Optimal Taxation when Adverse Selection Meets Moral Hazard*

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Abstract

This paper develops a comprehensive framework for optimal income taxation that simultaneously addresses both ability differences and income uncertainty as sources of inequality. By allowing tax schedules to be contingent on both ability and realized income, we decouple the design problem into distinct moral hazard and adverse selection components, enabling us to characterize the optimal balance between redistribution, insurance, and work incentives. We find several novel findings. First, among workers with identical realized incomes, lower-ability workers face higher marginal tax rates due to incentive effects. Second, workers may experience negative marginal tax rates when facing sufficiently large negative productivity shocks. Finally, our model reveals a lower redistributive effect than standard Mirrlees frameworks, while identifying an incentive effect that increases with ability and raises the *ex ante* marginal tax rate. These results demonstrate how optimal policy must account for the complex interplay between hidden information and hidden actions when addressing income inequality.

Keywords. Adverse Selection; Moral Hazard; Optimal Taxation
JEL. D82; H21; H22; H24

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1 Introduction

A substantial body of research highlights the innate ability and risk of earnings as key determinants of labor income (Storesletten et al., 2004; Huggett et al., 2011; Karahan and Ozkan, 2013; Heathcote et al., 2014),[1] resulting in an uncertain relationship between effective labor supply—determined by ability and effort—and income.[2] This raises the critical question of whether optimal redistributive labor income taxation should distinguish between income from effort and luck. However, existing literature on optimal redistributive tax often examines ability and luck independently, neglecting the intricate relationship between redistribution goals and work incentives.[3] This paper fills the gap by exploring optimal redistributive tax when labor income depends jointly on workers' ability and earnings risk.

We develop an optimal income tax schedule in a Mirrlees model incorporating both ability and luck as sources of income inequality. The model features a unit measure of workers who differ in ability and face idiosyncratic labor productivity shocks. Workers possess private information about their ability, labor supply, and productivity shock, none of which is observed by the tax authority.[4] Furthermore, Labor supply is chosen *ex ante*, before shocks are realized, while labor income is determined *ex post*. Productivity shocks decouple effective labor supply from income: workers with different abilities may earn the same, while identical labor supplies may yield different incomes.

To address the challenges posed by hidden information (from unobserved abilities) and hidden actions (from hidden efforts and productivity shocks) on income tax design, we decouple the problem using the method in Castro-Pires et al. (2024) by breaking down the relaxed government problem into a pure moral hazard problem

[1] Storesletten et al. (2004) report that individuals would forgo 26% of lifetime consumption to protect against life-cycle shocks, while Karahan and Ozkan (2013) find that workers under a zero borrowing constraint would sacrifice 18.20% of consumption to eliminate idiosyncratic risks. Meanwhile, Huggett et al. (2011) attribute 61.5% of lifetime earnings variation to initial conditions and 38.5% to shocks, and Heathcote et al. (2014) estimate that life-cycle productivity shocks account for about half of cross-sectional earnings inequality.

[2] Following the literature, labor income is modeled as the product of effective labor supply (determined by ability/wage and effort/hours) and luck.

[3] Mirrlees (1971) formalized optimal taxation with unobservable heterogeneity, a framework extended by Diamond (1998), Saez (2001); Farhi and Werning (2013), and Golosov et al. (2016), all emphasizing adverse selection. Meanwhile, Mirrlees (1974) examined tax schedules under earnings risk (moral hazard), later expanded by Varian (1980), Tuomala (1984), Low and Maldoom (2004), and Pirttilä and Tuomala (2007).

[4] It is also referred to as the government. We use "tax authority" and "government" interchangeably.

and an adverse selection problem.[5] For the moral hazard problem, the government redistributes across states for each type of worker, assigning consumption based on realized income to minimize expected costs while ensuring participation and incentive-compatible labor supply. Since hidden actions preclude full insurance (Holmström, 1979; Jewitt, 1988), the optimal allocation across states trades off risk-sharing and work incentives, with its shape determined by how income distribution signals unobserved worker effort. The resulting expected consumption function for each type of individual, which is determined by the reservation utility and the effective labor supply, then serves as an input to address the adverse selection problem. The government redistributes allocations across different types of workers and maximizes social welfare by choosing promised utility and effective labor supply, constrained by the truth-telling conditions. Solving these problems sequentially yields the optimal income tax schedule addressing both adverse selection and moral hazard.

The income tax schedule is both income-contingent and type-contingent.[6] This allows the government to implement direct "cross-subsidies" from high-ability to low-ability workers based on type information. We characterize the optimal tax schedule using two key measures: The first is the marginal tax rate on *ex post* income, which reflects how tax payments vary with realized income for workers of a given ability level; Compared to optimal tax in pure moral hazard senaros, optimal *ex post* taxation in our framework does not emphasize *ex post* equity concerns, as this function is fulfilled by the type-contingent component of the tax schedule.

The second is the *ex ante* marginal tax rate, which captures how expected tax payments change in response to a worker's labor effort. The optimal *ex ante* marginal tax rate extends the classical ABC formulas of Diamond (1998) and Saez (2001). Specifically, incorporating moral hazard introduces two novel components to the optimal *ex ante* marginal tax rate: The primal term is an information rent term arising from the pure moral hazard problem, which ensures that workers adopt the suggested effort level. This term balances the marginal tax burden against the worker's marginal gain from exerting effort under income uncertainty. Since this marginal gain is always

[5]In the traditional Mirrlees model, the government assigns each worker a bundle of required income and consumption to maximize social welfare, while respecting revenue and incentive compatibility constraints. When the income distribution conditional on effective labor supply satisfies the monotone likelihood ratio property, as shown in Milgrom (1981), optimal allocations of income and consumption increase with skill. This allows the government to infer ability, even if it is private information.

[6]Type-contingent tax schedules are common in practice. For example, the Earned Income Tax Credit (EITC) and Child Tax Credit adjust tax liabilities based on both income and family characteristics.

positive, it increases the *ex ante* marginal tax rate for all worker types and generates positive *ex ante* marginal tax rates even at the upper bound of the ability distribution. The second term captures the gap between the *ex ante* relative price and the marginal rate of substitution between labor effort and consumption, which reflects how uncertainty modifies the measurement of efficiency distortions caused by income taxation.

The decoupling method used to analyze the theoretical model also informs the numerical solution procedure. Numerically, we start with a zero co-state value at the lowest ability level and an initial utility guess, then solve for the control variables (consumption and labor supply). The Runge-Kutta method is applied to integrate over small sub-intervals in ascending order up to the highest ability, with the initial utility adjusted to ensure the co-state variable is zero at the highest ability. Castro-Pires et al. (2024) solves a similar numerical model in a principal-agent framework with both moral hazard and adverse selection, but their model assumes constant participation values and promised payments. In contrast, our approach incorporates more flexible assumptions, leading to a more complex yet widely applicable solution, highlighting the contribution of our algorithm.

Numerically, we show how optimal taxation should balance the insurance effect and the incentive effect on labor supply. While we confirm the standard result in the pure moral hazard problem that marginal tax rates on realized income increase with income due to the luck component, we uncover a previously unidentified ability gradient in taxation: when comparing workers with the same realized income, those with lower ability face higher marginal tax rates. Our findings thus highlight how optimal tax policy should account for both worker ability and realized income in providing insurance, extending beyond traditional frameworks that focus primarily on income levels.

The *ex ante* marginal tax rates follow a realistic inverse-U shape, which is robust when changing the risk attitude or the fiscal pressure. By decomposing it into a moral hazard part and an adverse selection part, we can see that the moral hazard contributes to a more progressive tax schedule. This is because, given the calibrated distribution of ability and uncertainty, the marginal benefit of providing insurance is positively correlated with ability.

We numerically examine the redistribution and social insurance roles by comparing our optimal tax schedule to those that solely address either pure moral hazard or adverse selection problems. Our tax schedule achieves better redistribution of expected consumption with minimal efficiency costs relative to the optimal tax schedule under pure moral hazard concerns. The adverse selection component of baseline *ex ante* tax rates is lower than in the Mirrlees tax results, indicating that income risk constrains the marginal benefits of redistribution across skill types.

Welfare analysis reveals that transitioning from a pure income-variant tax schedule of the HSV form to the baseline tax schedule generates welfare gains of approximately 0.658% in consumption-equivalent terms. The baseline tax schedule is more redistributive at the lower end of the ability distribution while imposing fewer disincentives on top earners. This demonstrates that the additional degree of freedom provided by allowing the tax schedule to be type-variant enables more effective redistribution at lower efficiency costs.

This paper is closely related to three strands of literature. The first emphasizes the social insurance role of optimal labor income taxation. Income tax serves as social insurance when workers' ability evolves stochastically without private insurance markets (Conesa and Krueger, 2006; Farhi and Werning, 2013; Golosov et al., 2016), or when private insurance is suboptimal, either because it covers only small groups of workers (Heathcote and Tsujiyama, 2021; Wu and Krueger, 2021) or because private insurance is impeded by information frictions (Ferey et al., 2023) or by workers' overconfidence (Chetty and Saez, 2010). When moral hazard exists in the private insurance market, a rise in income tax progressivity will crowd out private insurance and worsen moral hazard distortions as in Chetty and Saez (2010). It may also crowd in private insurance if pre-tax earnings risk is endogenous Doligalski et al. (2023). We contribute to the literature by characterizing optimal incomplete insurance provided by income tax contingent on not only income but also reported ability. In this way, our income tax schedule is incentive compatible in both encouraging truthful reporting of worker types as in standard Mirrlees taxation and exerting suggested efforts.

In the context of the optimal taxation literature, one strand addresses the moral hazard problem associated with social insurance, excluding private insurance considerations. When hidden actions manifest as labor effort, the risk of earnings forces individuals to engage in a precautionary labor supply so that income tax weakens work incentives (Mirrlees, 1974). Optimal income taxation balances the insurance effect, which favors higher marginal rates, with the precautionary labor supply that counters them (Varian, 1980; Low and Maldoom, 2004). The social insurance benefits of increasing marginal tax rates can be high for top income earners (Kindermann and Krueger, 2022). The commodity tax, levied on goods that are positively related to effort, helps motivate labor effort (Pirttilä and Tuomala, 2007). Unlike consumption, savings or investments are harder to observe. If assets are unobservable, Ábrahám et al. (2016) shows that optimal labor income taxes should be less progressive. When learning effort and human capital shocks are unobservable, Kapička and Neira (2019) find that optimal income tax should promote learning effort for high-ability workers, while providing more insurance for low-ability workers. Most of these studies above consider only the friction of moral hazard by assuming agents are *ex ante* identical, except that Kapička and Neira (2019) combine the private information problem in standard Mirrlees taxation with moral hazard friction in human capital investment. Our work contributes to the literature with a tractable discussion of the optimal income tax faced with both adverse selection and moral hazard by decoupling the influence of heterogeneous abilities and income shocks both theoretically and numerically.

Lastly, our work contributes to the literature on optimal contract design when adverse selection meets moral hazard. Early discussions of the necessary properties of an optimal contract can be found in (Baron and Besanko, 1987; Faynzilberg and Kumar, 1997). We adopt the decoupling approach, which has been developed and made tractable by (Fagart, 2002; Castro-Pires and Moreira, 2021; Gottlieb and Moreira, 2022; Castro-Pires et al., 2024), to analyze a more general case in which the principal (a social planner) has multidimensional objectives that extend beyond pure efficiency. In our model, the tax authority faces a trade-off between efficiency and equity, which intensifies the interaction between adverse selection and moral hazard. There is substantial research examining optimal contracts between a utilitarian social planner and consumers with heterogeneous, unobservable information who face unpredictable shocks. The contracts discussed in this literature include unemployment insurance (Hagedorn et al., 2010; Fuller, 2014), health insurance markets (Chade et al., 2022), and student loans (Gary-Bobo and Trannoy, 2015). Our work can be seen as complementary in introducing uncertainty into public policies, particularly in the context of optimal income taxation. While Gary-Bobo and Trannoy (2015) combine Mirrlees taxation with student loans to characterize second-best optima, they constrain income taxes to depend only on earnings, necessitating that optimal loan repayments be both income-contingent and education-contingent. Similarly, Boadway and Sato (2015) faced challenges achieving second-best outcomes under such constraints. In contrast, we allow the income tax to depend on both income and reported ability, enabling us to rely exclusively on the income tax to achieve second-best outcomes.

The rest of this paper is structured as follows. Section 2 models workers' labor supply under idiosyncratic labor productivity shocks and the government's problem. Section 3 outlines the approach to establish an incentive-compatible allocation, and Section 4 discusses the optimal nonlinear income tax schedule to implement this allocation. Section 5 calibrates the model parameters and provides numerical experiments. Finally, Section 6 concludes the paper.

2 The Model

The economy comprises a unit measure of workers with different innate abilities. The Workers' abilities, denoted by $\theta \in \Theta = \left[\underline{\theta}, \bar{\theta}\right]$, follow a probability density function $f(\theta)$, which is associated with the cumulative distribution function $F(\theta)$. Regardless of their abilities, all workers experience a common idiosyncratic shock to their labor productivity, denoted by χ, as outlined in Mirrlees (1974), Varian (1980), Low and Maldoom (2004), and Pirttilä and Tuomala (2007).

The timing of events is as follows. First, workers draw their ability type, θ, from $F(\theta)$. Second, they determine their labor supply or work effort *ex ante*, denoted as $l(\theta)$, before observing the realization of the productivity shock. The effective labor supply of a type-θ worker, $z(\theta)$, is given by $e^{\theta} l(\theta)$. Finally, the productivity shock is realized, and income is generated. This contrasts with the standard Mirrleesian framework, where income is determined after labor is supplied.

The worker's labor income, denoted as $y(\theta) = e^{\chi} z(\theta)$, depends on the worker's ability θ, the work effort $l(\theta)$, and the labor productivity shock χ. For a type-θ worker with effective labor supply $z(\theta)$, let $g(y \mid z(\theta))$ and $G(y \mid z(\theta))$ represent the density function and cumulative distribution function of y conditional on $z(\theta)$, respectively.[7] Assumption 1 shows the characteristics of $G(y \mid z(\theta))$.

Assumption 1. *(1) $G(y \mid z(\theta))$ is twice continuously differentiable in both arguments with partial derivatives G_z and G_y, where G_z is negative and G_y is nonnegative. Income is bounded by $[\underline{y}, \bar{y}]$, where $G(\bar{y} \mid z(\theta)) = 1$, $G(\underline{y} \mid z(\theta)) = 0$, and $G_y(y \mid z(\theta)) = 0$ for $y \geq \bar{y}$ for all θ.*

(2) The monotone likelihood ratio property (MLRP): Define the likelihood ratio of the distribution of y conditional on z by $h(y \mid z(\theta)) \equiv \frac{g_z(y \mid z(\theta))}{g(y \mid z(\theta))}$, its derivative on y satisfies $h_y(y \mid z(\theta)) > 0$.

[7]Formally, the distribution function should be denoted as $G(y \mid z, \theta)$, but for simplicity, we write it as $G(y \mid z(\theta))$, since z is a function of θ.

(3) The convexity distribution function condition (CDFC): $G_{zz}(y \mid z(\theta)) > 0$ for $y \leq \bar{y}$.

When the government cannot observe the effective labor supply of a type-θ worker, $z(\theta)$, MLRP in Assumption 1 implies, as has been proved by Milgrom (1981), that a high observed value of y likely indicates high effective labor supply $z(\theta)$.[8] CDFC helps to ensure that the second-order sufficient condition holds for a worker's optimization, which we will show in the worker decision part.

Given that a type-θ worker's income is determined by the product of wage rate, labor supply, and earnings risk, workers of different ability types may end up with the same income, while workers of the same ability may earn different incomes. Thus, the usual incentive constraints in the Mirrlees model, where taxes are based solely on income, are insufficient to distinguish workers by ability. To address this, we assume that the income tax $T(y,\theta)$ is defined based on both the realized income y and the worker's ability θ. Consequently, the after-tax income is given by $c(y,\theta) = y - T(y,\theta)$. In section 2.2, we will demonstrate that we can design an incentive-compatible tax schedule that not only induces workers to report their types truthfully, but also allows us to tax abilities and luck differently when we use θ as an instrument in the tax design.

Before we present the planning problem, we briefly describe workers' labor supply under idiosyncratic labor productivity shocks and the government's optimization.

