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Q1) $\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}(x-1)^n$

a)

Radius of convergence: $L = \lim_{n\to\infty}\left(\frac{n+1}{n}\right)$

$$\lim_{n\to\infty}\left|\frac{\frac{(n+1)!^2}{(2n+2)!}(x-1)^{n+1}}{\frac{(n!)^2}{(2n)!}(x-1)^n}\right| = \lim_{n\to\infty}\left|\frac{\frac{(n+1)!^2}{(2n+2)!}(x-1)(x-1)^n}{\frac{(n!)^2}{(2n)!}(x-1)^n}\right|$$

$$= |x-1| \lim_{n\to\infty}\left|\frac{(n+1)!^2(2n)!}{(n!)^2(2n+2)!}\right| = |x-1|\lim_{n\to\infty}\left|\left(\frac{(n+1)!}{(n!)}\right)^2\left(\frac{(2n)!}{(2n+2)!}\right)\right|$$

$$= |x-1|\lim_{n\to\infty}\left|\left(\frac{n!\times(n+1)}{n!}\right)^2\left(\frac{(2n)!}{(2n)!\times(2n+1)\times(2n+2)}\right)\right|$$

$$= |x-1|\lim_{n\to\infty}\left|(n+1)^2\times\frac{1}{(2n+1)\times(2n+2)}\right| = |x-1|\lim_{n\to\infty}\left|\frac{n^2+2n+1}{4n^2+6n+2}\right|$$

L'Hopitals

$$= |x-1|\lim_{n\to\infty}\left|\frac{2n+2}{8n+6}\right|$$

$$= |x-1|\lim_{n\to\infty}\left|\frac{2}{8}\right|$$

$$= |x-1|\times\frac{1}{4}$$

$\therefore |x-1|\frac{1}{4} < 1$

$\therefore -1 < |x-1|\frac{1}{4} < 1$

$-4 < |x-1| < 4$

$\therefore$ The radius of convergence is 4

b) $\therefore$ Centre $= +1$ as $0 = x-1$

$x = 1$

Interval: as the centre is at $x=1$ and the radius is 4: $(-3, 5)$ is the interval of convergence

Q2)

$$a_n = \int_0^{2\pi n} f(x) \sin x \, dx \qquad b_n = \int_0^{\pi + 2\pi n} f(x) \sin(x) \, dx$$

a) $f(x) = 1$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \int_0^{2\pi n} \sin(x) \, dx$$

$$= \lim_{n\to\infty} \left[-\cos x\right]_0^{2\pi n}$$

$$= \lim_{n\to\infty} \left(-\cos(2\pi n) + 1\right)$$

$\Rightarrow$ undefined

as $\lim_{n\to\infty} \cos(n)$ is undefined

$$\lim_{n\to\infty} b_n = \lim_{n\to\infty} \int_0^{\pi + 2\pi n} \sin(x) \, dx$$

$$= \lim_{n\to\infty} \left[-\cos(x)\right]_0^{\pi + 2\pi n}$$

$$= \lim_{n\to\infty} \left(-\cos(\pi + 2\pi n) + 1\right)$$

$\Rightarrow$ undefined

b) $\int_0^{\infty} \sin x \, dx$ will converge if it has a limit:

$$\lim_{x\to\infty} \int_0^{\infty} \sin x \, dx = \lim_{x\to\infty} \left[-\cos x\right]_0^{\infty}$$

$$= \lim_{x\to\infty} \left(-\cos\infty - 1\right)$$

this is undefined as $\cos\infty$ is undefined

$\therefore$ This integral does not converge

c) $f(x) = e^{-x}$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \int_0^{2\pi n} e^{-x} \sin x \, dx$$

LIATE

let $u = \sin x$ $\quad v' = e^{-x}$

$u' = \cos x$ $\quad v = -e^{-x}$

$$\int e^{-x} \sin x \, dx = -e^{-x} \sin x + \int +e^{-x} \cos x \, dx$$

let $u = \cos x$ $\quad v' = +e^{-x}$

$u' = -\sin x$ $\quad v = -e^{-x}$

$$= -e^{-x} \sin x - e^{-x} \cos x - \int e^{-x} \sin x \, dx$$

$$2 \int e^{-x} \sin x \, dx = -e^{-x} (\sin x + \cos x)$$

$$\int e^{-x} \sin x \, dx = \frac{-e^{-x} (\sin x + \cos x)}{2}$$

$\therefore$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \left[\frac{-e^{-x}(\sin(x) + \cos(x))}{2} \right]_0^{2\pi n}$$

$$= \lim_{n\to\infty} \left(\frac{-e^{-2\pi n}(\sin(2\pi n) + \cos(2\pi n))}{2} - -\frac{1(0+1)}{2} \right)$$

$$= \frac{1}{2}$$

$$\lim_{n\to\infty} b_n = \lim_{n\to\infty} \int_0^{2\pi n + \pi} e^{-x} \sin x \, dx$$

$$\therefore = \lim_{n\to\infty} \left[\frac{-e^{-x}(\sin x + \cos x)}{2} \right]_0^{\pi + 2\pi n}$$

$$= \lim_{n\to\infty} \left(\left(\frac{-e^{-(2\pi n + \pi)}(\sin(\pi + 2\pi n) + \cos(\pi + 2\pi n))}{2} \right) - -\frac{1(0+1)}{2} \right)$$

$$= \frac{1}{2}$$

d) $f(x) = e^{-x}\sin^3 x$

$\therefore$ for $\int_0^\infty f(x)\sin x \, dx$ to be convergent:

$\lim_{x\to\infty}\int_0^\infty e^{-x}\sin^4 x \, dx$ must have a solution.

