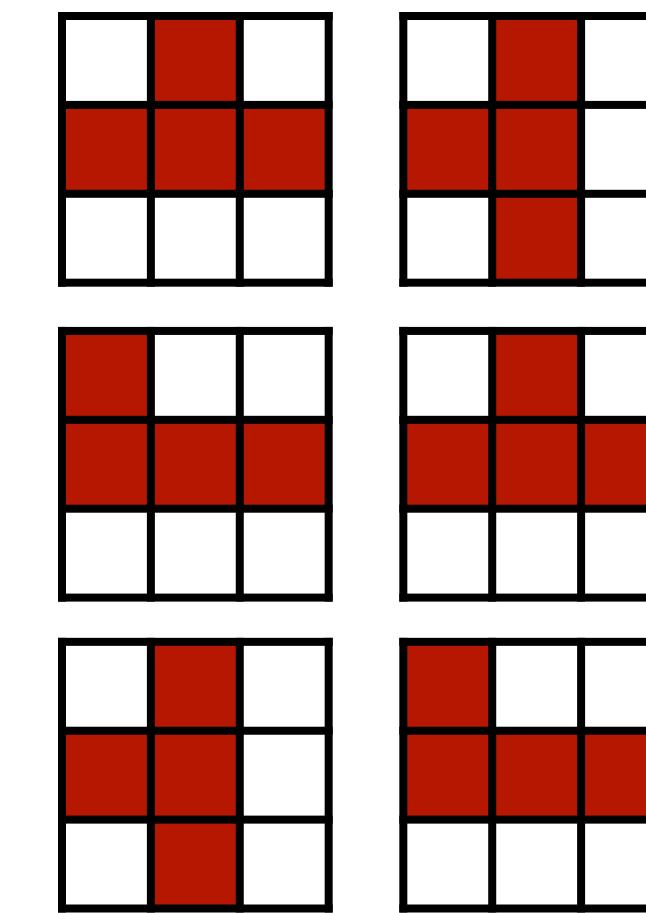
Pros? Cons?

Irregular

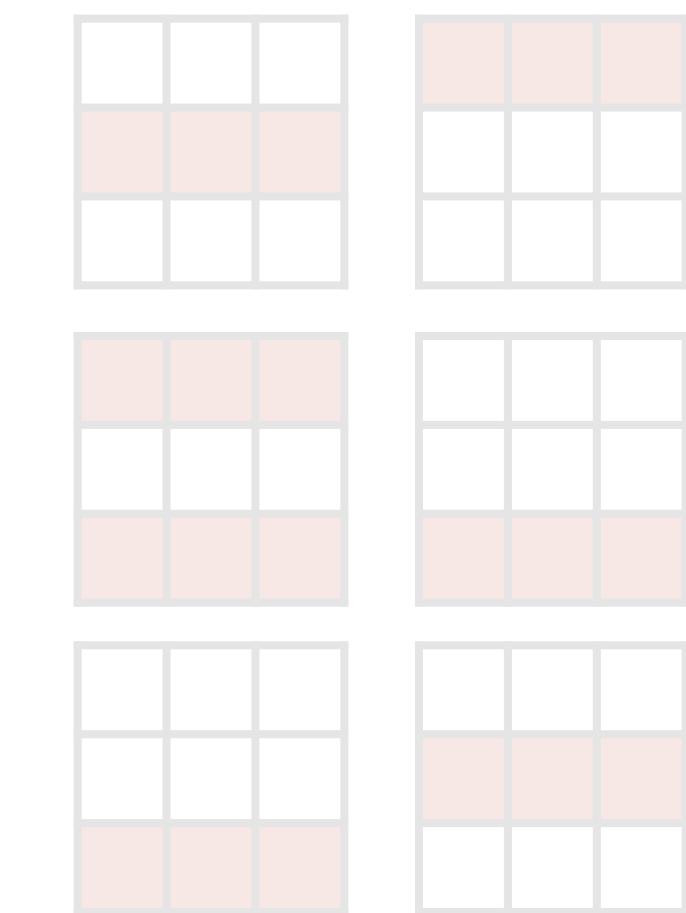
Regular



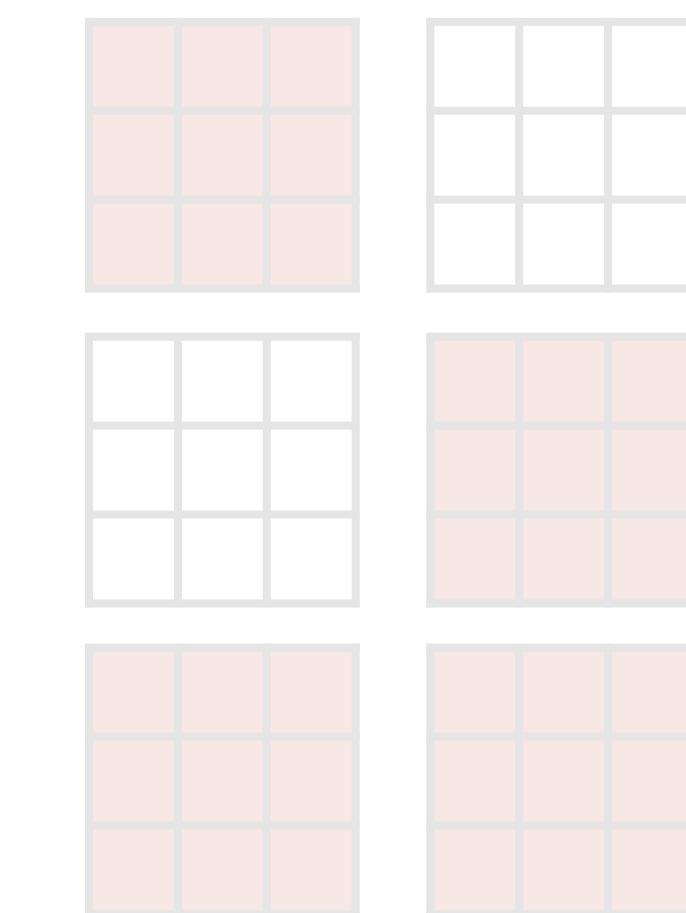
Fine-grained
Pruning



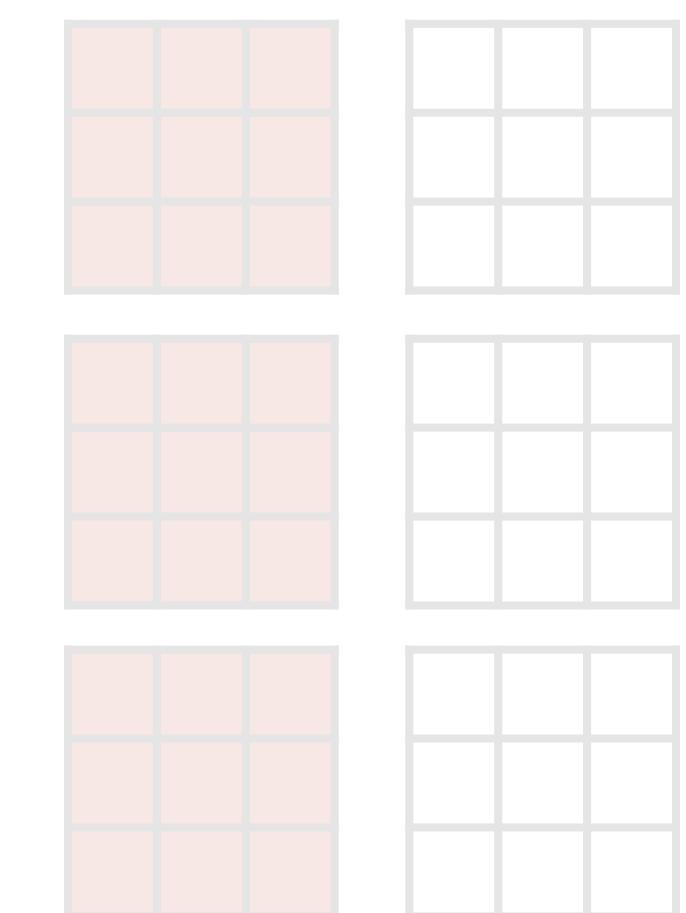
Pattern-based
Pruning



Vector-level
Pruning



Kernel-level
Pruning



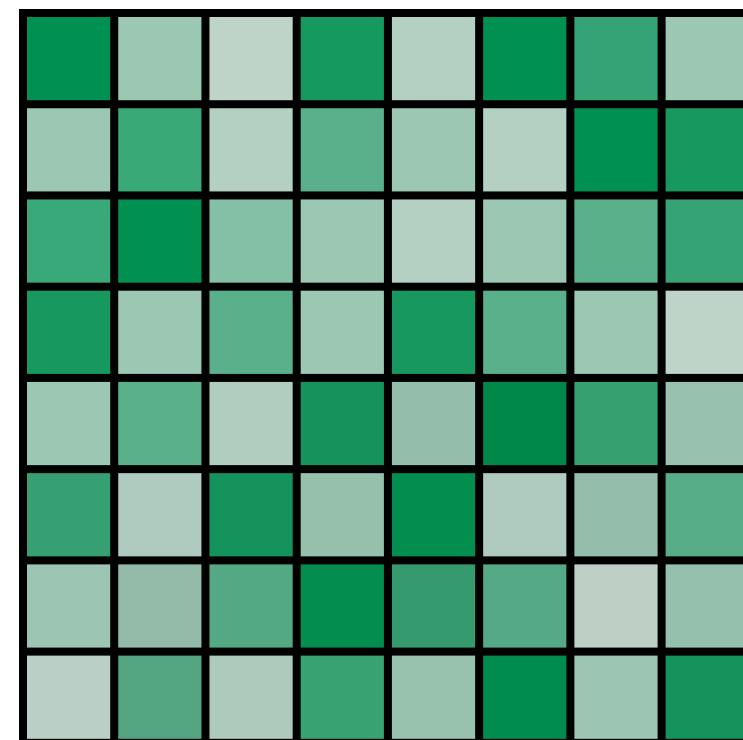
Channel-level
Pruning

Exploring the granularity of sparsity in convolutional neural networks [Mao et al., CVPR-W]

Pruning at Different Granularities

Let's look into some cases

- **Pattern-based Pruning: N:M sparsity**
 - N:M sparsity means that in each contiguous M elements, N of them is pruned

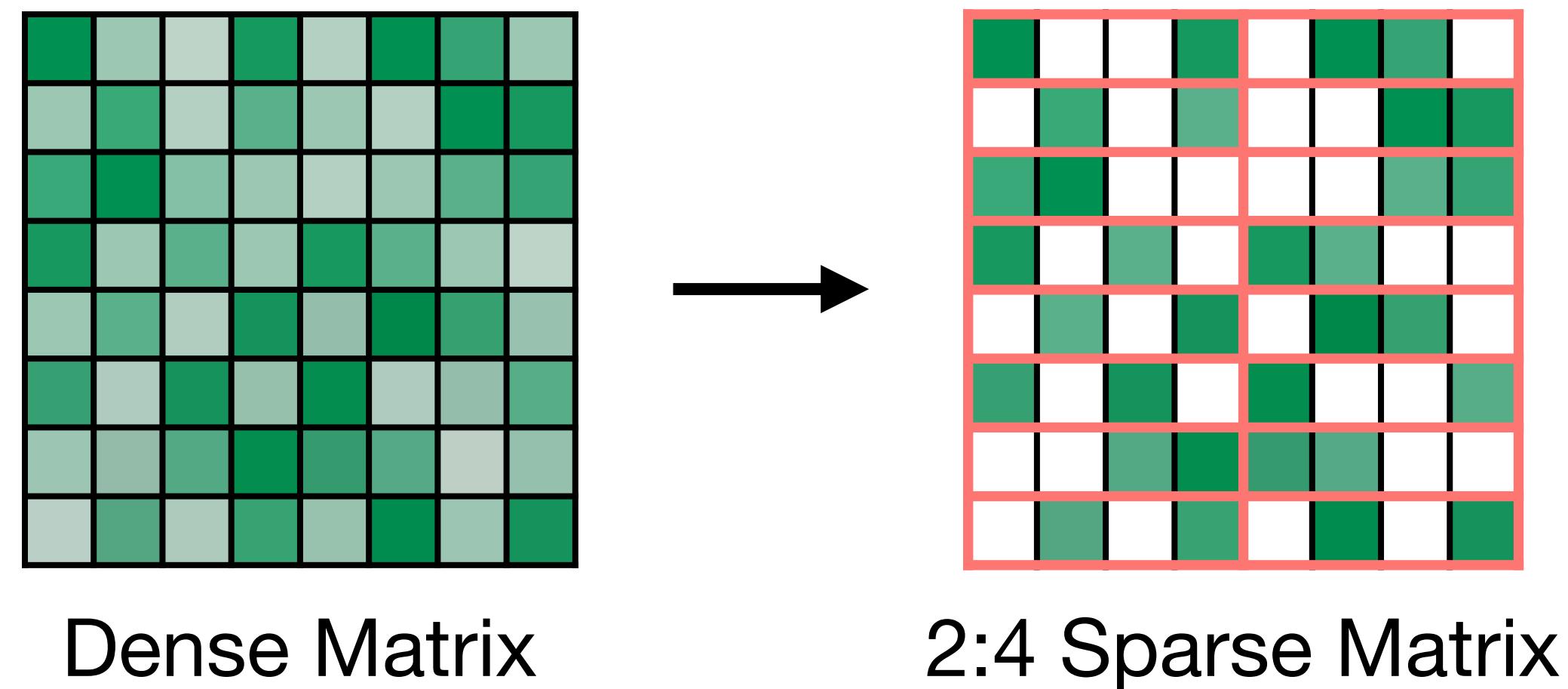


Dense Matrix

Pruning at Different Granularities

Let's look into some cases

- **Pattern-based Pruning: N:M sparsity**
 - N:M sparsity means that in each contiguous M elements, N of them is pruned
 - A classic case is 2:4 sparsity (50% sparsity)

