若 $E_{DC} = E_T$，则 I 为周期电流的有效值

$$I^2RT = \int_0^T i^2(t)R\mathrm{d}t \qquad \boxed{I_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2(t)\mathrm{d}t}}$$

同样，可定义电压有效值：

$$\boxed{V_{rms} \overset{\text{def}}{=} \sqrt{\frac{1}{T}\int_0^T v^2(t)\mathrm{d}t}}$$

有效值也称均方根值（root-mean-square，简记为 rms。）

$V_{rms} = \frac{1}{\sqrt{2}}V_m$，或 $V_m = \sqrt{2}V_{rms}$

正弦交流电路相量分析方法：

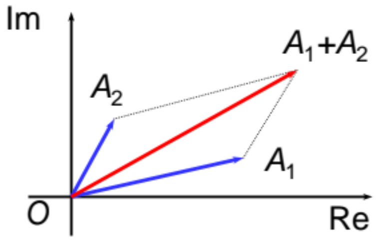
[复数的运算]

(1) 加减运算——直角坐标

若 $A_1 = a_1 + jb_1$，$A_2 = a_2 + jb_2$

则 $A_1 \pm A_2 = (a_1 \pm a_2) + j(b_1 \pm b_2)$

加减法可用图解法。



(2) 乘除运算——极坐标

若 $A_1 = |A_1|\angle\theta_1$，若 $A_2 = |A_2|\angle\theta_2$

则 $A_1 A_2 = |A_1||A_2|\angle\theta_1+\theta_2$

$$\frac{A_1}{A_2} = \frac{|A_1|\angle\theta_1}{|A_2|\angle\theta_2} = \frac{|A_1|e^{j\theta_1}}{|A_2|e^{j\theta_2}} = \frac{|A_1|}{|A_2|}e^{j(\theta_1-\theta_2)} = \frac{|A_1|}{|A_2|}\angle\theta_1-\theta_2$$

乘法：模相乘，角相加；　除法：模相除，角相减。

$$z = x + jy = r\angle\phi, \qquad z_1 = x_1 + jy_1 = r_1\angle\phi_1$$
$$z_2 = x_2 + jy_2 = r_2\angle\phi_2$$

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Addition:

$$z_1 + z_2 = (x_1 + x_2) + j(y_1 + y_2) \tag{9.18a}$$

Subtraction:

$$z_1 - z_2 = (x_1 - x_2) + j(y_1 - y_2) \tag{9.18b}$$

Multiplication:

$$z_1 z_2 = r_1 r_2 \angle\phi_1 + \phi_2 \tag{9.18c}$$

Division:

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}\angle\phi_1 - \phi_2 \tag{9.18d}$$

Reciprocal:

$$\frac{1}{z} = \frac{1}{r}\angle-\phi \tag{9.18e}$$

复数的运算

$z \quad r\underline{/-\phi}$ (9.18e)

Square Root:

$$\sqrt{z} = \sqrt{r}\,\underline{/\phi/2} \qquad \textbf{(9.18f)}$$

Complex Conjugate:

复数的共轭

$$z^* = x - jy = r\underline{/-\phi} = re^{-j\phi} \qquad \textbf{(9.18g)}$$

$v(t) = V_m\cos(wt+\theta) \longleftrightarrow A(t) = V_m e^{j(wt+\theta)} = V_m \cdot e^{j\theta} \cdot e^{jwt}$, $\dot{V} = V_m \cdot e^{j\theta} = V_m \angle\theta$

$\dot{I} = I_m \angle\theta$

$\Rightarrow v(t) = \mathrm{Re}(\dot{V}e^{jwt})$

$i(t) = \mathrm{Re}(\dot{I}e^{jwt})$

几何意义：在实轴上的投影

同频率： $i_1 \pm i_2 = i_3 \Longleftrightarrow \dot{I}_1 \pm \dot{I}_2 = \dot{I}_3$

1、同频率正弦量相加减

取实部

$$v_1(t) = \mathrm{Re}(\dot{V}_1 e^{j\omega t})$$

$$v_2(t) = \mathrm{Re}(\dot{V}_2 e^{j\omega t})$$

$$v_1(t)+v_2(t) = \mathrm{Re}\left(\dot{V}_1 e^{j\omega t}\right) + \mathrm{Re}\left(\dot{V}_2 e^{j\omega t}\right)$$

