When will the Last Total Solar Eclipse Take Place?

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Abstract. When the images of the Moon and the Sun in the sky intersect, a solar eclipse will occur. Depending on where the Moon is in its eccentric orbit when the intersection takes place, a solar eclipse can be either total or annular. However, tidal acceleration effects cause the Moon to slowly move away from Earth. If this orbital expansion continues, there will eventually be a final total solar eclipse. In this report, I investigate an upper limit for when this final total solar eclipse will occur. To accomplish this, I derive a simple theoretical model for the transfer of angular momentum between the Earth's spin and the Moon's orbit. This involved considering: Shadows cast by the Moon; Tidal forces due to the Moon's gravitational force on Earth; the orbital and spin angular momentum of the Earth-Moon system; the torque applied to Earth's tidal bulges by the Moon's gravity; how this torque affects the Moon's orbital angular momentum; and how this increase in orbital angular momentum changes the Moon's orbit. This model allowed a function for the rate of the Moon's recession to be found, which was used to calculate the time until the final eclipse: $t \approx 530$ million years.

1. Overview

Eclipses are spectacular astronomical events recorded as far back as 1223 BC [1]. They occur when the Sun, Earth and Moon align such that the Moon blocks light from the Sun over a region of Earth (solar eclipse) or the Earth blocks light from the Sun over the entire Moon (lunar eclipse) as seen in figure 1. During a lunar eclipse, the entire Moon lies within the Earth's shadow, making it visible to half the planet. On the other hand, during a solar eclipse, the Moon's shadow is only cast over a limited region of Earth's surface, making them much more uncommon since only those under the shadow's path observe them.

There are three types of solar eclipse: total, partial, and annular. These correspond to the different shadows cast by the Moon. In a total solar eclipse (TSE), all sunlight is blocked, corresponding to the Umbra of the Moon's shadow. In a partial solar eclipse, the Moon's image doesn't align with the Sun's, meaning part of the sunlight isn't blocked; this corresponds with the penumbra of the Moon's shadow. In an annular solar eclipse (ASE), the Moon's image aligns with the Sun's but is smaller than it, meaning even when aligned, part of the sunlight isn't blocked; this corresponds to the antumbra of the Moon's shadow.

The Moon's elliptical orbit around the Earth and the Earth's elliptical orbit around the Sun means that

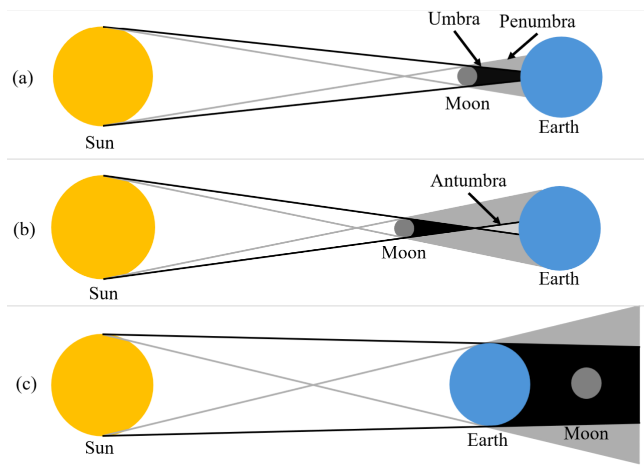

Figure 1. Schematic of (a) a total solar eclipse, (b) an annular solar eclipse, and (c) a lunar eclipse. In the Umbra (black), all sunlight is blocked, while in the penumbra (dark grey) and antumbra (light grey), only a fraction of it is blocked.

both TSEs and ASEs are observed depending on the distances between the Earth, Moon and Sun during a given eclipse. Therefore, it is obvious that if the Moon's orbit were slightly closer or further away, we would encounter only TSE or ASE, respectively.

Furthermore, by using a retroreflector array installed on the Moon during the Apollo 11 mission, the Moon's semi-major axis was measured to be expanding at a rate of about $3.8 cm/\text{year}$ [2, 3]. This expansion is explained by tidal acceleration described in the following steps:

(i) The Moon's gravitational influence generates tidal bulges on Earth, which the surface must travel through (since the Earth rotates faster than the Moon orbits).

