Optimal Mixed Taxation with Misperceptions of Prices

Xiaoyong Cui[a], Xiaoxiao Wang*[b], and Cheng Yuan[a]

[a] *Peking University, Beijing 100871, China*
[b] *Zhongnan University of Economics and Law, Wuhan 430073, China*

August 11, 2024

Abstract

This paper investigates how price misperceptions, which are pervasive and mainly ascribed to a complex system of mixed tax schedule, affect the design of optimal tax rules. Our theoretical results show that in the presence of price misperceptions indirect taxation exerts both a corrective role and a redistributive role even with the preference structure of Atkinson and Stiglitz (1976). This makes the linear commodity taxation no longer superfluous. In particular, the optimal income tax schedule can be more progressive if perceived marginal income tax rates are influenced by commodity prices. Moreover, taking income tax credit for electric vehicles as an example we simulate the optimal subsidy rate and income tax schedule when price misperceptions are considered in the design of optimal mixed taxation. Compared with conventional optimal taxation, optimal taxation in our model results in a rise in welfare by 8.79% to 36.34% under plausible values of labor supply elasticity.

Keywords: Misperceptions of prices; Optimal commodity taxation; Optimal income taxation

JEL Classification: D61, H21, H23

*Corresponding author at: School of Public Finance and Taxation, Zhongnan University of Economics and Law, Wuhan 430073, China. E-mail addresses: wangxiaoxiao9981@163.com.

I. Introduction

There is rich evidence that individuals are inattentive to various types of incentives and their perceptions of prices or tax rates are imperfect, which are further aggravated by the complexity and non-transparency of tax systems.[1] The influences of misperceptions on optimal linear commodity tax rules or on an optimal nonlinear income tax schedule have been studied extensively in the literature under the assumption that consumers only misperceive one commodity price or the marginal income tax rate.[2] This assumption, however, makes it impossible to study the influences of price changes of one commodity on the perceived prices of other commodities. It also excludes the possibility of designing optimal mixed taxation when both commodity prices and marginal income tax rates are misperceived.

Taking labor supply as a special kind of goods, we summarize the fact that price changes of one commodity influence the perceived prices of other commodities and perceived marginal tax rates to be "cross-commodity influences of actual prices on perceived prices". We notice three situations where such cross-influence makes sense for designing a mixed taxation.

First, the cross-influence can be considerable in the situation of tax incentives implemented to promote the popularization of electric vehicles (EVs).[3] When the purchase of an electric vehicle is associated with an income tax credit, the buyer would anticipate a lower marginal income tax rate since most taxpayers forecast their tax liability by applying the average tax rate to all incomes. Qualitatively, it is obvious that buyers' price misperceptions of electric vehicles will distort their labor supply and thus the design of optimal taxation. Quantitatively, tax incentives on price misperceptions might be considerable given the size of tax incentives.[4]

The cross-influence also emerges when using the tax-deferred saving account (TDA) to encourage retirement savings. As in the case of Individual Retirement Accounts (IRA) and traditional 401(k) in the United States, the interest in a TDA can be accumulated tax-free. This is actually a deduction of capital income tax, but it occurs when a taxpayer considers the contribution of his or her labor income. Therefore, the taxpayer might anticipate a lower marginal income tax rate. The gap between a perceived and the actual income tax rate depends on the perceived capital income tax rate. As almost all taxpayers need to decide on the amount and type of retirement savings, it is worth discussing how misperceptions of capital income tax rates might change the design of mixed taxation.

In the last, governments in most countries adopt income tax prepayements,[5] and allow taxpayers

---

[1] For commodity taxes, Chetty, Looney and Kroft (2009) find that consumers make systematic optimization errors concerning commodity taxes since these taxes are not fully salient. Taubinsky and Rees-Jones (2017) use an online shopping experiment to prove heterogeneity among consumers' under-reaction to not-fully-salient commodity taxes. Moreover, Rees-Jones and Taubinsky (2018) find that the non-linearity of income taxation has made it difficult for workers to calculate marginal income tax rates, let alone when there exists a wide variety of deductions and credits.

[2] See, e.g., Chetty, Looney and Kroft (2009); Goldin (2015); Allcott, Lockwood and Taubinsky (2018); Farhi and Gabaix (2020).

[3] For example, the federal government of the United States uses IRS 30D, the federal Plug-in Electric Drive Vehicle Credit. In California, buyers of a new electric car can get a $1,500 cash rebate. The federal government of Canada offers point-of-sale incentives for consumers who buy an electric vehicle.

[4] In the United States, buyers of qualifying plug-in electric vehicles may be able to claim a federal income tax credit of up to $7,500, which is 21.4% of the before-incentive price of a qualifying BYD 2012–17 e6 Electric Vehicle. According to Joint Committee on Taxation (JCT), the estimated cost of the revised electric vehicle purchase tax credit to the Treasury will total $9.2 billion from 2021 to 2030 the next decade.

[5] which means income taxes are collected when incomes realize. As there are generally various tax credits and deductions not taken into consideration at the time of tax prepaments, the total income taxes collected during a fiscal year may not equal to the taxpayer's actual tax burden. For instance, according to China's individual income tax law, individual income tax shall be calculated based on the cumulative withholding method, withheld and prepaid on a monthly basis, and annual tax clearance shall be arranged as per the actual income during the period from March

to claim tax refunds within specified periods. Due to limited attention or mental accounting, a taxpayer may overestimate the actual marginal income tax rate if his tax prepayment excludes tax credits or tax deductions he deserves. When the size of tax credits or deductions correlates with a taxpayer's consumption expenditure of certain commodities, such as children's education costs and continuing education costs in China, the cross-influence emerges and might alter the rule of optimal mixed taxation.

Inspired by-influence the above situations, we investigate how consumers' misperceptions of marginal income tax rates and commodity prices would change classical optimal tax rules by extending the framework of Atkinson and Stiglitz (1976) and Jacobs and Boadway (2014) to include cross-commodity influences of actual prices on perceived prices.[6] Theoretically, we find that a linear commodity tax is not superfluous even when consumers have preferences that are weakly separable between leisure and commodities. This breaks traditional optimal tax rules according to Atkinson and Stiglitz theorem, which asserts that indirect taxation only distorts consumption and thus no commodity tax is needed under such preferences.

In our model, price misperceptions change consumers' imaginary budget constraints.[7] Thus, besides a substitution effect and a redistribution effect in the case without price misperceptions,[8] a commodity tax also has a "behavioral wedge effect" to correct an additional loss of utility due to feeling an additional tightening of imaginary budget constraints. This is because consumers prefer to over-consume the substitutes at an increase in the commodity tax rate if they underestimate the prices of the substitutes to the commodity. This new effect validates the role played by commodity tax even with the preference structure of Atkinson and Stiglitz (1976). In a mixed taxation environment, the behavioral wedge effect requires a commodity tax to have a corrective role and a redistributive role. A commodity tax on the one hand corrects the inefficiency among commodity demands caused by misperceptions of commodity prices, on the other hand, it helps the government in redistribution.[9] We also examine within-group uniform tax rule, many-person Ramsey rule, and Corlett-Hague rule when there exist misperceptions.

We reveal price misperceptions as new origins of a redistributive role of indirect taxation. As long as a consumer's perceived marginal income tax rate is affected by commodity prices, at the optimum, distortions in commodity demands are tolerable as long as commodity taxes reduce the distortion caused by non-linear income taxation. This is possibly one of the reasons why some governments use a tax credit instead of a point-of-sale rebate for electric vehicles. As consumers tend to believe their marginal tax rates are given by their average tax rates, a tax credit makes it possible for the after-tax price of electric vehicles to influence perceived marginal tax rates. Therefore, a better redistribution can be achieved.

In the case of electric vehicles, our results reveal that an optimal income tax credit depends on two aspects. First, if buyers overestimate future energy costs, as the tax credit is a subsidy on

1 to June 30 of the following year.

[6]Though we take the tax credit on electric vehicles as an example, our theoretical framework is general and can be easily applied to analyze similar issues.

[7]When calculating the costs of a dollar increase in the price of a commodity, a fully rational consumer will find his budget constraint is tightened since each unit of that commodity costs one more dollar. However, a limited rational consumer will perceive an additional tightening of a budget constraint as he uses perceived prices in his imaginary budget constraint.

[8]First, an increase in a commodity tax decreases tax revenue since it suppresses compensated demands of commodities and compensated labor supply when the commodity is a complement to other commodities and a substitute to leisure. Second, it tightens consumers' budgets and transfers money from consumers to the government.

[9]From the perspective of information economics, misperceptions change the form of incentive compatibility constraint (IC constraint). If an increase in a commodity tax causes consumers to perceive lower marginal income tax rates, pretending to be a low-skill worker costs more, which enables the government to use a more progressive income tax schedule.

electric cars, it should correct the misperception. Second, since the worker also perceives a lower marginal income tax rate as the tax credit increases, the government can use a higher tax credit to achieve a more redistributive income tax schedule without discouraging labor supply too much. The two roles may also serve as an explanation that a government chooses an income tax credit for a kind of commodity at the cost of increasing tax complexity instead of directly using a point-of-sale rebate.

Quantitatively, we find that when the government takes the influence of price misperceptions into consideration, the optimal subsidy rate on electric vehicles is between 11.45% and 11.85% when labor supply elasticity increases from 0.2 to 0.4. This amount is slightly higher than the subsidy rate of the federal Plug-in Electric Drive Vehicle Credit on the most popular type of electric vehicles. Optimal marginal income tax rates are higher than the optimal values designed ignoring misperceptions across a large range of incomes (at least from 0 to 150 thousand dollars per year). Compared with the case not considering misperceptions, the income tax schedule is more progressive. The welfare improvement is also considerable. When labor supply elasticity decreases from 0.4 to 0.2, the additional welfare gain from considering misperceptions in the design of optimal mixed taxation as a share of welfare improvement of a conventional optimal policy ranges from 8.79% to 36.34%. A less redistributive social preference or a worse underestimation of actual EV prices will decrease the additional welfare gain.

Our work relates to research on optimal tax policy with inattentive agents. Liebman and Zeckhauser (2004) solve optimal income taxation when consumers observe a non-linear tax schedule with an ironing heuristic. In a general framework modeling inattention, Farhi and Gabaix (2020) use the differences between actual prices and marginal utility vectors expressed in a money metric to define behavioral wedges and then update optimal tax formulas. Boccanfuso and Ferey (2024) and Moore and Slemrod (2021) address the endogeneity of taxpayers' attention. The former research captures how endogenous attention to taxes can generate a time-inconsistency problem in the design of optimal income tax. The latter discusses the joint choice of rate and non-rate policy instruments like nudges may change both taxpayer incentives and when biases are endogenous. In summary, these works concentrated on adjustments of single tax instruments under misperceptions. Our paper studies the designs of optimal mixed taxation with misperceptions. This problem is of real-world significance since the tax categories in the real world are manifold and tax systems are complex. Misperceptions of one tax instrument can be changed by another tax instrument. As a result, optimal linear commodity tax may be influenced by misperceptions of income tax in the context of mixed taxation, and vice versa. We contribute to the literature by analyzing optimal linear commodity taxation mixed with non-linear income taxation when consumers misperceive commodity prices and marginal income tax rates. Optimal commodity tax formulas are shaped by both a commodity price misperception wedge and an income tax misperception wedge. An optimal income tax formula is modified by adding an income tax misperception wedge into the left-hand side of a Diamond's ABC formula as in Farhi and Gabaix (2020). Considering misperceptions in a mixed taxation environment also has considerable welfare improvement. Through a simple simulation, we show that the additional welfare gain from considering misperceptions in optimal taxation ranges from 7.17% to 36.18% as a share of welfare improvement of a conventional optimal policy.

This paper also contributes to the discussion about direct/indirect taxation. The role of indirect taxation has been widely discussed in the literature, but with fully-rational agents. One cornerstone is the Atkinson-Stiglitz theorem, which states that only non-linear income taxation is required to reach the optimum (Atkinson and Stiglitz, 1976). Similar spirits can be found in later discussions on uniform tax rules in Deaton (1979), Besley and Jewitt (1995), and so on. There are also papers emphasizing the role of indirect taxation. Mirrlees (1976) identifies that commodities tax should be greater on goods that people with high ability prefer. Christiansen (1984) points out that a commodity should be taxed if it is positively related to leisure. Jacobs and Boadway (2014) argue that government should tax/subsidize commodities if they are more/less complementary with leisure than a numerate good. Some literature focuses on indirect tax on a specific kind of good. For example, optimal inflationary tax in da Costa and Werning (2008), capital tax in Golosov et al. (2013), and sin tax in Allcott, Lockwood and Taubinsky (2019). da Costa and Werning (2008) find support for the Friedman rule, while the latter two papers imply that indirect taxation can be useful when consumers have heterogeneous preferences, as has been discussed in Saez (2002). Our work departs from these studies by highlighting a redistributive role of indirect taxation arising from a "cross-commodity influence of actual prices on perceived prices".

We also notice that commodity tax rules with misperceptions have drawn some attention in recent literature. For example, Allcott, Lockwood and Taubinsky (2018) find that if consumers are inattentive to commodity taxes when making labor supply decisions, an optimal commodity tax schedule should follow the classic "many-person Ramsey rule". By contrast, our framework differs from theirs as we keep the assumption in Chetty, Looney and Kroft (2009) and Taubinsky and Rees-Jones (2017) that consumers misperceive commodity tax rates or after-tax prices at the time of purchase. Therefore, while the scaling factor in Allcott, Lockwood and Taubinsky (2018) corresponds to a misperception wedge when people make labor supply choices and captures the extent to which consumers' labor supply is sensitive to the actual value of misperceived price, our model incorporates a behavioral wedge when people make commodity purchasing decisions so that the rescaling effects of a misperception wedge still exist if we preclude the sensitivity of labor supply on prices.

The remainder of this paper is organized as follows. Section II defines consumers' optimization problems. Section III derives optimal taxation formulas using a tax perturbation method to clarify the influence of misperceptions. Section IV discusses several commodity tax rules modified under misperceptions. Section V provides a numerical example with two goods to illustrate the effects of price misperceptions on optimal mixed taxation. Section VI discusses the connections between a tax perturbation method and a mechanism design approach under misperceptions and presents the optimal taxation under ironing heuristics. Section VII concludes. All proofs are contained in the appendix.

II. Individual Behavior under Misperceived Prices

We set up the model by adding misperceptions into the framework of Atkinson and Stiglitz (1976). Assume that there is a continuum of consumers indexed by skill $n \in N \equiv [\underline{n}, \bar{n}]$ with density $f(n)$. The cumulative distribution function of skill is denoted by $F(n)$. The consumer who works l_n hours with skill n earns a gross income $z_n = nl_n$. Use $\tilde{f}(z)$ and $\tilde{F}(z)$ to denote the density and the cumulative distribution of z. Use $\bar{z}$ and $\underline{z}$ to denote the maximum and minimum value of z. A type-n consumer's utility function $u(c_n, x_n, l_n)$ depends on the consumption of a numerate good c_n, general goods $x_n \equiv (x_{1n}, x_{2n}, \ldots, x_{In})$, and labor supply l_n. The partial derivatives of utility take the following signs: $\partial u(\cdot)/\partial c_n > 0, \partial u(\cdot)/\partial x_i > 0, \partial u(\cdot)/\partial l < 0$. The price vector on general goods is $q = (q_1, q_2, \ldots, q_I)$. Pre-tax prices of all commodities are normalized to unity so that the tax rate on commodity i is $t_i = q_i - 1$. Numerate good consumption is untaxed. The income tax function is given by $T(z_n)$, the derivative of which is assumed to be continuous and is denoted by $T'(z_n)$. Consumers cannot fully perceive their income tax function as well as the commodity prices. The perceived commodity prices vector and perceived income tax function are denoted by q^s and $T^s(z_n)$ separately.

To characterize an inattentive consumer's decision, we follow Jacobs and Boadway (2014) and Mirrlees (1976) by disaggregating the consumers' optimization problem into two stages. In the first stage, a type-n consumer chooses labor supply l_n and expenditure on consumption y_n. In the second stage, the consumer chooses c_n and x_n given the decision rules in the first stage.

A. Individual Optimization: Stage 2

In the second stage, an inattentive consumer chooses c_n and x_n, given labor supply l_n and disposable income $y_n \equiv z_n - T(z_n)$ to maximize utility function $u(c_n, x_n, l_n)$ under perceived prices. The actual budget constraint is $c_n + \sum_i q_i x_{in} = y_n$. We adopt the particular budget adjustment rule in Reck (2016), which allow consumers to choose c_n and x_n jointly based on perceived marginal rate of substitution conditions in (1), but with disposable income implicitly adjusted. In other words, we have

$$\frac{\partial u(c_n, x_n, l_n)}{\partial x_{in}} / \frac{\partial u(c_n, x_n, l_n)}{\partial c_n} = q_i^s, \forall i, n, \tag{1}$$

while making sure the choice of $\{c_n, x_n\}$ satisfies actual budget constraint with equality.

For perceived prices, we impose the following assumption:

Assumption 1. *q_j^s is a function of q and is differentiable with respect to its arguments.*

There are mainly two considerations behind this assumption. First, the assumption that q_j^s depends on q_j incorporates not only consumers' misperceptions of commodity taxes, as has been identified in Chetty, Looney and Kroft (2009) and Taubinsky and Rees-Jones (2017), but also many other psychological underpinnings of misperceptions of cost of purchasing one unit of a good in the real world. Empirical studies have found the existence of left-digit bias (Busse et al., 2013; Lacetera, Pope and Sydnor, 2012) and consumers' ignorance of accompanying expenses such as future costs of gasoline after buying an automobile or shipping charges associated with online shopping (Allcott and Wozny, 2014; Brown, Hossain and Morgan, 2010). Roger, Roger and Schatt (2018) points out that even financial analysts process small prices and large prices differently. All these phenomena imply that consumers' perception of the cost of purchasing is influenced by the actual price of one commodity.

Second, there is also evidence showing that q_k affects q_j^s when $k \neq j$. We define this characteristic to be a "cross-commodity influence of actual prices on perceived prices". Consumers may use the prices of some goods as reference prices to evaluate the cost of purchasing other goods. For instance, the experiments by Niedrich, Sharma and Wedell (2001) suggest that consumers use extreme prices of the category as external reference prices, which act as a purchasing stimuli.[10] Similar biases exist in portfolio choice. The increase in attention paid to certain assets can generate an increase in perceived volatility of the other assets delegated to the same specialist (Corwin and Coughenour, 2008). In addition, assuming q_j^s depends on q_k can characterize the phenomenon of nominal illusion, in which consumers' perception of commodities' prices depends on the price of money.[11] Moreover, the theory of endogenous attention helps us to understand why q_k affects q_j^s. Gabaix (2014) finds that paying attention to the price of one good increases with a consumer's expenditure share on that good, which depends on the whole price vector.

[10] External reference prices are observed as purchasing stimuli provided by point-of-purchase shelf tags that read "Compare at $X", or the actual prices of another product against which it can be compared (Mayhew and Winer, 1992).

[11] For example, investors are likely to think in nominal dollar terms rather than percentage changes when there might be a change in stock prices (Shue and Townsend, 2021).

The actual (rather than perceived) conditional demands are denoted by $c_n^s(q, y_n, l_n)$ and $x_{in}^s(q, y_n, l_n)$. The indirect utility function at a consumer's actual commodity demand is $v_n^s(q, y_n, l_n)$.[12]

B. Individual Optimization: Stage 1

In the first stage, a type-n consumer chooses labor supply l_n and consumption expenditure y_n to maximize conditional indirect utility $v_n^s(q, y_n, l_n)$. Denote $Q_n \equiv 1 - T'(z_n)$ as the actual marginal retention rate, and denote $R_n \equiv z_n T'(z_n) - T(z_n)$ as the generalized revenue. Then we have $y_n - Q_n z_n = R_n$, where we take z_n as a special kind of good with price $-Q_n$. As in Farhi and Gabaix (2020), we focus on cases where consumers' misperception of an income tax schedule is captured by a perceived marginal retention rate $Q_n^s \equiv 1 - dT^s(z_n)/dz_n$. The budget adjustment rule is the same with that of stage 1. We impose the following assumption on Q_n^s:

Assumption 2. *The perceived marginal retention rate of a type-n consumer $Q_n^s(Q_n, q, z_n)$ is a function of the actual marginal retention rate Q_n, a vector of commodity price q, and the consumer's income z_n.*

Misperceptions of income tax rates are documented in Brown (1969), Fujii and Hawley (1988), and Romich and Weisner (2000). Taxpayers not only find it hard to give their correct marginal tax rate in surveys, but also confuse a lump sum tax reform with marginal tax reforms (Feldman, KatuščÁk and Kawano, 2016). Assumption 2 corresponds to rich empirical research on what determines misperceptions of income tax rates. First, some studies point out the perceived marginal tax rate is a weighted average of one's actual marginal tax rate and of one's average tax rate (Liebman and Zeckhauser, 2004; Rees-Jones and Taubinsky, 2020).[13]

Second, prices of commodities can also influence Q_n^s. For instance, money illusion can confuse taxpayers over how much tax they have paid at the real price, thus inflation (the price of money) may influence the perception of the income tax rate as in Katseli-Papaefstratiou (1979). Moreover, the various tax deductions and tax credits terms make it rather difficult for taxpayers to understand marginal income tax rates. For instance, in the United States, an individual is eligible for a credit if he purchases a qualified plug-in electric drive motor vehicle. If the individual forecasts tax liability by applying his average tax rate to all incomes, he would perceive a lower marginal income tax rate due to the credit. In this way, the price of plug-in electric drive motor vehicles (taking into consideration the tax credit) might influence an individual's perception of marginal income tax rates. Taking labor income as a special kind of goods with price $-Q_n^s$, the above phenomenon can also be defined to be a "cross-commodity influence of actual prices on perceived prices".

Due to misperceptions, the marginal rate of substitution between z_n and y_n equals to relative perceived price though the actual budget constraint $y_n = n l_n Q_n + R_n$ still holds.

$$\frac{\partial v_n^s}{\partial l_n} / \frac{\partial v_n^s}{\partial y_n} = -n Q_n^s. \tag{2}$$

[12]Here we use superscript s to distinguish between a limited-rational consumer's actual demand x_n^s and an as-if rational consumer's demand x_n^r, which is defined in the Appendix A. We rely on the definition of x_n^r since we cannot directly apply the envelop theorem to the limited-rational consumer's optimization problem. In Appendix A, we provide detailed properties of a consumer's decision to see how misperceptions affect a consumer's behavior.

[13]Perceiving and responding to the average price instead of the actual price is defined as the ironing heuristic in Liebman and Zeckhauser (2004).de Bartolome (1995) uses a laboratory experiment to find that taxpayers tend to behave as if their marginal tax rate is given by their average tax rate. Rees-Jones and Taubinsky (2020) find out that 43% of the population irons. One example of a perceived marginal tax rate function is $Q_n^s = mQ(z_n) + (1-m)\frac{T(z_n)}{z_n}$. m denotes attention to the actual marginal tax rate. r_0 denotes a tax rebate at 0 income. To highlight the role of indirect taxation, we temporally do not consider the situations when perceptions of the marginal income tax rate are influenced by marginal tax rates at other income levels as in Farhi and Gabaix (2020). We extend our model to incorporate such "externalities" in VI as an extension.

Solving this problem gives the actual (rather than perceived) labor supply function $l_n(q, Q_n, R_n)$ and an indirect utility function $V_n^s(q, Q_n, R_n)$ of a limited-rational consumer. The actual labor income then satisfies $z_n = nl_n(q, Q_n, R_n)$.

III. Optimal Taxation

To gain some intuitions, we first use a Saez (2001) type of tax reform to decompose the bet welfare effects of changing marginal tax rates of income taxation as well as the commodity tax rates. Our decomposition will show how misperceptions influence optimal tax formulas by changing consumers' imaginary budget constraints. Different from an actual budget constraint, an imaginary budget constraint replaces actual prices with perceived prices and replaces actual revenue with imaginary revenue. Lemma 1 highlights how the imaginary budget constraints come into changing a limited-rational consumer's decision.

Lemma 1. *In stage 1 decision, a limited rational consumer feels like an imaginary generalized revenue* $\bar{R}_n \equiv y_n - z_n Q_n^s$*, which satisfies*

$$\bar{R}_n(q, Q_n, Q_n^s, R_n) = R_n - (Q_n^s - Q_n) z_n. \tag{3}$$

In stage 2 decision, an imaginary budget results in a modified Roy's identity:

$$\frac{\partial v_n^s}{\partial q_j} / \frac{\partial v_n^s}{\partial y_n} = -x_{jn}^s - w_{jn}^q, w_{jn}^q \equiv \sum_i (q_i - q_i^s) \frac{\partial x_{in}^{s*}}{\partial q_j}. \tag{4}$$

The proof is collected in Appendix A.

Equation (3) implies that any change in labor income, $\mathrm{d}z_n$, leads to an addition tightening of a z_n-earner's imaginary budget cosntraint by $(Q_n^s - Q_n) z_n$. To understand the modified Roy's identity in equation (4), notice that for a fully rational consumer, the traditional Roy's identity requires that the marginal cost of commodity j's price, $-x_{jn}^s$, equals its marginal benefit $\frac{\partial v_n^s}{\partial q_j} / \frac{\partial v_n^s}{\partial y_n}$. The cost $-x_{jn}^s$ arises because an increase in q_j mechanically tightens a consumer's actual budget constraint at x_{jn}^s dollars. However, when there exist misperceptions on commodities' prices, there is an additional cost measured by w_{jn}^q because the consumer evaluates marginal costs using an imaginary budget constraint with perceived prices. Specifically, an increase in q_j changes q_i^s and therefore the consumption of all commodities, resulting in a tightening of the consumer's imaginary budget constraint. This additional cost is captured by $(q_i - q_i^s) \frac{\partial x_{in}^{\partial s}}{q_j}$. Sum it up over j to get the expression for w_{jn}^q.

As in Saez (2002), assume that the optimal income tax schedule is regular and that there is no gap or bunching in the optimal schedule. The government maximizes social welfare taking as given consumers' behavior summarized in the previous section. The social welfare is the sum of non-decreasing concave social utilities $\Psi(V_n^s)$ on n:

$$\int_N \Psi\left(V_n^s(q, Q_n, R_n)\right) f(n) \mathrm{d}n.$$

A. Optimal Income Tax

Consider the effect of slightly raising the marginal income tax rate by $\mathrm{d}\tau$ in a narrow neighborhood ϕ around some income level $\hat{z}$. The effects of the tax reform will be decomposed into three parts: the mechanical effects, the behavioral effects, and the behavioral wedge effects.

