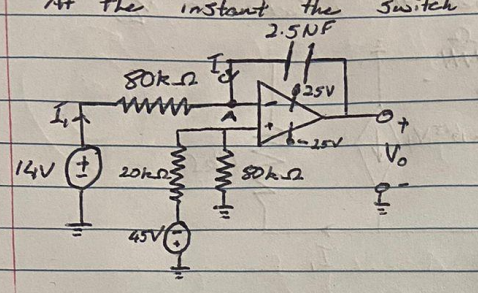


ELEC2004 Homework 2

By Samuel Allpass

Question 1)

At the instant the switch is closed, we find:



First we solve for the voltage of the op-amp terminals: Notice V_{+} is a simple voltage divider, and given it is ideal, we have no current or voltage over the terminals:

$$V_{+} = -45 \left(\frac{80000}{80000 + 20000} \right) = -36V$$

Applying KCL at node A we find:

$$I_1 = -I_c \quad \text{Notice that:}$$

$$I_1 = \frac{14 - (-36)}{80000} \quad \text{and} \quad I_c = C \frac{dV(t)}{dt}$$

$$6.25 \times 10^{-6} = - \left(2.5 \times 10^{-6} \right) \frac{dV(t)}{dt} \quad \text{thus:}$$

$$\frac{dV}{dt} = 250 \quad \text{and thus} \quad \int \frac{dV}{dt} dt = \int 250 dt$$

$$V = -250t + C$$

Initially we know $V(0) = 57V$ thus:

$$57 = 0 + C = C$$

$$V = -250t + 57$$

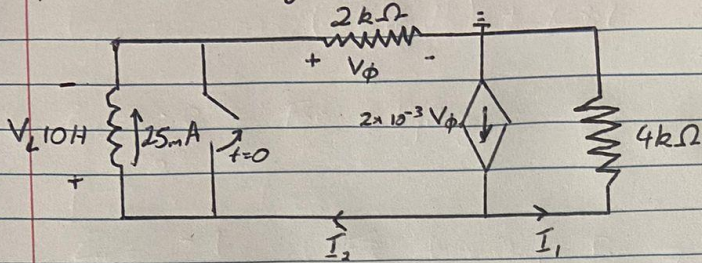
In order for V_o to be 0V, the capacitor voltage must negate the ~~57~~ -36V terminal thus:

$$36 = -250t + 57 \quad t = 0.084 \text{ seconds}$$

Thus, it took 84 milliseconds for the output voltage to equal 0.

Question 2)

Let the voltage over the inductor be V_L ;



From KCL we find:

$$2 \times 10^{-3} V_{\phi} = I_1 + I_2$$

We know $I_2 = \frac{V_{\phi}}{2000}$ and $I_1 = \frac{V_{\phi} + V_L}{4000}$ thus:

$$2 \times 10^{-3} V_{\phi} = \frac{V_{\phi} + V_L}{4000} + \frac{V_{\phi}}{2000}$$

The voltage over an inductor is given by: $V(t) = L \frac{di(t)}{dt}$

$$2 \times 10^{-3} V_{\phi} = \frac{V_{\phi} + 10 \frac{di(t)}{dt}}{4000} + \frac{V_{\phi}}{2000} \quad \text{Note } i(t) = i_2$$

We also know: $V_{\phi} = 2000 i_2$ thus:

$$4 i(t) \frac{di(t)}{dt} + \frac{1}{400} \frac{di(t)}{dt} + i(t) = 0$$

Thus we are left with the ODE:

$$0 = -\frac{5}{2} i(t) + \frac{1}{400} \frac{di(t)}{dt} \rightarrow \frac{di(t)}{dt} = 1000 i(t)$$

$$\int \frac{di(t)}{i(t)} = \int \frac{1}{1000} dt \Rightarrow t = \frac{1}{1000} \ln(i) + C$$

Given the initial conditions: $i(0) = 25 \times 10^{-3} \text{ A}$

$$C = -\frac{1}{1000} \ln(25 \times 10^{-3}) \Rightarrow t = \frac{1}{1000} \ln(i) - \frac{1}{1000} \ln(25 \times 10^{-3})$$

When $i = 5 \text{ A}$:

$$t = \frac{1}{1000} \ln(5) - \frac{1}{1000} \ln(25 \times 10^{-3}) = 0.0053 \text{ seconds}$$

Thus, it took 0.0053 seconds (53 milliseconds) after the switch was opened for the inductor to malfunction at the 5A limit.