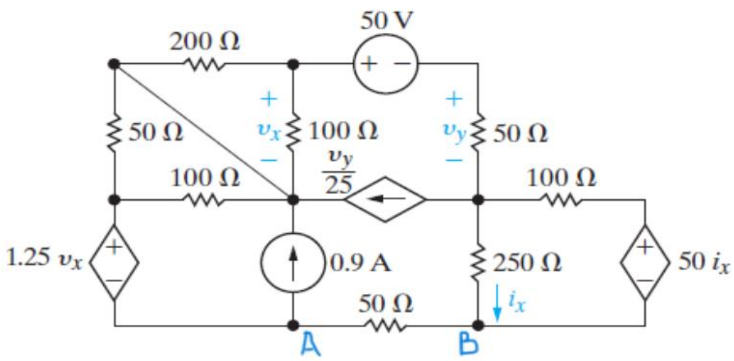
ELEC2300 Tutorial submission 2 - Samuel Allpass

Q1) (a)



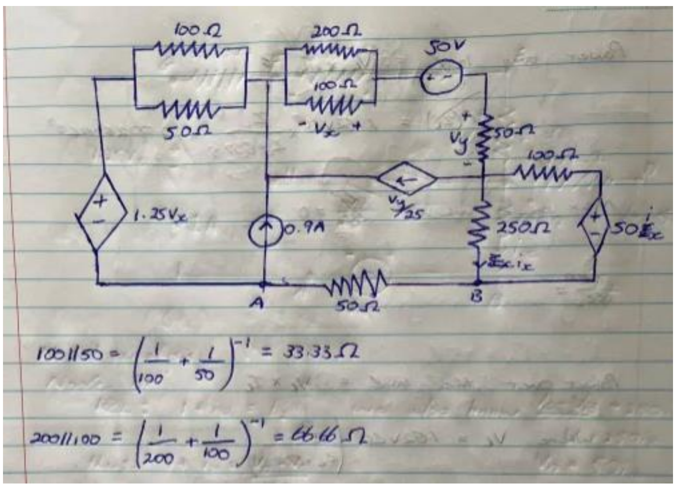
Strategy:

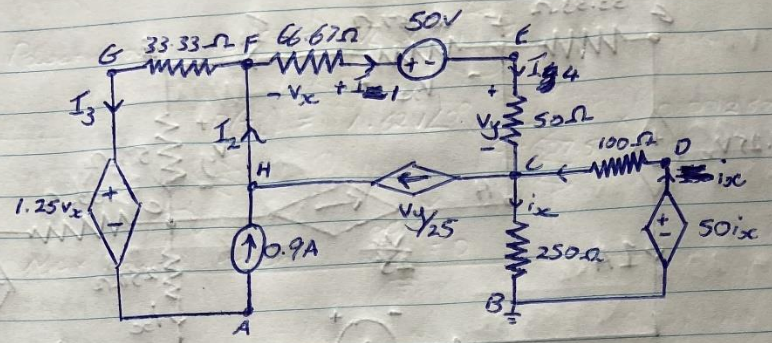
In order to calculate the Thevenin equivalent of the circuit across AB once the 50Ω resistor is removed, it is first vital to simplify the circuit and redraw it removing the 50Ω resistor.

After this, nodal analysis will be utilised in order to calculate the voltages at both A and B, such that the difference is the Thevenin voltage of an equivalent circuit.

In order to find the Thevenin equivalent resistance, we can employ a technique of zeroing the independent sources and placing a 1A current source over the AB terminals. The voltage across such a source would allow us to then calculate the Thevenin Resistance as $\frac{Voltage\ over\ source}{1A}$.

Solution:





Given loop on right: $\begin{cases} V_D = 50 i_x \\ V_C = 250 i_x \end{cases}$ and $\frac{V_D - V_C}{100} = i_x$

Thus: $\frac{50 i_x - 250 i_x}{100} = i_x$ only solution when $i_x = 0$

$V_A = 20\text{ V}$ $V_H = 50\text{ V}$
$V_B = 0\text{ V}$
$V_C = 0\text{ V}$
$V_D = 0\text{ V}$
$V_E = 0\text{ V}$
$V_F = 50\text{ V}$
$V_G = 20\text{ V}$

Nodal Analysis:

Node: C: KCL:

$i_x + \frac{V_y}{25} = i_x + I_4$

$\frac{V_y}{25} = \frac{V_y}{50}$ thus $V_y = 0$

$I_4 = 0$

From this we know $V_E = 0$ and $V_F = 50\text{ V}$ since $I_4 = 0$.

