Math 248 - HW 11

38.

$$\int_0^1 \underbrace{\sqrt{x}\, e^x}_{f(x)}\, dx \approx 1.2556300082551863 \quad \text{(True Val)}$$

Romberg:

n = 4
out = romberg(f, 0, 1, 4) = 1.2526188687212231
$error_1 = |true - out| = 0.0030111213383063$

n = 8
out = 1.2555827852443372
$error_2 = 0.0000472973074911$
$\dfrac{error_1}{error_2} = 63.665650126 \approx 64$

n = 16
out = 1.2556300070993618
$error_3 = 1.155824 5 \times 10^{-8}$
$\dfrac{error_2}{error_3} = 4092.08383202 \approx 4092$

(error going down rapidly) ↓

39. $y' = \frac{2}{t}y + t^2 e^t$, $1 \le t \le 2$, $y(1) = 0$

a) Show that the solution is $y = t^2(e^t - e)$

$y' = 2t(e^t - e) + t^2 e^t$

$2t(e^t - e) + t^2 e^t = \frac{2}{t}(t^2(e^t - e)) + t^2 e^t$

$2t(e^t - e) + t^2 e^t = 2t(e^t - e) + t^2 e^t$ ✓

The equation holds true, so $y = t^2(e^t - e)$ is a solution to the differential equation.

$0 = (1)^2(e^{(1)} - e)$

$0 = 1(e - e)$

$0 = 1(0)$

$0 = 0$ ✓

The initial condition $y(1) = 0$ is satisfied

b) Euler:

$n = 10$

$error_1 = 3.284861429107181$

$n = 20$

$error_2 = 1.7340838095350012$

$n = 40$

$error_3 = 0.8917325830515335$

$n = 80$

$error_4 = 0.4522816594036335$

b) $\dfrac{error_1}{error_2} = 1.89429219686 \approx 2$

$\dfrac{error_2}{error_3} = 1.94462313309 \approx 2$

$\dfrac{error_3}{error_4} = 1.97163109428 \approx 2$

Yes, the errors are approximately halving each time n is doubled.

c) Taylor:

$$y(t+h) = y(t) + hy'(t) + \frac{h^2}{2} y''(t) + \dots$$

2nd Order: $y(t+h) = y(t) + h y'(t) + \frac{h^2}{2} y''(t)$

4th Order: $y(t+h) = y(t) + hy'(t) + \frac{h^2}{2} y''(t) + \frac{h^3}{6} y'''(t)$
$\qquad\qquad\qquad + \frac{h^4}{24} y''''(t)$

$n = 10$
2nd Order $error_1 = 0.213102599324632$
4th Order $error_1 = 0.035186114604404$

$n = 20$
2nd Order $error_2 = 0.056143364896243$
4th Order $error_2 = 0.009323895048190$

$n = 40$
2nd Order $error_3 = 0.014412211428802$
4th Order $error_3 = 0.002413167802313$

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c) $n = 80$
   2nd Order $error_4 = 0.0036512675\underline{8}3550$
   4th Order $error_4 = 0.000613759773546\underline{7}182$

The errors are behaving as expected since they are decreasing and the 4th order approximations are always more accurate than the 2nd order approximations.