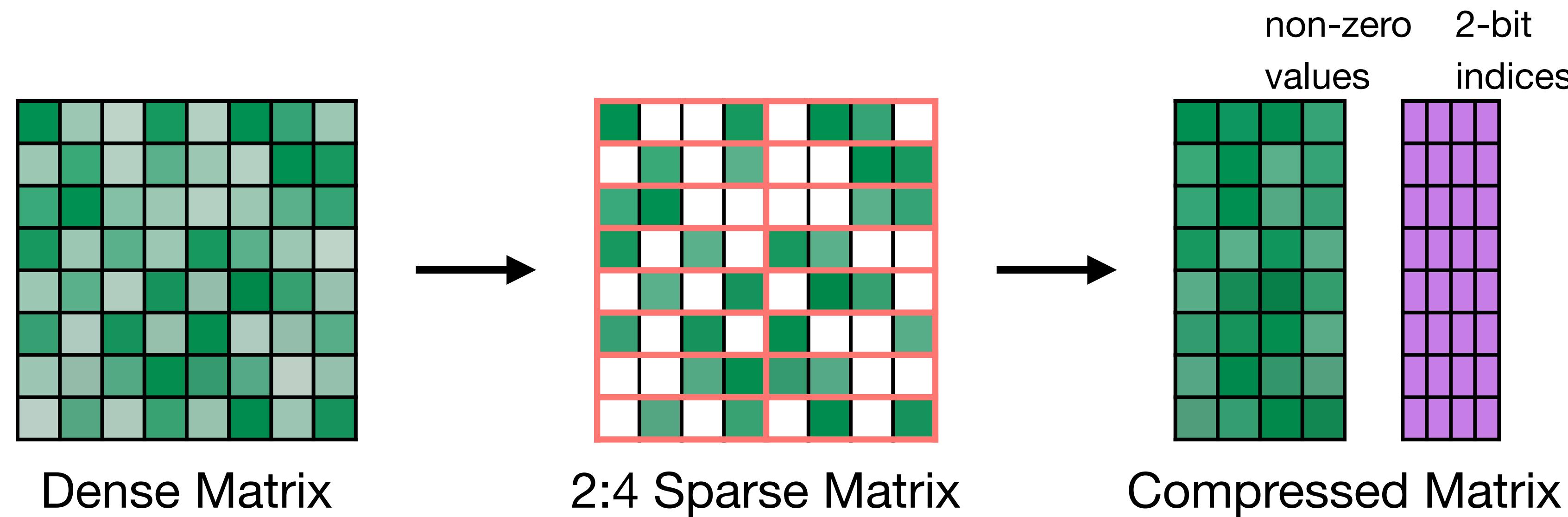


Pruning at Different Granularities

Let's look into some cases

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- It is supported by NVIDIA's Ampere GPU Architecture, which delivers up to 2x speed up



[Accelerating Inference with Sparsity Using the NVIDIA Ampere Architecture and NVIDIA TensorRT](#)

Pruning at Different Granularities

Let's look into some cases

- **Pattern-based Pruning: N:M sparsity**

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- A classic case is 2:4 sparsity (50% sparsity)
- It is supported by NVIDIA's Ampere GPU Architecture, which delivers ~2x speed up
- Usually maintains accuracy (tested on varieties of tasks)

Network	Data Set	Metric	Dense FP16	Sparse FP16
ResNet-50	ImageNet	Top-1	76.1	76.2
ResNeXt-101_32x8d	ImageNet	Top-1	79.3	79.3
Xception	ImageNet	Top-1	79.2	79.2
SSD-RN50	COCO2017	bbAP	24.8	24.8
MaskRCNN-RN50	COCO2017	bbAP	37.9	37.9
FairSeq Transformer	EN-DE WMT'14	BLEU	28.2	28.5
BERT-Large	SQuAD v1.1	F1	91.9	91.9

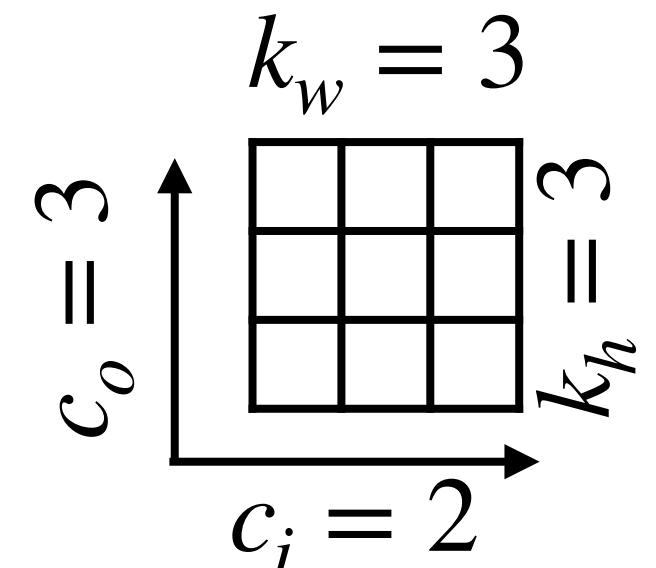
Accelerating Inference with Sparsity Using the NVIDIA Ampere Architecture and NVIDIA TensorRT

Pruning at Different Granularities

The case of convolutional layers

- Some of the commonly used pruning granularities

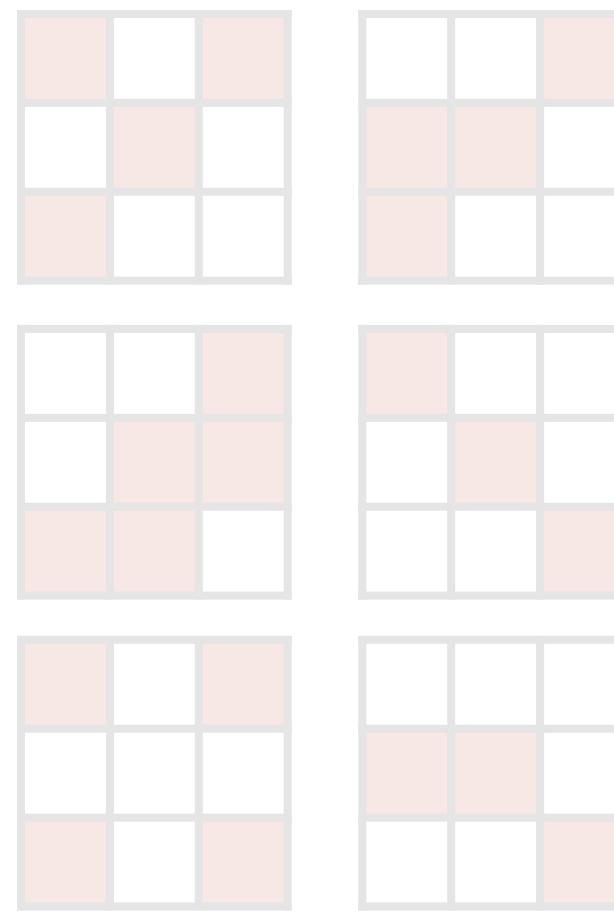
■ Preserved
□ Pruned



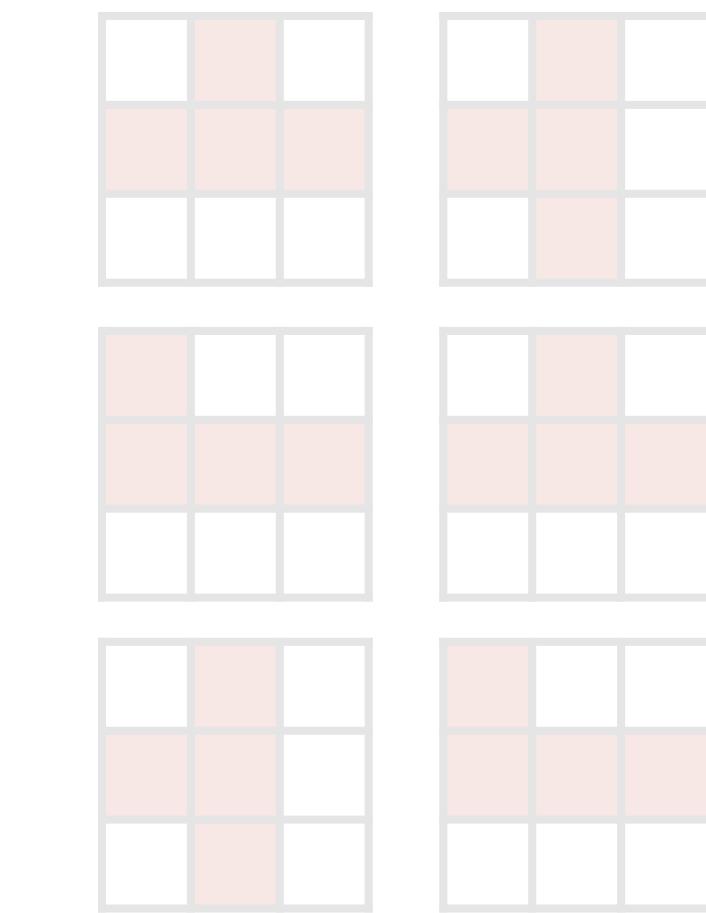
Pros? Cons?