In addition we know $V_x = I_4 \times 66.67 = 0$

Node H:

KCL: $0.9 + \frac{V_y}{25} = I_2$

$I_2 = 0.9\text{A}$

Node F:

KCL: $I_2 = I_1 + I_3$

$0.9 = \cancel{1.25 V_x} + I_3$

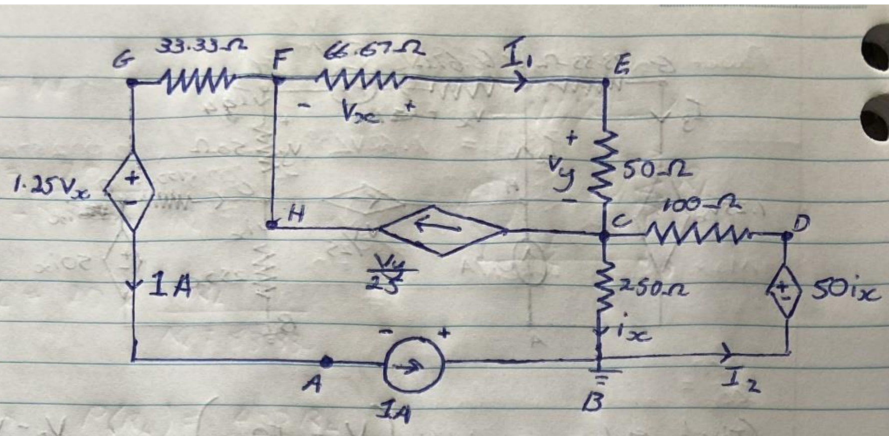
$I_3 = 0.9\text{A}$

~~Since~~ Now using KVL:

$\frac{V_F - V_G}{33.33} = 0.9 = \frac{50 - V_G}{33.33} \Rightarrow V_G = 20\text{V}$

Since $V_x = 0$: $V_G - V_A = 0$, thus $V_A = 20\text{ V}$.

Thus $V_{oc} = 20\text{V}$ and $V_{TH} = 20\text{V}$



Nodal analysis:

Node B:

KCL: $1 + i_x = I_2$ we also know $V_C - V_B = 250 i_x$

$V_C = 250 i_x$

$V_D = 50 i_x$

thus: $1 + i_x = \frac{V_D - V_C}{100}$

$= \frac{-200 i_x}{100}$

$i_x = -0.33A \rightarrow V_D = -16.67V$ $I_2 = 1 + i_x = 0.66A$

$V_C = -83.33V$

Node D:

KCL: $I_1 + I_2 = i_x + \frac{V_y}{25}$

$\frac{V_y}{50} + 0.66 = -0.33 + \frac{V_y}{25}$

$V_y = 50 V \rightarrow I_1 = \frac{V_y}{50} = 1A$

Thus we also know $-V_x = 1 \times 66.67 = 66.67$

$V_x = -66.67 V$

$V_E = V_C + V_y = 50 - 83.33 = -33.33V$

$V_F = V_E - V_x = -33.33 + 66.67 = 33.33V$

$V_G = V_F - 33.33 \times 1 = 0$ Then finally:

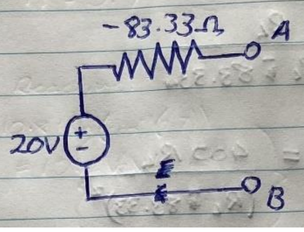
Since $V_x = -66.67V$

$V_A = V_G - 1.25(-66.67) = 83.33 V$

Note that the voltage from A to be is positive 83.33V, however, theory would dictate that the voltage over a current source should increase. It is because of this that we understand that the resistance value must be negative to uphold this theory.

We can then work out the thevenin resistance to be:

$$R_{TH} = \frac{-83.33\,V}{1\,A} = -83.33\,A$$



Significance:

It is clear from this analysis that the Thevenin equivalent circuit consists of a 20V supply with a -83.33Ω resistor. This implies that the resistor acts as a voltage supply.