2.1 The Worker's Decision

We formulate the optimization problem for a type-θ worker. This worker derives utility from consumption, $u(c)$, and experiences disutility from work. As the labor supply satisfies $l = \frac{z}{e^{\theta}}$, we denote by $v(z,\theta)$ the disutility function of work. u and v are twice continuously differentiable functions with $u'(c) > 0$, $u''(c) < 0$, $v_z > 0$, $v_{zz} > 0$, $v_{\theta}(z,\theta) < 0$, and $v_{z\theta}(z,\theta) < 0$. When the worker realizes income y, their consumption is represented as $c(y,\theta) \in \mathbb{R}_+$. The type-$\theta$ worker optimally chooses effective labor supply z before the realization of income to maximize the expected

[8] Since $h(y \mid z)$ equals the derivative of $\ln g(y \mid z)$ with respect to z, we have: $\frac{g(y|z_2)}{g(y|z_1)} = \exp\left(\int_{z_1}^{z_2} h(y \mid z)\, dz\right)$. This ratio increases with y for any $z_2 > z_1$ under the assumption of MLRP. Consequently, for any $y > x$ and $z_2 > z_1$: $\frac{g(y|z_2)}{g(y|z_1)} > \frac{g(x|z_2)}{g(x|z_1)}$. This implies that the posterior cumulative distribution of z satisfies $H(z \mid y) < H(z \mid x)$, which means $H(z \mid y)$ first-order stochastically dominates $H(z \mid x)$. Therefore, when a high output y is observed, it signals that the effort level z is also likely high.

utility below

$$\max_{z} \quad \int_{\underline{y}}^{\bar{y}} u(c(y,\theta))g(y \mid z,\theta)dy - v(z,\theta), \tag{1}$$

$$\text{s.t.} \quad c(y,\theta) = y - T(y,\theta). \tag{2}$$

Solving the worker's optimization problem gives Lemma 1:

Lemma 1. *Under the CDFC assumption that $G_{zz}(y \mid z(\theta)) > 0$, as long as the marginal tax rate for a type-θ worker with income y, defined by $T_y(y,\theta) \equiv \frac{\partial T(y,\theta)}{\partial y}$, satisfies $T_y(y,\theta) < 1$,*

(i) the first-order condition given by equation (3) is sufficient for solving the worker's optimization problem.

$$v_z(z,\theta) = \int_{\underline{y}}^{\bar{y}} u(c(y,\theta))g_z(y \mid z)dy. \tag{3}$$

(ii) $z'(\theta) > 0$ as long as $v_{\theta z} < 0$ and $\int_{\underline{y}}^{\bar{y}} u'(c)c_\theta g_z(y \mid z(\theta))dy > 0$.

The proof is contained in Appendix A. $z'(\theta) > 0$ indicates that the effective labor supply increases with the ability of workers.

2.2 The Government's Optimization

The benevolent government aims to maximize the following social welfare function

$$\int_{\underline{\theta}}^{\bar{\theta}} \beta(\theta)U(\theta)f(\theta)d\theta, \tag{4}$$

where $\beta(\theta) = \frac{e^{-\alpha\theta}}{\int_\theta e^{-\alpha\theta}dF(\theta)}$ and $U(\theta)$ is the utility of a type-θ worker. The parameter α represents the social planner's taste for redistribution. When $\alpha < 0$, the planner places a relatively higher weight on more productive workers; when $\alpha > 0$, the planner favors less productive workers. The case of $\alpha = 0$ corresponds to a utilitarian social welfare function with equal weights among all workers.

Denote by E the exogenous government's consumption. The government uses the labor income tax $T(y,\theta)$ to finance its expenditures. Therefore, the resource constraint is

$$\int_{\underline{\theta}}^{\bar{\theta}} \int_{\underline{y}}^{\bar{y}} (y - c(y,\theta))\, g(y \mid z(\theta))\, dy\, f(\theta)\, d\theta \geq E, \tag{5}$$

The government's optimization is actually a principle-agent problem with both moral hazard and adverse selection. Based on the revelation principle, the government's optimization is equivalent to assigning to each type of worker a menu of consumption allocation and recommended effective labor supply based on reported ability, which constitutes an incentive-compatible allocation aimed at maximizing social welfare. In this contract menu, $c(y, \hat{\theta})$ is the recommended consumption the agent receives if he reports $\hat{\theta}$ and y is observed. $z(\theta)$ is the recommended effective labor supply for a type-θ worker. The realized utility of a type-θ worker satisfies

$$U(\theta) = \int_{\underline{y}}^{\bar{y}} u(c(y, \theta))g(y \mid z(\theta))dy - v(z(\theta); \theta). \tag{6}$$

Given the set of menus provided by the government, the utility of a type-θ worker who reports ability $\hat{\theta}$ and exerts effort $\tilde{z}$ is defined as $U^r(\hat{\theta}, \tilde{z}; \theta)$. Thus, we have:

$$U^r(\hat{\theta}, \tilde{z}; \theta) \equiv \int_{\underline{y}}^{\bar{y}} u(c(y, \hat{\theta}))g(y \mid \tilde{z})dy - v(\tilde{z}; \theta) \tag{7}$$

Given $\hat{\theta}$, a worker who takes the government's menu chooses an effective labor supply $\tilde{z}$ to maximize utility. To induce the worker to take the recommended efforts (effective labor supply) z, we have

$$z = \tilde{z}(\hat{\theta}; \theta) \equiv \arg\max_{z} U^r(\hat{\theta}, z; \theta), \forall \theta, \hat{\theta} \tag{8}$$

To induce the workers to report truthfully about their ability, we have

$$U^r(\theta, \tilde{z}(\theta; \theta); \theta) \geq U^r(\hat{\theta}, \tilde{z}(\hat{\theta}; \theta); \theta), \forall \theta, \hat{\theta} \tag{9}$$

Combine conditions (8) and (9), we have

$$\{\theta, z(\theta)\} = \arg\min_{\tilde{z}, \hat{\theta}} \Psi(\tilde{z}, \hat{\theta}; \theta) = \arg\min_{\tilde{z}, \hat{\theta}} [U(\theta) - U^r(\hat{\theta}, \tilde{z}; \theta)], \tag{10}$$

Then, the government's optimization problem, indicated by $\mathcal{P}_1$, is to choose the menu to maximize the social welfare function (4), subject to the resource constraint (5), the participation constraint (6), and the incentive compatibility constraint (10). To make the incentive constraints (10) tractable, we next formulate a relaxed planning problem given by Lemma 2.

Lemma 2. *Under the assumption of CDFC and $u' > 0$, $u'' < 0$, $v_{zz} > 0$, and $v_{\theta z} < 0$,* *as long as* $c_y(y,\theta) \geq 0$ *and* $\int_{\underline{y}}^{\bar{y}} u'(c)c_\theta g_z dy > 0$ *, we have:*

(i) the relaxed planning problem can be formulated by replacing condition (10) *in the planning problem* $\mathcal{P}_1$ *with two of following three first-order incentive compatibility conditions:*

$$\int_{\underline{y}}^{\bar{y}} u(c(y,\theta))g_z(y \mid z(\theta))dy = v_z(z(\theta);\theta), \forall\theta; \tag{IC-MH}$$

$$U'(\theta) = -v_\theta(z(\theta);\theta), \forall\theta; \tag{IC-AS}$$

$$\int_{\underline{y}}^{\bar{y}} u'(c(y,\theta))c_\theta(y,\theta)g(y \mid z(\theta))dy = 0, \forall\theta. \tag{IC-D}$$

(ii) $z'(\theta) > 0$ *hold for all* θ *under the optimal contract.*

Equation (IC-MH) and (IC-D) come from the first-order condition on $\tilde{z}$ and $\hat{\theta}$ in optimization problem (10). Equation (IC-AS) can be obtained by applying (IC-MH) and (IC-D) to the expression of $U'(\theta)$. The proof is contained in the appendix A.

Equation (IC-MH) is the necessary condition in the moral hazard problem for a type-θ worker who is asked to supply $z(\theta)$. Equation (IC-AS) outlines the truth-telling condition. In the classical Mirrlees model, the two equations are equivalent, meaning that satisfying one equation guarantees that the other equation is also satisfied, thus either equation can be used without affecting the optimal solution. However, in the presence of moral hazard, both equations (IC-MH) and (IC-AS) must hold to establish the necessary conditions for incentive compatibility.

Equation (IC-D) means that, given that a worker takes the recommended labor effort, the utility pay-off will drop if the worker slightly but dishonestly announces his ability. In a pure Mirrlees model, second-order conditions require that consumption increases with ability. However, from (IC-D) holding fixed income y, there is no longer a monotonic correlation between consumption and ability. This is because we allow tax schedule to be ability-contingent. If $T_\theta(y,\theta) = 0$, we have $c_\theta = 0$ and (IC-D) for all θ. Therefore, equations (IC-MH) and (IC-AS) are again equivalent.

3 The Decoupling of Moral Hazard and Adverse Selection

According to Castro-Pires et al. (2024), the equivalent relaxed planning problem can be decoupled into a pure moral hazard problem and an adverse selection problem. This decoupling allows us to analyze hidden actions and hidden information independently.

3.1 The Pure Moral Hazard Problem

The equivalent relaxed planning problem simplifies to a pure moral hazard problem when workers' abilities are fixed at a specific value of θ, as discussed by Mirrlees (1974), Varian (1980), and Tuomala (1984).

The government's optimization problem involves selecting $c(y,\theta)$ to minimize spending on type-θ workers, represented as $\mathcal{C}(U(\theta), z(\theta))$:

$$\mathcal{C}(U(\theta), z(\theta)) = \min_{c(y,\theta)} \int_{\underline{y}}^{\bar{y}} c(y,\theta) g(y \mid z(\theta)) dy, \tag{11}$$

subject to the expected utility of each worker being above a threshold $U(\theta)$, captured by (6), and the first-order condition for effective labor supply (IC-MH).

Denote by $\mu(\theta)$ and $\gamma(\theta)$ the Lagrange multipliers for equation (6) and (IC-MH). By solving the cost-minimization problem, the marginal substitution rate between effective labor supply and consumption satisfies equation (12) in Proposition 1:

Proposition 1. *The cost-minimizing consumption scheme $c(y,\theta)$ satisfies the following first-order condition:*

$$\frac{v_z(z(\theta),\theta)}{u'(c(y,\theta))} = \underbrace{\tilde{\mu}(\theta)}_{\textit{Insurance}} + \underbrace{\tilde{\gamma}(\theta) h(y \mid z(\theta))}_{\textit{Incentive}}, \tag{12}$$

where $\tilde{\mu}(\theta)$ and $\tilde{\gamma}(\theta)$ are defined by

$$\tilde{\mu}(\theta) \equiv \mu(\theta) v_z(z(\theta),\theta) = E_y\left[\frac{v_z(z(\theta),\theta)}{u'(c(y,\theta))} \mid z(\theta)\right] > 0, \tag{13}$$

$$\tilde{\gamma}(\theta) \equiv \gamma(\theta) v_z(z(\theta),\theta) = cov_y\Big(u(c(y,\theta)), \frac{1}{u'(c(y,\theta))} \mid z(\theta)\Big) > 0. \tag{14}$$

$E_y(\cdot)$ and $cov_y(\cdot)$ denote the expectation operator and covariance operator.

See Appendix A for the proof. Equation (12) was initially explored in a principal-agent framework by Holmström (1979) and Jewitt (1988). It captures the optimal relative price between effective labor and consumption in a pure moral hazard problem. The left-hand side of equation (12), the marginal substitution rate between z and c (denoted by $MRS_{z,c}$), can be decomposed into an insurance term and an incentive term.

The insurance term is independent of the realized income y. If the government can observe a worker's labor efforts, it can provide full insurance and achieve first-best.

Full insurance implies consumption c is independent of the shock. In other words, $MRS_{z,c}$ is a constant. We use $\tilde{\mu}(\theta)$, the expected $MRS_{z,c}$, to measure the size of the insurance role.

However, the presence of a moral hazard hinders full insurance. Due to hidden action, the government cannot separate a worker's insufficient labor efforts from bad luck. Therefore, the government cannot clearly specify workers' actions when signing contracts. An optimal contract should also provide enough incentive to encourage the desired labor effort. To incentivize workers to take desired actions, the government adjusts work incentives by deviating from the optimal risk-sharing allocations, and assigning compensation based on observed income y. The incentive term in (12) has two components. The likelihood ratio $h(y|z(\theta))$ is the semi-elasticity of the density $g(y|z(\theta))$ with respect to $z(\theta)$ and measures the effectiveness of y as a signal of $z(\theta)$.[9] $\tilde{\gamma}(\theta)$ is the unit price of the effectiveness of the optimal contract. By definition, $\tilde{\gamma}(\theta)$ equals $\gamma(\theta)$, the information rent in the pure moral hazard problem, times the marginal utility of effective labor supply $z(\theta)$. As in equation (14), $\tilde{\gamma}(\theta)$ is positive, since the covariance between $u(c)$ and $\frac{1}{u'(c)}$ is positive. Both the likelihood ratio $h(y|z(\theta))$, and the price of ensuring the desired labor supply, $\tilde{\gamma}(\theta)$, increase the incentive term. The larger the incentive term, the larger the gap between consumption (or utility) under two different shocks.

Since $h_y > 0$, $u'' < 0$, we can infer that $c_y(y, \theta) > 0$. This implies, under the maximum likelihood ratio property, that the optimal contract should provide greater consumption to workers with higher observed income to encourage a type-θ worker to make efforts. This is consistent with Boadway and Sato (2015), which allowed income tax varies only with observed income.

In general, Proposition 1 shows that the government trades off some benefits from risk-sharing (the insurance effect) for increased work incentives (the incentive effect), because a contract designed to mitigate productivity shocks also influences actions and the resulting income distribution. This trade-off has been discussed by Varian (1980) and Tuomala (1984). We differ from them by allowing the magnitude of insurance and incentive term to vary with θ, thus enabling cross-ability redistribution, which will be discussed in 4.2.

Before reaching the adverse selection problem, we need to prepare the properties of the expected utility function $\mathcal{C}(U(\theta), z(\theta))$. Lemma 3 shows how the government

[9] As in Holmström (1979), $h(y \mid z(\theta))$ acts as the benefit-cost ratio for deviation from optimal risk sharing. A large value indicates that the worker is likely to deviate, necessitating higher compensation to ensure compliance, while a smaller value requires less.

should compensate for a worker's expected consumption when the desired utility and effort levels vary.

Lemma 3. *Denote by Ψ_{zz} the double derivative of Ψ on z at the optimal allocation. From the envelope conditions of the moral hazard problem and the optimization problem (7), the marginal influences of reservation utility and effective labor supply on expected consumption are:*

$$\frac{\partial \mathcal{C}(U(\theta), z(\theta))}{\partial U(\theta)} = \mu(\theta) = E_y\left[\frac{1}{u'(c(y,\theta))} \mid z(\theta)\right], \tag{15}$$

$$\frac{\partial \mathcal{C}(U(\theta), z(\theta))}{\partial z(\theta)} = \int_{\underline{y}}^{\bar{y}} c(y,\theta) g_z(y \mid z(\theta)) dy + \gamma(\theta)\Psi_{zz}. \tag{16}$$

See Appendix A for the proof. Equation (15) shows that the marginal cost of providing an additional unit of reservation utility equals the expected inverse marginal utility across income realizations, given labor supply $z(\theta)$. Intuitively, to increase utility by one unit at a specific income y, consumption must increase by $1/u'(c(y,\theta))$. Thus, the expected cost of raising the reservation utility is the probability-weighted average of these consumption increases, which equals $\mu(\theta)$.

Equation (16) captures the marginal effect of effective labor supply on contract cost, comprising two components. First, an increase in $z(\theta)$ alters the income distribution and thus the expected consumption, represented by $\int_{\underline{y}}^{\bar{y}} c(y,\theta) g_z(y \mid z(\theta)) dy$. Second, because $z(\theta)$ is unobservable, inducing a higher effort requires additional incentives, captured by $\gamma(\theta)\Psi_{zz}$. When the worker's second-order condition holds, $\Psi_{zz} = v_{zz} - \int_{\underline{y}}^{\bar{y}} u(c(y,\theta)) g_{zz}(y \mid z(\theta)) dy > 0$ in the second-best allocation. $\Psi_z = v_z - \int_{\underline{y}}^{\bar{y}} u(c(y,\theta)) g_z(y \mid z(\theta)) dy$ can be interpreted as the marginal net utility cost of efforts.[10] $\Psi_{zz} > 0$ indicates the net utility cost must increase as z increases, requiring additional consumption compensation to induce higher efforts. Thus, we call Ψ_{zz} the marginal incentive sensitivity to increasing labor efforts. The magnitude of this compensation in the form of expected consumption also depends on $\gamma(\theta)$, representing the price of ensuring workers choose the suggested $z(\theta)$. The term $\gamma(\theta)\Psi_{zz}$ under hidden action is crucial in shaping the tax schedule discussed in Section 4.2.

[10]By definition, we have $\Psi_z \equiv \frac{\Psi(z,\hat{\theta};\theta)}{\partial z}$ and $\Psi_{zz} \equiv \frac{\Psi^2(z,\hat{\theta};\theta)}{\partial z \partial z}$.

3.2 The Adverse Selection Problem

Based on the properties of the expected consumption functions, we can discuss the optimal allocation of $U(\theta)$ and $z(\theta)$ in a Mirrlees problem. The government aims to maximize net social welfare:

$$\max_{U(\theta),z(\theta)} \int_{\underline{\theta}}^{\bar{\theta}} \left[\frac{\beta(\theta)U(\theta)}{\lambda} + \int_{\underline{y}}^{\bar{y}} yg(y \mid z(\theta))dy - \mathcal{C}(U(\theta), z(\theta)) \right] f(\theta)d\theta. \tag{17}$$

$$\text{s.t.} \quad \text{the truth-telling conditions (IC-AS)}$$

λ denotes the social welfare value of a unit of government revenue.

By solving the adverse selection problem following the procedures in Appendix A, we can solve λ at the optimal allocations and connect it with Lagrange multipliers $\mu(\theta)$ in the pure moral hazard problem as in Lemma 4.

