AIC HW4

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NMOS: $I_{DS}=\frac{1}{2}\times \mu_n C_{OX} \times \frac{W}{L}(V_{GS}-|V_{th}|)^2$ ......①

$gm=\frac{2 \cdot Id}{Vov}$ ......②

$Vov=\sqrt{2u_n C_{ox}\frac{W}{L}Id}$ ......③

PART1:

I. 分析此電路的小訊號模型，得出 differential gain 為 $gm_{M1}(ro_{M1(M2)}||Rd)$，因此可知若要 gain > 20，則 gm_{M1} 要很大⟹ Iss 要大，i.e. W_{M3}/L_{M3} 要大 ⟹ choose W_{M3}/L_{M3} = 25um/1um。

II. 若 Vb 太大，會使 M1(M2) 的 source voltage 太大，造成 M1(M2) 操作在不飽和區，因此使 Vb 能超過 M3 的 Vth 就好，假設 Vth = 0.4V ⟹ choose Vb = 0.44V。

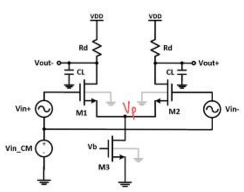
III. 若 Vin_CM 太大，根據電流①公式，為了維持電流相同，如果 Vov 不變，代表 Vs 因此提高，會容易造成 M3 操作在不飽和區，因此使 Vin_CM 能超過 M3 的 Vth，假設 Vth = 0.4V ⟹ choose Vin_CM = 0.5V。

IV. 因為 M1(M2) 的 $ro_{M1(M2)}$與 gain 有關係，而且此題要求的 gain 很大，預估 Rd 會取很大，所以盡量讓 $ro_{M1(M2)}$更大來減少 $ro_{M1(M2)}||Rd$ 的值，根據 hw1 ro ∝ L，i.e. $L_{M1(M2)}$不能太小 choose $L_{M1(M2)}$ = 4um。

V. 一開始假設讓 M3 和 M1(M2)的 Vov 相同，所以根據公式③，Vov_{M3} = $Vov_{M1(M2)}$，可以推得$(\frac{W}{L})_{M3}$=2$(\frac{W}{L})_{M1(M2)}$，取 $W_{M1(M2)}$ = 25 um (m = 2)。

VI. 大概推算一下 Iss =$\frac{1}{2}u_n C_{ox}(\frac{W}{L})_{M3}Vov_{M3}{}^2$ = 9.68 uA ⟹ $gm_{M1(M2)}$= $\frac{2(\frac{1}{2}Iss)}{Vov_{M1(M2)}}$ = 193 uA，假設 ro → ∞，Ad= $gm_{M1(M2)}$Rd >20 ⟹ Rd = 100kΩ。

VII. 驗證 mos region:



$I_{ss} \approx 9.68\mu A \quad V_{TH} \approx 0.4V \quad \mu_n C_{ox} \approx 300\times10^{-6}$

$\cancel{\frac{1}{2}} I_{ss} = \cancel{\frac{1}{2}}\mu_n C_{ox}(\frac{W}{L})_{M1(M2)}(V_{in\text{-}CM}-V_p-V_{TH})^2$

$V_{in\text{-}CM}-V_p-V_{TH} \approx 0.05V$

$\Rightarrow V_p \approx 0.05$

$\begin{cases} V_b > V_{TH} & \Rightarrow 0.44 > 0.4 \\ V_p > V_b - V_{TH} & \Rightarrow 0.05 > 0.44-0.4 \end{cases}$ for M3 on and sat.

$\begin{cases} V_{in\text{-}CM}-V_p > V_{TH} \Rightarrow 0.5-0.05 > 0.4 \\ V_{DD}-I_d R_d - \cancel{V_p} > V_{in\text{-}CM} - \cancel{V_p} - V_{TH} \end{cases}$ for $M_1(M_2)$ on and sat.

$\Rightarrow 1.5-9.68\mu\times100k > 0.5-0.4$

第 1 次測試:

Design parameter:

Vb	Vin_CM	W_{M3}/L_{M3}	$W_{M1(M2)}/L_{M1(M2)}$	Rd
0.44 V	0.5V	25um/1um	25 um/4um (m = 2)	100kΩ

Result:

```
subckt
element  0:mm1      0:mm2      0:mm3
model    0:n_18.1   0:n_18.1   0:n_18.1
region   Saturation Saturation Saturation
  id       10.7036u   10.7036u   21.4073u
  ibs    -354.7274a -354.7274a -3.270e-21
  ibd      -1.5775f   -1.5775f -177.3596a
  vgs     403.3866m  403.3866m  440.0000m
  vds     333.0237m  333.0237m   96.6134m
  vbs     -96.6134m  -96.6134m    0.
  vth     349.4255m  349.4255m  387.9827m
  vdsat    84.2819m   84.2819m   88.7119m
  vod      53.9611m   53.9611m   52.0173m
  beta      3.7593m    3.7593m    7.7755m
  gam eff 510.0843m  510.0843m  507.4460m
  gm      191.1409u  191.1409u  357.8438u
  gds       1.2720u    1.2720u   51.6959u
  gmb      37.3125u   37.3125u   74.5657u
  cdtot    71.7344f   71.7344f   45.7259f
  cgtot     1.2056p    1.2056p  161.5664f
  cstot     1.2176p    1.2176p  172.7527f
  cbtot   435.9652f  435.9652f   98.5710f
  cgs       1.0538p    1.0538p  133.7782f
  cgd      17.0470f   17.0470f   12.1195f
```

```
v(voutp,voutn)/vip                        =   16.9571
input resistance at            vip        =  1.000e+20
output resistance at v(voutp,voutn)       =  177.4316k
```

```
dcgain_in_db=  24.5870      at=    1.0000
            from=    1.0000       to= 100.0000g
bw=    1.6703x
```