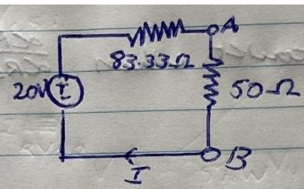
(b)

Strategy:

In order to calculate i_x from the original circuit using our findings, it can be observed that the current through the 50Ω resistor would be equal to the current that the Thevenin equivalent circuit experiences when the 50Ω resistor is placed on its terminals. After finding this current, KCL analysis on node B would provide enough information in order to calculate a value for i_x.

Solution:

$$I = \frac{20}{-83.33 + 50} = -0.6\,A$$



Consider Node B:

KCL: $I + i_x = I_{R2}$ where $I_{R2} = \frac{V_C - V_D}{100}$

$$= \frac{50 i_x - 250 i_x}{100}$$

$$= -2 i_x$$

$$-0.6 + i_x = -2 i_x$$

$$i_x = 0.2\,A$$

Significance:

It is clear from these calculations that the current i_x experienced in the original circuit is 0.2A. This is consistent with out other findings as node B has a lower voltage than that of node C, further verifying our findings in part a.

(c)

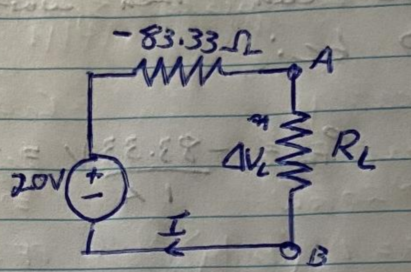
Strategy:

In order to find a resistor that absorbs the maximum amount of power, we can use our Thevenin equivalent to find power as a function of the load resistance. In doing this, we can conclude that the resistance where the derivative of the function is 0, is the resistance that provides maximum power absorption.

Solution:

$$P = VI$$

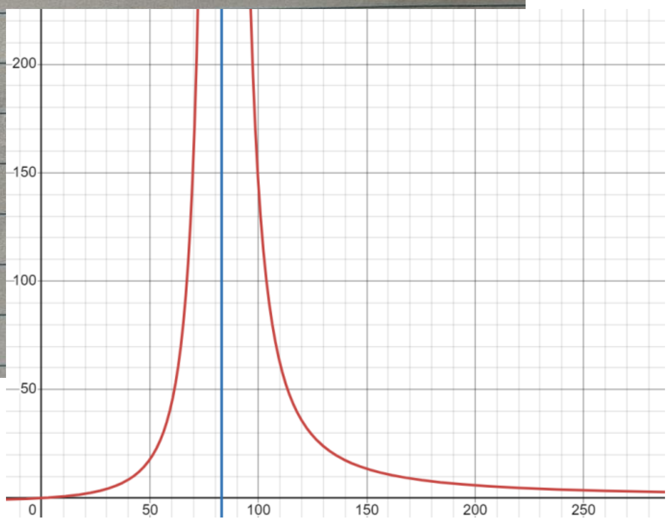
$$I = \frac{20}{-83.33 + R_L}$$



$$\Delta V_L = 20 \times \frac{R_L}{R_L - 83.33}$$

Thus $$P(R_L) = \frac{400 R_L}{(R_L - 83.33)^2}$$

Max power when $\frac{dP}{dR_L} = 0$



Significance:

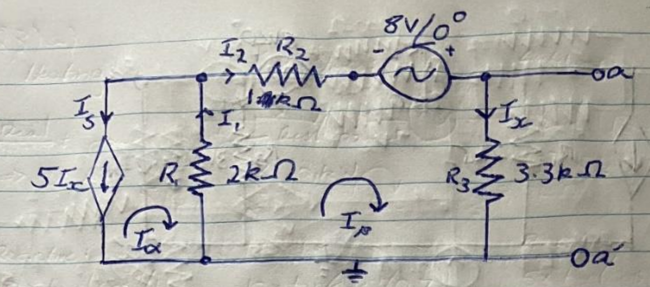
From this solution, we can see that the max power absorbed occurs when the resistance of the load is 83.33Ω.