Lemma 4. *At the optimal allocations, we have the following conditions:*

(1) The labor effort $z(\theta)$ satisfies

$$\left[\int_{\underline{y}}^{\bar{y}} yg_z(y \mid z(\theta))dy \right] f(\theta) - \frac{\partial \mathcal{C}(U(\theta), z(\theta))}{\partial z(\theta)} f(\theta) = q(\theta)v_{\theta z}. \tag{18}$$

(2) The co-state variable $q(\theta)$ of the truth-telling conditions satisfies

$$q(\theta) = -\int_{\theta}^{\bar{\theta}} \left[1 - \frac{\beta(\theta)/\mu(\theta)}{\lambda} \right] \mu(\theta)f(\theta)d\theta. \tag{19}$$

(3) The marginal value of public funds λ satisfies

$$\lambda = \left\{ \int_{\underline{\theta}}^{\bar{\theta}} \mu(\theta)f(\theta)d\theta \right\}^{-1} = \left\{ \int_{\underline{\theta}}^{\bar{\theta}} \mathbf{E}_y \left[\frac{1}{u'(c(y,\theta))} | z(\theta) \right] f(\theta)d\theta \right\}^{-1}. \tag{20}$$

Condition (18) captures the government's trade-off in designing desired labor effort $z(\theta)$. The items on the left-hand side correspond to the marginal increase in expected economic output minus the marginal change in expected consumption. The $q(\theta)v_{\theta z}$ on the right-hand side is the monetary value of the marginal decrease in $U'(\theta)$, which measures the temptation of the type-θ worker to misreport his ability if the suggested effort increases at $z(\theta)$.

The expression of the information rent $q(\theta)$ can be obtained by calculating the change in net social welfare induced by increasing the utility of all workers below θ

by $\delta_u(1-F(\theta))$ unit and decreasing the utility of all workers above θ by $\delta_u F(\theta)$ unit through redistribution. Thus, it captures the net social welfare cost of redistributing the utility from types above θ to types below θ. The form of information rent differs from that in the Mirrlees model by substituting the inverse of marginal utility with a marginal change in expected consumption for each worker in measuring marginal effects on expected consumption expenditure.

The marginal value of public funds equals the "cost" of transferring resources from workers through taxation, measured in utility terms. This cost is quantified as the expected consumption improvement needed to raise the utility by one unit in different income states and the distribution of θ.

4 Optimal Nonlinear Income Tax Schedule

This section analyzes an optimal non-linear income tax schedule to implement the previously established allocation. For a type-θ worker with realized income y, the tax payment is $T(y,\theta) = y - c(y,\theta)$. We define two types of marginal tax rate. The first is the marginal tax rate on ex post realized income y. The second is the *ex ante* marginal tax rate based on the effective labor supply, which measures how expected tax payment changes with effort $z(\theta)$.

4.1 Marginal Tax Rates on Ex Post Income

We first present the marginal tax rates on *ex post* income for a type-θ worker, $T_y(y,\theta)$, at the second-best optima. From a worker's budget constraint (2), we have $T_y(y,\theta) = 1 - c_y(y,\theta)$. By differentiating equation (12), which represents the decomposition of $MRS_{z,c}$, with respect to y, we obtain:

$$[\tilde{\mu}(\theta) + \tilde{\gamma}(\theta)h(y|z(\theta))]\, u''(c)c_y(y,\theta) + \tilde{\gamma}(\theta)h_y(y|z(\theta))u'(c) = 0. \tag{21}$$

Equation (21) reveals two key considerations in redistributing income across states. First, the extent to which a smoother distribution of y translates into a smoother consumption utility, given the insurance and incentive payments in (12). This is captured by the first term on the left-hand side of (21). Second, the variation in incentive payments across states, represented by differences in the likelihood ratio $h(y|z(\theta))$. This is captured by $\tilde{\gamma}(\theta)h_y(y|z(\theta))u'(c)$. Reorganizing (21) directly yields the optimal tax rates on *ex post* income in Proposition 2.

Proposition 2. *For a type-θ worker, the marginal tax rate on expost income y is*

$$1 - T_y(y,\theta) = \left(-\frac{u''(c(y,\theta))}{u'(c(y,\theta))}\right)^{-1} \gamma(\theta) h_y(y, z(\theta)) u'(c(y,\theta)). \tag{22}$$

The optimal marginal tax rates on *ex post* income in Proposition 2 share key characteristics with Low and Maldoom (2004) and Boadway and Sato (2015). Specifically, workers' absolute risk aversion (ARA) coefficients influence optimal marginal tax rates, with more risk-averse workers warranting more progressive taxation. Furthermore, marginal payments γh_y correlate negatively with optimal marginal tax rates.

Our results differ from pure moral hazard scenarios or income-only tax variation in that the marginal value of public funds λ does not appear in the expression for optimal T_y in equation (22).[11] This difference arises because our tax schedule is both income-contingent and type-contingent. The government can therefore collect revenue from high-ability workers to support low-ability workers through direct "cross-subsidies" based on ability. If we impose the restriction $T_\theta(y,\theta) = 0$, *ex post* equity concern reemerges in the expression of T_y, as demonstrated by Boadway and Sato (2015).

4.2 *Ex Ante* Marginal Tax Rate

The standard optimal tax literature typically focuses on tax rate on realized income. However, when income is subject to uncertainty, workers make labor supply decisions before knowing their exact income realization. To analyze how taxation affects these *ex ante* decisions, we need a measure that captures the relationship between expected tax burden and effort choice.

For a worker with ability θ who chooses effort level $z(\theta)$, we define the expected tax payment as:

$$\widetilde{T}(z(\theta),\theta) \equiv \mathbf{E}_y[T(y,\theta)|\theta] = \int_{\underline{y}}^{\bar{y}} T(y,\theta) g(y \mid z(\theta)) dy \tag{23}$$

The *ex ante* marginal tax rate with respect to effort is then:

$$\widetilde{T}_z(z(\theta),\theta) \equiv \frac{\partial \widetilde{T}(z(\theta),\theta)}{\partial z(\theta)} = \int_{\underline{y}}^{\bar{y}} T(y,\theta) g_z(y \mid z(\theta)) dy \tag{24}$$

[11] In the setting of Low and Maldoom (2004), optimal marginal income tax can be expressed as $1 - T_y(y) = (ARA)^{-1} \gamma h_y(y,z) \frac{u'(c)}{\lambda}$.

This rate quantifies how a worker's expected tax liability changes with incremental effort, before income uncertainty is resolved. Under an incentive-compatible tax schedule, $\widetilde{T}_z(z(\theta),\theta)$ also represents the ratio of the difference in expected tax burden to the difference in labor effort between adjacent ability types.

When the income shock parameter $\chi = 0$, our model reduces to the deterministic case where z equals realized income, and the *ex ante* marginal tax rate coincides with the conventional marginal tax rate in the Mirrlees model. This connection provides a natural benchmark for interpreting our results.

Denote by $\widetilde{y}(z(\theta))$ the expected income of a worker exerting effort $z(\theta)$ under the optimal tax schedule. The definition is $\widetilde{y}(z(\theta)) \equiv \mathbf{E}_y[y|z(\theta))] = \int_{\underline{y}}^{\bar{y}} yg(y \mid z(\theta))dy$. The derivative of $\widetilde{y}(z(\theta))$ on $z(\theta)$ is then the marginal expected income with respect to effort $z(\theta)$, which satisfies $\widetilde{y}'(z(\theta)) = \int_{\underline{y}}^{\bar{y}} yg_z(y \mid z(\theta))dy$.

The following proposition outlines the derivation of the *ex ante* marginal tax rate.

Proposition 3. *Assuming that* $v(z;\theta) = \frac{1}{1+\frac{1}{\varepsilon}}\left(\frac{z}{\exp(\theta)}\right)^{1+\frac{1}{\varepsilon}}$, *the ex ante marginal tax rate with respect to* $z(\theta)$ *is given by:*

$$\frac{\widetilde{T}_z(z(\theta),\theta)}{\widetilde{y}'(z(\theta)) - \widetilde{T}_z(z(\theta),\theta) + D_2(\theta)} = A \cdot B(\theta) \cdot C(\theta) + D_1(\theta). \tag{25}$$

where

$$\begin{aligned}
A =& \frac{1+\varepsilon}{\varepsilon}, \\
B(\theta) =& E_y\left[u'(c(y,\theta)) \mid z(\theta)\right] \\
& \int_{\theta}^{\bar{\theta}} \left[1 - \frac{1}{\lambda}\frac{\beta(\hat{\theta})}{E_y\left(\frac{1}{u'(c(y,\hat{\theta}))} \mid z(\hat{\theta})\right)}\right] E_y\Big[\frac{1}{u'(c(y,\hat{\theta}))} \mid z(\hat{\theta})\Big]\frac{f(\hat{\theta})}{1-F(\theta)}d\hat{\theta}, \\
C(\theta) =& \frac{1-F(\theta)}{f(\theta)}, \\
D_1(\theta) =& \frac{1}{\widetilde{y}'(z(\theta)) - \widetilde{T}_z(z(\theta),\theta) + D_2(\theta)}\gamma(\theta)\Psi_{zz}, \\
D_2(\theta) =& cov_y\left(\frac{u'(c(y,\theta))}{E_y\left[u'(c(y,\theta)) \mid z(\theta)\right]}, (1 - T_y(y,\theta))\frac{-G_z}{G_y} \mid z(\theta)\right).
\end{aligned}$$

See Appendix A for the proof. Equation (25) in Proposition 3 generalizes the Diamond ABC-formula to include income shocks and the moral hazard problem. The left-hand side of equation (25) shows the marginal efficiency distortion cased by income tax, indicated by the *ex ante* marginal income tax rate ($\widetilde{T}_z(z(\theta),\theta)$ the nominator) normalized by marginal substitution rate (MRS) between labor efforts and consumption (the denominator). Note that without risk, MRS is the ratio between the marginal utility of labor and the marginal utility of consumption, which equals $1 - T'$. In contrast, in our model with risk, since the worker cannot predict future consumption precisely before the risk, the MRS now becomes the ratio between the marginal utility of labor efforts and the average marginal utility of consumption.

From the perspective of mechanism design, the marginal efficiency distortion caused by income tax should equal to the marginal gain in keeping workers' incentive compatible, as indicated by the right-hand side of equation (25). The term $B(\theta)$ comes from the information rent to keep the truth-telling, while $D_1(\theta)$ comes from the information rent to keep the suggested efforts-taking. The terms A and $C(\theta)$ are equivalent to those in the formulation of the Mirrlees model in Diamond (1998), which need not be discussed further. The integral term in $B(\theta)$ is slightly modified by replacing the inverse of marginal utility in the formulation of the pure Mirrlees model with its expectation, in order to measure the marginal effects on the expected consumption expenditure, as implied in the explanations of equation (19).

The terms $D_1(\theta)$ and $D_2(\theta)$ reflect new roles of income taxation resulting from the moral hazard problem. Hence, we focus on explanations of them. $D_1(\theta)$ originates from the pure moral hazard problem. The terms γ and Ψ_{zz} in $D_1(\theta)$ are all positive, therefore making higher *ex ante* marginal tax rates appealing at all ability levels. Note that γ is the price in a pure moral hazard problem paid to induce the desired labor efforts to maintain a fixed expected utility. Ψ_{zz} captures the marginal incentive sensitivity to increasing efforts. From (16), the combination of $\gamma\Psi_{zz}$ corresponds to the increase in expected consumption as a compensation for increased efforts. Thus, $D_1(\theta)$ captures the marginal gain in increasing efforts. The optimal *ex ante* marginal tax rate should therefore balance the marginal tax burden, captured by $\tilde{T}_z(z(\theta), \theta)$, and the marginal gain $D_1(\theta)$, to deal with the moral hazard problem.

The term $D_2(\theta)$, which appears in the denominator on the left-hand side of equation (25), adjusts the marginal efficiency cost of the marginal income tax. Moreover, the denominator can be expressed as

$$\frac{v_z(z(\theta), \theta)}{E_y\left[u'(c(y, \theta))|z(\theta)\right]} = \widetilde{y}'(z(\theta)) - \widetilde{T}_z(z(\theta), \theta) + D_2(\theta) \tag{26}$$

which indicates that the risk-adjusted marginal rate of substitution between labor supply and consumption, negative in value and on the left-hand side of equation

(26), equals the marginal cost or relative price of labor effort when earnings risk is present. Specifically, the right-hand side consists of two components: expected retained earnings, $\widetilde{y}'(z(\theta)) - \widetilde{T}z(z(\theta), \theta)$, and the marginal improvement in *ex post* redistribution for type-θ workers. The latter is measured by the covariance between the marginal utility weight $u'(c(y,\theta))/E_y\left[u'(c(y,\theta))|z(\theta)\right]$ and the marginal increase in consumption of labor effort at a given y, $(1 - T_y(y,\theta))\frac{-Gz}{G_y}$.[12] In contrast, when earnings risk is excluded, equation (26) reduces to the classical equality, which states that the marginal rate of substitution between labor effort and consumption is equal to the relative price of labor.[13]

The components $C(\theta)$, along with $D_1(\theta)$ and $D_2(\theta)$, capture the interaction between adverse selection and moral hazard effects in determining the optimal *ex ante* marginal tax rate. The relative magnitudes of $C(\theta)$ and $D_1(\theta)$ indicate which of these economic forces dominates the optimal policy design. We provide a numerical decomposition of these effects in Section 5.2.

4.3 Marginal tax rates at the top and bottom

From the proposition 2, the marginal tax rates at the top and bottom are

$$T_y(y_i,\theta) = 1 - \left(-\frac{u''(c(y_i,\theta))}{u'(c(y_i,\theta))}\right)^{-1} \cdot \gamma(\theta) h_y(y_i, z(\theta)) u'(c(y_i,\theta)), \quad \text{for } y_i \in \{\underline{y}, \bar{y}\} \tag{27}$$

Equation (27) indicate that the marginal tax rates at the top and bottom are less than one and can be either positive or negative, depending on the positive second term on the right-hand side. Similar results are found in Mirrlees (1974), Varian (1980), and Tuomala (1984).

The *ex ante* marginal tax rates at the top and bottom are

$$\begin{aligned}
&\widetilde{T}_z(z(\theta_i),\theta_i) = \gamma(\theta_i)\Psi_{zz}\left(\theta_i, z; \theta_i\right) \\
=&\mathrm{cov}_y\left(u(c(y,\theta_i)), \frac{1}{u'(c(y,\theta_i))}|z(\theta_i)\right)\frac{\Psi_{zz}\left(\theta_i, z\left(\theta_i\right); \theta_i\right)}{v_z\left(z\left(\theta_i\right), \theta_i\right)}, \quad \theta_i \in \{\underline{\theta}, \overline{\theta}\}
\end{aligned} \tag{28}$$

Thus, the result "no distortion at the top" of Sadka (1976) and Seade (1977) does

[12] The term $1 - T_y(y,\theta) = c_y(y,\theta)$ measures the increase in consumption, $\frac{1}{G_y}$ represents the derivative of the quantile function at the quantile value y conditional on z, and $-G_z > 0$ captures the positive effect of effort on the quantile for a given value of y.

[13] Without earnings risk, the marginal rate of substitution between labor effort and consumption is identical across all type-θ workers. Since $\chi = 0$ implies $y(\theta) = z(\theta)$ from the equation $y(\theta) = e^{\chi}z(\theta)$, a worker's marginal income tax rate equals their *ex post* marginal income tax rate.

not apply whenever from the perspective of *ex ante* or ex post marginal tax rates.

Equation (28) show that the *ex ante* marginal tax rates at the top and bottom are shaped by effects due to moral hazard and primarily by $\gamma(\bar{\theta})$ and $\gamma(\underline{\theta})$. Inherited from the standard Mirrleeian taxation with only adverse selection, the information rent over the truth-telling constraint reaches zero at the end of ability's distribution. In other words, workers with the highest ability and workers with the lowest ability truthfully report their ability types. However, the moral hazard problem still deviates labor supply from efficient levels. Therefore, we still require distortion at the top and bottom. The right-hand side of (28) is positive. This means positive *ex ante* marginal tax rates might be suitable at the end of ability's distribution, as workers tend to over-supply labor efforts (like precautionary savings) in front of income risk. We numerically verify this point in the next section.

5 Numerical Model

To examine the interaction between moral hazard and adverse selection and to clarify the shape of the optimal tax schedule, we solve a numerical version of our theoretical model. This section has three parts: the first calibrates the model parameters; the second presents numerical results and analyzes the underlying mechanisms; the third performs a robustness check.

5.1 Calibration

We calibrate parameters for preferences, ability distributions, and productivity shock distributions, and social welfare weights. Following the optimal taxation literature, we use an additively separable utility function:

$$u(c) = \frac{c^{1-\rho}}{1-\rho}; \quad v\left(z, \theta\right) = \frac{1}{1+\frac{1}{\varepsilon}}\left(\frac{z}{\exp\left(\theta\right)}\right)^{1+\frac{1}{\varepsilon}}, \tag{29}$$

where ρ is the coefficient of relative risk aversion and ε is the Frisch labor supply elasticity. We set $\rho = 1$, yielding the commonly used log utility function in consumption. The value of ε is set to 0.50, consistent with empirical estimates ranging from 0.10 to 0.50 for men and 1.70 for women (MaCurdy, 1981; Greenwood et al., 1988; Keane, 2011).

For ability and productivity shock distributions, we follow a three-step calibration process. First, we adopt the HSV tax schedule from Feldstein (1969), which approximates the actual tax system as $T(y) = y - \phi y^{1-p}$, where p measures tax progressivity and ϕ is a level parameter. We set $p = 0.181$ and $\phi = 0.853$ to replicate the empirical patterns observed in the Panel Study of Income Dynamics (PSID) for working-age households (2000–2006), following Heathcote et al. (2017) and Heathcote and Tsujiyama (2021). As in Heathcote et al. (2017), p is estimated to match the linear relationship between $\log(y)$ and $\log(y - T(y))$, and ϕ is chosen to match the ratio of government purchases to GDP as 0.1816.