$$= \mathrm{Re}\left(\dot{V}_1 e^{j\omega t} + \dot{V}_2 e^{j\omega t}\right) = \mathrm{Re}\left(\left(\dot{V}_1+\dot{V}_2\right)e^{j\omega t}\right)$$

$$= \mathrm{Re}\left(\dot{V}e^{j\omega t}\right)$$

$$\therefore \quad \dot{V} = \dot{V}_1 + \dot{V}_2$$

时域 → 频域：

① 统一表示成 cos 形式

② 再用振幅和相位构成相量

正弦量的微分、积分运算：

$i \leftrightarrow \dot{I}$ $\qquad v \leftrightarrow \dot{V}$

$\frac{di}{dt} \Leftrightarrow jw\dot{I}$ $\qquad \int v\,dt \Leftrightarrow \frac{1}{jw}\cdot\dot{V}$

用相量法解常微分方程:

Using the phasor approach, determine the current $i(t)$ in a circuit described by the integrodifferential equation

on →

$$4i + 8\int i\,dt - 3\frac{di}{dt} = 50\cos(2t + 75^\circ)$$

先转换成频域: $4\dot{I} + 8\cdot\frac{1}{j\omega}\cdot\dot{I} - 3j\omega\cdot\dot{I} = 50\angle 75^\circ$

其中 $\omega = 2$

$\therefore 4\dot{I} - 4j\cdot\dot{I} - 6j\cdot\dot{I} = 50\angle 75^\circ$

故 $\dot{I} = \frac{50\angle 75^\circ}{4-10j} = \frac{50\angle 75^\circ}{10.77\angle -68.2^\circ} = 4.643\angle 143.2^\circ$

$\because 4 - 10j = 10.77\angle -68.2^\circ$

$\tan\alpha = -\frac{10}{4} \Rightarrow \alpha = -68.2^\circ$

$\therefore$ 转换回时域

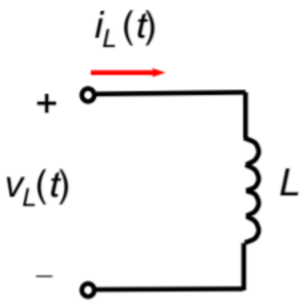
$i = 4.643\cos(143.2^\circ + 2t)\ A$

5.2.2 电路元件物理量描述.

① $R = \frac{v_R(t)}{i_R(t)} = \frac{R I_{m,R}\cos(\omega t+\phi_i)}{I_{m,R}\cos(\omega t+\phi_i)}$.

$\therefore \dot{V}_R = R\cdot\dot{I}_R$

② 2、电感



设 $i_L(t) = I_{m,L}\cos(\omega t + \phi_i)$

则 $v_L(t) = L\frac{\mathrm{d}i_L(t)}{\mathrm{d}t} = \underbrace{-\omega L I_{m,L}}_{V_{m,L}}\sin(\omega t + \phi_i)$

$= V_{m,L}\cos(\omega t + \phi_i + \frac{\pi}{2})$

特点: (1) v, i 同频

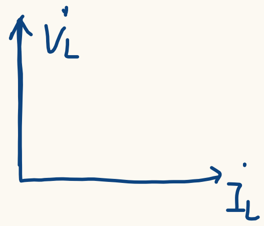
(2) 相位关系: $\phi_v = \phi_i + 90^\circ$ (v 超前 i 90°)

(3) 幅值关系: $V_{m,L} = \omega L I_{m,L}$

或 $I = \frac{V_{m,L}}{}$

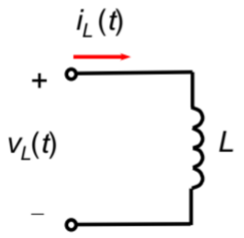
或 $I_{m,L} = \dfrac{\quad}{\omega L}$

$\dot{V}_L = j\omega L \dot{I}_L$



(由于 $v = L\cdot\frac{di}{dt}$. … v 超前 $\frac{\pi}{2}$)

