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Modeling the Metallic Reflection Using Appropriate BRDF Model

***Abstract**. BRDF is commonly used for modeling the reflection of light. Simulation of the light point figure requires modeling the reflection from the metallic surface. The paper presents review of the most important BRDF models and their features. He-Torrance-Sillion-Greenberg, Cook-Torrance, Embrechts, Ashikhmin-Shirley, Phong and Ward function are considered. The most useful model in a light point figure simulation is selected.*

***Streszczenie.** Do modelowania odbicia światła powszechnie jest używana funkcja rozkładu współczynnika odbicia dwukierunkowego (BRDF). Symulacja figury jasnych punktów wymaga modelowania odbicia światła od powierzchni metalu. Artykuł prezentuje przegląd najważniejszych modeli odbicia i ich właściwości. Rozpatrywane są modele He-Torrance'a-Silliona-Greenberga, Cooka-Torrance'a, Embrechtsa, Ashikhmina-Shirleya, Phonga i Warda. Analiza pozwala wybrać najbardziej przydatny model do symulacji figury jasnych punktów. (**Modelowanie odbicia od powierzchni metalu z wykorzystaniem funkcji BRDF**).*

Keywords: BRDF, reflection modeling, light point figure, ray tracing.
Słowa kluczowe: modelowanie odbicia, figura jasnych punktów, śledzenie promieni.

Introduction

Bidirectional Reflectance Distribution Function (BRDF) [1] is used for modeling the reflection and refraction of light. The BRDF $f(\vec{L},\vec{V})$ definition can be shown as formula (1).

(1) $$f(\vec{L},\vec{V}) = \frac{dL(\vec{V})}{dE(\vec{L})}$$

where $dL(\vec{V})$ is the radiance leaving the surface in direction $-\vec{V}$ (fig. 1.), and $dE(\vec{L})$ is the irradiance reaching the surface from direction $-\vec{L}$. BRDF is dimensionless but is expressed as sr^{-1}. Applied BRDF should fulfill the physical laws to describe the phenomena in a proper way: the Helmholtz Reciprocity Rule (BRDF should be symmetric) and the Energy Conservation Law (reflected energy cannot exceed incoming energy).

Simulation of the light point figure is a good example of the ray tracing application in lighting technology. The reflector surface has usually very good specular reflectance properties, most frequently it is a layer of metal. For this kind of surface the following properties are usually assumed [2]:

- Diffuse reflection is negligible. If needed, the Lambert's model can be simply used.
- BRDF is proportional to the $\cos^{-1}\theta_L$.
- Reflected color is determined by the Fresnel reflectivity factor $F(\theta)$ (Fresnel function).
- For the great incident angles, the maximum level of reflection (maximum of the reflection lobe) occurs at an angle that is grater than the incidence angle (off-specular phenomenon).

When the task of light point figure simulation is taken into consideration, it is worth analyzing the properties of different known models of BRDF and their usefulness in this task. It is especially important in calculation with application of commercial programs, where the choice of available forms of function may be limited.

BRDF and reflection models

The well-known models of light reflection and BRDF form can be divided into two groups: experimental worked out dependences and dependences having physical basis (physical grounds). The first group is formed by relations in which mathematical description has been selected to expected (or measured) effects. They do not have a theoretical reason, but they make a good approximation of the real phenomena. In this group, among other things the following models are mentioned: **Phong** [3], **Ward** [4], **Ashikhmin-Shirley** [5]. In the second group there are models that have been worked out according to a suitable physical theory that describes smoothness (roughness) of the surface: **Cook-Torrance** [6], **He-Torrance-Sillion-Greenberg** [7], **Embrechts** [8,9].

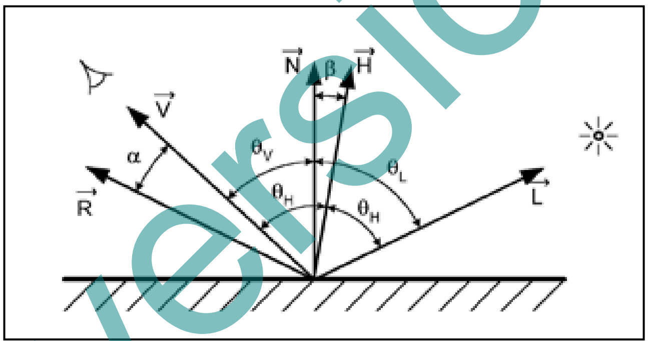

Fig.1. The set of vectors used to define BRDF, $\vec{N}$ - normal vector to the surface in reflection point, $\vec{V}$, $\vec{L}$ - vectors to observer and source of light, respectively, $\vec{R}$ - perfect specular reflection vector, $\vec{H}$ - vector that bisects the angle between $\vec{V}$ and $\vec{L}$, φ - angle indicated rotation about $\vec{N}$ (not marked in the picture)