Second, we assume that a worker's ability, θ, follows an exponentially modified Gaussian (EMG) distribution, i.e., $\theta \sim EMG(\mu_\theta, \sigma_\theta^2, \lambda_\theta)$, which is the sum of a normal distribution $\theta_N \sim \mathbf{N}(\mu_\theta, \sigma_\theta^2)$ and an exponential distribution $\theta_E \sim Exp(\lambda_\theta)$. We also introduce an idiosyncratic shock to labor productivity, $\chi \sim \mathbf{N}(-\frac{\sigma_\chi^2}{2}, \sigma_\chi^2)$, and the effective wage is $w = e^{\chi+\theta}$. Consequently, log earnings follow an EMG distribution: $\log y(\theta, \chi) \sim EMG(\mu_\theta - \frac{\sigma_\chi^2}{2} + \log(l(\theta)), \sigma_\theta^2 + \sigma_\chi^2, \lambda_\theta)$, where $l(\theta)$ is constant given our utility specification. The EMG distribution yields a fatter right tail compared to a log-normal wage distribution, with a negative association to λ_θ. [14]

Third, using 2007 Survey of Consumer Finance (SCF) data, we find that the empirical log income distribution is well-approximated by an EMG with normal variance $\sigma_\chi^2 + \sigma_\theta^2 = 0.412$ and tail parameter $\lambda_\theta = 2.20$, as in Heathcote and Tsujiyama (2021).[15] From this, we estimate $\sigma_\chi^2 = 0.21$ and $\sigma_\theta^2 = 0.21$, based on the fact that luck accounts for one-third of the observed earnings volatility. We set $\mu_\theta = -0.71$ to normalize the wage mean to 1, where log wage follows an $EMG(\mu_\theta - \frac{\sigma_\chi^2}{2}, \sigma_\theta^2 + \sigma_\chi^2, \lambda_\theta)$ distribution.

Finally, we set the redistribution parameter $\alpha = -0.37$ by assuming the observed U.S. tax schedule is optimal. Table 1 summarizes our benchmark parameterization.

5.2 Numerical Results

This subsection presents numerical results of the theoretical model, primarily focusing on the shape and level of optimal taxation and the underlying mechanisms. We first display simulated optimal marginal tax rates on *ex post* income and the *ex ante* marginal tax rates, then we discuss how the roles of redistribution, insurance and incentive are performed under the optimal tax schedule. We also compare the welfare improvements of adopting the optimal schedule.

[14] The density function of y conditional on $z(\theta)$ is $g(y \mid z(\theta)) = \frac{1}{y\sqrt{2\pi}\sigma_\chi} exp\left\{-\frac{[ln(y)-(-\frac{1}{2}\sigma_\chi^2+lnz(\theta))]^2}{2\sigma_\chi^2}\right\}$.

[15] This is derived through maximum likelihood estimation using the R package limma.

Table 1. Benchmark parameterization

Description	Parameters	Value
Preferences		
Relative risk aversion	ρ	1.00
Frisch labor supply elasticity	ε	0.50
Abilities and Shocks		
Tail parameter	λ_θ	2.20
Mean of ability	μ_θ	-0.71
Dispersion of ability	σ_θ^2	0.21
Dispersion of disutility	σ_χ^2	0.21
Social welfare		
Social weight	α	-0.37

5.2.1 Marginal Tax Rates on *Ex Post* Income

Panel A of Figure 1 shows how optimal marginal tax rates on *ex post* income given by Proposition 2 vary with income y and ability θ. Two key patterns emerge:

First, workers face progressive income taxation within ability groups. For any given ability level θ, the marginal tax rate increases with realized income y, starting negative at the lowest income realization $\underline{y}(\theta)$ and becoming positive but below unity at the highest realization $\bar{y}(\theta)$ (Panel B). This progressivity follows from the relationship between prudence and risk aversion in our preference specification,. The optimal progressivity satisfies:

$$T_{yy}(y,\theta) = -\left[1 - T_y(y,\theta)\right]\left\{\left[P(y,\tilde{\theta}) - 2A(y,\tilde{\theta})\right]\left[1 - T_y(y,\theta)\right] + \frac{h_{yy}(y,\theta)}{h_y(y,\theta)}\right\} \quad (30)$$

where $P(y,\theta) = -\frac{u'''(c(y,\theta))}{u''(c(y,\theta))}$ is prudence and $A(y,\theta) = -\frac{u''(c(y,\theta))}{u'(c(y,\theta))}$ is relative risk aversion. With log utility and Pareto log-normal income distribution conditional on θ, we have $T_{yy}(y,\theta) > 0$ when $T_y(y,\theta) < 1$ and $\rho \geq 0$.

Notably, marginal tax rates can be negative at sufficiently low incomes, indicating optimal subsidization of labor supply for workers experiencing significant negative productivity shocks. This contrasts with traditional optimal tax models suggesting non-negative marginal rates everywhere (Mirrlees, 1971; Seade, 1977), but aligns with Choné and Laroque (2010), which demonstrates that negative marginal tax rates can be optimal in the presence of multiple dimensions of heterogeneity. Our negative marginal tax result cannot be attributed to income shocks, as Heathcote and Tsu-

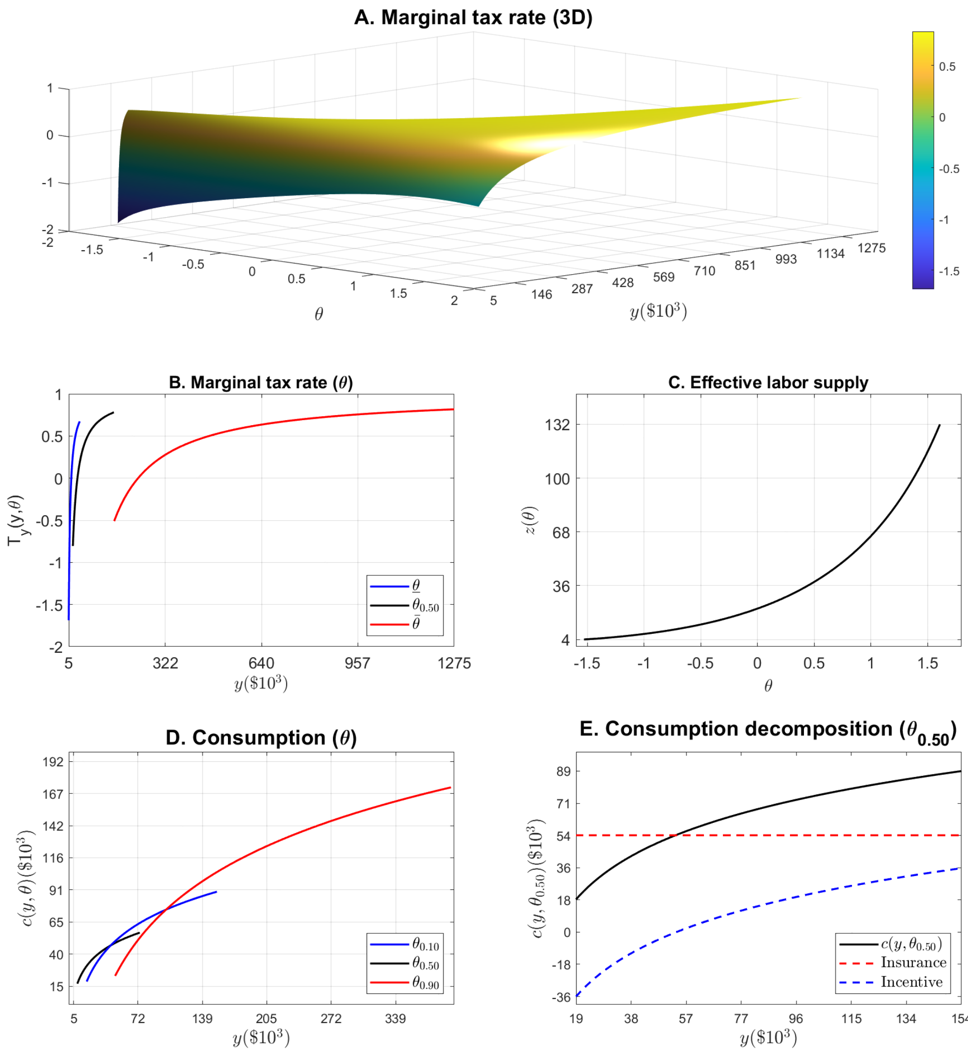

Figure 1. Optimal tax schedule and allocations. Panel A plots a three-dimensional surface illustrating how marginal tax rates vary with workers' types θ and realized income y onto a two-dimensional plane. The horizontal axis denotes θ, while the vertical axis indicates y. The color at each point (y, θ) represents the value of marginal tax rate $T_y(y, \theta)$, with a color bar on the right serving as a reference for the corresponding values of each color. Panel B displays the relationship between marginal tax rates and realized income for specific types of workers. The vertical axis shows marginal tax rates, and the horizontal axis depicts realized income. The blue, black, and dark lines illustrate the relationships for workers with the lowest ability $\underline{\theta}$, medium ability $0.5(\underline{\theta} + \bar{\theta})$, and highest ability $\bar{\theta}$, respectively.

jiyama (2021) find marginal income tax rate for low-income workers should be high when private insurance market is absent. The negative marginal tax result originates from an ability-contingent tax schedule. It allows distributional objectives to be achieved through *ex ante* marginal rates, while providing proper insurance through progressive marginal tax on *ex post* income. The high marginal tax rates for the top earners are consistent with Kindermann and Krueger (2022), and are driven by the social insurance rather than due to purely redistributional motives.

Second, optimal taxation is regressive across ability levels at fixed income. For any given income y, higher-ability workers face lower marginal tax rates than lower-ability workers (Panel B), meaning $T_y(y, \theta_1) > T_y(y, \theta_2)$ when $\theta_1 < \theta_2$. Under our preference specification, we can simplify to $1 - T_y(y, \theta) = \gamma(\theta) h_y(y, z(\theta))$, where $h_y(y, z(\theta)) = \frac{1}{yz(\theta)}$ given our distributional assumptions. Since $z'(\theta) > 0$ (Panel C), h_y decreases with θ at fixed y. We numerically find γ increases with θ (Panel B of Figure 2), suggesting the price of inducing labor supply rises with ability. The decrease of $T_y(y, \theta)$ with θ indicates the $\gamma(\theta)$ effect dominates.

Panels C to E in Figure 1 show the resulting allocations under optimal taxation: Specifically, Panel C verifies that effective labor supply increases with workers' ability, as a direct consequence of Assumption 1, which ensures that G_{zz} satisfies the convexity condition of the distribution function. Panel D depicts the relationship between realized income and consumption for workers at the 25th, 50th, and 75th percentiles of ability. We find that, for low realized income, high-ability workers consume less than low-ability workers. However, for high realized income, high-ability workers consume more. This pattern incentivizes effort from high-ability workers through lower consumption at low income realizations, while providing insurance to low-ability workers through higher consumption. Finally, Panel E decomposes consumption for median-ability workers into insurance and incentive effects following the logic in (12). The incentive effect reduces consumption at low income while increasing it at high income levels. For workers with median ability, the insurance effect dominates the incentive effect across all income levels.

5.2.2 The *Ex Ante* Marginal Tax Rates

Figure 2 displays the *ex ante* marginal tax rate as a function of worker type θ. Panel A shows that the *ex ante* marginal tax rate (EMTR, red line) follows an inverse-U shape with positive rates at both ends of the ability distribution. We decompose this rate into adverse selection (AS, blue line) and moral hazard (MH, dark line)

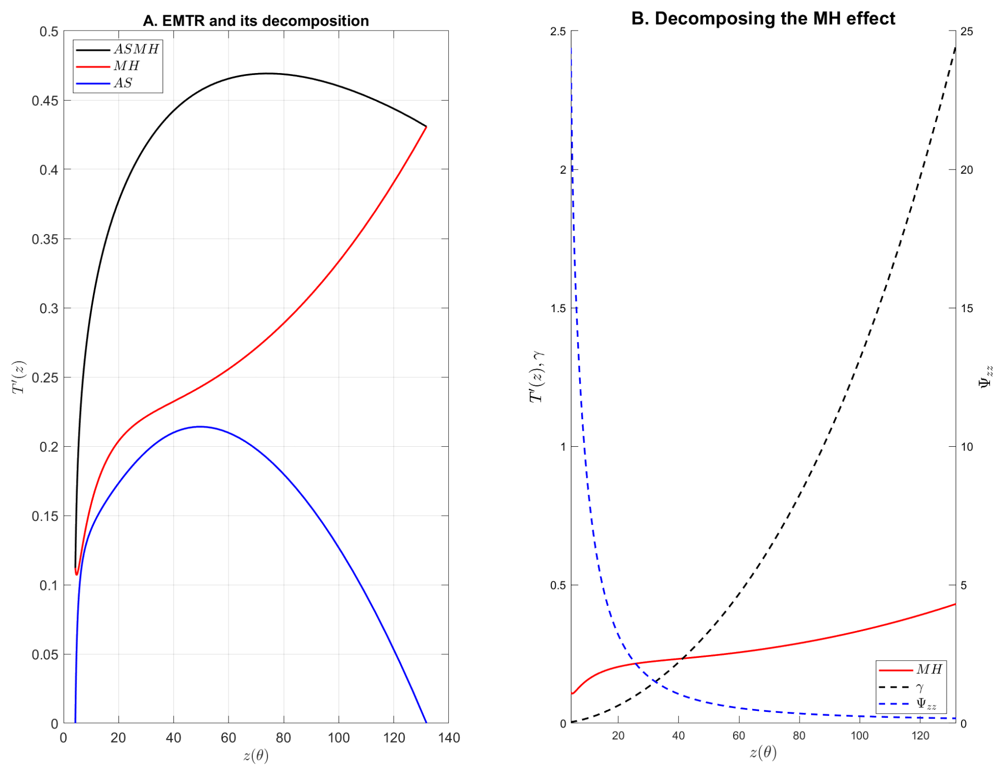

Figure 2. *Ex ante* marginal tax rate and its decompositions. The ASMH line represents the optimal *ex ante* marginal tax rate. The AS and MH lines illustrate the relative contributions of adverse selection and moral hazard concerns, with magnitudes proportional to $AB(\theta)C(\theta)$ and $D_1(\theta)$, respectively. The sum of AS and MH lines yields the ASMH line. Panel B employs a dual y-axis system. The left y-axis (labeled $T'(z), \gamma$) measures values from 0 to 2.5, while the right y-axis (labeled Ψ_{zz} ranges from 0 to 25.

effects based on Proposition 3. The adverse selection effect generates an inverse U-shaped *ex ante* marginal tax schedule with zero rates at both extremes of the ability distribution, while the moral hazard effect produces a positive *ex ante* marginal tax rate that increases with worker ability. Since MH increases with z, the EMTR is substantially more progressive than AS.

The inverse-U shape of the AS line, arising from adverse selection, aligns with optimal income tax schedules under partial private insurance and income risk as in Heathcote and Tsujiyama (2021), but contrasts with previous studies deriving U-shaped taxes (Saez, 2001; Mankiw et al., 2009; Golosov et al., 2016).[16] While

[16] According to Heathcote and Tsujiyama (2021), the U-shaped tax schedule in Saez (2001) results

our calibration matches realistic government spending ratios and ability distributions similar to Heathcote and Tsujiyama (2021), self-insurance mechanisms differ. In our model, labor supply is the only self-insurance channel, complementing their findings on optimal income tax schedule shapes.

To understand why MH is positive and increases with effective labor supply, Panel B of Figure 2 decomposes the moral hazard effect following $MH = \gamma\Psi_{zz}$, which forms the core of D_1 in Proposition 3. The price parameter in a pure moral hazard problem, $\gamma(\theta)$, is always positive by equation (14). The marginal incentive sensitivity, Ψ_{zz}, is also positive. Consequently, the MH term is consistently positive, contributing to higher *ex ante* marginal tax rates. Panel B shows that Ψ_{zz} decreases with effort, indicating a declining gap between marginal benefit and marginal cost of effort between neighboring effort levels as workers' ability increases. Meanwhile, γ increases with effort, reflecting higher costs of converting consumption into marginal utility changes for higher-ability workers. Overall, the positive correlation between γ and effort dominates the pattern of the MH effect.

Our numerical model produces positive *ex ante* marginal tax rates at both the bottom and top of the distribution, verifying our analysis in equation (28): although adverse selection from unobservable abilities does not distort labor supply at distribution extremes, income shocks induce these workers to supply excess labor. Kapička and Neira (2019) similarly find that "no distortion at the top" fails when adverse selection and moral hazard coexist, though through a different mechanism: positive marginal taxes on labor income increase workers' learning effort incentives when decreased labor supply reduces the marginal disutility of learning effort.

5.2.3 Mechanism Analysis

The optimal tax schedule serves multiple functions: redistribution, insurance, and incentive provision. In this subsection, we disentangle these mechanisms to understand how the optimal tax achieves its objectives.

To separate the redistribution and insurance roles in the progressive $T_y(y, \theta)$, we compare our baseline model with two variants. The first variant assumes the government has complete information about workers' abilities and prohibits redistribution across workers. Despite having multiple worker types, this variant only addresses

from high government spending ratios, while the U-shaped schedules in Mankiw et al. (2009) and Golosov et al. (2016) stem from assumptions of low productivity for most households and zero productivity for 5% of households, respectively.

moral hazard as in Varian (1980), Tuomala (1984), and Low and Maldoom (2004). We refer to this as the LM model.