Irregular

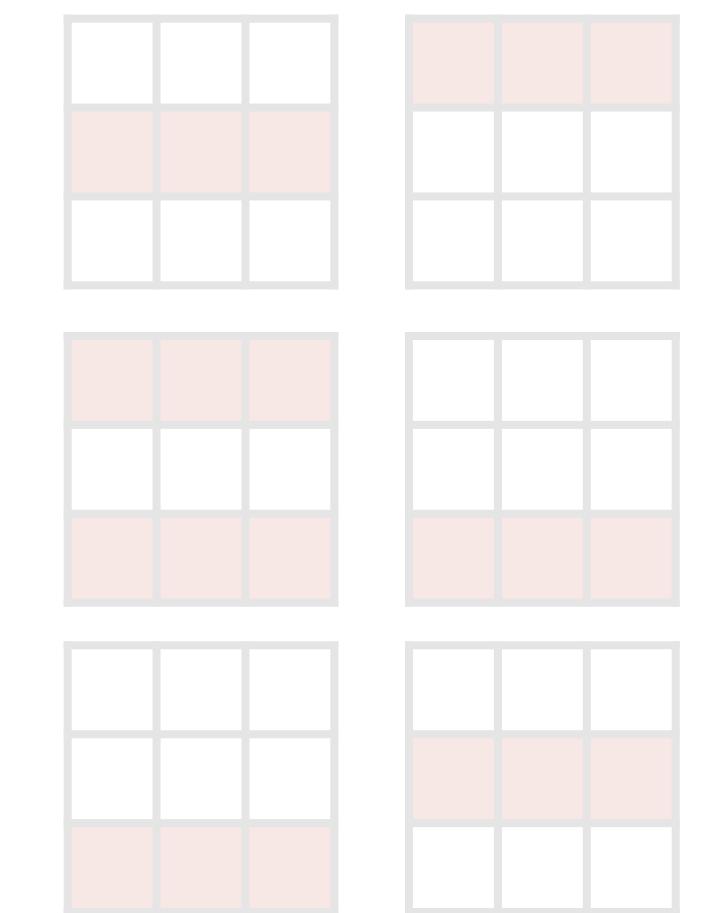
Regular



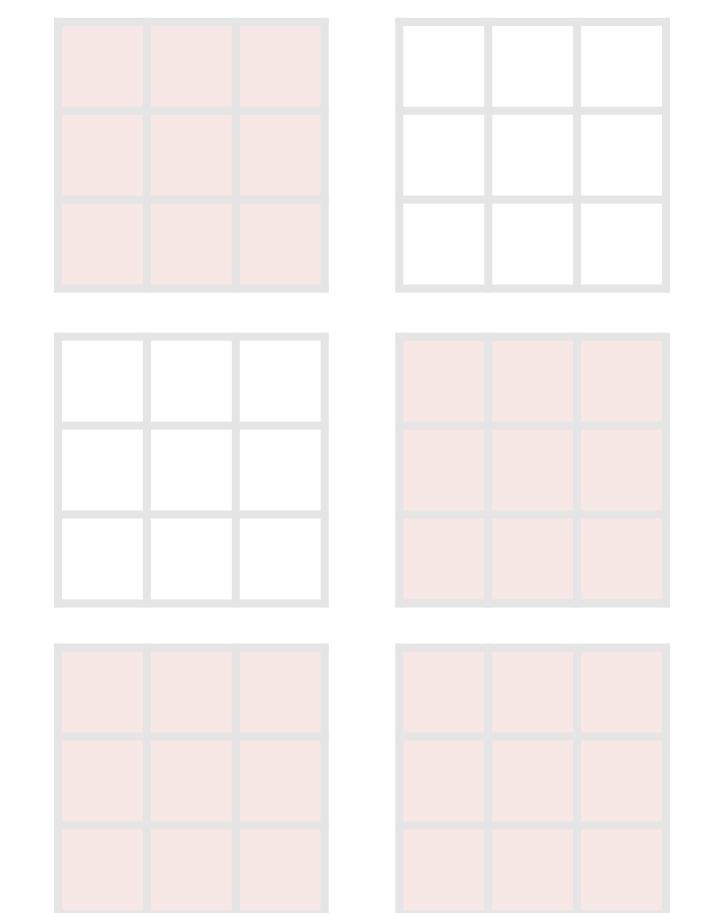
Fine-grained
Pruning



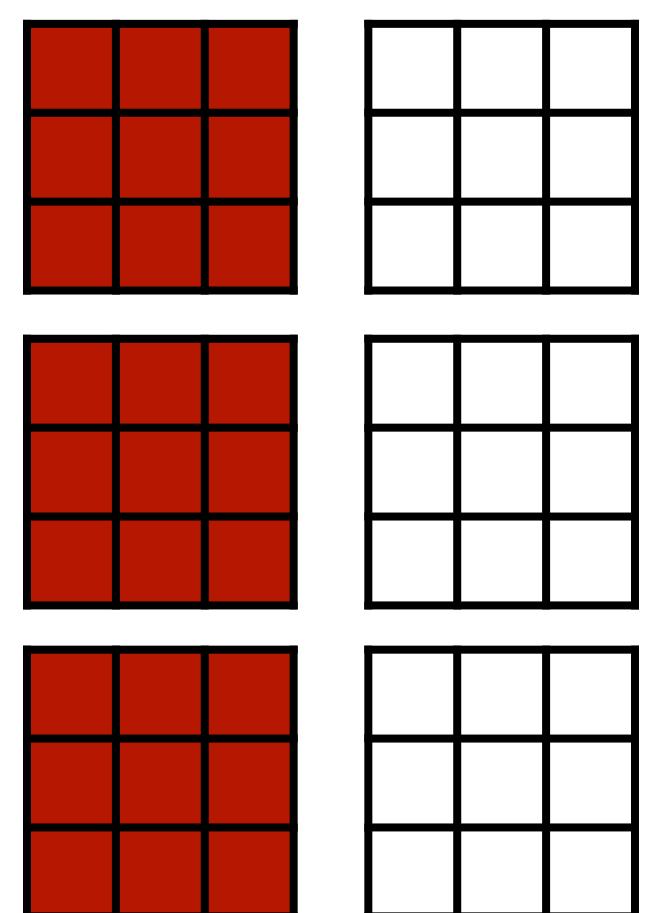
Pattern-based
Pruning



Vector-level
Pruning



Kernel-level
Pruning



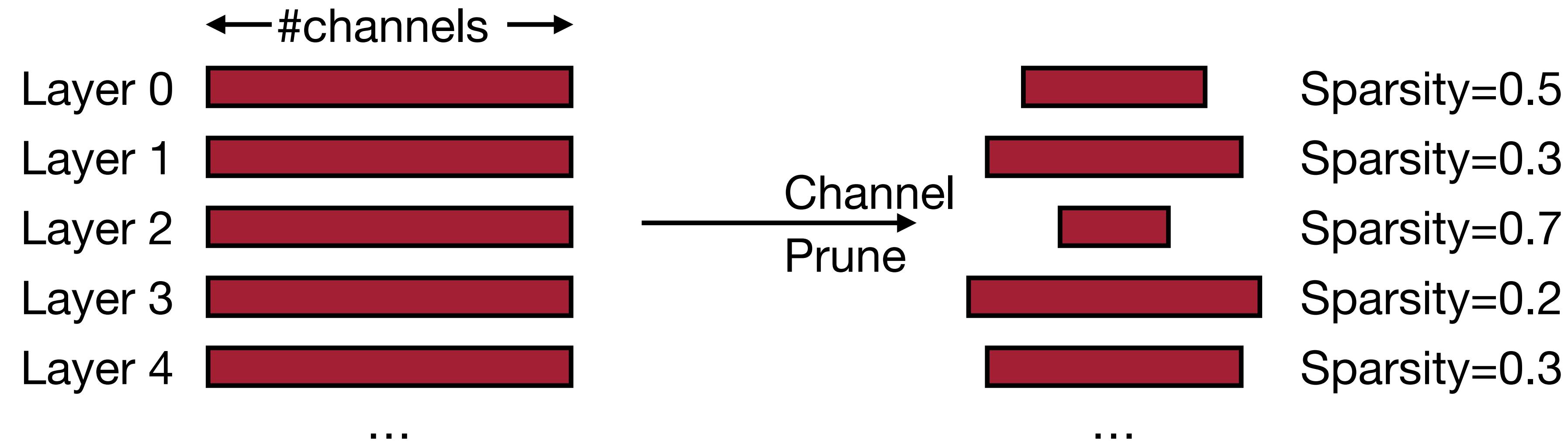
Channel-level
Pruning

Exploring the granularity of sparsity in convolutional neural networks [Mao et al., CVPR-W]

Pruning at Different Granularities

Let's look into some cases

- **Channel Pruning**
 - Pro: Direct speed up due to reduced channel numbers (leading to an NN with smaller #channels)
 - Con: smaller compression ratio

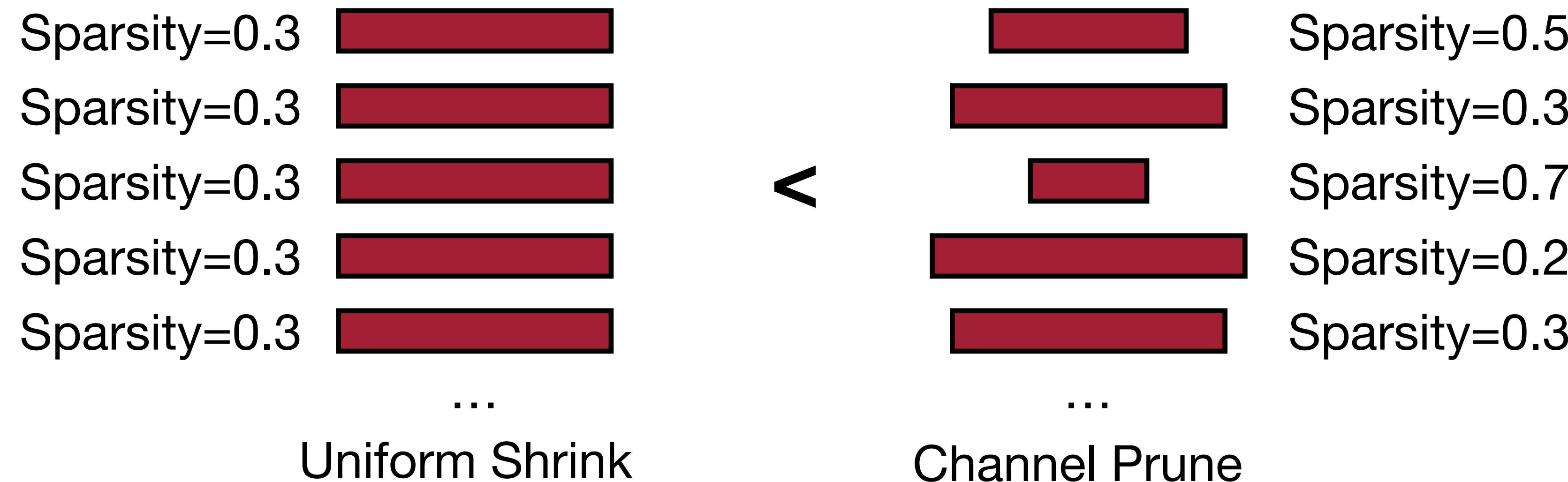


Pruning at Different Granularities

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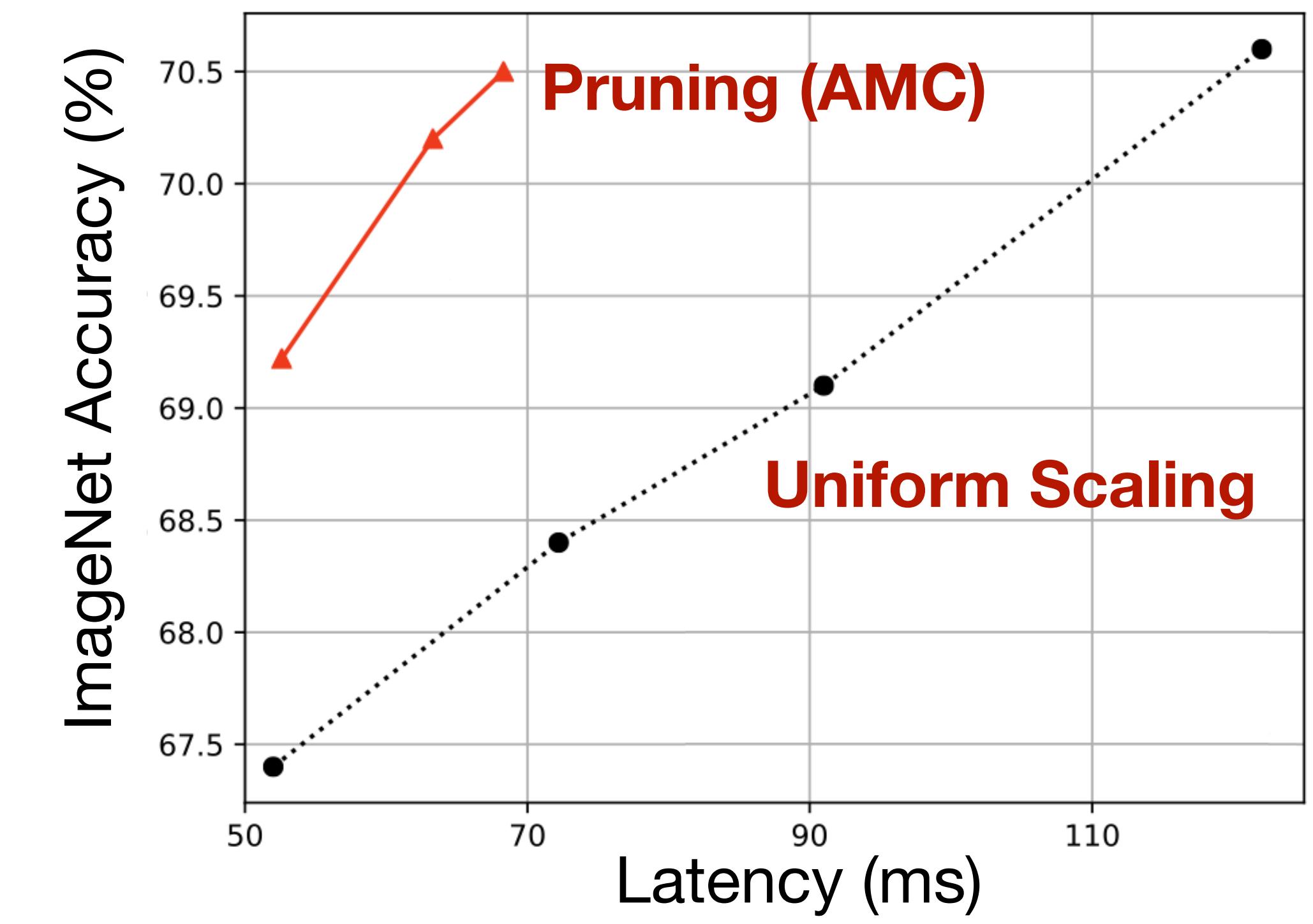
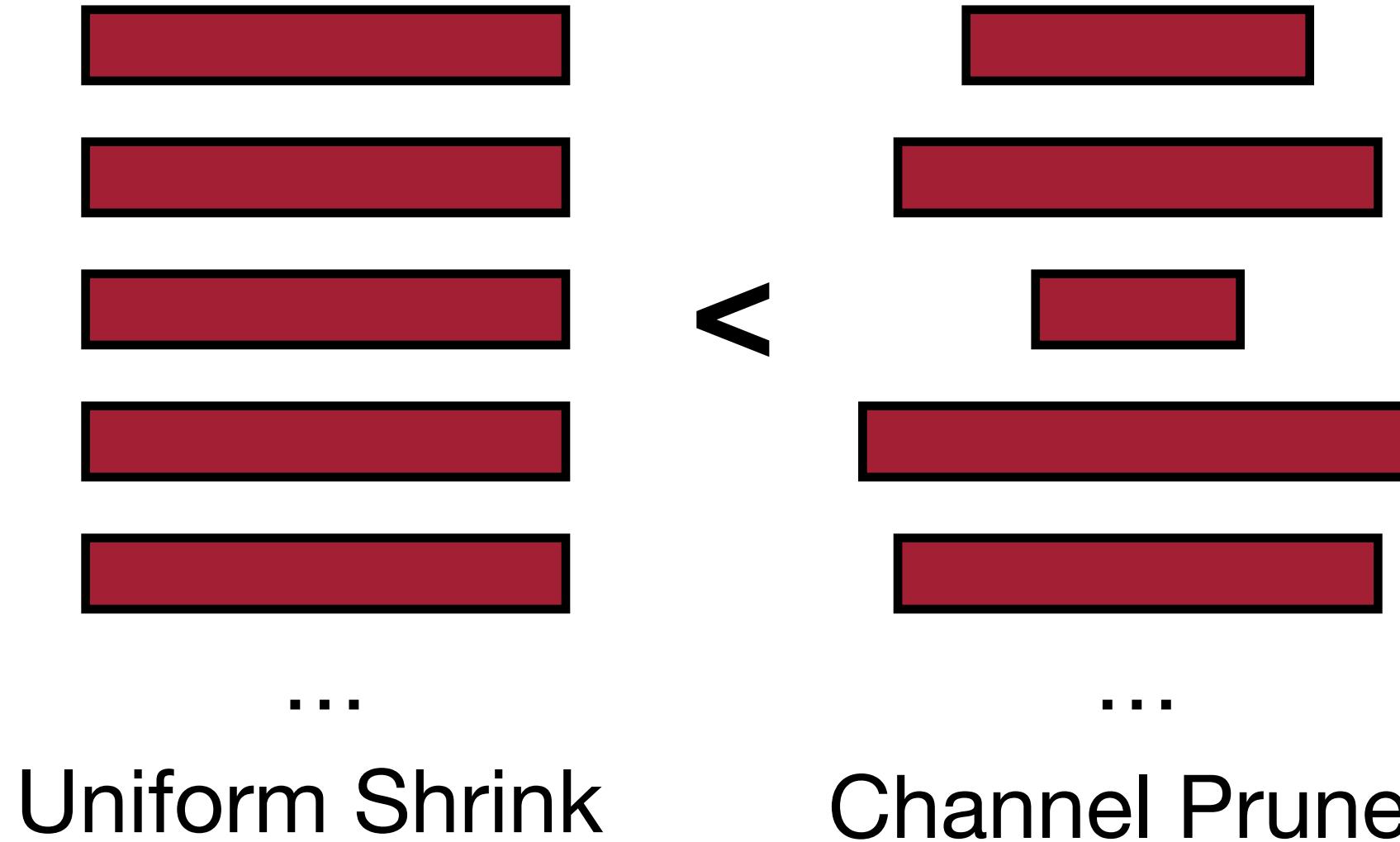
We will later discuss how to find sparsity ratios