A.1. The Mechanical Effects

If there was no behavioral response from consumers, this reform implies that the government can raise its income tax revenue by $\phi \mathrm{d}\tau(1-\tilde{F}(\hat{z}))$. This reform also reduces consumers' income from those earning more than $\hat{z}$. Use $g(z_n)$ to denote the social marginal welfare weight on an individual earning $z(n)$. Therefore, the "mechanical effect" of the reform can be expressed as

$$\mathrm{d}M = \phi \mathrm{d}\tau \int_{\hat{z}}^{\bar{z}} (1-g(z))\tilde{f}(z)\mathrm{d}z. \tag{5}$$

A.2. The Behavioral Effects

To analyze the behavioral response, we first ignore the change in labor supply: the reform results in a decrease in disposable income and directly changes the government's commodity tax revenue through income effects on commodity demands at the amount of $-\phi \mathrm{d}\tau \int_{\hat{z}}^{\bar{z}} \sum_i t_i \frac{\partial x_{in}^s}{\partial y_n} \tilde{f}(z)\mathrm{d}z$. Next, we consider the response of labor supply. As Saez (2001), the tax reform on the one hand increases the marginal income tax rate at $\hat{z}$, on the other hand, has income effects on all consumers earning above $\hat{z}$. One difference between our analysis with Saez (2001) is that one unit increase in labor income not only raises income tax by $T'(\hat{z})$, but also changes commodity demands x_n^s and therefore increases commodity tax by $\sum_i t_i \frac{x_{in}^s}{z_n}\varepsilon_{x_i z}$, where $\varepsilon_{x_i z} \equiv \frac{z_n}{x_{in}^s}\left(\frac{1}{n}\frac{\partial x_{in}^s}{\partial l_n} + Q_n \frac{\partial x_{in}^s}{\partial y_n}\right)$ denotes labor income elasticity of commodity i's conditional demand of a type-n consumer.

The change of labor income at different income levels may vary. Use $\xi_{zQ}^*(\hat{z})$ to denote the local compensated local elasticity of taxable income with respect to the marginal retention rate Q_n. for each of $\hat{z}$-earners, the change in labor income is $\mathrm{d}\hat{z} = -\mathrm{d}\tau \frac{\hat{z}}{1-T'(\hat{z})}\xi_{zQ}^*(\hat{z})$, the corresponding increase in tax revenue is $T'(\hat{z})\mathrm{d}\hat{z} + \sum_i t_i \frac{x_{in}^s}{z_n}\varepsilon_{x_i z}\mathrm{d}\hat{z}$. Multiply it by the size of $\hat{z}$-earners $\phi f(\hat{z})$ to get the total influence. For consumers earning higher than $\hat{z}$, the labor income decreases by $\frac{1}{Q_n}\phi \mathrm{d}\tau \xi_z^I$, where ξ_z^I denotes the local income elasticity of labor income. [14] The additional tax revenue collected from these consumers is $-\phi \mathrm{d}\tau \int_{\hat{z}}^{\bar{z}} \xi_z^I \left[T'(z) + \sum_i \frac{x_{in}^s}{z_n}\varepsilon_{x_i z}\right]\tilde{f}(z)\mathrm{d}z$. In summary, the "behavioral effect" of the reform can be written as

$$\begin{aligned}\mathrm{d}B = &-\phi \mathrm{d}\tau \int_{\hat{z}}^{\bar{z}} \sum_i t_i \frac{\partial x_{in}^s}{\partial y_n}\tilde{f}(z)\mathrm{d}z - \phi \tilde{f}(\hat{z})\left[T'(\hat{z}) + \sum_i \frac{x_{in}^s}{z_n}\varepsilon_{x_i z}\right]\frac{\hat{z}}{1-T'(\hat{z})}\xi_{zQ}^*(\hat{z})\mathrm{d}\tau \\ &-\phi \mathrm{d}\tau \int_{\hat{z}}^{\bar{z}} \frac{\xi_z^I}{Q(z)}\left[T'(z) + \sum_i \frac{x_{in}^s}{z_n}\varepsilon_{x_i z}\right]\tilde{f}(z)\mathrm{d}z\end{aligned} \tag{6}$$

A.3. The Behavioral Wedge Effects

Due to misperceptions, behavioral wedge effect should be concerned. Recall that a consumer uses an imaginary budget constraint $y_n - nl_nQ_n^s = \bar{R}_n$ to evaluate the cost and benefit of changes in $\mathrm{d}\hat{z}$. Since $\bar{R}_n = R_n - (Q_n^s - Q_n)z_n$, a $\hat{z}$-earner's imaginary budget constraint tightens by $[Q^s(\hat{z}) - Q(\hat{z})]\mathrm{d}\hat{z}$, leading to welfare change by $-g(\hat{z})[Q^s(\hat{z}) - Q(\hat{z})]\mathrm{d}\hat{z}$. For consumers earning higher than $\hat{z}$, we have shown their labor income will change by $-\frac{1}{Q_n}\phi \mathrm{d}\tau \xi_z^I$. Multiply it by $-g(\hat{z})[Q^s(\hat{z}) - Q(\hat{z})]$ to get

[14] By definition, we have $\xi_{zQ}^*(z_n) \equiv \frac{Q(z_n)}{z_n}\frac{\partial z_n}{\partial Q_n} - \xi_z^I(z_n)$ and $\xi_z^I(z_n) \equiv Q_n \frac{\partial z_n}{\partial R_n}$.

relevant welfare changes. In sum, the "behavioral wedge effect" can be expressed as

$$\begin{aligned}\mathrm{d}W =& \phi \tilde{f}(\hat{z})g(\hat{z})[Q^s(\hat{z})-Q(\hat{z})]\frac{\hat{z}}{1-T'(\hat{z})}\xi^*_{zQ}(\hat{z})\mathrm{d}\tau \\ &+\phi \mathrm{d}\tau \int_{\hat{z}}^{\bar{z}} \frac{\xi^I_z}{Q\left(z\right)} g(z)\left[Q^s(z)-Q(z)\right]\tilde{f}(z)\mathrm{d}z\end{aligned} \tag{7}$$

A.4. Optimal Labor Income Tax Formulas

Use the condition $\mathrm{d}M+\mathrm{d}B+\mathrm{d}W=0$, proposition 1 characterizes optimal income tax schedule.

Proposition 1. *The non-linear income tax $T(z)$ satisfies the following expression at all points of differentiability:*

$$J(\hat{z}) = \frac{1}{\hat{z}f\left(\hat{z}\right)}\frac{1}{\xi^*_{zQ}(\hat{z})}\int_{\hat{z}}^{\bar{z}}\left(1-g(z)-\sum_i t_i\frac{\partial x^s_{in}}{\partial y_n}-J(z)\xi^I_z(z)\right)\tilde{f}\left(z\right)\mathrm{d}z \tag{8}$$

with

$$J(z_n)\equiv\frac{T'\left(z_n\right)}{1-T'\left(z_n\right)}+\sum_i\frac{t_i}{Q_n}\frac{x^s_{in}}{z_n}\varepsilon_{x_iz}-g(z_n)\tau^b(z_n) \tag{9}$$

and $\tau^b(z_n)\equiv\frac{Q^s_n-Q_n}{Q_n}$.

Overall, equation (8) can be seen as a modified version of the ABC-formula of Diamond (1998), in which linear commodity taxes and a misperception wedge τn^b are included. τ^b, the misperception wedge of labor income tax, represents the degree of deviation of perceived marginal retention rate from the actual one. The definition is consistent with Farhi and Gabaix (2020).

Solving the first-order linear differential equation of $J(z^*)$ in (8) gives:

$$\frac{T'(z)}{1-T'(z)}+\sum_i\frac{t_i}{Q_n}\frac{x^s_{in}}{z}\varepsilon_{x_iz}-g(z)\tau^b(z)=\frac{1}{\varepsilon^*_{zQ}}\frac{1}{z\tilde{f}(z)}\Theta_z \tag{10}$$

ε^*_{zQ} and ε^I_z are the global compensated tax elasticity and income elasticity of labor income. [15] Θ_z is the marginal social value of redistributing one unit of income from all consumers above income level z to the government. It can also be interpreted as the net social welfare loss caused by increasing $T'(z)$ by one unit. The expression of Θ_z is given by

$$\Theta_z=\int_z^{\bar{z}} e^{-\int_z^{z'}\rho(s)\mathrm{d}s}\left(1-g(z)-\sum_i t_i\frac{x^s_{in}}{y_n}\varepsilon^I_{x_i}\right)\tilde{f}(z')\mathrm{d}z';\rho(z)=-\frac{\varepsilon^I_z}{\varepsilon^*_{zQ}}\frac{1}{z}. \tag{11}$$

The form and interpretation of equation (10) are similar to those of (8). The only two differences are Θ_z is a conditional average of $1-g(z)-\sum_i t_i\frac{\partial x^s_i}{\partial y}$, and that the income effects on labor supply are extracted to formulate the exponential term.

[15] The local and global elasticity of labor income satisfy

$$\varepsilon^*_{zQ}(z_n)=-\frac{\xi^*_{zQ}(z_n)}{1+T''\left(z_n\right)\frac{z_n}{Q_n}\xi^*_{zQ}(z_n)};\varepsilon^I_z(z_n)=-\frac{\xi^I_z(z_n)}{1+T''\left(z_n\right)\frac{z_n}{Q_n}\xi^*_{zQ}(z_n)}.$$

As in Farhi and Gabaix (2020), the optimal tax rates depend on the sum of two terms. One is the standard optimal tax formula for rational consumers, and the other is $g(z)\tau^b(z)$, which reflects the value of misperceptions of marginal income tax rates. Since our model also considers the design of optimal commodity tax rates, the term of standard optimal tax formula is the same as in Jacobs and Boadway (2014).

Compared with Farhi and Gabaix (2020), we assume that a type-n consumer's perception of his marginal income tax rate is not influenced by the marginal tax rates of consumers at other income levels. As a result, there is neither a behavioral "cross-influence" nor an externality in equation (8). The optimal tax formula incorporating such an externality is presented in section VI as an extension.

At the endpoints of the skill distribution, price misperceptions depart optimal marginal tax rate from zero, which contrasts the classic result in Saad (2014) and Seade (1977).[16]

B. Optimal Commodity Tax

Assume that the income tax schedule is optimal and smooth.[17] Consider the effect of slightly increasing the commodity tax rate on commodity j by $\mathrm{d}\tau$.

B.1. The Mechanical Effects

If we do not consider consumers' behavioral changes, the "mechanical effects" of the reform can be expressed as

$$\mathrm{d}M = \mathrm{d}\tau \int_Z [1 - g(z)] x_{jn}^s \tilde{f}(z) \mathrm{d}z. \tag{12}$$

B.2. The Behavioral Effects

We separate the behavioral effects into two parts. The first part ignores any effects on the labor supply: the reform results in a decrease in disposable income and directly changes the government's commodity tax revenue through income effects on commodity demands at the amount of $\sum_i t_i \frac{x_{in}^s}{y_n} \varepsilon_{x_i}^I$ and substitution effects at the amount of $\sum_i t_i \frac{x_{in}^s}{q_j} \varepsilon_{x_i q_j}^*$ from each consumer. $\varepsilon_{x_i}^I$ and $\varepsilon_{x_i q_j}^*$ denote income elasticity and compensated price elasticity of commodity i's conditional demand. [18]

The second part considers the response of labor supply. The reform changes the labor supply by reducing disposable income by x_{jn}^s and by substitution effects. Notice that one unit increase in labor income raises both income tax by $T'(z_n)$ and commodity tax by $\sum_i t_i \frac{x_{in}^s}{z_n} \varepsilon_{x_i z}$.

[16] From first-order conditions on $v_{\bar{n}}$ and $v_{\underline{n}}$ of government's problem, we have $\theta_{\bar{n}} = \theta_{\underline{n}} = 0$. Therefore, marginal income tax rates at the endpoints of the skill distribution satisfy:

$$T'(z_n) = g_n \left(T'(z_n) - T^{s\prime}(z_n) \right) - \sum_i t_i \frac{x_{in}^s}{z_n} \varepsilon_{x_i z}, n \in \{\bar{n}, \underline{n}\}.$$

If we refer to the left-hand side of equation (8) as a total labor wedge, at the endpoints, as long as the sum of the part caused by misperceptions of income tax $-g_n \tau_n^b$ and the part caused by commodity tax $\sum_i \frac{t_i}{Q} \frac{x_{in}^s}{z} \varepsilon_{x_i z}$ are non zero, a direct tax wedge is required to ensure the total labor wedge be at zero at the endpoints.

[17] Although an optimal commodity tax formula derived by tax perturbation methods is valid for any existing income taxation since we aim to analyze the optimal mixed tax system, we assume that the income tax rates are at the optimum in the analysis.

[18] By denition, we have $\varepsilon_{x_i}^I(z_n) \equiv \frac{y_n}{x_{in}^s} \frac{\partial x_{in}^s}{\partial y_n}$ and $\varepsilon_{x_i q_j}^*(z_n) \equiv \frac{q_j}{x_i} \frac{\partial x_n^{s*}}{\partial q_j}$. x_n^{s*} is the compensated conditional demand of a type-n consumer.

Denote by $\xi^*_{zq_i}$ and $\varepsilon^*_{zq_i}$ the local and global compensated commodity tax elasticity of labor income separately.[19] The additional tax revenue from each consumer through income effects is thus $-\left[T'(z_n)+\sum_i t_i \frac{x^s_{in}}{z_n}\varepsilon_{x_i z}\right]\frac{\varepsilon^I_z}{Q_n}x^s_{jn}$; the additional tax revenue through substitution effects is $\left[T'(z_n)+\sum_i t_i \frac{x^s_{in}}{z_n}\varepsilon_{x_i z}\right]\frac{z_n}{q_j}\varepsilon^*_{zq_j}$. In summary, the "behavioral effects" of the commodity reform can be written as

$$\begin{aligned}\mathrm{d}B = \mathrm{d}\tau \int_Z \left[\sum_i t_i \frac{x^s_{in}}{q_j}\varepsilon^*_{x_i q_j} + T'(z_n)\frac{z_n}{q_j}\varepsilon^*_{zq_j} + \sum_i t_i \frac{x^s_{in}}{z_n}\varepsilon_{x_i z}\frac{z_n}{q_j}\varepsilon^*_{zq_j}\right]\tilde{f}(z)\mathrm{d}z \\ -\mathrm{d}\tau \int_Z \left[\sum_i t_i \frac{x^s_{in}}{y_n}\varepsilon^I_{x_i} + T'(z_n)\frac{\varepsilon^I_z}{Q_n} + \sum_i t_i \frac{x^s_{in}}{z_n}\varepsilon_{x_i z}\frac{\varepsilon^I_z}{Q_n}\right]x^s_{jn}\tilde{f}(z)\mathrm{d}z \end{aligned} \quad (13)$$

B.3. The Behavioral Wedge Effects

The behavioral wedge effects emerge under price misperceptions due to differences between actual and imaginary budget constraints, or more rigorously, the traditional Roy's identity and Slutsky Equation being inapplicable. Depending on whether to consider responses in labor supply, the behavioral wedge effects can also be separated into two parts.

The first part includes the disutility caused by an additional tightening of a consumer's imaginary budget constraint. From lemma 1, an increase in q_j brings an additional cost at w^q_{jn} because the consumer evaluates marginal costs using an imaginary budget constraint constructed with perceived prices. Intuitively, if $w^q_{jn} > 0$, which can be the case where a consumer underestimates the prices of the substitutes to commodity j, he prefers to over-consume the substitutes when q_j increases compared with a rational counterpart. This aggravates his feeling of a tight budget and results in a disutility $\mathrm{d}\tau g(z_n)w^q_{jn}$ under the tax reform.

The adoption of an imaginary budget constraint also modifies the decomposition of commodity j's price elasticities. The conventional income elasticity and the substitution elasticity still exist, but the income elasticity should multiply $x^s_{jn}+w^q_{jn}$ in the expression of total price elasticity. Therefore, the commodity tax reform raises additional tax revenue by $\mathrm{d}\tau\sum_i t_i \frac{x^s_{in}}{y_n}\varepsilon^I_{x_j}w^q_{jn}$ from each consumer.

The second part considers changes in labor supply. In the stage-1 decision of a type-n consumer, taking y_n and z_n as two kinds of goods with prices 1 and $-Q_n$ separately, the consumer will feel like his imaginary budget constraint additionally tightens by $(Q^s_n - Q_n)\mathrm{d}z_n$ if z_n increases by $\mathrm{d}z_n$. Thus, the tax reform causes disutility $\mathrm{d}\tau g(z_n)(Q^s_n - Q_n)\frac{\mathrm{d}z_n}{\mathrm{d}q_j}$ from each consumer.

Changes in labor income can further modify the government's tax revenue through imaginary budget constraints. The measure of tightening in imaginary budget constraint $\mathrm{d}I$ can be expressed as $w^q_{jn}+(Q^s_n - Q_n)\frac{z_n}{q_j}\xi^*_{zq_j}$. For labor income tax, the influence on labor income tax collected should include $\mathrm{d}\tau\frac{\varepsilon^I_z}{Q_n}T'(z_n)\mathrm{d}I$. For commodity tax collected, the influence of the tax reform should include $\mathrm{d}\tau\sum_i t_i \frac{x^s_{in}}{z_n}\varepsilon_{x_i z}\frac{\varepsilon^I_z}{Q_n}\mathrm{d}I$.

[19]By definition, we have $\xi^*_{zq_i}(z_n) \equiv \frac{q_i}{z_n}\frac{\partial z^*_n}{\partial q_i}, \varepsilon^*_{zq_i} \equiv \frac{\xi^*_{zq_i}(z_n)}{1+T''(z_n)\frac{z_n}{Q_n}\xi^*_{zQ}(z_n)}$. $z^*_n \equiv n l^*_n$ and l^*_n is the compensated labor supply of a type-n consumer.

In summary, the "behavioral wedge effects" of the commodity tax reform can be expressed as

$$\begin{aligned}\mathrm{d}W=&-\mathrm{d}\tau\int_Z\left[g\left(z\right)+\sum_i t_i\frac{x_{in}^s}{y_n}\varepsilon_{x_j}^I\right]w_{jn}^q\tilde{f}(z)\mathrm{d}z-\mathrm{d}\tau\int_Z g\left(z\right)\frac{\mathrm{d}z_n}{\mathrm{d}q_j}\left(Q_n^s-Q_n\right)\tilde{f}(z)\mathrm{d}z\\&-\mathrm{d}\tau\int_Z\left[\left(T'\left(z\right)+\sum_i t_i\frac{x_{in}^s}{z_n}\varepsilon_{x_iz}\right)\frac{\varepsilon_z^I}{Q_n}\right]\left[w_{jn}^q+\left(Q_n^s-Q_n\right)\frac{z_n}{q_j}\xi_{zq_j}^*\right]\tilde{f}(z)\mathrm{d}z\end{aligned}$$

B.4. Optimal Commodity Tax Formulas

Commodity taxes are set optimally under the condition that $\mathrm{d}M+\mathrm{d}B+\mathrm{d}W=0$ for all kinds of commodities. Proposition 2 provides one possible form of optimal commodity tax equation:

Proposition 2. *Optimal commodity tax derived by the tax perturbation approach: for $\forall j\in\{1:I\}$, the optimal linear commodity tax t_j satisfies:*

$$\begin{aligned}&\overbrace{-\int_Z\left[\sum_i t_i\frac{x_{in}^s}{q_j}\varepsilon_{x_iq_j}^*+\left(T'\left(z\right)+\sum_i t_i\frac{x_{in}^s}{z_n}\varepsilon_{x_iz}\right)\frac{z_n}{q_j}\varepsilon_{zq_j}^*\right]\tilde{f}(z)\mathrm{d}z}^{\textit{Substitution Effects}}\\&=\overbrace{\int_Z\left(1-\gamma(z)\right)x_{jn}^s\tilde{f}(z)\mathrm{d}z}^{\textit{Redistribution Effects}}\qquad\overbrace{-\int_Z\gamma\left(z\right)w_{jn}^q\tilde{f}(z)\mathrm{d}z}^{\textit{Commodity Prices Misperception Effects}}\\&\overbrace{-\int_Z\left[g\left(z\right)\frac{\mathrm{d}z}{\mathrm{d}q_j}+\left(T'\left(z\right)+\sum_i t_i\frac{x_{in}^s}{z_n}\varepsilon_{x_iz}\right)\frac{\varepsilon_z^I}{Q_n}\frac{z_n}{q_j}\xi_{zq_j}^*\right]\left(Q_n^s-Q_n\right)\tilde{f}(z)\mathrm{d}z}^{\textit{Income Tax Misperception Effects}},\end{aligned}\tag{14}$$

in which $\gamma(z_n)$ denotes the social marginal utility of income of a type-n consumer and satisfies

$$\gamma\left(z_n\right)=g\left(z_n\right)+\sum_i t_i\frac{x_{in}^s}{y_n}\varepsilon_{x_i}^I+T'\left(z_n\right)\frac{\varepsilon_z^I}{Q_n}+\sum_i t_i\frac{x_{in}^s}{z_n}\varepsilon_{x_iz}\frac{\varepsilon_z^I}{Q_n}.\tag{15}$$

Equation (14) reorganizes the welfare effects of a change in q_j in the spirit of the Diamond-Mirrlees rule by putting in the left-hand side the substitution effects, and by putting in the right-hand side the redistribution effects. It also clarifies two additional forces due to misperceptions of commodity prices and marginal income tax rates.

By considering a small perturbation of a commodity tax schedule, the influence of misperceptions is obtained in an intuitive way. From the decomposition of welfare effects of a commodity tax increase, misperceptions of prices lead to behavioral wedge effects, which should be offset by other effects, as in fully-rational agent models. However, from equation (14), we can hardly tell how misperceptions influence the sign of optimal commodity tax rates. We follow Ferey, Lockwood and Taubinsky (2024) to derive an alternative expression of optimal commodity taxation. We use a combination of commodity tax reform and income tax reforms which neutralize all lump-sum changes in tax liability. The proof is contained in appendix E.

Proposition 3. *Denote by ε_{zn} the uncompensated elasticity of labor income on ability. For $\forall j \in \{1 : I\}$, optimal linear commodity tax rates at the optimal non-linear income tax satisfy:*

$$\int_Z \sum_i t_i \frac{\partial x_{in}^{s*}}{\partial q_j} \tilde{f}(z)\mathrm{d}z - \int_Z w_{jn}^q \tilde{f}(z)\mathrm{d}z$$
$$= \int_Z \Theta_z \left[\frac{\partial \left(x_{jn}^s + w_{jn}^s \right)}{\partial l} \frac{1}{n\varepsilon_{zn}} + \frac{\partial \left(x_{jn}^s + w_{jn}^s \right)}{\partial y} \left(Q_n^s - Q_n \right) - \frac{\partial Q_n^s}{\partial q_j} \right] \mathrm{d}z. \tag{16}$$

The expression of equation (16) is quite different from equation (14). While equation (14) is derived by comparing marginal benefits and losses between raising government's revenue and improving the social welfare, equation (16) captures the trade-off between distortions (measured in government's income) caused by commodity tax and by labor income tax. Without misperceptions, (16) would reduce to the result in Jacobs and Boadway (2014), which highlights the role of linear commodity tax in releasing the distortion caused by labor income taxation. Commodity tax rates are set optimally if the marginal cost from increasing distortions on consumption equates to the marginal benefit from decreasing distortions in labor supply. Detailed interpretations of equation (16) and influences of misperceptions are presented as follows.

The first item on the left-hand side corresponds to marginal distortions on commodity demands caused by substitution effects, or the marginal excess burden of commodity demand distortions as in Jacobs and Boadway (2014). The second item captures marginal distortions on commodity demands through commodity prices misperceptions, as w_{jn}^s measures the degree of excessive spending on commodities for a type n consumer when there is an increase in t_j if $w_{jn}^s > 0$.

The right-hand side of equation (16) shows how changing q_j changes marginal distortions from non-linear income taxation at all income levels. The terms in the brackets measure how will the marginal income tax rate increase to replicate the influence of increasing q_j on labor supply l and disposable income y at income level z. By improving t_j, we get and increase in q_j, as well as those influence on labor supply l and disposable income y, which means we can reduce marginal income tax rates by the magnitude of items in the brackets. By multiplying the items by Θ_z, the net social welfare loss due to a slight increase in the marginal tax rate at z, we get marginal distortions of income tax rates that can be eliminated at z after introducing an increase in t_j. Through summing over z, we get the total marginal gain from decreasing distortions in labor supply.[20] The composition of the items in the square brackets is much more complex than in Jacobs and Boadway (2014), which can be decomposed in the following way.

Substitution effects with labor supply l_n. In the analysis of Christiansen (1984) and Jacobs and Boadway (2014), a linear commodity tax changes conditional demand of good j, which correlates with labor supply through complementary effects or substitution effects. If $\frac{\partial x_{jn}^s}{\partial l_n} < 0$, a decrease in x_{jn} mitigates the distortions of the income tax caused by discouraging labor supply, motivating the adoption of a positive commodity tax. Misperceptions enter into this effect through the term $\frac{\partial w_{jn}^q}{\partial l_n}$. This term arises because elasticity that matters in measuring marginal decrease in labor supply distortions, actually, is between $\frac{\partial e_n^s}{\partial q_j}$ and l_n, where e_n^s is a consumer's conditional expenditure function in a stage-2 decision. $\frac{\partial e_n^s}{\partial q_j}$ measures the additional consumption expenditure

[20]From the perspective of mechanism design, the right-hand side of equation (16) shows how t_j changes marginal distortions resulting from non-linear income taxation through relaxing or tightening the incentive compatible (IC) constraints. The items in the square brackets measure the impacts of an increase in q_j on a type-n consumer's IC constraint. By multiplying the items by Θ_z, the net social welfare loss due to a slight increase in the marginal tax rate at z, impacts on IC constraints are transformed into impacts on utilities. We solve optimal taxation under misperceptions using the mechanism design approach in appendix EE.2.

after an increase in q_j, holding fixed the consumer's labor supply. When consumers are fully rational, $\frac{\partial e_n^s}{\partial q_j} = x_{jn}^s$. When there exist misperceptions, we have $\frac{\partial e_n^s}{\partial q_j} = x_{jn}^s + w_{jn}^q$. This is because an increase in q_j changes not only the conditional demands on x_{jn}, but also conditional demands on other commodities since perceived prices of these commodities change with q_j. Such changes in commodity demands, summarized by w_{jn}^q, also correlates with labor supply. This explains the elasticity between w_{jn}^q and labor supply in equation (16). Intuitively, the additional consumption expenditure to maintain ones utility after the tax increase can be larger if a consumer underestimates prices of substitutes of the taxed commodity. When the correlation between additional consumption expenditure and labor supply is negative, a consumer with high skill finds it less attractive to pretend to be a low-skill worker because supplying less labor leads to more additional consumption expenditure to keep previous utility level.

An imaginary budget constraint effect. To replicate earnings changes caused by q_j, we need to consider the imaginary budget constraint effect due to income tax misperceptions. Denote by $\mathrm{d}z_n$ the change in labor income resulted from an increase in q_j. When a type-n consumer perceives a lower marginal income tax rate, we have $Q_n^s - Q_n > 0$. Then the consumer will find imaginary budget constraint tightens by $(Q_n^s - Q_n)\mathrm{d}z_n$. Therefore, the distortions caused by labor income tax will be amplified, resulting in higher marginal income tax rates to reproduce earnings changes caused by q_j. For the design of optimal commodity tax rates, this implies we can lower the optimal t_j on a substitute to labor supply if consumers under-estimate marginal labor income tax rates, which is consistent with our intuition.