(ii) Friction between the tidal bulges and the Earth's surface drags the tides out of alignment with the Moon.

(iii) The Moon's gravitational influence exerts some torque on the now misaligned tidal bulges, decreasing Earth's spin angular momentum.

(iv) The conservation of momentum means the Moon's orbital angular momentum must increase in response.

(v) The increase in angular momentum results in an increase in the Moon's orbital radius.

As this process continues, the Moon will drift further from Earth until it reaches a geosynchronous orbit. It's obvious then that there will come a day when a final TSEs takes place, after which they will no longer be possible.

In this report, I derive an upper bound for when the final TSE will occur. To do this, the problem is divided into several sections. In section 2, I derive the maximum distance in which a TSE is possible based on the optics of the Umbra. In section 3, I derive how the Moon's gravitational force influences the Earth's tides. Finally, in section 4, I derive the angular momentum of the Earth-Moon system, the torque applied to the Earth by the Moon, how the orbit of the Moon expands as a result, and evaluate the time until the final TSE. All astrophysical constants used for evaluations are listed in table 1 in the appendix.

2. Maximum Distance Where a TSE is Possible

TSE are possible so long as the largest image of the Moon is bigger than the smallest image of the Sun as seen from Earth. The largest image of the Moon occurs at its closest approach to Earth (its perigee), d_{perigee}. Whereas the smallest image of the Sun occurs at the Earth's farthest distance from the Sun (its aphelion), d_{aphelion}.

Therefore, the upper bound on when the final TSE will take place is when the extent of the Moon's umbra, d_{umbra}, is the same as the Moon's distance from Earth: when the Moon is at its perigee and the Earth is at its aphelion. Ignoring inclination in the Moon's orbit for simplicity, this also implies the distance between the Moon and the Sun is: $d_{\text{aphelion}} - d_{\text{perigee}}$.

The extent of the Moon's umbra, d_{umbra}, is found by considering the geometry shown in figure 2. Since

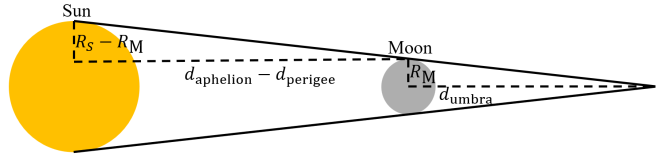

Figure 2. Schematic showing the geometry of the Sun and the Moon.

the two triangles are similar, the extent of the umbra is obtained as:

$$\frac{d_{\text{umbra}}}{R_M} = \frac{d_{\text{aphelion}} - d_{\text{perigee}}}{R_S - R_M}, \tag{1}$$

where R_M is the radius of the Moon, and R_S is the radius of the Sun.

To check this equation 1 is evaluated at the current value of the Moon's perigee, $d_{\text{perigee},0}$. This gives the extent of the umbra:

$$d_{\text{umbra}} = 0.38002 \times 10^6 km > d_{\text{perigee},0},$$

implying TSEs are currently possible as expected. If the current value of the Moon's apogee (farthest distance from Earth), $d_{\text{apogee},0}$, is used instead, then the extent of the umbra is calculated to be:

$$d_{\text{umbra}} = 0.37991 \times 10^6 km < d_{\text{apogee},0},$$

implying that even when the Earth is at its aphelion, ASEs are possible today.

For the upper bound on the last TSE $d_{\text{umbra}} = d_{\text{perigree}}$ meaning equation 1

$$d_{\text{umbra}} = \frac{R_M}{R_S} d_{\text{aphelion}}. \tag{2}$$

Evaluating this:

$$d_{\text{umbra}} = 0.3800 \times 10^6 km,$$

giving the upper bound for the last TSE. The question, therefore, simplifies into deriving when $d_{\text{perigee}} \geq 0.3800 \times 10^6 km$.