Q2) (a)

Strategy:

In order to calculate the Thevenin equivalent circuit for the network to the left of the a and a' terminals, we can first employ mesh analysis. Mesh analysis will allow us to calculate a value for i_x and in turn a voltage across the 3.3kΩ resistor. As the resistor is in parallel with the output terminals, this calculated voltage will be the open circuit voltage.

Solution:



Using mesh analysis.

$I_s = -I_\alpha = 5I_x$

$I_1 = I_\beta - I_\alpha = I_\beta + 5I_x$ ①

$I_2 = I_\beta$ ②

$I_x = I_\beta$

The voltage over ~~the whole~~ all the components I_β loop must be 0.
Thus:

$0 = 2000\,I_1 + 1000\,I_2 + 3300\,I_x - 8$

Given ① and ②

$0 = 2000(I_\beta + 5I_x) + 1000\,I_\beta + 3300\,I_x - 8$

Since $I_\beta = I_x$

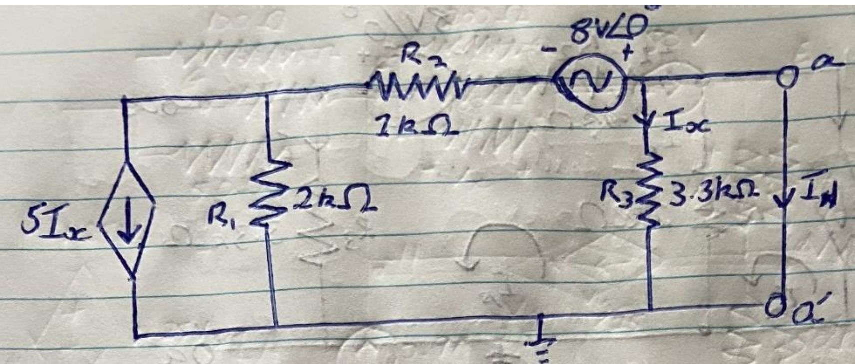
$0 = 2000\,(6I_x) + 1000\,I_x + 3300\,I_x - 8$

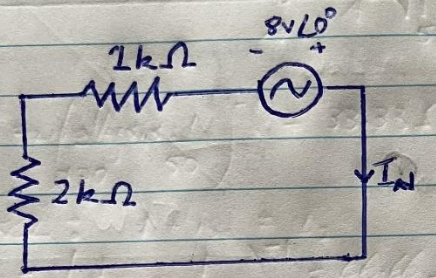
Thus: $I_x = 0.00049\,A\angle 0°$

~~Thus~~ Since the voltage over a-a' will be the same as over the 3.3kΩ resistor,

$V = IR = 0.00049A\angle 0° \times 3300$
$= 1.62\,V\angle 0°$

Thus, the thevenin equivalent voltage is $1.62\,V\angle 0°$.





Total resistance $= 2k\Omega + 1k\Omega = 3000\Omega$

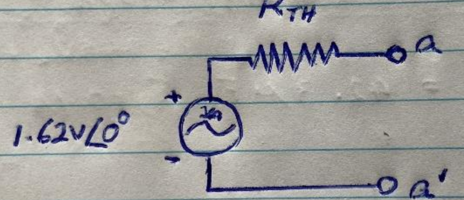
Thus $I = \frac{V}{R} = \frac{8V\angle 0^\circ}{3000} = 0.00267\angle 0^\circ$

Thus thevenin equivalent:

where $R_{TH} = \frac{V_{TH}}{I_N}$

$= \frac{1.62V\angle 0^\circ}{0.00267\angle 0^\circ}$

$= 607.5\Omega$



Significance:

It is clear from this method that the Thevenin equivalent circuit to the original circuit consists of a 1.62V source and 607.5Ω resistor.

(b)

Strategy:

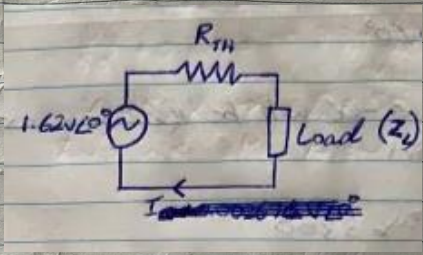
In a similar method to question 1, the impedance load that absorbs the maximum power from the circuit when placed across the a-a' terminals can be found using by studying the power as a function of impedance.