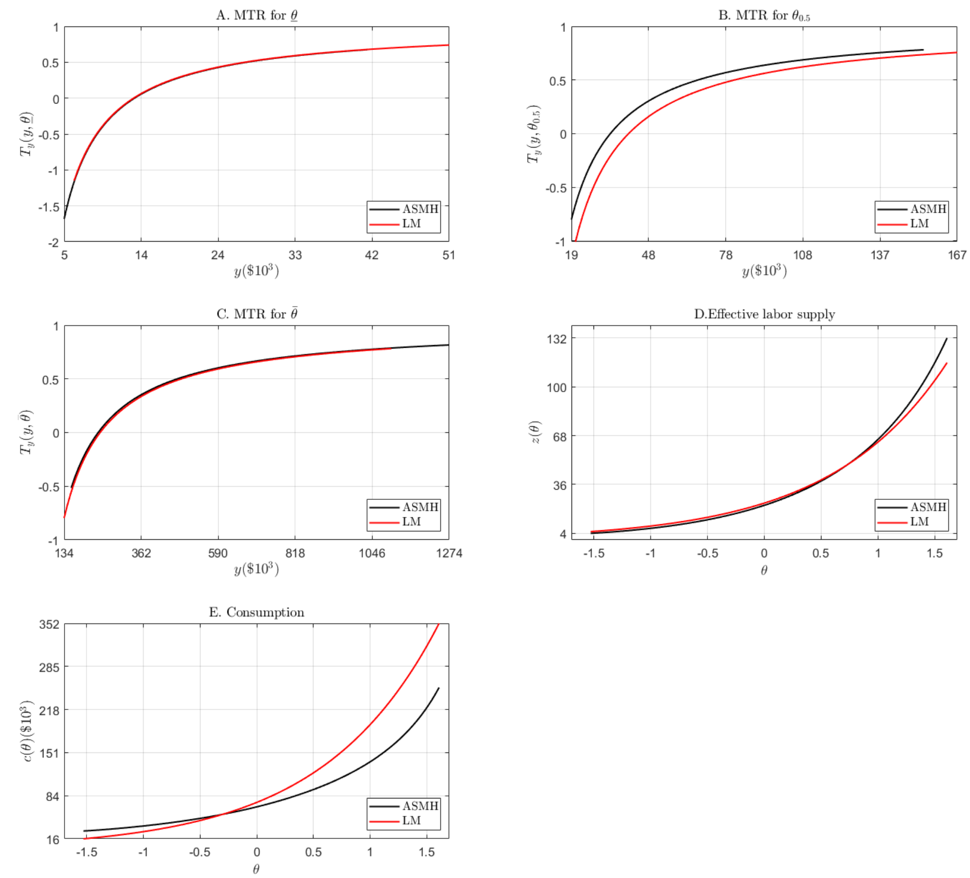

Figure 3. Baseline results vs. LM model. In the LM model, the government has complete information about workers' abilities. Redistribution across workers is prohibited. The distributions of risk and ability are identical in both models.

Figure 3 compares the marginal tax rates on *ex post* income between the benchmark model (dark line) and the LM model (blue line). Panels A, B, and C show results for low-skilled, medium-skilled, and high-skilled workers, respectively. In panels A and C, marginal tax rates at the extremes of the ability distribution are identical across both models. This confirms our theoretical analysis in section 4.3: at the ability distribution endpoints, workers have no incentive to misreport their types. Therefore, optimal taxation for the lowest and highest ability workers only needs to address moral hazard, as in the LM model.

Panel B demonstrates that marginal tax rates for medium-skilled workers are higher in the benchmark model at all realized income levels. Since the LM model prohibits redistribution across workers, this tax rate gap indicates that the government achieves redistribution by imposing higher burdens on the middle and upper classes to benefit the poor. The gains in redistribution are displayed through changes in expected consumption in Panel E. Low-skilled workers' expected consumption in the benchmark model significantly exceeds that in the LM model, while high-skilled workers' consumption is lower in the benchmark model. Notably, the effective labor supply of low-skilled workers remains almost identical between models. For high-skilled workers, z is higher than in the LM model, suggesting redistribution can be achieved with minimal efficiency costs.

It is important to note that the adverse selection effect in our benchmark model, as decomposed in Figure 2, does not exclude the influence of unobservable shocks.[17] To isolate the role of risk in the adverse selection effect, we examine a second variant that eliminates earnings risk, aligning with the standard Mirrleesian model of pure adverse selection. We refer to this as the Mirrlees model.

Figure 4 compares marginal tax rates between the benchmark model (dark solid line) and the Mirrlees model (red dashed line). The blue line represents the adverse selection (AS) effect decomposed from the *ex ante* marginal tax rate in our baseline model. In panel A, both the Mirrlees model results and AS effect display inverse-U shapes, though tax rates in the Mirrlees model are approximately twice as high as the AS effect.

This gap between the AS effect and Mirrlees results indicates that income risk constrains the marginal benefit of redistribution across skill types. When income risk is introduced, optimal taxation provides social insurance at the cost of distorting labor supply. As shown in panel B, effective labor supply is higher for all workers under the Mirrlees model, leading to greater expected consumption for each worker. Since marginal utility decreases with consumption, the marginal benefit of transferring income from workers with abilities above θ to the government is consistently larger in the Mirrlees model. Intuitively, when income shocks are introduced, the social insurance function is fulfilled at the cost of downward labor supply distortions, thus reducing the marginal benefit of transferring income from workers to the government.

[17]This occurs because our optimal tax schedule is obtained by first solving the moral hazard problem and then addressing adverse selection, allowing these mechanisms to interact.

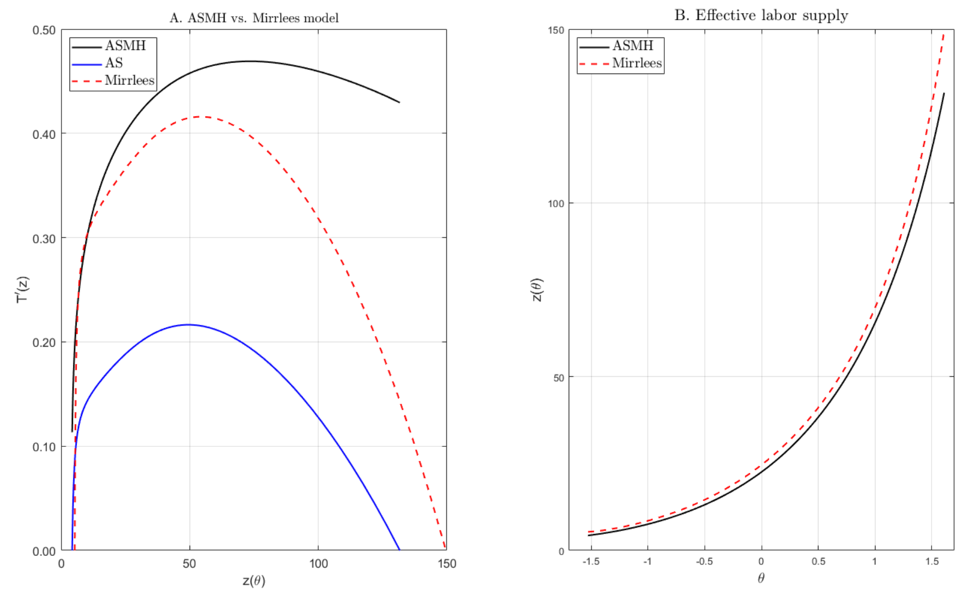

Figure 4. The baseline results vs. Mirrlees model. The distributions of ability are the same in both models, but there is no labor productivity shock in the Mirrlees model.

5.2.4 Welfare Analysis

Allowing the tax schedule to be both income-variant and type-variant is crucial in our model. After characterizing the optimal schedule, we examine the welfare improvements when transitioning from a pure income-variant tax schedule (e.g., HSV form) to our optimal schedule.

This section compares allocations under our baseline tax schedule and the HSV tax schedule across worker ability percentiles. The comparison in Table 2 highlights the redistributional effects, incentive impacts, and welfare implications of the baseline schedule, revealing three key findings.

First, labor income increases more sharply under a type-variant income tax. Under the baseline schedule, labor supply $l^b(\theta)$ increases with workers' innate abilities θ. In contrast, labor supply under HSV, $l^c(\theta)$, remains constant due to the additively separable utility function and logarithmic consumption utility. Consequently, average income $y^b(\theta)$ under the baseline schedule rises with θ because both work effort and wages increase with ability, while under HSV, average income $y^h(\theta)$ increases solely due to the rising wage rate $w(\theta)$.

Second, the baseline tax schedule is more redistributive at the lower end of the ability distribution and imposes fewer disincentives on top earners. The average marginal tax rate on *ex post* income under the baseline schedule, $E_y(T^b_y(y,\theta)|\theta)$, is lower than that under HSV, $E_y(T^h_y(y)|\theta)$, for workers below the 25th percentile and above the 75th percentile. However, for workers near the median, $E_y(T^b_y(y,\theta)|\theta)$ exceeds $E_y(T^h_y(y)|\theta)$, indicating higher average marginal tax rates around the middle of the ability distribution. Additionally, average consumption for low-ability workers (below the 50th percentile) is higher under the baseline schedule than under HSV, with this relationship reversing for workers at or above the 75th percentile.

Finally, the baseline tax schedule achieves higher social welfare. Measured in consumption terms, social welfare under the baseline model is $48,593.85$, compared to $48,275.99$ under the HSV schedule, representing a welfare gain of approximately 0.658% when transitioning from HSV to the baseline tax schedule.

Taking the above together, an additional degree of freedom by allowing the tax schedule to be type-variant helps to redistribute at a lower cost of efficiency.

Table 2. Baseline versus HSV Taxation

Percentile	1	25	50	75	99	mean
Panel A: Baseline						
$l^b(\theta)$	2762.00	2973.21	3074.77	3178.51	3682.99	3075.00
$y^b(\theta)$	15747.92	39909.09	58499.39	94269.65	484880.90	77326.00
$c^b(\theta)$	28719.37	43632.85	53725.15	71506.03	250616.03	61778.83
$E_y(T^b_y(y,\theta)\|\theta)$	-2.64%	24.79%	31.01%	38.01%	42.32%	30.77%
Welfare (c^b)	24406.82	35395.27	42471.45	54916.45	159911.44	48593.85
Panel B: HSV						
$l^h(\theta)$	3438.82	3438.82	3438.82	3438.82	3438.82	3438.82
$y^h(\theta)$	19606.90	46158.83	65425.57	101989.75	452734.83	83796.08
$c^h(\theta)$	21419.11	43186.02	57466.63	82665.88	280191.31	68350.49
$E_y(T^h_y(y)\|\theta)$	7.58%	20.84%	25.69%	31.43%	47.64%	26.10%
Welfare (c^h)	15367.34	30946.03	41357.13	59278.38	200787.02	48275.99

5.3 Robustness Analysis

This subsection assesses whether our results remain consistent under alternative parameter values. We first explore the impact of the social preference parameter, which reflects the government's redistribution objective, a crucial element in designing optimal taxation under adverse selection, as in the Mirrlees model. Next, we investigate how variations in relative risk aversion, earnings risk, and Frisch labor supply elasticity—factors that primarily affect workers' responses—shape the results. Finally, we examine how the tax schedule's shape changes with different levels of fiscal pressure.

Our analysis focuses on *ex ante* marginal tax rates for two reasons. First, they provide a direct measure of how alternative parameters influence workers' behavior. Second, while *ex post* marginal tax rates depend on both ability and realized income, their presentation requires a three-dimensional plot, which complicates interpretation. Moreover, the robustness of the results is clearly demonstrated by the *ex ante* marginal tax rate plots.

5.3.1 Social Preference

This section explores the *ex ante* marginal tax rate under two different social preference values, i.e., $\alpha \in \{-1, 0\}$, besides the empirically motivated value of -0.37. While $\alpha = -1$ corresponds to the laissez-faire case, $\alpha = 0$ is the utilitarian case.

Figure 5 presents the sensitivity of optimal tax schedules to social preferences.

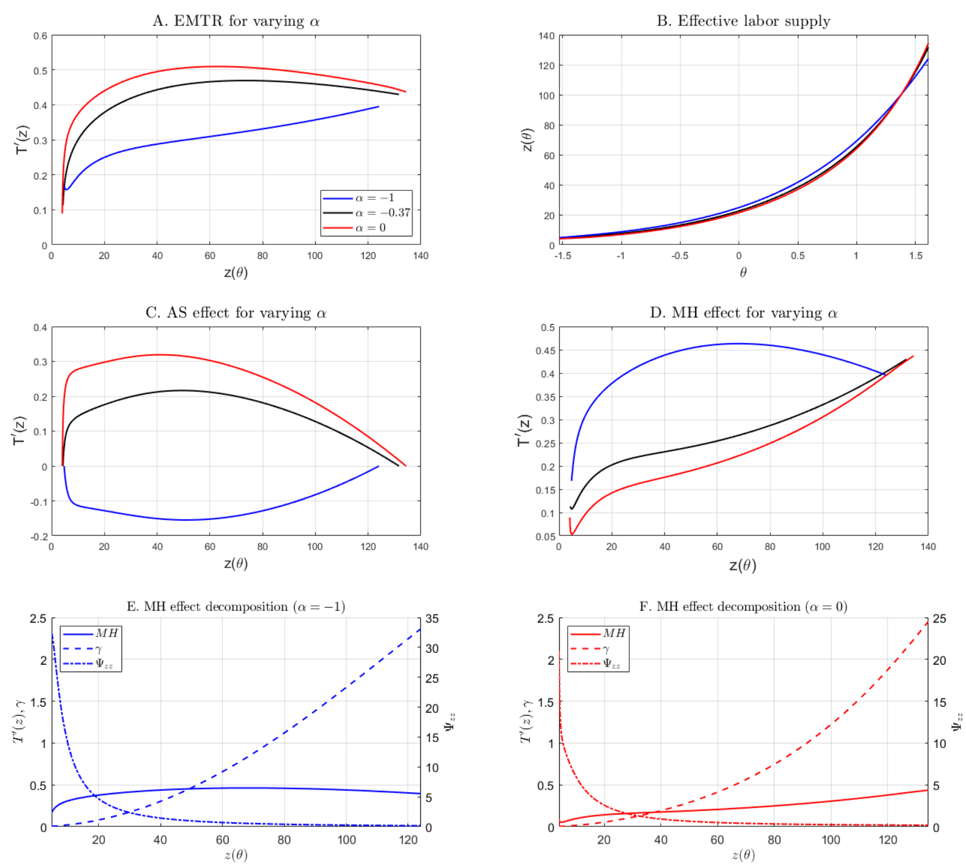

Figure 5. EMTR and its decompositions under alternative social preference. In Panels E and F, the left y-axis (labeled $T'(z), \gamma$) measures values from 0 to 2.5. The right y-axis (labeled Ψ_{zz}) ranges from 0 to 40 in panel E and 0 to 25 in panel F.

Panel A shows that *ex ante* marginal tax rates increase with stronger redistributive preferences for any given worker ability. Under baseline and utilitarian social preferences, these rates are positive with an inverse-U shape, but become negative and U-shaped in the laissez-faire case.

To understand these differences, Panel C isolates the adverse selection (AS) component of *ex ante* marginal tax rates. In the laissez-faire case, this component exhibits a U-shape, contrary to the inverse-U shape observed with larger values of α. This pattern emerges because the laissez-faire government, deriving limited distributional gains from redistributing to lower-income workers, prioritizes efficiency instead. Higher efficiency is achieved through generally declining marginal tax rates in low and middle income groups, resulting in lower average marginal tax rates, consistent with Heathcote and Tsujiyama (2021).

Panel D shows that the moral hazard (MH) component of *ex ante* marginal tax rates is negatively correlated with the social preference parameter α for most workers, except those with the highest abilities. The combined evidence from Panels C and D indicates that the adverse selection effect drives the distinctive U-shaped tax schedule in the laissez-faire case shown in Panel A.

The inverse relationship between the MH effect and α can be explained by Panels E and F, which decompose the moral hazard effect. The information rent $\gamma(\theta)$ in the pure moral hazard problem (at given effort levels) is lower when $\alpha = 0$ than when $\alpha = -1$. This occurs because under stronger redistributive preferences, low to the middle ability workers supply less labor effort, as demonstrated in Panel B. Consequently, the government can use lower information rents to induce desired effort levels. Intuitively, the pronounced adverse selection effect under stronger redistributive preferences already constrains precautionary labor supply under income risk, thereby reducing the magnitude of the moral hazard effect.