A direct incentive by q_j. This term shows how cross-commodity influences of actual prices on perceived income tax rates come into changing optimal commodity tax formula. Under assumption 2, an increase in q_j directly changes consumers perceived marginal income tax rates. In this way, the perceived loss from pretending to be a low-skilled individual of a type-n consumer still changes holding fixed conditional commodity demands. When $\frac{\partial Q_n^s}{\partial q_j} < 0$, which means the perceived marginal income tax rate positively correlates with q_j, an decrease in q_j directly increase the incentive to supply labor because consumers perceive lower marginal income tax rates. Therefore, a commodity subsidy contributes to decreasing distortions in labor supply. As an illustration, the government can use an income tax credit for electric motor cars to substitutes fossil fuel vehicles. This subsidy may not depend solely on the reduced externality costs. It can be higher if consumers perceive marginal tax rates using ironing heuristics. A higher subsidy decreases the perceived income tax rates and motivates these consumers to supply labor.

Compared with the optimal taxation expressions in equation (14), equation (16) has an advantage in bringing out the roles of bias-correction and redistribution of indirect taxation. The sin goods tax in Allcott, Lockwood and Taubinsky (2019) also plays the two roles. However, the mechanisms behind them are different from ours. The redistributive motive in Allcott, Lockwood and Taubinsky (2019) arises from preference heterogeneity, which disappears when variations in consumption are purely caused by variations in income. However, in our model, consumers of the same skill levels have the same preference. The redistributive motive comes from complementarity between preference for commodities and labor supply as well as misperceptions of both commodity prices and marginal income tax rates. The bias-correction term is also different due to different causes of behavioral bias. Commodity tax needs to correct a misperception wedge on commodity price in our model. By contrast, the sin tax in their research aims to correct the negative externality of sin goods consumption and internality caused by a mismatch between decision utility and experienced utility.

IV. Revisit Commodity Tax Rules

Based on Proposition 3, we can examine classical commodity tax rules, including Atkinson and Stiglitz theorem, within-group uniform tax rule, many-person Ramsey rule, and Corlett-Hague rule when there exist misperceptions.

Atkinson and Stiglitz theorem revisited. We first revisit the Atkinson and Stiglitz theorem to see whether linear commodity taxes are superfluous when preferences are weakly separable between commodities and labor. We find that commodity tax should still be considered to alleviate labor-supply distortions.

Corollary 1. *When the utility function takes the form $u(h(c_n, x_n), l_n)$, differential linear commodity taxes can be deployed to reduce labor-supply distortions as long as misperceptions of marginal labor income tax rates are influenced by price of commodity j. Define $t_i^s \equiv q_i^s - 1$. For $\forall j \in \{1 : I\}$, at the optimum, we have:*

$$\int_Z \sum_i t_i^s \frac{\partial x_{in}^{s*}}{\partial q_j} \tilde{f}(z) \mathrm{d}z = -\int_Z \Theta_z \left[\frac{\partial Q_n^s}{\partial q_j} - \frac{\partial \left(x_{jn}^s + w_{in}^s \right)}{\partial y} \left(Q_n^s - Q_n \right) \right] \tag{17}$$

If $\frac{\partial Q_n^s}{\partial q_j} = 0$ and $Q_n^s = Q_n$ for $\forall n$, then it is obvious that $t_i^s = 0$ for $\forall i$ is a sufficient condition of (17). When consumers perceive pre-tax prices correctly but not the commodity taxes, t_i^s can be interpreted as a perceived commodity tax rate. The Atkinson and Stiglitz theorem should be modified to require all perceived commodity taxes to be zero. Since actual tax rates might not coincide with perceived tax rates, the government still needs non-uniform commodity taxes; Otherwise, it should make additional efforts (e.g., highlight the tax-inclusive price below the original pre-tax price tags) to eliminate such misperceptions. By intuition, without regard of labor-supply distortions, the government would always prefer consumers to ignore the existence of taxation. If consumers cannot correctly observe pre-tax prices, the role of commodity taxes is to compensate for the gaps in price perception. For instance, generally, consumers fail to foresee the energy costs in the future when purchasing lightbulbs (Allcott and Taubinsky, 2015). This means consumers may over-consume energy-inefficient lightbulbs, which induces sizeable unexpected energy costs in the long run. Therefore, a subsidy for energy-efficient lightbulbs can be used to correct such biases and to improve welfare.

If $\frac{\partial Q_n^s}{\partial q_j} \neq 0$, misperceptions on commodity prices allow for a more progressive income tax system. This is another channel through which commodity taxes play a role. The integral on the right-hand side of equation (17) measures the overall influence of q_j on distortions caused by income tax schedule. The left-hand side measures distortions on consumption caused by substitution effects. At the optimum, the two effects cancel out. Therefore, the government uses commodity taxes to adjust the relative size of the two distortions. In other words, the government can make use of individuals' misperceptions to redistribute income.

To illustrate this point, we take money as a special kind of goods and assume that consumers use ironing heuristics in understanding an income tax schedule. The price of money will affect a consumer's perception of marginal income tax rate if the government sets income tax credits to keep up with the inflation rate. This is because the average tax rates get lower due to tax credits, which are perceived as marginal tax rates due to ironing heuristics. Corollary 1 indicates that, at least in partial equilibrium, setting a positive inflation rate and an inflation-based personal exemption for income tax schedule (e.g., the basic personal amount in Canada) may achieve higher social welfare than just keeping a zero inflation rate.

Within group uniform tax rule revisited. The same intuition applies when we revisit the uniform taxation rule for any separable subgroup of commodities in Deaton (1979). When individuals are rational, the within-group uniform taxation requires that within-group Engel curves are linear, or that the sub-utility function of commodities in this group is homothetic. Corollary 2 describes how a within-group uniform taxation rule is broken by misperceptions.

Corollary 2. *Neither a homothetic sub-utility function nor linear within-group Engel curves are sufficient for a uniform taxation on commodities in that group when consumers misperceive commodity prices.*

Without misperceptions, all the commodities in the subgroup constitute a composite good with linear price, so a linear tax on the composite good is equivalent to taxing all goods in that subgroup at the same rate. By intuition, a uniform tax within the group is superior because it avoids distortions within the composite good. However, if there are misperceptions of commodity prices, we cannot aggregate commodities within that group into a composite commodity with constant marginal cost.[21] Misperceptions cause distortions among the consumption decisions over the subgroup. Therefore, differential within-group commodity taxes are required to correct such distortions. We use the subsidies on energy-saving TVs to illustrate this point. When consumers fully understand that the prices of TVs include both the cost of purchasing (prices on the tag) and the electricity cost, the tax rates on all types of TVs should be uniform. No subsidy should apply without regard of any externality. However, if consumers only consider the prices on the tags at the time of purchasing, they will underestimate the real cost of buying an energy-saving TV. Then the government will find it optimal to subsidize more on the more energy-efficient types.

From the perspective that agents misperceive utilities, Farhi and Gabaix (2019) point out that uniform ad valorem commodity taxes are not optimal in general. Our work supplements their study by showing how misperceptions on prices depart commodity taxation from uniformity.

Many-person Ramsey rule revisited. We find that a bias-correction and a bias-motivated redistribution term should be added to the traditional "many-person Ramsey rule" in our setting.

Corollary 3. *The many-person Ramsey rule with non-linear income tax and misperceptions should be modified as*

$$\begin{aligned}-\frac{1}{\overline{w_j^q}+\overline{x_j^s}}\overline{\sum_i t_i\frac{\partial x_i^{s*}}{\partial q_j}} &= 1-\overline{\Gamma}-\operatorname{cov}\left(\Gamma,\frac{e_{q_i}}{\overline{w_j^q}+\overline{x_j^s}}\right)\\ &\quad-\frac{\overline{w_j^q}}{\overline{w_j^q}+\overline{x_j^s}}+\frac{1}{\overline{w_j^q}+\overline{x_j^s}}\int_Z \Theta_z\frac{\partial Q_n^s}{\partial q_j}\mathrm{d}z.\end{aligned} \tag{18}$$

The "bar" indicates an integral on z. $\Gamma(z)$ is the social marginal utility of increasing one unit of income z and is defined by

$$\mathit{\Gamma}(z)=g(z)+\sum_i t_i\frac{\partial x_{in}^s}{\partial y_n}+\frac{\mathit{\Theta}_z}{f(z)}\frac{\varepsilon_z^I}{\varepsilon_{zQ}^*}+\frac{\mathit{\Theta}_z}{f(z)}\frac{1}{e_{q_i}}\left(Q_n^s\frac{\partial\left(x_{jn}^s+w_{jn}^q\right)}{\partial y_n}+\frac{\partial\left(x_{jn}^s+w_{jn}^q\right)}{\partial z_n}\right) \tag{19}$$

The first line of (18) is the familiar form in the traditional "many-person Ramsey rule" as in Diamond (1975) with multiple goods and continuous agent type. $\bar{\Gamma}$ is the average social marginal utility of income, and $1-\bar{\Gamma}$ captures the government's revenue-raising motive.[22] The covariance

[21]Due to misperceptions, the price of the composite commodity is no longer a weighted average of the price of each commodity in the subgroup.

[22]The equivalence of Γ and γ and be proved using the same logic in proof of Proposition 4.

term is slightly modified as we replace the conventional term x_{jn} with $x^s_{jn} + w^q_{jn}$ so that the right-hand side is smaller if people of higher marginal social utility of income consume more of commodity j and have a higher positive misperception wedge w^q_{jn}. Thus, at the optimum, the government tends to use a smaller tax on commodity j.

The second line of (18) reflects the bias-correction motive of commodity taxation. The first item in the second line of (18) corresponds to *bias-correcting term* in Farhi and Gabaix (2020). Good j is more discouraged if the ratio of an average misperception wedge to an average consumption of commodity j is higher. The second item in the second line is new, which corrects the bias caused by the influence of q_j on the perceived marginal income tax rate. Intuitively, if the price of commodity j contributes to a higher perception of the marginal income tax rate, which means $\frac{\partial Q^s_n}{\partial q_j} < 0$, then commodity j should be more encouraged by commodity subsidy at the optimum.

Allcott, Lockwood and Taubinsky (2018) point out that if commodity taxes are not fully salient, optimal commodity taxes essentially follow the "many-person Ramsey rule" scaled by the degree of inattention. Our work differs from theirs because we assume that consumers misperceive commodity price when they make purchase decisions, while in their setting, commodity tax rates are salient at the time of purchasing but are misunderstood when people choose labor supply. Therefore, we have an additional bias-correcting term. Besides, our scaling factor is different. While the scaling factor in Allcott, Lockwood and Taubinsky (2018) captures the extent to which consumers' labor supply is sensitive to the actual value of misperceived price, our scaling factor in the covariance term reflects the behavioral wedge at the time people purchase goods, which exists even if labor supply is fixed.

Corlett-Hague rule revisited. Jacobs and Boadway (2014) find that the classical Corlett–Hague rule is implied in the Mirrlees framework with optimal linear commodity taxes. This point is also embedded in (16) in our model. With rational consumers, the Corlett–Hague rule in the Mirrlees framework means a high complementarity of one commodity to leisure pushes tax on this commodity upward.[23] If a higher conditional commodity demand corresponds to a lower labor supply and more leisure time, commodity tax tends to be positive.

When there exist misperceptions, from (16) we learn that the item $\frac{\partial x^s_{jn}}{\partial l_n}$ still plays an important role in shaping optimal commodity taxation, while additional items arise as has been explained for the right-hand side of (16).

V. An Illustrative Example

The quantitative importance of misperceptions to optimal mixed taxation is analyzed in this section.

A. The Case of EV credit

We apply our theory to the situation at the beginning of our introduction where a worker is about to purchase an electrical motor car. We show how optimal income tax schedule as well as optimal commodity tax rate differs from conventional analysis when we take into consideration misperceptions. Under reasonable parameters, compared with the case where the government believes that consumers are rational, we find that once the government incorporates misperceptions in tax policies, there will be a higher subsidy for electrical motor cars, a higher marginal income tax rates, and a considerable increase in social welfare.

[23] Such complementarity is measured using the partial derivative of conditional commodity demands on labor supply. As has been stressed in Christiansen (1984), the correlation between labor supply and commodity demand is akin to but not identical to compensated elasticity concepts used by Corlett and Hague (1953).

A.1. Parameters

Assume that there is only one kind of general good x, the electric motor car. We adopt the following utility function for a type-n consumer

$$u\left(c_n^s, x_n^s, l_n\right) = (c_n^s)^{\beta}(x_n^s)^{(1-\beta)} - \frac{1}{\sigma}(l_n)^{\sigma}. \tag{20}$$

β and σ are parameters about consumer's preference. This specification for utility implies a zero commodity tax when consumers are fully rational. Therefore, a non-zero subsidy for electrical motor cars in our simulation totally results from misperceptions. The parameter σ controls the labor supply elasticity. Chetty (2012) estimates the preferred intensive margin elasticity to be 0.33. We set $\sigma = 4$ to match this elasticity. We calibrate β using the average share of consumer spending on new vehicle purchases from the Consumer Expenditure Survey in the United States.[24]

The government uses an income tax credit to accelerate electric vehicle purchases. The federal governments have provided various incentives in an effort to promote the adoption of electric vehicles. (e.g. the Alternative Motor Vehicle Credit, or AMVC; the Qualified Plug-In Electric Drive Motor Vehicle Credit, or PEDVC; and the federal Plug-in Electric Drive Vehicle Credit, or IRS 30D.) Since we use the 2010 Current Population Survey (CPS) to calibrate the income distribution and current income tax schedule, we focus on PEDVC, a credit for electric vehicles and plug-in hybrid vehicles purchased starting from 2009. The size of the PEDVC varies with the battery capacity of the vehicle. For example, the credit for the Chevrolet Volt is $7500, while for the Toyota Prius the credit is $2500. We compute the subsidy rate using tax credit and before tax price of the Toyota Prius, the most popular model of qualifying vehicles according to the National Household Travel Survey in the United States. We adopt a subsidy rate of 0.086 in our baseline simulation so that the after-tax price of electric motor cars is $q = 0.9127$.

As the pre-tax price of an electric motor car is normalized to 1, $(1-q)x_n^s$ represents the total amount of subsidies a consumer receives from purchasing electric motor cars. The pre-tax price consists of a tag price of an electric motor car and all the discounted fuel costs. To calibrate the share of the two compositions, we use the National Household Travel Survey (NHTS) in the United States.[25] (See Appendix D for details.)

We assume a type-n consumer perceive his marginal income tax rate $T_n^{s'}$ as

$$T_n^{s'} = k_1 T'(z_n) - k_2 \frac{(1-q)x_n^s}{z_n}. \tag{21}$$

The parameter k_1 represents a consumer's inattention to actual income tax. We set $k_1 = 1$ in our baseline simulation to highlight how the influence of commodity price on income tax misperceptions shape optimal income tax schedule. The second item on the right-hand side of equation (21) reflects an "ironing heuristic", a common type of misperceptions of non-linear price schedule. First proposed in Liebman and Zeckhauser (2004), an "ironing heuristic" implies that an individual believes that the marginal incentive is equal to his average incentive. For example, a taxpayer may behave as if their marginal tax rates are given by their average tax rates. Misperceptions on income tax rate are documented in Brown (1969), Fujii and Hawley (1988) and Romich and Weisner (2000). Taxpayers not only find it hard to give their correct marginal tax rate in surveys but also confuse a lump

[24]We compute the average ratio of new vehicle purchases to after-tax income between 2000 to 2020 from the Consumer Expenditure Survey. The expression of that ratio derived from our model is a function of β and prices. We choose the unique β to equate the result from our model to the data.

[25]NHTS provides information about travel by US residents. The information relevant to our calibration includes fuel types, annual fuel expenditures, makers, and models of each vehicle owned by each household surveyed.

sum tax reform with marginal tax reform (Feldman, KatuščÁk and Kawano, 2016). de Bartolome (1995) uses a laboratory experiment to find that taxpayers tend to behave as if their marginal tax rate is given by their average tax rate. Rees-Jones and Taubinsky (2020) find out that 43% of the population irons. When a consumer with ironing heuristic enjoys a federal income tax credit at an amount of $(1-q)x_n^s$ for his electric car, he will find his average labor income tax rate is lowered, which decreases his perceived marginal tax rate by $k_2(1-q)x_n^s/z_n$. The parameter k_2 reflects a consumer's inattention to his income tax credit. It can also be interpreted as the degree of adopting "ironing heuristic". When consumers pay no attention to the income tax credit, or there is no ironing heuristic, the second item disappears in equation (21). To clearly separate the influence of misunderstanding income tax credit, we do not take into consideration how marginal tax rates at other income levels affect a consumer's perception.

The actual price of an electric motor car q is perceived as q^s. There is a large empirical research on estimating consumers' undervaluation of future energy costs on the automobile market. Based on consumer responses to changes in fuel costs, Allcott and Wozny (2014) find that consumers are indifferent between 76 cents in the purchase price and one dollar of discounted future gas cost. Leard, Linn and Zhou (2017) find a more severe underestimation of future energy costs, where the willingness to pay for one dollar of discounted future fuel cost savings is 54 cents. However, Busse, Knittel and Zettelmeyer (2013) and Sallee, West and Fan (2016) find full estimation of future energy costs. We adopt a modest undervaluation of 76% of future energy cost identified in Allcott and Wozny (2014). The result of a 54% perception is also reported. Since we have specified that preferences are weakly separable between commodities and labor, optimal commodity taxes are not aimed at correcting consumers' misperceptions on commodity prices but reflect a motivation to change commodity prices in order to boost labor supply through changing perceived marginal income tax rates.

Following Lockwood (2020), we calibrate the income distribution and current income tax schedule using the 2010 Current Population Survey (CPS) and the National Bureau of Economic Research's TAXSIM calculator (Feenberg and Coutts, 1993). We invert the first-order condition for an individual's labor supply to obtain the ability distribution under the prevailing income distribution and the tax code.

Finally, we check the robustness of the results by adopting different marginal welfare weights g_n. In our baseline simulation, as in Lockwood (2020), we adopt g-weights such that consumers with the lowest income are weighted by 10% more than the median household, while top earners receive a weight 40% less than the median. $g(z)$ is linearly interpolated between income percentiles 0, 50, and 100. These weights are more redistributive than tastes embodied in existing tax policies, and much less redistributive than a logarithmic redistributive preference conventionally used in the optimal taxation literature.[26] The results under social preferences inferred from an inverse optimal analysis are also reported. The parameters selected are summarized in table 1.

A.2. Optimal Tax Simulations

To see the sensitivity of the optimal tax schedule to different assumptions, we simulate the optimal mixed tax schedule under eight different specifications. Figure 1 presents the shape of marginal income tax. Table 2 report optimal subsidy rate and optimal average subsidy expenditure on electric motor cars. The welfare implications of considering misperceptions in optimal tax design are also presented.

[26] Under a logarithmic redistributive preference, the welfare weight of a type-n consumer is equal to the inverse of his utility.

Symbol	Sufficient Statistics	Value
β	Preference parameter	0.9807
σ	Preference parameter	4
q	After tax price of electric motor cars	0.9119
q^s	Perceived price of electric motor cars	0.8780
k_1	Attention to marginal income tax rates	1
k_2	Attention to income tax credits	1

Table 1: Parameters for Baseline Optimal Tax Calculation

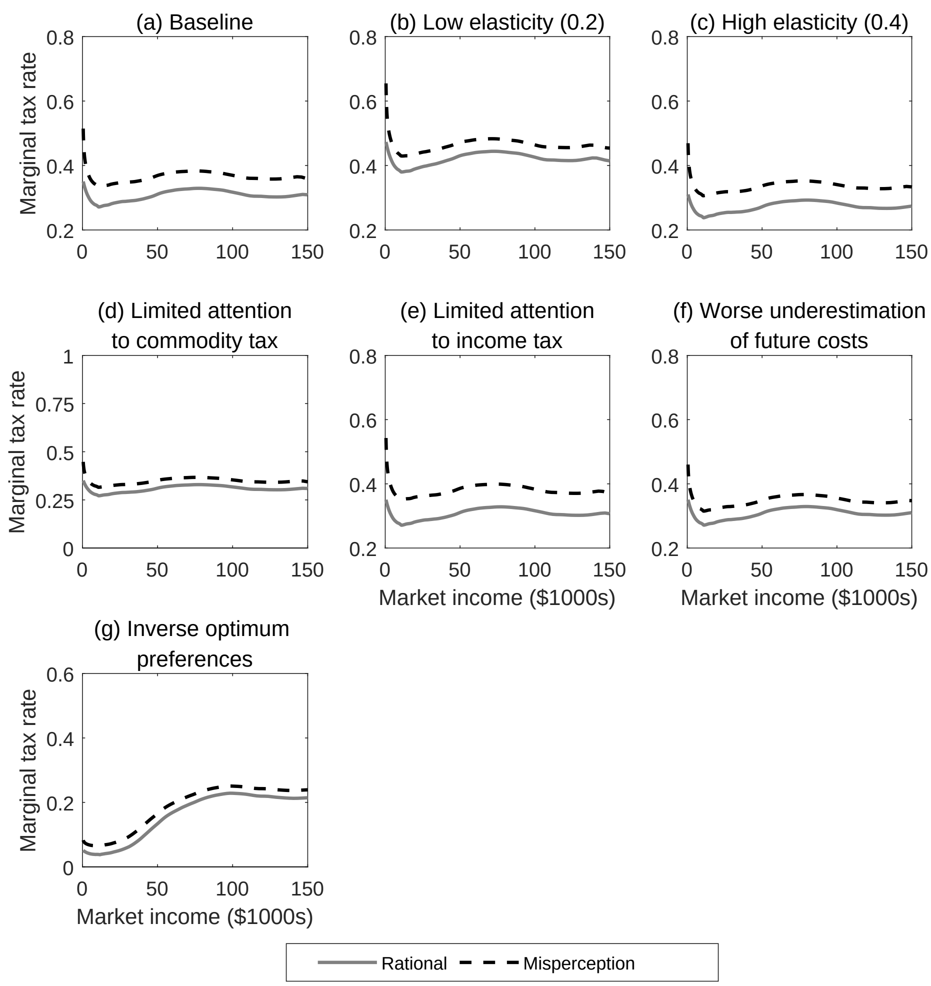

Figure 1: Optimal Labor Income Taxation

Panel (a) in figure 1 plots the schedule of optimal marginal income tax rates in the baseline model economy with and without misperceptions, under a reasonable set of parameters collected in table **??**. Misperceptions tend to increase optimal marginal income tax rates across a large range of incomes. Panel (b) and (c) present results under lower and higher labor supply elasticities. Panel

	Optimal subsidy rate (%)	Average subsidies (current: 145.82$)	Welfare increase: relative to welfare level in conventional optimum (%)	Welfare increase: a share of improvement in conventional optimum (%)
(a) Baseline	11.73	196.85	0.16	21.56
(b) Low elasticity (0.2)	11.45	183.57	0.24	36.34
(c) High elasticity (0.4)	11.85	203.72	0.10	8.79
(d) Underestimate subsidy	8.74	142.99	0.15	20.33
(e) Underestimate income tax rates	11.25	187.26	0.66	150.44
(f) Perceive less future costs	8.16	136.29	0.15	7.18
(g) Inverse optimum preferences	4.37	73.44	0.03	8.25

Table 2: Optimal Subsidies and Welfare Increase Under Optimal Taxation

(d) assumes that $k_2 = 0.8$, which means consumers only perceive 80% of commodity tax rates. Panel (e) plots optimal tax rates assuming $k_1 = 0.95$, which means consumers only perceive 95 percent of marginal tax rates. Panel (f) corresponds to the estimation in Leard, Linn and Zhou (2017) that a consumer's willingness to pay for one dollar of discounted future fuel cost savings is 54 cents. Panel (g) adopts a redistributive preference that rationalizes U.S. income tax. The specifications (a) to (g) in Table 2 correspond to these cases separately.

The simulation results show the following key lessons for tax policy when consumers misperceive prices. First, the optimal commodity tax rates are no longer zero when consumers' preferences are separable between commodities and labor. Under the specific case of electric vehicle purchasing, optimal subsidy rates in the form of an income tax credit range from 4.37% to 11.85% depending on parameters selected. This verifies our analysis in Corollary 1. The optimal tax credit should be positive because consumers will perceive lower marginal income tax rates, making it easier for the government to redistribute resources. The size of the optimal tax credit is sensitive to consumers' attention to both types of taxes. Compared to the baseline case, the optimal tax credit should decrease when a consumer pays less attention to either his tax credit or his marginal income tax rate. In addition, the average subsidies under the optimal tax schedule are larger if the government has a stronger taste for redistribution.

The second lesson relates to the design of optimal income tax. In simulation results from all these specifications, we find that misperceptions, or more specifically the influence of commodity prices on perceived marginal income tax rates, leads to higher optimal marginal income tax rates across an extensive range of incomes, especially the middle incomes. This means a more progressive income tax schedule is preferred due to misperceptions, which is consistent with our analysis in Corollary 1.

Third, once taken into account misperceptions, the increase in aggregate social welfare can be considerable. The last two columns of Table 2 report the welfare changes from considering misperceptions in the design of optimal tax schedule. Welfare is measured as the level of consumption that must be given to an average worker to make her indifferent between consuming this without having to work at all, and participating in the labor market as in the model.[27] Column (3) reports the percentage of welfare changes compared with an economy adopting a conventional optimal policy, or in other words, an optimal tax schedule without considering misperceptions. We also compute

[27]Since there are two kinds of consumption goods that are not perfect substitutes, we construct a composite good to measure the amount of consumption. The utility brought by consumption of composite good equals to the utility brought by c_n and x_n.

separately the welfare increase of a conventional optimum relative to existing policy and the welfare increase of an optimum considering misperceptions relative to conventional optimum. The share of the latter value to the former value is reported in column (4).

In the baseline case, compared with a conventional optimum, our optimal tax schedule provides welfare of 0.16% higher. The additional welfare improvement due to considering misperceptions is nearly one-fifth of welfare improvement brought by a conventional optimal policy. With a lower labor supply elasticity, the optimal tax schedule in our framework will provide a larger welfare improvement of 0.24% relative to the welfare level under conventional optimal tax schedule, or more than one-third of a welfare improvement brought by conventional optimum. A less redistributive social preference, a higher labor supply elasticity, or a worse underestimation of future energy costs can lower the share of additional welfare improvement attributed to considering misperceptions to less than one-tenth. In a word, while a conventional optimal tax schedule can increase the welfare of the economy under existing policies, considering misperceptions in the design of optimal taxation further improves welfare at a range from less than one-tenth to over one-third as a share of welfare improvement brought by conventional optimal policies, depending on model parameters.