It will be clear that the maximum power absorbed by an impedance occurs when the gradient of such a function is 0.

Solution:

Power over load $= V_L \times I_N$

$$P(Z_L) = 1.62V\angle 0^\circ \times \frac{Z_L}{Z_L + 607.5} \times 0.00267A\angle 0^\circ$$



Max power when $\frac{dP}{dZ_L} = 0$

$\frac{dP}{dZ_L} =$

Power over the load $= V_L \times I_L$

where $V_L = 1.62V\angle 0^\circ \left(\frac{Z_L}{Z_L + 607.5}\right)$

$$I_L = \frac{1.62V\angle 0^\circ}{Z_L + 607.5}$$

Thus power out: $P = \frac{2.6244 Z_L}{(Z_L + 607.5)^2}$

Max power when $\frac{dP}{dZ_L} = 0$

$$\frac{dP}{dZ_L} = \frac{6561}{2500\left(Z_L + \frac{1215}{2.5}\right)^2} - \frac{6561 Z_L}{1250\left(Z_L + \frac{1215}{2}\right)^3} = \frac{-6561(2Z_L - 1215)}{625(2Z_L + 1215)^2}$$

$$\frac{dP}{dZ_L} = 0 = \frac{-6561(2Z_L - 1215)}{625(2Z_L + 1215)^2} \quad \text{thus } Z_L = 607.5\Omega$$

Thus the max power output occurs when the load is a 607.5Ω resistor.

Significance:

This solution clearly demonstrates that the maximum power absorption occurs when the load resistance is equal to 607.5Ω. This supports the theoretical notion that for a positive Thevenin resistance, the maximum power is absorbed when the load resistance is equal to that of the Thevenin resistance.

(c)

The reactive power is calculatable by studying P = IV such that:

$$P = IV = \frac{(2.6244\angle 0^{\circ}) * 607.5}{(607.5 + 607.5)^2}$$

$$P = (0.0011\ +\ 0i)VA$$

This is significant as it implies that there is reactive power and only active power. This is consistent with theoretical understandings as there is no capacitive or inductive load to create reactive power.

Q3) (a)

Strategy:

As the components of the system are all in parallel, we can calculate the total power triangle as the sum of the individual component power triangles. The given powers are all active, and thus we first need to find the reactive power for each component. This can be done using the given power factors:

$$PowerFactor\ (PF) = \cos(\theta)$$

Where θ is the angle between the active (real) and reactive powers.

