Math 1AA3/1ZB3

1st Sample Test #1

Name:____
(Last Name) (First Name)

Student Number:____________________ **Tutorial Number:**____________________

This test consists of 20 multiple choice questions worth 1 mark each (no part marks), and 1 question worth 1 mark (no part marks) on proper computer card filling. All questions must be answered on the COMPUTER CARD with an HB PENCIL. Marks will not be deducted for wrong answers (i.e., there is no penalty for guessing). You are responsible for ensuring that your copy of the test is complete. Bring any discrepancy to the attention of the invigilator. Calculators are NOT allowed.

1. Evaluate the following integral,

$$\int_0^{\sqrt[4]{\pi}} x^7 \sin x^4 \, dx$$

(a) $\frac{\pi}{4}$ **(b)** $\frac{\pi}{3}$ **(c)** $\frac{\pi}{2}$ **(d)** π **(e)** $\frac{3\pi}{2}$

2. Which of the following series converge?

(i) $\sum_{n=1}^{\infty} \frac{e^{-\sqrt{n}}}{\sqrt{n}}$ (ii) $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^3}$

(a) (i) only **(b)** (ii) only **(c)** (i) and (ii) **(d)** neither

3. What is the minimum number of terms needed in order to estimate the sum

$$\sum_{n=1}^{\infty} \frac{1}{(3n+5)^4}$$

correct to within .001?

(a) 1 **(b)** 2 **(c)** 3 **(d)** 4 **(e)** 5

4. Evaluate the following improper integral.

$$\int_{-\infty}^{0} x^4 e^{x^5} dx$$

(a) 1 **(b)** $\frac{1}{4}$ **(c)** $\frac{1}{5}$ **(d)** 0 **(e)** Divergent

5. Evaluate the following integral.

$$\int \frac{x-9}{x^2+3x-10}dx$$

(a) $\ln|x+5| - 2\ln|x-2|$ **(b)** $2\ln|x+5| - \ln|x-2|$
(c) $\ln|x+5| + 2\ln|x-2|$ **(d)** $-\ln|x+5| - 2\ln|x-2|$
(e) $2\ln|x+5| + \ln|x-2|$

6. Which of the below integrals is equal to

$$\int \frac{\sqrt{x^2-4}}{x^6}dx$$

(with an appropriately defined θ)?
(a) $\int \frac{1}{16}\cos^3\theta \sin^2\theta \, d\theta$ **(b)** $\int \frac{1}{32}\cos^5\theta \sin\theta \, d\theta$ **(c)** $\int \frac{1}{16}\cos^3\theta \sin^3\theta \, d\theta$
(d) $\int \frac{1}{16}\frac{\cos^2\theta}{\sin^6\theta}d\theta$ **(e)** $\int \frac{1}{32}\frac{\cos\theta}{\sin^6\theta}d\theta$

7. Using the comparison theorem, which of the following integrals is convergent?
(i) $\int_1^\infty \frac{x\sin^2 x}{\sqrt[3]{1+x^7}}dx$ (ii) $\int_1^\infty \frac{dx}{x+e^{2x}}$ (iii) $\int_2^\infty \frac{x^2}{\sqrt{x^6-1}}dx$
(a) (i) only **(b)** (ii) only **(c)** (i) and (ii) only **(d)** (i) and (iii) only **(e)** (ii) and (iii) only

8. Consider the sequence defined by $a_1 = 2,\ a_{n+1} = \frac{1}{2}(a_n + 6)$. Which of the following statements is correct?
(a) $\{a_n\}$ is increasing and bounded above by 3
(b) $\{a_n\}$ converges to 5
(c) $\{a_n\}$ is increasing and bounded above by 5
(d) $\{a_n\}$ is increasing and bounded above by 6
(e) $\{a_n\}$ diverges

9. Determine whether the following sequences are convergent or divergent. When convergent, find the limit.
(i) $a_n = \dfrac{(-1)^n n^3}{n^3+2n^2+1}$
(ii) $a_n = n\sin(n\pi)$
(a) diverges, diverges **(b)** diverges, 0 **(c)** 0, 0 **(d)** 1, diverges **(e)** 1, 0

10. If you were to use use Mathematical Induction to show that

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6},$$

which of the following would be the second step?