Using Identities:

$$\lim_{x\to\infty}\int_0^\infty e^{-x}\sin^4 x \, dx = \lim_{x\to\infty}\int_0^\infty e^{-x}\left(\frac{1}{2}(1-\cos(2x))\right)^2 dx$$

$$= \lim_{x\to\infty}\int_0^\infty e^{-x}\left(\frac{1}{2} - \frac{1}{2}\cos(2x)\right)^2 dx$$

$$= \lim_{x\to\infty}\int_0^\infty e^{-x}\left(\frac{1}{4} - \frac{1}{2}\cos(x) + \frac{1}{4}\cos^2(2x)\right) dx$$

$$= \lim_{x\to\infty}\int_0^\infty \frac{e^{-x}}{4} - \frac{e^{-x}}{2}\cos(x) + \frac{e^{-x}}{4}\left(\frac{1}{2}(\cos(4x)+1)\right) dx$$

$$= \lim_{x\to\infty}\int_0^\infty \frac{e^{-x}}{4} - \frac{e^{-x}}{2}\cos(x) + \frac{e^{-x}}{8}\cos(4x) + \frac{e^{-x}}{8} \, dx$$

$$= \lim_{x\to\infty}\int_0^\infty \frac{e^{-x}\cos(4x)}{8} - \frac{e^{-x}\cos(2x)}{2} + \frac{3e^{-x}}{8} \, dx$$

let $e^{-x} = v'$ $\quad u = \cos(4x)$

$v = -e^{-x}$ $\quad u' = -4\sin(4x)$

$$\int \frac{e^{-x}\cos(4x)}{8} dx = -e^{-x}\cos(4x) - \int -e^{-x} \times -4\sin(4x)$$

let $u = -4\sin(4x)$ $\quad v' = e^{-x}$

$u' = -16\cos(4x)$ $\quad v = -e^{-x}$

$$\int e^{-x}\cos(4x)\,dx = -e^{-x}\cos(4x) + e^{-x}\sin(4x) - \int -16e^{-x}\cos(4x)$$

$$\int e^{-x}\cos(4x)\,dx = \frac{e^{-x}(4\sin(4x) - \cos(4x))}{17} + C$$

let $e^{-x} = v'$ $\quad u = \cos(2x)$

$v = -e^{-x}$ $\quad u' = -2\sin(2x)$

$$\int \frac{e^{-x}\cos(2x)}{2} dx = -e^{-x}\cos(2x) - \int -e^{-x} \times -2\sin(2x)$$

let $u = -2\sin(2x)$ $\quad v' = e^{-x}$

$u' = -4\cos(2x)$ $\quad v = -e^{-x}$

$$\int e^{-x}\cos(2x)\,dx = -e^{-x}\cos(2x) + 2\sin(2x)e^{-x} - \int e^{-x} \times 4\cos(2x)\,dx$$

$$= \frac{e^{-x}(2\sin(2x) - \cos(2x))}{5} + C$$

$\therefore \lim_{x\to\infty} \int_0^\infty e^{-x} \sin^4(x)\, dx$

$$= \lim_{x\to\infty} \left[\frac{1}{8}\left(\frac{e^{-x}(4\sin(4x) - \cos(4x))}{17}\right) - \frac{1}{2}\left(\frac{e^{-x}(2\sin(2x) - \cos(2x))}{5}\right) - \frac{3e^{-x}}{8} \right]_0^\infty$$

$$= \lim_{x\to\infty} \left(\frac{e^{-\infty}(4\sin(\infty) - \cos(\infty)}{136} - \frac{e^{-\infty}(2\sin(\infty) - \cos(\infty))}{10} - \frac{3e^{-\infty}}{8} \right)$$

$$- \left(\frac{e^{-0}(4\sin(0) - \cos(0)}{136} - \frac{e^{-0}(2\sin(0) - \cos(0))}{10} - \frac{3e^{0}}{8} \right)$$

$$= \frac{24}{85} \approx 0.28 = \lim_{x\to\infty} \int_0^\infty e^{-x} \sin^4(x)\, dx$$

$\therefore$ as the limit has a solution, the improper integral $\int_0^\infty f(x) \sin^3(x)\, dx$ converges.

