An Exploration of the Art Gallery Problem with Small Satellites

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Overview

The Art Gallery problem seeks to place a minimum number of guards within an art gallery such that they can collectively maximize, or completely cover, the total area they can see. The Watchman routing problem is similar, only it seeks to give each guard a path and set of orientations along that path which maximize the coverage of the art gallery. There is little research on the Art Gallery and Watchman Routing Problems as they relate to swarms of small spacecraft. In this context, the problem is not exactly a watchman routing problem. Here the satellites move along slightly modifiable trajectories rather than fixed paths. The goal is either to find the best modified path or best series of orientations along a fixed path.

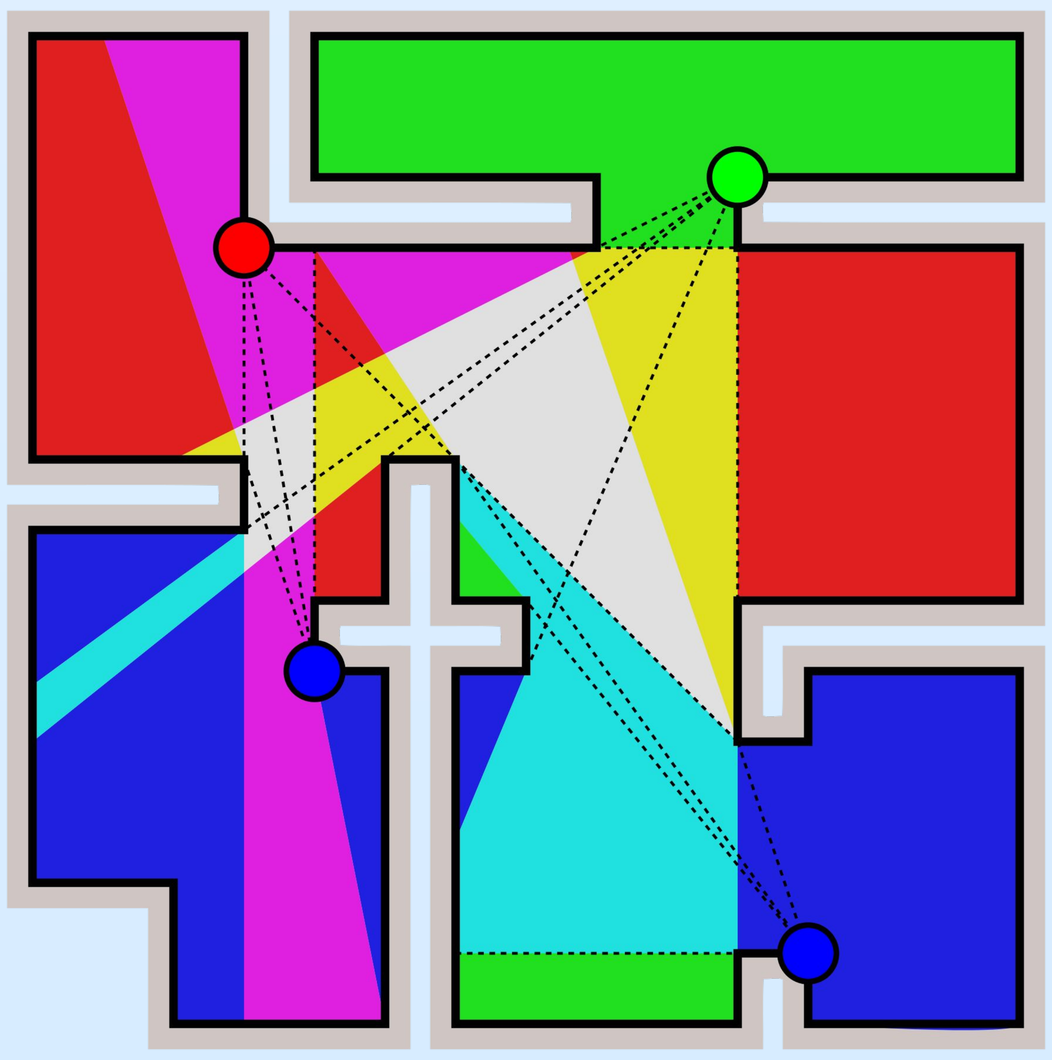


Fig 1. Example of an Art Gallery which is fully guarded

Problem Definition

- There are several satellites S_n with momentum p_n defining their orbits.
- Satellite sensor has a field of view of θ_n and their minimum distance from the target is hn.
- The diagram is a simplification and the satellites are not necessarily orbiting within the same plane.
- Note that figure 5 is at some instantaneous time tn, where the satellites continue on their trajectory to some new location.
- Assume that the satellites are not continually observing, that they are taking measurements / observations at some time step Δt.
- A single spacecraft generates a series of circular observations, which may overlap if Δt is small enough.
- The goal is to describe a method which can be used as an optimization function.
- Past coverage should weigh less to the optimization function, revisit should be incentivized
- Overlapping circles represent the duel coverage area, we only count most recent coverage.

