Problem Similar to 2.3.5: Evaluate (be careful if $n = m \neq 0$ and if $n = m = 0$)

$$\int_{-L}^{L} \sin\frac{n\pi x}{L} \sin\frac{m\pi x}{L} dx,$$

and

$$\int_{-L}^{L} \cos\frac{n\pi x}{L} \cos\frac{m\pi x}{L} dx,$$

for $n > 0$, $m > 0$.

Problem 2.4.3: Solve the eigenvalue problem

$$\frac{d^2\phi}{dx^2} = -\lambda\phi,$$

subject to $\phi(0) = \phi(2\pi)$, and $\frac{d\phi}{dx}(0) = \frac{d\phi}{dx}(2\pi)$. *Note: This is an ODE, not a PDE so there is no time dependence here.* You must consider whether $\lambda > 0$, $\lambda = 0$, or $\lambda < 0$.

Problem 2.4.6: Determine the equilibrium temperature distribution for the thin circular ring of Section 2.4.2:

(a) directly from the equilibrium problem (see Section 1.4), and

(b) by computing the limit as $t \to \infty$ of the time-dependent problem.

Problem 2.5.1(a): Solve Laplace's equation inside a rectangle $0 \leq x \leq L$, $0 \leq y \leq H$, with the following boundary conditions:

(a) $\frac{\partial u}{\partial x}(0, y) = 0$, $\frac{\partial u}{\partial x}(L, y) = 0$, $u(x, 0) = 0$, $u(x, H) = f(x)$.

Problem 2.5.3(a): Solve Laplace's equation *outside* a circular disk ($r \geq a$) subject to the boundary conditions

(a) $u(a, \theta) = \ln 2 + 4\cos\theta$

Problem 2.5.5(a): Solve Laplace's equation inside the quarter circle of radius 1 ($0 \leq \theta \leq \pi/2$, $0 \leq r \leq 1$) subject to the boundary conditions

(a) $\frac{\partial u}{\partial \theta}(r, 0) = 0$, $u(r, \pi/2) = 0$, and $u(1, \theta) = f(\theta)$.

Problem 2.5.6(a): Solve Laplace's equation inside a semicircle of radius a ($0 < r < a$, $0 < \theta < \pi$) subject to the boundary conditions

(a) $u(r, \theta) = 0$ on the diameter and $u(a, \theta) = g(\theta)$.

Problem 2.5.15(a): Solve Laplace's equation inside a semi-infinite strip ($0 < x < \infty$, $0 < y < H$) subject to the boundary conditions

(a) $\frac{\partial u}{\partial y}(x, 0) = 0$, $\frac{\partial u}{\partial y}(x, H) = 0$, and $u(0, y) = f(y)$.