觀察與微調:

M1、M2、M3 都進入了飽和區，且如預期三者的 Vov 接近，很明顯可以發現 A_{DM} 還差一些，是因為 ro 把 output resistance 分掉了，有兩個做法，第一個是將 Rd 繼續提高，但這樣做會讓 bw 降低，於是我選擇第二個方法，增加 $gm_{M1(M2)}$，根據公式②，如果將 $Vov_{M1(M2)}$降低就可以增加 $gm_{M1(M2)}$的值，再觀察公式①，可以發現如果提高$(\frac{W}{L})_{M3}$的值，就可以降低 $Vov_{M1(M2)}$，試取$(\frac{W}{L})_{M3}$= $\frac{50um}{4um}$。

第 2 次測試:

Design parameter:

Vb	Vin_CM	W_{M3}/L_{M3}	$W_{M1(M2)}/L_{M1(M2)}$	Rd
0.44 V	0.5V	25um/1um	50um/4um (m = 2)	100kΩ

Result:

```
subckt
element  0:mm1      0:mm2      0:mm3
model    0:n_18.1   0:n_18.1   0:n_18.1
region   Saturation Saturation Saturation
  id       11.1959u   11.1959u   22.3918u
  ibs    -898.3297a -898.3297a -3.420e-21
  ibd      -2.7380f   -2.7380f -229.1217a
  vgs     375.1906m  375.1906m  440.0000m
  vds     255.5985m  255.5985m  124.8094m
  vbs    -124.8094m -124.8094m    0.
  vth     355.0247m  355.0247m  387.7696m
  vdsat    67.9273m   67.9273m   88.8316m
  vod      20.1660m   20.1660m   52.2304m
  beta      7.5168m    7.5168m    7.7754m
  gam eff 510.8295m  510.8295m  507.4460m
  gm      230.8634u  230.8634u  386.1202u
  gds       1.6559u    1.6559u   23.0819u
  gmb      44.8729u   44.8729u   80.2472u
  cdtot   144.6724f  144.6724f   41.5689f
  cgtot     2.0169p    2.0169p  160.0552f
  cstot     1.9074p    1.9074p  173.2041f
  cbtot   859.7789f  859.7789f   98.0280f
  cgs       1.6330p    1.6330p  133.4956f
  cgd      35.4268f   35.4268f   10.5633f
```

```
v(voutp,voutn)/vip                        =   19.8065
input resistance at            vip        =  1.000e+20
output resistance at v(voutp,voutn)       =  171.5873k
```

```
dcgain_in_db=  25.9362      at=    1.0000
            from=    1.0000       to= 100.0000g
bw=    1.6170x
```

觀察與微調:

發現 $Vov_{M1(M2)}$降低使得 A_{DM} 提高了，繼續調高$(\frac{W}{L})_{M3}$的值。試取$(\frac{W}{L})_{M3}$= $\frac{50um}{4um}$。

第 3 次測試:

Design parameter:

Vb	Vin_CM	W_{M3}/L_{M3}	$W_{M1(M2)}$ /$L_{M1(M2)}$	Rd
0.44 V	0.5V	25um/1um	55um/4um (m = 2)	100kΩ

Result:

```
subckt
element  0:mm1      0:mm2      0:mm3
model    0:n_18.1   0:n_18.1   0:n_18.1
region   Saturation Saturation Saturation
 id        11.2366u   11.2366u   22.4733u
 ibs       -1.0155f   -1.0155f -3.433e-21
 ibd       -2.9741f   -2.9741f -235.8914a
 vgs      371.5030m  371.5030m  440.0000m
 vds      247.8396m  247.8396m  128.4970m
 vbs     -128.4970m -128.4970m    0.
 vth      355.7577m  355.7577m  387.7417m
 vdsat     66.0495m   66.0495m   88.8472m
 vod       15.7453m   15.7453m   52.2583m
 beta       8.2683m    8.2683m    7.7754m
 gam eff  510.9262m  510.9262m  507.4460m
 gm       235.5891u  235.5891u  388.3616u
 gds        1.7098u    1.7098u   21.1271u
 gmb       45.7584u   45.7584u   80.6950u
 cdtot    159.1119f  159.1119f   41.2243f
 cgtot      2.1392p    2.1392p  159.9281f
 cstot      1.9924p    1.9924p  173.2585f
 cbtot    943.5317f  943.5317f   97.9681f
 cgs        1.7008p    1.7008p  133.4636f
 cgd       39.1976f   39.1976f   10.4400f
```

```
v(voutp,voutn)/vip                      =   20.1189
input resistance at              vip    =  1.000e+20
output resistance at v(voutp,voutn)     =  170.7975k
```

```
dcgain_in_db=  26.0721      at=    1.0000
           from=    1.0000      to= 100.0000g
bw=    1.6042x
```