3. The Moon's Influence on Earth's Tides

Earth's tides result from the variation in the gravitational field of the Moon across Earth's diameter, as well as a centrifugal force due to the Earth-Moon system's co-orbit around their centre of mass. In this section, I derive the height of the tides by considering these two forces acting together, ignoring the effect of the tides being dragged due to Earth's rotation. This relation between the Moon's distance and the height/mass of the tides will be necessary for the subsequent section to derive the torque being applied to Earth.

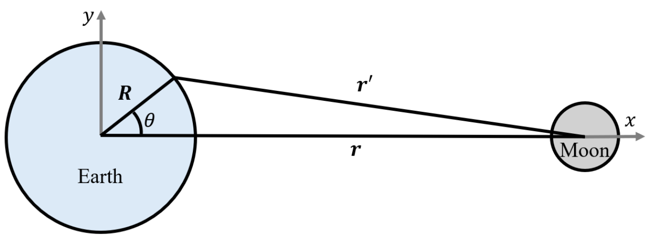

Figure 3. Schematic of the tides in the Earth-Moon system.

The system is considered in a rotating reference frame, shown in figure 3, with the origin at the Earth and Moon's centre of mass. I start by deriving the gravitational force acting on Earth's oceans parallel and perpendicular to the Moon's position. As shown in figure 3: $\boldsymbol{R}$ is the vector pointing from the barycentre of the Earth to a point on its surface; $\boldsymbol{r}$ is the vector pointing from the barycentre of the Earth to the centre of the Moon; $\boldsymbol{r\prime} = |\boldsymbol{r} - \boldsymbol{R}|$ is the vector pointing from a point on Earth's surface to the barycentre of the Moon. Along x and y in figure 3, it is geometrically determined that:

$$(\boldsymbol{r} - \boldsymbol{R})_x = r - R\cos\theta \tag{3a}$$
$$(\boldsymbol{r} - \boldsymbol{R})_y = -R\sin\theta, \tag{3b}$$

and that

$$|\boldsymbol{r} - \boldsymbol{R}| = \sqrt{r^2 + R^2 - 2rR\cos\theta}. \tag{4}$$

Newton's law of gravitation gives the force due to the Moons gravity on a test mass m of ocean water, at Earth's surface and pointed to by $\boldsymbol{R}$:

$$\boldsymbol{F_g} = \frac{GmM_M}{(r\prime)^3}\boldsymbol{r\prime}. \tag{5}$$

The fictitious force due to the accelerating reference frame is [4]:

$$\boldsymbol{F_f} = -\frac{GmM_M}{r^3}\boldsymbol{r}. \tag{6}$$

The total tidal force is therefore:

$$\boldsymbol{F_T} = GmM_M\left(\frac{\boldsymbol{r\prime}}{(r\prime)^3} - \frac{\boldsymbol{r}}{r^3}\right). \tag{7}$$

Decomposing this into acceleration along x and y by using equation 3 and 4 gives:

$$(\boldsymbol{F_T})_x = \frac{GmM_M}{r^2}\left[\frac{1 - \frac{R}{r}\cos\theta}{(1 + \frac{R^2}{r^2} - 2\frac{R}{r}\cos\theta)^{3/2}} - 1\right], \tag{8}$$

and

$$(\boldsymbol{F_T})_y = -\frac{GmM_M}{r^3}R\sin\theta. \tag{9}$$

Since $r \gg R$, equation 8 is simplified by treating $\frac{R^2}{r^2}$ as negligible and using the binomial approximation on the remaining $(1 - 2\frac{R}{r}\cos\theta)^{3/2}$ in the denominator:

$$(\boldsymbol{F_T})_x \approx \frac{2GmM_M}{r^3}R\cos\theta. \tag{10}$$

Using this result, the height of the tides can now be derived. For the oceans to be in equilibrium, the potential energy at the surface of the ocean must remain approximately constant throughout. Therefore, the potential energy at high and low tide must be equal. At high tide, $\theta = 0$, this means:

$$\boldsymbol{F_{T,}}_{\text{ high}} = \frac{2GmM_M}{r^3}R, \tag{11}$$

and at low tide, $\theta = \pi/2$, this means:

$$\boldsymbol{F_{T,}}_{\text{ low}} = -\frac{GmM_M}{r^3}R. \tag{12}$$

The potential energy at heights h_{high} and h_{low} from the Earth's barycentre for high and low tide, respectively, is, therefore:

$$U_{\text{high}} = \int_0^{h_{\text{high}}} mg - \frac{2GmM_M}{r^3}hdh, \tag{13}$$
$$U_{\text{low}} = \int_0^{h_{\text{low}}} mg + \frac{GmM_M}{r^3}hdh, \tag{14}$$

where g is the (approximately constant) gravitational acceleration due to Earth's gravity at its surface. Equating these, rearranging and taking the integrals yields:

$$g(h_{\text{high}} - h_{\text{low}}) = \frac{GM_M}{r^3}(h_{\text{high}}^2 + \frac{h_{\text{low}}^2}{2}),$$

To simplify this, I use the fact that $h = R + \delta R$ where δR is some height above the Earth's surface. I then have: $h^2 = R^2 + 2R\delta R + \delta R^2$ but since $R^2 \gg 2R\delta R \gg \delta R^2$ this can be approximated as $h^2 \approx R^2$, therefore:

$$g\Delta h = \frac{3GM_M}{2r^3}R^2,$$

where $\Delta h = (h_{\text{high}} - h_{\text{low}})$ is the difference in height between high and low tide. Finally, using the fact that:

$$g = \frac{GM_E}{R^2},$$

where M_E is the mass of Earth, yields the needed result that the change in tidal height is:

$$\Delta h = \frac{3}{2}\frac{M_M}{M_E}\frac{R^4}{r^3}. \tag{15}$$

To put this in terms of the semi-major axis of the Moon, a, the radius of its elliptical orbit is given by:

$$r = \frac{a(1 - e^2)}{1 + e\cos\phi}, \tag{16}$$

where e is the eccentricity, and ϕ is the angle from the perigee. Combining this with equation 15 the change in height is now given by:

$$\Delta h = \frac{3}{2}\frac{M_M}{M_E}\frac{(1+e\cos\phi)^3}{(1-e^2)^3}\frac{R^4}{a^3}. \tag{17}$$

This is evaluated, ignoring eccentricity ($e \approx 0$), at the current value of the Moon's semi-major axis, a_0, giving the average tidal height:

$$\Delta h = 53.5cm.$$

It is also noted that for a given area, assuming constant ocean water density, the mass of the tides is:

$$m \propto h \propto \frac{1}{a^3}. \tag{18}$$

4. Tidal Acceleration Model

4.1. Overview & Assumptions

Having obtained an expression for the mass of the tides, it is now possible to derive the expansion of the Moon's orbit over time. In this section, I derive: the Earth-Moon system's total angular momentum; the torque applied by the Moon on Earth's tidal bulges; the exchange of angular momentum from Earth's spin to the Moon's orbit (resulting from this Torque), and the resulting rate of expansion in the Moon's orbit. The rate of expansion is then evaluated to find the upper bound for when the final TSE will occur.

The following derivations are simplified by assuming the following:

- The Earth and the Moon are an isolated system. This may impact the result since the Sun's tidal forces are ignored. However, I expect any impact to be small since tidal forces due to the Sun are noticeably smaller than those due to the Moon. Additionally, the increase in torque experienced during a spring tide should balance with the decrease in torque experienced during a neap tide.
- The semi-major axis of the Earth and the Moon changes slowly enough that the Moon's orbit may be considered an ellipse.
- The Earth's axis of rotation is perpendicular to the Moon's orbital plane. In reality, the Moon's inclination to Earth's equator is between 18.28° and 28.58° . This assumption will, therefore, limit the precision of the derivation's result.
- The line connecting the Earth's tidal bulges forms a fixed angle with the line connecting the Earth and the Moon. In reality, as Earth's rotation slows down, this angle will decrease. However, over the timescale being investigated, the decrease in Earth's rotation will be small, meaning the change in this angle is negligible.
- The eccentricity of the Moon's orbit remains approximately constant throughout its orbit expansion.