The authors of the paper [10] conducted comparative research for reflection from the surface of real materials and appropriate simulation with different BRDF models. The conclusion was that **He-Torrance-Sillion-Greenberg, Ashikhmin-Shirley and Cook-Torranc** are the best well-known models. However, special cases showing differences between these three models were not analyzed, neither was the **Embrechts** model taken into consideration in this work.

Discussed reflection models in chronological order.

The Phong model. The oldest one, practically still used in computer graphic. Bui Tuong Phong introduced this model in 1975 [3]. The following formula (BRDF) describes specular reflection in it.

(2) $$f_S(\vec{L},\vec{V}) = k_S \times \cos^N(\alpha)$$

k_S - is the specular reflection factor, $N \geq 1$ defines the smoothness of a surface - enlarging this parameter brings the surface closer to the ideal mirror. The Phong model is an experimental one, physically groundless. It does not fulfill the Energy Conservation Law. Despite this, it is, probably, the most often applied model of reflection in computer graphic. It is so popular because it allows quick obtaining pictures with sufficiently realistic colors. The methods of improvement of the Phong model are described in literature [11]. This improvement allows meeting the Energy Conservation Law.

There exists Blinn version of this model [12], in which β is used instead of α angle – formula (3) for specular reflection.

(3) $$f_S(\vec{L},\vec{V}) = k_S \cdot \cos^N(\beta)$$

The Cook-Torrance model, proposed in 1981 [6]. This model was worked out on the basis of theoretical works of Torrance and Sparrow concerning the physical description of smoothness (roughness) of the surface as well as on the Blinn's considerations [12]. Specular reflection in this model is described in formula (4).

(4) $$f_S(\vec{L},\vec{V}) = \frac{F(\theta_H)\cdot G\cdot D}{\pi\cdot(\vec{N}\circ\vec{V})\cdot(\vec{N}\circ\vec{L})}$$

where

$F(\theta)$ - the Fresnel reflectivity factor (Fresnel function) describes the reflection dependence on an angle of incidence and on a length of wave.

D - microfacet distribution function describes dependence of light reflection on surface roughness. Cook and Torrance proposed use of the Beckmann microfacet distribution function as the most corresponding to the polyhedral character of surface roughness for different materials.

G - geometrical factor describes attenuation (shadowing and masking) of the surface microelements. In the Cook-Torrance model the surface of material is polyhedron that consists of micro mirrors. Their size and position decide about roughness (smoothness) of the surface.

The Cook-Torrance model is physically grounded, fulfills the Helmholtz Reciprocity Rule and the Energy Conservation Law. It was the first physical model used in computer graphics.

The He-Torrance-Sillion-Greenberg model was worked out as an extension of the Cook-Torrance model in 1991 [7]. This is probably the most complicated model from known reflection models. It takes into account almost all known physical phenomena connected with the reflection: polarization, diffraction, interference, but not anisotropy. It fulfills certainly the Helmholtz Reciprocity Rule and the Energy Conservation Law. The specular component of BRDF in the version for non-polarized light is given in formula (8). It is a similar form to the Cook-Torrance description (4).

(8) $$f_S(\vec{L},\vec{V}) = \frac{F(\theta_H)\cdot S\cdot G\cdot D}{\pi\cdot(\vec{N}\circ\vec{V})\cdot(\vec{N}\circ\vec{L})}$$

where

$F(\theta)$ - the Fresnel reflectivity factor.

D - microfacet distribution function describes roughness of the surface for different materials.

G - geometrical factor describes attenuation of the surface microelements.

S - extended geometrical factor for shadowing and masking attenuation

The He-Torrance-Sillion-Greenberg model has very important disadvantages. It is entirely impractical. The time of its realization is 100 - 300 times longer than realization of the other different models that is described here [2]. Additionally for the reason of the conducted operation (the necessity of nonlinear equation solution) this model cannot be used in Monte Carlo algorithms e.g. stochastic ray tracing.