觀察與微調:

發現已達到題目要求，A_{DM} > 20(V/V)，bw > 1.5M(Hz)

(b)

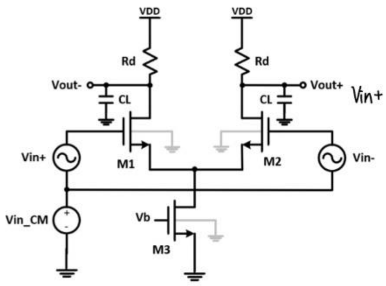

```
subckt
element  0:mm1      0:mm2      0:mm3
model    0:n_18.1   0:n_18.1   0:n_18.1
region   Saturation Saturation Saturation
 id        11.2366u   11.2366u   22.4733u
 ibs       -1.0155f   -1.0155f -3.433e-21
 ibd       -2.9741f   -2.9741f -235.8914a
 vgs      371.5030m  371.5030m  440.0000m
 vds      247.8396m  247.8396m  128.4970m
 vbs     -128.4970m -128.4970m    0.
 vth      355.7577m  355.7577m  387.7417m
 vdsat     66.0495m   66.0495m   88.8472m
 vod       15.7453m   15.7453m   52.2583m
 beta       8.2683m    8.2683m    7.7754m
 gam eff  510.9262m  510.9262m  507.4460m
 gm       235.5891u  235.5891u  388.3616u
 gds        1.7098u    1.7098u   21.1271u
 gmb       45.7584u   45.7584u   80.6950u
 cdtot    159.1119f  159.1119f   41.2243f
 cgtot      2.1392p    2.1392p  159.9281f
 cstot      1.9924p    1.9924p  173.2585f
 cbtot    943.5317f  943.5317f   97.9681f
 cgs        1.7008p    1.7008p  133.4636f
 cgd       39.1976f   39.1976f   10.4400f
```

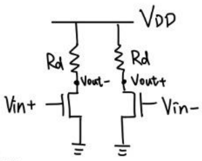

(c)

```
****      small-signal transfer characteristics

     v(voutp,voutn)/vip                      =   20.1189
     input resistance at              vip    =  1.000e+20
     output resistance at v(voutp,voutn)     =  170.7975k
```

(d)





$V_{id} = V_{in1} - V_{in2}$; $V_{od} = (V_{out-}) - (V_{out+})$; $A_d = \frac{V_{od}}{V_{id}}$

$V_{in1} = V_{in\_CM} + V_{in+} \Rightarrow V_{out-} = A_{v1} \cdot V_{in1} = A_{v1}(V_{in\_CM} + V_{in+})$

$V_{in2} = V_{in\_CM} + V_{in-} \Rightarrow V_{out+} = A_{v2} \cdot V_{in2} = A_{v2}(V_{in\_CM} + V_{in-})$

if $A_{v1} = A_{v2} = -g_m(R_D // r_o)$

$\Rightarrow A_D = \frac{V_{od}}{V_{id}} = \frac{A_{v1} + A_{v2}}{2} = -g_m(R_D // r_o)$

$= -235.5891 \times 10^{-6} \times (100k // \frac{1}{1.7098 \times 10^{-6}}) = 20.118968$ (V/V)

誤差 $= \frac{20.1189 - 20.118968}{20.118968} \times 100\% = -3.416 \times 10^{-4}$

(e)

```
****** ac analysis tnom=  25.000 temp=  25.000 ******
acm_in_db=  -1.4903
```

CMRR $_{\text{simulated}} = 20 \log(20.1189) - (-1.4903) = 27.562\text{dB}$

(f)

$gm1$	$gm2$	R_D	r_{o1}	r_{o3}
$235.5891u\Omega^{-1}$	$235.5891u\Omega^{-1}$	$100k\Omega$	584863Ω	47332Ω
C_L	C_p	w		
1pF	4.026 pF	10kHz		

$R_D \,\|\|\, r_{o1} \,\|\|\, \frac{1}{C_L S}$	$r_{o3} \,\|\|\, \frac{1}{C_p S}$
85398.72Ω	47332.49Ω

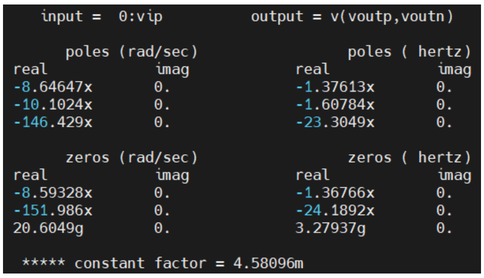
$$|A_{C,CM}| = \frac{gm1\,[R_D \,||\, r_{o1} \,||\, \frac{1}{C_L S}]}{(gm1 + gm2)\left[r_{o3} \,||\, \frac{1}{C_p S}\right] + 1} = \frac{235.5891 \cdot 10^{-6} \cdot 85398.72}{2 \cdot 235.5891 \cdot 10^{-6} 47332.49 + 1} = 0.86340126 \text{ (V/V)}$$

$$\text{CMRR}_{\text{hand}} = 20 \cdot \log\left(\left|\frac{A_{DM}}{A_{V,CM}}\right|\right) = 27.34786044 \text{ dB}$$

$$\text{CMRR}_{\text{error}} = \frac{\text{CMRR}_{\text{simulated}} - \text{CMRR}_{\text{hand}}}{CMRR_{hand}} \cdot 100\% = \left|\frac{27.562\, dB - 27.34786044\, dB}{27.34786044\, dB}\right| \cdot 100\%$$

$$= 0.783\%$$

(g)

