Altair Paper Remix

Calc II

February 2, 2025

1 questions

1. Evaluate the following integral,

$$\int_0^{\sqrt[4]{2\pi}} x^7 \cos(x^4) dx$$

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{8}$ (c) $\frac{3\pi}{2}$ (d) π (e) $\frac{3}{2}$

2. Which of the following series converge?

(i) $\sum_{n=1}^{\infty} \frac{e^{-\sqrt{2n}}}{\sqrt{n}}$

(ii) $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$

(a) (i) only (b) (ii) only (c) (i) and (ii) (d) neither

3. What is the minimum number of terms needed in order to estimate the sum

$$\sum_{n=1}^{\infty} \frac{1}{(2n+3)^4}$$

correct to within .0005?
(a) 1 (b) 2 (c) 3 (d) 4 (e) 5

4. Evaluate the following improper integral.

$$\int_{-\infty}^{0} x^2 e^{2x} dx$$

(a) 1 (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) $\frac{1}{8}$ (e) Divergent

1. Evaluate the following integral.

$$\int \frac{x-1}{x^2+2x-8}\,dx$$

(a) $3\ln|x+4|-2\ln|x-2|$ (b) $2\ln|x+4|-\ln|x-2|$ (c) $2\ln|x+4|+\ln|x-2|$ (d) $\ln|x+4|-2\ln|x-2|$ (e) $\ln|x+4|+2\ln|x-2|$

2. Which of the below integrals is equal to

$$\int \frac{\sqrt{x^2-9}}{x^5}\,dx$$

(with an appropriately defined θ)?

(a) $\int \frac{1}{27}\cos^3\theta\sin^2\theta\,d\theta$ (b) $\int \frac{1}{81}\cos^3\theta\sin\theta\,d\theta$ (c) $\int \frac{1}{27}\cos^3\theta\sin^3\theta\,d\theta$ (d) $\int \frac{1}{27}\frac{\cos^2\theta}{\sin^5\theta}\,d\theta$ (e) $\int \frac{1}{81}\frac{\cos\theta}{\sin^5\theta}\,d\theta$

3. Using the comparison theorem, which of the following integrals is convergent?

(i) $\int_1^\infty \frac{x\cos^2 x}{\sqrt{1+x^5}}\,dx$ (ii) $\int_1^\infty \frac{1}{x+e^{3x}}\,dx$ (iii) $\int_2^\infty \frac{x^2}{\sqrt{x^4-1}}\,dx$

(a) (i) only (b) (ii) only (c) (i) and (ii) only (d) (i) and (iii) only (e) (ii) and (iii) only

4. Consider the sequence defined by $a_1 = 1$, $a_{n+1} = \frac{1}{3}(a_n + 8)$. Which of the following statements is correct?

(a) $\{a_n\}$ is increasing and bounded above by 4 (b) $\{a_n\}$ converges to 7 (c) $\{a_n\}$ is increasing and bounded above by 7 (d) $\{a_n\}$ is increasing and bounded above by 8 (e) $\{a_n\}$ diverges

5. Determine whether the following sequences are convergent or divergent. When convergent, find the limit.

(i) $a_n = \frac{(-1)^n n^2}{n^2+3n+2}$ (ii) $a_n = n\cos(n\pi)$

(a) diverges, diverges (b) diverges, 0 (c) 0, 0 (d) 1, diverges (e) 1, 0

1. If you were to use use Mathematical Induction to show that

$$\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4},$$

which of the following would be the second step?

(a) Assume $\sum_{i=1}^{k} i^3 = \frac{k^2(k+1)^2}{4}$, and show that $\sum_{i=1}^{k+1} i^3 = \frac{(k+1)^2(k+2)^2}{4}$

(b) Assume $\sum_{i=1}^{k} i^3 = \frac{k^2(k+1)^2}{4}$, and show that $\sum_{i=1}^{k-1} i^3 = \frac{(k-1)^2k^2}{4}$

(c) Assume $\sum_{i=1}^{k} i^3 = \frac{k^2(k+1)^2}{4}$, and show that $\sum_{i=1}^{k-1} i^3 = \frac{(k-1)^2k^2}{4}$

(d) Assume $\sum_{i=1}^{k} i^3 = \frac{k^2(k+1)^2}{4}$, and show that $\sum_{i=1}^{k+1} i^3 = \frac{(k+1)^2(k+2)^2}{4}$

(e) Assume $\sum_{i=1}^{k} i^3 = \frac{k^2(k+1)^2}{4}$, and show that $\sum_{i=1}^{k-1} i^3 = \frac{(k-1)^2k^2}{4}$

2. Find the sum of the following series

$$\sum_{n=1}^{\infty} \frac{\cos^n x}{2^n}$$

(a) $\frac{\cos x - 2}{2}$

(b) $\frac{2}{\cos x}$

(c) $\frac{\cos x}{2}$

(d) $\frac{\cos x}{2 - \cos x}$

(e) $\frac{2}{2 - \cos x}$

3. If the n^{th} partial sum of the series $\sum_{n=0}^{\infty} a_n$ is $s_n = \frac{3n+2}{5n+1} - \frac{2}{\sqrt{n}}$, find $\sum_{n=0}^{\infty} a_n$.

