ELEC2004 Homework 2

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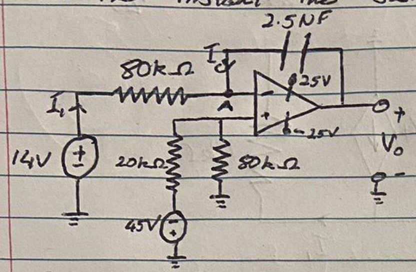
Question 1)

At the instant the switch is closed, we find:



First we solve for the voltage of the op-amp terminals: Notice V_- +Ve is a simple voltage divider, and given it is ideal, we have no current or voltage over the terminals:

$$V_- = -45\left(\frac{80000}{80000+20000}\right) = -36V$$

Applying KCL at node A we find:

$I_1 = -I_c$ Notice that:

$I_1 = \frac{14--36}{80000}$ and $I_c = C\frac{dV(t)}{dt}$

$625\times10^{-6} = -(2.5\times10^{-6})\frac{dV(t)}{dt}$ thus:

$\frac{dV}{dt} = 250$ and thus $\int\frac{dV}{dt}\,dt = -\int 250\,dt$

$V = -250t + C$

Initially we know $V(0) =$ 56V thus:

56 $= 0 + C = C$

$V = -250t +$ 56

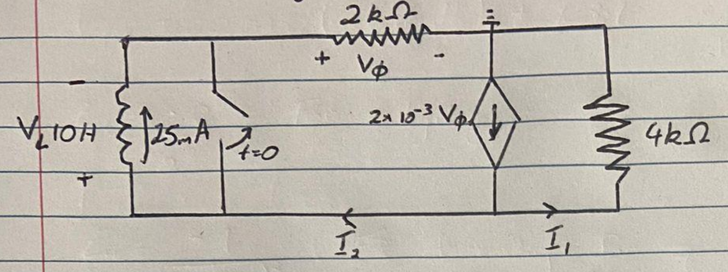
In order for V_o to be 0V, the capacitor voltage must negate the ~~-~~ -36V terminal thus:

$36 = -250t +$ 56 $t =$ 0.080 seconds

Thus, it took 80 milliseconds for the output voltage to equal 0.

Question 2)

Let the voltage over the inductor be V_L;



From KCL we find:

$$2\times10^{-3}V_\phi = I_1 + I_2$$

We know $I_2 = \frac{V_\phi}{2000}$ and $I_1 = \frac{V_\phi + V_L}{4000}$ thus:

$$2\times10^{-3}V_\phi = \frac{V_\phi + V_L}{4000} + \frac{V_\phi}{2000}$$

The voltage over an inductor is given by: $V(t) = L\frac{di(t)}{dt}$

$$2\times10^{-3}V_\phi = \frac{V_\phi + 10\frac{di(t)}{dt}}{4000} + \frac{V_\phi}{2000}$$ Note $i(t) = i_2$

We also know: $V_\phi = 2000 i_2$ thus:

$$4i(t) = \frac{i(t)}{2} + \frac{1}{400}\frac{di(t)}{dt} + i(t)$$

Thus we are left with the ODE:

$$0 = -\frac{5}{2}i(t) + \frac{1}{400}\frac{di(t)}{dt} \rightarrow \frac{di(t)}{dt} = 1000i(t)$$

$$\int\frac{dt}{di(t)}di(t) = \int\frac{1}{1000i}di \Rightarrow t = \frac{1}{1000}\ln(i) + C$$

Given the inital conditions: $i(0) = 25\times10^{-3}A$

$$C = -\frac{1}{1000}\ln(25\times10^{-3}) \Rightarrow t = \frac{1}{1000}\ln(i) - \frac{1}{1000}\ln(25\times10^{-3})$$

When $i = 5A$:

$$t = \frac{1}{1000}\ln(5) - \frac{1}{1000}\ln(25\times10^{-3}) = 0.0053 \text{ seconds}$$

Thus, it took 0.0053 seconds (53 milliseconds) after the switch was opened for the inductor to malfunction at the 5A limit.