2、电感



设 $i_L(t) = I_{m,L}\cos(\omega t + \phi_i)$

则 $v_L(t) = L\dfrac{di_L(t)}{dt} = -\underbrace{\omega L I_{m,L}}_{V_{m,L}}\sin(\omega t + \phi_i)$

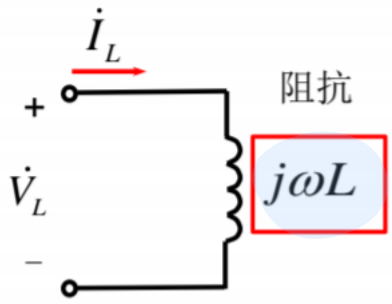
$= V_{m,L}\cos(\omega t + \phi_i + \dfrac{\pi}{2})$

特点：(1) v, i 同频

(2) 相位关系：$\phi_v = \phi_i + 90^\circ$ （v 超前 i 90° ）

(3) 幅值关系：$V_{m,L} = \omega L I_{m,L}$

或 $I_{m,L} = \dfrac{V_{m,L}}{\omega L}$

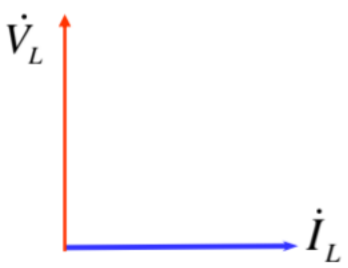


相量表示：

$$\dot{V}_L = V_L \angle \phi_v$$

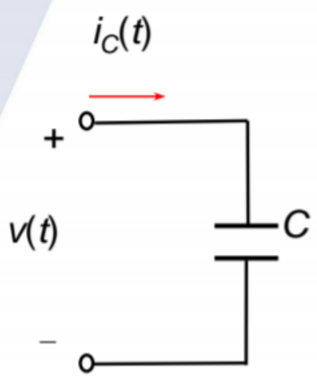
$$\dot{I}_L = \frac{\dot{V}_L}{j\omega L}$$

$$\dot{V}_L = j\omega L \dot{I}_L$$



相量图

3、电容



已知 $v_C(t) = V_{m,C}\cos(\omega t + \phi_v)$

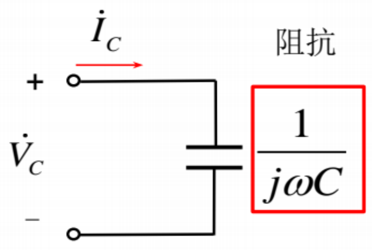
则 $i_C(t) = C\dfrac{dv_C(t)}{dt} = -\omega C V_{m,C}\sin(\omega t + \phi_C)$