e) Example of nonconstant $f(x)$ for which $\int_0^{\infty} f(x)\sin(x)\,dx$ diverges implies that:

$$\lim_{x\to\infty}\int_0^{\infty} f(x)\sin(x)\,dx \text{ has no solution}$$

Example: $f(x) = x$

$$\lim_{x\to\infty}\int_0^{\infty} x\sin(x)\,dx$$

let $x = u$ $\quad V' = \sin x$

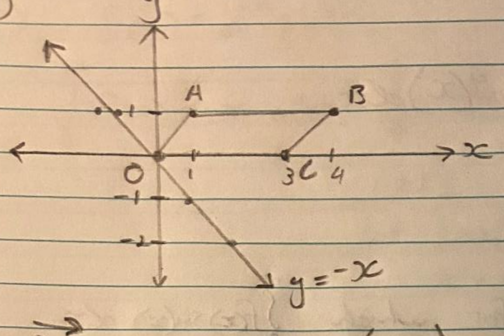
$u' = 1$ $\quad V = -\cos x$

$$= \lim_{x\to\infty}\left[-x\cos x - \int -\cos x\,dx\right]_0^{\infty}$$

$$= \lim_{x\to\infty}\left[-x\cos x + \sin x\right]_0^{\infty}$$

as $\cos(\infty)$ has no solution, it cannot have a limit and must therefore be divergent.

Q3)



As parallelogram, B must be at $(4, 1)$

$$\vec{OB} = OA + AB = (4, 1) = \begin{pmatrix}4\\1\end{pmatrix}$$

$y = -x$ for the sake of this question can be Vector f where: $\vec{f} = \begin{pmatrix}4\\-4\end{pmatrix}$

$$\text{Proj}_f\,\vec{OB} = \frac{\vec{OB}\cdot\vec{f}}{|f|} \times \frac{\vec{f}}{|f|} = \frac{(4\times4)+(1\times-4)}{\sqrt{4^2+4^2}} \times \frac{\begin{pmatrix}4\\-4\end{pmatrix}}{\sqrt{4^2+4^2}}$$

$$\text{Proj}_f\,\vec{OB} = \begin{pmatrix}4\\-4\end{pmatrix} \times \frac{3}{8} = \begin{pmatrix}\frac{3}{2}\\-\frac{3}{2}\end{pmatrix}$$

$\therefore$ The projection is line from the origin to $\left(\frac{3}{2}, -\frac{3}{2}\right)$ following $y = -x$.

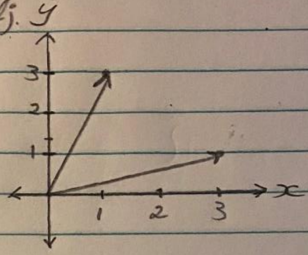
Q4)

$$A = \begin{pmatrix} 1 & 3 \\ 3 & 1 \end{pmatrix}$$

a) Considering the coloumns, the two vectors are using standard basis of i and j.

$$\begin{pmatrix} 1 & 3 \\ 3 & 1 \end{pmatrix} \times \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} \text{ and } \begin{pmatrix} 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 3 \\ 3 & 1 \end{pmatrix} \times \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

these are displayed as:

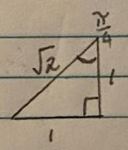


b) Rotating by some angle anticlockwise: The formula is:

$$B = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

as $\theta = 45°$ or $\frac{\pi}{4}$ radians.

$$\therefore B = \begin{pmatrix} \cos\frac{\pi}{4} & -\sin\frac{\pi}{4} \\ \sin\frac{\pi}{4} & \cos\theta \end{pmatrix}$$



$$B = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix}$$

c) $$AB = \begin{pmatrix} 1 & 3 \\ 3 & 1 \end{pmatrix} \begin{pmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{2}}{2} + 3\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} + 3\frac{\sqrt{2}}{2} \\ 3\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} & -3\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \end{pmatrix} = \begin{pmatrix} 2\sqrt{2} & \sqrt{2} \\ 2\sqrt{2} & -\sqrt{2} \end{pmatrix}$$

$$BA = \begin{pmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix} \begin{pmatrix} 1 & 3 \\ 3 & 1 \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{2}}{2} - 3\frac{\sqrt{2}}{2} & 3\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} + 3\frac{\sqrt{2}}{2} & 3\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \end{pmatrix} = \begin{pmatrix} -\sqrt{2} & \sqrt{2} \\ 2\sqrt{2} & 2\sqrt{2} \end{pmatrix}$$

No they are not the same.

d) $$\begin{pmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix} C = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$ → This is the case when B is transformed to $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

∴ Clockwise by $45° \left(\frac{\pi}{4}\right)$ ← when $\theta = 0$.

$$\therefore C = \begin{pmatrix} \cos\left(-\frac{\pi}{4}\right) & -\sin\left(-\frac{\pi}{4}\right) \\ \sin\left(-\frac{\pi}{4}\right) & \cos\left(-\frac{\pi}{4}\right) \end{pmatrix}$$

Proof: $$\overset{B}{\begin{pmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix}} \times \overset{C}{\begin{pmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix}} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$