Pruning at Different Granularities

Let's look into some cases

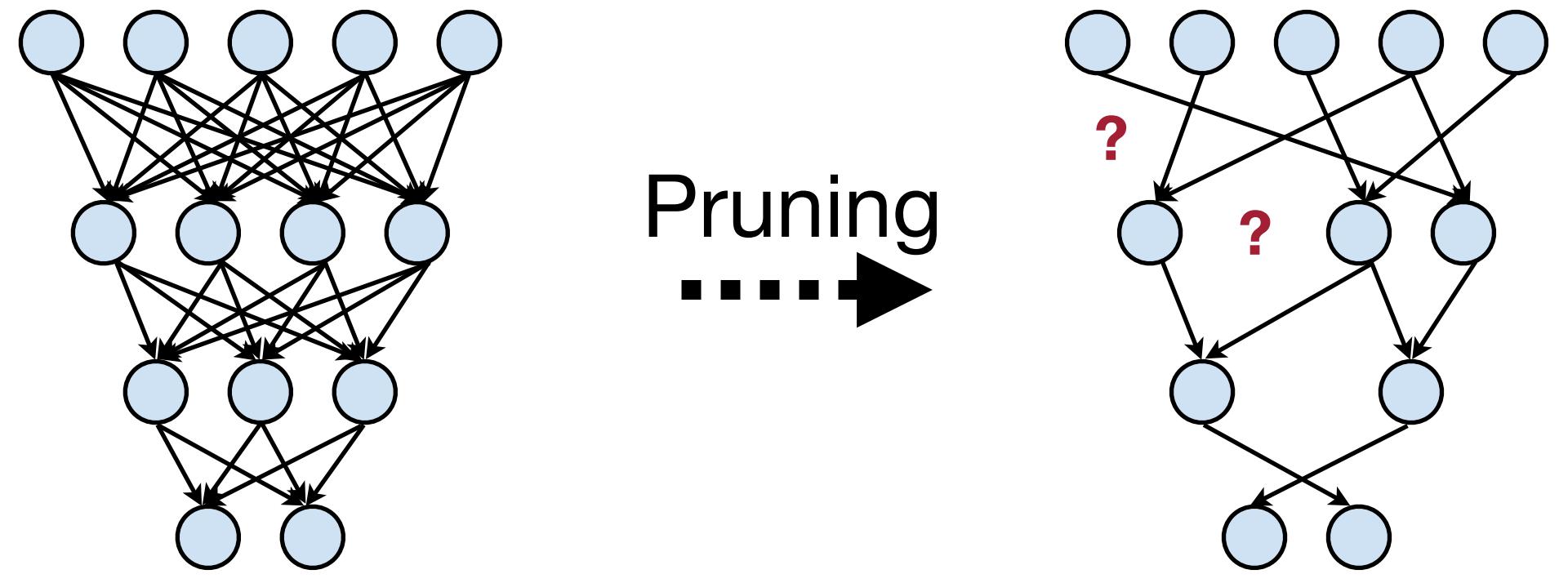
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AMC: Automl for Model Compression and Acceleration on Mobile Devices [He et al., ECCV 2018]

Neural Network Pruning

- **Introduction to Pruning**
 - What is pruning?
 - How should we formulate pruning?
- **Determine the Pruning Granularity**
 - In what pattern should we prune the neural network?
- **Determine the Pruning Criterion**
 - What synapses/neurons should we prune?
- **Determine the Pruning Ratio**
 - What should target sparsity be for each layer?
- **Fine-tune/Train Pruned Neural Network**
 - How should we improve performance of pruned models?



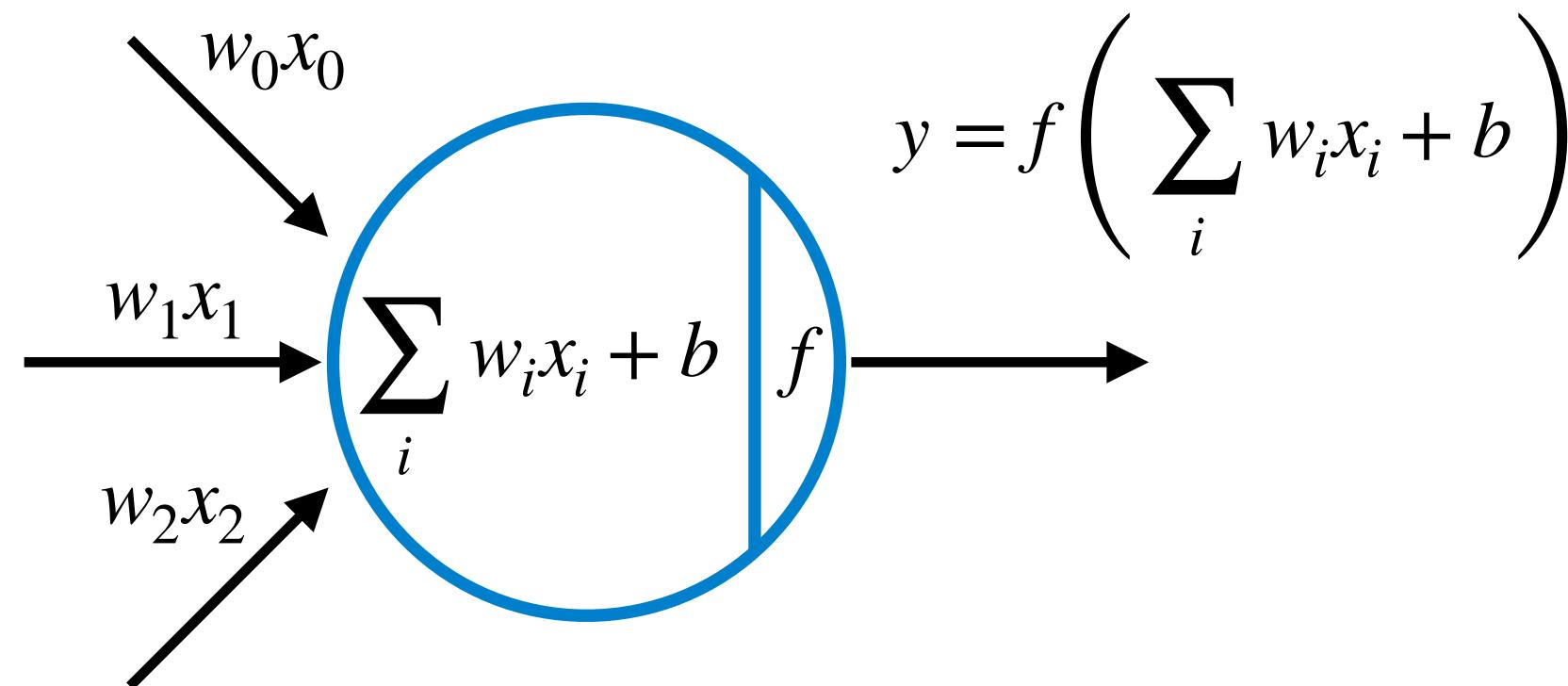
which synapses?
which neurons?

Section 3: Pruning Criterion

What synapses and neurons should we prune?

Selection of Synapses to Prune

- When removing parameters from a neural network model,
 - ***the less important*** the parameters being removed are,
 - the better the performance of pruned neural network is.



Example

$$f(\cdot) = \text{ReLU}(\cdot), \quad W = [10, -8, 0.1]$$

$$\rightarrow y = \text{ReLU}(10x_0 - 8x_1 + 0.1x_2)$$

- If one weight will be removed, which one?

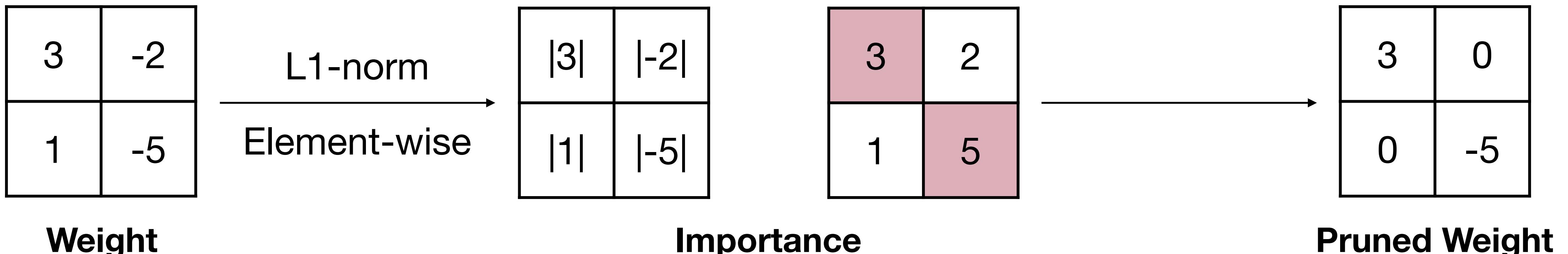
Magnitude-based Pruning

A heuristic pruning criterion

- Magnitude-based pruning considers weights with ***larger absolute values*** are more important than other weights.
 - For element-wise pruning,

$$\text{Importance} = |W|$$

- **Example**



Learning Both Weights and Connections for Efficient Neural Network [Han et al., NeurIPS 2015]

