ARS: Analisi Reti Sociali

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Note: Whenever an exercise requires the application of a known formula both the formula and its solution must be reported and discussed.

Exercise 1: Graph Modeling [5 points]
Given the matrix $\mathcal{G}$

$$\mathcal{G} = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

- Draw $\mathcal{G}$;
- Represent $\mathcal{G}$ using the edge list format;
- Synthetically characterize the graph $\mathcal{G}$ describes (directedness, number of nodes/edges, density, components, max/min/avg degrees...).

Solution
Edge List:
$\{(1,2),(1,3),(1,4),(2,5),(2,7),(3,1),(3,2),(3,3),(4,6),(5,1),(5,4),(6,1),(6,2),(6,4),(7,6),(7,7)\}$

Graph description:

- $|V| = 7$, $|E| = 16$, Directed, un-weighted,
  single component, with self loops
- Density $= \frac{14}{42} = \frac{1}{3}$
  (directed $a \to b \neq b \to a$, self-loops play no role in density)
- $k_{in} = k_{out}$: max = 3, min = 1, avg = 2,28

Exercise 2: Random Graphs [5 points]
Let $\mathcal{G}$ be a ER graph having N=1500 and p=$8 * 10^{-4}$:

- How many edges we should expect in $\mathcal{G}$?
- What will be the average degree? and the graph density?
- Describe the regime of $\mathcal{G}$;

Solution

- $L = p\frac{N(N-1)}{2} \simeq 899$
- $< k >= \frac{2L}{N} \simeq 1,19$
- Density $= p$
- Regime: supercritical $p = 8 * 10^{-4} > \frac{1}{N} = 6 * 10^{-4}$

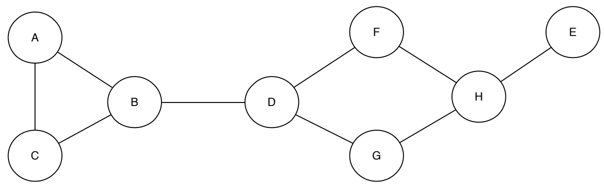

Figure 1:

Exercise 3: Paths [6 points]
Given the graph $\mathcal{G}$ shown in Figure 1:

- Compute the diameter of $\mathcal{G}$;
- List all the shortest paths among the pairs [A,E], [B,H] and [F,D];
- Compute the edge betweenness of (B,D), (G,H) and (A,C);
- Add the minimum number of edges such that an Hamiltonian cycle can be identified on $\mathcal{G}$.

Solution

- Diameter = 5
- $[A, E] = \{ABDFHE, ABDGHE\}$,
  $[B, H] = \{BDFH, BDGH\}$,
  $[F, D] = \{FD\}$
- $BE(BD) = 15$
  $BE(GH) = 6,5$
  $BE(AC) = 1$
- Minimum number of edges for an Hamiltonian **cycle**: 2
  (example EF and FA, cycle $EFACBDGHE$)

Exercise 4: Indicators [6 points]
Given the graph $\mathcal{G}$ shown in Figure 1 compute:

- Degree Centrality of all nodes;
- Closeness Centrality of A, D, E, H;
- Betweenness Centrality of B, C, E, F;
- Local Clustering Coefficient of B, C, G, H.

Solution

- $d_A = d_C = d_F = d_G = 2$
  $d_B = d_D = d_H = 3$
  $d_E = 1$
- $CC(A) = \frac{19}{7}$
  $CC(D) = \frac{12}{7}$
  $CC(E) = \frac{22}{7}$
  $CC(H) = \frac{16}{7}$
- $BC(B) = 10$
  $BC(C) = BC(E) = 0$
  $BC(F) = 4$
- $C_B = \frac{1}{3}$
  $C_C = 1$
  $C_G = C_H = 0$

Exercise 5: Graph Construction [6 points]
Given 10 nodes - identified with letters - and, at most, 18 edges build a graph such that all the following conditions hold:

- The graph is composed by two separated components;
- There exists a path of length 3 between nodes A and C;
- Node E has a clustering coefficient of $\frac{1}{2}$;
- The shortest path among B and F is equal to 2;
- Node I has the highest Degree Centrality;
- Node H has the lowest Closeness Centrality;
- Edge (A,L) has the highest betweenness centrality.

Solution
Several solutions are possible, among them

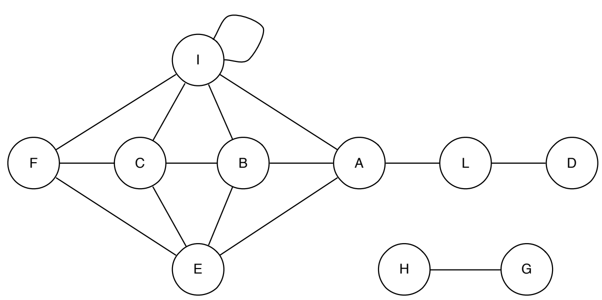

Figure 2:

NB: Betweenness centrality is defined only on connected graphs, thus where accepted as correct both the following answers: (i) $BE(A \to L)$ max in its component (ii) all BE equal to ∞ in a disconnected graph.

Exercise 6: Preferential Attachment[5 points]
Let $\mathcal{G}$ be a BA graph with $N = 1500$ and $m = 10$:

- How many edges in $\mathcal{G}$?
- What is the expected degree of the largest hub?
- What fraction of edges is incident on the largest hub?

Solution

- $|E| = mN$
- $k_{max} = k_{min} N^{\frac{1}{\gamma-1}} = 10 * 1500^{\frac{1}{2}} \simeq 387$ ($K_{min} = m$, in BA $\gamma = 3$)
- ratio $= \frac{k_{max}}{|E|} \simeq 0,0026$