Then we add two more misperceptions separately into the model. When consumers pay less attention to income credits for electric vehicles, the welfare under conventionally designed optimal tax policies has a slightly smaller room for improvement, which is around 0.15% relative to welfare level of conventional optimum, or slightly over one fifth as a share of welfare improvement of conventional optimum. When consumers underestimate their marginal income tax rates, the welfare improvement relative to the conventional optimum will be much larger, reaching 0.66%. If we compare welfare improvement by considering misperceptions in optimal policies with welfare improvement by conventional optimal policies, the former is one and a half times larger than the latter.

VI. Discussion

A. Connecting Results from the Two Methods

The mechanism design approach and tax perturbation approach are two widely used methods in solving optimal non-linear income taxation when individuals' characteristics are hidden.[28]. The mechanism design approach lends itself to the toolbox of optimal control theory and solves the allocation problem with a mathematically well-defined procedure. It has been broadly applied not only in static (Mirrlees, 1971) but also in dynamic settings (Golosov et al., 2006)

The tax perturbation approach is more intuitive as it directly presents the social welfare effects of changes in taxes. Saez (2001) firstly heuristically applied the tax perturbation approach and showed the expression of optimal income tax corresponds to results obtained using the mechanism design approach. Soon more research followed (Saez, 2002; Jacquet, Lehmann and Van der Linden, 2013) relying on heuristics until Gerritsen (2024) provided a rigorous procedure for solving the class of problems.

Most research on optimal non-linear income taxation with inattentive consumers uses only a perturbation method to solve optimal tax formulas (Farhi and Gabaix, 2020; Allcott, Lockwood and Taubinsky, 2019). Gerster and Kramm (2019) and Lockwood (2020) use the mechanism design approach when consumers misperceive utilities. However, there is hardly any research that uses the mechanism design approach when there exist misperceptions of prices.

[28]The mechanism design approach is also called the primal approach. The tax perturbation approach is also named the dual approach.

It can be easily verified that the expression of optimal income tax from the tax perturbation approach can be transformed into the result from the mechanism design approach: Equation (8) is a first-order linear differential equation of $J(z^*)$, which can be integrated to get an expression of optimal income tax from the mechanism design approach.

However, even with fully rational agents, the expressions of optimal linear commodity tax from the two approaches look quite different, and there is hardly any discussion on their relationships. Furthermore, as has been reviewed in section I, the discussion of the connection between optimal income tax formulas by the tax perturbation and mechanism design methods is limited. In this subsection, we find that the seemingly different expressions of optimal linear commodity tax from the two approaches are equivalent, which applies with and without price misperceptions. We interpret such equivalence by linking the commodity tax rules behind the two expressions together.

Proposition 4. *Given that the non-linear income tax schedule is at its optimum, an optimal commodity tax formula derived from the tax perturbation method can be transformed into an optimal commodity tax formula solved by the mechanism design approach.*

To the best of our knowledge, proposition 4 is the first to establish equivalence between a mechanism design and a tax perturbation method concerning commodity tax in a model with heterogeneous agents and mixed taxation. The equivalence still exists when consumers are fully rational. The tedious proof is provided in the appendix EE.4.

B. Ironing Heuristics

The perceived marginal income tax rates may also depend on marginal tax rates at other income levels. For example, a taxpayer may behave as if their marginal tax rates are given by their average tax rates. This kind of understanding of non-linear price schedule is called "ironing heuristic"(Liebman and Zeckhauser, 2004).de Bartolome (1995) uses a laboratory experiment to find that taxpayers tend to behave as if their marginal tax rate is given by their average tax rate. Rees-Jones and Taubinsky (2020) find out that 43% of the population irons. One example of perceived marginal income tax rate is discussed in Farhi and Gabaix (2020), where Q_n^s satisfies

$$Q_n^s = mQ(z_n) + (1-m)\frac{r_0 + \int_0^{z_n} Q\left(z\right)dz}{z_n}.$$

m denotes attention to the actual marginal tax rate. r_0 denotes a tax rebate at 0 income.

Farhi and Gabaix (2020) provide optimal income tax schedule for this setting. To compare our results with theirs, we extend our model by assuming Q_n^s is a function of q, Q_n, z_n and vector of marginal retention rates $\mathbf{Q}$. This assumption on Q_n^s also captures other patterns of income tax misperceptions like overconfidence.[29] This assumption implies that the marginal tax rates at other income levels exert some externalities on the perceived marginal income tax rate at a certain income level.

Denote by ε^*_{mQn} the compensated tax elasticity of z_m when marginal tax rate at z_n changes. Denote by δ_z the Dirac distribution at point z_n. Optimal income tax schedule is then characterized in proposition 5.

[29]Bénabou and Tirole (2006) and Alesina, Stantcheva and Teso (2018) propose and verify that people are overconfident of achieving high incomes. It is then reasonable that people use marginal tax rates of high incomes to shape their actual tax rates.

Proposition 5. *When Q_n^s is determined by $Q_n^s(q, Q_n, \mathbf{Q}, z_n)$, the optimal income tax rate satisfies*

$$\begin{aligned} \frac{T'(z)}{1-T'(z)} + \sum_i \frac{t_i}{Q}\frac{x_{in}^s}{z}\varepsilon_{x_i z} - g_n \tau_n^b = &\frac{1}{\varepsilon_{zQ}^*}\frac{1}{z\tilde{f}(z)}\Theta_z \\ &+ \frac{T''(z)}{1-T'(z)}\frac{1}{z\tilde{f}(z)}\frac{n}{\varepsilon_{zn}}\int_Z \left[\frac{\varepsilon_{mQ_n}^*}{\varepsilon_{mQ_m}^*}\Theta_m(1-\delta_z)\right]\mathrm{d}m \end{aligned} \tag{22}$$

with

$$\Theta_z = \int\limits_{z}^{\bar{z}} e^{-\int\limits_{z}^{z'}\rho(s)\mathrm{d}s}\left(1 - g - \sum_i t_i \frac{\partial x_i^s}{\partial y}\right)\tilde{f}(z')\mathrm{d}z'; \rho = -\frac{\varepsilon_z^I}{\varepsilon_{zQ}^*}\frac{1}{z} \tag{23}$$

This ABCD formula is a combination of results in Jacobs and Boadway (2014) which explore optimal mixed taxation with fully rational individuals and results in Farhi and Gabaix (2020), which derive optimal income tax formula with inattentive agents. The introduction of externalities accounts for the second line of equation (22), implying that income tax should correct for such externalities.

VII. Conclusion

This paper explores optimal linear commodity tax mixed with non-linear labor income tax formulas when consumers misperceive commodity prices and marginal income tax rates. Due to misperceptions, consumers perform imperfect optimization. The results from a tax perturbation approach show that misperceptions change optimal tax formulas through changing consumers' imaginary budget constraints. The optimal commodity tax formulas are modified by adding a term of marginal loss of utility caused by misperceptions to the Diamond-Mirrlees Rule. As in Farhi and Gabaix (2020), the optimal income tax rates depend on the sum of the standard optimal tax formula for rational consumers, and a term reflecting income tax misperceptions. Since commodity taxation has both a corrective role and a redistributive role when there exist misperceptions, a linear commodity tax may not be superfluous with the typical preference structure in the Atkinson and Stiglitz theorem. Similarly, the within-group uniform tax rule no longer holds. A bias-correction and a bias-motivated redistribution term should be added to the traditional "many-person Ramsey rule".

This analytical framework can be helpful in dealing with a "cross-commodity influence of actual prices on perceived prices", especially when perceived marginal income tax rates are influenced by commodity prices (i.e., a tax credit from purchasing specific commodities). Commodity taxes play a redistributive role, and a more progressive income tax schedule can be adopted. This possibly explains why a government chooses an income tax credit for a kind of commodity at the cost of increasing tax complexity instead of directly using a point-of-sale rebate. In a simple specification with two commodities, we simulate the optimal tax system when consumers mistakenly believe their marginal income tax rates are lower if they can enjoy a higher income tax credit for electric vehicles. The welfare gain from considering misperceptions in the design of optimal mixed taxation is considerable under reasonable parameters.

One should be cautious about advocating our optimal tax estimate too strongly as our setting has some simplifications. A more precise quantitative analysis can be performed considering the reduction in externality costs, the influence of marginal tax rates at other income levels on perceived income tax rates, or the heterogeneity in misperceptions.

References

Alesina, Alberto, Stefanie Stantcheva, and Edoardo Teso. 2018. "Intergenerational Mobility and Preferences for Redistribution." *American Economic Review*, 108(2): 521–554.

Allcott, Hunt, and Dmitry Taubinsky. 2015. "Evaluating Behaviorally Motivated Policy: Experimental Evidence from the Lightbulb Market." *American Economic Review*, 105(8): 2501–38.

Allcott, Hunt, and Nathan Wozny. 2014. "Gasoline Prices, Fuel Economy, and the Energy Paradox." *Review of Economics and Statistics*, 96(5): 779–795.

Allcott, Hunt, Benjamin B. Lockwood, and Dmitry Taubinsky. 2018. "Ramsey Strikes Back: Optimal Commodity Tax and Redistribution in the Presence of Salience Effects." *AEA Papers and Proceedings*, 108: 88–92.

Allcott, Hunt, Benjamin B. Lockwood, and Dmitry Taubinsky. 2019. "Regressive Sin Taxes, with an Application to the Optimal Soda Tax." *The Quarterly Journal of Economics*, 134(3): 1557–1626.

Atkinson, Anthony B., and Joseph E. Stiglitz. 1976. "The Design of Tax Structure: Direct Versus Indirect Taxation." *Journal of Public Economics*, 6(1-2): 55–75.

Bénabou, Roland, and Jean Tirole. 2006. "Belief in a Just World and Redistributive Politics." *Quarterly Journal of Economics*, 121(2): 699–746.

Besley, Timothy, and Ian Jewitt. 1995. "Uniform Taxation and Consumer Preferences." *Journal of Public Economics*, 58(1): 73–84.

Boccanfuso, Jérémy, and Antoine Ferey. 2024. "Inattention and the Taxation Bias." *Journal of the European Economic Association*, 22(3): 1452–1494.

Brown, C. V. 1969. "Misconceptions About Income Tax and Incentives." *Scottish Journal of Political Economy*, 16(2): 1–21.

Brown, Jennifer, Tanjim Hossain, and John Morgan. 2010. "Shrouded Attributes and Information Suppression: Evidence from the Field." *Quarterly Journal of Economics*, 125(2): 859–876.

Busse, Meghan R, Christopher R Knittel, and Florian Zettelmeyer. 2013. "Are Consumers Myopic? Evidence from New and Used Car Purchases." *American Economic Review*, 103(1): 220–56.

Busse, Meghan R., Nicola Lacetera, Devin G. Pope, Jorge Silva-Risso, and Justin R. Sydnor. 2013. "Estimating the Effect of Salience in Wholesale and Retail Car Markets." *American Economic Review*, 103(3): 575–579.

Chetty, Raj. 2012. "Bounds on Elasticities with Optimization Frictions: A Synthesis of Micro and Macro Evidence on Labor Supply." *Econometrica*, 80(3): 969–1018.

Chetty, Raj, Adam Looney, and Kory Kroft. 2009. "Salience and Taxation: Theory and Evidence." *American Economic Review*, 99(4): 1145–1177.

Christiansen, Vidar. 1984. "Which Commodity Taxes Should Supplement the Income Tax?" *Journal of Public Economics*, 24(2): 195–220.

Corlett, W. J., and D. C. Hague. 1953. "Complementarity of the Excess Burden." *The Review of Economic Studies*, 21(1): 21–30.

Corwin, Shane A., and Jay F. Coughenour. 2008. "Limited Attention and the Allocation of Effort in Securities Trading." *The Journal of Finance*, 63(6): 3031–3067.

da Costa, Carlos E., and Iván Werning. 2008. "On the Optimality of the Friedman Rule with Heterogeneous Agents and Nonlinear Income Taxation." *Journal of Political Economy*, 116(1): 82–112.

Deaton, Angus. 1979. "Optimally Uniform Commodity Taxes." *Economics Letters*, 2(4): 357–361.

de Bartolome, Charles A.M. 1995. "Which Tax Rate do People Use: Average or Marginal?" *Journal of Public Economics*, 56(1): 79–96.

Diamond, Peter A. 1975. "A Many-Person Ramsey Tax Rule." *Journal of Public Economics*, 4(4): 335–342.

Diamond, Peter A. 1998. "Optimal Income Taxation: An Example with a U-Shaped Pattern of Optimal Marginal Tax Rates." *American Economic Review*, 88(1): 83–95.

Farhi, Emmanuel, and Xavier Gabaix. 2019. "Optimal Taxation with Behavioral Agents." NBER Working Paper 21524.

Farhi, Emmanuel, and Xavier Gabaix. 2020. "Optimal Taxation with Behavioral Agents." *American Economic Review*, 110(1): 298–336.

Feenberg, Daniel, and Elisabeth Coutts. 1993. "An Introduction to the TAXSIM Model." *Journal of Policy Analysis and Management*, 12(1): 189–194.

Feldman, Naomi E., Peter KatuščÁk, and Laura Kawano. 2016. "Taxpayer Confusion: Evidence from the Child Tax Credit." *American Economic Review*, 106(3): 807–835.

Ferey, Antoine, Benjamin Lockwood, and Dmitry Taubinsky. 2024. "Sufficient Statistics for Nonlinear Tax Systems with General Across-Income Heterogeneity." *American Economic Review (Conditional Accept).*

Fujii, Edwin T., and Clifford B. Hawley. 1988. "On the Accuracy of Tax Perceptions." *The Review of Economics and Statistics*, 70(2): 344.

Gabaix, Xavier. 2014. "A Sparsity-Based Model of Bounded Rationality." *The Quarterly Journal of Economics*, 129(4): 1661–1710.

Gerritsen, Aart. 2024. "Optimal nonlinear taxation: a simpler approach." *International Tax and Public Finance*, 31(2): 486–510.

Gerster, Andreas, and Michael Kramm. 2019. "Correct Me if You Can-Optimal Non-Linear Taxation of Internalities." CESifo Working Paper No.7732.

Goldin, Jacob. 2015. "Optimal Tax Salience." *Journal of Public Economics*, 131: 115–123.

Golosov, Mikhail, Aleh Tsyvinski, Ivan Werning, Peter Diamond, and Kenneth L Judd. 2006. "New Dynamic Public Finance: A User's Guide [with Comments and Discussion]." *NBER macroeconomics annual*, 21: 317–387.

Golosov, Mikhail, Maxim Troshkin, Aleh Tsyvinski, and Matthew Weinzierl. 2013. "Preference Heterogeneity and Optimal Capital Income Taxation." *Journal of Public Economics*, 97(1): 160–175.

Jacobs, Bas, and Robin Boadway. 2014. "Optimal Linear Commodity Taxation under Optimal Non-linear Income Taxation." *Journal of Public Economics*, 117: 201–210.

Jacquet, Laurence, Etienne Lehmann, and Bruno Van der Linden. 2013. "Optimal Redistributive Taxation with both Extensive and Intensive Responses." *Journal of Economic Theory*, 148(5): 1770–1805.

Katseli-Papaefstratiou, Louka T. 1979. "Nominal Tax Rates and the Effectiveness of Fiscal Policy." *National Tax Journal*, 32(1): 77–82.

Lacetera, Nicola, Devin G. Pope, and Justin R. Sydnor. 2012. "Heuristic Thinking and Limited Attention in the Car Market." *American Economic Review*, 102(5): 2206–2236.

Leard, Benjamin, Joshua Linn, and Yichen Christy Zhou. 2017. "How Much Do Consumers Value Fuel Economy and Performance? Evidence from Technology Adoption." *The Review of Economics and Statistics*, 1–45.

Liebman, Jeffrey B., and Richard J. Zeckhauser. 2004. "Schmeduling." https://sites.hks.harvard.edu/fs/rzeckhau/Schmeduling_Oct172004.pdf (accessed January 24, 2019).

Lockwood, Benjamin B. 2020. "Optimal Income Taxation with Present Bias." *American Economic Journal: Economic Policy*, 12(4): 298–327.

Mankiw, N Gregory, Matthew Weinzierl, and Danny Yagan. 2009. "Optimal Taxation in Theory and Practice." *Journal of Economic Perspectives*, 23(4): 147–74.

Mayhew, Glenn E., and Russell S. Winer. 1992. "An Empirical Analysis of Internal and External Reference Prices Using Scanner Data." *Journal of consumer Research*, 19(1): 62–70.

Mirrlees, James A. 1971. "An Exploration in the Theory of Optimum Income Taxation." *The Review of Economic Studies*, 38(2): 175–208.

Mirrlees, James A. 1976. "Optimal Tax Theory: A Synthesis." *Journal of Public Economics*, 6(4): 327–358.

Moore, Dylan T, and Joel Slemrod. 2021. "Optimal Tax Systems with Endogenous Behavioral Biases." *Journal of Public Economics*, 197: 104384.

Niedrich, Ronald W., Subhash Sharma, and Douglas H. Wedell. 2001. "Reference Price and Price Perceptions: A Comparison of Alternative Models." *Journal of Consumer Research*, 28(3): 339–354.

Reck, Daniel. 2016. "Taxes and mistakes: what's in a sufficient statistic?" *Available at SSRN 2268617.*

Rees-Jones, Alex, and Dmitry Taubinsky. 2018. "Taxing Humans: Pitfalls of the Mechanism Design Approach and Potential Resolutions." *Tax Policy and the Economy*, 32(1): 107–133.

Rees-Jones, Alex, and Dmitry Taubinsky. 2020. "Measuring "Schmeduling"." *The Review of Economic Studies*, 87(5): 2399–2438.

Roger, Tristan, Patrick Roger, and Alain Schatt. 2018. "Behavioral Bias in Number Processing: Evidence from Analysts' Expectations." *Journal of Economic Behavior & Organization*, 149(0): 315–331.

Romich, Jennifer L., and Thomas Weisner. 2000. "How Families View and Use the EITC: Advance Payment versus Lump Sum Delivery." *National Tax Journal*, 53(4): 1245–1266.

Saad, Natrah. 2014. "Tax Knowledge, Tax Complexity and Tax Compliance: Taxpayers' View." *Procedia-Social and Behavioral Sciences*, 109: 1069–1075.

Saez, Emmanuel. 2001. "Using Elasticities to Derive Optimal Income Tax Rates." *Review of Economic Studies*, 68: 205–229.

Saez, Emmanuel. 2002. "The Desirability of Commodity Taxation Under Non-linear Income Taxation and Heterogeneous Tastes." *Journal of Public Economics*, 83(2): 217–230.

Sallee, James M, Sarah E West, and Wei Fan. 2016. "Do Consumers Recognize the Value of Fuel Economy? Evidence from Used Car Prices and Gasoline Price Fluctuations." *Journal of Public Economics*, 135: 61–73.

Seade, Jesús. 1977. "On the Shape of Optimal Tax Schedules." *Journal of Public Economics*, 7(2): 203–235.

Shue, Kelly, and Richard R Townsend. 2021. "Can the Market Multiply and Divide? Non-Proportional Thinking in Financial Markets." *The Journal of Finance*, 76(5): 2307–2357.

Taubinsky, Dmitry, and Alex Rees-Jones. 2017. "Attention Variation and Welfare: Theory and Evidence from a Tax Salience Experiment." *The Review of Economic Studies*, 85(4): 2462–2496.

Appendix

Appendix A provides details on an individual's behavior. Appendix B uses the tax perturbation method to get optimal tax formulas. Appendix C derives expressions of elasticities using the shift function method.

Appendix D provides data sources, calibration procedure and computing algorithm for the simulation. Appendix E contains proofs not included in the main paper.

Appendix A Complements on an individual's behavior

A.1 Optimizations of an as-if rational consumer

To see how misperceptions affect a consumer's decision, define the following utility-maximization problem of an "as-if rational" consumer, whose consumption demands are the same as an inattentive type-n consumer:

$$\max_{c_n, x_n} u\left(c_n, x_n, l_n\right), s.t. c_n + \sum_i q_i^s x_{in} = \bar{y}_n\left(q, q^s, y_n, l_n\right). \tag{A.1}$$

Obviously, the first-order conditions are exactly the same as equations in (1). $\bar{y}_n$ is defined as a virtual disposable income, which ensures the scale of the consumption bundle is the same as the choice of an inattentive consumer. $\bar{y}_n$ is non-linear in y_n, and it depends on both actual prices and perceived prices, which indicates income effects on commodity demands can be distorted by misperceptions.[30] The introduction of an as-if rational consumer provides an initial intuition on how a consumer's decisions are distorted by misperceptions. Misperceptions not only make the marginal rate of substitutions deviate from actual relative prices, but also create an "imaginary budget constraint" which is non-linear in actual prices. The solution yields conditional demands $x_{in}^r(q^s, \bar{y}_n, l_n)$, $c_n^r(q^s, \bar{y}_n, l_n)$. We can use the envelop theorem in this maximization problem.

It would be useful to define the conditional expenditure-minimization problem which is dual to the utility-maximization problem of an as-if rational consumer:

$$\min_{c_n, x_n} \left(c_n + \sum_i q_i^s x_{in}\right), s.t. u\left(c_n, x_n, l_n\right) \geq v_n^s. \tag{A.2}$$

The solution yields compensated conditional demands $x_{in}^{r*}\left(q^s, v_n^s, l_n\right), c_n^{r*}\left(q^s, v_n^s, l_n\right)$ and an expenditure function $e_n^r\left(q^s, v_n^s, l_n\right) \equiv c_n^{r*} + \sum_i q_i^s x_{in}^{r*}$. A superscript $*$ is used to denote that demands are generated by an expenditure-minimization problem. Based on these demand functions, define real expenditure as $e_n^s\left(q, v_n^s, l_n\right) = c_n^{r*}\left(q^s, v_n^s, l_n\right) + \sum_i q_i x_{in}^{r*}\left(q^s, v_n^s, l_n\right)$.[31]

A consumer's decision in stage 1 also has an "as-if rational counterpart". If we take y_n and l_n as two kinds of goods with prices 1 and $-nQ^n$ separately, then for a type-n consumer with

[30] Our characterization in this part is a special case of Gabaix (2014) using price misperceptions as the bias. $\bar{y}_n$ corresponds to the as-if budget of a type-n consumer in choosing commodities given labor supply as in Gabaix (2014). $\bar{y}_n$ ensures the value of the as-if rational consumer's consumption evaluated by actual prices is exactly the disposable income y_n. A thorough behavioral price theory to characterize the decisions of consumers with behavioral biases has been developed in Farhi and Gabaix (2020).

[31] e_n^r does not equal to real expenditure e_n^s of an inattentive agent since we define e_n^s using real prices instead of perceived prices in e_n^r.

misperception Q_n^s, we can again define an as-if rational consumer who makes the same decisions on y_n and l_n in the following way:

$$\max_{y_n, l_n} v_n^s(q, y_n, l_n); s.t. y_n = n l_n Q_n^s + \bar{R}_n(q, Q_n, Q_n^s, R_n).$$

$\bar{R}_n$ is virtual generalized revenue satisfying $\bar{R}_n(q, Q_n, Q_n^s, R_n) = R_n - (Q_n^s - Q_n) z_n$[32]. Similar to $\bar{y}_n$ which is non-linear in y_n, $\bar{R}_n$ is non-linear in R_n.

A.2 Properties of an individual's behavior in stage 2

Properties of conditional demand functions. Take partial derivatives of a bounded rational agent's budget constraint $c_n + \sum\limits_i q_i x_{in} = y_n$ on y_n, l_n and q_j separately to get

$$\frac{\partial c_n^s}{\partial y_n} + \sum_i q_i \frac{\partial x_{in}^s}{\partial y_n} = 1; \frac{\partial c_n^s}{\partial l_n} = -\sum_i q_i \frac{\partial x_{in}^s}{\partial l_n}; \frac{\partial c_n^s}{\partial q_j} + \sum_i q_i \frac{\partial x_{in}^s}{\partial q_j} + x_{jn}^s = 0, \forall j. \tag{A.3}$$

Take partial derivatives of an as-if rational agent's budget constraint $c_n^r + \sum\limits_i q_i^s x_{in}^r = \bar{y}_n (q, q^s, y_n, l_n)$ on $\bar{y}_n$, l_n and q_j separately to get

$$\frac{\partial c_n^r}{\partial \bar{y}_n} + \sum_i q_i^s \frac{\partial x_{in}^r}{\partial \bar{y}_n} = 1; \frac{\partial c_n^r}{\partial l_n} + \sum_i q_i^s \frac{\partial x_{in}^r}{\partial l_n} = 0; \frac{\mathrm{d}c_n^r}{\mathrm{d}q_j} + \sum_i q_i^s \frac{\mathrm{d}x_{in}^r}{\mathrm{d}q_j} + \sum_i x_{in}^r \frac{\mathrm{d}q_i^s}{\mathrm{d}q_j} = \frac{\mathrm{d}\bar{y}_n}{\mathrm{d}q_j}, \forall j. \tag{A.4}$$

Since $c_n^s(q, y_n, l_n) = c_n^r(q^s, \bar{y}, l_n), x_n^s(q, y_n, l_n) = x_n^r(q^s, \bar{y}, l_n)$, we have

$$\begin{aligned} \frac{\partial x_{in}^s}{\partial y_n} &= \frac{\partial x_{in}^r}{\partial \bar{y}_n} \frac{\partial \bar{y}_n}{\partial y_n}; \frac{\partial x_{in}^s}{\partial l_n} = \frac{\partial x_{in}^r}{\partial \bar{y}_n} \frac{\partial \bar{y}_n}{\partial l_n} + \frac{\partial x_{in}^r}{\partial l_n}; \frac{\partial x_{in}^s}{\partial q_j} = \frac{\partial x_{in}^r}{\partial \bar{y}_n} \frac{\mathrm{d}\bar{y}_n}{\mathrm{d}q_j} + \frac{\mathrm{d}x_{in}^r}{\mathrm{d}q_j}; \\ \frac{\partial c_{in}^s}{\partial y_n} &= \frac{\partial c_{in}^r}{\partial \bar{y}_n} \frac{\partial \bar{y}_n}{\partial y_n}; \frac{\partial c_{in}^s}{\partial l_n} = \frac{\partial c_{in}^r}{\partial \bar{y}_n} \frac{\partial \bar{y}_n}{\partial l_n} + \frac{\partial c_{in}^r}{\partial l_n}; \frac{\partial c_{in}^s}{\partial q_j} = \frac{\partial c_{in}^r}{\partial \bar{y}_n} \frac{\mathrm{d}\bar{y}_n}{\mathrm{d}q_j} + \frac{\mathrm{d}c_{in}^r}{\mathrm{d}q_j}. \end{aligned} \tag{A.5}$$

Equations in (A.5) build up relationships between (A.3) and (A.4).