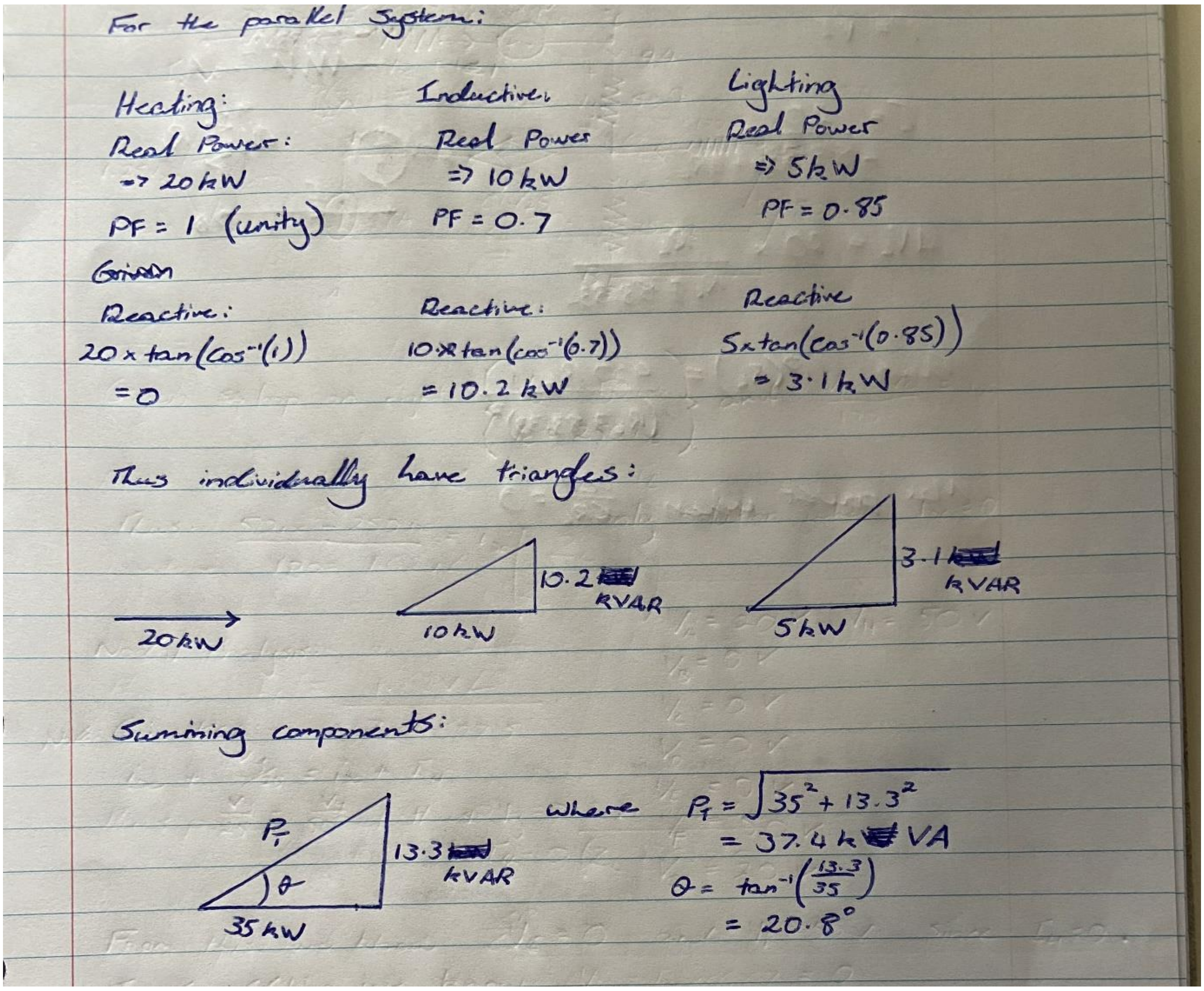
$$\tan(\theta) = \frac{reactive}{active}$$

Thus, combining the two we get:

$$reactive\ power = \tan\ (\arccos(PF))$$

We can then sum the active and reactive powers for each component to find the total power supplied by the voltage supply.

Solution:



Significance:

We can therefore see that the power triangle of total loading on the supply consists of 35kW active power, 13.3kVAR reactive power for a total 37.4kVA apparent power. This results in an angle of 20.8°.

(b)

Strategy:

In order to produce a unity power factor, we need can use the following derivation:

$$Qtotal = VAR$$

$$Qcap = \frac{V^2}{x} \; thus \; x = \frac{Qcap}{V^2} \; (Note \; that \; V \; given \; in \; question \; is \; RMS)$$

$$And \; given \; x = \frac{1}{wc} \; we \; have \; C = \frac{1}{xw} = \frac{1}{2\pi f * \frac{V^2}{Qcap}}$$

Solution:

b) $\Delta Q = 13.3\,kVAR$ $\omega = 2\pi f = 100\pi$ $(f = 50\,Hz)$

$$C = \frac{\Delta Q}{\omega V_{rms}^2} = \frac{13.3 \times 10^3}{100\pi (1000)^2} = 42.3\,\mu F$$

Significance:

We therefore find that to correct the power factor to unity, we can implement a capacitor of 42.3μF.

(c)

Strategy:

The apparent power of a system is given by:

$$P = V_{rms} * I_{rms}$$

Initially the apparent power was 37.4kVA, and once the capacitor was implemented, the apparent power becomes equal to magnitude of the active power, 35kVA. In addition to this we know that the system runs of 1000V rms. This makes solving for I_{rms} easy.

Solution:

c)

Initially: $P = VI$: $I = \frac{37.4 \times 10^3}{1000} = 37.4A$

After Capacitor Correction : $P = VI$: $I = \frac{35 \times 10^3}{1000} = 35A$

Significance:

It is clear from this that once the capacitor is implemented, the current changes by 2.4A. This indicates a 2.4A difference between the uncompensated and compensated systems.