CS 4830/6830
Final Examination

1) (25 pts - 5 pts each) Random Number Generators

   a) Is the following $rng(s_i) = (13 \cdot s_i + 13)\ mod\ 16$ a full period LCG generator?

The LCG generator is full period m/4=16/4=4 is whole and (a-1)/4=(13-1)/4=3 is whole, and m and c are relatively prime. Also…

$rng(s_i) = (13 \cdot s_i + 13)\ mod\ 16$
$S_0 = 2$
$s_1 = (13 \cdot 2 + 13)\ mod\ 16 = 7$
$s_2 = (13 \cdot 7 + 13)\ mod\ 16 = 8$
$s_3 = (13 \cdot 8 + 13)\ mod\ 16 = 5$
$s_4 = (13 \cdot 5 + 13)\ mod\ 16 = 14$
$s5 = (13 \cdot 14 + 13)\ mod\ 16 = 3$
$s_6 = (13 \cdot 3 + 13)\ mod\ 16 = 4$
$s_7 = (13 \cdot 4 + 13)\ mod\ 16 = 1$
$s_8 = (13 \cdot 10 + 13)\ mod\ 16 = 15$
$s_9 = (13 \cdot 15 + 13)\ mod\ 16 = 0$
$s_{10} = (13 \cdot 0 + 13)\ mod\ 16 = 13$
$s_{11} = (13 \cdot 13 + 13)\ mod\ 16 = 6$
$s_{12} = (13 \cdot 6 + 13)\ mod\ 16 = 11$
$s_{13} = (13 \cdot 11 + 13)\ mod\ 16 = 12$
$s_{14} = (13 \cdot 12 + 13)\ mod\ 16 = 9$
$s_{15} = (13 \cdot 9 + 13)\ mod\ 16 = 2$
$s_{16} = (13 \cdot 2 + 13)\ mod\ 16 = 7$

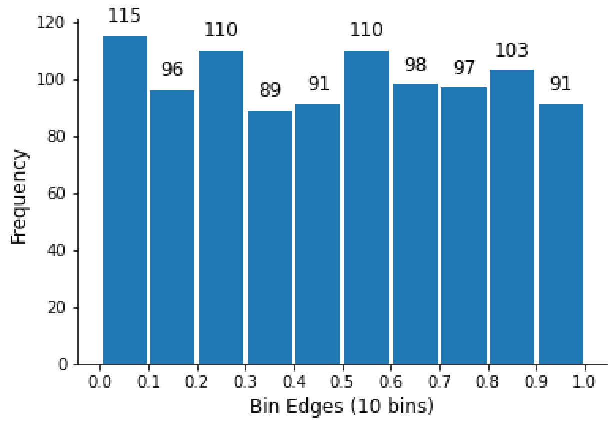
   b) Suppose we created a novel pseudo-random number generator. Then we picked an initial seed value (e.g. 5634781290) and then generated the ten pseudo-random numbers shown below:

      5634781290
      4263748710
      6722158713
      6721558719
      6720958725
      6720358731
      6719758737
      6719158743
      6718558749
      6717958755

      Without focusing on the algorithmic details of the generator, discuss the performance of this generator relative to the desirable properties of a "good" pseudo-random number generator.

I don't think that this would be a "good" random number generator. It has way too short of a period even if it otherwise appeared to have no correlation between values and was evenly distributed the period is so short that we can't see any clear indication of the other desirable properties.

   c) Suppose we used a new pseudo-random number generator to generate 1000 values on the real interval [0.0, 1.0]. We then binned values to create the histogram shown below. Using an empirical test, demonstrate whether or not this sample of 1000 values satisfies the test for uniformity.



Test:

d) When statistical tests for "randomness" are applied to new pseudo-random number generators, we often apply (1) serial tests, (2) runs tests, and (3) correlation tests. In a few sentences, explain how each of these tests is conducted.

The serial test is done by … The runs test is done by… The correlation test is done by

e) Suppose we used the following fragment of code to write 100,000 uniformly distributed random numbers on the interval [0.0,1.0] to a binary file named randomData.bin.

```
from scipy import stats
import pickle

fileHandle = open('randomData.bin', 'wb')
pickle.dump(stats.uniform.rvs(size=100000), fileHandle)
fileHandle.close()
```

If we tried to compress randomData.bin using a zip application, what level of compression would you expect? Explain your answer.

2) (10 pts) Generating Random Variates

a) (4 pts) What are the four methods we discussed in class that is used to generate random variates?

The four methods we discussed are: Inverse transform, composition, convolution, and acceptance-rejection.

b) (2 pts) What type of method is used to generate an exponential distributed random variate?

The inverse-transform method is used for exponentially distributed random variates.

c) (2 pts) What type of method is used to generate a normal distributed random variate?

The text discusses many forms used for generating normally distributed random variates but of them the one that we discussed would be the inverse-transform method.

d) (2 pts) What type of method is used to generate a binomial distributed random variate?

The convolution method is used for exponentially distributed random variates.

3) (20 pts) Experimental Design

a) (5 pts) Give one example of a quantitative and qualitative factor used in project 1.

