ELEC2400 A2 – Samuel Allpass s4803050

(a) Assuming that $R_{C1} = R_{C2} = 10\ k\Omega$, and the internal resistance REB of the current source is infinite, calculate DC bias points (Q-points) for all the transistors in Figure 1. Classify the region of operation of each transistor as *active*, *cut-off* or *saturation*. Will your classification change if REB = 470 kΩ? You should state and justify any assumptions that you make.

a) with $V_1 = V_2 = 0\,V$ we find:
and assume Q_1, Q_2 and Q_3 are in the active region.

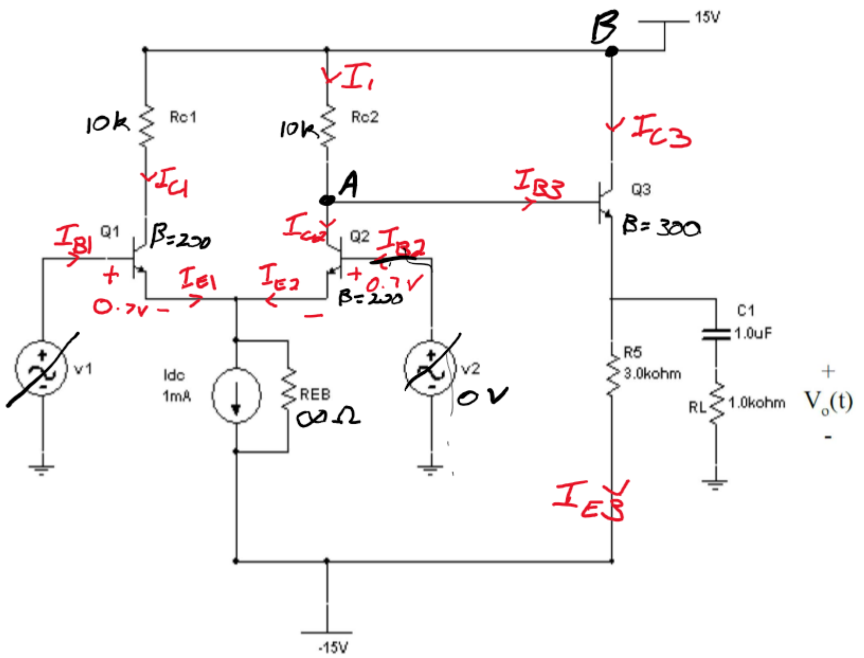


Figure 1: Circuit for Question 1

Calculating the DC operating points, first observe:

As $V_{BE1} = V_{BE2} \Rightarrow I_{E1} = I_{E2}$

$\therefore$ KCL at current source:

$I_{E1} + I_{E2} = 1\,mA$

$\therefore I_{E1} = I_{E2} = 0.5\,mA$

And so follows:

$$I_{c1} = I_{c2} = \left(\frac{\beta}{\beta+1}\right) \times 0.5mA \qquad (\beta = 200)$$

$$= 0.498mA$$

We notice as β of Q3 is high, I_{B3} will be extremely small, thus:

$I_1 \approx I_{c2}$ thus

$$V_{B3} = 15 - I_{c2} \times 10\,000$$
$$= 10.02V$$

Thus finally:

$$V_{E3} = 10.02 - 0.7$$
$$= 9.32V$$

And so: $$I_{E3} = \frac{9.32 - -15}{3000}$$
$$= 0.008A$$
$$= 8mA$$

Therefore we can conclude:

Q_1:

$V_c = 15 - 10000 I_{c1} = 10.02V$

$V_B = 0V$

$V_E = -0.7V$

$I_c = 0.498mA$

Q_2:

