Direct kinematic solution

$$P1 = \begin{bmatrix} L_1 * \cos(\theta_1) \\ L_1 * \sin(\theta_1) \\ 0 \end{bmatrix}$$

If P2proj is the projection of P2 on the (x, y) plan, and d12 is the distance between P1 and P2proj then:

$$P2 = \begin{bmatrix} (L_1 + d_{12}) * \cos(\theta_1) \\ (L_1 + d_{12}) * \sin(\theta_1) \\ 0 + L_2 * \sin(\theta_2) \end{bmatrix}$$

With:

$$d_{12} = L_2 * \cos(\theta_2)$$

=>

$$P2 = \begin{bmatrix} P_1.x + \cos(\theta_1) * L_2 * \cos(\theta_2) \\ P_1.y + \sin(\theta_1) * L_2 * \cos(\theta_2) \\ P_1.z + L_2 * \sin(\theta_2) \end{bmatrix}$$

If P3proj is the projection of P3 on the (x, y) plan, and d23 is the distance between P2proj and P3proj then :

$$P3 = \begin{bmatrix} (L_1 + d_{12} + d_{23}) * \cos(\theta_1) \\ (L_1 + d_{12} + d_{23}) * \sin(\theta_1) \\ P_2 \cdot z + L_3 * \sin(\theta_2 + \theta_3) \end{bmatrix}$$

With:

$$d_{23} = L_3 * \cos(\theta_2 + \theta_3)$$

If we call:

planContribution=
$$L_1+d_{12}+d_{23}$$

planContribution= $L_1+L_2*\cos(\theta_2)+L_3*\cos(\theta_2+\theta_3)$

Final result:

$$P3 = \begin{bmatrix} planContribution*\cos(\theta_1) \\ planContribution*\sin(\theta_1) \\ L_2*\sin(\theta_2) + L_3*\sin(\theta_2 + \theta_3) \end{bmatrix}$$