Calibration and Hedging LAB

Computational Finance 2025/2026

You are provided with a 6-month dataset of S&P 500 options, covering all quoted bid and ask prices, along with the time series of the S&P 500 spot price. The dataset spans from **8 June to 13 December 2017**. You are acting as a market maker on **8 December 2017**.

1. **Calibrate the discount factor and forward rates** as of 8 December 2017 using option prices. Apply appropriate filtering techniques to exclude illiquid options.

2. **Calibrate the equity volatility surface.** Choose a model for the forward suitable for the hedging tasks of a market maker (you may use one discussed during lectures or select another model). Report the Calibration error in term of RMSE and MAPE on the entire volatility surface.

3. **Implement an efficient simulation algorithm** for the chosen model and justify your selection. Verify that the algorithm correctly prices European options across different maturities (i.e., the Monte Carlo price converges to the model price with a sufficiently large number of simulations).

4. **Price the certificate** described in **Annex A** as of 8 December 2017.

5. Assume the market maker is **short the certificate**. Define an **automatic hedging strategy** for the contract based on your selected model. You may hedge the risk factors you consider relevant (e.g., spot, volatility, or model parameters). You are allowed to trade synthetic forwards and any options included in the dataset. Account for bid-ask spreads when executing trades. You may borrow and lend cash, with the remuneration rate derived from the shortest maturity in the option market.

6. **Justify your hedging strategy** by backtesting it on the dataset. Report the strategy's performance from **11 to 13 December 2017** (additional testing dates). On each testing day:

   o Recalibrate your model and reprice the instrument.

   o Evaluate the P&L of your portfolio (certificate + hedging instruments + interest rates paid or received) using market prices for quoted instruments and computing the certificate price.

o Reapply your hedging strategy, tracking borrowed/lent cash and its remuneration.

For example: You hedge on 8 December with a €10 million call option, borrowing €10 million. On 11 December, you recalibrate. The certificate has gained €30k, the call €25k, and you paid €1k in interest. Your P&L is therefore -€6k. You then adjust the hedge to a €9.5 million call, sell the previous call at market price, update the cash position, and continue accordingly.

Deliver a report detailing all your analysis (maximum 15 pages) and a MATLAB replication package via email to michele.azzone@polimi.it and luca.russo@polimi.it.

After delivering the assignment you will receive an additional dataset of six months option data. You should test your hedging strategy for the six months (every day recalibrate the model and report the P&L). You should deliver a one-page document reporting the plot of your portfolio value and a plot of the daily P&L squared. Report also the quantity $\frac{1}{T}\sum_{t=1}^{T} P\&L_t^2$ for the testing set (starting from the 14 of December) which will be the metric to evaluate your hedging strategy performance. The winner is the group with the **lowest** P&L squared. Send also the code to replicate your results.

In the paths in which the certificate is liquidated close all open positions and assume the P&L value zero in all days after liquidation.

Evaluations

1 Point: Quality of Reports and Analysis

Extra Point for best Hedgers

1 Point for the best two groups

0.5 For third and fourth groups

Certificate Miami Crocodile

Emission Date: 8th of December 2017

Notional: 15 million USD.

Reference Value: S&P 500 Spot

Additional final amount = 23%

Protection: 0%

Factor: 90%

Strike=100%

Barrier Level: 90%;

Trigger Level: 120%;

Final Liquidation date: 9th of December 2019

Liquidation:

- In case at liquidation date the certificate has not been liquidated yet the Settlement Amount is computed with the following formula. In case at valuation date the Reference Value is above the trigger level Settlement Amount will be: (Liquidation price+Additional final amount)*Notional.
- In the event that on the Valuation Date the Reference Value is below the Trigger Level and the Barrier Event has not occurred, the Settlement Amount will be equal to: (Liquidation price)*Notional.
- In the event that on the Valuation Date the Reference Value is below the Trigger Level and the Barrier Event has occurred, the Settlement Amount will be equal to Max(Protection, Factor*(Reference Value/Strike))
- In the event that on the Valuation Date the Reference Value is above the Trigger Level and the Barrier Event has occurred, the Settlement Amount will be equal to 110%.

If on any of the Observation Dates (for the Automatic Early Settlement) the Automatic Early Settlement Condition is met, on the corresponding Automatic Early Settlement Date, the Holder will receive the Automatic Early Settlement Amount. 'Automatic Early Settlement Condition' means the occurrence of the circumstance in which, on any of the Observation Dates (for the Automatic Early Settlement), the Reference Value of the Underlying is equal to or above than the Trigger Level.

Observation Dates and Liquidation Price (for Automatic Early Settlement and for barrier)

1. 9 April 2018, Liquidation Price=102%*Notional
2. 8 August 2018, Liquidation Price=103%*Notional
3. 10 December 2018, Liquidation Price=105%*Notional
4. 8 April 2019, Liquidation Price=110%*Notional
5. 8 August 2019, Liquidation Price=115%*Notional
6. 9 December 2019,   Liquidation Price=120%*Notional