$= \omega C V_{m,C}\cos(\omega t + \phi_C + \dfrac{\pi}{2})$

特点：(1) v, i 同频

(2) 相位关系：i 超前 v 90°

(3) 幅值关系：$I_{m,C} = \omega C\, v_{m,C}$

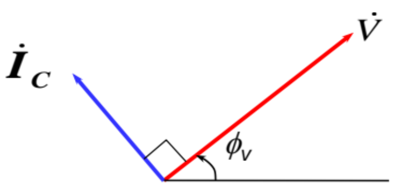


相量形式：

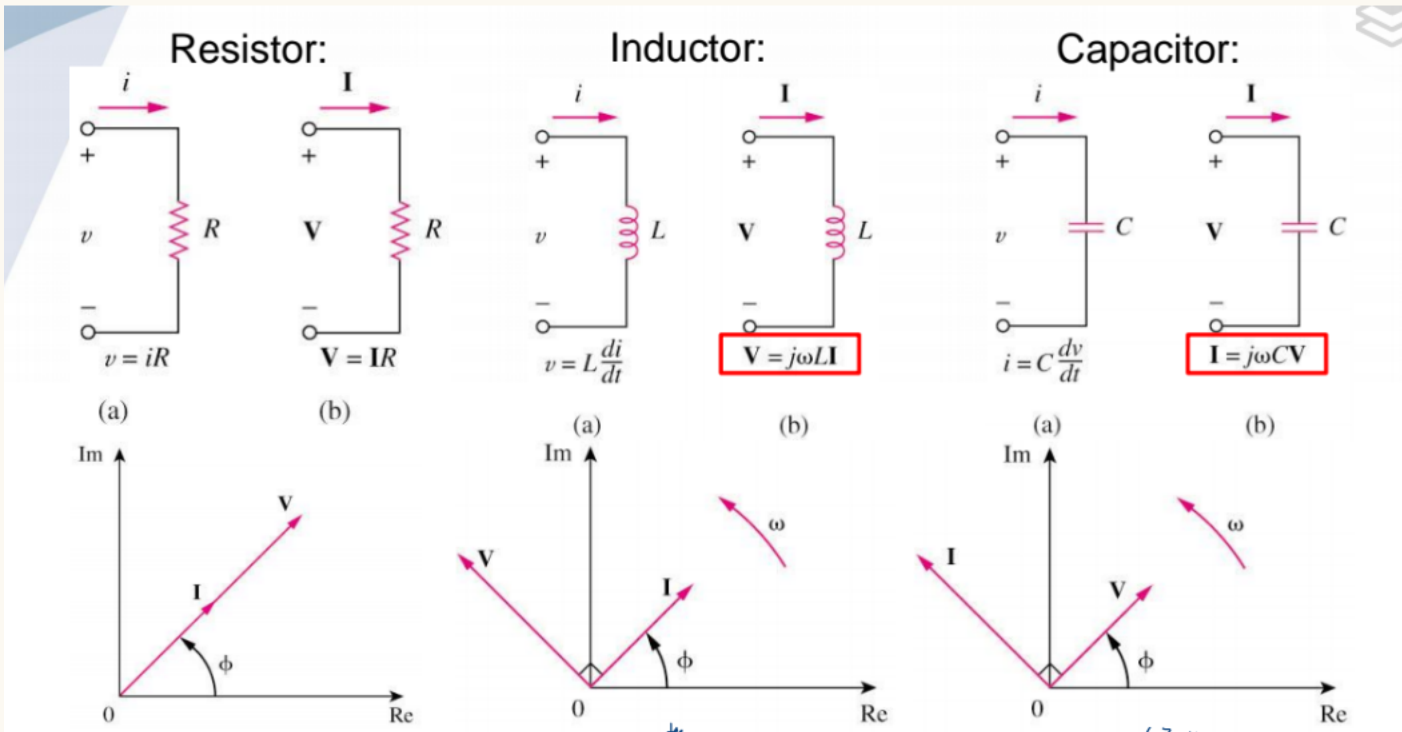
$$\dot{V}_C = V_C \angle \phi_v$$

$$\dot{I}_C = j\omega C \dot{V}_C$$

$$\dot{V}_C = \frac{1}{j\omega C} \dot{I}_C$$



相量图



滞后　　　　　　领先

电感：**I** lags **V**　　　电容：**I** leads **V**

因为一般是施加电压，求电流，所以在描述相位关系时，常用“电流领先/滞后电压”

TABLE 9.2

Summary of voltage-current relationships.

Element	Time domain	Frequency domain
R	$v = Ri$	$\mathbf{V} = R\mathbf{I}$
L	$v = L\frac{di}{dt}$	$\mathbf{V} = j\omega L\mathbf{I}$
C	$i = C\frac{dv}{dt}$	$\mathbf{V} = \frac{\mathbf{I}}{j\omega C}$

阻抗: $Z=\frac{\dot{V}}{\dot{I}}$, 单位: 欧姆

阻抗也可表示为　　　　阻抗是一个复数

$$Z = R + jX$$

其中R为电阻(resistance)，X为电抗(reactance)。

当$X>0$时，称为感性阻抗或滞后阻抗。j

当$X<0$时，称为容性阻抗或超前阻抗。$-j$

电感　$Z_L = j\omega L$

电容　$Z_C = \frac{1}{j\omega C}$ $= \frac{1}{wC}\cdot(-j)$

阻抗也可表示为极坐标形式

$$Z = |Z|\angle\theta$$

$$|Z| = \sqrt{R^2+X^2}, \quad \theta = \tan^{-1}\frac{X}{R}$$

E.g. 求 $v(t)$, $i(t)$

Example 9.12

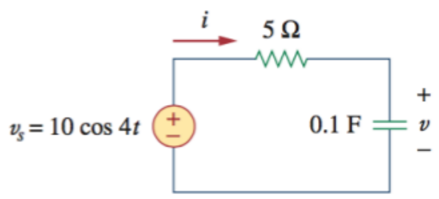


Figure 9.16
For Example 9.9.