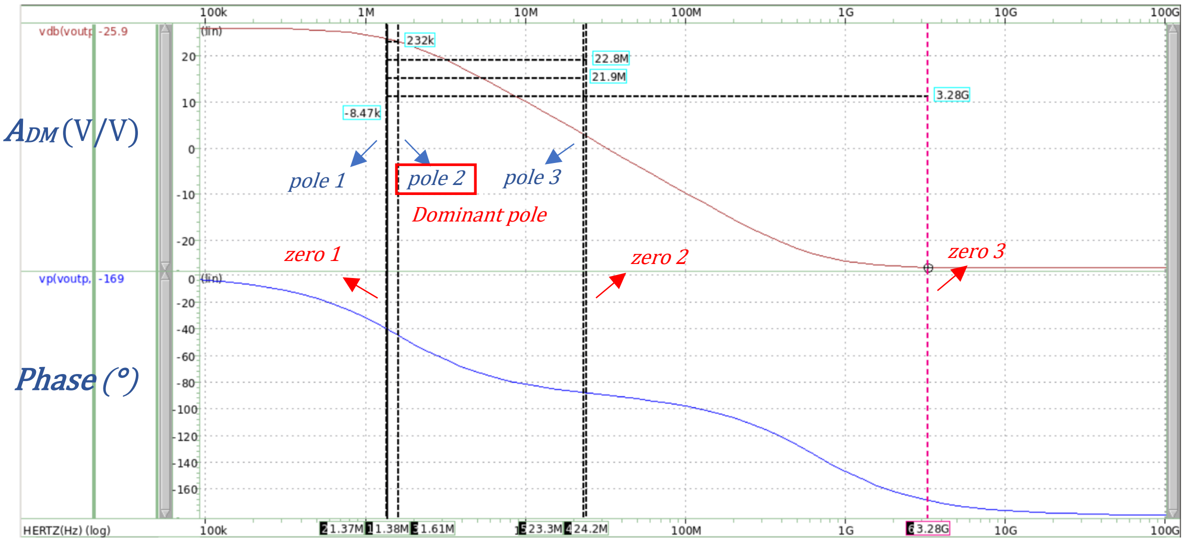


```
input =  0:vip                output = v(voutp,voutn)

        poles (rad/sec)                poles ( hertz)
real                imag         real                imag
-8.64647x           0.           -1.37613x           0.
-10.1024x           0.           -1.60784x           0.
-146.429x           0.           -23.3049x           0.

        zeros (rad/sec)                zeros ( hertz)
real                imag         real                imag
-8.59328x           0.           -1.36766x           0.
-151.986x           0.           -24.1892x           0.
20.6049g            0.           3.27937g            0.

 ***** constant factor = 4.58096m
```

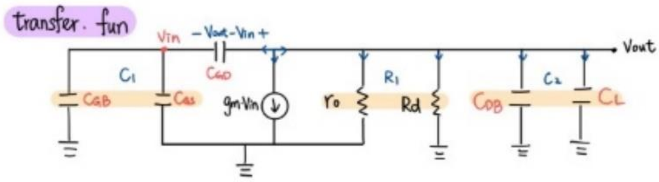


Frequency (Hz)

從圖中可以發現 pole 1 和 zero 1 的值非常相近，所以其實會互相抵銷，使得 dominant pole 出現在 pole 2 的位置也幾乎就是-3dB bandwidth 的位置。

(h)

transfer. fun



$C_1 = C_{GB} \parallel C_{GS} = C_{GB} + C_{GS}$

$C_2 = C_{DB} \parallel C_L = C_{DB} + C_L$

$R_1 = r_o \parallel R_d$

$(V_{out} - V_{in})\, s\, C_{GD} + g_m V_{in} + \frac{V_{out}}{R_1} + V_{out} \cdot s C_2 = 0$

$\Rightarrow V_{out} \cdot \left(s C_{GD} + \frac{1}{R_1} + s C_2 \right) = V_{in} \left(s C_{GD} - g_m \right)$

$\Rightarrow \frac{V_{out}}{V_{in}} = \frac{s C_{GD} - g_m}{s C_{GD} + \frac{1}{R_1} + s C_2} = \frac{s \underbrace{C_{GD}}_{a} - \underbrace{g_m}_{b}}{s \underbrace{(C_{GD} + C_2)}_{c} + \underbrace{\frac{1}{R_1}}_{d}} = \frac{as - b}{cs + d}$

zero : $as - b = 0 \Rightarrow s = \frac{b}{a}$

pole : $cs + d = 0 \Rightarrow s = \frac{-d}{c}$

-3dB pole

$\frac{gain, -3dB}{gain, max} = \frac{1}{\sqrt{2}} = \left| \frac{as + b}{cs + d} \right| \cdot \frac{1}{|gain, max|}$

$\Rightarrow \frac{1}{\sqrt{2}} |gain, max| = \sqrt{\frac{b^2 + w^2 a^2}{d^2 + w^2 c^2}} = A$