4.2. Angular Momentum the Earth-Moon System

I start by considering the Earth and Moon's co-orbit around the centre of mass, shown in figure 4 to find an expression for the orbital angular momentum L_{orb}. From the assumptions made over a single orbital cycle L_{orb} is approximately constant, L_{orb} is therefore evaluated when the Earth and Moon are separated by the semi-major axis of the Moon's orbit, $a = r_E + r_M$, with r_E is the distance from Earth to the centre of mass (C), and r_M the distance from the Moon to the centre of mass, as shown in figure 4. From the definition of the centre of mass:

$$r_E = a\frac{M_M}{M_E+M_M},$$
$$r_M = a\frac{M_E}{M_E+M_M}.$$

The orbital moment of inertia for this system is therefore:

$$I_{\text{orb}} = M_E r_E^2 + M_M r_E^2 = a^2\frac{M_E M_M}{M_E+M_M}, \tag{19}$$

meaning the orbital angular momentum is:

$$L_{\text{orb}} = I_{\text{orb}}\omega = \frac{M_E M_M}{M_E+M_M}a^2\frac{2\pi}{T},$$

where ω is the orbital frequency, and T is the orbital period. To reduce this further, Keplers 3rd law of planetary motion:

$$a^3 = \frac{G(M_E+M_M)}{4\pi}T^2 \implies \frac{2\pi}{T} = \frac{G(M_E+M_M)^{1/2}}{a^{3/2}},$$

is used, leading to:

$$L_{\text{orb}} = \frac{M_E M_M}{\sqrt{M_E+M_M}}\sqrt{Ga}. \tag{20}$$

Evaluating this with the current value of the Moons semi-major axis, a_0 gives:

$$L_{\text{orb},0} = 2.859\times10^{34}kgm^2s^{-1}.$$

The total angular momentum of the Earth-Moon system is given by

$$L_{\text{tot}} = L_{\text{orb}} + S_E + S_M,$$

where S_E is the spin angular momentum of Earth's rotation, and S_M is the spin angular momentum of the Moon's rotation. S_E and S_M are evaluated using the current values of the Earth and Moon's spin angular

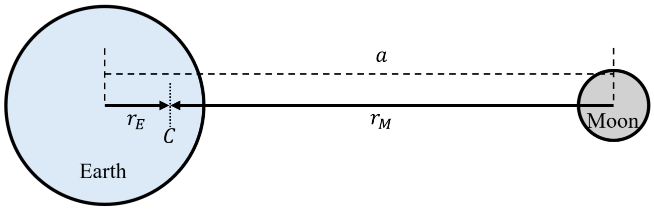

Figure 4. Schematic of the Earth and the Moon co-orbiting their centre of mass.

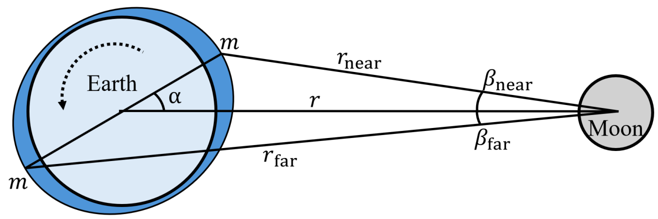

Figure 5. Schematic of the forces experienced by Earth's tidal bulges due to the Moon's gravitational attraction.

velocity, $\omega_{E,0}$ and $\omega_{M,0}$, and their respective moment of inertia, I_E and I_M, therefore:

$$S_{E,0} = I_E\omega_{E,0} = 5.86 \times 10^{33} kgm^2s^{-1}$$
$$S_{M,0} = I_M\omega_{M,0} = 2.35 \times 10^{29} kgm^2s^{-1}.$$

This evaluation shows S_M to be 4-5 orders of magnitude smaller than S_E and L_{orb}, meaning L_{tot} simplifies to:

$$L_{\text{tot}} = I_E\omega_E + \frac{M_E M_M}{\sqrt{M_E + M_M}}\sqrt{Ga}, \tag{21}$$

by neglecting S_M.

4.3. Torque Acting on Earth's Tidal Bulges

To calculate the torque the Earth is considered to have two tidal bulges of equal mass, m, at its surface. These are rotated out of alignment with the Moon by some angle, α, as shown in figure 5. The Moon's gravitational force acts on each bulge in Earth's tides generating the torque.