Properties of conditional indirect utility functions. The solution to an as-if rational consumer's utility maximization problem (A.1) yields an indirect utility function $v_n^r(q^s, \bar{y}_n, l_n) \equiv u(c_n^r, x_n^r, l_n)$. Use the envelop theorem to get

$$\frac{\partial v_n^r}{\partial \bar{y}_n} = \frac{\partial u(c_n, x_n, l_n)}{\partial c_n}; \frac{\partial v_n^r}{\partial l_n} = \frac{\partial u(c_n, x_n, l_n)}{\partial l_n}; \frac{\partial v_n^r}{\partial q_k^s} = -\frac{\partial u(c_n, x_n, l_n)}{\partial c_n} x_{kn}^r, \forall k \in \{1, \ldots, I\}. \tag{A.6}$$

Since $v_n^s = v_n^r$, we have the following links between derivatives of indirect utilities of a bounded rational consumer and his as-if rational counterpart:

$$\frac{\partial v_n^s}{\partial y_n} = \frac{\partial v_n^r}{\partial \bar{y}_n} \frac{\partial \bar{y}_n}{\partial y_n}; \frac{\partial v_n^s}{\partial l_n} = \frac{\partial v_n^r}{\partial l_n} + \frac{\partial v_n^r}{\partial \bar{y}_n} \frac{\partial \bar{y}_n}{\partial l_n}; \frac{\partial v_n^s}{\partial q_i} = \frac{\partial v_n^r}{\partial q_i} + \frac{\partial v_n^r}{\partial \bar{y}_n} \frac{\mathrm{d}\bar{y}_n}{\mathrm{d}q_i}. \tag{A.7}$$

Similarly, we can derive the connections between expenditure functions of the two kinds of agents.

[32] $\bar{R}_n$ ensures the after-tax income of an as-if rational agent is the same as y. In other words, $\bar{R}_n$ is set so that $y_n = z_n - T(z_n)$ still holds for the as-if rational agent.

Properties of conditional compensated demand functions. Take partial derivatives of equation $e_n^r = c_n^{r*} + \sum_i q_i^s x_{in}^{r*}$ on v_n, z_n and q_j^s separately to get

$$\frac{\partial e_n^r}{\partial l_n} = \frac{\partial c_n^{r*}}{\partial l_n} + \sum_i q_i^s \frac{\partial x_{in}^{r*}}{\partial l_n}; \tag{A.8}$$

$$\frac{\partial e_n^r}{\partial v_n^r} = \frac{\partial c_n^{r*}}{\partial v_n^r} + \sum_i q_i^s \frac{\partial x_{in}^{r*}}{\partial v_n^r}; \tag{A.9}$$

$$\frac{\partial e_n^r}{\partial q_j^s} = x_{jn}^r + \sum_i q_i^s \frac{\partial x_{in}^{r*}}{\partial q_j^s} + \frac{\partial c_n^{r*}}{\partial q_j^s}. \tag{A.10}$$

Properties of conditional expenditure functions. For an as-if rational consumer, we can use the envelop theorem in his expenditure-minimization problem described in (A.2) to get

$$\frac{\partial e_n^r}{\partial l_n} = -\frac{\partial u\left(c_n, x_n, l_n\right)}{\partial l_n} / \frac{\partial u\left(c_n, x_n, l_n\right)}{\partial c_n}; \tag{A.11}$$

$$\frac{\partial e_n^r}{\partial v_n^r} = 1 / \frac{\partial u\left(c_n, x_n, l_n\right)}{\partial c_n}; \tag{A.12}$$

$$\frac{\partial e_n^r}{\partial q_j^s} = x_{jn}^r. \tag{A.13}$$

In addition, dual analysis shows $e_n^r\left(q^s, v_n^r, l_n\right) = \bar{y}_n\left(q, q^s, y_n, l_n\right)$.

Properties of virtual disposable income $\bar{y}_n$. The virtual disposable income $\bar{y}_n$ has the following properties:

$$\frac{\partial \bar{y}_n}{\partial l_n} = \sum_i \left(q_i^s - q_i\right) \frac{\partial x_{in}^s}{\partial l_n}; \frac{\partial \bar{y}_n}{\partial y_n} = 1 + \sum_i \left(q_i^s - q_i\right) \frac{\partial x_{in}^s}{\partial y_n}. \tag{A.14}$$

Proof. Equations in (A.5) build up relationships between (A.3) and (A.4). Therefore, we have

$$\sum_i \left(q_i^s - q_i\right) \frac{\partial x_{in}^r}{\partial \bar{y}_n} = 1 - \left(\frac{\partial \bar{y}_n}{\partial y_n}\right)^{-1}; \tag{A.15}$$

$$\sum_i \left(q_i^s - q_i\right) \frac{\partial x_{in}^r}{\partial l_n} = \left(\frac{\partial \bar{y}_n}{\partial y_n}\right)^{-1} \frac{\partial \bar{y}_n}{\partial l_n}; \tag{A.16}$$

$$\sum_i \left(q_i^s - q_i\right) \frac{\mathrm{d} x_{in}^r}{\mathrm{d} q_j} + \sum_i x_{in}^r \frac{\partial q_i^s}{\partial q_j} - x_{jn}^r = \frac{\mathrm{d} \bar{y}_n}{\mathrm{d} q_j}. \tag{A.17}$$

These relationships can also be expressed in derivatives of x_{in}^s as:

$$\sum_i \left(q_i^s - q_i\right) \frac{\partial x_{in}^s}{\partial y_n} = \frac{\partial \bar{y}_n}{\partial y_n} - 1; \tag{A.18}$$

$$\sum_i \left(q_i^s - q_i\right) \frac{\partial x_{in}^s}{\partial l_n} = \frac{\partial \bar{y}_n}{\partial l_n}; \tag{A.19}$$

$$\sum_i \left(q_i^s - q_i\right) \frac{\partial x_{in}^s}{\partial q_j} + \sum_i x_{in}^s \frac{\mathrm{d} q_i^s}{\mathrm{d} q_j} - x_{jn}^s = \frac{\mathrm{d} \bar{y}_n}{\mathrm{d} q_j}: \tag{A.20}$$

□

Links between derivatives of compensated and uncompensated conditional demands. Combine (A.5) with (A.11) to (A.13) to get links between derivatives of compensated and uncompensated conditional demands.

$$\begin{aligned}\frac{\partial x_n^{r*}}{\partial v_n^r} =& \frac{\partial x_{in}^r}{\partial \bar{y}_n} / \frac{\partial u\left(c_n, x_n, l_n\right)}{\partial c_n};\\ \frac{\partial x_{in}^{r*}}{\partial l_n} =& -\frac{\partial x_n^r}{\partial \bar{y}_n} \frac{\partial u\left(c_n, x_n, l_n\right)}{\partial l_n} / \frac{\partial u\left(c_n, x_n, l_n\right)}{\partial c_n} + \frac{\partial x_n^r}{\partial l_n};\\ \frac{\partial x_{in}^{r*}}{\partial q_k^s} =& \frac{\partial x_{in}^r}{\partial q_k^s} + \frac{\partial x_{in}^r}{\partial \bar{y}_n} x_{kn}^r.\end{aligned} \tag{A.21}$$

Properties of real expenditure functions. The real expenditure function e_n^s has the following properties:

$$\frac{\partial e_n^s}{\partial v_n^s} = \left(\frac{\partial v_n^s}{\partial y_n}\right)^{-1}; \frac{\partial e_n^s}{\partial l_n} = nQ_n^s; \frac{\partial e_n^s}{\partial q_j} = \sum_i \left(q_i - q_i^s\right) \sum_k \frac{\partial x_{in}^{r*}}{\partial q_k^s} \frac{\partial q_k^s}{\partial q_j} + x_{jn}^{r*}. \tag{A.22}$$

The first equation arises from the fact that $e_n^s = y_n$ for any inattentive agent. The second equation indicates that an one dollar increase in labor supply leads to nQ_n^s dollars increase of real expenditure. The third equation implies a failure of symmetry of the Slutsky matrix due to misperceptions on commodity prices.

Proof. Recall that e_n^s is defined as

$$e_n^s\left(q, v_n^s, l_n\right) = c_n^{r*}\left(q^s, v_n^s, l_n\right) + \sum_i q_i x_{in}^{r*}\left(q^s, v_n^s, l_n\right). \tag{A.23}$$

Take partial derivatives of e_n^s on l_n, v_n and q_j separately:

$$\frac{\partial e_n^s}{\partial l_n} = \frac{\partial c_n^{r*}\left(q^s, v_n^s, l_n\right)}{\partial l_n} + \sum_i q_i^s \frac{\partial x_{in}^{r*}\left(q^s, v_n^s, l_n\right)}{\partial l_n} + \sum_i \left(q_i - q_i^s\right) \frac{\partial x_{in}^{r*}\left(q^s, v_n^s, l_n\right)}{\partial l_n}; \tag{A.24}$$

$$\frac{\partial e_n^s}{\partial v_n^s} = \frac{\partial c_n^{r*}\left(q^s, v_n^s, l_n\right)}{\partial v_n^s} + \sum_i q_i^s \frac{\partial x_{in}^{r*}\left(q^s, v_n^s, l_n\right)}{\partial v_n^s} + \sum_i \left(q_i - q_i^s\right) \frac{\partial x_{in}^{r*}\left(q^s, v_n^s, l_n\right)}{\partial v_n^s}; \tag{A.25}$$

$$\begin{aligned}\frac{\partial e_n^s}{\partial q_j} =& \sum_k \frac{\partial c_n^{r*}\left(q^s, v_n^s, l_n\right)}{\partial q_k^s} \frac{\partial q_k^s}{\partial q_j} + \sum_i q_i \sum_k \frac{\partial x_{in}^{r*}\left(q^s, v_n^s, l_n\right)}{\partial q_k^s} \frac{\partial q_k^s}{\partial q_j} + x_{jn}^{r*}\\ =& \sum_k \left(\frac{\partial c_n^{r*}\left(q^s, v_n^s, l_n\right)}{\partial q_k^s} \frac{\partial q_k^s}{\partial q_j} + \sum_i q_i^s \frac{\partial x_{in}^{r*}\left(q^s, v_n^s, l_n\right)}{\partial q_k^s} \frac{\partial q_k^s}{\partial q_j}\right)\\ &+ \sum_i \left(q_i - q_i^s\right) \sum_k \frac{\partial x_{in}^{r*}\left(q^s, v_n^s, l_n\right)}{\partial q_k^s} \frac{\partial q_k^s}{\partial q_j} + x_{jn}^{r*}.\end{aligned} \tag{A.26}$$

Use (A.9), (A.12) and (A.21) to transform $\frac{\partial e_n^s}{\partial v_n^s}$ in (A.25) into

$$\frac{\partial e_n^s}{\partial v_n^s} = \frac{1}{u_c}\left[1 + \sum_i \left(q_i - q_i^s\right) \frac{\partial x_n^r\left(q^s, \bar{y}_n, l_n\right)}{\partial \bar{y}_n}\right] = \frac{1}{u_c}\left(\frac{\partial \bar{y}_n}{\partial y_n}\right)^{-1} = \left(\frac{\partial v_n^s}{\partial y_n}\right)^{-1}.$$

The last step is due to the relationship between v_n^s and v_n^r in (A.7). Use (A.8), (A.11) and (A.21) to transform $\frac{\partial e_n^s}{\partial l_n}$ in (A.24) into

$$\frac{\partial e_n^s}{\partial l_n} = -\frac{u_l}{u_c}\left[1 + \sum_i \left(q_i - q_i^s\right) \frac{\partial x_n^r}{\partial \bar{y}_n}\right] + \sum_i \left(q_i - q_i^s\right)\left(\frac{\partial x_n^r}{\partial l_n}\right).$$

Use (A.15) and (A.16) to further simplify it into

$$\frac{\partial e_n^s}{\partial l_n} = -\left(\frac{\partial \bar{y}_n}{\partial y_n}\right)^{-1} \frac{\partial u}{\partial l_n} / \frac{\partial u}{\partial c_n} - \left(\frac{\partial \bar{y}_n}{\partial y_n}\right)^{-1} \frac{\partial \bar{y}_n}{\partial l_n} = nQ_n^s.$$

The last step is due to the first-order condition of stage-1 optimization in (A.34).

From (A.10) and (A.13) we get $\sum_i q_i^s \frac{\partial x_{in}^{r*}}{\partial q_j^s} + \frac{\partial c_n^{r*}}{\partial q_j^s} = 0$. Therefore, we can transform (A.26) into

$$\frac{\partial e_n^s}{\partial q_j} = \sum_i (q_i - q_i^s) \sum_k \frac{\partial x_{in}^{r*}(q^s, v_n^s, l_n)}{\partial q_k^s} \frac{\partial q_k^s}{\partial q_j} + x_{jn}^{r*}. \tag{A.27}$$

An alternative expression of $\frac{\partial e_n^s}{\partial q_j}$ is derived as follows:

$$\begin{aligned}
\frac{\partial e_n^s}{\partial q_j} =& \sum_i (q_i - q_i^s) \left(\frac{\partial x_{in}^r}{\partial q_j} + \frac{\partial x_{in}^r}{\partial \bar{y}_n} \sum_k \frac{\partial q_k^s}{\partial q_j} x_{kn}^r\right) + x_{jn}^{r*} \\
=& \sum_i (q_i - q_i^s) \left(\frac{\partial x_{in}^s}{\partial q_j} + \frac{\partial x_{in}^r}{\partial \bar{y}_n} \left(\sum_k \frac{\partial q_k^s}{\partial q_j} x_{kn}^r - \frac{\mathrm{d}\bar{y}_n}{\mathrm{d}q_j}\right)\right) + x_{jn}^{r*} \\
=& \sum_i (q_i - q_i^s) \left(\frac{\partial x_{in}^s}{\partial q_j} - \frac{\partial x_{in}^r}{\partial \bar{y}_n} \sum_k (q_k^s - q_k) \frac{\partial x_{kn}^s}{\partial q_j} + \frac{\partial x_{in}^r}{\partial \bar{y}_n} x_{jn}^s\right) + x_{jn}^{r*} \\
=& \sum_i (q_i - q_i^s) \left(\frac{\partial x_{in}^s}{\partial q_j} + \frac{\partial x_{in}^r}{\partial \bar{y}_n} x_{jn}^s\right) - \sum_k (q_k^s - q_k) \frac{\partial x_{kn}^s}{\partial q_j} \sum_i \frac{\partial x_{in}^r}{\partial \bar{y}_n} (q_i - q_i^s) + x_{jn}^{r*} \\
=& \sum_i (q_i - q_i^s) \left(\frac{\partial x_{in}^s}{\partial q_j} + \frac{\partial x_{in}^r}{\partial \bar{y}_n} x_{jn}^s\right) + \sum_i (q_i^s - q_i) \frac{\partial x_{in}^s}{\partial q_j} \left(1 - \left(\frac{\partial \bar{y}_n}{\partial y_n}\right)^{-1}\right) + x_{jn}^{r*} \\
=& \left(x_{jn}^s - \sum_i (q_i^s - q_i) \frac{\partial x_{in}^s}{\partial q_j}\right) \left(\frac{\partial \bar{y}_n}{\partial y_n}\right)^{-1}.
\end{aligned} \tag{A.28}$$

The first line is obtained from (A.27) by applying (A.21). The second and third line are obtained by using (A.5) and (A.20) separately. The fifth line is derived by using (A.15). □

Modified Roy's identity and Slutsky equation. $\forall j$, the modified Roy's identity is

$$\frac{\partial v_n^s}{\partial q_j} / \frac{\partial v_n^s}{\partial y_n} = -x_{jn}^s - w_{jn}^q, w_{jn}^q \equiv \sum_i (q_i - q_i^s) \frac{\partial x_{in}^{s*}}{\partial q_j}. \tag{A.29}$$

The modified Slutsky equation is:

$$\frac{\partial x_{in}^s}{\partial q_j} = \sum_k \frac{\partial x_{in}^{r*}(q^s, v_n^s, l_n)}{\partial q_k^s} \frac{\partial q_k^s}{\partial q_j} + \frac{\partial x_{in}^s}{\partial y_n} \left(\frac{\partial v_n^s}{\partial q_j} / \frac{\partial v_n^s}{\partial y_n}\right). \tag{A.30}$$

Proof. Use (A.7) and (A.6) to transform $\frac{\partial v_n^s}{\partial q_j} / \frac{\partial v_n^s}{\partial y_n}$ into

$$\begin{aligned}
\frac{\partial v_n^s}{\partial q_j} / \frac{\partial v_n^s}{\partial y_n} =& \left(\sum_k \frac{\partial v_n^r}{\partial q_k^s} \frac{\partial q_k^s}{\partial q_i} + \frac{\partial v_n^r}{\partial \bar{y}_n} \frac{\mathrm{d}\bar{y}_n}{\mathrm{d}q_i}\right) / \frac{\partial v_n^r}{\partial \bar{y}_n} \frac{\partial \bar{y}_n}{\partial y_n} \\
=& -\left(\sum_k \frac{\partial q_k^s}{\partial q_j} x_{kn}^r - \frac{\mathrm{d}\bar{y}_n}{\mathrm{d}q_j}\right) \left(\frac{\partial \bar{y}_n}{\partial y_n}\right)^{-1}.
\end{aligned}$$

Use (A.20) to get an alternative expression as:

$$\frac{\partial v_n^s}{\partial q_j}/\frac{\partial v_n^s}{\partial y_n}=\left[\sum_i\left(q_i^s-q_i\right)\frac{\partial x_{in}^s}{\partial q_j}-x_{jn}^s\right]\left(\frac{\partial\bar{y}_n}{\partial y_n}\right)^{-1}.$$

The right-hand side is just the expression of $\frac{\partial e_n^s}{\partial q_j}$ in equation (A.28) of the opposite sign. Therefore, we have

$$\frac{\partial e_n^s}{\partial q_j}=-\frac{\partial v_n^s}{\partial q_j}/\frac{\partial v_n^s}{\partial y_n},\tag{A.31}$$

as well as

$$\frac{\partial v_n^s}{\partial q_j}/\frac{\partial v_n^s}{\partial y_n}=-x_{jn}^s+\sum_i\left(q_i^s-q_i\right)\sum_k\frac{\partial q_k^s}{\partial q_j}\frac{\partial x_{in}^{r*}}{\partial q_k^s}$$

by applying the third equation in (A.22). Take partial derivatives on z_n and y_n to get additional properties about e_{q_j} as:

$$v_{zq_j}+e_{q_i}v_{yz}+\frac{\mathrm{d}e_{q_i}(q,v(q,y_n,l_n),l_n)}{\mathrm{d}z_n}v_y^s=0;\,v_{yq_j}+e_{q_i}v_{yy}+\frac{\partial e_{q_j}}{\partial v_n^s}\left(v_y^s\right)^2=0.\tag{A.32}$$

where $e_{q_j}=x_{jn}^s+w_{in}^s$.

To get (A.30), use (A.21) to change the following expression

$$\frac{\partial x_{in}^s}{\partial q_j}=\frac{\partial x_{in}^{r*}\left(q^s,v_n^s,l_n\right)}{\partial q_j}+\frac{\partial x_{in}^{r*}\left(q^s,v_n^s\left(q,y_n,l_n\right),l_n\right)}{\partial v_n^s}\frac{\partial v_n^s}{\partial q_i}$$

into

$$\frac{\partial x_{in}^s}{\partial q_j}=\sum_k\frac{\partial x_{in}^{r*}\left(q^s,v_n^s,l_n\right)}{\partial q_k^s}\frac{\partial q_k^s}{\partial q_j}+\frac{\partial x_{in}^s}{\partial y_n}\left(\frac{\partial v_n^s}{\partial q_j}/\frac{\partial v_n^s}{\partial y_n}\right).$$

□

A.3 Properties of an individual's behavior in stage 1

Properties of conditional indirect utility functions. Use (A.7) to transform (2) into

$$\left(\frac{\partial v_n^r}{\partial\bar{y}_n}\frac{\partial\bar{y}_n}{\partial l_n}+\frac{\partial v_n^r}{\partial l_n}\right)/\left(\frac{\partial v_n^r}{\partial\bar{y}_n}\frac{\partial\bar{y}_n}{\partial y_n}\right)=-nQ_n^s.\tag{A.33}$$

Combine it with (A.6) to get

$$\left(\frac{\partial\bar{y}_n}{\partial y_n}\right)^{-1}\frac{\partial u}{\partial l_n}/\frac{\partial u}{\partial c_n}+\left(\frac{\partial\bar{y}_n}{\partial y_n}\right)^{-1}\frac{\partial\bar{y}_n}{\partial l_n}=-nQ_n^s.\tag{A.34}$$

Properties of unconditional indirect utility functions. We first apply the envelop theorem to an as-if rational agent's first-stage maximization problem to get

$$\frac{\partial V_n^r}{\partial q_j}=\frac{\partial v_n^s}{\partial q_j};\frac{\partial V_n^r}{\partial Q_n^s}=\frac{\partial v_n^s}{\partial y}z_n;\frac{\partial V_n^r}{\partial\bar{R}_n}=\frac{\partial v_n^s}{\partial y}.\tag{A.35}$$

For a bounded rational consumer, the indirect utility function satisfies:

$$\begin{aligned}\frac{\partial V_n^s}{\partial q_j}=&\frac{\partial v_n^s}{\partial q_j}-n\left(Q_n^s-Q_n\right)\frac{\partial l_n}{\partial q_j}\frac{\partial v_n^s}{\partial y};\\\frac{\partial V_n^s}{\partial Q_n}=&\frac{\partial v_n^s}{\partial y}\left(z_n-n\left(Q_n^s-Q_n\right)\frac{\partial l_n}{\partial Q_n}\right);\\\frac{\partial V_n^s}{\partial R_n}=&\frac{\partial v_n^s}{\partial y}\frac{\partial\bar{R}_n}{\partial R_n}.\end{aligned}\tag{A.36}$$

Proof. To simplify the expressions of derivatives of V_n^r, define

$$\bar{R}_n^*(q, Q_n, R_n) = \bar{R}_n(q, Q_n, Q_n^s(q, Q_n, nl_n(q, Q_n, R_n)), R_n)$$
$$Q_n^{s*}(q, Q_n, R_n) = Q_n^s(q, Q_n, nl_n(q, Q_n, R_n))$$

to get

$$\bar{R}_n^*(q, Q_n, R_n) = R_n - n\left[Q_n^{s*}(q, Q_n, R_n) - Q_n\right] l_n(q, Q_n, R_n)$$

Take partial derivatives of the above equation on q_j, R_n, Q_n:

$$\begin{aligned}
&\frac{\partial \bar{R}_n^*}{\partial q_j} + \frac{\partial Q_n^{s*}}{\partial q_j} z_n = n\left(Q_n - Q_n^s\right)\frac{\partial l_n}{\partial q_j} \\
&\frac{\partial \bar{R}_n^*}{\partial R_n} + \frac{\partial Q_n^{s*}}{\partial R_n} z_n = 1 - n\left(Q_n^s - Q_n\right)\frac{\partial l_n}{\partial R_n} = \frac{\partial \bar{R}_n}{\partial R_n} \\
&\frac{\partial \bar{R}_n^*}{\partial Q_n} + \frac{\partial Q_n^{s*}}{\partial Q_n} z_n = z_n - n\left(Q_n^s - Q_n\right)\frac{\partial l_n}{\partial Q_n}.
\end{aligned} \tag{A.37}$$

Since $V_n^s(q, Q_n, R_n) = V_n^r\left(q, Q_n^{s*}(q, Q_n, R_n), \bar{R}_n^*(q, Q_n, R_n)\right)$, using properties of V_n^r in (A.35) relationships in (A.37), we have equations in (A.36). □

Properties of labor supply functions. Denote by $l_n^r(q, Q_n^s, \bar{R}_n)$ the labor supply function of a rational consumer. The relationship between labor supply functions of a rational and a limited rational consumer is

$$\frac{\partial l_n}{\partial Q_n} = z_n \frac{\partial l_n}{\partial R_n} + \left[\frac{\partial l_n}{\partial R_n}\left(\frac{\partial l_n^r}{\partial \bar{R}_n}\right)^{-1}\left(\frac{\partial l_n^r}{\partial Q_n^s}\right) - z_n \frac{\partial l_n}{\partial R_n}\right]\frac{\mathrm{d}Q_n^s}{\mathrm{d}Q_n}. \tag{A.38}$$

Proof. Since

$$\bar{R}_n(q, Q_n, Q_n^s, R_n) = R_n - n\left(Q_n^s - Q_n\right) l_n^r(q, Q_n^s, \bar{R}_n(q, Q_n, Q_n^s, R_n)),$$

the partial derivatives of $\bar{R}_n$ satisfy

$$\begin{aligned}
&\frac{\partial \bar{R}_n}{\partial R_n} = \left(1 + n\left(Q_n^s - Q_n\right)\frac{\partial l_n^r}{\partial \bar{R}_n}\right)^{-1} = 1 - n\frac{Q_n^s - Q_n}{Q_n}\frac{\partial l_n}{\partial R_n} Q_n \\
&\frac{\partial \bar{R}_n}{\partial Q_n} = z_n \frac{\partial \bar{R}_n}{\partial R_n}; \\
&\frac{\partial \bar{R}_n}{\partial Q_n^s} = \left[-z_n - n\left(Q_n^s - Q_n\right)\left(\frac{\partial l_n^r}{\partial Q_n^s}\right)\right]\frac{\partial \bar{R}_n}{\partial R_n} \\
&\frac{\partial \bar{R}_n}{\partial q_j} = -n\left(Q_n^s - Q_n\right)\frac{\partial l_n^r}{\partial q_j}\frac{\partial \bar{R}_n}{\partial R_n}.
\end{aligned}$$

Since $l_n(q, Q_n, R_n) = l_n^r(q, Q_n^s, \bar{R}_n(q, Q_n, Q_n^s, R_n))$, we get relationships between derivatives of l_n and l_n^r as:

$$\begin{aligned}
&\frac{\partial l_n}{\partial R_n}\left(1 - \frac{\partial l_n^r}{\partial Q_n^s}\frac{\partial Q_n^s}{\partial l_n} - \frac{\partial l_n^r}{\partial \bar{R}_n}\frac{\partial \bar{R}_n}{\partial Q_n^s}\frac{\partial Q_n^s}{\partial l_n}\right) = \frac{\partial l_n^r}{\partial \bar{R}_n}\frac{\partial \bar{R}_n}{\partial R_n}; \\
&\frac{\partial l_n}{\partial Q_n}\left(1 - \frac{\partial l_n^r}{\partial Q_n^s}\frac{\partial Q_n^s}{\partial l_n} - \frac{\partial l_n^r}{\partial \bar{R}_n}\frac{\partial \bar{R}_n}{\partial Q_n^s}\frac{\partial Q_n^s}{\partial l_n}\right) = \frac{\partial l_n^r}{\partial Q_n^s}\frac{dQ_n^s}{dQ_n} + \frac{\partial l_n^r}{\partial \bar{R}_n}\left(\frac{\partial \bar{R}_n}{\partial Q_n} + \frac{\partial \bar{R}_n}{\partial Q_n^s}\frac{dQ_n^s}{dQ_n}\right); \\
&\frac{\partial l_n}{\partial q_j}\left(1 - \frac{\partial l_n^r}{\partial Q_n^s}\frac{\partial Q_n^s}{\partial l_n} - \frac{\partial l_n^r}{\partial \bar{R}_n}\frac{\partial \bar{R}_n}{\partial Q_n^s}\frac{\partial Q_n^s}{\partial l_n}\right) = \frac{\partial l_n^r}{\partial q_j} + \frac{\partial l_n^r}{\partial Q_n^s}\frac{\partial Q_n^s}{\partial q_j} + \frac{\partial l_n^r}{\partial \bar{R}_n}\frac{d\bar{R}_n}{dq_j},
\end{aligned}$$

Combine the first and second equations as well as expressions of partial derivatives of $\bar{R}_n$ to get (A.38). □

Appendix B Derive optimal taxation by tax perturbation method

To write the expression of optimal tax rates concisely, define the following notation:

- $\varepsilon_{x_i z} \equiv \frac{z_n}{x_{in}^s}\left(\frac{1}{n}\frac{\partial x_{in}^s}{\partial l_n} + Q_n \frac{\partial x_{in}^s}{\partial y_n}\right)$: conditional labor income elasticity of commodity demand, which captures the direct influence of labor supply as well as the indirect influence of z_n through y_n.
- $\tau_n^b \equiv \frac{Q_n^s - Q_n}{Q_n}$: misperception wedge of labor income tax, representing the degree of deviation of perceived marginal retention rate from the actual one. The definition is consistent with Farhi and Gabaix (2020).
- $\xi_z^I \equiv Q_n \frac{\partial z_n}{\partial R_n}$: income elasticity of labor income (does not account for the non-linearity of income taxation).