$\Rightarrow A^2 d^2 + w^2 \cdot A^2 c^2 = b^2 + w^2 a^2$

$\Rightarrow w^2 (A^2 c^2 - a^2) = b^2 - A^2 d^2$

$\Rightarrow w = \left(\frac{b^2 - A^2 d^2}{A^2 c^2 - a^2} \right)^{\frac{1}{2}}$

CL	CGS	CDB	CGD	C2
1E-12	1.7008E-12	1.199143 E-13	39.1976E-15	1.1199143E-12
R1	gm			
85398.55506	388.3616E-6			

$a = C_{GD}$ ， $b = gm$ ， $c = C_{GD} + C_2$ ， $d = \frac{1}{ro||Rd}$ ， $A = \frac{1}{\sqrt{2}} \cdot |gain_{max}|$

a	b	c	d	A
3.91976E-14	2.355891E-4	1.1591119 E-12	1.17098E-5	14.22621062

from transfer function ： as + b = 0

$$\Longrightarrow s = zero = \frac{b}{a} = \frac{2.355891 \cdot 10^{-4}}{3.91976 \cdot 10^{-14}} = 6010293998\ (\frac{rad}{sec}) = 956567999.2\ (Hz)$$

from transfer function ： cs + d = 0

$$\Longrightarrow s = pole = \frac{-d}{c} = \frac{-1.17098 \cdot 10^{-5}}{1.1591119 \cdot 10^{-12}} = 10099801.41\ (\frac{rad}{sec}) = 1607433.318(Hz)$$

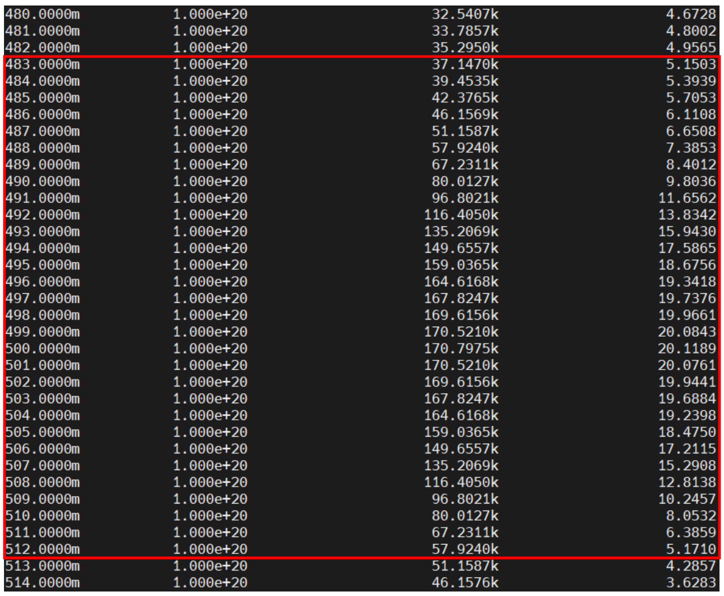
$$-3dB\ BW = \omega = \left(\frac{b^2 - A^2 d^2}{A^2 c^2 - a^2}\right)^{\frac{1}{2}} = \left(\frac{(2.355891 \cdot 10^{-4})^2 - (14.22621062)^2 (1.17095 \cdot 10^{-5})^2}{(14.22621062)^2 (1.1591119 \cdot 10^{-12})^2 - (3.91976 \cdot 10^{-14})^2}\right)^{\frac{1}{2}}$$

$$= 1.60786077\ (MHz)$$

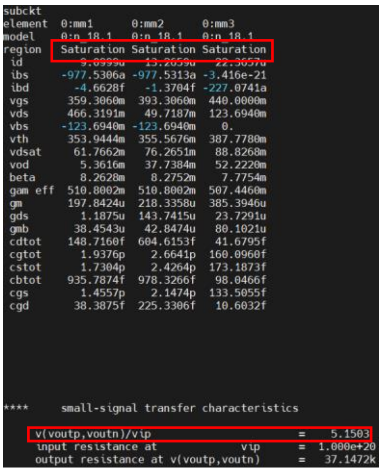
	hand calculation	hspice	error
pole	1.607433318 MHz	1.60780 MHz	0.0228%
-3dB Bandwidth	1.607860770 MHz	1.60420 MHz	0.2275%

推測造成誤差的原因為電路中的寄生電容, 且因此式較為複雜, 故計算時, 各值的四捨五入也會造成最終計算結果有些許誤差 。

(i)