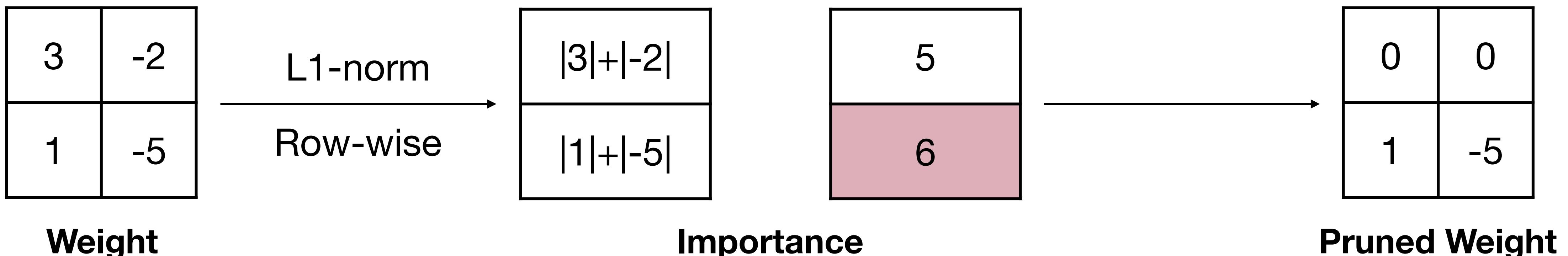
Magnitude-based Pruning

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 - For row-wise pruning, the L1-norm magnitude can be defined as,

$$Importance = \sum_{i \in S} |w_i|, \text{ where } \mathbf{W}^{(S)} \text{ is the structural set } S \text{ of parameters } \mathbf{W}$$

- **Example**



Learning Both Weights and Connections for Efficient Neural Network [Han et al., NeurIPS 2015]

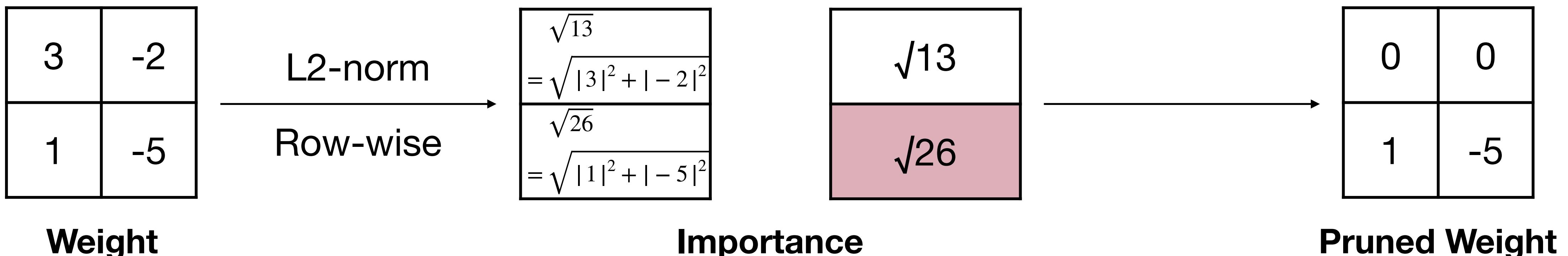
Magnitude-based Pruning

A heuristic pruning criterion

- Magnitude-based pruning considers weights with ***larger absolute values*** are more important than other weights.
 - For row-wise pruning, the L2-norm magnitude can be defined as,

$$Importance = \sqrt{\sum_{i \in S} |w_i|^2}, \text{ where } \mathbf{W}^{(S)} \text{ is the structural set } S \text{ of parameters } \mathbf{W}$$

- Example



Learning Both Weights and Connections for Efficient Neural Network [Han et al., NeurIPS 2015]

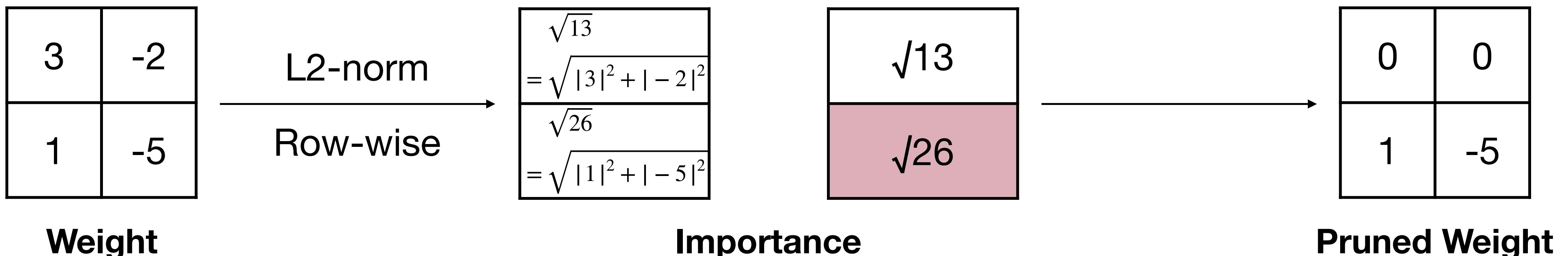
Magnitude-based Pruning

A heuristic pruning criterion

- Magnitude-based pruning considers weights with ***larger absolute values*** are more important than other weights.
- Magnitude is also known as L_p -norm defined as,

$$\|\mathbf{W}^{(S)}\|_p = \left(\sum_{i \in S} |w_i|^p \right)^{\frac{1}{p}}, \text{ where } \mathbf{W}^{(S)} \text{ is a structural set of parameters}$$

- **Example**

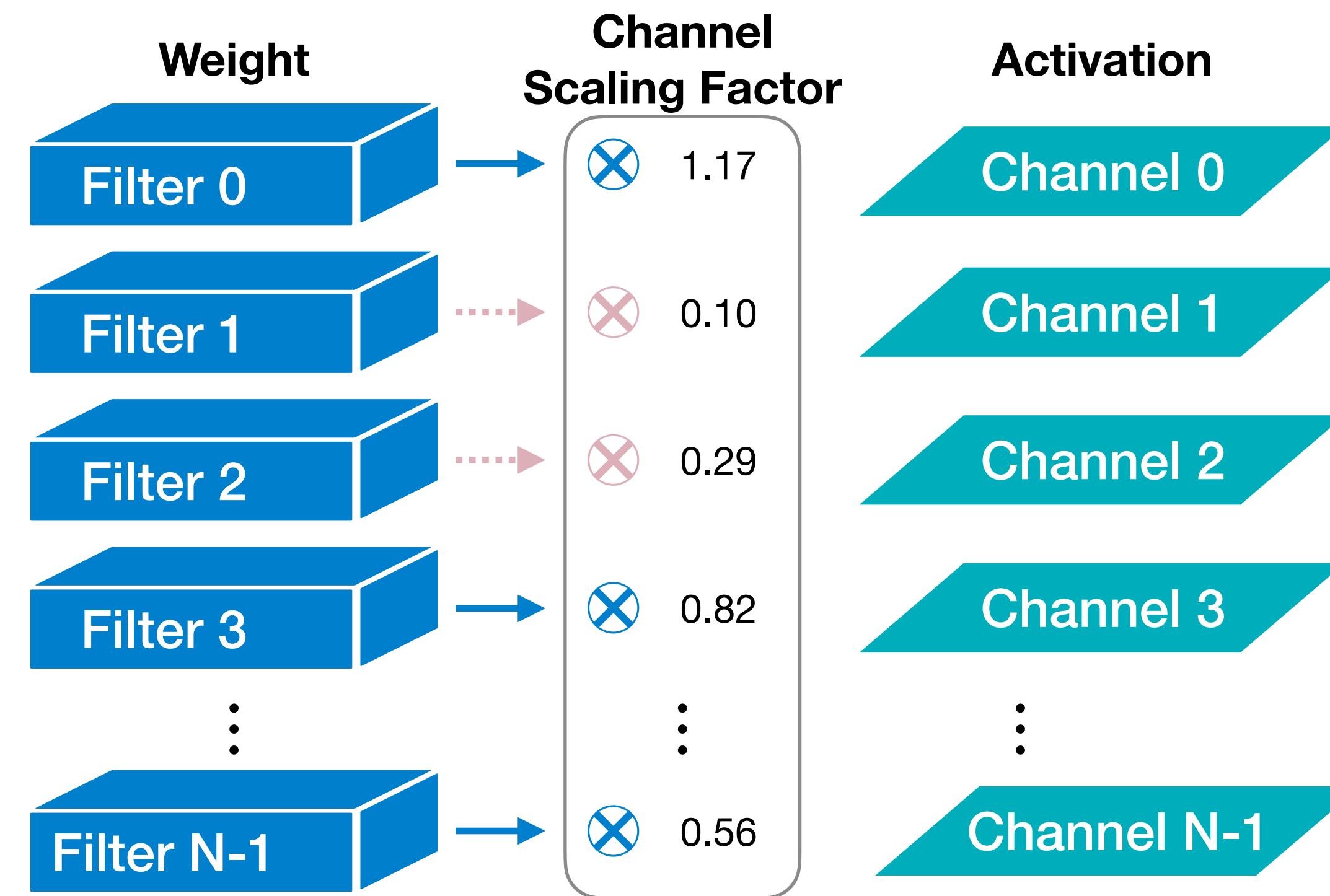


Learning Structured Sparsity in Deep Neural Networks [Wen et al., NeurIPS 2016]

Scaling-based Pruning

Pruning criterion for filter pruning

- A scaling factor is associated with each filter (*i.e.*, output channel) in convolutional layers
 - The scaling factor is multiplied to the output of that channel
 - The scaling factors are trainable parameters

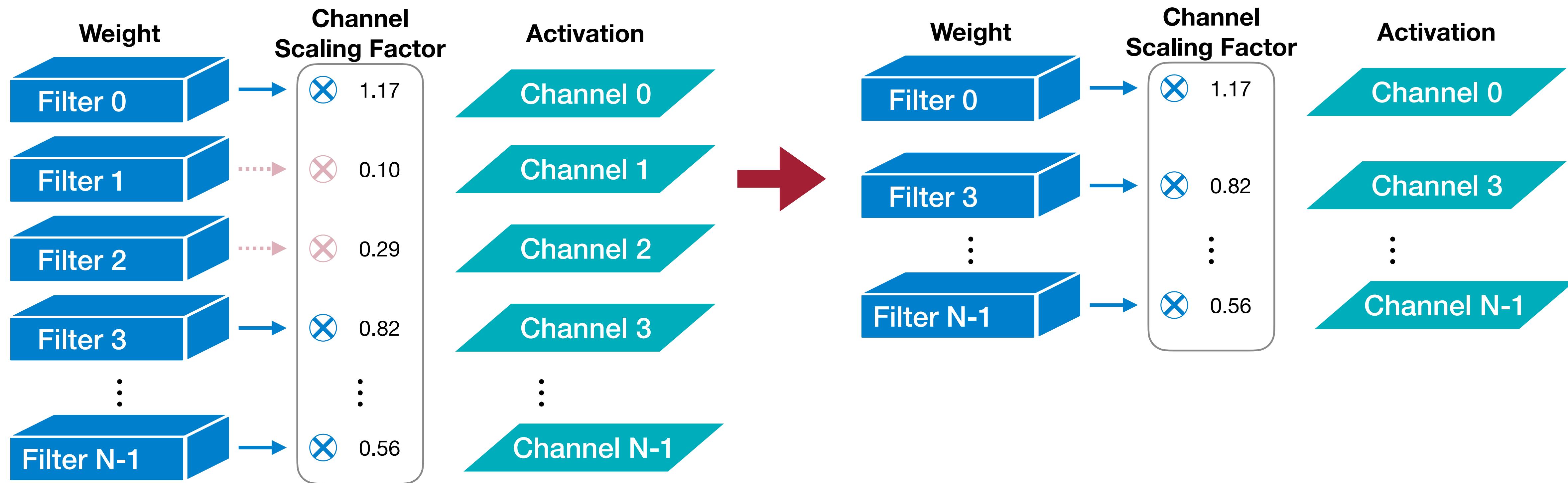


Learning Efficient Convolutional Networks through Network Slimming [Liu et al., ICCV 2017]

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Pruning criterion for filter pruning

- A scaling factor is associated with each filter (i.e., output channel) in convolutional layers
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 - The scaling factors are trainable parameters
- The filters/output channels with small scaling factor magnitude will be pruned