B.1 Optimal commodity tax

The proof of optimal commodity tax formula in proposition 2:

Proof. Set a commodity tax reform which slightly increase commodity j's price by $\mathrm{d}q_j$. The marginal effects of the tax reform is

$$\frac{1}{\mu}\frac{\mathrm{d}W}{\mathrm{d}q_j} + \frac{\mathrm{d}B}{\mathrm{d}q_j} = \overbrace{\int_N x_{jn}^s f(n)\,\mathrm{d}n}^{\text{changes in tax income ignoring behavioral effects}} + \overbrace{\int_N \sum_i t_i \frac{\mathrm{d}x_{in}^s}{\mathrm{d}q_j} f(n)\,\mathrm{d}n + \int_N T' \frac{\mathrm{d}z_n}{\mathrm{d}q_j} f(n)\,\mathrm{d}n}^{\text{changes in tax income due to behavioral effects}} \tag{B.1}$$

$$+ \overbrace{\int_N \frac{\Psi'}{\mu}\frac{\mathrm{d}V_n^s}{\mathrm{d}q_j} f(n)\,\mathrm{d}n}^{\text{changes in social welfare}}.$$

We first derive expressions of $\frac{\mathrm{d}z_n}{\mathrm{d}q_j}$, $\frac{\mathrm{d}x_{in}^s}{\mathrm{d}q_j}$ and $\frac{\mathrm{d}V_n^s}{\mathrm{d}q_j}$. Since

$$\begin{aligned}\frac{\mathrm{d}z_n}{\mathrm{d}q_j} - \frac{\partial z_n}{\partial q_j} =& \frac{\partial z_n}{\partial Q_n}\frac{\mathrm{d}Q_n}{\mathrm{d}q_j} + \frac{\partial z_n}{\partial R_n}\frac{\mathrm{d}R_n}{\mathrm{d}q_j} \\ =& -\xi_Q^s \frac{z_n}{Q_n} T''(z_n)\frac{\mathrm{d}z_n}{\mathrm{d}q_j} + \eta \frac{z_n}{Q_n} T''(z_n)\frac{\mathrm{d}z_n}{\mathrm{d}q_j} \\ =& -T''(z_n)\xi_Q^{cs}\frac{z_n}{Q_n}\frac{\mathrm{d}z_n}{\mathrm{d}q_j},\end{aligned}$$

we have

$$\frac{\mathrm{d}z_n}{\mathrm{d}q_j} = \frac{1}{1 + T''(z_n)\xi_Q^{cs}\frac{z_n}{Q_n}}\frac{\partial z_n}{\partial q_j}. \tag{B.2}$$

We rewrite $\frac{\mathrm{d}V_n^s/\mathrm{d}q_j}{\mathrm{d}V_n^s/\mathrm{d}R_n}$ as

$$\begin{aligned}\frac{\mathrm{d}V_n^s/\mathrm{d}q_j}{\mathrm{d}V_n^s/\mathrm{d}R_n} =& \frac{\partial V_n^s/\partial q_j}{\mathrm{d}V_n^s/\mathrm{d}R_n} + \frac{\mathrm{d}R_n}{\mathrm{d}q_j} + \frac{\partial V_n^s/\partial Q_n}{\mathrm{d}V_n^s/\mathrm{d}R_n}\frac{\mathrm{d}Q_n}{\mathrm{d}q_j} \\ =& \frac{\partial v_n^s}{\partial q_j}/\frac{\partial v_n^s}{\partial y}\left(\frac{\partial \bar{R}_n}{\partial R_n}\right)^{-1} - (Q_n^s - Q_n)\frac{\partial z_n}{\partial q_j}\left(\frac{\partial \bar{R}_n}{\partial R_n}\right)^{-1} \\ & + z_n T''(z_n)\xi_Q^{cs}\frac{Q_n^s - Q_n}{Q_n}\frac{\mathrm{d}z_n}{\mathrm{d}q_j}\left(\frac{\partial \bar{R}_n}{\partial R_n}\right)^{-1}.\end{aligned}$$

We simplify $\frac{\mathrm{d}V_n^s/\mathrm{d}q_j}{\mathrm{d}V_n^s/\mathrm{d}R_n}$ into

$$\frac{\mathrm{d}V_n^s/\mathrm{d}q_j}{\mathrm{d}V_n^s/\mathrm{d}R_n} = \frac{\partial v_n^s}{\partial q_j} / \frac{\partial v_n^s}{\partial y} \left(\frac{\partial \bar{R}_n}{\partial R_n} \right)^{-1} - \left(Q_n^s - Q_n\right) \frac{\mathrm{d}z_n}{\mathrm{d}q_j} \left(\frac{\partial \bar{R}_n}{\partial R_n} \right)^{-1}. \tag{B.3}$$

The left-hand-side of (B.3) corresponds to the pure welfare effect of a commodity tax reform. When there is no misperception of the income tax schedule, equation (B.3) would reduce to $\frac{\mathrm{d}V_n^s/\mathrm{d}q_j}{\mathrm{d}V_n^s/\mathrm{d}R_n} = \frac{\partial v_n^s}{\partial q_j} / \frac{\partial v_n^s}{\partial y}$, because the marginal influence on indirect utility of an increase in q_j in stage-1 optimization is the same as in stage 2. Misperception distorts this relationship through changing the imaginary budget constraint in two ways. First, the consumer evaluates the marginal cost of an additional unit of consumption in commodity j with the tightening of his imaginary budget constraint. This explains the appearance of $\left(\frac{\partial \bar{R}_n}{\partial R_n} \right)^{-1}$ on the right hand side of (B.3). Second, as long as $Q_n^s \neq Q_n$, an increase in q_j changes z_n and therefore affects the consumer's imaginary budget constraint, which corresponds to the second term on the right hand side of (B.3). The influences of misperception of commodity prices on the pure welfare effect are embedded in the term $\frac{\partial v_n^s}{\partial q_j} / \frac{\partial v_n^s}{\partial y_n}$, as has been explained in relation to equation (A.29).

$\frac{\mathrm{d}x_{in}^s}{\mathrm{d}q_j}$ could be decomposed into

$$\frac{\mathrm{d}x_{in}^s}{\mathrm{d}q_j} = \frac{\partial x_{in}^s}{\partial q_j} + \frac{\partial x_{in}^s}{\partial y_n} \frac{\mathrm{d}y_n}{\mathrm{d}q_j} + \frac{\partial x_{in}^s}{\partial z_n} \frac{\mathrm{d}z_n}{\mathrm{d}q_j}.$$

Define compensated conditional commodity demands as $x_{in}^{s*}(q, v_n^s, l_n) \equiv x_{in}^s\left(q, e_n^s\left(q, v_n^s, l_n\right), l_n\right)$, then we have

$$\frac{\partial x_{in}^{s*}}{\partial q_i} = \frac{\partial x_{in}^s}{\partial q_i} + \frac{\partial x_{in}^s}{\partial y_n} \frac{\partial e_n^s}{\partial q_i} = \frac{\partial x_{in}^s}{\partial q_i} + \frac{\partial x_{in}^s}{\partial y_n} \left(w_{jn}^q + x_{jn}^s \right); \tag{B.4}$$

$$\frac{\partial x_{in}^{s*}}{\partial l_n} = \frac{\partial x_{in}^s}{\partial l_n} + \frac{\partial x_{in}^s}{\partial y_n} \frac{\partial e_n^s}{\partial l_n} = \frac{\partial x_{in}^s}{\partial l_n} + \frac{\partial x_{in}^s}{\partial y_n} n Q_n^s. \tag{B.5}$$

Use the above equations to transform $\frac{\mathrm{d}x_{in}^s}{\mathrm{d}q_j}$ into

$$\frac{\mathrm{d}x_{in}^s}{\mathrm{d}q_j} = \frac{\partial x_{in}^{s*}\left(q, v_n^s, l_n\right)}{\partial q_j} - \left(x_{jn}^s + w_{jn}^q + n\left(Q_n^s - Q_n\right) \frac{\mathrm{d}l_n}{\mathrm{d}q_j} \right) \frac{\partial x_{in}^s}{\partial y_n} + \frac{\partial x_{in}^{s*}}{\partial l_n} \frac{\mathrm{d}l_n}{\mathrm{d}q_j}. \tag{B.6}$$

The first item on the right-hand side denotes the substitution effect of commodities, and the last item corresponds to the substitution effect of l_n. The second item reflects income effects. An increase in q_j not only tightens the actual budget constraint by x_{jn}^s, but also additionally tightens the imaginary budget constraint by w_{jn}^q in a consumer's stage-2 decision. When $Q_n^s \neq Q_n$, such a change in q_j also additionally tightens the imaginary budget constraint by $n\left(Q_n^s - Q_n\right) \frac{\mathrm{d}z_n}{\mathrm{d}q_j}$ in the stage-1 decision.

Using the expressions of $\frac{\mathrm{d}x_{in}^s}{\mathrm{d}q_j}$, $\frac{\mathrm{d}z_n}{\mathrm{d}q_j}$ and $\frac{\mathrm{d}V_n^s}{\mathrm{d}q_j}$, the marginal effects of a tax reform could be transformed into

$$\begin{aligned}
\frac{1}{\mu} \frac{\mathrm{d}W}{\mathrm{d}q_j} + \frac{\mathrm{d}B}{\mathrm{d}q_j} &= \int_N g_n \left[-x_{jn}^s - w_{jn}^q - \left(Q_n^s - Q_n\right) \frac{\mathrm{d}z_n}{\mathrm{d}q_j} \right] f\left(n\right) \mathrm{d}n + \int_N x_{jn}^s f\left(n\right) \mathrm{d}n \\
&+ \int_N \sum_i t_i \left[\frac{\partial x_{in}^{s*}}{\partial q_j} - \left(x_{jn}^s + w_{jn}^q + \left(Q_n^s - Q_n\right) \frac{\mathrm{d}z_n}{\mathrm{d}q_j} \right) \frac{\partial x_{in}^s}{\partial y_n} + \frac{\partial x_{in}^{s*}}{\partial z_n} \frac{\mathrm{d}z_n}{\mathrm{d}q_j} \right] f\left(n\right) \mathrm{d}n \\
&+ \int_N T'\left(z_n\right) \frac{1}{1 + T''(z_n) \xi_Q^{cs} \frac{z_n}{Q_n}} \frac{\partial z_n}{\partial q_j} f\left(n\right) \mathrm{d}n.
\end{aligned}$$

To introduce the compensated price effects $\frac{\partial z_n^*}{\partial q_j}$, define $c^*\left(q, Q_n, V_n^s\right) \equiv c^{s*}(q, V_n^s, l_n^*(q_n, Q_n, V_n^s))$, $x_{in}^*\left(q, Q_n, V_n^s\right) \equiv x_{in}^{s*}(q, V_n^s, l_n^*(q_n, Q_n, V_n^s))$. Since $l_n^*\left(q, Q_n, V_n^s\right) = l_n(q, Q_n, c^* + \sum_i q_i x_{in}^* - Q_n l_n^*)$, we have

$$\begin{aligned}\frac{\partial l_n^*}{\partial q_j} &= \frac{\partial l_n}{\partial q_j} + \frac{\partial l_n}{\partial R_n} x_{jn} + \frac{\partial l_n}{\partial R_n}\left(\frac{\partial c_n^*}{\partial q_j} + \sum_i \frac{\partial x_{in}^*}{\partial q_j} - nQ_n \frac{\partial l_n^*}{\partial q_j}\right) \\ &= \frac{\partial l_n}{\partial q_j} + \frac{\partial l_n}{\partial R_n} x_{jn} + \frac{\partial l_n}{\partial R_n}\left[w_{jn}^q + n\left(Q_n^s - Q_n\right)\frac{\partial l_n^*}{\partial q_j}\right]\end{aligned}$$

Therefore, we have

$$\frac{\partial l_n}{\partial q_j} = \frac{\partial l_n^*}{\partial q_j}\left[1 - \frac{\partial l_n}{\partial R_n} n\left(Q_n^s - Q_n\right)\right] - \frac{\partial l_n}{\partial R_n}\left(x_{jn} + w_{jn}^q\right) \tag{B.7}$$

Using expressions of $\frac{\mathrm{d}l_n}{\mathrm{d}q_j}$ and $\frac{\partial l_n^*}{\partial q_j}$, the definition that $z_n^* \equiv n l_n^*(q, Q_n, V_n^s)$, and the optimum condition that the marginal effects of a tax reform is zero, we finally have

$$\begin{aligned}&\frac{1}{\mu}\frac{\mathrm{d}W}{\mathrm{d}q_j} + \frac{\mathrm{d}B}{\mathrm{d}q_j} = \int_N \left(1-\gamma_n\right) x_{jn}^s f\left(n\right)\mathrm{d}n \\ &+ \int_N \left[\sum_i t_i \frac{\partial x_{in}^{s*}}{\partial q_j} + \sum_i t_i \frac{x_{in}^s}{z_n}\varepsilon_{x_i z}\frac{1}{1+T''(z_n)\xi_Q^{cs}\frac{z_n}{Q_n}}\frac{\partial z_n^*}{\partial q_j} + T'\left(z_n\right)\frac{1}{1+T''(z_n)\xi_Q^{cs}\frac{z_n}{Q_n}}\frac{\partial z_n^*}{\partial q_j}\right]\mathrm{d}n \\ &+ \int_N \left(-\gamma_n\right) w_{jn}^q f\left(n\right)\mathrm{d}n \\ &- \int_N \left[g_n \frac{\mathrm{d}z_n}{\mathrm{d}q_j} + T'\left(z_n\right)\frac{\varepsilon_z^I}{Q_n}\frac{\partial z_n^*}{\partial q_j} + \sum_i t_i \frac{x_{in}^s}{z_n}\varepsilon_{x_i z}\frac{\varepsilon_z^I}{Q_n}\frac{\partial z_n^*}{\partial q_j}\right](Q_n^s - Q_n) f\left(n\right)\mathrm{d}n,\end{aligned}$$

in which $\gamma_n = g_n + \sum_i t_i \frac{\partial x_{in}^s}{\partial y_n} - T'\left(z_n\right)\frac{\varepsilon_z^I}{Q_n} - \sum_i t_i \frac{x_{in}^s}{z_n}\varepsilon_{x_i z}\frac{\varepsilon_z^I}{Q_n}$.

By having $\xi_{zq_i}^* \equiv \frac{q_i}{z_n}\frac{\partial z_n^*}{\partial q_i}, \varepsilon_{zq_i}^* \equiv \frac{\xi_{zq_i}^*}{1+T''(z_n)\frac{z_n}{Q_n}\xi_{zQ}^*}$ we get the same optimal commodity tax formula as in proposition 2. □

B.2 Optimal income tax

The proof of optimal income tax formula in proposition 1:

Proof. As in Gerritsen (2024), write the income tax as a function of labor income and a reform parameter ϕ: $\tilde{T}(z,\phi) = T(z) + \phi\tau(z)$. $\tau(z)$ is the reform function. Then we can analyze any non-linear marginal reform of the tax schedule. Assume that z is differentiable in ϕ. The tilde over a variable indicates that it is realized after the tax reform. At $\phi = 0$, the marginal impacts of ϕ on net social welfare can be decomposed into four parts:

$$\begin{aligned}\frac{\mathrm{d}\tilde{W}}{\mathrm{d}\phi}/\mu + \frac{\mathrm{d}\tilde{B}}{\mathrm{d}\phi} = &\overbrace{\int \tau(z_n) f\left(n\right)\mathrm{d}n}^{\text{Mechanical revenue effect}} + \overbrace{\int T'\left(z_n\right)\frac{\mathrm{d}z_n}{\mathrm{d}\phi} f\left(n\right)\mathrm{d}n + \int \sum_i t_i \frac{\mathrm{d}\tilde{x}_{in}^s}{\mathrm{d}\phi} f\left(n\right)\mathrm{d}n}^{\text{Behavioral revenue effect}} \\ &+ \overbrace{\int \frac{\Psi'(\tilde{V}_n^s)}{\mu}\frac{\partial \tilde{V}_n^s}{\partial \tilde{R}_n}\frac{\mathrm{d}\tilde{V}_n^s/\mathrm{d}\phi}{\partial \tilde{V}_n^s/\partial \tilde{R}_n} f\left(n\right)dn}^{\text{Effect on social welfare}},\end{aligned}$$

First, the reform mechanically raises an amount of resource $\tau(z_n)\mathrm{d}\phi$ from each individual $n \in N$. Second, it affects individuals' labor supply, resulting in a change in income tax revenue. Third, it changes individuals' commodity demands and therefore influences commodity tax revenue. Lastly, the reform changes individuals' utilities and therefore social welfare.

Income taxes are set optimally if no reform of the income tax schedule can raise net social welfare, which requires that $\frac{\mathrm{d}\tilde{W}}{\mathrm{d}\phi}/\mu + \frac{\mathrm{d}\tilde{B}}{\mathrm{d}\phi} = 0$ at $\phi = 0$.

After the tax reform, an individual's marginal retention rate, generalized revenue and disposable income satisfy:

$$\tilde{Q}_n = 1 - T'(z_n) - \phi\tau'(z_n); \tilde{R}_n = z_n\left(T'(z_n) + \phi\tau'(z_n)\right) - T(z_n) - \phi\tau(z_n); \tilde{y}_n = z_n - T(z_n) - \phi\tau(z_n).$$

At $\phi = 0$, the marginal impacts of ϕ on income tax, marginal retention rate and generalized revenue are

$$\frac{\mathrm{d}\tilde{T}(z_n)}{\mathrm{d}\phi} = \tau(z_n) + T'(z_n)\frac{\mathrm{d}z_n}{\mathrm{d}\phi}; \tag{B.8}$$

$$\frac{\mathrm{d}\tilde{Q}_n}{\mathrm{d}\phi} = -T''(z_n)\frac{\mathrm{d}z_n}{\mathrm{d}\phi} - \tau'(z_n); \tag{B.9}$$

$$\frac{\mathrm{d}\tilde{R}_n}{\mathrm{d}\phi} = \left(T''(z_n)\frac{\mathrm{d}z_n}{\mathrm{d}\phi} + \tau'(z_n)\right)z_n - \tau(z_n). \tag{B.10}$$

The reform also has the following influences on an individual's commodity demand:

$$\frac{\mathrm{d}x_{in}^s(q,\tilde{y},l_n)}{\mathrm{d}\phi} = \frac{\partial x_{in}^s}{\partial y_n}\frac{\mathrm{d}y_n}{\mathrm{d}\phi} + \frac{\partial x_{in}^s}{\partial l_n}\frac{\mathrm{d}l_n}{\mathrm{d}\phi} - \frac{\partial x_{in}^s}{\partial y_n}\tau(z) = \frac{x_{in}^s}{z_n}\varepsilon_{x_i z}\frac{\mathrm{d}z_n}{\mathrm{d}\phi} - \frac{\partial x_{in}^s}{\partial y_n}\tau(z), \tag{B.11}$$

and on indirect utility function:

$$\begin{aligned}\frac{\mathrm{d}\tilde{V}_n^s}{\mathrm{d}\phi} =& \frac{\partial V_n^s}{\partial \tilde{Q}_n}\frac{\mathrm{d}\tilde{Q}_n}{\mathrm{d}\phi} + \frac{\partial V_n^s}{\partial \tilde{R}_n}\frac{\mathrm{d}\tilde{R}_n}{\mathrm{d}\phi}\\ =& -\tau(z_n)\frac{\partial V_n^s}{\partial R_n} + \frac{\partial V_n^s}{\partial R_n}\left(\frac{\partial \bar{R}_n}{\partial R_n}\right)^{-1} z_n\tau_n^b\xi_{zQ}^*\left(T''(z_n)\frac{\mathrm{d}z_n}{\mathrm{d}\phi} + \tau'(z_n)\right).\end{aligned} \tag{B.12}$$

The impact on labor income z_n has been presented in (C.11).

Optimal income tax requires that marginal influence of ϕ on $\tilde{W}/\mu - \tilde{B}$ at $\phi = 0$ should be zero. The marginal influence of ϕ on net social welfare is

$$\begin{aligned}\frac{\mathrm{d}\tilde{W}}{\mathrm{d}\phi}/\mu + \frac{\mathrm{d}\tilde{B}}{\mathrm{d}\phi} =& \int \frac{\Psi'(\tilde{V}_n^s)}{\mu}\frac{\partial \tilde{V}_n^s}{\partial \tilde{R}_n}\frac{\mathrm{d}\tilde{V}_n^s/\mathrm{d}\phi}{\partial \tilde{V}_n^s/\partial \tilde{R}_n} f(n)\,dn\\ &+ \int \frac{\mathrm{d}\tilde{T}(z_n)}{\mathrm{d}\phi} f(n)\,\mathrm{d}n + \int \sum_i t_i \frac{\mathrm{d}x_{in}^s}{\mathrm{d}\phi} f(n)\,\mathrm{d}n.\end{aligned}$$

Substituting (B.8) to (B.12) into $\frac{\mathrm{d}\tilde{W}}{\mathrm{d}\phi}/\mu + \frac{\mathrm{d}\tilde{B}}{\mathrm{d}\phi} = 0$ yields

$$\begin{aligned}&\int \left(1 - Q_n + \sum_i t_i \frac{x_{in}^s}{z_n}\varepsilon_{x_i z}\right)\frac{\mathrm{d}z_n}{\mathrm{d}\phi} f(n)\,\mathrm{d}n\\ =& -\int \tau(z_n)\left(1 - g_n - \sum_i t_i \frac{\partial x_{in}^s}{\partial y_n}\right) f(n)\,\mathrm{d}n - \int g_n\tau_n^b\eta\tau(z_n) f(n)\,\mathrm{d}n\\ &- \int g_n z_n \tau_n^b\left[\xi_{zQ}^*\left(T''\frac{\mathrm{d}z_n}{\mathrm{d}\phi} + \tau'(z_n)\right)\right] f(n)\,\mathrm{d}n.\end{aligned} \tag{B.13}$$

Since (C.11) could be transformed into

$$\frac{z_n}{Q_n}\xi^*_{zQ}\left(T''\left(z_n\right)\frac{\mathrm{d}z_n}{\mathrm{d}\phi}+\tau'(z_n)\right)=-\frac{\tau(z_n)}{Q_n}\eta_n-\frac{\mathrm{d}z_n}{\mathrm{d}\phi},$$

we can simplify (B.13) into

$$\begin{aligned}&\int_N J_n z_n \varepsilon^*_{zQ}\tau'(z_n)f\left(n\right)\mathrm{d}n\\ =&-\int_N\left[1-g_n-\sum_i t_i\frac{\partial x^s_{in}}{\partial y_n}-J_n\varepsilon^I_z\right]\tau(z_n)f\left(n\right)\mathrm{d}n,\end{aligned}\tag{B.14}$$

in which J_n is defined as

$$J_n\equiv\frac{1-Q_n}{Q_n}+\sum_i\frac{t_i}{Q_n}\frac{x^s_{in}}{z_n}\varepsilon_{x_iz}-g_n\tau^b_n.$$

To solve J_n, we apply a specific tax reform as $\tau(z_n)=1$ if $z_n\geq z^*$, $\tau(z_n)=0$ if $z_n<z^*$, leading to $\tau'(z^*)\mathrm{d}n=\mathrm{d}n/\mathrm{d}z^*$ at $z=z^*$. Together with (C.11) and (B.14), the specific tax reform yields solution for J_n at $z_n=z^*$, which is equivalent to optimal nonlinear income tax formula in (10) derived by the mechanism design approach. $\square$

Appendix C Derivation of elasticities

We defined several elasticities and Table 3 collects the notations and definitions.

There are two sets of elasticities of labor income in Table 3.

Elasticities in one set (ξ^I_z and ξ^*_{zQ}) do not consider the none-linearity of income taxation, while elasticities in another set (ε^I_z, ε^*_{zQ} and ε_{zn}) do. The expressions of ε^I_z, ε^*_{zQ} and ε_{zn} are obtained by a shift function method.

