Math 248 - HW 9

35.
a) $\int_a^b f(x)\,dx = M(n) + (b-a)\frac{h^2}{24}f''(c)$ (midpoint)

$+ \int_a^b f(x)\,dx = T(n) - (b-a)\frac{h^2}{12}f''(c)$ (trapezoidal)

$2\int_a^b f(x)\,dx = M(n) + T(n) + \left((b-a)\frac{h^2}{24}f''(c) - (b-a)\frac{2h^2}{24}f''(c)\right)$

$2\int_a^b f(x)\,dx = M(n) + T(n) - \frac{h^2}{24}$

$\int_a^b f(x)\,dx = \frac{M(n) + T(n) - \frac{h^2}{24}}{2}$

Trap Error >
Midpoint Error >
B.I. Error

Arbitrary $f(x) = x^3 + 4x^2 - 10$

True Value: $\int_0^{10} f(x)\,dx = 3733.3333333\overline{3}$

Midpoint: out = mid(f, 0, 10, 100000)
= 3733.333333175029

Midpoint Error = |True - out| = $1.5830391931 \times 10^{-7}$

Trapezoidal: out = trap(f, 0, 10, 100000)
= 3733.333333649982

Trapezoidal Error = |True - out| = $3.1664876587 \times 10^{-7}$

Better Integration: out = betterint(f, 0, 10, 100000)
= 3733.333333412297

B.I. Error = |True - out| = $7.8964148997 \times 10^{-8}$

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35.
b) $\int_0^2 e^x dx = e^2 - 1 = 6.38905609893$ (True Val)

Midpoint:
$n = 100$
out $= 6.38894961590463 6$
$error_1 = 1.06483026014 \times 10^{-4}$

$\frac{error_1}{error_2} \approx 4$

$n = 200$
out $= 6.389029477941217$
$error_2 = 2.66209894333 \times 10^{-5}$

$\frac{error_2}{error_3} \approx 4$

$n = 400$
out $= 6.389049443668735$
$error_3 = 6.65526191579 \times 10^{-6}$

Trapezoidal:
$n = 100$
out $= 6.389269066047504$
$error_1 = 2.12967116854 \times 10^{-4}$

$\frac{error_1}{error_2} \approx 4$

$n = 200$
out $= 6.389109340976075$
$error_2 = 5.32420454249 \times 10^{-5}$

$\frac{error_2}{error_3} \approx 4$

$n = 400$
out $= 6.389069409458643$
$error_3 = 1.33105279927 \times 10^{-5}$

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35. c) $\int_0^2 e^x dx = e^2 - 1 = 6.38905609893$ (True Val)

Better Int:

$n = 100$
out $= 6.389101007642736$
$\text{error}_1 = 4.49087120866 \times 10^{-5}$

$\frac{\text{error}_1}{\text{error}_2} \simeq 4$

$n = 200$
out $= 6.389067326125312$
$\text{error}_2 = 1.12271946662 \times 10^{-5}$

$\frac{\text{error}_2}{\text{error}_3} \simeq 4$

$n = 400$
out $= 6.389058905730356$
$\text{error}_3 = 2.80679970555 \times 10^{-6}$

Ratio of errors is the same as midpoint and trapezoidal approximations (~ 4) but on average the integral approximations were more accurate.

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36.
a) $\int_0^1 \sqrt{x}\, e^x\, dx \simeq 1.2556300825518 64$ (True Val)

Midpoint:
$n = 100$
out $= 1.255674146223198$
$\text{error}_1 = 4.40636713341 \times 10^{-5}$

$n = 200$
out $= 1.255647391656979$
$\text{error}_2 = 1.73091051152 \times 10^{-5}$

$\dfrac{\text{error}_1}{\text{error}_2} \simeq 2.546$

$n = 400$
out $= 1.255636636927137$
$\text{error}_3 = 6.55437527319 \times 10^{-6}$

$\dfrac{\text{error}_2}{\text{error}_3} \simeq 2.641$

The ratio of errors is not 4 as expected, it is closer to 2.5

b) Since the midpoint approximation needs the second derivative f'' and since f'' is not bounded at $x=0$, the function is not well-behaved and the error blows up as $x \to 0$.

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36. c) $\int_0^1 \sqrt{x}\, e^x\, dx \rightarrow \int_0^1 2u^2 e^{u^2} du$

$u = \sqrt{x}$
$x = u^2$
$dx = 2u\,du$

Midpoint:
$n = 100$
out = 1.2555394776500 41
$\text{error}_1 = 9.060490 18228 \times 10^{-5}$

$n = 200$
out = 1.255607430484084
$\text{error}_2 = 2.26520677797 \times 10^{-5}$

$\frac{\text{error}_1}{\text{error}_2} \approx 4$

$n = 400$
out = 1.2556244194822 71
$\text{error}_3 = 5.66306959282 \times 10^{-6}$

$\frac{\text{error}_2}{\text{error}_3} \approx 4$

It's working well this time because the second derivative of the new integrand $2u^2e^{u^2}$ is well-behaved on $[0,1]$, so the error doesn't blow up as $x \to 0$.