The distance from the Moon to the nearest and furthest bulges are r_{near} and r_{far} respectively, each of which is found by considering the geometry of the system:

$$r_{\text{near}} = \sqrt{r^2 + R^2 - 2rR\cos\alpha}, \tag{22a}$$
$$r_{\text{far}} = \sqrt{r^2 + R^2 + 2rR\cos\alpha}. \tag{22b}$$

The magnitude of the force acting on each bulge is:

$$F_{\text{near}} = \frac{GmM_M}{r_{\text{near}}^2} = \frac{GmM_M}{r^2 + R^2 - 2rR\cos\alpha}, \tag{23a}$$
$$F_{\text{far}} = \frac{GmM_M}{r_{\text{far}}^2} = \frac{GmM_M}{r^2 + R^2 + 2rR\cos\alpha}. \tag{23b}$$

From this, the torque acting on each bulge is found:

$$\tau_{\text{near}} = |\boldsymbol{r} \times \boldsymbol{F}_{\text{near}}| = r\frac{GmM_M}{r_{\text{near}}^2}\sin\beta_{\text{near}},$$
$$\tau_{\text{far}} = |\boldsymbol{r} \times \boldsymbol{F}_{\text{far}}| = r\frac{GmM_M}{r_{\text{far}}^2}\sin\beta_{\text{far}},$$

where β_{near} and β_{far} are angles shown in figure 5, geometrically determined to be:

$$\sin\beta_{\text{near}} = \frac{R\sin\alpha}{r_{\text{near}}},$$
$$\sin\beta_{\text{far}} = -\frac{R\sin\alpha}{r_{\text{far}}}.$$

The torque can therefore be expressed as:

$$\tau_{\text{near}} = \frac{GmM_M rR\sin\alpha}{(r^2 + R^2 - 2rR\cos\alpha)^{3/2}}, \tag{25a}$$
$$\tau_{\text{far}} = -\frac{GmM_M rR\sin\alpha}{(r^2 + R^2 + 2rR\cos\alpha)^{3/2}}. \tag{25b}$$

The overall torque acting against Earth's rotation is $\tau = \tau_{\text{near}} + \tau_{\text{far}}$, or:

$$\tau = \mu[(1+\frac{R^2}{r^2}-2\frac{R}{r}\cos\alpha)^{-\frac{3}{2}}-(1+\frac{R^2}{r^2}+2\frac{R}{r}\cos\alpha)^{-\frac{3}{2}}],$$

where:

$$\mu = \frac{GmM_M\sin\alpha}{r^2}.$$

Since $r \gg R$ the binomial approximation can be used to simplify this expression to:

$$\tau = \mu\frac{6R\cos\alpha}{r} = \frac{6mM_M R\sin\alpha\cos\alpha}{r^3},$$

using equation 16 this then becomes:

$$\tau = \frac{6mM_M R\sin\alpha\cos\alpha(1+\cos\phi)^3}{(1-e^2)^3a^3} \propto \frac{m}{a^3}. \tag{26}$$

Combining this with the result from section 3: $m \propto a^{-3}$, gives a proportionality between the torque and the Moon's semi-major axis:

$$\tau \propto \frac{1}{a^6}. \tag{27}$$

4.4. Expansion of the Moon's Orbit

Finally, to derive an expression for the expansion of the Moon's orbit, I combine the previous derivations of the angular momentum and torque. The torque is related to the angular momentum of Earth's spin by:

$$\tau = \frac{dS_E}{dt} \propto \frac{1}{a^6},$$

and the conservation of angular momentum then implies:

$$\frac{dL_{\text{orb}}}{dt} = -\frac{dS_E}{dt} \propto -\frac{1}{a^6}.$$

It was also previously seen (in equation 20) that:

$$L_{\rm orb} \propto a^{1/2},$$

which is used to obtain a relation between the semi-major axis and its corresponding rate of change:

$$\frac{da^{1/2}}{dt} \propto -a^{-6} \implies \frac{da}{dt} = Ca^{-11/2}, \qquad (28)$$

where a constant of proportionality C has been introduced. To determine C, consider the current value of the Moon's semi-major axis a_0 and the current value of the Moon's recession [2, 3]:

$$\left.\frac{da}{dt}\right|_{t=0} = 3.8cm/\text{year}.$$

This gives the constant of proportionality:

$$C = a_0^{11/2}\left.\frac{da}{dt}\right|_{t=0} = 1.98 \times 10^{38} m^{13/2} s^{-1}.$$