$V_c = 15 - 10000 I_1 = 10.02V$

$V_B = 0V$

$V_E = -0.7V$

$I_c = 0.498mA$

Q_3

$V_c = 15V$

$V_B = V_{c2} = 10.02$

$V_E = V_B - 0.7 = 9.32V$

$I_c = I_E \times \left(\frac{\beta}{\beta+1}\right) \approx 8mA$

If $REB = 470k\Omega$

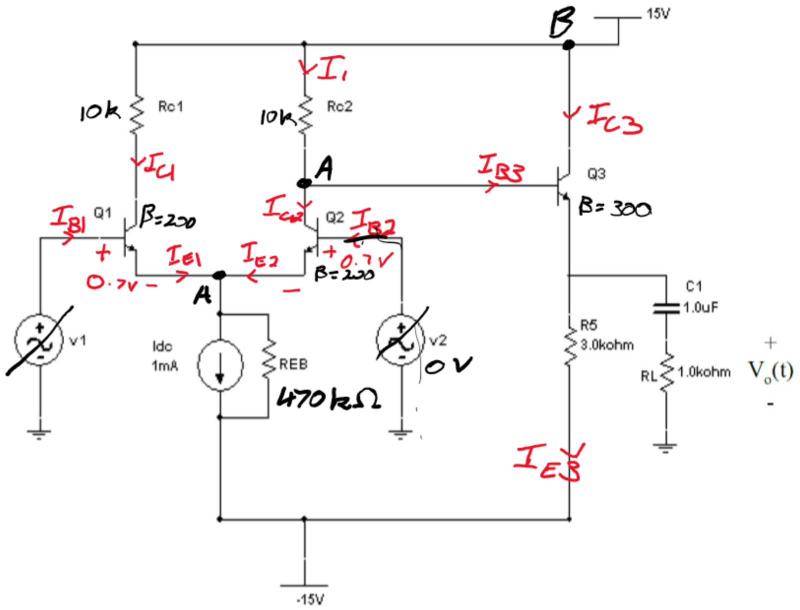


Figure 1: Circuit for Question 1

We understand

$$V_{REB} = \frac{-0.7 - -15}{470k\Omega} = 0.03mA$$

Thus KCL at A:

$$I_{E1} + I_{E2} = 1mA + 0.03mA$$

$$I_{E1} = I_{E2} = 0.5mA$$

$$\therefore I_{C1} = I_{C2} = 0.5m \times \left(\frac{200}{201}\right) = 0.498mA$$

Thus as Q_3 is large

$$V_{C1} \approx V_{C2} \approx 15 - 0.498 \times 10^{-3} \times 10\,000$$
$$= 10.02\ V$$

And so again:

Q_1:

$V_C = 15 - 10000\, I_{C1} = 10.02V$

$V_B = 0V$

$V_E = -0.7V$

$I_C = 0.498mA$

Q_2:

$V_C = 15 - 10000\, I_1 = 10.02V$

$V_B = 0V$

$V_E = -0.7V$

$I_C = 0.498mA$

Q_3

$V_C = 15V$

$V_B = V_{C2} = 10.02$

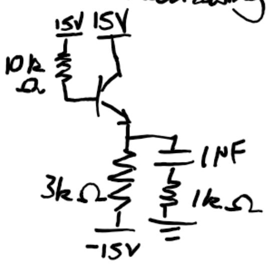
$V_E = V_B - 0.7 = 9.32V$

$I_C = I_E \times \left(\frac{\beta}{\beta+1}\right) \approx 8mA$

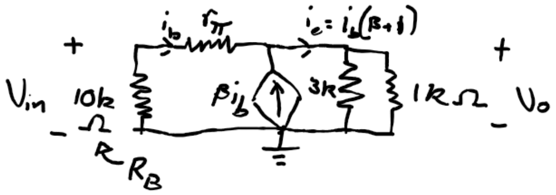
As for both REB= ∞ and REB= 470kΩ $V_C > V_B > V_E$, we understand all the transistors are in the active region.

(b) Draw the mid-band small signal equivalent circuit of the common-collector stage (Q3), taking the load RL into account. Derive symbolic expressions for the input impedance Z_{In} and the voltage gain A_v of the common collector (CC) stage. Substitute values for circuit components as given in Figure 1 and calculate numerical figures for Z_{In} and A_v.

Redrawing Q3 in its equivalent circuit:



Zero DC Voltage Sources
Short Capacitors.



$$R_L \| R_E = \frac{3 \times 1}{3+1} k\Omega = 750\,\Omega$$

Thus $\quad V_o = 750 \times i_b(\beta+1)$

KVL: $\quad V_{in} = i_b r_\pi + i_b(\beta+1) \times 750$

$$\therefore \quad \frac{V_{in}}{V_o} = \frac{(1+\beta) \times 750}{r_\pi + (1+\beta) \times 750}$$

given $\quad \beta = 300$

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{300 \times 0.026}{8mA} = 975\,\Omega$$

$$\therefore \quad \frac{V_o}{V_{in}} = A_v = 0.996$$

We also know:

$$Z_{in} = \frac{V_{in}}{I_{in}} = \frac{i_{B3}\left(r_{\pi 3} + R_E(\beta+1)\right)}{i_{B3}}$$

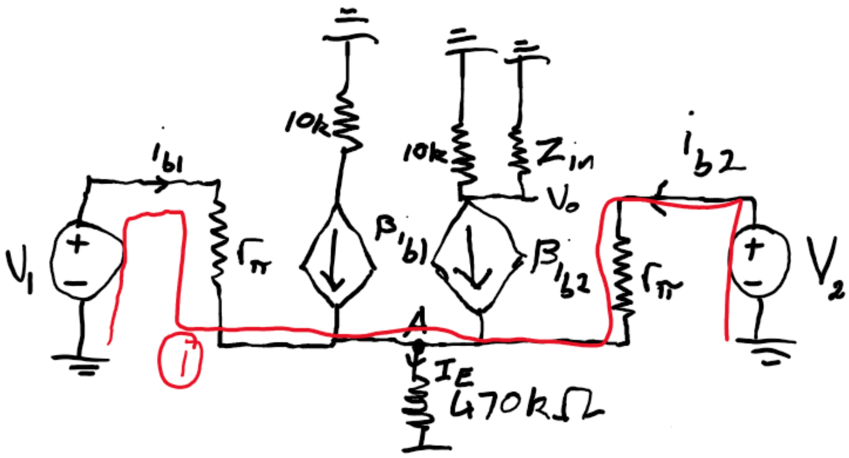
$$= r_{\pi 3} + R_E(\beta+1) = 226713\,\Omega$$

(c) Draw the differential mode half circuit of the amplifier in Figure 1, and derive an expression for the differential mode voltage gain (A_{vds}) of the differential amplifier stage, i.e., gain from the differential-mode input to the collector of Q2. The loading effect from the amplifier formed by Q3 should be taken into account. Rc1 = Rc2 = 10 kΩ and REB = 470 kΩ. Calculate A_{vds}.

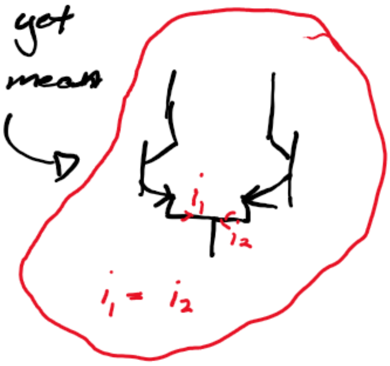
(c) Draw the differential mode half circuit of the amplifier in Figure 1, and derive an expression for the differential mode voltage gain (A_{vds}) of the differential amplifier stage, i.e., gain from the differential-mode input to the collector of Q2. The loading effect from the amplifier formed by Q3 should be taken into account. Rc1 = Rc2 = 10 kΩ and REB = 470 kΩ. Calculate A_{vds}.

In differential mode: $V_{dm} = V_1 - V_2$

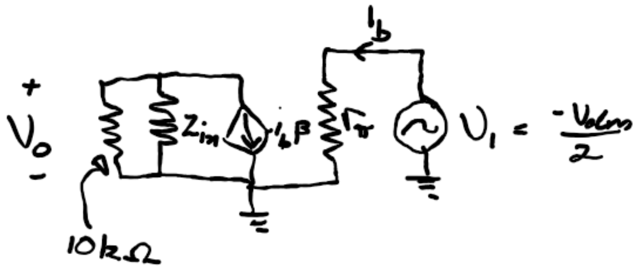
and thus $V_o = -V_1 = \frac{V_{dm}}{2}$:



We first notice $I_E = 0$ as the emmitter symmetry and equal yet opposite V_1 and V_2 polarities mean the currents are equal.



We notice the following half circuit:



$$0 = i_{b1}(1+\beta) + i_{b2}(1+\beta)$$

$$\therefore i_{b1} = -i_{b2}$$

Sub into ①

$$V_1 - V_2 = r_\pi(-2i_{b2})$$

Notice:

$$V_o = -\beta i_{b2}\,(10k \,||\, Z_{in})$$

Thus

$$A_d = \frac{V_o}{V_1 - V_2} = \frac{-\beta i_{b2}(10k \,||\, Z_{in})}{-2 r_\pi i_{b2}}$$

$$r_\pi = \frac{200 \times 0.026}{0.498 \times 10^{-3}} = 10441.8\,\Omega$$

$$10k \,||\, Z_{in} = 9577.5\,\Omega$$

Thus:

$$A_d = 91.7$$

(d) The common-mode voltage gain from the common-mode input to the collector of Q2 has been calculated to be $A_{vcm} = -0.01$. The two components $v_{sig}(t)$ and $v_{unw}(t)$ appear at the two inputs v1(t) and v2(t) of the amplifier as described by Eq. (1) and (2):

$$v1(t) = v_{sig}(t) + v_{unw}(t) \quad (1)$$
$$v2(t) = 0.01\, v_{sig}(t) + 0.99\, v_{unw}(t) \quad (2)$$

Write an expression for the voltage at the collector of Q2, when signals v1(t) and v2(t) are applied as shown in Figure 1. Has the amplifier succeeded in selectively amplifying the signal of interest $v_{sig}(t)$?

$$V_{vcm} = -0.01$$

$$V_1(t) = V_{sig}(t) + V_{unw}(t)$$
$$V_2(t) = 0.01 V_{sig}(t) + 0.99 V_{unw}(t)$$

We know the total amplification to be:

$$V_o(t) = A_{dm} V_{dm} + A_{cm} V_{cm}$$

$$V_{dm} = V_1(t) - V_2(t) = 0.99 V_{sig}(t) + 0.01 V_{unw}(t)$$

$$V_{cm} = \frac{V_1 + V_2}{2} = \frac{1.01}{2} V_{sig}(t) + \frac{1.99}{2} V_{unw}(t)$$

$$V_o(t) = 91.7 \times 0.99 V_{sig} + 91.7 \times 0.01 V_{unw} - 0.01 \times \frac{1.01}{2} V_{sig} - 0.01 \times \frac{1.99}{2} V_{unw}$$

$$= 90.78 V_{sig}(t) + 0.91 V_{unw}$$

By observation we notice Vsig(t) was amplified by a factor of 90.78, whilst Vunw was reduced by a factor of 0.91.

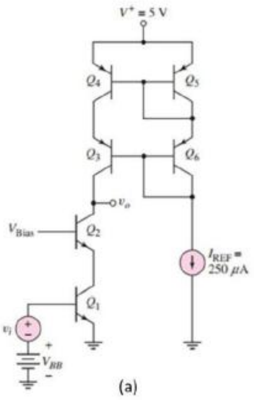
We can further describe this relationship by observing the common-mode rejection ratio:

$$CMRR = 20 \log \frac{|A_d|}{|A_{cm}|} = 79.24\,dB$$

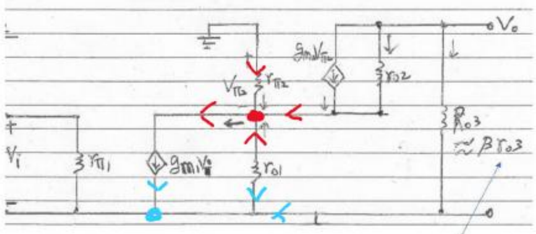
This is lies within the typical low frequency CMRR range of 70dB to 100dB, indicating it was very successful at amplifying the $V_{sig}(+)$ signal.

2. (40 marks)

Figure 2 (a) shows a BJT cascode amplifier with a cascode active load. Figure 2 (b) is the corresponding small signal equivalent circuit for Q1 and Q2 in Figure 2 (a). The transistor parameters are $\beta = 120$ and $V_A = 80$ V. Assume $g_m \gg \frac{1}{r_o}$, $V_T = 0.026$ V. The V_{BB} voltage is such that all transistors are biased in the active region. Determine the small signal voltage gain $Av = \frac{v_o}{vi}$.



(a)



Please refer to Neamen's book Page 697 for R_{03} derivation

(b)

Begining with diagram b:

KCL at red:

$$g_{m1} V_i = \frac{V_{\pi 2}}{r_{\pi 2}} + \frac{V_{\pi 2}}{r_{o1}} + g_{m2} V_{\pi 2} + \frac{V_o - -V_{\pi 2}}{r_{o2}}$$

KCL at Blue:

$$\frac{V_o}{R_{o3}} + \frac{V_o - -V_{\pi 2}}{r_{o2}} + g_{m2} V_{\pi 2} = 0$$

$$g_{m1} V_i = V_{\pi 2}\left(\frac{1}{r_{\pi 2}} + \frac{1}{r_{o1}} + g_{m2} + \frac{1}{r_{o2}}\right) + \frac{V_o}{r_{o2}}$$

$$V_o\left(\frac{1}{R_{o3}} + \frac{1}{r_{o2}}\right) + V_{\pi 2}\left(\frac{1}{r_{o2}} + g_{m2}\right) = 0$$

given $g_m >> \frac{1}{r_o}$

$$g_{m1} V_i = V_{\pi 2}\left(\frac{1+\beta}{r_{\pi 2}}\right) + \frac{V_o}{r_{o2}} \quad (1)$$

$$V_o\left(\frac{1}{R_{o3}}+\frac{1}{r_{o2}}\right)+V_{\pi 2}\,g_{m2}=0$$

$$V_{\pi 2}=-\frac{V_o}{g_{m2}}\left(\frac{1}{R_{o3}}+\frac{1}{r_{o2}}\right)$$

Subbing $V_{\pi 2}$ into ①

$$g_{m1}V_i=-\frac{V_o}{g_{m2}}\left(\frac{1}{R_{o3}}+\frac{1}{r_{o2}}\right)\left(\frac{1+\beta}{r_{\pi 2}}\right)+\frac{V_o}{r_{o2}}$$

Notice $g_{m2}\times r_{\pi 2}=\beta$

$$g_{m1}V_i=-V_o\left(\left(\frac{1}{R_{o3}}+\frac{1}{r_{o2}}\right)\left(\frac{1+\beta}{\beta}\right)+\frac{1}{r_{o2}}\right)$$

Given r_{o2} is so small:

$$g_{m1}V_i\simeq-\frac{V_o}{R_{o3}}\left(\frac{1+\beta}{\beta}\right)$$

$$\frac{V_o}{V_i}=-g_{m1}R_{o3}\left(\frac{\beta}{\beta+1}\right)$$

Which given $R_o\simeq\beta r_{o3}$

$$\frac{V_o}{V_i}=\frac{-g_{m1}r_{o3}\beta^2}{1+\beta}$$

We now make the assumption that the base currents of the transitors are very small. Given the transistors are all of equal β, from KCL we find $I_{E6} \approx I_{C6} = I_{E5} \approx I_{C5}$ and so on. Following this we can justify $I_{C1} \approx I_{REF}$

We understang g_{m1} is given by:

$$g_{m1} = \frac{I_{ref}}{V_T} = \frac{250 \times 10^{-3}}{0.026} = 9.62 \frac{mA}{V}$$

and $$r_{o3} = \frac{V_A}{I_{ref}} = 320\,K$$

$$\therefore \frac{V_o}{V_i} = A_v = \frac{-9.615 \times 320 \times 120^2}{121}$$

$$= -366\,164.63$$