The Ward model proposed in 1992 [4] was one of the first models taking into account the anisotropy of reflection. The specular reflection is described by equation (6). This is one of the most popular models. It is still very often used in commercial software.

(6) $$f_S(\vec{L},\vec{V}) = \frac{k_S\cdot \exp\left(-tg^2\beta\cdot\left(\frac{\cos^2\phi}{m_x^2}+\frac{\sin^2\phi}{m_y^2}\right)\right)}{4\cdot\pi\cdot m_x\cdot m_y\cdot\sqrt{\cos\theta_L\cdot\cos\theta_V}}$$

where k_S - is the specular reflection factor, m_x and m_y are the material coefficients (Cook-Torrance like) that describe smoothness independently along the perpendicular directions of the surface. Unfortunately this model does not take into account the Fresnel reflectivity factor, which does not fully allow taking the angular dependences of the material reflectance properties into consideration. This model fulfills the Helmholtz Reciprocity Rule but there is a problem with the Energy Conservation Law. The author states that the balance of energy is correct if $m_x, m_y < 0.2$. The greater angle of incidence and the larger values of coefficients describing the smoothness, the more visible this problem is.

The Embrechts model. Worked out in 1995 [8], improved (partly altered) in 1999 [9]. This model functions practically only in lighting technique literature. It fulfills the Helmholtz Reciprocity Rule and the Energy Conservation Law. It is one of the most complex models of BRDF. The specular reflection is described by formula (7).

(7) $$f_S(\vec{L},\vec{V}) = \frac{a_{SC}\cdot F(\theta_H)\cdot C(\theta_L,\theta_V,\phi)\cdot \exp(-\sigma^2\cdot tg^2\beta)}{\cos\theta_L\cdot\cos\theta_V\cdot\cos^4\beta}$$

where a_{SC} is the material coefficient, σ describes smoothness (roughness) of the surface, $C(\theta_L,\theta_V,\phi)$, describes geometrical attenuation (additionally dependent on material parameters). Analyzing this equation it could be stated that Embrechts dependences of the specular reflection are practically similar to the Cook-Torrance formula (4), in which as the microfacet distribution function the Beckmann like function is applied, but in Embrechts model a more advanced function of geometrical attenuation is used. This function also influences angular relationship depending on material parameters. Embrechts model contains also the advanced description of the diffuse reflection and an attempt to take the ideal (theoretical) directional reflection into account.

The Ashikhmin-Shirley model introduced in 2000 [5]. This is an extension of the Phong model, but it fulfills the Helmholtz Reciprocity Rule and the Energy Conservation Law. This model also takes into account the anisotropy of reflection as well as the Fresnel reflectivity factor. Specular reflection in this model is described by equation (5).

$$(5)\quad f_S(\vec{L},\vec{V}) = \frac{\sqrt{(n_u+1)\cdot(n_v+1)}\cdot F(\theta_H)\cdot \cos^p \beta}{8\cdot\pi\cdot\cos\theta_H\cdot max(\cos\theta_L,\cos\theta_V)}$$

where $p = n_u \cdot \cos^2\varphi + n_v \cdot \sin^2\varphi$, n_u and n_v coefficients (Phong like) describes smoothness independently along the suitable perpendicular directions on the surface. It is worth noticing that authors applied in this model also their own description of the diffuse reflection (nonlambertian).

The Ashikhmin-Shirley model takes into account the correction introduced by **Neumann-Neumann-Szirmay-Kalos** [2] in their model of metallic reflection. They suggested, on the basis of investigations, the factor $1/max(cos(\theta_L),cos(\theta_V))$ that should be added to BRDF in such a case. This factor guarantees the correctness of BRDF for the large incidence angles.

Properties of the reflection models

All the considered models were realized in the MATLAB environment, which gives a possibility of analyzing their properties. Simultaneously all the models except the **He-Torrance-Sillion-Greenberg** (on account of a very long time of calculation) were implemented in the program that determines the light point figure.

All the considered models allow simulating both diffuse and specular reflection. There exists the possibility of continuous change of proportion between these reflections. It is highly significant in a simulation of real (not theoretical – ideal) reflections.