A qualitative factor from project 1 would be whether the order station is able to move or is stationary. A quantitative factor from project 1 would be how many cars can fit in the order station line or how many order stations there are.

b) (5 pts) Give one example of a controllable and uncontrollable factor used in project 1.

A controllable factor from project 1 was the mean interarrival time, while an uncontrollable factor was the order that interactions happened in as this was decided by the project handout.

c) (10 pts) A simulation experiment was performed to determine the best combination of the number of order stations and the type of service at the stations to minimize customer waiting time. A simple k-factorial experiment was created and the results are tabulated below.

Factor	**Low Setting (-1)**	**High Setting (+1)**
Number of Order Stations	1	2
Type of Order Station	Electronic Order Window	Human Server Standing Outside

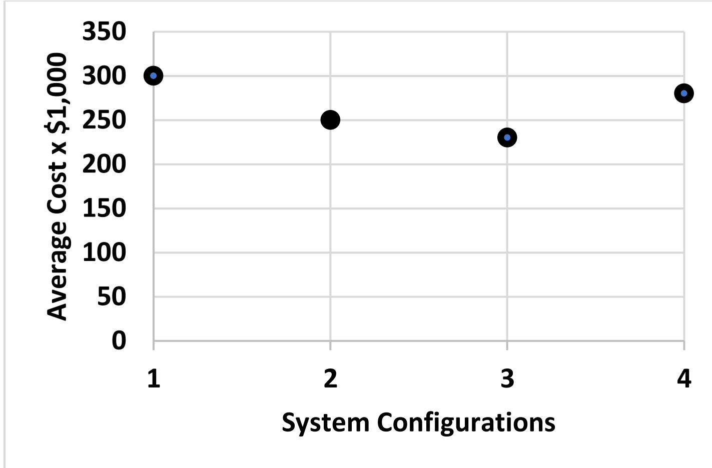
Number of Order Stations	**Type of Order Stations**	**Customer Waiting Time (minutes)**
-1 – 1	-1 – electronic	5.9
-1 – 1	+ 1– human	4.2
+1 – 2	-1 – electronic	3.7
+1 – 2	+1 – human	2.5

What is the average effect of changing the number of order stations from the low setting to the high setting on customer waiting time? What does it mean?

Going from the low setting to high setting almost cuts customer wait time in half for either setting for the type of order station. This means that no matter the type of order station, customer wait times will always be shorter with two order station compared to a single order station.

4) (20 pts) Analysis of Output

a) (10 pts) A consultant conducted a series of simulation experiments to evaluate the cost of four different configurations of a system. One hundred replicates of each simulation experiment were performed and the cost of each configuration was averaged across the replicates. These costs are summarized in the plot shown below. Based on this information, the consultant recommended the use of configuration 3. Assuming the simulation is based on a behavioral valid model of the real system and choice of input distributions and parameters are valid, and the simulation is properly implemented would you agree or disagree with this recommendation or would you need additional information before making your recommendation? Briefly explain your answer.



Assuming that all that was said to be done properly and it truly was I would likely agree with the recommendation, I might want to know kind of variance there is between replicates but other than that I can't think of any additional information.

b) (10 pts) Suppose we conduct ten replicates of an experiment and computed the average customer waiting time for each replicate as follows: [125, 98, 112, 130, 90, 118, 107, 100, 137, 88]

Replicate Number	1	2	3	4	5	6	7	8	9	10
Average Waiting Time	125	98	112	130	90	118	107	100	137	88

What are the sample mean, sample variance, and 95% confidence interval for these results?

The sample mean is 110.5, sample variance is 284.056, and the confidence interval for 95% is ±10.446

5) (15 pts) Continuous System Simulation

a) (10 pts) We discussed one simple approach to continuous system simulation that uses Euler's Method to estimate the behavior of a continuous system of equations. Euler's Method simulates the behavior of the equation f(t) given an equation for the derivative of the equation f'(t). The simulation uses the following simple update rule: x(t+dt) = x(t) + dt * f'(t) where x(t) is the simulated value of the function at time t and dt is the size of the simulation time step. The computational cost of using Euler's Method increases as the size of the time step decreases. Yet, we still want to use the smallest possible value of dt that we can afford from a computational point of view. Why? Briefly explain your answer.

We want to use the smallest time-delta or dt because that is decreasing the amount of time before each simulated step leading to a more continuous simulation.

b) (5 pts) In practice, we often use a Runge-Kutta Method to simulate continuous systems. Briefly explain the rationale for using the 4^{th}-order Runge-Kutta Method instead of Euler's Method.

We use 4^{th}-order Runge-Kutta because it averages more slops, giving it reduced error

6) (10 pts) M/M/1 Queueing Systems

a) (5 pts) If the arrival rate into the system is 5 customers per minute and the service rate is 10 customers per minute, what is the expected utilization of the server?

In an M/M/1 queueing system expected utilization of the server would be half that of the potential utilization of the server. This would result in short wait times for customers.

b) (5 pts) What is the expected number of customers in a system with the parameters described in the first part of this question?

The expected number of customers in the system from part one is 5 per minute.