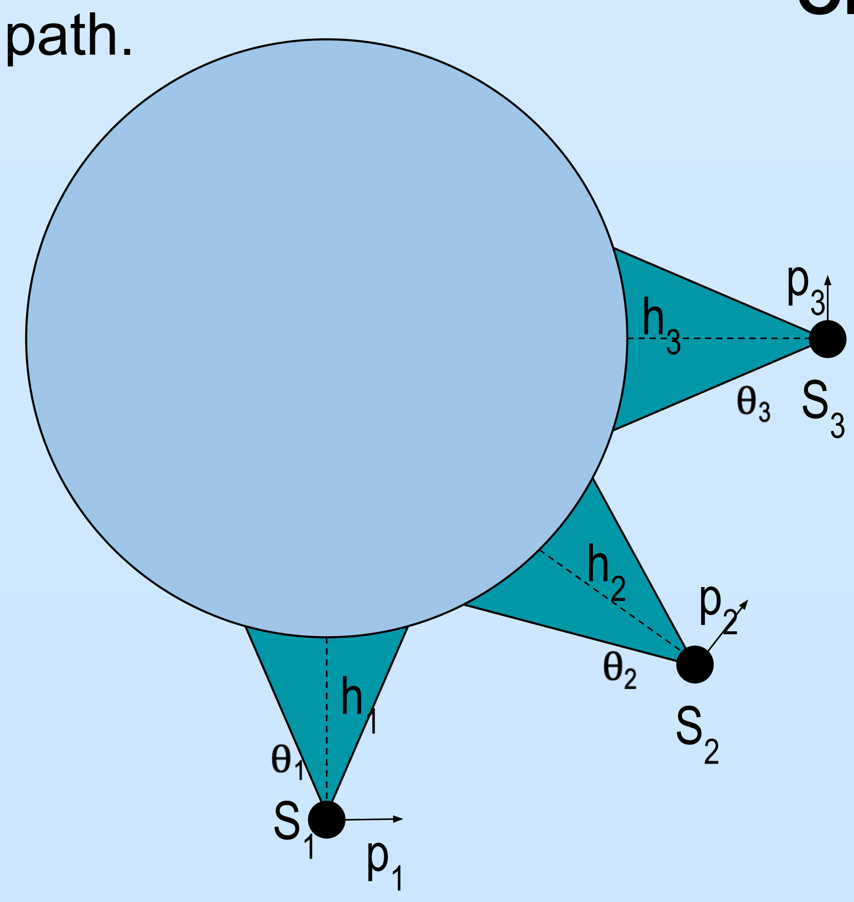
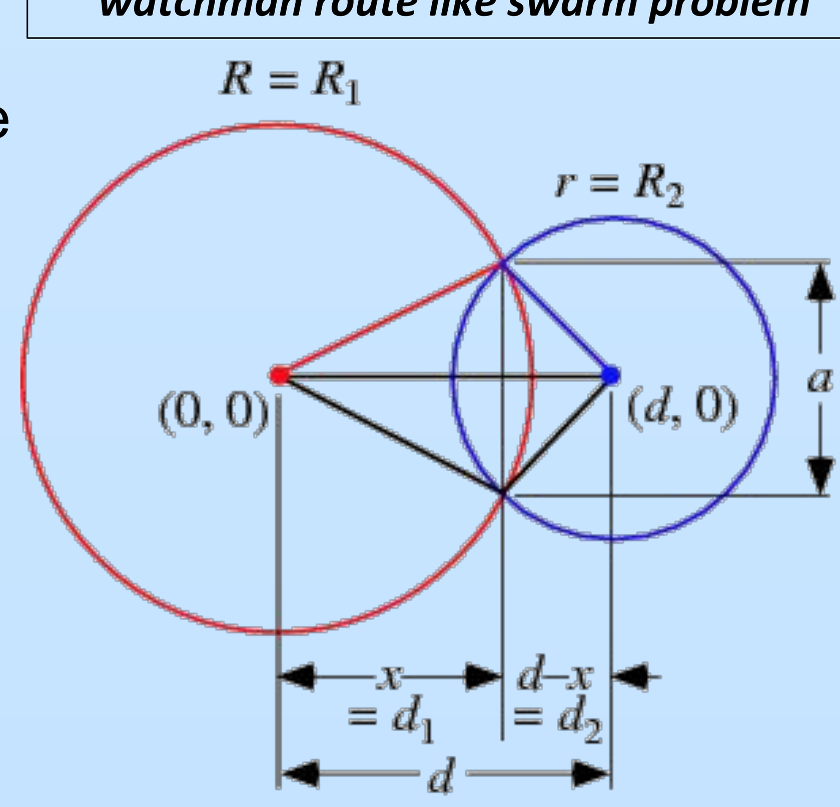