All the considered models give the possibility of simulating the smoothness (roughness) changes in a wide range. There exist some methods [12] to recalculate parameters for replacing one model with another one. Such a method was used in this paper to compare properties of the models. The values of parameters were chosen in the individual model to obtain half of the normalized value of BRDF for the same β angle in all models.

Cook-Torrance, Ashikhmin-Shirley, He-Torrance-Sillion-Greenberg and Embrechts models show the similar shapes of BRDF function for different incidence angles (fig.2). There are different proportions of angular dependences in the **Ward model** because this model does not take into account Fresnel reflectivity factor. The **Phong model** does not fit to the others because in this model the angle of incidence is not taken into account at all.

The examples of function describing the reflection from surface of metal for the large incidence angles are shown in

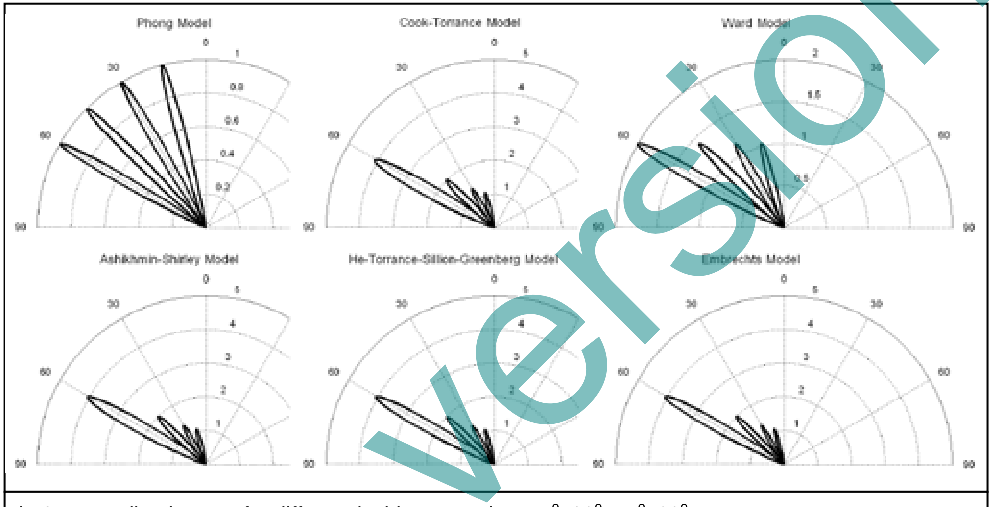

Fig.2. Normalized BRDF for different incidence angles: 15°, 30°, 45°, 60°.

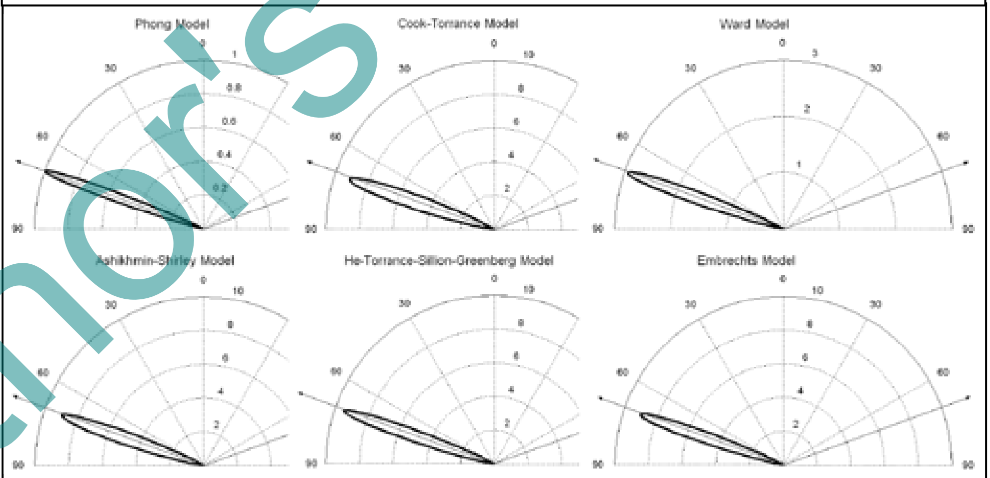

Fig.3. Normalized BRDF for metal reflection. The large incidence angles (70°) The marked vectors according to fig.1. The off-specular phenomenon is correct simulated by all the discussed BRDF except Phong and Ward models.