To find the time until the final TSE, t_f, equation 28 is integrated giving:

$$t_f = \frac{2}{13C}\left[a_f^{13/2} - a_0^{13/2}\right], \qquad (29)$$

where a_f is the semi-major axis of the Moon's orbit during the final TSE. The value for a_f is retrieved from the evaluation in section 2 in which it was determined that:

$$r_{\text{perigee},f} \approx 0.3800 \times 10^6 km,$$

was the upper bound for the final TSE. This corresponds to a semi-major axis of:

$$a_f = \frac{r_{\text{perigee},f}}{(1-e)},$$

using the eccentricity of the Moon's orbit the final value of the semi-major axis is then:

$$a_f = 0.4021 \times 10^6 km.$$

Plugging into equation 29 the upper bound for the time until the final TSE is:

$$t_f \approx 530 \text{ million years.}$$

5. Conclusion

In conclusion, I have successfully derived from first principles a model for the Moon's tidal acceleration and used it to predict an upper bound for when the final TSE will occur. A simpler model in which: $\frac{da}{dt} = const = 3.8cm/\text{year}$ would give a result of $t_f \approx 470$ million years. Our derived model impacts this result by ∼11%.

This model is applicable for predicting a number of other phenomena, for example:

- Projection bak in time: to find when the first ASE took place.
- Other planet-moon or star-planet systems: such as Mars and one of its moons, Phobos. In this example, the reverse process, tidal deceleration, occurs since Phobos' orbital period is less than the duration of a day on Mars, meaning Phobos' semi-major axis is shrinking.
- Tidal locking: by using Keplers 3rd law to replace a with T, this model can predict when the orbital period and spin period of a body will match (when it is tidally locked).

A more advanced model might remove a number of assumptions made here. For example, the Moon's orbital plane and the Earth's axis of rotation are not perpendicular; this misalignment causes the Earth's axis of rotation to cycle. Additionally, these tidal forces do change the eccentricity of the Moon's orbit in a process known as circularisation. This circularization may slightly increase the upper bound for the final TSE.

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Appendix

Table 1. Collection of astrophysical constants used throughout this report. **Astrophysical constants with no citation provided are taken from the University of Bath formula booklet.*

Description	Constant	Value	Citation
Radius of the Sun	R_S	$6.957 \times 10^8 m$	*
Radius of Earth	R	$6.371 \times 10^6 m$	*
Radius of the Moon	R_M	$1.738 \times 10^6 m$	*
Mass of Earth	M_E	$5.972 \times 10^{24} kg$	*
Mass of the Moon	M_M	$7.348 \times 10^{22} kg$	*
Current semi-major axis Moon's orbit	a_0	$0.3844 \times 10^6 km$	[5]
Current Perigee of Moon's orbit	$d_{\text{perigee},0}$	$0.3633 \times 10^6 km$	[5]
Current apogee of Moon's orbit	$d_{\text{apogee},0}$	$0.4055 \times 10^6 km$	[5]
Eccentricity of Moon's orbit	e	0.0549	[5]
Aphelion of Earth's Orbit	d_{aphelion}	$1.521 \times 10^8 km$	[6]
Spin angular velocity of Earth	$\omega_{E,0}$	$7.29 \times 10^{-5} s^{-1}$	[7]
Spin angular velocity of the Moon	$\omega_{M,0}$	$2.66 \times 10^{-6} s^{-1}$	[5]
Moment of inertia of Earth	I_E	$8.04 \times 10^{37} kgm^2$	[8]
Moment of inertia of the Moon	I_M	$8.83 \times 10^{34} kgm^2$	[9]