Math 248 - HW7

31. $ax_1 + x_2 = 1$

a) $x_1 + x_2 = 2$

$(x_1 + x_2) - (ax_1 + x_2) = 2 - 1$

$x_1 - ax_1 = 1$

$x_1(1 - a) = 1$

$$x_1 = \frac{1}{1-a}$$

$x_1 + x_2 = 2$

$$\frac{1}{1-a} + x_2 = 2$$

$$x_2 = 2 - \frac{1}{1-a}$$

b) $ax_1 + x_2 = 1$
$x_1 + x_2 = 2$ $\rightarrow \left[\begin{array}{cc|c} a & 1 & 1 \\ 1 & 1 & 2 \end{array}\right]$

$a = 10^{-n}$

$n = 4, 8, 12, 16$

$a_1 = 0.0001 = 10^{-4}$

$a_2 = 0.00000001 = 10^{-8}$

$a_3 = 0.000000000001 = 10^{-12}$

$a_4 = 0.0000000000000001 = 10^{-16}$

$x_1 = \frac{1}{1-10^{-4}} = 1.00010001$ $\quad error_1 = 0$

$x_2 = 2 - \frac{1}{1-10^{-4}} = 0.99989998$ $\quad error_2 = 0$

$\left[\begin{array}{cc|c} 10^{-4} & 1 & 1 \\ 1 & 1 & 2 \end{array}\right] \rightarrow \left[\begin{array}{cc|c} 1 & 0 & 1.00010001 \\ 0 & 1 & 0.99989998 \end{array}\right]$

Little to no error for a_1

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$x_1 = \frac{1}{1-10^{-8}} = 1.00000001$ $\quad error_1 = 0.00000001$

$x_2 = 2 - \frac{1}{1-10^{-8}} = 0.99999999$ $\quad error_2 = 0$

$$\begin{bmatrix} 10^{-8} & 1 & 1 \\ 1 & 1 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 1.00000000 \\ 0 & 1 & 0.99999999 \end{bmatrix}$$

More error than a, but it is still very small

$x_1 = \frac{1}{1-10^{-12}} = 1.000000000001$

$x_2 = 2 - \frac{1}{1-10^{-12}} = 0.999999999999$

$$\begin{bmatrix} 10^{-12} & 1 & 1 \\ 1 & 1 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0.999977878279 \\ 0 & 1 & 0.999999999999 \end{bmatrix}$$

$error_1 = |1 - 0.999977878279| = 0.000022121721$

$error_2 = 0$

Error is rapidly increasing (~2000x more error than a_2)

$x_1 = \frac{1}{1-10^{-16}} = 1$

$x_2 = 2 - \frac{1}{1-10^{-16}} = 1$

$$\begin{bmatrix} 10^{-16} & 1 & 1 \\ 1 & 1 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 2.220446049250313 \\ 0 & 1 & 1.000000000000000 \end{bmatrix}$$

$error_1 = |1 - 2.220446049250313| = 1.220446049250$

$error_2 = 0$

Error has completely blown up (~55000x more error than a_3)

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31.

c) $x = A \backslash b$; $a_1 = 10^{-4}$

$x_1 = 1.0001\,0001$

$x_2 = 0.99989998$

Same answers as gauss.m (just as good)

$a_2 = 10^{-8}$

$x_1 = 1.0000\,0001$

$x_2 = 0.9999\,9999$

Slightly more accurate answers than gauss.m (backslash is slightly better)

$a_3 = 10^{-12}$

$x_1 = 1.0000\,0000\,0001$

$x_2 = 0.9999\,9999\,9999$

Notably more accurate answers than gauss.m (backslash is definitely better here)

$a_4 = 10^{-16}$

$x_1 = 1$

$x_2 = 1$

Way more accurate answers than gauss.m (backslash is also definitely better here)

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31. Reverse equation order:

d)

$$x_1 + x_2 = 2$$
$$ax_1 + x_2 = 1 \rightarrow \left[\begin{array}{cc|c} 1 & 1 & 2 \\ a & 1 & 1 \end{array}\right]$$

$a_1 = 10^{-4}$
$x_1 = 1.00010001$
$x_2 = 0.99989998$ (good)

$a_2 = 10^{-8}$
$x_1 = 1.00000001$
$x_2 = 0.99999999$ (good)

$a_3 = 10^{-12}$
$x_1 = 1.000000000001$
$x_2 = 0.999999999999$ (good)

$a_4 = 10^{-16}$
$x_1 = 1$
$x_2 = 1$ (good)