5.3.2 Relative Risk Aversion

We examine alternative values of relative risk aversion ρ (3.0 and 5.0), alongside the log utility function in our baseline model, following the precautionary saving literature such as Aiyagari (1994). Panel A of Figure 6 reveals that higher relative risk aversion leads to increased *ex ante* marginal tax rates for low-ability workers but decreased rates for high-ability workers. This pattern primarily stems from the adverse selection effect. Panel C shows that this effect intensifies with higher values of ρ. Under greater risk aversion, the marginal utility of consumption diminishes more rapidly for high-

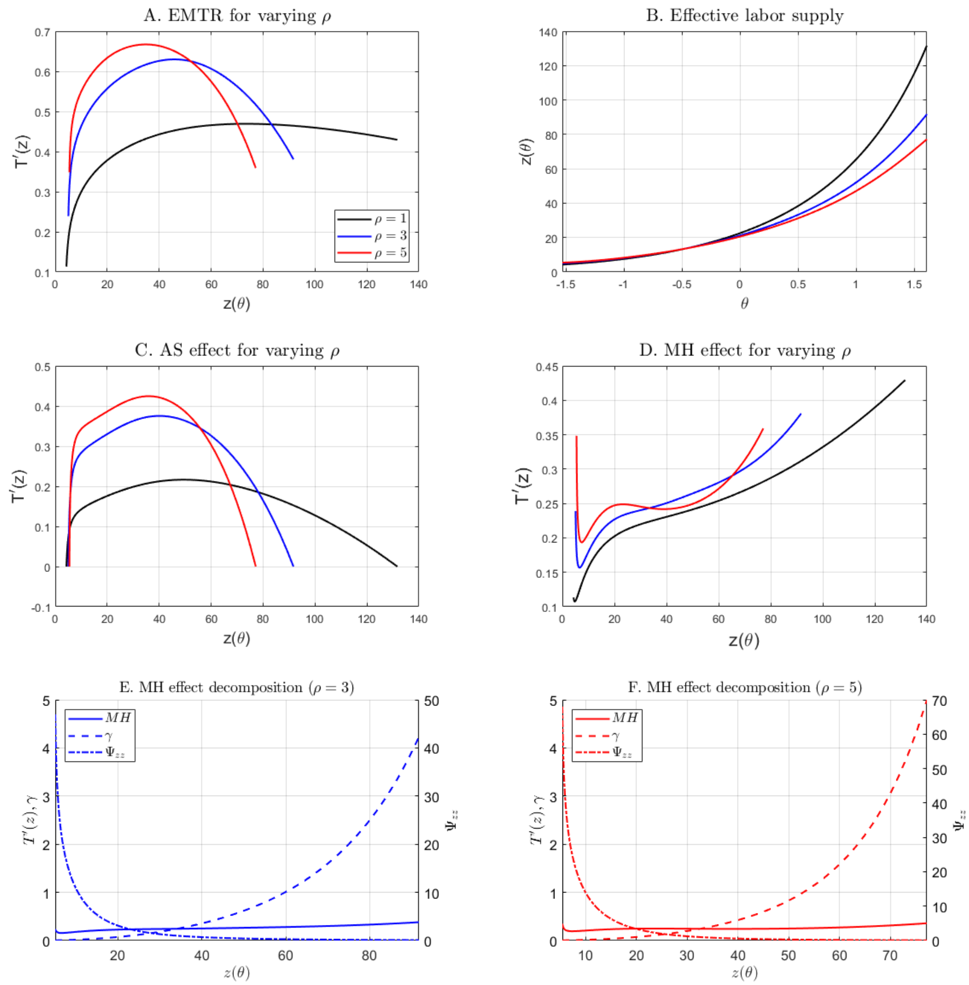

Figure 6. EMTR and its decompositions under alternative risk preference. In Panels E and F, the left y-axis (labeled $T'(z), \gamma$) measures values from 0 to 2.5. The right y-axis (labeled Ψ_{zz}) ranges from 0 to 50 in Panel E and 0 to 80 in Panel F.

income workers, enhancing social welfare gains from redistribution. Thus, for any given worker ability θ, the *ex ante* marginal tax rate is higher under increased risk aversion compared to the baseline case, consistent with Heathcote and Tsujiyama (2021).

Our novel finding concerns how higher risk aversion (ρ) transforms the moral hazard effect's profile. Panel D demonstrates that this effect decreases for low-ability workers as ρ increases. As shown in Panels E and F, this pattern results from a higher initial value and steeper decline in Ψ_{zz} (the marginal incentive sensitivity to increasing z), despite Panel B showing minimal changes in low-ability workers' labor effort.

Intuitively, greater risk aversion substantially reduces low-ability workers' expected utility given future consumption distributions, necessitating larger compensation to induce desired expected utility levels.

Conversely, high-ability workers increase their labor effort under second-best allocations as ρ rises, requiring higher information rent γ and amplifying the moral hazard effect. Panel B clearly illustrates this increased labor effort among high-ability workers, which aligns with the substantial drops in *ex ante* marginal tax rates observed for these workers in Panel A.

5.3.3 Earnings Risk

This section discusses the results under two alternative values of earnings risk, which are $\sigma_\chi^2 = 0.144$, a 4.87% decrease in variance of wage rate for all workers relative to the baseline model; and $\sigma_\chi^2 = 0.268$, a 6.18% increase in variance of wage rate for all workers. [18]

Panel A in Figure 7 demonstrates that *ex ante* marginal tax rates increase with earnings risk. As shown in Panels C and D, this relationship is primarily driven by changes in the moral hazard (MH) effect.

The adverse selection (AS) effect responds differently to earnings risk compared to the MH effect. Panel C illustrates that the *ex ante* marginal tax rate attributable to the adverse selection effect decreases with earnings risk across all ability levels. This aligns with our comparison between the AS effect and the Mirrlees model in Figure 4, which shows that introducing income risk reduces the marginal benefit of redistribution. Greater volatility leads to larger decreases in expected consumption for all workers, further diminishing the marginal benefit of redistribution.

Regarding the MH effect, the decompositions in Panels E and F explain its positive relationship with earnings risk. The price γ of inducing workers to exert desired labor efforts increases because higher earnings risk makes realized income less informative about actual labor effort.

Additionally, Panel B shows that effective labor supply decreases with earnings risk, though these changes are modest. Intuitively, as earnings risk increases, workers tend to increase their labor supply *ex ante* as a buffer against heightened uncertainty. This allows the government to impose higher marginal tax rates without substantially distorting labor supply. Consequently, earnings risk has only a limited impact on

[18] As $w = e^{\chi+\theta}$, $\chi \sim \mathbf{N}(-\frac{\sigma_\chi^2}{2}, \sigma_\chi^2)$, the variance of wage rate is $(e^{\sigma_\chi^2} - 1) \cdot e^{2\theta}$.

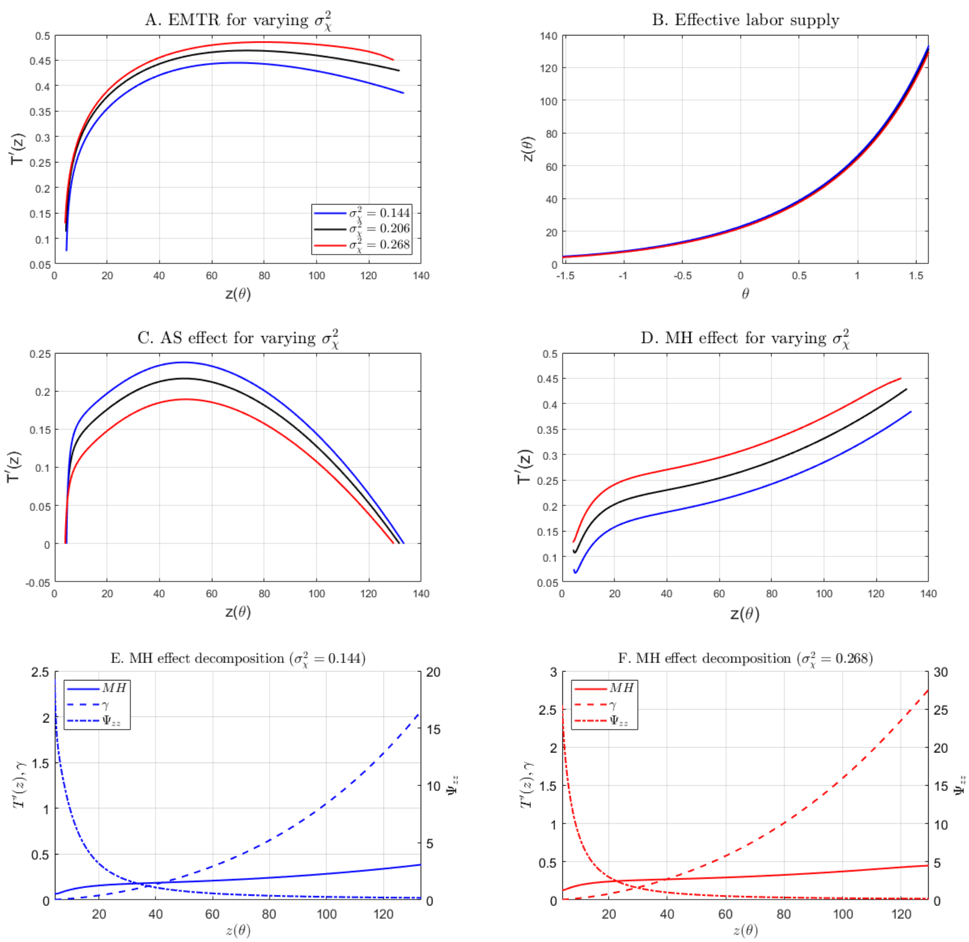

Figure 7. EMTR and its decompositions under alternative earnings risk In Panels E and F, the left y-axis (labeled $T'(z), \gamma$) ranges from 0 to 2.5 in Panels E and 0 to 3 in Panel F. The right y-axis (labeled Ψ_{zz}) ranges from 0 to 50 in Panel E and 0 to 80 in Panel F.

effective labor supply.

5.3.4 Fiscal Pressure

Fiscal pressure significantly influences optimal marginal income tax rates. Heathcote and Tsujiyama (2021) demonstrates that high fiscal pressure causes planners to impose disproportionately high marginal tax rates at relatively low productivity levels, transforming the optimal tax schedule from an inverse U-shape to a U-shape. To see whether this effect also exits when faced with moral hazard problem, Panel A of Figure 8 compares optimal *ex ante* marginal tax schedules under varying government expenditure-to-output ratios (g), ranging from 0 to our baseline value of 0.20 and a high value of 0.50.

Our findings diverge from Heathcote and Tsujiyama (2021). While fiscal pressure increases *ex ante* marginal tax rates across all ability levels, it minimally affects the shape of the optimal tax schedule in our model. Intuitively, higher fiscal pressure reduces lump-sum transfers, diminishing the deterministic component of disposable income for low-ability workers. This substantially raises the marginal benefit of insurance for these workers and amplifies the moral hazard effect, as shown in Panel D, where the MH effect increases dramatically at the left tail of the ability distribution when fiscal pressure rises. When g reaches 0.5, this amplification becomes so pronounced that the MH curve first decreases and then increases.

As a result, the planner increases marginal income taxes on low-ability workers. Additionally, these increases in MH effects reduce the gains from sacrificing redistribution to the middle class to focus on the very poorest. Consequently, unlike Heathcote and Tsujiyama (2021), we do not observe the adverse selection effect exhibiting a disproportionate increase at relatively low ability levels as fiscal pressure rises.

6 Conclusion

In this paper, we study optimal income taxation that simultaneously addresses both ability differences and income uncertainty as sources of inequality. By introducing income risk into the classical Mirrlees framework, our model confronts the government with a dual challenge: adverse selection from hidden information about abilities and moral hazard from unobservable effort choices.

We develop a framework where taxation can be both income-contingent and type-contingent, enabling the government to achieve second-best outcomes through *ex* *ante* redistribution across types while providing optimal insurance against income shocks. This approach effectively decouples the impacts of heterogeneous abilities and income uncertainty. The government employs *ex post* marginal tax rates to provide social insurance against income shocks, while using *ex ante* marginal tax rates for redistribution across ability types. We identify two novel components in the optimal *ex ante* marginal tax formula beyond the classical ABC formula: a primal term arising from moral hazard that balances tax burden against effort incentives under uncertainty, and a term that recalibrates the measurement of efficiency distortions from taxation.

Our quantitative analysis reveals that workers with lower ability face higher marginal tax rates on *ex post* income, even when realizing identical incomes as higher-ability

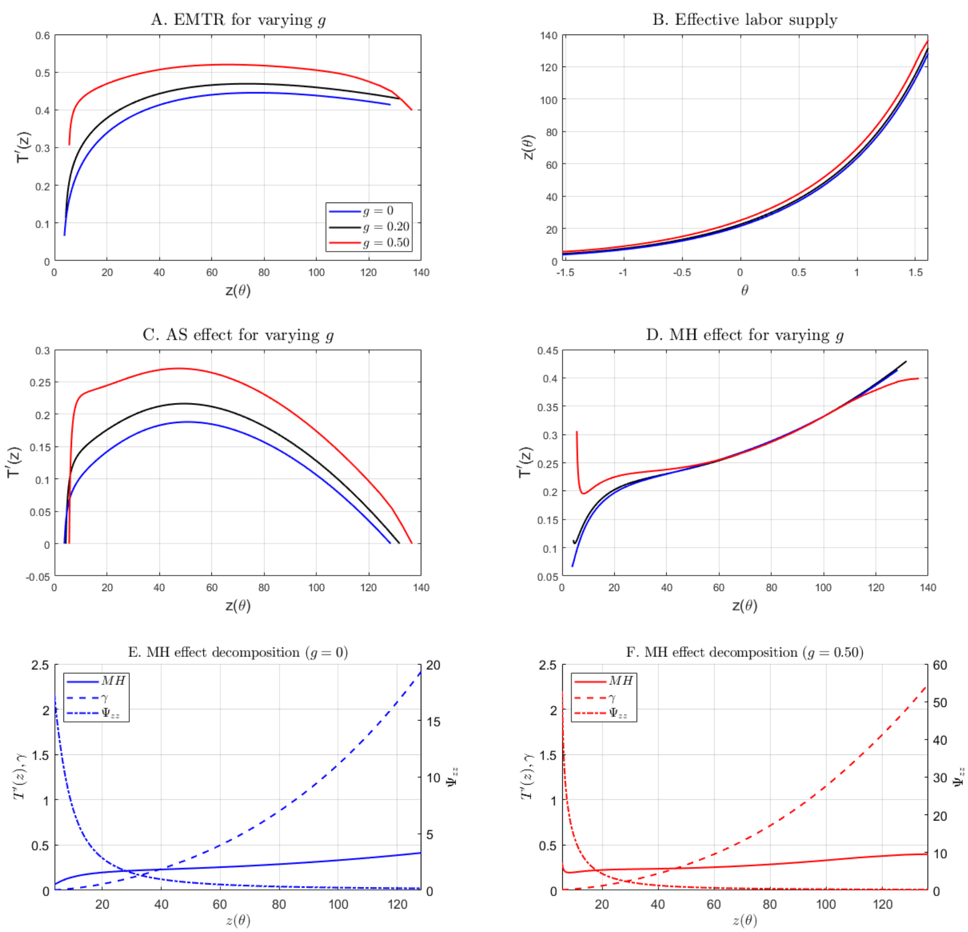

Figure 8. EMTR and its decompositions under alternative fiscal pressure g. In Panels E and F, the left y-axis (labeled $T'(z), \gamma$) measures values from 0 to 2.5. The right y-axis (labeled Ψ_{zz}) ranges from 0 to 50 in Panel E and 0 to 80 in Panel F.

workers. The *ex ante* marginal tax rates exhibit a robust inverse-U shape, with moral hazard considerations contributing to greater progressivity. Importantly, the type-variant tax schedule enables more effective redistribution at lower efficiency costs compared to type-invariant alternatives. The welfare gain from moving from a pure income-variant HSV tax schedule to our baseline type-variant schedule is approximately 0.658%.

A Proofs

A.1 Proof of Lemma 1

The first-order condition of the worker's optimization problem is

$$v_z(z,\theta) = \int_{\underline{y}}^{\bar{y}} u(c(y,\theta))g_z(y \mid z)dy = -\int_{\underline{y}}^{\bar{y}} u'(c(y,\theta))(1-T_y(y,\theta))G_z(y \mid z(\theta))dy, \tag{A.1}$$

where $T_y(y,\theta) = \frac{\partial T(y,\theta)}{\partial y}$ is the marginal tax rate for a type-θ worker with income y. The second equality follows from integration by parts and using the conditions $G(\bar{y}|z(\theta)) = 1, G(\underline{y}|z(\theta)) = 0$ for all θ.

The second-order condition is given by

$$\int_{\underline{y}}^{\bar{y}} u(c(y,\theta))g_{zz}(y \mid z(\theta))dy - v_{zz} < 0, \text{ or } -\int_{\underline{y}}^{\bar{y}} u_c(c(y,\theta))c_y(y,\theta)G_{zz}(y \mid z(\theta))dy - v_{zz} < 0, \tag{A.2}$$

where the second inequality is derived using integration by parts. Under the CDFC assumption that $G_{zz}(y \mid z(\theta)) > 0$, as long as $c_y(y,\theta) > 0$, the second-order condition holds, which means the first-order condition is sufficient for solving the worker's optimization problem.

Differentiating the first-order condition (3) with respect to θ, we obtain the following result under the assumption of an interior solution

$$z'(\theta) = \frac{v_{\theta z} - \int_{\underline{y}}^{\bar{y}} u'(c)c_\theta g_z(y \mid z(\theta))dy}{\int_{\underline{y}}^{\bar{y}} u(c(y))g_{zz}(y \mid z(\theta))dy - v_{zz}}. \tag{A.3}$$

Equation (A.3) describes how effective labor supply varies with workers' ability. Under the assumption that $v_{\theta z} < 0$, as long as the second-order condition of a worker's optimization (A.2) holds, we have $z'(\theta) > 0$. This indicates the effective labor supply increases with workers' ability.

A.2 Proof of Lemma 2

Proof. Solving the first-order conditions on $\tilde{z}$ and $\hat{\theta}$ leads to (IC-MH) and (IC-D). Since $U'(\theta) = \int_{\underline{y}}^{\bar{y}} u'c_\theta g(y \mid z(\theta))dy + \left[\int_{\underline{y}}^{\bar{y}} ug_z(y \mid z(\theta)) - v_z(z(\theta);\theta)\right] z'(\theta) - v_\theta$, applying (IC-MH) and (IC-D) generates (IC-AS) .

To ensure the necessary conditions are sufficient, the Hessian matrix must be positive definite. This is satisfied when $u' > 0$, $u'' < 0$, $v_{zz} > 0$, $v_{\theta z} < 0$, $c_y(y, \theta) \geq 0$, $\int_{\underline{y}}^{\bar{y}} u' c_\theta g_z dy > 0$ and CDFC hold. Under these conditions, we get $z'(\theta) > 0$ based on equation (A.3).