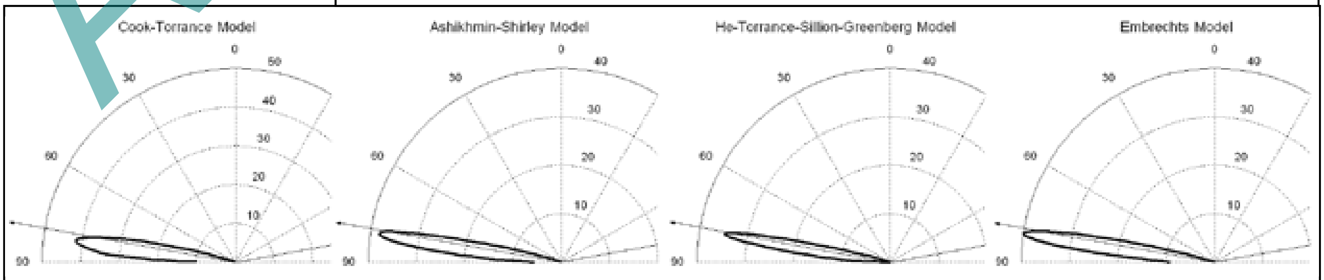

Fig.4. Normalized BRDF for metal reflection. Simulation of the off-specular phenomenon. The large incidence angles (80°). The marked vectors according to fig.1.

Table 1. Properties of the selected BRDFs. The He-Torrance-Sillion-Greenberg model (HTSG), the Embrechts model (E), the Cook-Torrance model (CT) and the Ashikhmin-Shirley model (AS)

	HTSG	E	CT	AS
Taking Fresnel reflectivity factor into consideration	+	+	+	+
Helmholtz Reciprocity Rule and Energy Conservation Law	+	+	+	+
Off-specular reflection	+	+	+	+ Neumann,Neumann, Szirmay-Kalos factor
Microfacet distribution function	based on the Beckmann Spizzichino theory	Beckmann like	Beckmann	Phong like
Conformity with HTSG		+	+	++
Ideal (theoretical) directional reflection	+ Kronecker delta	+ Dirac delta	-	-
Interference, diffraction, polarization	+	-	-	-
Anisotropy	-	-	-	+
Nonlambertian diffuse reflection	-	+	-	+
Computational properties, speed	- ! requires nonlinear equation solving	+	++	+++

figure 3. The **Phong model** does not show changes of function's extremum for the large incidence angles at all. The **Ward model** requires correction [2]. All the other considered models take into accoumt the off-specular phenomenon. The maximum level of reflection occurs at an angle that is greater than the incidence angle. It is worth noticing, however, that if the **He-Torrance-Sillion-Greenberg model** is assumed as the point of reference (which is most often acted), then the closer one to it is the **Ashikhmin-Shirley model** (not the **Cook-Torrance model**) – it is visible in figures 2 and 3. It is a bit surprising when considering the fact, that the **Cook-Torrance model** was built on the basis of the physical theory of surface roughness, while the **Ashikhmin-Shirley model** is the experimental study. For larger incidence angles (fig.4 - 80°) the differences between these models are unfortunately larger.

The **Embrechts model** allows taking comparatively good properties of BRDF in a wide range of incidence angles but this model is more complicated and is less effective in computations. Nevertheless it is worth noticing the advanced microfacet distribution function used in this model, which allows obtaining the proper angular dependences. The **Ward model** shows large differences in comparison to other models (it does not take into account Fresnel reflectivity factor). The **Phong model** is practically entirely useless in this context. The comparative list of properties for **He-Torrance-Sillion-Greenberg**, **Ashikhmin-Shirley**, **Cook-Torrance** and **Embrechts models** is shown in Table 1.

Conclusion

The analysis of properties for the most often applied BRDFs was conducted. The comparison was made with the assumption that for all functions half of the normalized value of BRDF are for the same b angle. The **Ashikhmin-Shirley model** turned out the most useful one in simulation of the light point figure. It fulfills all preliminary assumptions and from among the discussed forms of function it is the closer one to the **He-Torrance-Sillion-Greenberg model,** which is most often assumed as the point of reference. The **Ashikhmin-Shirley model** is also computationally the most effective one.

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