$\dot{V}_s = 10\angle 0°$.

$Z_c = -j\cdot 2.5$

$\therefore Z_{总} = 5-2.5j$

$\therefore \dot{I} = \frac{\dot{V}_s}{Z} = \frac{10\angle 0°}{\frac{5}{2}\sqrt{5}\angle -26.57°} = 1.79\angle 26.57°$ A

$\dot{V} = \dot{I}\cdot Z_c = 1.79\angle 26.57° \cdot 2.5\angle -90°$

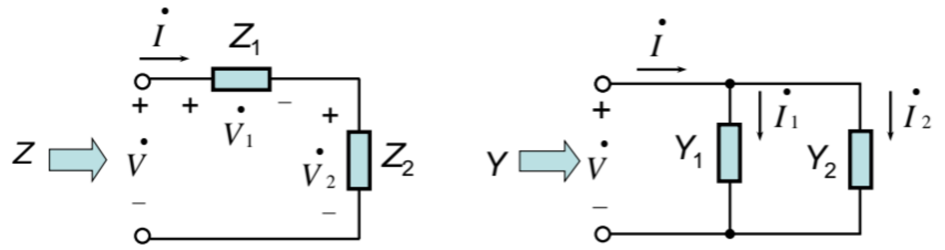
$= 4.48\angle -63.43°$

故 $i(t) = 1.79\cos(4t+26.57°)$

$v(t) = 4.48\cos(4t-63.43°)$

串/并联、Y-Δ：与直流类似：

三、串联、并联和Y-Δ电阻变换



$$\begin{cases} Z = Z_1 + Z_2 \\ \dot{V}_1 = \dfrac{Z_1}{Z}\dot{V} \end{cases} \qquad \begin{cases} Y = Y_1 + Y_2 \\ \dot{I}_1 = \dfrac{Y_1}{Y}\dot{I} \end{cases} \Rightarrow \begin{cases} Z = \dfrac{Z_1 Z_2}{Z_1 + Z_2} \\ \dot{I}_1 = \dfrac{Z_2}{Z_1 + Z_2}\dot{I} \end{cases}$$

同直流电路相似：

串联： $Z = \sum Z_k, \quad \dot{V}_k = \dfrac{Z_k}{\sum Z_k}\dot{V}$

并联： $Y = \sum Y_k, \quad \dot{I}_k = \dfrac{Y_k}{\sum Y_k}\dot{I}$

Y-Δ电阻变换的公式也类似。

若阻值相等，则称为**平衡**，平衡时delta的阻值是wye的3倍

A delta or wye circuit is said to be **balanced** if it has equal impedances in all three branches.

一般：

$$\boxed{\mathbf{Z}_\Delta = 3\mathbf{Z}_Y \quad \text{or} \quad \mathbf{Z}_Y = \frac{1}{3}\mathbf{Z}_\Delta}$$

w = 50 rad/s. 求总的阻抗!

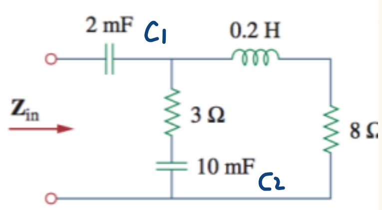


Figure 9.23
For Example 9.10.

$Z_{C_1} = \dfrac{1}{wC}\cdot(-j) = -10j \qquad Z_{L_1} = j\cdot wL = 10j$

$Z_{C_2} = -2j$

$\therefore Z_{in} = -10j + \dfrac{(3-2j)\cdot(10j+8)}{3-2j+10j+8} = (3.22 - 11.07j)\,\Omega.$

Eg. 9.11. 求 $V_o(t)$

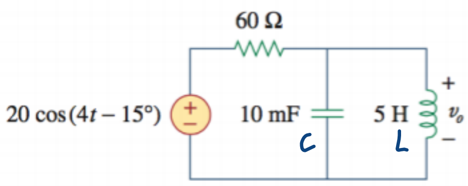


Figure 9.25
For Example 9.11.