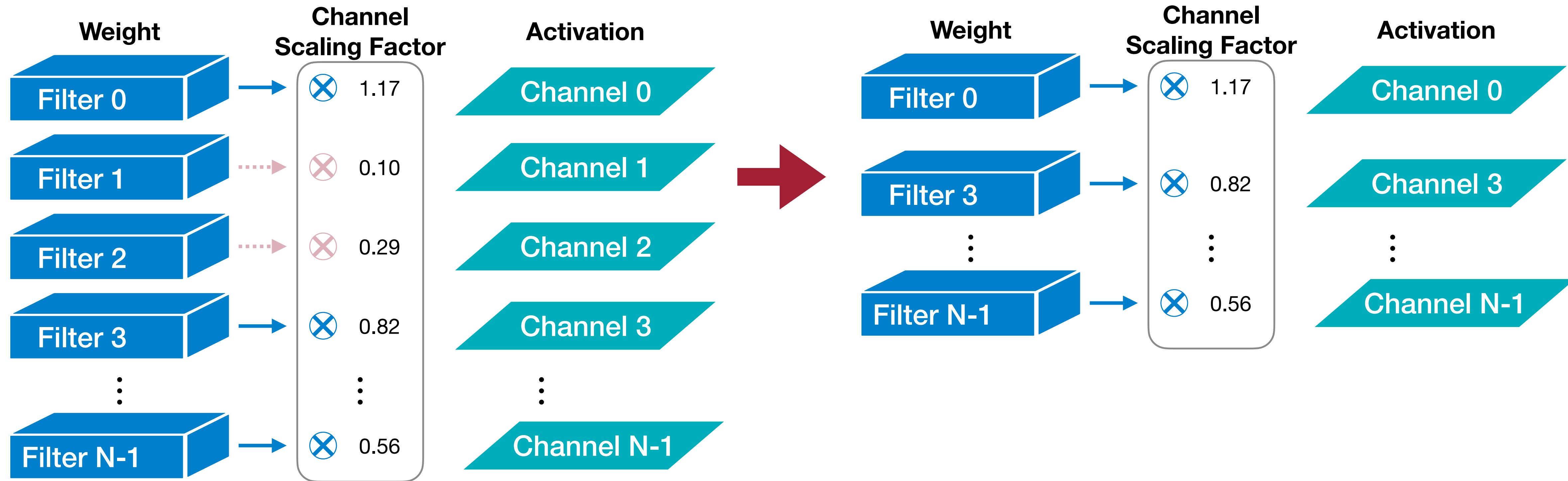
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Scaling-based Pruning

Pruning criterion for filter pruning

- A scaling factor is associated with each filter (i.e., output channel) in convolutional layers
- The scaling factors can be reused from batch normalization layer

$$\mathbf{z}_o = \gamma \frac{\mathbf{z}_i - \mu_{\mathcal{B}}}{\sqrt{\sigma_{\mathcal{B}}^2 + \epsilon}} + \beta$$



Learning Efficient Convolutional Networks through Network Slimming [Liu et al., ICCV 2017]

Second-Order-based Pruning

Minimize the error on loss function introduced by pruning synapses

- The induced error can be approximated by a Taylor series.

$$\delta L = L(\mathbf{x}; \mathbf{W}) - L(\mathbf{x}; \mathbf{W}_P = \mathbf{W} - \delta\mathbf{W}) = \sum_i g_i \delta w_i + \frac{1}{2} \sum_i h_{ii} \delta w_i^2 + \frac{1}{2} \sum_{i \neq j} h_{ij} \delta w_i \delta w_j + O(\|\delta\mathbf{W}\|^3)$$

where

$$g_i = \frac{\partial L}{\partial w_i}, \quad h_{i,j} = \frac{\partial^2 L}{\partial w_i \partial w_j}$$

- Optimal Brain Damage assumes that

Optimal Brain Damage [LeCun et al., NeurIPS 1989]

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 - The objective function L is nearly quadratic: the last term is neglected

Optimal Brain Damage [LeCun et al., NeurIPS 1989]

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Optimal Brain Damage [LeCun et al., NeurIPS 1989]

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Minimize the error on loss function introduced by pruning synapses

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- The synapses with smaller induced error $|\delta L_i|$ will be removed; that is to say,

$$importance_{w_i} = |\delta L_i| = \frac{1}{2} h_{ii} w_i^2$$

* h_{ii} is non-negative

Hessian Matrix H is difficult to compute.

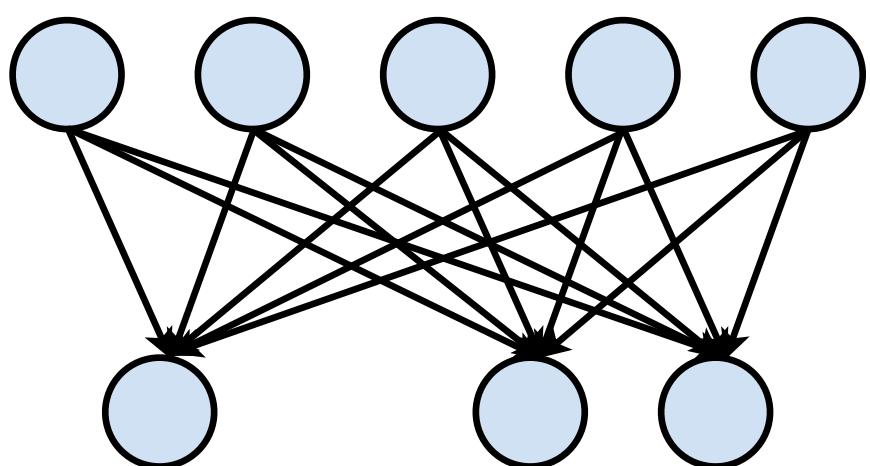
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Selection of Neurons to Prune

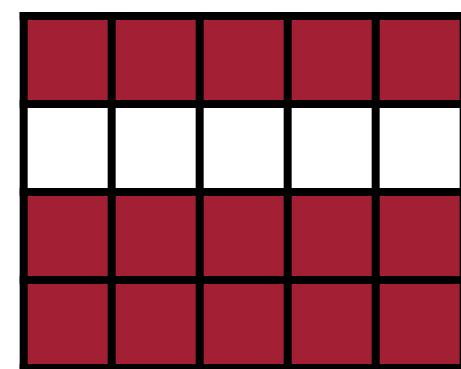
- When removing neurons from a neural network model,
 - ***the less useful*** the neurons being removed are,
 - the better the performance of pruned neural network is.

Neuron pruning is coarse-grained weight pruning

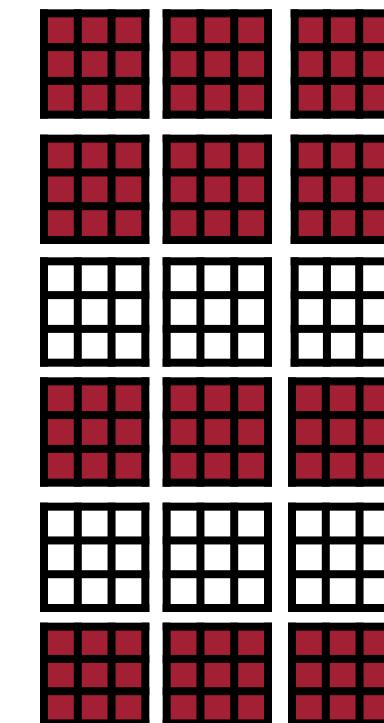
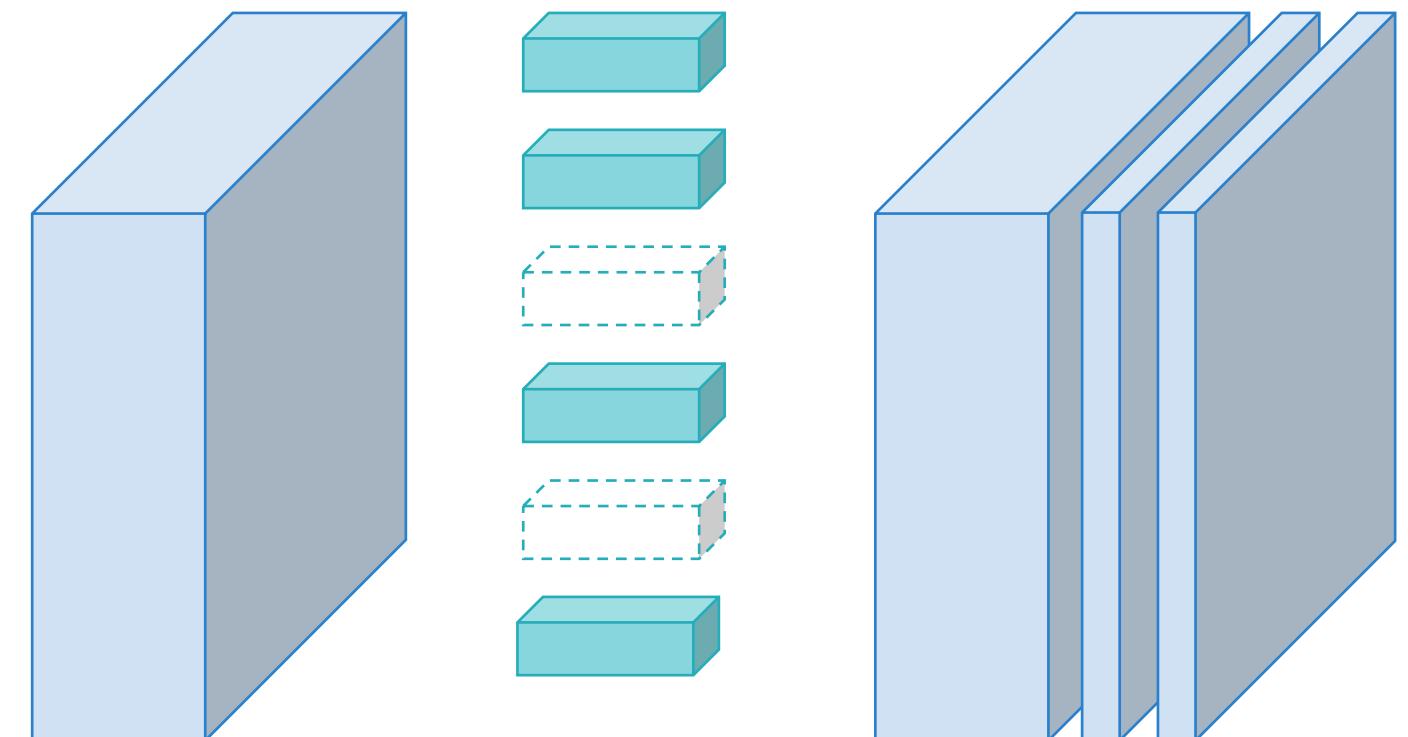
**Neuron Pruning
in Linear Layer**



Weight Matrix

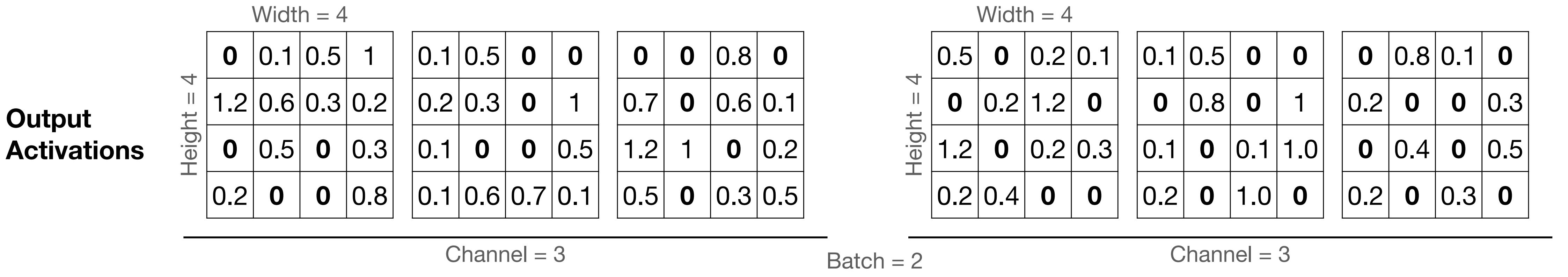


**Channel Pruning
in Convolution Layer**



Percentage-of-Zero-Based Pruning

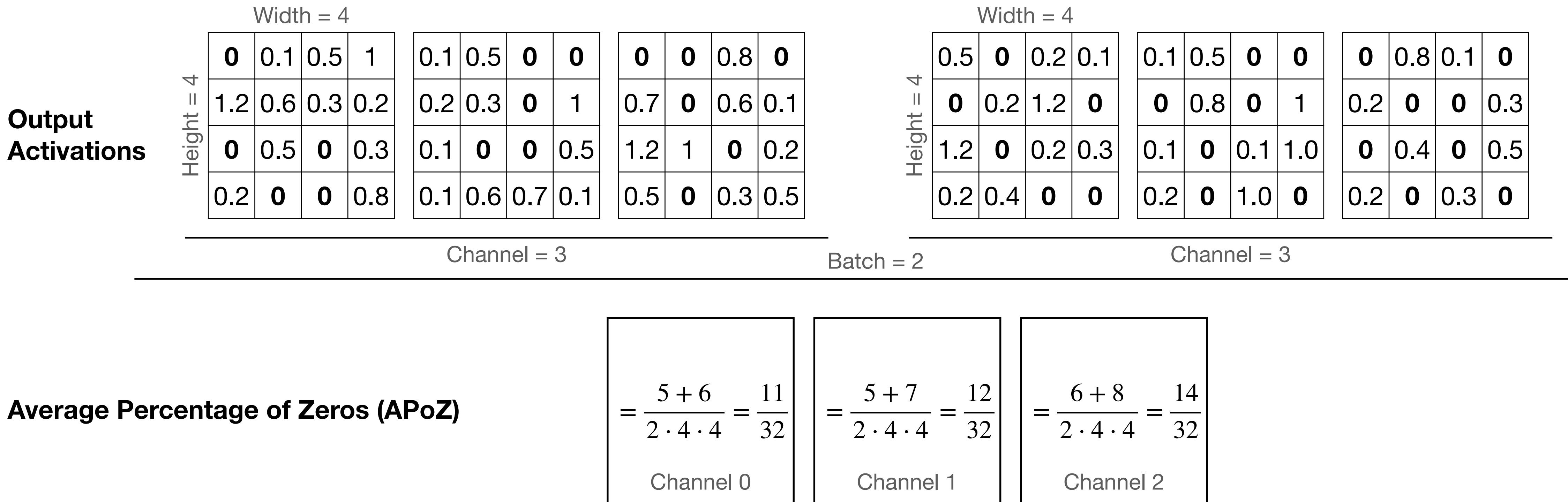
- ReLU activation will generate zeros in the output activation.



