RUBRIC:

Questions	Points	Score
Total		

Problem 1.3.1: Consider a one-dimensional rod, $0 \leq x \leq L$. Assume that the heat energy flowing out of the rod at $x = L$ is proportional to the temperature difference between the end temperature of the bar and the known external temperature. Derive (1.3.5); briefly, physically explain why $H > 0$.

Problem 1.3.2: Two one-dimensional rods of different materials joined at $x = x_0$ are said to be in **perfect thermal contact** if the temperature is continuous at $x = x_0$:

$$u(x_{0-}, t) = u(x_{0+}, t)$$

and no heat energy is lost at $x = x_0$. What mathematical equation represents the later condition at $x = x_0$? Under what special condition is $\frac{\partial u}{\partial x}$ continuous at $x = x_0$?

Problem 1.4.1a,d, e-f: Determine the equilibrium temperature distribution for a one-dimensional rod with constant thermal properties with the following sources and boundary conditions:

(a) $Q = 0$, $u(0) = 0$, and $u(L) = T$

(d) $Q = 0$, $u(0) = T$, and $\frac{\partial u}{\partial x}(L) = \alpha$

(e) $\frac{Q}{K_0} = 1$, $u(0) = T_1$, and $u(L) = T_2$

(f) $\frac{Q}{K_0} = x^2$, $u(0) = T$, and $\frac{\partial u}{\partial x}(L) = 0$

Problem 1.4.2a-b: Consider the equilibrium temperature distribution for a uniform one-dimensional rod with sources $\frac{Q}{K_0} = x$ of thermal energy subject to the boundary conditions $u(0) = 0$ and $u(L) = 0$.

(a) Determine the heat energy generated per unit time inside the entire rod.

(b) Determine the heat energy flowing out of the rod per unit time at $x = 0$ and at $x = L$ (remember, this is at equilibrium).

Problem 1.4.3: Determine the equilibrium temperature distribution for a one-dimensional rod composed of two different materials in perfect thermal contact at $x = 1$. For $0 < x < 1$, there is one material ($c\rho - 1$, $K_0 = 1$) with a constant source ($Q = 1$), whereas for the other $1 < x < 2$, there are no sources ($Q = 0$, $c\rho = 2$, $K_0 = 2$ with $u(0) = 0 = u(2)$.

Problem 1.4.5: Consider a one-dimensional rod $0 \leq x \leq L$ of known length and constant thermal properties without sources or sinks. Suppose that the temperature is an *unknown* constant T at $x = L$. Determine T if we known (in the steady state) for the temperature and the heat flow at $x = 0$.

Problem 1.4.7a-b: For the following problems, determine an equilibrium temperature distribution (if one exists). For what values of β are there solutions? Explain physically.

(a) $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + 1$, $u(x, 0) = f(x)$, $\frac{\partial u}{\partial x}(0, t) = 1$, and $\frac{\partial u}{\partial x}(L, T) = \beta$

(b) $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, $u(x, 0) = f(x)$, $\frac{\partial u}{\partial x}(0, t) = 1$, and $\frac{\partial u}{\partial x}(L, T) = \beta$

Problem 1.4.10: Suppose $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + 4$, $u(x, 0) = f(x)$, $\frac{\partial u}{\partial x}(0, t) = 5$, and $\frac{\partial u}{\partial x}(L, t) = 6$. Calculate the total thermal energy in the one-dimensional rod (as a function of time).