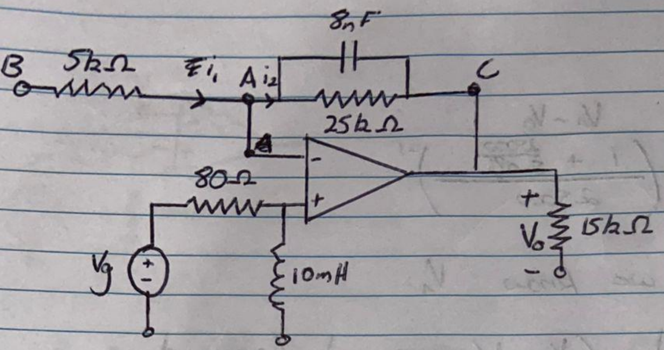

As it is an ideal op amp, we understand $V_{-} = V_{+}$ thus:

$$V_A = V_g \frac{Z_I}{Z_I + R_1}$$ where $R_1 = 80\Omega$, $Z_I = (10\times10^{-3})\omega j$

Further, KCL at A we understand no current passes through the op amp:

$i_1 = i_2$: $$\frac{V_A}{R_2} = \frac{V_A - V_C}{Z_C \| R_3}$$ where $R_2 = 5k\Omega$, $Z_C = \frac{1}{j\omega(8\times10^{-9})}$, $R_3 = 25k\Omega$

Additionally we know $V_C = V_o$ thus:

$$\frac{V_A}{R_2} = \frac{V_A - V_o}{Z_C \| R_3}$$ thus: $$V_g\left(\frac{Z_I}{Z_I+R_1}\right) = \frac{V_g\left(\frac{Z_I}{Z_I+R_1}\right)}{Z_C \| R_3} - \frac{V_o}{Z_C \| R_3}$$

$$Z_C \| R_3 = \left(\frac{1}{Z_C} + \frac{1}{R_3}\right)^{-1} = \frac{Z_C R_3}{Z_C + R_3}$$

Thus finally: $$V_g\left(\frac{Z_I}{Z_I+R_1}\right) = V_g\left(\frac{Z_I}{Z_I+R_1}\right)\left(\frac{Z_C+R_3}{Z_C R_3}\right) - V_o\left(\frac{Z_C+R_3}{Z_C R_3}\right)$$

$$\frac{V_o}{V_g} = \frac{\left(\frac{Z_I}{Z_I+R_1}\right)\left(\frac{Z_C+R_3}{Z_C R_3}\right) - \left(\frac{Z_I}{Z_I+R_1}\right)}{\frac{Z_C+R_3}{Z_C R_3}}$$

Subbing in these values we find:

$$\frac{V_g}{V_o} = \frac{1}{\left(\frac{10^{-2} j\omega}{10^{-2} j\omega + 80}\right)\left(\frac{25000 \frac{1}{8\times10^{-9} j\omega}}{\left(\frac{1}{8\times10^{-9} j\omega} + 25000\right)\frac{1}{5\times10^{3}}} - 1\right)}$$

$$= \left(1 + \frac{80}{10^{-2} j\omega}\right)\left(\frac{25\,000}{\frac{1}{5\times10^{3}} + \frac{5}{8\times10^{-9} j\omega}} - 1\right)^{-1}$$

$$= \frac{1 - \frac{8000}{\omega} j}{\frac{25000}{\frac{1}{5\times10^{3}} - \frac{625\times10^{8} j}{\omega}} - 1}$$

Or alternatively:

$$\frac{V_o}{V_g} = \frac{\frac{25000}{(1/5\times10^{3}) - \frac{625\times10^{9} j}{\omega}} - 1}{1 - \frac{8000 j}{\omega}} \xrightarrow{\text{Long simplification}} = \left(\frac{(j\omega\times10^{-2})}{8\times10^{3} + j\omega}\right)\left(\frac{j\omega + 30\,000}{j\omega + 5000}\right)$$

