Problem 3.2.2(d): For the following functions, sketch the Fourier series of $f(x)$ (on the interval $-L \leq x \leq L$) and determine the Fourier coefficients where

$$f(x) = \begin{cases} 0 & x < 0 \\ x & x > 0 \end{cases}$$

Problem 3.2.3: Show that the Fourier series operation is linear. That is, show that the Fourier series of $c_1 f(x) + c_2 g(x)$ is the sum of c_1 times the Fourier series of $f(x)$ and c_2 times the series of $g(x)$.

Problem 3.3.1(b): For the following functions, sketch $f(x)$, the Fourier series of $f(x)$, the Fourier sine series of $f(x)$, and the Fourier cosine series of $f(x) = 1 + x$

Problem 3.3.2(b): For the following functions, sketch the Fourier sine series of $f(x)$ and determine its Fourier coefficients:

$$f(x) = \begin{cases} 1 & x < L/6 \\ 3 & L/6 < x < L/2 \\ 0 & x > L/2 \end{cases}$$

Problem 3.3.3(b): For the following functions, sketch the Fourier sine series of $f(x)$. Also, roughly sketch the sum of a finite number of nonzero terms (at least the first two)of the Fourier sine series:

(b) $f(x) = \begin{cases} 1 & x < L/2 \\ 0 & x > L/2 \end{cases}$

Problem 3.3.5(b): For the following functions, sketch the Fourier cosine series of $f(x)$ and determine its Fourier coefficients:

$$f(x) = \begin{cases} 1 & x < L/6 \\ 3 & L/6 < x < L/2 \\ 0 & x > L/2 \end{cases}$$

Problem 3.3.6(b): For the following function, sketch the Fourier cosine series of $f(x)$. Also roughly sketch the sum of a finite number of nonzero terms (at least the first two) of the Fourier cosine series:

(b) $f(x) = \begin{cases} 0 & x < L/2 \\ 1 & x > L/2 \end{cases}$

Problem 3.3.7: Show that e^x is the sum of an even and an odd function.

Problem 3.3.8(a-c):

(a) Determine the formulas for the even extension of any function $f(x)$. Compare to the formula for the even part of $f(x)$.

(b) Do the same for the odd extension of $f(x)$ and the odd part of $f(x)$.

(c) Calculate and sketch the four functions of parts (a) and (b) if

$$f(x) = \begin{cases} x & x > 0 \\ x^2 & x < 0. \end{cases}$$

Problem 3.4.4(b): Suppose that $f(x)$ and df/dx are piecewise smooth. Prove that the Fourier cosine series of a continuous function $f(x)$ can be differentiated term by term.

Problem 3.4.8: Consider

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$$

subject to $\frac{\partial u}{\partial x}(0,t) = 0 = \frac{\partial u}{\partial x}(L,t)$ and $u(x,0) = f(x)$. Solve in the following way. Look for the solution as a Fourier cosine series. Assume that u and $\partial u/\partial x$ are continuous and that $\partial^2 u/\partial x^2$ and $\partial u/\partial t$ are piecewise smooth. Justify all differentiations of infinite series.

Problem 3.5.2(a-b):

(a) Using (3.3.11) and (3.3.12), obtain the Fourier cosine series of x^2.

(b) From part (a), determine the Fourier sine series of x^3.