(a) $\frac{3}{5}$

(b) $\frac{2}{5}$

(c) $\frac{1}{5}$

(d) $\frac{4}{5}$

(e) divergent

4. Which of the following series converge?

(i) $\sum_{n=1}^{\infty} \frac{\cos^2 n}{n^4+1}$

(ii) $\sum_{n=1}^{\infty} \frac{n^4+2n^3+2n+1}{\sqrt{n^6+2n^2+n+3}}$

(a) (i) only

(b) (ii) only

(c) (i) and (ii)

(d) neither

1. Evaluate the following integral,

$$\int_0^{\sqrt[3]{\pi}} x^5 \cos x^3 \, dx$$

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$ (c) $\frac{\pi}{6}$ (d) π (e) $\frac{2\pi}{3}$

2. Which of the following series converge?

(i) $\sum_{n=1}^{\infty} \frac{e^{-n^2}}{n^2}$

(ii) $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$

(a) (i) only (b) (ii) only (c) (i) and (ii) (d) neither

3. What is the minimum number of terms needed in order to estimate the sum

$$\sum_{n=1}^{\infty} \frac{1}{(2n+3)^5}$$

correct to within .0001? (a) 1 (b) 2 (c) 3 (d) 4 (e) 5

4. Evaluate the following improper integral.

$$\int_{-\infty}^{0} xe^{2x} \, dx$$

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{1}{8}$ (d) 0 (e) Divergent

1. Evaluate the following integral.

$$\int \frac{x-13}{x^2+2x-15} \, dx$$

   (a) $\ln|x+5| - 2\ln|x-3|$ (b) $2\ln|x+5| - \ln|x-3|$ (c) $\ln|x+5| + 2\ln|x-3|$ (d) $-\ln|x+5| - 2\ln|x-3|$ (e) $2\ln|x+5| + \ln|x-3|$

2. Which of the below integrals is equal to

$$\int \frac{\sqrt{x^2-9}}{x^5} \, dx$$

   (with an appropriately defined θ)?

   (a) $\int \frac{1}{27} \cos^3\theta \sin^2\theta \, d\theta$ (b) $\int \frac{1}{81} \cos^5\theta \sin\theta \, d\theta$ (c) $\int \frac{1}{27} \cos^3\theta \sin^3\theta \, d\theta$ (d) $\int \frac{1}{27} \frac{\cos^2\theta}{\sin^5\theta} \, d\theta$ (e) $\int \frac{1}{81} \frac{\cos\theta}{\sin^5\theta} \, d\theta$

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5. Determine whether the following sequences are convergent or divergent. When convergent, find the limit.

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   (a) diverges, diverges (b) diverges, 0 (c) 0, 0 (d) 1, diverges (e) 1, 0

1. If you were to use Mathematical Induction to show that

$$\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4},$$

   which of the following would be the second step?

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   (b) Assume $\sum_{i=1}^{k} i^3 = \frac{k^2(k+1)^2}{4}$, and show that $\sum_{i=1}^{k-1} i^3 = \frac{(k-1)^2k^2}{4}$
   (c) Assume $\sum_{i=1}^{k} i^3 = \frac{k^2(k+1)^2}{4}$, and show that $\sum_{i=1}^{k+1} i^3 = \frac{(k+1)(k+2)^2}{4}$
   (d) Assume $\sum_{i=1}^{k} i^3 = \frac{k^2(k+1)^2}{4}$, and show that $\sum_{i=1}^{k-1} i^3 = \frac{(k-1)k(2k-1)}{4}$
   (e) Assume $\sum_{i=1}^{k} i^3 = \frac{k^2(k+1)^2}{4}$, and show that $\sum_{i=1}^{k-1} i^3 = \frac{(k-1)^2(k+1)^2}{4}$

2. Find the sum of the following series

$$\sum_{n=1}^{\infty} \frac{\cos^n x}{2^n}$$

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   (b) $\frac{2}{\cos x}$
   (c) $\frac{\cos x}{2}$
   (d) $\frac{\cos x}{2-\cos x}$

(e) $\frac{2}{2-\cos x}$

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(a) $\frac{1}{2}$

(b) $\frac{3}{2}$

(c) $\frac{2}{3}$

(d) $\frac{3}{4}$

(e) divergent

4. Which of the following series converge?

(i) $\sum_{n=1}^{\infty} \frac{\cos^2 n}{n^4+1}$

(ii) $\sum_{n=1}^{\infty} \frac{n^2+2n+1}{\sqrt{n^5+3n^2+n+1}}$

(a) (i) only

(b) (ii) only

(c) (i) and (ii)

(d) neither