

5. Evaluate the following integral.

$$\int \frac{x-9}{x^2+3x-10} dx$$

- (a) $\ln|x+5| - 2\ln|x-2|$ (b) $2\ln|x+5| - \ln|x-2|$
 (c) $\ln|x+5| + 2\ln|x-2|$ (d) $-\ln|x+5| - 2\ln|x-2|$
 (e) $2\ln|x+5| + \ln|x-2|$

6. Which of the below integrals is equal to

$$\int \frac{\sqrt{x^2-4}}{x^6} dx$$

(with an appropriately defined θ)?

- (a) $\int \frac{1}{16} \cos^3 \theta \sin^2 \theta d\theta$ (b) $\int \frac{1}{32} \cos^5 \theta \sin \theta d\theta$ (c) $\int \frac{1}{16} \cos^3 \theta \sin^3 \theta d\theta$
 (d) $\int \frac{1}{16} \frac{\cos^2 \theta}{\sin^6 \theta} d\theta$ (e) $\int \frac{1}{32} \frac{\cos \theta}{\sin^6 \theta} d\theta$

7. Using the comparison theorem, which of the following integrals is convergent?

- (i) $\int_1^\infty \frac{x \sin^2 x}{\sqrt[3]{1+x^7}} dx$ (ii) $\int_1^\infty \frac{dx}{x+e^{2x}}$ (iii) $\int_2^\infty \frac{x^2}{\sqrt{x^6-1}} dx$

- (a) (i) only (b) (ii) only (c) (i) and (ii) only (d) (i) and (iii) only (e) (ii) and (iii) only

8. Consider the sequence defined by $a_1 = 2$, $a_{n+1} = \frac{1}{2}(a_n + 6)$. Which of the following statements is correct?

- (a) $\{a_n\}$ is increasing and bounded above by 3
 (b) $\{a_n\}$ converges to 5
 (c) $\{a_n\}$ is increasing and bounded above by 5
 (d) $\{a_n\}$ is increasing and bounded above by 6
 (e) $\{a_n\}$ diverges

9. Determine whether the following sequences are convergent or divergent. When convergent, find the limit.

(i) $a_n = \frac{(-1)^n n^3}{n^3 + 2n^2 + 1}$

(ii) $a_n = n \sin(n\pi)$

- (a) diverges, diverges (b) diverges, 0 (c) 0, 0 (d) 1, diverges (e) 1, 0