Network Trimming: A Data-Driven Neuron Pruning Approach towards Efficient Deep Architectures [Hu et al., ArXiv 2017]

Percentage-of-Zero-Based Pruning

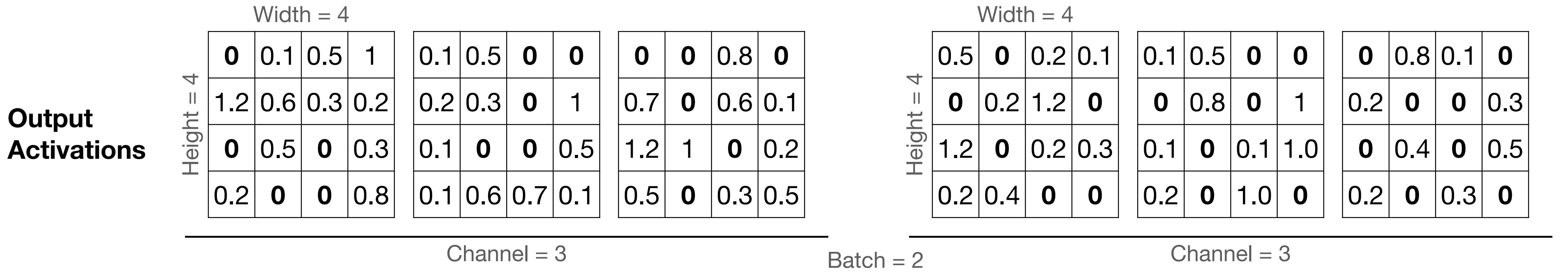
- ReLU activation will generate zeros in the output activation.
- Similar to magnitude of weights, the Average Percentage of Zero activations (APoZ) can be exploited to measure the importance of the neurons.



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Percentage-of-Zero-Based Pruning

- ReLU activation will generate zeros in the output activation.
- Similar to magnitude of weights, the Average Percentage of Zero activations (APoZ) can be exploited to measure the importance of the neurons.
- The smaller APoZ is, the more importance the neuron has.



Average Percentage of Zeros (APoZ)

$$= \frac{5+6}{2 \cdot 4 \cdot 4} = \frac{11}{32}$$

Channel 0

$$= \frac{5+7}{2 \cdot 4 \cdot 4} = \frac{12}{32}$$

Channel 1

~~$$= \frac{6+8}{2 \cdot 4 \cdot 4} = \frac{14}{32}$$~~

Channel 2

Network Trimming: A Data-Driven Neuron Pruning Approach towards Efficient Deep Architectures [Hu et al., ArXiv 2017]

Regression-based Pruning

Minimize reconstruction error of the corresponding layer's outputs

- Instead of considering the pruning error of the objective function $L(\mathbf{x}; \mathbf{W})$, regression-based pruning minimizes the reconstruction error of the corresponding layer's outputs.

The diagram shows the forward pass of a neural network layer. An input matrix \mathbf{X} (with b rows and c_i columns) is multiplied by a weight matrix \mathbf{W}^T (with c_i columns and c_o rows). The result is a matrix \mathbf{Z} (with b rows and c_o columns). The matrices are represented as grids of numbers, and the multiplication is indicated by a large cross symbol between the \mathbf{X} and \mathbf{W}^T terms.

Regression-based Pruning

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$$\begin{array}{c} c_i \\ b \end{array} \times \begin{array}{c} c_o \\ c_i \end{array} = \begin{array}{c} c_o \\ b \end{array}$$

$\mathbf{X} \quad \mathbf{W}^T \quad \mathbf{Z}$

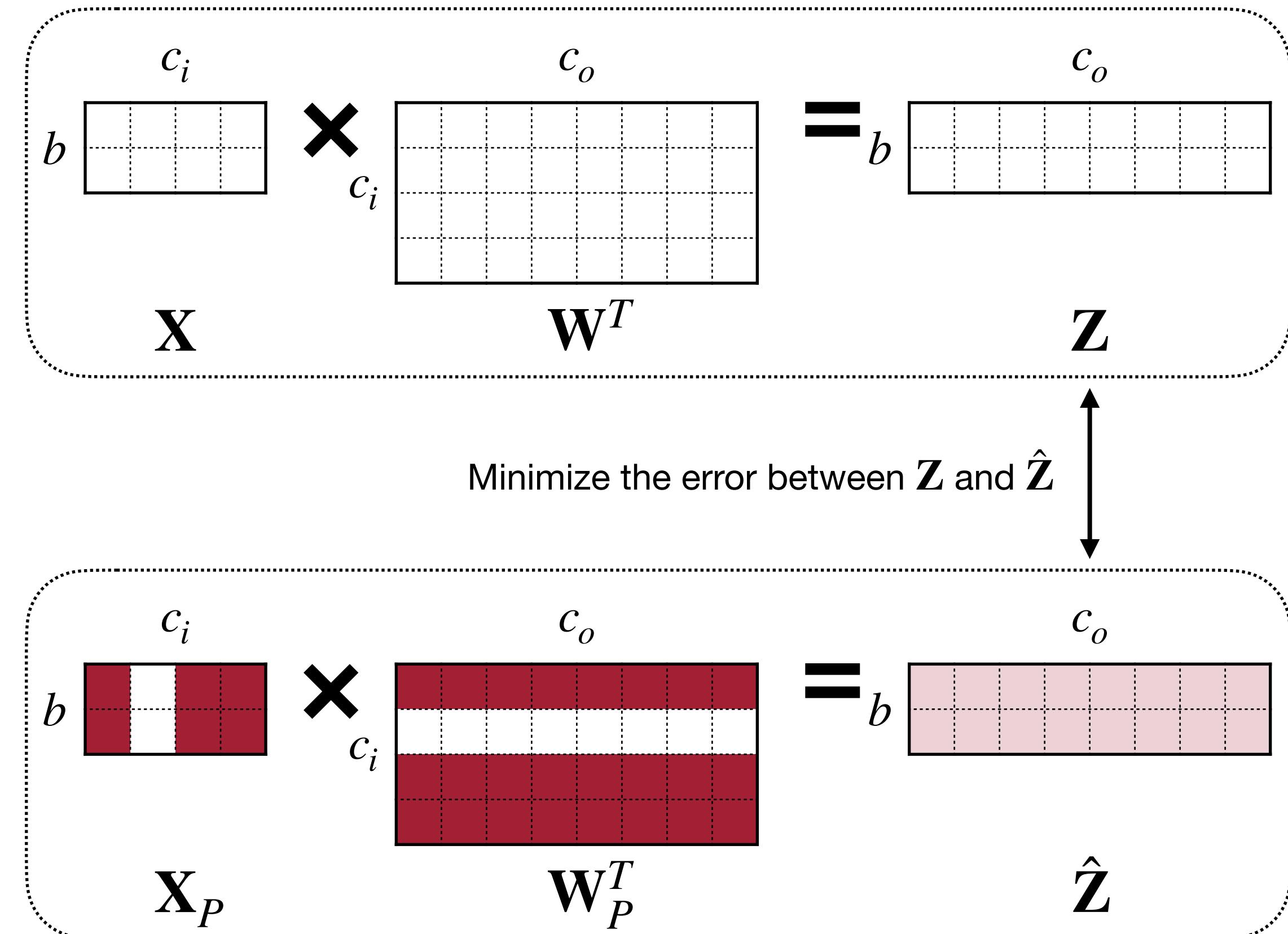
$$\begin{array}{c} c_i \\ b \end{array} \times \begin{array}{c} c_o \\ c_i \end{array} = \begin{array}{c} c_o \\ b \end{array}$$

$\mathbf{X}_P \quad \mathbf{W}_P^T \quad \hat{\mathbf{Z}}$

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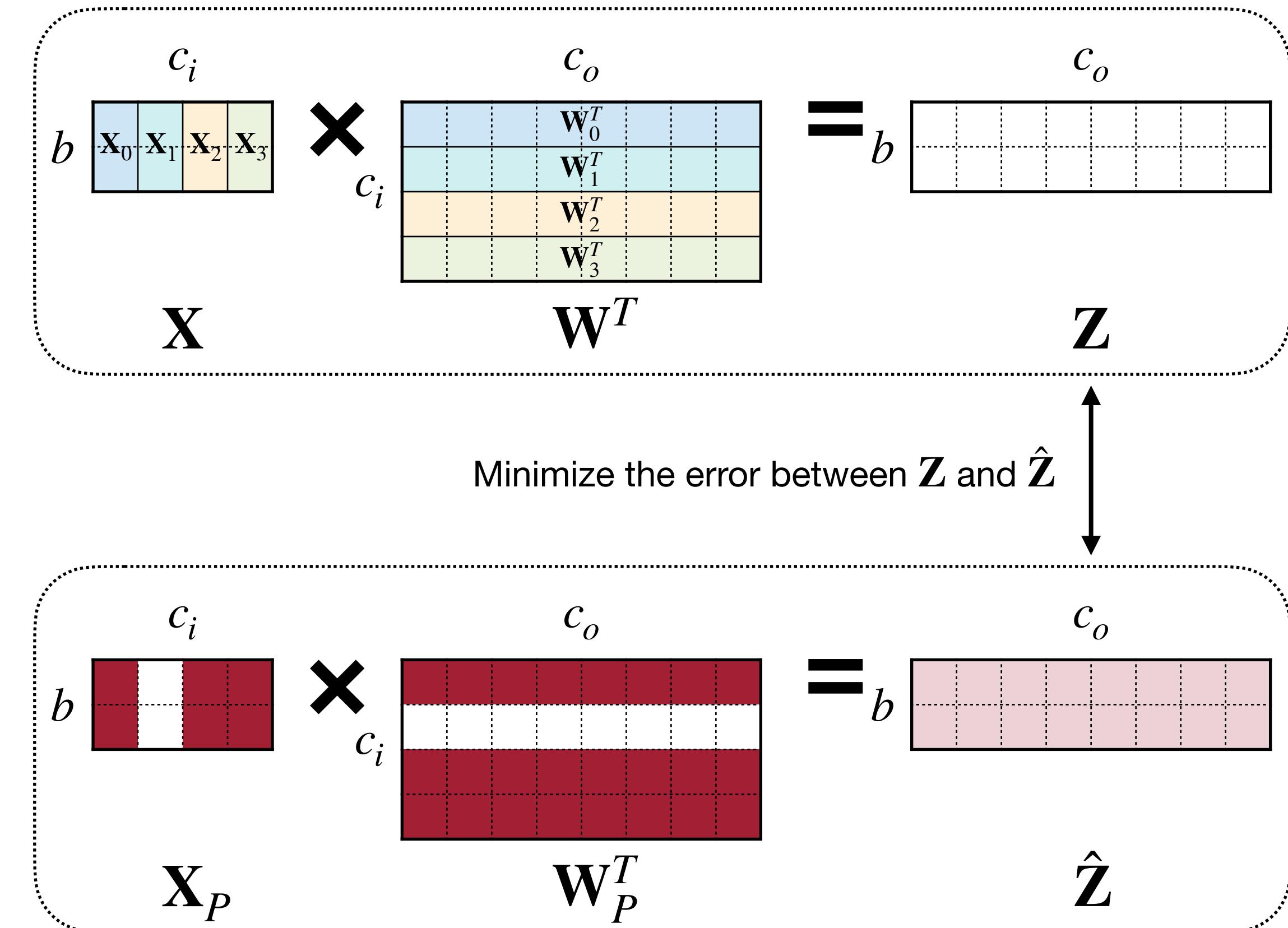


Regression-based Pruning

Minimize reconstruction error of the corresponding layer's outputs

- Let

$$\mathbf{Z} = \mathbf{X}\mathbf{W}^T = \sum_{c=0}^{c_i-1} \mathbf{X}_c \mathbf{W}_c^T$$