Now given $s = j\omega + \sigma$ we find for $\sigma = 0$:

$$\frac{V_o(s)}{V_g(s)} = \left(\frac{j\omega}{8\times10^{3} + j\omega}\right)\left(\frac{j\omega + 30\,000}{j\omega + 5000}\right) = \frac{s(s + 30\,000)}{(s + 8000)(s + 5000)}$$

b) In order to change $V_g(t)$ into a frequency dependent function, we use a Laplace transform.

$$\mathcal{L}\{0.6u(t)\} = 0.6\,\mathcal{L}\{u(t)\}$$

which by definition $= 0.6 \times \frac{1}{s} = V_g(s)$

Given $\frac{V_o}{V_g}$ we find

$$V_o = V_g \frac{s(s+30\,000)}{(s+8000)(s+5000)} = \frac{0.6}{s}\frac{s(s+30\,000)}{(s+8000)(s+5000)}$$

$$= \frac{0.6(s+30\,000)}{(s+8000)(s+5000)}$$

Using partial fractions:

$$V_o = \frac{A}{s+8000} + \frac{B}{s+5000}$$

We solve to find $A = \frac{-22}{5}$ and $B = 5$

Thus finally $V_o = \frac{-22}{5}\frac{1}{(s+5000)} + \frac{5}{(s+8000)}$

With inverse laplace $\frac{k}{s+a} \rightarrow ke^{-at}$ from lectures

$$V_o(t) = \frac{-22}{5}e^{-5000t} + 5e^{-8000t}$$

Q2)

Given we know the RLC will follow: $i(t) = C\sin(\omega_d t)e^{-\alpha t}$
and:

$$T \approx \frac{10.9-0.9}{8} = \frac{\text{First Peak} - \text{second Peak}}{\text{Num peaks}} = 1.25s$$

we know since the graph follows $\sin(\omega_d t)$:

$$\omega_d = \frac{2\pi}{T} = \frac{2\pi}{1.25} = 5 \text{ rad/s}$$

We also see a decrease current ~~factor~~:

$$\alpha = \frac{-\ln\left(\frac{A_{11}}{A_1}\right)}{t} = \frac{-\ln\left(\frac{0.2}{0.83}\right)}{10} = 0.142$$

Thus given:

$$L = \frac{R}{2\alpha} \Rightarrow L = \frac{100}{2 \times 0.14} = 351.4H$$

$$C = \frac{1}{L\omega_0^2} \quad \text{where } \omega_0 = \sqrt{\omega_n^2 + \alpha^2} = \sqrt{5^2 + 0.14^2} = 5 \text{ rad/s}$$

$$= \frac{1}{351.4 \times 5^2} = 112.5\mu F$$

b) Ideally only power should be lost in resistor:

$$P = I^2R = i(t)^2R = A^2e^{-2\alpha t}R$$

Given $E_{(P\,lost)}(t) = \int_0^t P_{lost}\, d\alpha = R\int_0^t A^2 e^{-2\alpha t}\, d\alpha$

$$= RA^2\left[\frac{1}{-2\alpha} e^{-2\alpha t}\right]_0^t = \frac{A^2Re^{-2t^2}}{-2\alpha} + \frac{A^2R}{2\alpha}$$

And finally, since $E_{99\%} = 0.99\, E_{Initially}$

we find

$$E_I = \lim_{t \to \infty} \left(\frac{A^2 R}{-2\alpha} e^{-2\alpha t} + \frac{A^2 R}{2\alpha} \right) = \frac{A^2 R}{2\alpha}$$

we find: $E_{99\%} = 0.99 \frac{A^2 R}{2\alpha}$

The time at which this occurs we then find by subbing it into our time varying equation:

$$0.99 \frac{A^2 R}{2\alpha} = \frac{A^2 R}{-2\alpha} e^{-2\alpha t} + \frac{A^2 R}{2\alpha}$$

$t = 16.18$ seconds