Fig 2. A cross sectional example of the type of "swarm" that is considered in the watchman route like swarm problem



$$A = A(R, d_1) + A(r, d_2) = r^2 \cos^{-1}\left(\frac{d^2+r^2-R^2}{2dr}\right) + R^2 \cos^{-1}\left(\frac{d^2+R^2-r^2}{2dR}\right) - \frac{1}{2}\sqrt{(-d+r+R)(d+r-R)(d-r+R)(d+r+R)}.$$

Fig 3. calculating the area of intersection of two circles. These areas are projected using estimated surface geometry.

Experiment configure

Experiment will take place on real spacecraft hardware Used so each component can be individually tested.

The "Flatsat" will contain:

- Custom GPU/SoC system [2] similar to the MOCI mission [1]
- A google Coral dev board or google Coral mini PCIe accelerator
- Simplified OBC (On Board Computer),
- EPS (Electrical Power System)
- Antenna system with Transceiver
- Simulated imager. The simulated imaging system will use existing SSRL 3D model observation simulations [1][4].

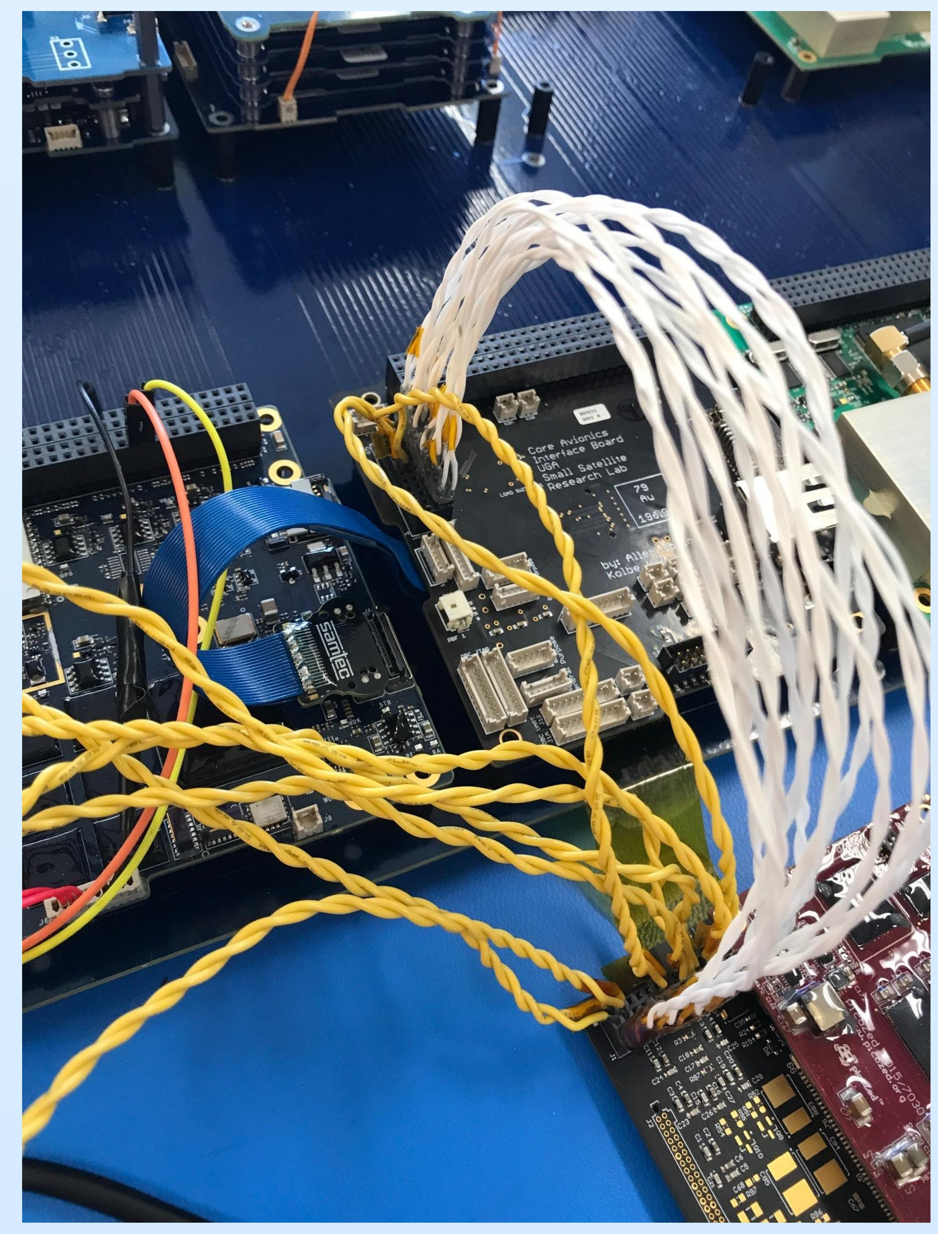
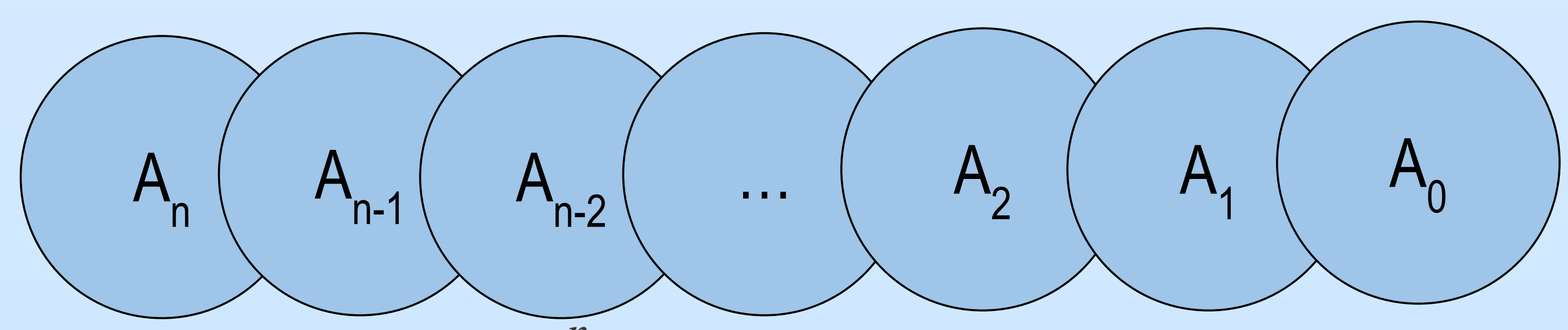


Fig 4. The "flat sat" for the SPOC satellite [3]

Objective Function



$$A_0 + \sum_{i=1}^{n} \frac{1}{di}\left(A_i - |\, A_{i-1} \cap A_i \,|\right)$$

Fig 5. A potential objective function for the swarm, a total weighted calculation for quality of area covered

The area within projected circles (at a fixed resolution), instead of the estimated surface area of the encircled region, is used for two reasons:

1) The estimated surface within the projected circle is not entirely accurate, therefor using its surface area would add more noise to the optimization function.
2) The estimated surface area may increase with better resolution, due to the coastline paradox.

The optimization function is the sum of most recent observations by weight. The goal is to maximize this function.

Citations

[1] C. Adams, "A near real time space based computer vision system for accurate terrain mapping,"32nd Annual AIAA/USU Conference on Small Satellites, 2018.

[2] C. Adams, "Towards an Integrated GPU Accelerated SoC as a Flight Computer for Small Satellites," IEEE Aerospace Conference 2019.

[3] D. Cotton, C. Adams, et al. "The Spectral Ocean Color Imager (SPOC) - An Adjustable Multispectral Imager,"33nd Annual AIAA/USU Conference on Small Satellites, 2019.

[4] C. Adams, "The Feasibility of Structure from Motion over Planetary Bodies with Small Satellites" 32nd Annual AIAA/USU Conference on Small Satellites, 2018.