MATH440 PA Thursday Week 2

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August 28, 2025

1 Determine the equilibrium temperature distribution for a one-dimensional rod with constant thermal properties with the following sources and boundary conditions:

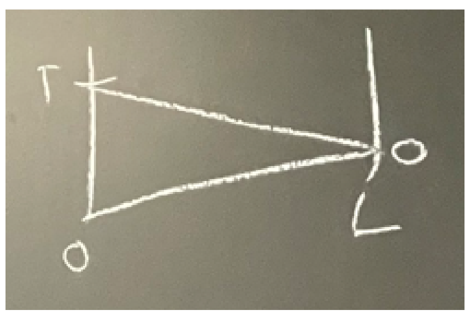
1.1 (1.4.2b) $Q = 0, u(0) = T, u(L) = 0$

Heat Equation: $\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2} + Q$
Constant thermal properties ($\frac{\partial u}{\partial t} = 0$) and no sources/sinks ($Q = 0$)
$\Rightarrow k\frac{\partial^2 u}{\partial x^2} = 0$
$\Rightarrow \frac{\partial^2 u}{\partial x^2} = 0$
$\Rightarrow \frac{\partial u}{\partial x} = c_1$
$\Rightarrow u(x) = c_1 x + c_2$

$u(0) = c_1(0) + c_2 = T$
$\Rightarrow c_2 = T$

$u(L) = c_1(L) + c_2 = 0$
$\Rightarrow c_1(L) + T = 0$
$\Rightarrow c_1 L = -T$
$\Rightarrow c_1 = -\frac{T}{L}$

$u(x) = c_1 x + c_2 = -\frac{T}{L}x + T$
Heat distribution: $u(x) = -\frac{T}{L}x + T$
Below is a diagram of this distribution:



1.2 (1.4.2c) $Q = 0, \frac{\partial u}{\partial x}(0) = 0, u(L) = T$

Heat Equation: $\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2} + Q$
Constant thermal properties ($\frac{\partial u}{\partial t} = 0$) and no sources/sinks ($Q = 0$)
$\Rightarrow k\frac{\partial^2 u}{\partial x^2} = 0$
$\Rightarrow \frac{\partial^2 u}{\partial x^2} = 0$

$\frac{\partial u}{\partial x}(0) = c_1 = 0$
$\Rightarrow c_1 = 0$

$u(x) = c_1 x + c_2$
$\Rightarrow u(x) = c_2$

$u(L) = T = c_2$
$\Rightarrow c_2 = T$

$u(x) = c_2 = T$
Heat distribution: $u(x) = T$
Below is a diagram of this distribution: