Q1 (Total 30 marks)

For the 2-stage amplifier shown in Figure 1, the circuit parameters are given below.

$R_1 = 67.3\ k\Omega$, $R_2 = 12.7\ k\Omega$, $R_{C1} = 10\ k\Omega$, $R_{E1} = 2\ k\Omega$

$R_3 = 15\ k\Omega$, $R_4 = 45\ k\Omega$, $R_{E2} = 1.6\ k\Omega$

$R_L = 250\Omega$

C_1, C_2, C_3 are coupling capacitors and have very large values. C_E is an emitter bypass capacitor.

$\beta = 120$ *for both transistors*, Q_1 *and* Q_2, α=0.99 and V_T=0.026V

For this amplifier,

(a) Draw the DC model of the circuit and calculate the Q-point parameters, I_{CQ} and V_{CEQ} for both transistors. **(14 marks)**

(b) Draw the small signal (AC) model of the circuit and calculate the small-signal parameters, g_m and r_π for both transistors. **(6 marks)**

and calculate:

(c) The overall small-signal gain, $A_V = v_o/v_s$. **(6 marks)**

(d) The input resistance, R_{is}. **(1.5 marks)**

(e) The output resistance, R_o. **(2.5 marks)**

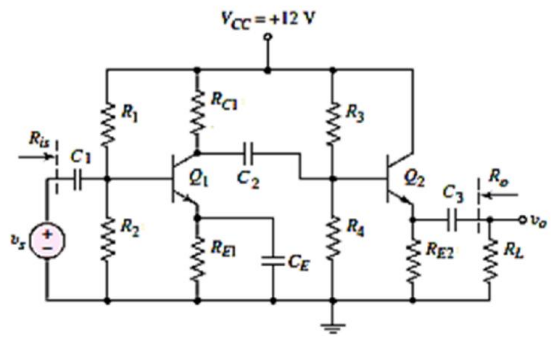
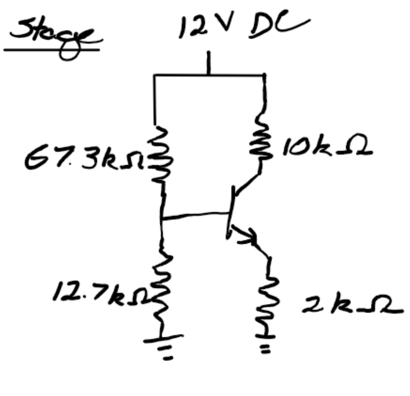


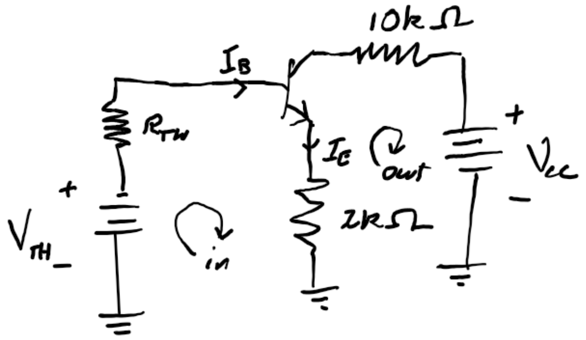
Figure 1.

a) First analysing in DC, we understand the capacitors are open sources leaving us with a first and second side amplifier.

First Stage



OR equivalently:



where $R_{TH} = R_1 || R_2$

$$= \frac{R_1 R_2}{R_1 + R_2} = 10.68k\Omega$$

$$V_{TH} = \frac{R_2}{R_1 + R_2} V_{cc} = \frac{12.7k\Omega}{(12.7+67.3)k\Omega} \times 12$$

$$= 1.905V$$

KVL of input loop:

$V_{TH} = R_{TH} I_B + R_E I_E + V_{BE}$ Given $I_E = (\beta+1) I_B$

and $\beta = 120$

$V_{BE} = 0.7V$

$$1.905 = 10680 I_B + 2000 \times 121 I_B + 0.7$$

$$I_B = 4.8\mu A$$

$$I_C = \beta I_B = 576\mu A$$

$$\therefore I_E = 580.8\mu A \quad (I_B + I_C)$$

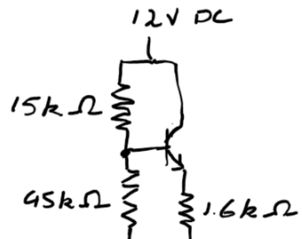
KVL at output loop:

$$V_{cc} = 10\,000 I_C + V_{CE} + 2000 I_E$$

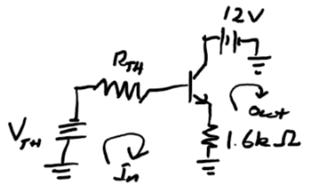
$$V_{CE} = 12 - 10000(576 \times 10^{-6}) - 2000(580.8 \times 10^{-6})$$

$$\approx 5.08V$$

For the second stage:



Equivalently:



$R_{TH} = 15k\Omega || 45k\Omega$

$$= 11.25k\Omega$$

$$V_{TH} = \frac{45k\Omega}{(45+15)k\Omega} \times V_{cc}$$

$$= 9V$$

KCL on input loop:

$$V_{TH} = R_{TH} I_B + V_{BE} + R_E \underbrace{I_E}_{(\beta+1)I_B} \qquad \text{note } I_E = 121 I_B$$

$$V_{BE} = 0.7V$$

$$I_B = \frac{9 - 0.7}{11.25\times10^3 + 1.6\times10^3(121)}$$

$$= 4.05\times10^{-5} \text{ A}$$

$$\therefore I_E = 121 I_B = 0.0049 \text{ A}$$

$$I_C = 120 I_B = 0.00486 \text{ A}$$

KCL on output loop:

$$V_{CC} = V_{CE} + R_E I_E$$

$$V_{CE} = 12 - 1600 \times 0.0049$$

$$= 4.16 \text{ V}$$

b) We know $g_m = \dfrac{I_C}{V_T}$

For the first transistor

$$g_{m_1} = I_{CQ}/V_T \qquad r_{\pi_1} = \frac{\beta V_T}{I_{CQ}}$$

$$= \frac{576\times10^{-6}}{0.026}$$

$$= 0.022$$

$$= \frac{120 \times 0.026}{576\times10^{-6}}$$

$$= 5416.67$$

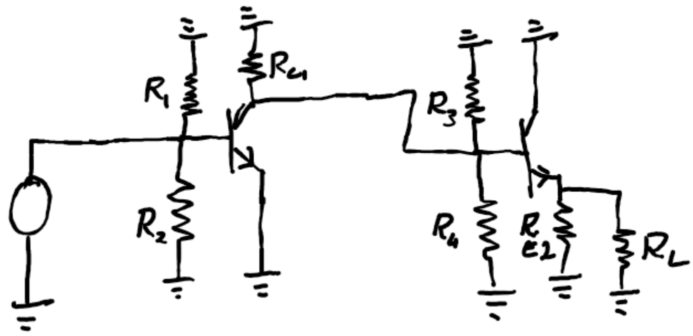
For the second:

$$g_{m2} = I_q / V_T = \frac{0.00486}{0.026} = 0.19$$

$$r_{\pi 2} = \frac{\beta V_T}{I_{cq}} = \frac{120 \times 0.026}{0.00486} = 641.98$$

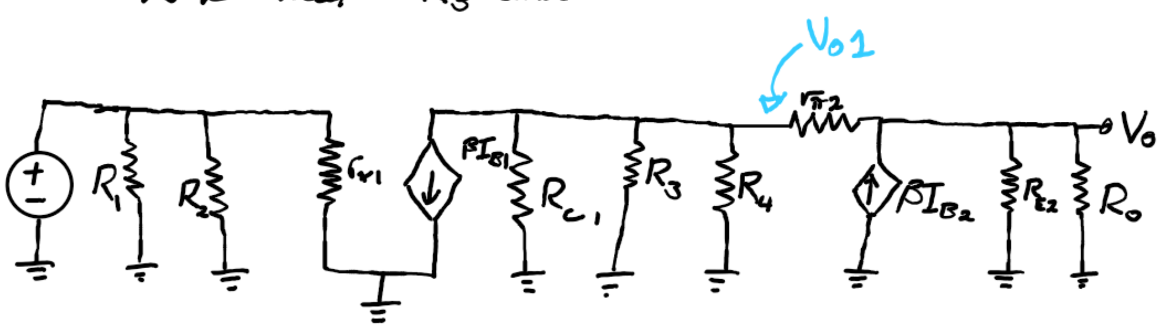
Now drawing the diagram:

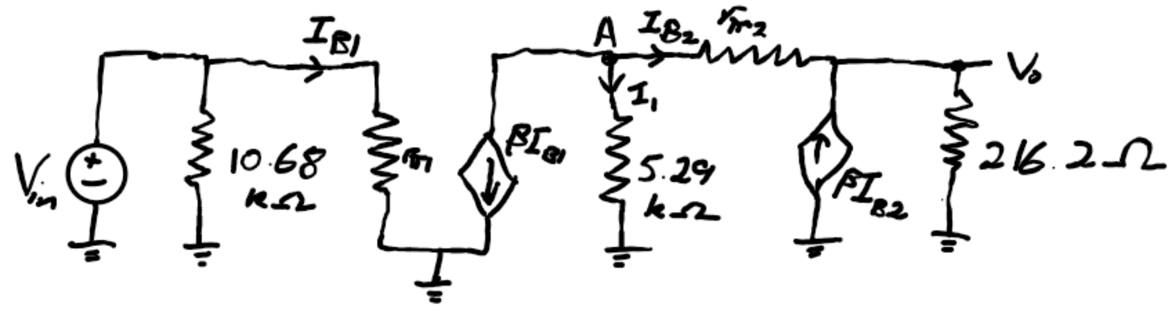
Shorting the capacitors and zeroing the DC source we find:



Now drawing the equivalent transistor diagrams:

Note that R_3 and





For stage 1:

For common collector

$R_{B1} = R_1 || R_2 = 10.683\,k\Omega$

$R'_{L_1} = R_{C1} || R_3 || R_4 = 5.294\,k\Omega$

$$A_V = \frac{R'_L \beta}{r_\pi} \quad \therefore \quad A_{V_1} = \frac{5.294 \times 10^3 \times 120}{541.67}$$

$$= -117.28$$

Now for second stage:

For common emitter

$R'_{L2} = R_L || R_E = 216.22\,\Omega$

$$A_{V_2} = \frac{(1+\beta)R'_L}{r_\pi + (1+\beta)R'_L} = 0.976$$

Finally, the total gain A_v:

$$A_v = A_{v_1} \times A_{v_2} = -114.47$$

Thus $\quad V_0 = -114.47 V_{in}$

d) The total input resistance is for that of the common emitter.

$$Z_{in} = \frac{V_{in}}{I_{in}} = \frac{1}{\frac{1}{R_B} + \frac{1}{r_\pi}}$$

$R_{B_1} = 10.683 k\Omega$

$r_{\pi_1} = 5416.67\Omega$

$$= 3945.35\Omega$$

e) The total output resistance is for that of the common collector.

$$Z_0 = \frac{V_0}{I_0} = \frac{1}{(1+\beta)(R_S' + r_\pi) + 1/R_E}$$

Given $\quad R_S' = \left(\frac{1}{R_S} + \frac{1}{R_3} + \frac{1}{R_4}\right)^{-1}$

$R_S = 0$

$r_{\pi_2} = 641.98$

$\beta = 120$

$R_E = 1600$

$Z_o = 6.9 \times 10^{-7}\Omega$

Q2 (Total 30 marks)

For the Figure 2 sketch v_o versus v_{in} to scale when v_{in} changes from -10 V to +10 V. Assume that diodes are ideal. Explain possible states of the diode when v_{in} changes in the above range.

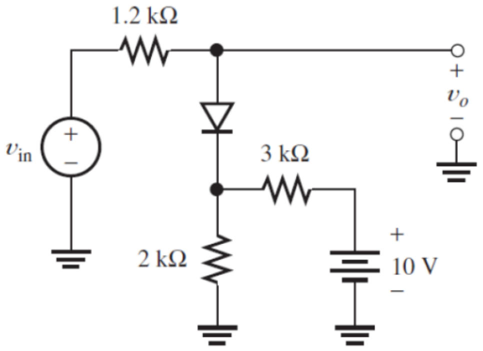
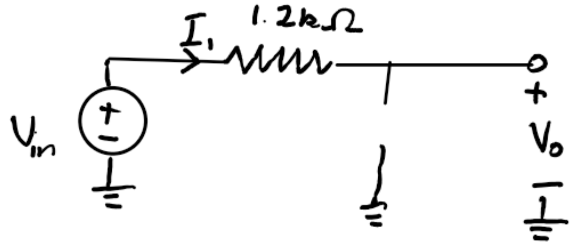


Figure 2

We must consider three cases:
Case 1, Diode reverse bias. During this State, no current flows through the diode. Thus it is an open circuit:



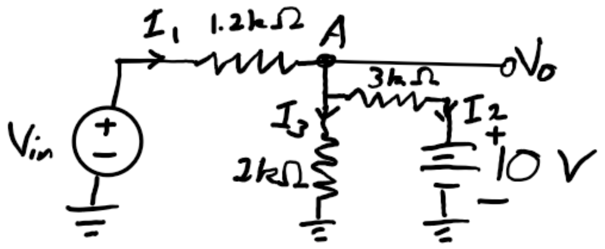
During this stage, $I_1 = 0$ as no ground is connected, and thus no voltage drop over the 1.2kΩ resistor.

Thus $V_o = V_{in}$

Case 2: The diode is on, but the current through it is almost 0A. As a result, V_{in} will still equal V_o, but, this will mark the first V_{in} which creates a forward bias.

$\therefore$ $V_{in} = V_o$ $\quad I_1 = 0.0000\ldots1 \approx 0$

Diode becomes a short:



KVL: $V_{in} = 1200 I_1 + 2000 I_3$ $(I_1 = 0)$

$V_{in} = 2000 I_3$

$10 + 3000 I_2 = 2000 I_3$

Notice $I_2 + I_3 = 0 \rightarrow I_3 = -I_2$

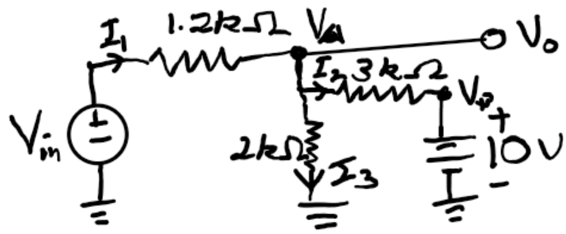
Thus:

$10 - 3000 I_3 = 2000 I_3$

$$I_3 = \frac{10}{5000} = \frac{1}{500}$$

$$\therefore V_{in} = 2000 \times \frac{1}{500} = 4\ V$$

Case 3: V_{in} is over 4V and thus $I_1 > 0$:



KCL at V_A:

$I_1 = I_2 + I_3$

$$\frac{V_{in} - V_A}{1200} = \frac{V_A - V_B}{3000} + \frac{V_A - V_G}{2000}$$

$$\frac{V_{in} - V_A}{1200} = \frac{V_A - 10}{3000} + \frac{V_A - 0}{2000}$$

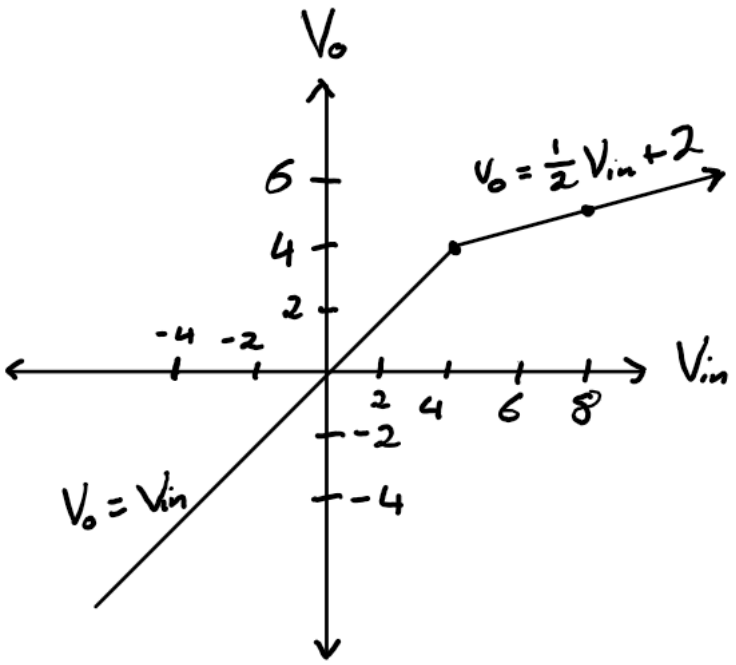
$$\frac{V_{in}}{1200} = \frac{V_A}{600} - \frac{10}{3000}$$

$$V_{in} = 2V_A - 4$$

Given $V_A = V_o$ we can rearrange:

$$V_{in} + 4 = 2V_o \rightarrow V_o = \frac{1}{2}V_{in} + 2$$

Finally plotting this:



Q3 (Total 40 marks)

Figure 3 shows an equivalent circuit of a Darlington pair configuration. The transistor parameters are $\beta_1 = 120$, $\beta_2 = 80$, $V_{A1} = 80$ V and $V_{A2} = 50$ V. Assume $V_T = 0.026$ V. Draw the small signal equivalent circuit and determine the output resistance R_o for $I_{C2} = I_{\text{Bias}} = 1$ mA. (V_{A1} and $V_{A1}$are Early voltages, which have been discussed in Lecture 5 "Introduction to Multistage Amplifiers".)

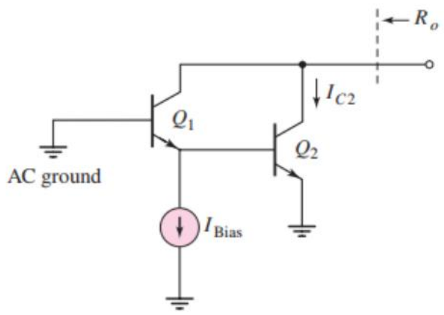
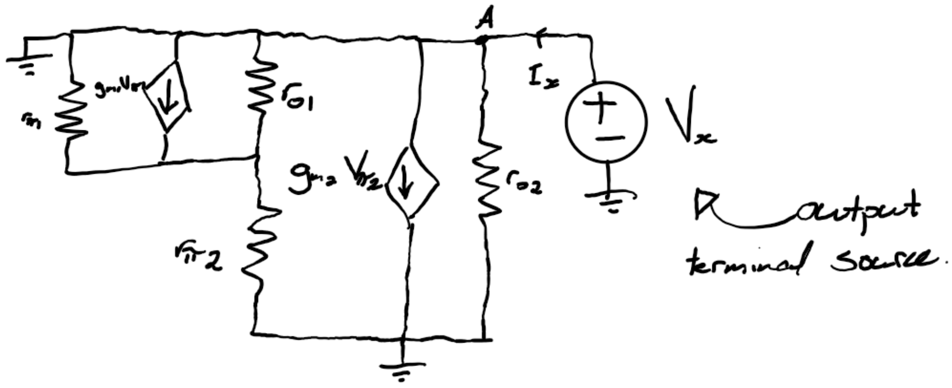


Figure 3

Drawing the small signal equivalent circuit



Given we know $I_{c2} = I_{Bias} = 1mA$

$$I_{c1} = \left(\frac{\beta_2 + 1}{\beta_2}\right)\left(\frac{\beta_1 + 1}{\beta_1}\right) I_{c2} \qquad \beta_1 = 120 \quad \beta_2 = 80$$

$$= 1.004mA$$

Given $g_m = \frac{I_{cQ}}{V_T}$ and $V_T = 0.026$

$$g_{m1} = \frac{1.004 \times 10^{-3}}{0.026} \qquad g_{m2} = \frac{1 \times 10^{-3}}{0.026}$$

$$= 0.03862 \text{ A/V} \qquad = 0.03846 \text{ A/V}$$

And given $r_\pi = \frac{\beta V_T}{I_{cQ}}$

$$r_{\pi 1} = \frac{120 \times 0.026}{1.004 \times 10^{-3}} \qquad r_{\pi 2} = \frac{80 \times 0.026}{1 \times 10^{-3}}$$

$$= 3.108k\Omega \qquad = 2.08k\Omega$$

And finally:

$$r_o = \frac{V_A}{I_{cQ}} \qquad V_{A1} = 80V \quad V_{A2} = 50V$$

$$r_{o1} = 79.68k\Omega \qquad r_{o2} = 50k\Omega$$

KCL at A:

$$I_x = \frac{V_x}{r_{o2}} + g_{m2} V_{\pi 2} + \frac{V_x - V_{\pi 2}}{r_{o1}} + g_{m1} V_{\pi 1}$$

Given we also notice $V_{\pi 1} = -V_{\pi 2}$ (KVL)

$$I_x = \frac{V_x}{r_{o2}} + g_{m2} V_{\pi 2} + \frac{V_x - V_{\pi 2}}{r_{o1}} - g_{m1} V_{\pi 2}$$

$$I_x = \left(\frac{1}{r_{o2}} + \frac{1}{r_{o1}}\right) V_x + \left(g_{m2} - \frac{1}{r_{o1}} - g_{m1}\right) V_{\pi 2} \quad (2)$$

We also know $V_{\pi 2} = I\, r_{\pi 2}$

using KCL to find I through $r_{\pi 2}$:

$$I_{\pi 2} = \frac{V_{\pi 1}}{r_{\pi 1}} + g_{m1} V_{\pi 1} + \frac{V_x - V_{\pi 2}}{r_{o1}}$$

Thus $$V_{\pi 2} = \left(\frac{V_{\pi 1}}{r_{\pi 1}} + g_{m1} V_{\pi 1} + \frac{V_x - V_{\pi 2}}{r_{o1}}\right) r_{\pi 2}$$

$$V_{\pi 2} = \frac{-V_{\pi 2}}{r_{\pi 1}} r_{\pi 2} - g_{m1} V_{\pi 2} r_{\pi 2} + \frac{V_x}{r_{o1}} r_{\pi 2} - \frac{V_{\pi 2}}{r_{o1}} r_{\pi 2}$$

$$\therefore \quad V_{\pi 2} = \frac{V_x\, r_{\pi 2}}{r_{o1}\left(1 + \frac{r_{\pi 2}}{r_{\pi 1}} + r_{\pi 2} g_{m1} + \frac{r_{\pi 2}}{r_{o1}}\right)}$$

$$V_{\pi 2} = 0.00032\, V_x$$

Thus subbing into (2)

$$I_x = \left(\frac{1}{r_{o1}} + \frac{1}{r_{o2}}\right)V_x + \left(g_{m2} - \frac{1}{r_{o1}} - g_{m1}\right) \times 0.00032\, V_x$$

Thus $\frac{V_x}{I_x} = R_o$

$$= \frac{1}{\left(\frac{1}{r_{o1}} + \frac{1}{r_{o2}}\right) + \left(g_{m2} - \frac{1}{r_{o1}} - g_{m1}\right) \times 0.00032}$$

$= 30.7\,k\Omega$

Thus the output impedance is $30.7\,k\Omega$