$$Z_C = \frac{1}{wC}\cdot(-\vec{j}) = -25\vec{j}$$

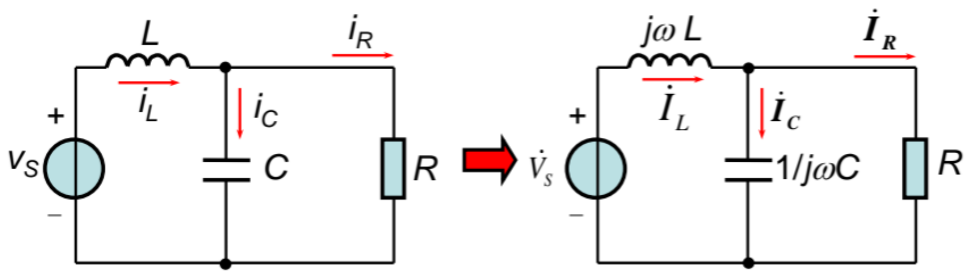
$$Z_L = wL\cdot\vec{j} = 20\vec{j}$$

$$Z_{总} = 60 + \frac{-25j\cdot 20j}{-5j} = 60+100j.$$

$$\frac{100j}{60+100j}\,20\cdot\angle-15^\circ = 20\cdot\angle-15^\circ\cdot\frac{100\angle 90^\circ}{116.62\angle 59.04^\circ} = 17.15\angle 15.96^\circ\ V$$

$\therefore V_o(t) = 17.15\cos(4t+15.96^\circ)\ V.$

一、电路的相量模型（phasor model ）



时域电路　　　　相量模型

$$\begin{cases} i_L = i_C + i_R \\ L\dfrac{\mathrm{d}i_L}{\mathrm{d}t} + \dfrac{1}{C}\displaystyle\int i_C\mathrm{d}t = v_S \\ R i_R = \dfrac{1}{C}\displaystyle\int i_C\mathrm{d}t \end{cases} \Rightarrow \begin{cases} \dot{I}_L = \dot{I}_C + \dot{I}_R \\ j\omega L\dot{I}_L + \dfrac{1}{j\omega C}\dot{I}_C = \dot{V}_S \\ R\dot{I}_R = \dfrac{1}{j\omega C}\dot{I}_C \end{cases}$$

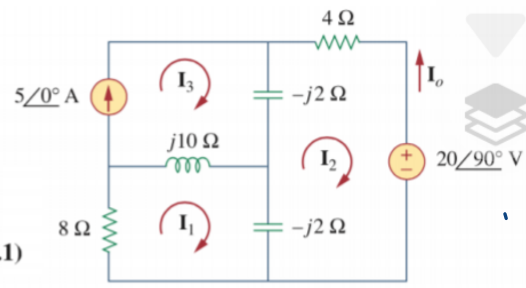
时域列写微分方程　　　　相量形式代数方程

相量模型：电压、电流用相量；元件用复数阻抗或导纳。

网孔电流法 & 节点电压法.

Determine current $\mathbf{I}_o$ in the circuit of Fig. 10.7 using mesh analysis. **Example 10.3**

频域网孔电流法



Solution:
Applying KVL to mesh 1, we obtain

$$(8 + j10 - j2)\mathbf{I}_1 - (-j2)\mathbf{I}_2 - j10\mathbf{I}_3 = 0 \qquad \textbf{(10.3.1)}$$

$$(8 + j10 - j2) I_1 - \cancel{j10 I_3}^{\,50j} + j2 I_2 = \cancel{0}\ 50j$$

$$(4 - j4) I_2 + j2 I_1 + \cancel{j2 I_3}^{\,10j} = -20\angle 90^\circ = \cancel{20j} - 30j$$

$$I_3 = 5\angle 0^\circ \text{ A}$$

$$\therefore (8j + 8) I_1 + 2j I_2 = 50j$$

$$2j I_1 + (4 - 4j) I_2 = -30j$$

$$\therefore \begin{bmatrix} 8j+8 & 2j \\ 2j & 4-4j \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 50j \\ -30j \end{bmatrix}$$

$\therefore$