Regression-based Pruning

Minimize reconstruction error of the corresponding layer's outputs

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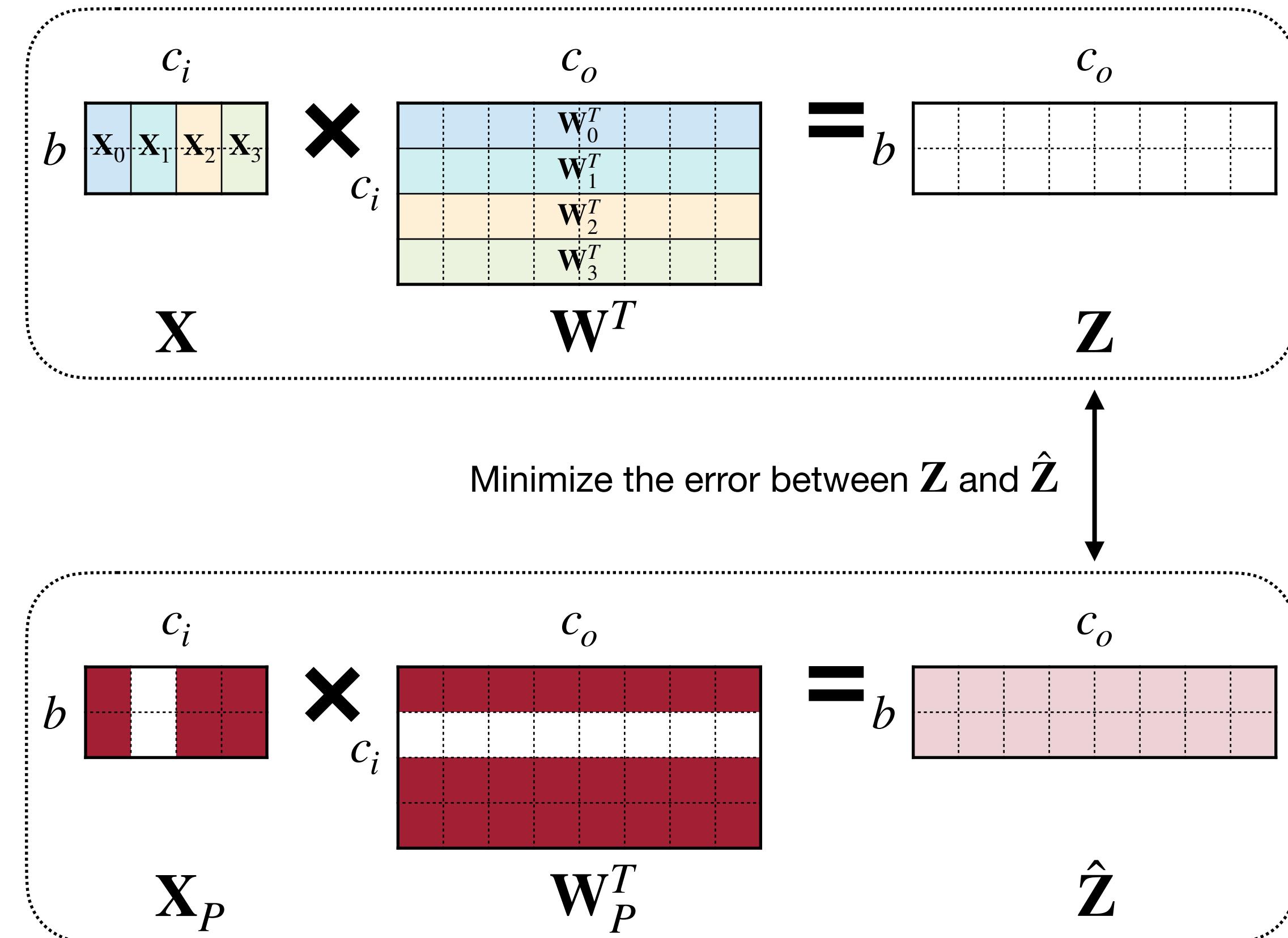
- The problem can be formulate as

$$\arg \min_{\mathbf{W}, \beta} \|\mathbf{Z} - \hat{\mathbf{Z}}\|_F^2 = \|\mathbf{Z} - \sum_{c=0}^{c_i-1} \beta_c \mathbf{X}_c \mathbf{W}_c^T\|_F^2$$

subject to

$$\|\beta\|_0 \leq N_c$$

- β is coefficient vector of length c_i for channel selection. $\beta_c = 0$ means channel c is pruned.
- N_c is the number of nonzero channels.



Regression-based Pruning

Minimize reconstruction error of the corresponding layer's outputs

- Let

$$\mathbf{Z} = \mathbf{X}\mathbf{W}^T = \sum_{c=0}^{c_i-1} \mathbf{X}_c \mathbf{W}_c^T$$

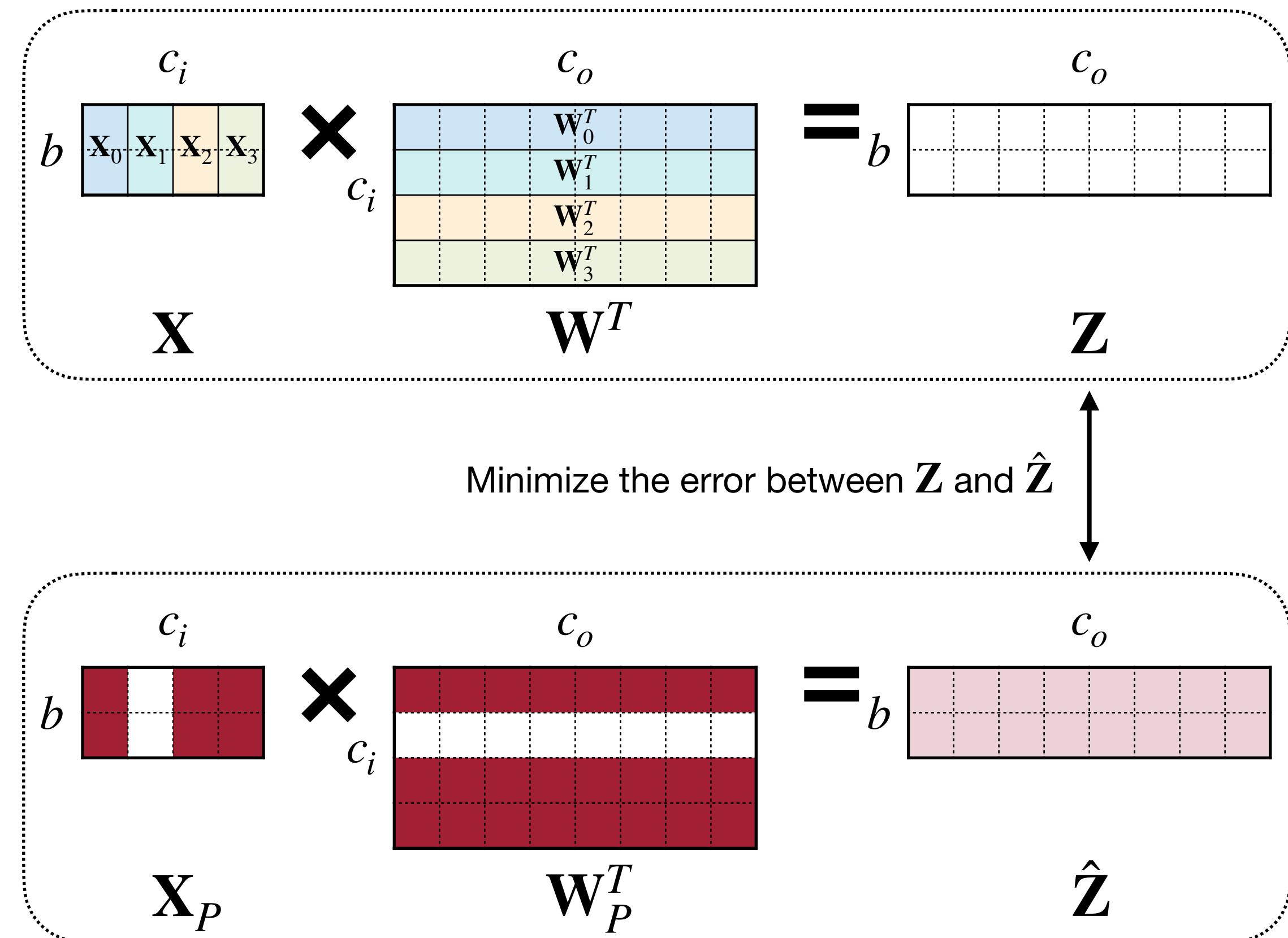
- The problem can be formulate as

$$\arg \min_{\mathbf{W}, \beta} \|\mathbf{Z} - \hat{\mathbf{Z}}\|_F^2 = \|\mathbf{Z} - \sum_{c=0}^{c_i-1} \beta_c \mathbf{X}_c \mathbf{W}_c^T\|_F^2$$

subject to

$$\|\beta\|_0 \leq N_c$$

- β is coefficient vector of length c_i for channel selection. $\beta_c = 0$ means channel c is pruned.
- N_c is the number of nonzero channels.
- Solve the problem by:
 - Fix \mathbf{W} , solve β for channel selection
 - Fix β , solve \mathbf{W} to minimize reconstruction error

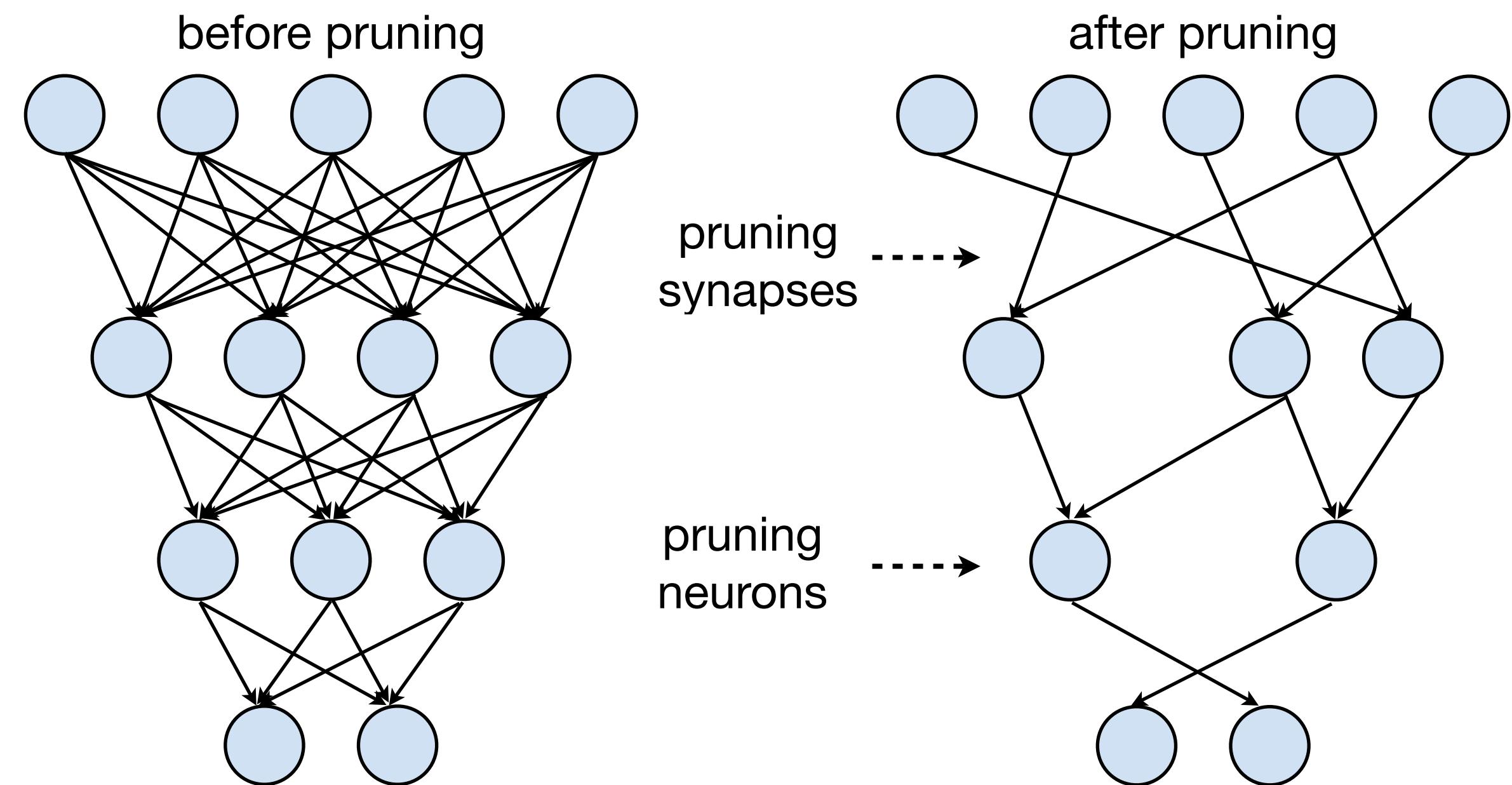


Summary of Today's Lecture

Pruning Demo

In this lecture, we introduced:

- What is pruning
- Granularities of pruning
- Criteria to select weights to prune
- **We will cover in the next lecture:**
 - How to find pruning ratio for each layer
 - How to train/fine-tune the pruned layer
 - Automated ways to find pruning ratios
 - System support for different granularities



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