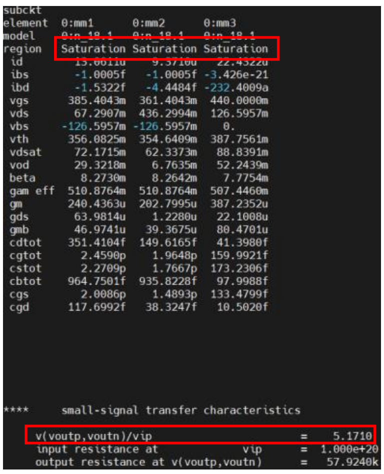
480.0000m	1.000e+20	32.5407k	4.6728
481.0000m	1.000e+20	33.7857k	4.8002
482.0000m	1.000e+20	35.2950k	4.9565
483.0000m	1.000e+20	37.1470k	5.1503
484.0000m	1.000e+20	39.4535k	5.3939
485.0000m	1.000e+20	42.3765k	5.7053
486.0000m	1.000e+20	46.1569k	6.1108
487.0000m	1.000e+20	51.1587k	6.6508
488.0000m	1.000e+20	57.9240k	7.3853
489.0000m	1.000e+20	67.2311k	8.4012
490.0000m	1.000e+20	80.0127k	9.8036
491.0000m	1.000e+20	96.8021k	11.6562
492.0000m	1.000e+20	116.4050k	13.8342
493.0000m	1.000e+20	135.2069k	15.9430
494.0000m	1.000e+20	149.6557k	17.5865
495.0000m	1.000e+20	159.0365k	18.6756
496.0000m	1.000e+20	164.6168k	19.3418
497.0000m	1.000e+20	167.8247k	19.7376
498.0000m	1.000e+20	169.6156k	19.9661
499.0000m	1.000e+20	170.5210k	20.0843
500.0000m	1.000e+20	170.7975k	20.1189
501.0000m	1.000e+20	170.5210k	20.0761
502.0000m	1.000e+20	169.6156k	19.9441
503.0000m	1.000e+20	167.8247k	19.6884
504.0000m	1.000e+20	164.6168k	19.2398
505.0000m	1.000e+20	159.0365k	18.4750
506.0000m	1.000e+20	149.6557k	17.2115
507.0000m	1.000e+20	135.2069k	15.2908
508.0000m	1.000e+20	116.4050k	12.8138
509.0000m	1.000e+20	96.8021k	10.2457
510.0000m	1.000e+20	80.0127k	8.0532
511.0000m	1.000e+20	67.2311k	6.3859
512.0000m	1.000e+20	57.9240k	5.1710
513.0000m	1.000e+20	51.1587k	4.2857
514.0000m	1.000e+20	46.1576k	3.6283



```
subckt
element   0:mm1        0:mm2        0:mm3
model     0:n_18.1     0:n_18.1     0:n_18.1
region    Saturation   Saturation   Saturation
 id       [illegible]  [illegible]  [illegible]
 ibs      -977.5306a   -977.5313a   -3.416e-21
 ibd      -4.6628f     -1.3704f     -227.0741a
 vgs      359.3060m    393.3060m    440.0000m
 vds      466.3191m    49.7187m     123.6940m
 vbs      -123.6940m   -123.6940m   0.
 vth      353.9444m    355.5676m    387.7780m
 vdsat    61.7662m     76.2651m     88.8268m
 vod      5.3616m      37.7384m     52.2220m
 beta     8.2628m      8.2752m      7.7754m
 gam eff  510.8002m    510.8002m    507.4460m
 gm       197.8424u    218.3358u    385.3946u
 gds      1.1875u      143.7415u    23.7291u
 gmb      38.4543u     42.8474u     80.1021u
 cdtot    148.7160f    604.6153f    41.6795f
 cgtot    1.9376p      2.6641p      160.0960f
 cstot    1.7304p      2.4264p      173.1873f
 cbtot    935.7874f    978.3266f    98.0466f
 cgs      1.4557p      2.1474p      133.5055f
 cgd      38.3875f     225.3306f    10.6032f

****     small-signal transfer characteristics

   v(voutp,voutn)/vip                         =     5.1503
   input resistance at                  vip   =  1.000e+20
   output resistance at v(voutp,voutn)        =   37.1472k
```



```
subckt
element   0:mm1        0:mm2        0:mm3
model     0:n_18.1     0:n_18.1     0:n_18.1
region    Saturation   Saturation   Saturation
 id       [illegible]  [illegible]  [illegible]
 ibs      -1.0005f     -1.0005f     -3.426e-21
 ibd      -1.5322f     -4.4484f     -232.4009a
 vgs      385.4043m    361.4043m    440.0000m
 vds      67.2907m     436.2994m    126.5957m
 vbs      -126.5957m   -126.5957m   0.
 vth      356.0825m    354.6409m    387.7561m
 vdsat    72.1715m     62.3373m     88.8391m
 vod      29.3218m     6.7635m      52.2439m
 beta     8.2730m      8.2642m      7.7754m
 gam eff  510.8764m    510.8764m    507.4460m
 gm       240.4363u    202.7995u    387.2352u
 gds      63.9814u     1.2280u      22.1008u
 gmb      46.9741u     39.3675u     80.4701u
 cdtot    351.4104f    149.6165f    41.3980f
 cgtot    2.4590p      1.9648p      159.9921f
 cstot    2.2709p      1.7667p      173.2306f
 cbtot    964.7501f    935.8228f    97.9988f
 cgs      2.0086p      1.4893p      133.4799f
 cgd      117.6992f    38.3247f     10.5020f

****     small-signal transfer characteristics

   v(voutp,voutn)/vip                         =     5.1710
   input resistance at                  vip   =  1.000e+20
   output resistance at v(voutp,voutn)        =   57.9240k
```

(j)

$$\text{FoM} = \frac{total\ current(uA)}{input\ range(mV) * -3dB\ bandwidth\ (MHz)} = \frac{22.4733\ (uA)}{29\ (mV) * 1.6042(MHz)} = 0.4830703025$$

優化:

題目說明 $\text{FoM} = \frac{total\ current(uA)}{input\ range(mV) \cdot -3dB\ bandwidth\ (MHz)}$ 越小越好，根據測試發現如果調小 Vin_CM，可以大幅提升 input range 的範圍，但是一旦調太小有可能會使得 V_{GS} 變得比 Vth 小，mos 進入 cutoff，所以非常需要注意 $Vov_{M1(M2)}$ 的值，不能因為為了讓 A_{DM} 增加，而不斷去調高 $(\frac{W}{L})_{M3}$ 的值，使得 $Vov_{M1(M2)}$ 太小，而需要適時的調高 Rd 來達到 A_{DM} 增加的目的，當然這麼做就會降低 bw 的頻寬，所以需要作出取捨，同時我也注意到如果降低 Iss，雖然會降低 $gm_{M1(M2)}$ 造成需要更大的 Rd，但 bw 下降的幅度比 Iss 還多，也有助於降低 Fom。至於 A_{CM} 根據公式 $A_{C,CM} = \frac{gm1\ [R_D \,||\, r_{o1} \,||\, \frac{1}{C_L s}]}{(gm1+gm2)\left[\ r_{o3}\ ||\ \frac{1}{C_p s}\right]+1}$ 如果提高 r_{o3} 有助於降低 A_{CM}，根據 hw1 $r_o \propto L$，L_{M3} 不能取太小，且電阻不能太大，代表說為了要達到 A_{DM} >20(V/V)的要求，I_D 不能太小。

大致流程:

1. 先大概取一個 L_{M3} 的值，取 L_{M3}=3um，再跑看看 hspice 發現 Vth 大約等於 0.34V，所以取一個較接近的 Vb 值，取 Vb=0.35 V。Vin_CM 取 0.45V。
2. 經過幾次測試後發現電流不太能夠小於 20 uA，我取電流大約為 21uA，經過公式①計算後和實際跑 hspice 後大概推得 W_{M3}/L_{M3} 大約要等於 60，所以我設 5.2um/3um(m=35)。
3. 跑看看 hspice 發現電流不夠大，於是加大 Vb 值，取 0.352 V。
4. 經過幾次測試後發現如果 range 要 39mV 的話，$Vod_{M1(M2)}$不能太小，要大於 15.6mV 左右。
5. 先隨意代一個 $W_{M1(M2)}$ /$L_{M1(M2)}$值和 Rd 值，再來從 gain 和 4.來決定最終值。
6. 跑看看 A_{CM} 發現值蠻大的，根據經驗如果升高 Vin_CM 和減少 $Vod_{M1(M2)}$可以降低 A_{CM}，但因為不希望動到電流，所以加大 Vin_CM，大約要取到 Vin_CM=0.4565V。
7. 將 $Vod_{M1(M2)}$控制到約 15.6mV 左右，發現 gain 超過 20V/V。
   根據經驗發現降低電流減少 A_{DM} 比增加電阻可以得到更好的 Fom 值，所以我打算降低電流使得 A_{DM} 接近 20V/V。經過測試可以得到 W_{M3}/L_{M3}=5.15um/3um(m=35)，要同時注意 $Vod_{M1(M2)}$有沒有小於約 15.6mV，不然到時候測 input range 會進到 cut off，所以在這個過程中需要

不斷嘗試 $W_{M1(M2)}$ /$L_{M1(M2)}$和 Rd，最後得到

$W_{M1(M2)}$ /$L_{M1(M2)}$= 14.6um/4.4um (m = 8)，Rd =101.5kΩ

8. 再跑看看 A_{CM} 發現值還大了一點點，所以稍稍提升 Vin_CM 值，Vin_CM=0.4566V。
9. 重複回到步驟 7.，因為 Vin_CM 也會影響到 $Vod_{M1(M2)}$。
10. 跑看看 input range，發現在邊界的地方 M2 進到 linear，推測應該是電阻太大造成 V_{DS} 值過小，但觀察發現離飽和區不遠了，所以稍微減少 Rd 值，取 Rd =100.6kΩ。
11. 重複步驟 7.、10.。
12. 經過多次嘗試後我取得 Fom 最小的設計。

第 4 次測試:

Design parameter

Vb	Vin_CM	W_{M3}/L_{M3}	$W_{M1(M2)}$ /$L_{M1(M2)}$	Rd
0.352 V	0.4566V	5.166um/3um(m=35)	14.685um/4.4um (m = 8)	100.690kΩ

Result

```
subckt
element  0:mm1      0:mm2      0:mm3
model    0:n_18.1   0:n_18.1   0:n_18.1
region   Saturation Saturation Saturation
 id       10.7673u   10.7673u   21.5346u
 ibs     -828.6809a -828.6809a -3.790e-21
 ibd       -3.6873f   -3.6873f   -1.4297f
 vgs      363.1444m  363.1444m  352.0000m
 vds      322.3845m  322.3845m   93.4556m
 vbs      -93.4556m  -93.4556m    0.
 vth      347.4730m  347.4730m  339.9587m
 vdsat     65.1735m   65.1735m   63.3044m
 vod       15.6714m   15.6714m   12.0413m
 beta       8.0034m    8.0034m   18.0797m
 gam eff  510.0001m  510.0001m  507.4459m
 gm       225.7648u  225.7648u  446.9981u
 gds        1.3566u    1.3566u   28.4525u
 gmb       44.5799u   44.5799u   93.6443u
 cdtot    167.0234f  167.0234f  341.5147f
 cgtot      2.5128p    2.5128p    2.6471p
 cstot      2.3345p    2.3345p    2.4190p
 cbtot      1.1000p    1.1000p    1.3501p
 cgs        1.9944p    1.9944p    2.0341p
 cgd       40.8139f   40.8139f   90.0559f
```

```
v(voutp,voutn)/vip                        =   20.0001
input resistance at             vip       =  1.000e+20
output resistance at v(voutp,voutn)       =  177.1775k
```

```
dcgain_in_db=   26.0207      at=    1.0000
             from=    1.0000      to= 100.0000g
bw=    1.5355x
```

