Math 248 - HW10

37.
a) $\int_0^1 \ln(x) f(x)\,dx$

You can't use Trapezoidal or Simpson's rules to approximate the above integral because there is a singularity at the left endpoint ($x=0$). Since $\ln(0)$ does not exist, the integrand ($\ln(x)f(x)$) is undefined at $x=0$, therefore making the approximations inaccurate.

b) $\int_0^1 \ln(x) f(x)\,dx \approx af(0)+bf\left(\frac{1}{2}\right)+cf(1)+kf^{(4)}(c)$

Exact for: $f(x)=1, x, x^2$

$$\int_0^1 x^n \ln(x)\,dx = \frac{-1}{(n+1)^2}$$

$f(x)=1$:

$$\int_0^1 \ln(x)\cdot 1\,dx = \frac{-1}{(0+1)^2} = -1$$

$f(x)=x$:

$$\int_0^1 x\ln(x)\,dx = \frac{-1}{(1+1)^2} = \frac{-1}{4}$$

$f(x)=x^2$:

$$\int_0^1 x^2\ln(x)\,dx = \frac{-1}{(2+1)^2} = \frac{-1}{9}$$

$f(x)=x^3$:

$$\int_0^1 x^3\ln(x)\,dx = \frac{-1}{(3+1)^2} = \frac{-1}{16}$$

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37. $-1 = a(1) + b(1) + c(1) = a + b + c$

b) $-\frac{1}{4} = a(0) + b(\frac{1}{2}) + c(1) = \frac{1}{2}b + c$

$-\frac{1}{9} = a(0) + b(\frac{1}{4}) + c(1) = \frac{1}{4}b + c$

$(\frac{1}{2}b + c) - (\frac{1}{4}b + c) = -\frac{1}{4} + \frac{1}{9} \Rightarrow \frac{1}{4}b = \frac{-5}{36} \Rightarrow b = \frac{-5}{9}$

$\frac{1}{2}(\frac{-5}{9}) + c = \frac{-1}{4} \Rightarrow \frac{-5}{18} + c = \frac{-1}{4} \Rightarrow c = \frac{1}{36}$

$a - \frac{5}{9} + \frac{1}{36} = -1 \Rightarrow a - \frac{19}{36} = -1 \Rightarrow a = \frac{-17}{36}$

$k = \frac{-1}{288}$ (not exact: for x^n with $n > 2$)

$\int_0^1 \ln(x) f(x)\,dx = \frac{-17}{36} f(0) + \frac{-5}{9} f(\frac{1}{2}) + \frac{1}{36} f(1) + \frac{-1}{288} f^{(4)}(0)$

c) $\int_0^1 \ln(x)\cos(x)\,dx \approx -0.946\,083\,070\,4$ (True Val)

$f(x) = \cos(x)$

$f(0) = 1$

$f(\frac{1}{2}) = 0.877\,582\,561\,89$

$f(1) = 0.540\,302\,305\,868$ (approx.)

$\frac{-17}{36} f(0) + \frac{-5}{9} f(\frac{1}{2}) + \frac{1}{36} f(1) = -0.944\,759\,692\,554$

error $= |\text{true} - \text{approx}| = 1.323\,377\,813\,31 \times 10^{-3}$

(fairly accurate approx.)

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37. $f(x) = \ln(x)\cos(x)$

d) $\int_0^1 \ln(x) f(x)\,dx \approx -0.9460830704$

Midpoint:

$n = 50$:

$approx = -0.9391605118677891$

$error_1 = 6.922558499291 \times 10^{-3}$

$n = 100$:

$approx = -0.9426195742816777$

$error_2 = 3.463496085551 \times 10^{-3}$

$n = 200$:

$approx = -0.9443507638019066$

$error_3 = 1.732306565528 \times 10^{-3}$

$error_{prev} = 1.323377813331 \times 10^{-3}$

$error_3 \approx error_{prev}$ with $n = 200$