(a) Assume $\sum_{i=1}^{k} i^2 = \frac{k(k+1)(2k+1)}{6}$, and show that $\sum_{i=1}^{k} (i+1)^2 = \frac{(k+1)(k+2)(2k+3)}{6}$

(b) Assume $\sum_{i=1}^{k} i^2 = \frac{k(k+1)(2k+1)}{6}$, and show that $\sum_{i=1}^{k+1} i^2 = \frac{(k-1)k(2k-1)}{6}$

(c) Assume $\sum_{i=1}^{k} i^2 = \frac{k(k+1)(2k+1)}{6}$, and show that $\sum_{i=1}^{k-1} i^2 = \frac{(k-1)k(2k-1)}{6}$

(d) Assume $\sum_{i=1}^{k} i^2 = \frac{k(k+1)(2k+1)}{6}$, and show that $\sum_{i=1}^{k+1} i^2 = \frac{(k+1)(k+2)(2k+3)}{6}$

(e) Assume $\sum_{i=1}^{k} i^2 = \frac{k(k+1)(2k+1)}{6}$, and show that $\sum_{i=1}^{k-1} i^2 = \frac{(k+1)(k+2)(2k+3)}{6}$

11. Find the sum of the following series

$$\sum_{n=1}^{\infty} \frac{\sin^n x}{3^n}$$

(a) $\dfrac{\sin x - 3}{3}$ **(b)** $\dfrac{3}{\sin x}$ **(c)** $\dfrac{\sin x}{3}$ **(d)** $\dfrac{\sin x}{3 - \sin x}$ **(e)** $\dfrac{3}{3 - \sin x}$

12. If the n^{th} partial sum of the series $\sum_{n=0}^{\infty} a_n$ is $s_n = \frac{2n+1}{4n+3} - \frac{n}{\ln n}$, find $\sum_{n=0}^{\infty} a_n$.

(a) $\frac{1}{2}$ **(b)** $\frac{1}{3}$ **(c)** $\frac{1}{4}$ **(d)** $\frac{2}{3}$ **(e)** divergent

13. Which of the following series converge?

(i) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin^2 n}{n^3+2}$ (ii) $\displaystyle\sum_{n=1}^{\infty} \frac{n^3+3n^2+n+2}{\sqrt{n^7+4n^4+n+1}}$

(a) (i) only **(b)** (ii) only **(c)** (i) and (ii) **(d)** neither

14. What is the minimum number of terms needed in order to estimate the following sum to within .001?

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{(n+1)!}$$

(a) 2 **(b)** 3 **(c)** 4 **(d)** 5 **(e)** 6

15. Which of the following series converge?

(i) $\displaystyle\sum_{n=1}^{\infty} \frac{n+1}{2^n}$ (ii) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{1 \cdot 3 \cdot 5 \cdot 7 \cdot \cdots \cdot (2n-1)}{(2n+1)!}$

(a) (i) only **(b)** (ii) only **(c)** (i) and (ii) **(d)** neither

16. Which of the following series are absolutely convergent?

(i) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \left(\frac{n^2+2}{n^3+1}\right)^n$ (ii) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n+3}$

(a) (i) only **(b)** (ii) only **(c)** (i) and (ii) **(d)** neither

17. Find the interval of convergence of

$$\sum_{n=1}^{\infty} \frac{n(3x+2)^n}{n^2+1}$$

(a) $(-\infty, \infty)$ **(b)** $[-\frac{1}{3}, -\frac{2}{3}]$ **(c)** $[-\frac{1}{3}, -\frac{2}{3})$ **(d)** $[-1, -\frac{1}{3}]$ **(e)** $[-1, -\frac{1}{3})$

18. If $\displaystyle\sum_{n=0}^{\infty} c_n 2^n$ is convergent, what can you conclude about the convergence of the following series?

(i) $\displaystyle\sum_{n=0}^{\infty} c_n(-2)^n$ (ii) $\displaystyle\sum_{n=0}^{\infty} c_n(-3)^n$

(a) convergent, nothing **(b)** convergent, convergent **(c)** nothing, divergent
(d) nothing, nothing **(e)** convergent, divergent

19. Find the radius of convergence of

$$\sum_{n=1}^{\infty} (n+1)!(3x-1)^n$$

(a) ∞ **(b)** 0 **(c)** 1 **(d)** $\frac{1}{3}$ **(e)** $\frac{2}{3}$

20. Find the radius of convergence of the following power series.

$$\sum_{n=0}^{\infty}(-1)^n \frac{1 \cdot 4 \cdot 7 \cdot \cdots \cdot (3n-2)}{n!3^n} x^{3n}$$

(a) $\sqrt[3]{3}$ **(b)** $\frac{1}{\sqrt[3]{3}}$ **(c)** 1 **(d)** 0 **(e)** ∞

21. Correctly fill out the bubbles corresponding to all 9 digits of your student number, as well as the version number of your test in the correct places on the computer card. (Use the below computer card for this sample test.)