□

A.3 Proof of Proposition 1 and Lemma 3

Proof. Define the Lagrangian function $\mathcal{L}(U(\theta), z(\theta))$ to solve the constrained cost-minimization problem. The first-order condition on $c(y, \theta)$ gives

$$\frac{1}{u'(c(y,\theta))} = \mu(\theta) + \gamma(\theta)\frac{g_z(y \mid (\theta))}{g(y \mid z(\theta))}. \tag{A.4}$$

Multiplying both sides of the above equation by $g(y \mid z(\theta))$ and integrating over y yields

$$\mu(\theta) = \int_{\underline{y}}^{\bar{y}} \frac{1}{u'(c(y,\theta))} g(y \mid z(\theta))dy - \gamma(\theta) \int_{\underline{y}}^{\bar{y}} g_z(y \mid z(\theta))dy = E_y\left[\frac{1}{u'(c(\tilde{y},\theta))} \mid z(\theta)\right].$$

The second equal sign uses $\int_{\underline{y}}^{\bar{y}} g_z(y \mid z(\theta))dy = 0$ when $\underline{y} \to 0$ and $\bar{y} \to +\infty$. Multiply $\mu(\theta)$ by $v_z(z(\theta), \theta)$ to get the expression of $\tilde{\mu}(\theta)$ in proposition 1.

Using (A.4) again, we find

$$\gamma(\theta)\frac{g_z(y \mid z(\theta))}{g(y \mid z(\theta))} = \frac{1}{u'(c(y,\theta))} - E_y\left[\frac{1}{u'(c(\tilde{y},\theta))} \mid z(\theta)\right].$$

Multiplying both sides by $u(c(y))g(y \mid z(\theta))$ and integrating over y yields

$$\gamma(\theta) \int_{\underline{y}}^{\bar{y}} u(c(y,\theta))g_z(y \mid z(\theta))dy = cov_y\left(u(c(\tilde{y},\theta)), \frac{1}{u'(c(\tilde{y},\theta))} \mid z(\theta)\right).$$

Rearranging the above equation gives the expression of $\gamma(\theta)$ and $\tilde{\gamma}(\theta)$.

The envelop conditions are obtained by taking derivatives of $\mathcal{L}(U(\theta), z(\theta))$ on $U(\theta)$ and $z(\theta)$, and using the expression of $\Psi_{zz} = \frac{\partial^2 U^r(r,z;\theta)}{\partial z \partial z}|_{r=\theta, z=z(\theta)} = v_{zz} - \int_{\underline{y}}^{\bar{y}} u(c(y,\theta))g_{zz}(y \mid z(\theta))dy$ derived from (7). □

A.4 Proof of Lemma 4

Proof. We define the Hamiltonian $\mathcal{H}$ for this adverse selection problem. The optimality conditions are given by

$$q'(\theta) = -\frac{\partial \mathcal{H}}{\partial U(\theta)} = \left[\frac{\beta(\theta)}{\lambda} - \mu(\theta)\right] f(\theta), \tag{A.5}$$

$$\frac{\partial \mathcal{H}}{\partial z(\theta)} = \left[\int_{\underline{y}}^{\bar{y}} y g_z(y \mid z(\theta)) dy\right] f(\theta) - \frac{\partial \mathcal{C}(U(\theta), z(\theta))}{\partial z(\theta)} f(\theta) - q(\theta) v_{\theta z} = 0. \tag{A.6}$$

The boundary conditions are $q(\underline{\theta}) = q(\bar{\theta}) = 0$.

Integrating equation (A.5) to obtain $q(\theta) = -\int_{\theta}^{\bar{\theta}} \left[1 - \bar{\beta}(\theta)\right] \mu(\theta) f(\theta) d\theta$, where $\bar{\beta}(\theta) = \frac{\beta(\theta)/\mu(\theta)}{\lambda}$ is the social welfare weights on type-θ workers. Use the boundary condition $q(\underline{\theta}) = 0$ and the condition that $\mu(\theta) = E_y\left[\frac{1}{u'(c(y,\theta))} \mid z(\theta)\right]$ to solve the expression of λ in lemma 4. □

A.5 Proof of Proposition 3

Proof. Under the specified form of $v(z;\theta)$ in proposition 3, we have $\frac{v_{\theta z}}{v_z} = -\frac{1+\varepsilon}{\varepsilon}$. Using $T(y,\theta) = y - c(y,\theta)$ and the condition (16), equation (A.6) can be rewritten as:

$$\begin{aligned}
\widetilde{T}_z(z(\theta),\theta) &= \gamma(\theta)\Big(v_{zz} - \int_{\underline{y}}^{\bar{y}} u(c(y,\theta)) g_{zz}(y \mid z(\theta)) dy\Big) + v_{\theta z}\frac{q(\theta)}{f(\theta)} \\
&= \gamma(\theta)(v_{zz} - \int_{\underline{y}}^{\bar{y}} u(c(y,\theta)) g_{zz}(y \mid z(\theta)) dy) + \left(1 + \frac{1}{\varepsilon}\right) v_z \frac{q(\theta)}{f(\theta)}.
\end{aligned}$$

The second line is obtained using properties of $v(z;\theta)$.

Using the first-order condition (3), we have

$$\begin{aligned}
v_z =& -\int_{\underline{y}}^{\bar{y}} u'(c(y,\theta))(1 - T_y(y,\theta)) \frac{G_z(y \mid z(\theta))}{g(y \mid z(\theta))} g(y \mid z(\theta)) dy \\
=& E_y\left[u'(c(y,\theta)) \mid z(\theta)\right]\left[\widetilde{y}'(z(\theta)) - \widetilde{T}_z(z(\theta),\theta) \mid z(\theta)\right] - cov_y[u'(c(y,\theta)), (1 - T_y(y,\theta))\frac{G_z}{G_y} \mid z(\theta)]
\end{aligned}$$

Substitute the expression of $\gamma(\theta)$, $q(\theta)$ and $v_z(\theta)$ into the above equation and rearrange to get (25).

□

B Numerical Procedure

Calibration. We calibrate the actual U.S. tax progressivity parameter p following Heathcote et al. (2017). The social preference parameter and tax level parameter are determined by optimal tax progressivity and the ratio of $\frac{\mathbf{E}(c)}{\mathbf{E}(y)}$, assuming the current income tax schedule is optimal.

Simulation. We solve the second-best allocation using a system of ordinary differential equations for $U(\theta)$ and $q(\theta)$, with boundary conditions $q(\underline{\theta}) = q(\bar{\theta}) = 0$. The corresponding algorithm is:

1. Discrete ability distribution. Create 100 ability bins and approximate the true probability density function $\theta \sim EMG(\mu_\theta, \sigma_\theta^2, \lambda_\theta)$ with a discrete probability mass function. Set $\underline{\theta}, \bar{\theta}$ to correspond to 0.01 and 0.99 CDF values. Following Heathcote and Tsujiyama (2021), construct evenly spaced log wage values with corresponding probabilities. Adjust the value of σ_θ^2 to ensure that the variance of the discretely distributed variable $log(\theta_i)$ remains exactly equal to 0.348.

2. Start with an initial guess value of λ.

3. For given $\{\theta, z(\theta), U(\theta)\}$, express $\mu(\theta)$ and $\gamma(\theta)$ in terms of θ, $z(\theta)$ and $U(\theta)$ by solving equation systems containing (1), (3) and (A.6).

4. Solve the system represented by (A.5) and (IC-AS) using Runge-Kutta method. Check for potential corner solutions for $\mu(\theta)$ and $\gamma(\theta)$.

5. Given $\{z(\theta), \mu(\theta), \gamma(\theta), U(\theta)\}$, derive $\{c(y, \theta)), T[y(z(\theta))]\}$. Specifically, we have $\underline{y}(z(\theta))$ and $\bar{y}(z(\theta))$ for each θ using CDF of log-normal distribution of $y \sim \mathbf{LN}(-\frac{\sigma_\chi^2}{2} + lnz(\theta), \sigma_\chi^2)$. $\underline{y}, \bar{y}$ correspond to 0.01 and 0.99 CDF values. Create 100 grids of y for each θ between the 0.01 and 0.99 CDF values, and calculate $c(y, \theta)$ using Proposition 1.

6. Verify the resource constraint. If aggregate income is larger (smaller) than aggregate consumption, go back to step 2 and adopt a lower (higher) initial guess of λ. When the gap between consumption and income is very trivial, use (22) to get $T_y(y, \theta)$. Use (25) to get $\widetilde{T}_z(z(\theta), \theta)$.

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Online Appendix for "Optimal Taxation when Adverse Selection Meets Moral Hazard"

OA.1 Supplementary Results of Numerical Analysis

In the baseline model, we set the value of Frisch labor supply elasticity ε to 0.5. This section discusses the results under two alternative values, $\varepsilon = 0.25$ and $\varepsilon = 0.75$, respectively. These values are consistent with the empirical findings in MaCurdy (1981) and Greenwood et al. (1988).

Panels A, C, and D in Figure OA.1 show that both AS effect and MH effect positively correlate with the Frisch labor supply elasticity. Therefore, the EMTR also positively correlates with elasticity. The inverse relationship between the MH effect and ε can be explained from Panels E and F. Under a high labor supply elasticity, the information rent $\gamma(\theta)$ paid to generate desired labor efforts in a pure moral hazard problem is larger, although the realized labor efforts are almost indistinguishable in Panel B. The AS effect positively correlates with ε, because a higher elasticity amplifies the reduction in marginal benefits of transferring income from workers to the government caused by income risk.

OA.2 Calibrate the Government's Preference for Redistribution

We use the HSV tax schedule to approximate the actual tax system as $T(y) = y - \phi y^{1-p}$, where p measures tax progressivity and ϕ is a level parameter. We use the actual tax progressivity in U.S. to get p. The social preference parameter and tax level parameter are determined by the rule of optimal progressivity in Lemma 5 and the ratio of $\frac{\mathbf{E}(c)}{\mathbf{E}(y)}$ in Lemma 6.

We need the following two properties to derive Lemma 5 and 6.

(1) The moment-generating function for the normal distribution, $\chi \sim \mathbf{N}(-\frac{\sigma_\chi^2}{2}, \sigma_\chi^2)$, for $t \in R$ is given by

$$MGF_\chi(t) = \int e^{t\chi} dF_\chi(\theta) = e^{-\frac{1}{2}t\sigma_\chi^2 + \frac{1}{2}t^2\sigma_\chi^2}.$$

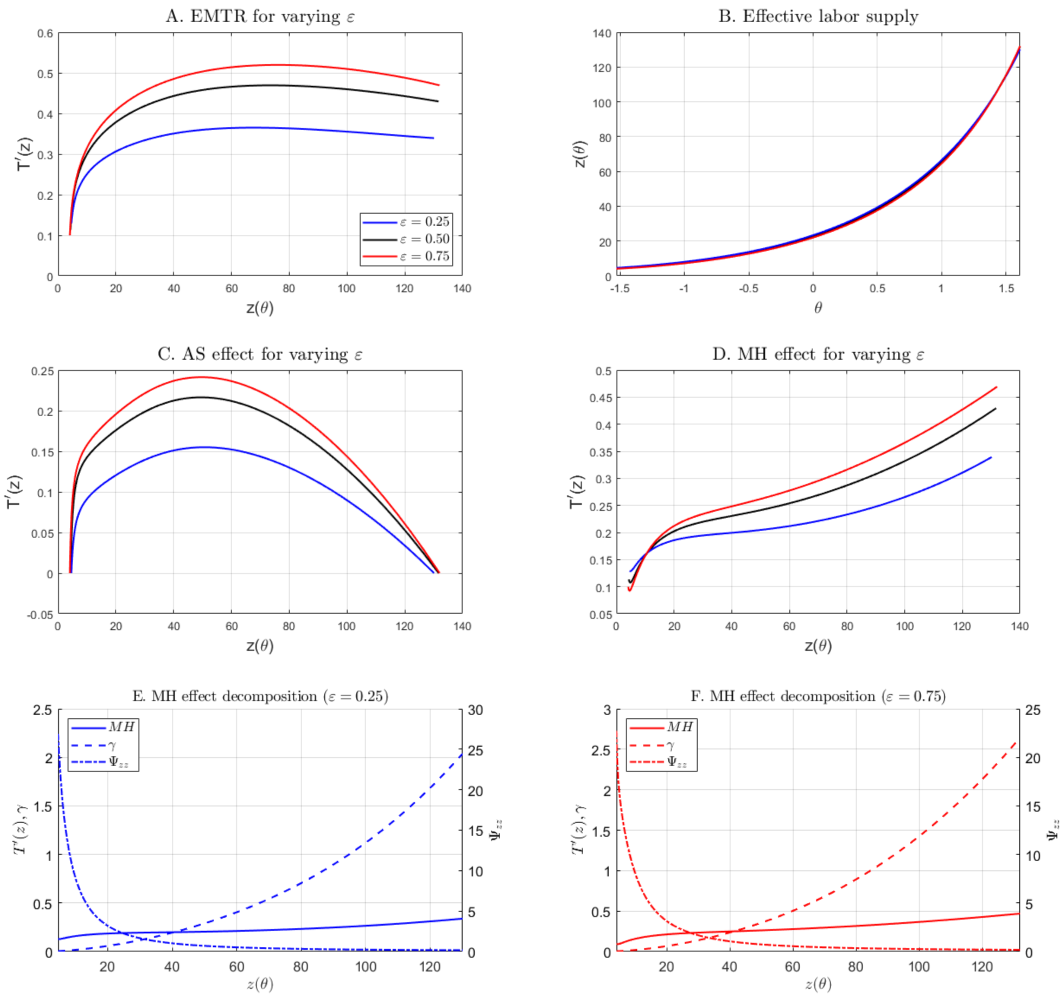

Figure OA.1. EMTR and its decompositions under alternative ε. In Panels E and F, the left y-axis (labeled $T'(z), \gamma$) measures values from 0 to 2.5 in Panel E and 0 to 3 in Panel F. The right y-axis (labeled Ψ_{zz}) ranges from 0 to 30 in Panel E and 0 to 25 in Panel F.

Also, we have

$$\frac{\partial MGF_\chi(t)}{\partial t} = \int \chi e^{t\chi} dF_\chi(\theta) = \left(-\frac{1}{2}\sigma_\chi^2 + t\sigma_\chi^2\right) e^{-\frac{1}{2}t\sigma_\chi^2 + \frac{1}{2}t^2\sigma_\chi^2}.$$

(2) The moment-generating function for the EMG distribution, $\theta \sim EMG(\mu_\theta, \sigma_\theta^2, \lambda_\theta)$,

for $t \in R$ is given by

$$MGF_\theta(t) = \int e^{t\theta} dF_\theta(\theta) = \frac{\lambda_\theta}{\lambda_\theta - t} e^{\mu_\theta t + \frac{\sigma_\theta^2 t^2}{2}}. \tag{OA.2.1}$$

Also, we have

$$\frac{\partial MGF_\theta(t)}{\partial t} = \int \theta e^{t\theta} dF_\theta(\theta) = \left(\mu_\theta + \sigma_\theta^2 t - \frac{1}{\lambda_\theta - t}\right) \frac{\lambda_\theta}{\lambda_\theta - t} e^{\mu_\theta t + \frac{\sigma_\theta^2 t^2}{2}}. \tag{OA.2.2}$$

Lemma 5. *Assume income tax function follows a HSV form $T(y) = y - \phi y^{1-p}$. $\chi \sim \mathbf{N}(-\frac{\sigma_\chi^2}{2}, \sigma_\chi^2)$ and $\theta \sim \mathbf{EMG}(\mu_\theta, \sigma_\theta^2, \lambda_\theta)$, the optimal progressivity p satisfies:*

$$\begin{aligned}
&\phi l^{1-p} \left[\frac{\sigma_\chi^2}{2} - \frac{\log(1-p)}{1+\frac{1}{\varepsilon}} - \left(\mu_\theta - \sigma_\theta^2 \alpha - \frac{1}{\lambda_\theta + \alpha}\right)\right] - \frac{l}{1+\frac{1}{\varepsilon}} \frac{1}{1-p} \\
&+\phi l^{1-p} \left[\mu_\theta + \sigma_\theta^2 (1-p) - \frac{1}{\lambda_\theta - (1-p)} + \log l + (\frac{1}{2} - p)\sigma_\chi^2 + \frac{1}{1+\frac{1}{\varepsilon}}\right] \frac{\lambda_\theta - 1}{\lambda_\theta - (1-p)} \\
&\times \exp\left(-p\mu_\theta + \frac{\sigma_\theta^2 (1-p)^2}{2} - \frac{\sigma_\theta^2}{2} - (1-p)p\frac{\sigma_\chi^2}{2}\right) = 0
\end{aligned}$$

Proof. We use the perturbation method to derive the optimal condition for p. Since a type-θ worker's consumption is $c(y(\theta,\chi)) = y(\theta,\chi) - T(y(\theta,\chi))$ and $y(\theta,\chi) = e^{\theta+\chi} l(\theta)$, the expected utility is

$$\begin{aligned}
U(\theta) &= \int_\chi \log\left[y(\theta,\chi) - T(y(\theta,\chi))\right] dF_\chi(\chi) - \left(1 + \frac{1}{\varepsilon}\right)^{-1} l(\theta)^{1+\frac{1}{\varepsilon}} \\
&= \log\phi + (1-p)\theta - (1-p)\frac{\sigma_\chi^2}{2} + (1-p)\log l(\theta) - \left(1 + \frac{1}{\varepsilon}\right)^{-1} l(\theta)^{1+\frac{1}{\varepsilon}}.
\end{aligned} \tag{OA.2.3}$$