C.1 Elasticities obtained by shift function method

Define the shift function as

$$L(z_n,n,\phi)\equiv n\tilde{Q}^s_n\frac{\partial v^s_n\left(q,\tilde{y}_n,z_n/n\right)}{\partial y_n}+\frac{\partial v^s_n\left(q,\tilde{y}_n,z_n/n\right)}{\partial l_n},$$

in which $\tilde{Q}^s_n\equiv Q^s_n-\phi\tau'\left(z_n\right)$ is the perceived marginal retention rate after tax reform and $\tilde{y}_n\equiv z_n-T(z_n)-\phi\tau(z_n)$ is disposable income after the reform. The shift function captures the shift in the first-order condition for labor income when one of the variables z_n,n,ϕ changes. The partial derivatives of L at $\phi=0$ are

$$\begin{aligned}\frac{\partial L(z_n,n,0,q)}{\partial z_n}=&nv^s_y\left(\frac{\partial Q^s_n}{\partial Q_n}\right)Q'\left(z_n\right)+nv^s_y\frac{\partial Q^s_n}{\partial z_n}+nQ^s_nQ_nv^s_{yy}+nQ^s_nv^s_{yz}+nQ_nv^s_{yz}+nv^s_{zz};\\ \frac{\partial L(z_n,n,0,q)}{\partial n}=&-\left[\left(v^s_{zz}+v^s_{zy}Q^s_n\right)z_n+v^s_z\right];\\ \frac{\partial L(z_n,n,0,q)}{\partial\phi}=&-nv^s_y\tau'\left(z_n\right)-nQ^s_nv^s_{yy}\tau\left(z\right)-nv^s_{zy}\tau\left(z\right);\\ \frac{\partial L(z_n,n,0,q)}{\partial q_j}=&nQ^s_nv^s_{yq_j}+nv^s_{zq_j}+nv^s_y\frac{\partial Q^s_n}{\partial q_j}.\end{aligned}$$

Economic meaning	Symbol	Notes
Elasticities of consumption demand		
Conditional compensated commodity tax elasticity of commodity i's demand	$\varepsilon^*_{x_i q_j}$	The influence of commodity j's price on commodity j's conditional demand
Conditional income elasticity of commodity i's demand	$\varepsilon^I_{x_j}$	The influence of disposable income on commodity j's conditional demand
Labor income elasticity of commodity demand	$\varepsilon_{x_i z}$	
Elasticities of labor income		
Local compensated commodity tax elasticity of labor income	$\xi^*_{zq_j}$	Ignore the non-linearity of income taxation
Local income elasticity of labor income	ξ^I_z	Ignore the non-linearity of income taxation
Local compensated tax elasticity of labor income	ξ^*_{zQ}	Ignore the non-linearity of income taxation
Compensated commodity tax elasticity of labor income	$\varepsilon^*_{zq_j}$	
Income elasticity of labor income	ε^I_z	
Compensated tax elasticity of labor income	ε^*_{zQ}	
Elasticity of labor income on ability	ε_{zn}	
Wedges and weights		
Misperception wedge of commodity tax	w^q_{jn}	$w^q_{jn} \equiv \frac{1}{q_j} \sum_i (q_i - q^s_i)\, x^{s*}_{in} \varepsilon^*_{x_i q_j}$
Misperception wedge of labor income tax	τ^b_n	$\tau^b_n \equiv \frac{Q^s_n - Q_n}{Q_n}$
Social marginal welfare weight on a z-earner	$g(z)$	

Table 3: Notation

Then we can use the implicit function theorem to express elasticities of labor income in derivatives of indirect utility function, which is useful in deriving optimal tax formula under mechanism design. The implicit function theorem gives $\frac{\mathrm{d}z_n}{\mathrm{d}\phi}|_{\phi=0} = -\frac{\partial L(z_n,z,n,0)/\partial\phi}{\partial L(z_n,z,n,0)/\partial z_n}|_{\phi=0}$. Thus, we get overall impact of the tax reform on consumer's labor income as

$$\frac{\mathrm{d}z_n}{\mathrm{d}\phi} = D_n^{-1} v_y^s \tau'(z_n) + D_n^{-1}\left(Q_n^s v_{yy}^s + v_{zy}^s\right)\tau(z_n), \tag{C.1}$$

in which D_n is defined as

$$D_n \equiv v_y^s \left(\frac{\partial Q_n^s}{\partial Q_n}\right) Q'(z_n) + v_y^s \frac{\partial Q_n^s}{\partial z_n} + Q_n^s Q_n v_{yy}^s + Q_n^s v_{yz} + Q_n v_{yz} + v_{zz}. \tag{C.2}$$

From (C.1) we can decompose the impact of a tax reform into two parts: (1) Income effect, which captures the change in disposable income. (2) Price effect. This corresponds to the case when a reform changes marginal income tax rate at z_n, which means $\tau(z_n) = 0$ and $\tau'(z_m)|_{m\neq n} = 0$.

Denote the compensated tax elasticity of labor income by $\varepsilon_{zQ^s}^*$, and denote the income elasticity of labor income by ε_z^I. Then we have

$$\varepsilon_{zQ}^* = \frac{Q_n}{z_n} D_n^{-1} v_y^s; \tag{C.3}$$

$$\varepsilon_z^I = Q_n D_n^{-1}(Q_n^s v_{yy}^s + v_{zy}^s). \tag{C.4}$$

These elasticities take into account the non-linearity of income tax schedule. Using these elasticities the expression of $\frac{\mathrm{d}z_n}{\mathrm{d}\phi}$ could then be transformed into

$$\frac{\mathrm{d}z_n}{\mathrm{d}\phi} = \frac{z_n}{Q_n}\left[\tau'(z_n)\,\varepsilon_{zQ}^* + \tau(z_n)\,\varepsilon_z^I / z_n\right] \tag{C.5}$$

Using similar logic, denote by ε_{zn} the uncompensated elasticity of labor income on ability. Then we have

$$\varepsilon_{zn} \equiv \frac{n}{z}\frac{\partial z}{\partial n} = \frac{1}{z_n} D_n^{-1}\left[\left(v_{zz}^s + v_{zy}^s Q_n^s\right) z_n + v_z^s\right]. \tag{C.6}$$

In addition, we have

$$\frac{\mathrm{d}z_n}{\mathrm{d}q_j} = -\frac{Q_n^s v_{yq_j}^s + v_{zq_j}^s + v_y^s \frac{\partial Q_n^s}{\partial q_j}}{D_n}. \tag{C.7}$$

C.2 Connections between two sets of income elasticities

Although the two sets of income elasticities have different expressions, they are connected in the following way:

$$\varepsilon_{zQ}^* = -\frac{\xi_{zQ}^*}{1 + T''(z_n)\frac{z_n}{Q_n}\xi_{zQ}^*}; \varepsilon_z^I = -\frac{\xi_z^I}{1 + T''(z_n)\frac{z_n}{Q_n}\xi_{zQ}^*}. \tag{C.8}$$

Proof. From (A.38), we have:

$$\begin{aligned} \xi_{zQ}^s &= \xi_z^I + \frac{Q_n}{z_n}\left[n\frac{\partial l_n}{\partial R_n}\left(\frac{\partial l_n^r}{\partial \bar{R}_n}\right)^{-1}\frac{\partial l_n^r}{\partial Q_n^s} - \frac{z_n}{Q_n}\xi_z^I\right]\frac{\mathrm{d}Q_n^s}{\mathrm{d}Q_n} \\ \xi_{zQ}^* &= \frac{Q_n}{z_n}\left[n\frac{\partial l_n}{\partial R_n}\left(\frac{\partial l_n^r}{\partial \bar{R}_n}\right)^{-1}\frac{\partial l_n^r}{\partial Q_n^s} - \frac{z_n}{Q_n}\xi_z^I\right]\frac{\mathrm{d}Q_n^s}{\mathrm{d}Q_n} \end{aligned} \tag{C.9}$$

Define the following income tax reform as

$$\tilde{T}(z,\phi) = T(z) + \phi\tau(z), \tag{C.10}$$

$\tau(z)$ is the reform function. Then we can analyze any nonlinear marginal reform of the tax schedule. Assume that z is differentiable in ϕ . Individual's labor income and indirect utility after the reform are $z_n(q, \tilde{Q}_n, \tilde{R}_n)$ and $V_n^s(q, \tilde{Q}_n, \tilde{R}_n)$. $\tilde{Q}_n$ is marginal retention rate after the tax reform satisfying

$$\tilde{Q}(z_n) = Q(z) - \phi\tau'(z).$$

Generalized revenue after the tax reform $\tilde{R}_n$ satisfies

$$\tilde{R}_n = z - \tilde{T}(z,\phi) - \tilde{Q}_n z = zT'(z) - T(z) - \phi\tau(z) + z\phi\tau'(z).$$

Therefore, the impact of the tax reform on z_n could be expressed as

$$\begin{aligned} \frac{\mathrm{d}z_n}{\mathrm{d}\phi_n} = \frac{n\mathrm{d}l_n(q,\tilde{Q}_n,\tilde{R}_n)}{\mathrm{d}\phi} &= n\frac{\partial l_n}{\partial Q_n}\frac{\mathrm{d}\tilde{Q}_n}{\mathrm{d}\phi} + n\frac{\partial l_n}{\partial R_n}\frac{\mathrm{d}\tilde{R}_n}{\mathrm{d}\phi} \\ &= n\left(-\frac{\partial l_n}{\partial Q_n} + z_n\frac{\partial l_n}{\partial R_n}\right)\left(T''(z_n)\frac{\mathrm{d}z_n}{\mathrm{d}\phi} + \tau'(z_n)\right) - n\tau(z_n)\frac{\partial l_n}{\partial R_n}. \end{aligned} \tag{C.11}$$

Using expressions of elasticities defined in (C.9), we could transform the equation above into

$$\frac{\mathrm{d}z_n}{\mathrm{d}\phi}\left(1 + T''(z_n)\frac{z_n}{Q_n}\xi_{zQ}^*\right) = -\frac{z_n}{Q_n}\left[\tau'(z_n)\xi_{zQ}^* + \frac{\tau(z_n)}{z_n}\xi_z^I\right].$$

By defining

$$\tilde{\xi}_{zQ}^* \equiv \frac{\xi_{zQ}^*}{1 + T''(z_n)\frac{z_n}{Q_n}\xi_{zQ}^*}; \tilde{\xi}_z^I \equiv \frac{\xi_z^I}{1 + T''(z_n)\frac{z_n}{Q_n}\xi_{zQ}^*};$$

we finally get

$$\frac{\mathrm{d}z_n}{\mathrm{d}\phi} = -\frac{z_n}{Q_n}\left[\tau'(z_n)\tilde{\xi}_{zQ}^* + \frac{\tau(z_n)}{z_n}\tilde{\xi}_z^I\right].$$

Comparing the above equation with (C.5), we have the following relationships between two sets of elasticities:

$$\tilde{\xi}_{zQ}^* = -\varepsilon_{zQ}^*; \tilde{\xi}_z^I = -\varepsilon_z^I.$$

The signs of two kinds of elasticities are opposite because we define tax elasticities of labor income on changes in ϕ in shift function method, while elasticities in Saez's form are defined on changes in marginal retention rate. The directions of impacts of ϕ and Q_n are opposite by definition. $\square$

Appendix D Simulations of Optimal Mixed Taxation

D.1 Optimal Tax Formulas in the Case of EV credits

Under the utility function specified in (20), the demand functions of a type-n consumer satisfy

$$c_n = \frac{q^s}{\frac{1-\beta}{\beta}q + q^s}y_n; x_n = \frac{1}{q + \frac{\beta}{1-\beta}q^s}y_n. \tag{D.1}$$

Therefore, we have

$$\frac{\partial x_n^{s*}}{\partial q} = -\left(\frac{1}{q+\frac{\beta}{1-\beta}q^s}\right)^2\left(1+\frac{\beta}{1-\beta}\frac{\partial q^s}{\partial q}\right)y_n + \frac{1}{q+\frac{\beta}{1-\beta}q^s}\left(x_n + w_n^q\right). \tag{D.2}$$

And

$$v_n^s = \left(\frac{1-\beta}{\beta}\right)^{1-\beta}\frac{(q^s)^\beta}{\frac{1-\beta}{\beta}q+q^s}y_n - \frac{1}{\sigma}\left(z_n/n\right)^\sigma. \tag{D.3}$$

His misperception wedge of commodity price satisfies

$$w_n^q = (q-q^s)\frac{\partial x_n^{s*}}{\partial q}. \tag{D.4}$$

The optimality condition (2) indicates that

$$n = \left[\left(q+\frac{\beta}{1-\beta}q^s\right)^{1-\beta}\left(\frac{1-\beta}{\beta}\frac{q}{q^s}+1\right)^\beta\frac{1}{Q_n^s}z_n^{\sigma-1}\right]^{1/\sigma}. \tag{D.5}$$

As $Q_n^s = 1-k_1T'(z_n)+k_2\frac{(1-q)x_n^s}{z_n} = 1-k_1+k_1Q_n+k_2(1-q)\frac{x_n}{z_n}$, we have

$$\frac{\partial Q_n^s}{\partial q} = -k_2\left(1+\frac{1-q}{q+\frac{\beta}{1-\beta}q^s}\left(1+\frac{\beta}{1-\beta}\frac{\partial q^s}{\partial q}\right)\right)\frac{x_n}{z_n}; \tag{D.6}$$

$$\frac{\partial Q_n^s}{\partial Q_n} = k_1; \tag{D.7}$$

$$\frac{\partial Q_n^s}{\partial z_n} = -k_2\left(\frac{1}{z_n}\right)^2(1-q)x_n^s + k_2(1-q)\frac{1}{z_n}Q_n\frac{\partial x_n}{\partial y_n}. \tag{D.8}$$

Using optimal taxation formulas in Proposition 3, we have the following expressions for optimal marginal income tax rates and the perceived price for electrical vehicles.

$$\frac{T'(z_n)}{1-T'(z_n)} = \left[\frac{\partial Q_n^s}{\partial Q_n}Q'(z_n)+\frac{\partial Q_n^s}{\partial z_n}+\frac{v_{zz}^s}{v_y^s}\right]\frac{1}{Q_n\tilde{f}(z)}\Theta_z - (q-1)\frac{\partial x_n}{\partial y_n} + g_n\tau_n^b \tag{D.9}$$

$$(q^s-1)\int_Z\frac{\partial x_{in}^{s*}}{\partial q_j}\tilde{f}(z)\mathrm{d}z = \int_Z\mathit{\Theta}_z\left[-\frac{\partial\left(x_{jn}^s+w_{jn}^s\right)}{\partial y}(Q_n-Q_n^s)-\frac{\partial Q_n^s}{\partial q_j}\right]\mathrm{d}z. \tag{D.10}$$

where $\Theta = \int_z^{\bar{z}}(1-g-\frac{q-1}{q+\frac{\beta}{1-\beta}q^s})\tilde{f}(z)dz$. Denote by p_1 and p_2 the share of tag price and the future energy cost for an electric car. Since consumers are assumed to overestimate future energy cost, we can write the perceived price q^s as $q^s = p_1+sp_2-(1-q)$, in which s measures the degree of overestimation and $1-q$ denotes the subsidy on purchasing an electric car.

D.2 Data Sources, Calibration Procedure and Computing Algorithm

The simulation process takes the following steps. First, we calibrate actual price and perceived price for electric cars using the 2017 National Household Travel Survey. Second, we calibrate consumers' preference parameter β using the average expenditure share of on vehicle purchases from the Consumer Expenditure Survey in the United States. Third, we draw the income distribution from the 2010 Current Population Survey (CPS), and use kernel density estimation to calibrate the density across incomes. We also compute marginal tax rates for each income level to get the implicit skill distribution. Then, we use the fixed-point algorithm as in Mankiw, Weinzierl and Yagan (2009) to find the optimal mixed tax schedule. More details are provided in the below.

The National Household Travel Survey (NHTS) is the main national source of data on the travel behavior of the American public. The data are directly collected from a stratified random sample of U. S. households. The 2017 NHTS collected information on model, fuel type, annual fuel expenditure and age for each car owned by an interviewed family, which can be used to estimate actual price for each model of electric cars. We adopt the most popular model to do the analysis. Among vehicles using hybrid, electric or alternative fuel, those made by Toyota accounts for 54.75 percent, while the second most popular maker, Ford, only occupies a share of 9.76 percent. 78.22 percent of surveyed Toyota cars using hybrid, electric or alternative fuel are Toyota Prius. Therefore, we adopt a price of 24,370 US dollars posted on https://www.caranddriver.com/toyota/prius-2017 as the un-normalized tag price for electric vehicles. We also find from the data that the average annual fuel expenditure for hybrid, electric or alternative fuel vehicles is 787 dollars and the average age of them is 5.8 years. In this way, we get the share of tag price and the future energy cost as 0.84 and 0.16 separately. The Qualified Plug-In Electric Drive Motor Vehicle Credit, or PEDVC is an income tax credit for the purchase of a new qualified plug-in electric drive motor vehicle starting from 2009. The credit for these types of vehicles ranges from \$2,500 to \$7,500, depending on the capacity of the electric battery. Since the credit is \$2500 for a Toyota Prius Plug-in Electric Drive Vehicle whose model year ranges from 2012 to 2015, we calibrate the subsidy rate to be 0.086, as a share of the sum of tag price and future energy cost. In the baseline simulation, we use the estimation in Allcott and Wozny (2014) that consumers are indifferent between 76 cents in vehicle purchase price and one dollar of discounted future gas cost to get a normalized perceived price of 0.962. We compute the average ratio of new vehicle purchases to after-tax income between 2000 to 2020 from the Consumer Expenditure Survey. Using calibrated actual and perceived prices, we get β from equation (D.1). Using the condition (D.5), we can obtain the ability distribution.

We use a bandwidth of \$5000 for the density estimation of income distribution. To compute marginal income tax rates under current tax schedule, we follow Lockwood (2020) by using CPS and the National Bureau of Economic Research's TAXSIM calculator. The bandwidth for the computation of marginal tax rates is \$2000. Given the initial mixed tax schedule and the initial income distribution, we can compute a new income tax schedule using equation (D.9) and a new q^s using (D.10). Then consumers choose labor supply and consumption demands. This loop is repeated until a fixed-point optimal tax schedule is found.

Appendix E Proofs

E.1 The solution of optimal commodity tax following Ferey, Lockwood and Taubinsky (2024)

Lemma 2. *A small increase* $\mathrm{d}\tau$ *in the commodity tax rate* j *faced by an individual earning* z *induces the same earnings change as a small increase* $\mathrm{d}\tau^l$ *in the marginal tax rate on* z *satisfying*

$$\mathrm{d}\tau^l = \left[\left(\frac{\partial x_{jn}^s}{\partial y_n} + \frac{\partial w_{jn}^q}{\partial y_n} \right) Q_n^s + \frac{1}{n} \left(\frac{\partial x_{jn}^s}{\partial l_n} + \frac{\partial w_{jn}^q}{\partial l_n} \right) - \frac{\partial Q_n^s}{\partial q_j} \right] \mathrm{d}\tau \tag{E.1}$$

Proof. To construct such an equivalent marginal income tax reform, use the expression of elasticities defined in section II to construct the following relationship:

$$\frac{\mathrm{d}z_n}{\mathrm{d}q_j} - e_{q_i}\varepsilon_z^I = \frac{z_n}{Q_n}\left[\left(Q_n^s \frac{\partial e_{q_i}}{\partial y_n} + \frac{\mathrm{d}e_{q_i}}{\mathrm{d}z_n}\right) - \frac{\partial Q_n^s}{\partial q_j}\right]\varepsilon_{zQ}^*. \tag{E.2}$$

Therefore, we have

$$\frac{\mathrm{d}z_n}{\mathrm{d}q_j}\mathrm{d}\tau - e_{q_i}\varepsilon_z^I\mathrm{d}\tau = \frac{z_n}{Q_n}\varepsilon_{zQ}^*\mathrm{d}\tau^l, \tag{E.3}$$

where $\mathrm{d}\tau^l$ satisfies (E.1). As ε_{zQ}^* measures the effects of decreasing Q_n by definition, (E.3) implies that a small increase $\mathrm{d}\tau$ in the commodity tax rate j changes earnings z_n. If we deduct income effects $e_{q_i}\varepsilon_z^I\mathrm{d}\tau$ from the changes, the remaining can be replicated by reducing Q_n by $\mathrm{d}\tau^l$ through compensation effects. □

We use a combination of commodity tax reform and income tax reforms which neutralize all lump-sum changes in tax liability. In this way, all utility changes can be offset.

First, we introduce a small increase in commodity j's tax rate $\mathrm{d}\tau$. The mechanical effect of the reform at $\underline{z}$ is $x_{jn}(\underline{z})\mathrm{d}\tau$. We then adjust the labor income tax with a lump-sum decrease by $(x_{jn}(\underline{z}) + w_{jn}^q(\underline{z}))\mathrm{d}\tau$ to offset the mechanical effect of the reform at $\underline{z}$. The marginal change of the government objective is

$$\begin{aligned}
\frac{\mathrm{d}L}{\mu} = &\int_{\underline{z}}^{\bar{z}} (1-\gamma(z))\left(x_{jn}(z) - x_{jn}(\underline{z})\right)\mathrm{d}\tau \tilde{f}(z)\,\mathrm{d}z - \int_{\underline{z}}^{\bar{z}} (1-\gamma(z))\, w_{jn}^q(\underline{z})\,\mathrm{d}\tau \tilde{f}(z)\,\mathrm{d}z \\
&+ \int_{\underline{z}}^{\bar{z}} \sum_i t_i \frac{x_{in}^s}{q_j}\varepsilon_{x_i q_j}^* \mathrm{d}\tau \tilde{f}(z)\,\mathrm{d}z \\
&- \int_{\underline{z}}^{\bar{z}} \left(T'(z_n) + \sum_i t_i \frac{x_{in}^s}{z_n}\varepsilon_{x_i z}\right)\frac{z_n}{Q_n}\varepsilon_{zQ}^*\mathrm{d}\tau^l \tilde{f}(z)\,\mathrm{d}z \\
&- \left(x_{jn}(\underline{z}) + w_{jn}^q(\underline{z})\right)\mathrm{d}\tau \int_{\underline{z}}^{\bar{z}} \frac{\varepsilon_z^I}{Q(z)} g(z)\left[Q^s(z) - Q(z)\right]\tilde{f}(z)\,\mathrm{d}z \\
&- \mathrm{d}\tau \int_{\underline{z}}^{\bar{z}} \left[\gamma(z)\, w_{jn}^q + g(z_n)\frac{\mathrm{d}z_n}{\mathrm{d}q_j}\left(Q_n^s - Q_n\right)\right]\tilde{f}(z)\,\mathrm{d}z
\end{aligned}$$

Reorganize it to get

$$
\begin{aligned}
\frac{\mathrm{d}L}{\mu} =& \int_{\underline{z}}^{\bar{z}} (1-\gamma(z))\left(x_{jn}(z) - x_{jn}(\underline{z})\right)\mathrm{d}\tau \tilde{f}(z)\mathrm{d}z - \int_{\underline{z}}^{\bar{z}} \gamma(z_n)\left(w_{jn}^{q}(z) - w_{jn}^{q}(\underline{z})\right)\mathrm{d}\tau\tilde{f}(z)\mathrm{d}z \\
&- \int_{\underline{z}}^{\bar{z}} w_{jn}^{q}(\underline{z})\mathrm{d}\tau\tilde{f}(z)\mathrm{d}z \\
&+ \int_{\underline{z}}^{\bar{z}} \sum_i t_i \frac{x_{in}^{s}}{q_j}\varepsilon_{x_i q_j}^{*}\mathrm{d}\tau\tilde{f}(z)\,\mathrm{d}z \\
&- \int_{\underline{z}}^{\bar{z}} \left(T'(z_n) + \sum_i t_i \frac{x_{in}^{s}}{z_n}\varepsilon_{x_i z}\right)\frac{z_n}{Q_n}\varepsilon_{zQ}^{*}\mathrm{d}\tau^{l}\tilde{f}(z)\,\mathrm{d}z \\
&- \left(x_{jn}(\underline{z}) + w_{jn}^{q}(\underline{z})\right)\int_{\underline{z}}^{\bar{z}} \varepsilon_{z}^{I} g(z)\tau^{b}(z)\mathrm{d}\tau\tilde{f}(z)\mathrm{d}z - \int_{\underline{z}}^{\bar{z}} z g(z_n)\frac{\mathrm{d}z_n}{\mathrm{d}q_j}\left(Q_n^{s} - Q_n\right)\mathrm{d}\tau\tilde{f}(z)\mathrm{d}z
\end{aligned}
$$

Next, we construct a sequence of income tax reforms to cancel out lump-sum changes in tax liabilities at all income levels. In a small income bandwidth $[\underline{z}, \underline{z} + \Delta z]$, introduce a decrease in marginal income tax rate by

$$\dot{x}_{jn}^{s}(\underline{z})\,\mathrm{d}\tau + \dot{w}_{jn}^{q}(\underline{z})\,\mathrm{d}\tau,$$

where $\Delta z = (\bar{z} - \underline{z})/K$. The dot over x_{jn}^{s} and w_{jn}^{q} denotes total derivative on z. For instance,

$$\dot{x}_{jn}^{s} = \frac{\partial x_{jn}^{s}}{\partial y_n}Q_n + \frac{1}{n}\frac{\partial x_{jn}^{s}}{\partial l_n} - \frac{l_n}{n}\frac{\partial x_{jn}^{s}}{\partial l_n}\frac{1}{\dot{z}_n}$$

This additional reform indicates a lump-sum transfer of $\left[\dot{x}_{jn}^{s}(\underline{z})\,\mathrm{d}\tau + \dot{w}_{jn}^{q}(\underline{z})\,\mathrm{d}\tau\right]\Delta z$ within the bandwidth $[\underline{z}, \underline{z} + \Delta z]$. Since

$$\left(x_{jn}(\underline{z} + \Delta z) - x_{jn}(\underline{z})\right)\mathrm{d}\tau + \left(w_{jn}^{q}(\underline{z} + \Delta z) - w_{jn}^{q}(\underline{z})\right) = \left[\dot{x}_{jn}^{s}(\underline{z})\,\mathrm{d}\tau + \dot{w}_{jn}^{q}(\underline{z})\,\mathrm{d}\tau\right]\Delta z$$

the new reform cancels out redistribution effects caused by previous reforms within $[\underline{z}, \underline{z} + \Delta z]$. It also induces behavioral wedge effects and behavioral wedge effects. The overall effects of these reforms are

$$\begin{aligned}
\frac{\mathrm{d}L}{\mu} &= \int_{\underline{z}+\Delta z}^{\bar{z}} (1-\gamma(z))\left(x_{jn}(z) - x_{jn}(\underline{z}) - \dot{x}_{jn}(\underline{z})\Delta z\right)\mathrm{d}\tau \tilde{f}(z)\mathrm{d}z \\
&-\mathrm{d}\tau \int_{\underline{z}+\Delta z}^{\bar{z}} \gamma(z_n)\left(w_{jn}^q(z) - w_{jn}^q(\underline{z}) - \dot{w}_{jn}^q(\underline{z})\Delta z\right)\mathrm{d}\tau \tilde{f}(z)\mathrm{d}z \\
&- \int_{\underline{z}}^{\bar{z}} w_{jn}^q(\underline{z})\mathrm{d}\tau \tilde{f}(z)\mathrm{d}z - \int_{\underline{z}}^{\bar{z}} \dot{w}_{jn}^q(\underline{z})\mathrm{d}\tau \tilde{f}(z)\mathrm{d}z \\
&+ \int_{\underline{z}}^{\bar{z}} \sum_i t_i \frac{x_{in}^s}{q_j} \varepsilon_{x_i q_j}^* \mathrm{d}\tau \tilde{f}(z)\mathrm{d}z \\
&- \int_{\underline{z}}^{\bar{z}} \left(T'(z_n) + \sum_i t_i \frac{x_{in}^s}{z_n}\varepsilon_{x_i z}\right)\frac{z_n}{Q_n}\varepsilon_{zQ}^* \left[\left(\frac{\partial x_{jn}^s}{\partial y_n} + \frac{\partial w_{jn}^q}{\partial y_n}\right)Q_n^s + \frac{1}{n}\left(\frac{\partial x_{jn}^s}{\partial l_n} + \frac{\partial w_{jn}^q}{\partial l_n}\right) - \frac{\partial Q_n^s}{\partial q_j}\right]\mathrm{d}\tau \tilde{f}(z)\mathrm{d}z \\
&- \left(x_{jn}(\underline{z}) + w_{jn}^q(\underline{z})\right)\mathrm{d}\tau \int_{\underline{z}}^{\bar{z}} \varepsilon_z^I g(z)\tau^b(z)\tilde{f}(z)\mathrm{d}z \\
&- \left(\dot{x}_{jn}(\underline{z}) + \dot{w}_{jn}^q(\underline{z})\right)\mathrm{d}\tau \Delta z \int_{\underline{z}}^{\bar{z}} \varepsilon_z^I g(z)\tau^b(z)\tilde{f}(z)\mathrm{d}z \\
&+ \left(\dot{x}_{jn}(\underline{z}) + \dot{w}_{jn}^q(\underline{z})\right)\mathrm{d}\tau \int_{\underline{z}}^{\underline{z}+\Delta z} \left[T'(z) + \sum_i t_i \frac{x_{in}^s}{z_n}\varepsilon_{x_i z} - g(z)[Q^s(z) - Q(z)]\right]\frac{z}{Q_n}\varepsilon_{zQ}^*(z)\tilde{f}(z)\mathrm{d}z
\end{aligned}$$

The following sequence of income tax reforms are: (1) In the bandwith $[\underline{z}+\Delta z, \underline{z}+2\Delta z]$, decrease the marginal income tax rate by $\dot{x}_{jn}^s(\underline{z}+\Delta z)\,\mathrm{d}\tau + \dot{w}_{jn}^q(\underline{z}+\Delta z)\,\mathrm{d}\tau$; ... (k) In the bandwith $[\underline{z}+k\Delta z, \underline{z}+(k+1)\Delta z]$, decrease the marginal income tax rate by $\dot{x}_{jn}^s(\underline{z}+k\Delta z)\,\mathrm{d}\tau + \dot{w}_{jn}^q(\underline{z}+k\Delta z)\,\mathrm{d}\tau$; ... ; (K-1) In the bandwith $[\underline{z}+(K-1)\Delta z, \underline{z}+K\Delta z]$, decrease the marginal income tax rate by $\dot{x}_{jn}^s(\underline{z}+(K-1)\Delta z)\,\mathrm{d}\tau + \dot{w}_{jn}^q(\underline{z}+(K-1)\Delta z)\,\mathrm{d}\tau$. At the limit $K \to \infty$, the total effects can be expressed as

$$\begin{aligned}
\frac{\mathrm{d}L}{\mu} &= \int_{\underline{z}}^{\bar{z}} \sum_i t_i \frac{x_{in}^s}{q_j}\varepsilon_{x_i q_j}^* \mathrm{d}\tau \tilde{f}(z)\mathrm{d}z - \int^{\bar{z}} w_{jn}^q(z)\mathrm{d}\tau \tilde{f}(z)\mathrm{d}z \\
&+\mathrm{d}\tau \int_{\underline{z}}^{\bar{z}} \left(\frac{T'(z)}{Q_n} + \sum_i \frac{t_i}{Q_n}\frac{x_{in}^s}{z_n}\varepsilon_{x_i z} - g(z)\tau^b(z)\right) z\varepsilon_{zQ}^* \times \\
&\quad \left[\dot{x}_{jn}^s(z) + \dot{w}_{jn}^q(z) - \left(\frac{\partial x_{jn}^s}{\partial y_n} + \frac{\partial w_{jn}^q}{\partial y_n}\right)Q_n^s - \frac{1}{n}\left(\frac{\partial x_{jn}^s}{\partial l_n} + \frac{\partial w_{jn}^q}{\partial l_n}\right) + \frac{\partial Q_n^s}{\partial q_j}\right]\tilde{f}(z)\mathrm{d}z
\end{aligned}$$

Use equation (10) and let $\frac{\mathrm{d}L}{\mu} = 0$ to get equation (16).