By solving the optimal labor supply, we have

$$l = l(\theta) = (1-p)^{\frac{1}{1+\frac{1}{\varepsilon}}} \tag{OA.2.4}$$

We then analyze the incidence of a tax reform. First, consider a tax reform that marginally raises the rate of progressivity p by a small amount $\kappa \to 0$. The result tax function can be written as

$$\tilde{T}(y) = T(y) + \kappa\tau(y),$$

where $\tau(y) = \phi y^{1-p} \log y$ and $\tau'(y) = \phi(1-p)y^{-p}\log y + \phi y^{-p}$. The expected utility of a type-θ worker under the new tax schedule $\tilde{T}(y(\theta,\chi))$ is

$$U(\theta) = \int_\chi \log\left[e^{\theta+\chi}l(\theta) - T(e^{\theta+\chi}l(\theta)) - \kappa\tau(e^{\theta+\chi}l(\theta))\right] dF_\chi(\chi) - \frac{l(\theta)^{1+\frac{1}{\varepsilon}}}{1+\frac{1}{\varepsilon}}$$

Effect on labor supply. Following a tax reform, the perturbed level of effort satisfies

$$\int_\chi \frac{e^{\theta+\chi}\left(1 - T'(e^{\theta+\chi}l(\theta)) - \kappa\tau'(e^{\theta+\chi}l(\theta))\right)}{e^{\theta+\chi}l(\theta) - T(e^{\theta+\chi}l(\theta)) - \kappa\tau(e^{\theta+\chi}l(\theta))} dF_\chi(\chi) = l(\theta)^{\frac{1}{\varepsilon}} \qquad \text{(OA.2.5)}$$

Taking the derivative of this expression with respect to κ evaluated at $\kappa = 0$ gives

$$\frac{dl(\theta)}{d\kappa} = \mathbf{SE}(\phi,p,\theta) + \mathbf{IE}(\phi,p,\theta). \qquad \text{(OA.2.6)}$$

where

$$\mathbf{SE}(\phi,p,\theta) = -\frac{l(\theta)}{1+\frac{1}{\varepsilon}}\left[\log l(\theta) + \theta - \frac{\sigma_\chi^2}{2} + \frac{1}{1-p}\right];$$
$$\mathbf{IE}(\phi,p,\theta) = \frac{l(\theta)}{1+\frac{1}{\varepsilon}}\left[\log l(\theta) + \theta - \frac{\sigma_\chi^2}{2}\right].$$

Effect on individual utility. The impact of tax perturbation on individual utility is

$$\frac{dU(\theta)}{d\kappa} = -\int_\chi \frac{\tau(e^{\theta+\chi}l(\theta))}{[e^{\theta+\chi}l(\theta) - T(e^{\theta+\chi}l(\theta))]} dF_\chi(\chi) \qquad \text{(OA.2.7)}$$

Under HSV tax function, it can be written as

$$\frac{dU(\theta)}{d\kappa} = -\int_\chi [\theta + \log l(\theta) + \chi] dF_\chi(\chi) = -\theta - \log l(\theta) + \frac{\sigma_\chi^2}{2}$$

Effect on social welfare. Given welfare weight $\beta(\theta) = \frac{e^{-\alpha\theta}}{\int_{\theta'} e^{-\alpha\theta'} dF_\theta(\theta')}$, the impact of tax reform on social welfare is

$$\frac{dW}{d\kappa} = \frac{1}{\int_{\theta'} e^{-\alpha\theta'} dF_\theta(\theta')}\int_\theta e^{-\alpha\theta}\frac{dU(\theta)}{d\kappa} dF_\theta(\theta) = \frac{1}{\int_{\theta'} e^{-\alpha\theta'} dF_\theta(\theta')}\int_\theta e^{-\alpha\theta}[\frac{\sigma_\chi^2}{2} - \theta - \log l(\theta)] dF_\theta(\theta)$$

Using the properties of the moment-generating function for the EMG distribution in (OA.2.1) and (OA.2.2), we have

$$\frac{dW}{d\kappa} = \frac{\sigma_\chi^2}{2} - \frac{\log(1-p)}{1+\frac{1}{\varepsilon}} - \left(\mu_\theta - \sigma_\theta^2\alpha - \frac{1}{\lambda_\theta + \alpha}\right)$$

Effect on government revenue. Government tax revenue after the reform is

$$G = \int_\theta \int_\chi \left[e^{\theta+\chi} l(\theta) - \phi[e^{\theta+\chi} l(\theta)]^{1-p} + \kappa\tau(e^{\theta+\chi} l(\theta))\right] dF_\chi(\chi) dF_\theta(\theta)$$

The impact of tax reform on government revenue is then

$$\frac{dG}{d\kappa} = \int_\theta \int_\chi \left[\left[e^{\theta+\chi} - \phi(1-p)e^{(1-p)(\theta+\chi)} l(\theta)^{-p}\right] \frac{dl(\theta)}{d\kappa} + \tau(e^{\theta+\chi} l(\theta))\right] dF_\chi(\chi) dF_\theta(\theta)$$

Apply expression (OA.2.6) to simplify the expression of $\frac{dG}{d\kappa}$ into:

$$\begin{aligned}\frac{dG}{d\kappa} =& \phi l^{1-p} \left[\left(\mu_\theta + \sigma_\theta^2(1-p) - \frac{1}{\lambda_\theta - (1-p)}\right) + \left[\log l + (\frac{1}{2} - p)\sigma_\chi^2\right] + \frac{1}{1+\frac{1}{\varepsilon}}\right] \frac{\lambda_\theta}{\lambda_\theta - (1-p)} \\ & \times \exp\left(\mu_\theta(1-p) + \frac{\sigma_\theta^2(1-p)^2}{2} - (1-p)p\frac{\sigma_\chi^2}{2}\right) \\ & - \frac{l}{1+\frac{1}{\varepsilon}} \frac{1}{1-p} \frac{\lambda_\theta}{\lambda_\theta - 1} \exp\left(\mu_\theta + \frac{\sigma_\theta^2}{2}\right)\end{aligned}$$

Effect on total welfare and optimal progressivity. From equation (20), the social welfare of one unit government revenue λ satisfies

$$\lambda = \left\{\mathbf{E}\left(\frac{1}{u'}\right)\right\}^{-1}.$$

Then we have

$$1/\lambda = \int_\theta \int_\chi \left[e^{\theta+\chi} l(\theta) - T(e^{\theta+\chi} l(\theta))\right] dF_\chi(\chi) dF_\theta(\theta) = \phi l^{1-p} \frac{\lambda_\theta}{\lambda_\theta - 1} \exp\left(\mu_\theta + \frac{\sigma_\theta^2}{2}\right)$$

Assume that ϕ and p are optimal in reality. Then we have $\frac{1}{\lambda}\frac{dW}{d\kappa} + \frac{dG}{d\kappa} = 0$. This implies

$$\phi l^{1-p} \frac{\lambda_\theta}{\lambda_\theta - 1} \exp\left(\mu_\theta + \frac{\sigma_\theta^2}{2}\right) \frac{dW}{d\kappa} + \frac{dG}{d\kappa} = 0$$

Substitute the expression of $\frac{dW}{d\kappa}$, $\frac{dG}{d\kappa}$ into the above equation and rearrange it to get the expression in Lemma 5.

□

Lemma 6. *The ratio of average consumption to average income satisfies*

$$\log\left(\frac{\mathbf{E}\left(c\right)}{\mathbf{E}\left(y\right)}\right) = \log\left(\frac{1}{\log\left(1-p\right)}\left(1+\frac{1}{\varepsilon}\right)\phi\frac{\lambda_\theta - 1}{\lambda_\theta-(1-p)}\right) + (1-p)\,\frac{\log\left(1-p\right)}{1+\frac{1}{\varepsilon}}$$
$$+\left(\sigma_\theta^2\left(\frac{1}{2}-p\right)-\frac{1}{\lambda_\theta-(1-p)}-p(1-p)\frac{\sigma_\chi^2}{2}\right).$$

Proof. Given $y(\theta,\chi)=e^{\theta+\chi}l(\theta)$ and the HSV tax form, we have $c[y(\theta,\chi)]=\phi[e^{\theta+\chi}l(\theta)]^{1-p}$. The expected income for a type-θ worker is

$$\mathbf{E}_y[y(\theta,\chi)]=\mathbf{E}_y\left[e^{\theta+\chi}l(\theta)\right]=l(\theta)e^\theta\mathbf{E}_y\left(e^\chi\right)=l(\theta)e^\theta.$$

The last equal is gained by using the assumption that $\chi\sim\mathbf{N}(-\frac{\sigma_\chi^2}{2},\sigma_\chi^2)$. Therefore, the average income for all workers satisfies

$$\mathbf{E}[y(\theta,\chi)]=\mathbf{E}_\theta[l(\theta)e^\theta]=l\frac{\lambda_\theta}{\lambda_\theta-1}\exp\left(\mu_\theta+\frac{\sigma_\theta^2}{2}\right).$$

The expected consumption for a type-θ worker is

$$\begin{aligned}\mathbf{E}_y\left\{c[y(\theta,\chi)]\right\}&=\mathbf{E}_y\left[\phi[e^{\theta+\chi}l(\theta)]^{(1-p)}\right]=\phi l(\theta)^{(1-p)}e^{(1-p)\theta}\mathbf{E}_y\left[e^{(1-p)\chi}\right]\\&=\phi l(\theta)^{(1-p)}e^{-p(1-p)\frac{\sigma_\chi^2}{2}}e^{(1-p)\theta}.\end{aligned}$$

To get the last line, we use the assumption that $\chi\sim\mathbf{N}(-\frac{\sigma_\chi^2}{2},\sigma_\chi^2)$. Therefore, the average consumption of all workers is

$$\mathbf{E}\left\{c[y(\theta,\chi)]\right\}=\mathbf{E}_\theta[\phi l(\theta)^{(1-p)}e^{-p(1-p)\frac{\sigma_\chi^2}{2}}e^{(1-p)\theta}].$$

Using the properties of the moment-generating function for the normal distribution and the EMG distribution, the ratio of average consumption to average income thus satisfies

$$\frac{\mathbf{E}\left[c\right]}{\mathbf{E}\left[y\right]}=\phi l^{-p}\frac{\lambda_\theta-1}{\lambda_\theta-(1-p)}exp\left[-p(1-p)\frac{\sigma_\chi^2}{2}-p\mu_\theta+[(1-p)^2-1]\frac{\sigma_\theta^2}{2}\right].$$

Take logrithm on both sides to get Lemma 6. □

Since from data, $p = 0.181$ and $1 - \frac{\mathbf{E}[c]}{\mathbf{E}[y]} = 0.1816$, we can solve for $\alpha = -0.37$ and $\phi = 0.853$ using Lemma 5 and 6.

OA.3 Solving Variants of the Benchmark Model

OA.3.1 Pure Mirrlees Model

Following Mirrlees (1971), the government levies a non-linear income tax $T(y)$. A consumer consumes all disposable income, so $c(y) = y - T(y)$. The pure Mirrlees model disregards risk.

A type-θ worker chooses effective labor $z = e^{\theta}l$ and consumption c to maximize

$$u(c) - v(z;\theta),$$

where $u_c > 0$, $u_{cc} < 0$, $v_z > 0, v_{zz} > 0, v_\theta < 0, v_{z\theta} < 0$.

Let $C(U(\theta), z(\theta), \theta)$ denote the consumption function of a type-θ consumer who achieves utility $U(\theta)$ with labor supply $z(\theta)$. We have $\frac{\partial C(U(\theta),z(\theta),\theta)}{\partial U(\theta)} = \frac{1}{u'(c)}$; $\frac{\partial C(U(\theta),z(\theta),\theta)}{\partial z(\theta)} = \frac{v_z(z;\theta)}{u'(c)}$.

The first-order incentive compatibility constraint is

$$U'(\theta) = -v_\theta(z(\theta);\theta). \tag{OA.3.1}$$

With λ denoting the marginal value of government revenues, the optimal tax problem is

$$\max_{U(\theta),z(\theta)} \int_{\underline{\theta}}^{\bar{\theta}} \left[\frac{\beta(\theta)U(\theta)}{\lambda} + z(\theta) - C(U(\theta), z(\theta), \theta)\right] f(\theta)d\theta$$
$$\text{s.t. the truth-telling condition (OA.3.1)}$$

The Hamiltonian is

$$H = \left[\frac{\beta(\theta)U(\theta)}{\lambda} + z(\theta) - C\left(U\left(\theta\right), z\left(\theta\right), \theta\right)\right] f\left(\theta\right) - q\left(\theta\right)\left[v_\theta(z(\theta);\theta)\right].$$

The optimality conditions are

$$q'(\theta)=-\frac{\partial\mathcal{H}}{\partial U(\theta)}=\left[\frac{\beta(\theta)}{\lambda}-\frac{1}{u'(c)}\right]f(\theta), \tag{OA.3.2}$$

$$\frac{\partial\mathcal{H}}{\partial z(\theta)}=\left[1-\frac{v_z(z,\theta)}{u'(c)}\right]f(\theta)-q(\theta)v_{\theta z}=0. \tag{OA.3.3}$$

Solving the equations yields

$$T'(z(\theta))=v_{\theta z}\frac{q(\theta)}{f(\theta)}=-v_{\theta z}\frac{1}{f(\theta)}\int_{\theta}^{\bar{\theta}}[1-\frac{\beta(\tilde{\theta})u'(c(\tilde{\theta}))}{\lambda}]\frac{1}{u'(c(\tilde{\theta}))}f(\tilde{\theta})d\tilde{\theta}, \tag{OA.3.4}$$

where $\lambda=[\int_{\underline{\theta}}^{\bar{\theta}}u'(c(\theta))^{-1}f(\theta)d\theta]^{-1}$.

OA.3.2 The Low-Maldoom Model Without Redistribution Across Ability Types

When the government has full information about worker types, income tax $T(y(\theta))$ is defined on realized income and pools idiosyncratic income risks within the same type, similar to Low and Maldoom (2004). Income tax has two effects: a social insurance effect that helps workers share idiosyncratic income risk, and an incentive effect that reduces precautionary motives and labor supply incentives. Optimal income taxation balances these effects.

The optimal income tax problem for a type-θ worker is:[19]

$$U=\max_{c(\cdot),z^*}\int_{\underline{y}}^{\bar{y}}u(c(y))g(y\mid z^*)dy-v(z^*) \tag{OA.3.5}$$

$$s.t.\qquad z^*=arg\max_{z}\int_{\underline{y}}^{\bar{y}}u(c(y))g(y\mid z)dy-v(z) \tag{OA.3.6}$$

$$\int_{\underline{y}}^{\bar{y}}c(y)g(y\mid z^*)dy=\int_{\underline{y}}^{\bar{y}}yg(y\mid z^*)dy \tag{OA.3.7}$$

where effective labor supply satisfies $z^*\equiv e^{\theta}l^*(\theta)$. Equations (OA.3.6) and (OA.3.7) are the incentive compatibility and resource constraints. Equation (OA.3.6) can be rewritten as

$$\int u[c(y)]g_z(y\mid z^*)dy=v'(z^*). \tag{OA.3.8}$$

Let q and λ be the Lagrange multipliers for (OA.3.8) and (OA.3.7), where λ

[19]We drop θ whenever there is no confusion.

measures deviation from complete insurance ($\lambda = 0$). The Lagrangian is

$$L = \int \{u(c(y)) + \lambda [y - c(y)]\} g(y \mid z^*)dy - v(z^*) + q \left\{ \int u(c(y)) g_z(y \mid z^*)dy - v'(z^*) \right\}$$

The first-order condition for c is

$$u'[c(y)] [g(y \mid z) + q g_z(y \mid z)] - \lambda g(y \mid z) = 0.$$

With $u(c) = \frac{1}{1-\rho} c^{1-\rho}$, we get $c(y) = [\frac{1+q\frac{g_z(y|z)}{g(y|z)}}{\lambda}]^{1/\rho}$.

The first-order condition for z is

$$\int \{u(c(y)) + \lambda [y - c(y)]\} g_z(y \mid z)dy - v'(z) + q \left\{ \int u(c(y)) g_{zz}(y \mid z)dy - v''(z) \right\} = 0.$$

Replace $v'(z)$ in the above equation using (OA.3.8) to get

$$\lambda \int y g_z(y \mid z)dy - \lambda \int c(y) g_z(y \mid z)dy + q \left\{ \int u(c(y)) g_{zz}(y \mid z)dy - v''(z) \right\} = 0.$$

Define $h(y \mid z) \equiv \frac{g_z(y|z)}{g(y|z)}$. The marginal tax rate, $T_y = 1 - c'(y)$, can be found by differentiating the expression of $c(y)$ and rearranging:

$$1 - T_y = \left(-\frac{u''}{u'} \right)^{-1} \frac{q}{1 + qh} \frac{\partial h}{\partial y}.$$

To compare results with (22) in the baseline model, the above equation can be reformulated as

$$1 - T_y(y, \theta) = \left(-\frac{u''\left(c(y)\right)}{u'\left(c(y)\right)} \right)^{-1} \gamma\left(\theta\right) u'\left(c(y)\right) h_y,$$

where

$$\gamma\left(\theta\right) \equiv \frac{cov_y\left(u(c(y,\theta)), \frac{1}{u'(c(y,\theta))} \mid z(\theta) \right)}{v'(z^*)}.$$