E.2 The solution using mechanism design approach

We use mechanism design approach to get the results, as a comparison with Farhi and Gabaix (2020), who used tax perturbation method instead.

The government provides contracts over $\{y_n, z_n\}$ to consumers, who will perceive y_n as y_n^s. The incentive compatible constraint requires that

$$v^s(q, y_n^s, z_n/n) \geqslant v^s(q, y_{\tilde{n}}^s, z_{\tilde{n}}/n)$$

While the first-order IC is

$$\dot{v}_n^s = -\frac{z_n}{n}\frac{1}{n}\frac{\partial v_n^s\left(q, e_n, z_n/n\right)}{\partial l_n} + \frac{\partial v_n^s\left(q, e_n, z_n/n\right)}{\partial y_n}\left(Q_n - Q_n^s\right)\kappa_n$$

where $\kappa_n \equiv \dot{z}_n$.

The Lagrangian for this optimal control problem is

$$\begin{aligned}
L \equiv & \int_N \left[\Psi\left(v_n^s\right) + \mu\left(z_n - c_n^{s*}\left(q, v_n^s, z_n/n\right) - \sum x_n^{s*}\left(q, v_n^s, z_n/n\right) - R\right)\right] f(n)\mathrm{d}n \\
& + \int_N \left[\theta_n \frac{z_n}{n}\frac{1}{n}\frac{\partial v_n^s\left(q, e_n\left(q, v_n^s, z_n/n\right), z_n/n\right)}{\partial l_n} - \theta_n \frac{\partial v_n^s\left(q, e_n\left(q, v_n^s, z_n/n\right), z_n/n\right)}{\partial y_n}\left(Q_n - Q_n^s\right)\kappa_n - v_n^s\dot{\theta}_n\right]\mathrm{d}n \\
& - \int_N \left(\lambda_n\kappa_n + z_n\dot{\lambda}_n\right)\mathrm{d}n \\
& + \theta_{\bar{n}}v_{\bar{n}} - \theta_{\underline{n}}v_{\underline{n}} + \lambda_{\bar{n}}z_{\bar{n}} - \lambda_{\underline{n}}z_{\underline{n}}.
\end{aligned}$$

Define $\Theta_n \equiv \theta_n v_y^s/\mu$. By solving the first-order conditions, we have

$$\frac{1 - Q_n}{Q_n} - g_n\tau_n^b + \sum_i \frac{t_i}{Q_n}\frac{x_{in}^s}{z_n}\varepsilon_{x_i z} = -\frac{\Theta_n}{nf(n)}\frac{\varepsilon_{zn}}{\varepsilon_{zQ}^*}, \tag{E.4}$$

Optimal income tax rates satisfy

$$\frac{T'(z)}{1 - T'(z)} + \sum_i \frac{t_i}{Q_n}\frac{x_{in}^s}{z}\varepsilon_{x_i z} - g_n\tau_n^b = \frac{1}{\varepsilon_{zQ}^*}\frac{1}{z\tilde{f}(z)}\Theta_z. \tag{E.5}$$

Θ_z is given by

$$\Theta_z = \int_z^{\bar{z}} e^{-\int_z^{z'}\rho(s)\mathrm{d}s}\left(1 - g(z) - \sum_i t_i\frac{x_{in}^s}{y_n}\varepsilon_{x_i}^I\right)\tilde{f}(z')\mathrm{d}z'; \rho(z) = -\frac{\varepsilon_z^I}{\varepsilon_{zQ}^*}\frac{1}{z}. \tag{E.6}$$

By solving first-order conditions on q_j, We find that optimal commodity tax rates satisfy

$$\begin{aligned}
& \int_Z \left[\sum_k t_k\frac{\partial x_{kn}^{s*}\left(q, v_n^s, z_n/n\right)}{\partial q_i} - w_{in}^s\right]\tilde{f}(z)\mathrm{d}z \\
= & \int_Z \mathit{\Theta}_z\frac{\mathrm{d}e_{q_j}}{\mathrm{d}z}\frac{1}{\varepsilon_{zn}}\mathrm{d}z - \int_N \mathit{\Theta}_z\left[\frac{\mathrm{d}e_{q_j}}{\mathrm{d}y}\left(Q_n - Q_n^s\right)\right]\mathrm{d}z - \int_Z \mathit{\Theta}_z\frac{\partial Q_n^s\left(Q\left(z\right), q, z\right)}{\partial q_j}\mathrm{d}z.
\end{aligned}$$

Since $e_{q_j} = x_{jn}^s + w_{jn}^s$, we can rewrite the above equation as the form in (16).

E.3 Proof of proposition 5

The Lagrangian for this optimal control problem is the same with subsection E.1. Notice that Q_m^s might be influenced by Q_n, then the first-order condition on z_n changes into:

$$\begin{aligned}
\frac{\partial L}{\partial z_n} =& \mu \left(1 - \frac{\partial c_n^{s*}}{\partial z_n} - \frac{\sum_i \partial x_{in}^{s*}}{\partial z_n}\right) f(n) + \frac{\theta_n}{n}\frac{1}{n}\frac{\partial v_n^s}{\partial l_n} \\
&+\theta_n \frac{z_n}{n}\frac{1}{n}\left[\frac{\partial^2 v_n^s\left(q, y_n, z_n/n\right)}{\partial l_n \partial l_n}\frac{1}{n} + Q_n^s \frac{\partial^2 v_n^s\left(q, y_n, z_n/n\right)}{\partial y_n \partial l_n}\right] \\
&-\theta_n \left[\frac{\partial^2 v_n^s\left(q, y_n, z_n/n\right)}{\partial y_n \partial l_n}\frac{1}{n} + Q_n^s \frac{\partial^2 v_n^s\left(q, y_n, z_n/n\right)}{\partial y_n \partial y_n}\right]\left(Q_n - Q_n^s\right)\kappa_n \\
&-\theta_n \frac{\partial v_n^s\left(q, y_n, z_n/n\right)}{\partial y_n}\left[Q\prime(z_n) - \frac{\partial Q_n^s}{\partial Q_n} Q\prime(z_n) - \frac{\partial Q_n^s}{\partial z_n}\right]\kappa_n \\
&-Q\prime(z_n)\int_N \left(\theta_{\tilde{n}} v_{\tilde{z}} \frac{Q_{\tilde{n}}}{\left(Q_{\tilde{n}}^s\right)^2}\frac{\partial Q_{\tilde{n}}^s}{\partial Q_n}\kappa_{\tilde{n}}\right)\mathrm{d}\tilde{n} - \dot{\lambda}_n = 0.
\end{aligned}$$

Use the shift function method to get the expression of $\frac{\mathrm{d}z_n}{\mathrm{d}\phi}$:

$$\frac{\mathrm{d}z_n}{\mathrm{d}\phi} = \frac{z_n}{Q_n}\left[\tau'\left(z_n\right)\varepsilon_{zQ}^* + \tau\left(z_n\right)\varepsilon_z^I/z_n + \int_N \varepsilon_{zQm}^*\left(\tau'(z_m) - Q'(z_m)\frac{dz_m}{d\phi}\right)(1-\delta_n)\mathrm{d}m\right],$$

in which ε_{zQm}^* is the compensated tax elasticity when marginal tax rate at z_m changes, and δ_n is the Dirac distribution at skill level n. Taking the similar approach as in section EE.2 to get optimal tax formulas in proposition 5.

E.4 Proof of proposition 3 and 4

Proof. Since $\frac{\mathrm{d}y_n}{\mathrm{d}q_j} = Q_n \frac{\mathrm{d}z_n}{\mathrm{d}q_j}$, we use modified Roy's identity (A.29) as well as Slutsky equation (A.30) to get

$$\frac{\mathrm{d}x_{in}^s}{\mathrm{d}q_j} = \frac{\partial x_{jn}^{s*}}{\partial q_j} - \left(w_{jn}^q + x_{jn}^s\right)\frac{\partial x_{in}^s}{\partial y_n} + \left(Q_n \frac{\partial x_{in}^s}{\partial y_n} + \frac{\partial x_{in}^s}{\partial z_n}\right)\frac{\mathrm{d}z_n}{\mathrm{d}q_j}. \tag{E.7}$$

Then we could transform the right-hand side of (B.1) by substituting $\frac{\mathrm{d}V_n^s/\mathrm{d}q_j}{\mathrm{d}V_n^s/\mathrm{d}R_n}$ and $\frac{\mathrm{d}x_{in}^s}{\mathrm{d}q_j}$ with expression in (B.3) and (E.7). Using the definition that $g_n \equiv \frac{\Psi' v_y^s}{\mu}$ and $\xi_{zq_j} \equiv \frac{q_j}{z_n}\frac{\mathrm{d}z_n}{\mathrm{d}q_j}$ and the relationship $\int_n^{\bar{n}} af(n)\mathrm{d}n = \int_z^{\bar{z}} a\tilde{f}(z)\mathrm{d}z$ implied by $F(n) \equiv \tilde{F}(z_n)$, we could express the marginal impact of commodity tax on net social welfare as

$$\begin{aligned}
\frac{\mathrm{d}W}{\mathrm{d}q_j}/\mu + \frac{\mathrm{d}B}{\mathrm{d}q_j} =& \int_Z \sum_i t_i \frac{\partial x_{in}^{s*}}{\partial q_j}\tilde{f}(z)\mathrm{d}z - \int_Z w_{jn}^q \tilde{f}(z)\mathrm{d}z \\
&+ \int_Z \left(1 - g_n - \sum_i t_i \frac{\partial x_{in}^s}{\partial y_n}\right)\left(w_{jn}^q + x_{jn}^s\right)\tilde{f}(z)\mathrm{d}z \\
&- \int_Z \left(Q_n^s - Q_n\right) g_n \frac{\mathrm{d}z_n}{\mathrm{d}q_j}\tilde{f}(z)\mathrm{d}z \\
&+ \int_Z \left[-Q_n \sum_i t_i \frac{\partial x_{in}^s}{\partial y_n} - \sum_i t_i \frac{\partial x_{in}^s}{\partial z_n} + T'(z_n)\right]\frac{\mathrm{d}z_n}{\mathrm{d}q_j}\tilde{f}(z)\mathrm{d}z.
\end{aligned}$$

Applying $\frac{\mathrm{d}W}{\mathrm{d}q_j}/\mu+\frac{\mathrm{d}B}{\mathrm{d}q_j}=0$ gives an optimal linear commodity tax formula. Transform this formula using $e_{q_j}=w^q_{jn}+x^s_{jn}$ into

$$
\begin{aligned}
\int_N \sum_i t_i \frac{\partial x^{s*}_{in}}{\partial q_j} f(n)\mathrm{d}n - \int_N w^q_{jn} f(n)\mathrm{d}n & \\
= -\int_N \left(1-g_n-\sum_i t_i \frac{\partial x^s_{in}}{\partial y_n}\right) f\left(n\right) e_{q_j} dn & \\
-\int_N \left(\frac{1-Q_n}{Q_n}-g_n\tau^b_n+\sum_i \frac{t_i}{Q_n}\frac{x^s_{in}}{z_n}\varepsilon_{x_i z}\right) Q_n \frac{\mathrm{d}z_n}{\mathrm{d}q_j} f(n)\mathrm{d}n. &
\end{aligned} \tag{E.8}
$$

The first item on the right-hand side could be found in the expression of Θ_z in (11) and the second item of the right-hand side relates to the left-hand side of optimal income tax formula. Thus we could transform the right-hand side of (E.8) into

$$
\begin{aligned}
&\int_N e_{q_j}\mathrm{d}\Theta_n - \int_N \Theta_n \frac{1}{z_n}\frac{\varepsilon^I_z}{\varepsilon^*_{zQ}}\frac{\mathrm{d}z_n}{\mathrm{d}n} e_{q_j}\mathrm{d}n \\
&+\int_N \Theta_n \frac{\mathrm{d}z_n}{\mathrm{d}n}\left(\frac{Q_n}{z_n}\frac{-1}{\varepsilon^*_{zQ}}\right)\frac{\mathrm{d}z_n}{\mathrm{d}q_j}\mathrm{d}n
\end{aligned} \tag{E.9}
$$

Then we perform the following transformations on separate parts of (E.9). Firstly, since

$$
\int_N e_{q_j}\mathrm{d}\Theta_n = -\int_N \Theta_n \frac{\mathrm{d}e_{q_j}}{\mathrm{d}n}\mathrm{d}n = \int_N \Theta_n \frac{z_n}{n}\frac{\mathrm{d}e_{q_j}}{\mathrm{d}z_n}\mathrm{d}n - \int_N \Theta_n \left(\frac{\partial e_{q_j}}{\partial v^s_n}\frac{\partial v^s_n}{\partial z_n}+\frac{\partial e_{q_j}}{\partial z_n}+\frac{\partial e_{q_j}}{\partial v^s_n}v^s_y Q_n\right)\frac{\mathrm{d}z}{\mathrm{d}n}\mathrm{d}n,
$$

and income elasticities could be expressed with derivatives of indirect utility function as in the main text, we have

$$
\begin{aligned}
\int_N e_{q_j}\mathrm{d}\Theta_n - \int_N \Theta_n \frac{1}{z_n}\frac{\varepsilon^I_z}{\varepsilon^*_{zQ}}\frac{\mathrm{d}z_n}{\mathrm{d}n} e_{q_j}\mathrm{d}n & \\
= \int_N \Theta_n \frac{z_n}{n}\frac{\mathrm{d}e_{q_j}}{\mathrm{d}z_n} dn - \int_N \Theta_n \frac{\mathrm{d}z_n}{\mathrm{d}n}\left(\frac{\mathrm{d}e_{q_i}}{\mathrm{d}z_n}+\frac{\partial e_{q_j}}{\partial v^s_n}\frac{\partial v^s_n}{\partial y_n}Q_n\right)\mathrm{d}n & \\
-\int_N \Theta_n \frac{\mathrm{d}z_n}{\mathrm{d}n}\frac{Q^s_n v^s_{yy}+v^s_{zy}}{v^s_y} e_{q_j}\mathrm{d}n. &
\end{aligned} \tag{E.10}
$$

Next, using relationship between elasticities in (C.8) and the expression of $\mathrm{d}z_n/\mathrm{d}q_j$ in (B.2), we have

$$
\begin{aligned}
&\int_N \Theta_n \frac{\mathrm{d}z}{\mathrm{d}n}\left(\frac{Q_n}{z}\frac{-1}{\varepsilon^*_{zQ}}\right)\frac{\mathrm{d}z_n}{\mathrm{d}q_j}\mathrm{d}n \\
&= \int_N \Theta_n \frac{\mathrm{d}z}{\mathrm{d}n}\left(\frac{Q_n}{z}\frac{1}{\frac{Q_n}{z_n}D^{-1}_n v^s_y}\right)\frac{Q^s_n v^s_{yq_j}+v^s_{zq_j}+v^s_y\frac{\partial Q^s_n}{\partial q_j}}{D_n}\mathrm{d}n \\
&= \int_N \Theta_n \frac{\mathrm{d}z}{\mathrm{d}n}\frac{Q^s_n v^s_{yq_j}+v^s_{zq_j}+v^s_y\frac{\partial Q^s_n}{\partial q_j}}{v^s_y}\mathrm{d}n
\end{aligned} \tag{E.11}
$$

The first to the second line is gained by using (C.3) and (C.7). Taking all these transformations together, we could reorganize the right-hand side of (E.8) into

$$\begin{aligned}&\int_N \Theta_n \frac{z_n}{n}\frac{\mathrm{d}e_{q_j}}{\mathrm{d}z_n}\mathrm{d}n\\ &-\int_N \Theta_n \frac{\mathrm{d}z_n}{\mathrm{d}n}\left[\frac{\mathrm{d}e_{q_i}}{\mathrm{d}z_n}+\frac{\partial e_{q_j}}{\partial v_n^s}v_y^s Q_n+\left(\frac{Q_n^s v_{yy}^s+v_{zy}^s}{v_y^s}e_{q_j}+\frac{Q_n^s v_{yq_j}^s+v_{zq_j}^s}{v_y^s}+\frac{\partial Q_n^s}{\partial q_j}\right)\right]\mathrm{d}n\end{aligned} \tag{E.12}$$

While we could use properties of e_{q_j} in (A.32), to simplify the first line plus the second line of (E.12) into (transformed into integrals on z)

$$\begin{aligned}&\int_Z \mathit{\Theta}_z \frac{\partial\left[x_{jn}^s(q,y,z)+w_{in}^s(q,y,z)\right]}{\partial z}\frac{1}{\varepsilon_{zn}}\mathrm{d}z\\ &-\int_Z \mathit{\Theta}_z \frac{\partial\left[x_{jn}^s(q,y,z)+w_{in}^s(q,y,z)\right]}{\partial y}(Q_n-Q_n^s)\mathrm{d}z-\int_Z \mathit{\Theta}_z\frac{\partial Q_n^s(Q(z),q,z)}{\partial q_j}\mathrm{d}z\end{aligned}$$

Therefore, we exactly get the right-hand side of (16). When there is no misperception, the proof is similar. □

E.5 Proof of corollary 1

Proof. From the first-order condition (1) and budget constraint in second stage optimization problem, we find that when individual's utility is weakly separable between commodities and labor, conditional commodity demands x_n^s and c_n^s are independent of z_n so that $\frac{\partial x_{jn}^s}{\partial z_n}=0$ for $\forall j\in\{1:I\}$. Moreover, since $v_n^s(q,y_n,l_n)=u(c_n^s,x_n^s,l_n)=u(h(c_n^s,x_n^s),l_n)$, we also find that $\frac{\partial v_n^s}{\partial q_j}/\frac{\partial v_n^s}{\partial y_n}$ is independent to z_n. Therefore, in Slutsky equation (A.30), all items except the first item on the right-hand side are irrelevant to z_n, which means $\frac{\partial w_{jn}^q}{\partial z_n}=0$. Use these conditions to simplify (16) to get (17). □

E.6 Proof of corollary 3

Proof. We first transform optimal commodity tax formula (E.8) into:

$$\begin{aligned}&\int_N \sum_i t_i \frac{\partial x_{in}^{s*}}{\partial q_j}f(n)\mathrm{d}n-\int_N w_{jn}^q f(n)\mathrm{d}n\\ =&-\int_N\left(1-g_n-\sum_i t_i\frac{\partial x_{in}^s}{\partial y_n}\right)\left(x_{jn}^s+w_{jn}^q\right)f(n)\mathrm{d}n\\ &-\int_N\left(\frac{1-Q_n}{Q_n}-g_n\tau_n^b+\sum_i\frac{t_i}{Q_n}\frac{x_{in}^s}{z_n}\varepsilon_{x_iz}\right)Q_n\frac{\mathrm{d}z_n}{\mathrm{d}q_j}f(n)\mathrm{d}n.\end{aligned} \tag{E.13}$$

Use (E.4) to substitute expression in the brackets in the second item on the right-hand side of this formula. Use expression of $\mathrm{d}z_n/\mathrm{d}q_j$ in (B.2) to decompose the influence of q_j on z_n. The right-hand side of (E.13) could be transformed into

$$\begin{aligned}RHS=&-\int_N\left(1-g_n-\sum_i t_i\frac{\partial x_{in}^s}{\partial y_n}\right)\left(x_{jn}^s+w_{jn}^q\right)f(n)\mathrm{d}n\\ &+\int_N\left(\frac{\Theta_n}{n}\frac{\varepsilon_{zn}}{\varepsilon_{zQ}^*}\right)Q_n\frac{\mathrm{d}z_n}{\mathrm{d}q_j}\mathrm{d}n.\end{aligned} \tag{E.14}$$

Using the expression of elasticities defined in section II, we could have

$$\frac{Q_n \frac{\mathrm{d}z_n}{\mathrm{d}q_j}}{\varepsilon^*_{zQ}} = z_n \left[Q^s_n \frac{\partial e_{q_i}}{\partial y_n} + \frac{\mathrm{d}e_{q_i}}{\mathrm{d}z_n} \right] + e_{q_i} \frac{\varepsilon^I_z}{\varepsilon^*_{zQ}} - z_n \frac{\partial Q^s_n}{\partial q_j} \tag{E.15}$$

Saez (2002) also makes similar decomposition (in Lemma 1) to compute influence of commodity tax reform on labor income. However, he uses the first-order Taylor expansion to get an approximate result, while our decomposition is more precise taken into consideration the non-linearity of income tax schedule.

Using (E.15), we could transform the right-hand side of equation (E.13) for a step further as:

$$\begin{aligned} RHS = &- \int_N (1-\Gamma_n) e_{q_i} f(n) \mathrm{d}n \\ &- \int_N \left(\frac{\Theta_n}{n} z_n \varepsilon_{zn} \right) \frac{\partial Q^s_n}{\partial q_j} \mathrm{d}n \end{aligned} \tag{E.16}$$

Γ_n is the marginal social utility of one unit of government transfer to consumer n which is defined by

$$\mathit{\Gamma}_n = g_n + \sum_i t_i \frac{\partial x^s_{in}}{\partial y_n} + \frac{\mathit{\Theta}_n}{f(z_n)} \frac{\varepsilon^I_z}{\varepsilon^*_{zQ}} + \frac{\mathit{\Theta}_n}{f(z_n)} \frac{1}{e_{q_i}} \left(Q^s_n \frac{\partial \left(x^s_{jn} + w^q_{jn} \right)}{\partial y_n} + \frac{\partial \left(x^s_{jn} + w^q_{jn} \right)}{\partial z_n} \right)$$

To interpret Γ_n, the first item measures a direct gain of the transfer on consumer's welfare, the second item measures the effect on commodity tax income, and the sum of the third and the forth items measures social welfare gain from using lump-sum transfer instead of using distortionary income tax. Then we express (E.13) with integral on z and rearrange it into

$$\begin{aligned} -\int_Z \sum_i t_i \frac{\partial x^{s*}_{in}}{\partial q_j} \tilde{f}(z) \mathrm{d}z = &- \int_Z w^q_{jn} \tilde{f}(z) \mathrm{d}z + \int_Z (1-\Gamma_z) e_{q_i} \tilde{f}(z) \mathrm{d}z \\ &+ \int_Z \Theta_n \frac{\partial Q^s_n}{\partial q_j} \mathrm{d}z. \end{aligned} \tag{E.17}$$

It is then straightforward to transform the above equation into

$$\begin{aligned} -\frac{1}{\overline{w^q_j} + \overline{x^s_j}} \overline{\sum_i t_i \frac{\partial x^{s*}_i}{\partial q_j}} = &\, 1 - \overline{\Gamma} - \operatorname{cov}\left(\Gamma, \frac{e_{q_i}}{\overline{w^q_j} + \overline{x^s_j}} \right) \\ &- \frac{\overline{w^q_j}}{\overline{w^q_j} + \overline{x^s_j}} + \frac{1}{\overline{w^q_j} + \overline{x^s_j}} \int_Z \Theta_z \frac{\partial Q^s_n}{\partial q_j} \mathrm{d}z. \end{aligned} \tag{E.18}$$

The "bar" indicates an integral on z. For example, $\overline{w^q_j} \equiv \int_Z w^q_{jn} \tilde{f}(z) \mathrm{d}z, \overline{x^s_j} \equiv \int_N x^s_{jn} \tilde{f}(z) \mathrm{d}z$. □

G. Acknowledgements

We would like to thank the Associate Editors and all anonymous referees for their helpful comments and suggestions on earlier versions of the paper.