觀察與微調:

發現達到題目要求，A_{DM} > 20(V/V)，bw > 1.5M(Hz)。

Input range:

```
432.0000m      1.000e+20      37.6000k      5.0433
433.0000m      1.000e+20      38.5897k      5.1331
434.0000m      1.000e+20      39.8134k      5.2467
435.0000m      1.000e+20      41.3348k      5.3905
436.0000m      1.000e+20      43.2425k      5.5736
437.0000m      1.000e+20      45.6629k      5.8090
438.0000m      1.000e+20      48.7794k      6.1155
439.0000m      1.000e+20      52.8640k      6.5212
440.0000m      1.000e+20      58.3188k      7.0674
441.0000m      1.000e+20      65.7333k      7.8155
442.0000m      1.000e+20      75.8994k      8.8485
443.0000m      1.000e+20      89.6267k     10.2534
444.0000m      1.000e+20     107.0080k     12.0474
445.0000m      1.000e+20     126.1945k     14.0503
446.0000m      1.000e+20     143.5414k     15.8921
447.0000m      1.000e+20     156.3746k     17.2916
448.0000m      1.000e+20     164.6472k     18.2318
449.0000m      1.000e+20     169.6606k     18.8367
450.0000m      1.000e+20     172.6676k     19.2297
451.0000m      1.000e+20     174.4955k     19.4931
452.0000m      1.000e+20     175.6272k     19.6754
453.0000m      1.000e+20     176.3359k     19.8039
454.0000m      1.000e+20     176.7759k     19.8940
455.0000m      1.000e+20     177.0341k     19.9546
456.0000m      1.000e+20     177.1579k     19.9900
457.0000m      1.000e+20     177.1688k     20.0022
458.0000m      1.000e+20     177.0686k     19.9909
459.0000m      1.000e+20     176.8401k     19.9534
460.0000m      1.000e+20     176.4419k     19.8845
461.0000m      1.000e+20     175.7966k     19.7742
462.0000m      1.000e+20     174.7665k     19.6054
463.0000m      1.000e+20     173.1081k     19.3475
464.0000m      1.000e+20     170.3899k     18.9472
465.0000m      1.000e+20     165.8649k     18.3121
466.0000m      1.000e+20     158.3596k     17.2983
467.0000m      1.000e+20     146.5059k     15.7406
468.0000m      1.000e+20     129.9318k     13.6013
469.0000m      1.000e+20     110.8033k     11.1589
470.0000m      1.000e+20      92.8346k      8.8773
471.0000m      1.000e+20      78.3394k      7.0398
472.0000m      1.000e+20      67.5182k      5.6656
473.0000m      1.000e+20      59.6223k      4.6575
```

```
subckt
element  0:mm1       0:mm2       0:mm3
model    0:n_18.1    0:n_18.1    0:n_18.1
region   Saturation  Saturation  Saturation
  id        7.7784u    13.5770u    21.3554u
  ibs    -778.4346a  -778.4355a  -3.758e-21
  ibd      -6.3559f    -1.1787f    -1.3431f
  vgs     345.2110m   392.4110m   352.0000m
  vds     629.0072m    45.1401m    87.7890m
  vbs     -87.7890m   -87.7890m    0.
  vth     345.2109m   347.3161m   339.9812m
  vdsat    58.9548m    79.0165m    63.2952m
  vod      96.1623n    45.0949m    12.0188m
  beta      7.9956m     8.0132m    18.0798m
  gam eff 509.8488m   509.8488m   507.4459m
  gm      172.0187u   209.5318u   441.3017u
  gds     938.6565n   181.5911u    35.0938u
  gmb      34.0047u    41.8985u    92.4931u
  cdtot   155.6146f   914.4145f   352.2538f
  cgtot     2.1570p     3.2971p     2.6516p
  cstot     1.8597p     2.9894p     2.4169p
  cbtot     1.0887p     1.1542p     1.3513p
  cgs       1.5586p     2.6326p     2.0353p
  cgd      41.2163f   351.5168f    94.2886f

****     small-signal transfer characteristics

     v(voutp,voutn)/vip                   =     5.1331
     input resistance at            vip   =  1.000e+20
     output resistance at v(voutp,voutn)  =   38.5898k
```

```
subckt
element  0:mm1       0:mm2       0:mm3
model    0:n_18.1    0:n_18.1    0:n_18.1
region   Saturation  Saturation  Saturation
  id       13.2747u     8.2260u    21.5007u
  ibs    -818.3326a  -818.3318a  -3.784e-21
  ibd      -1.4486f    -5.9563f    -1.4119f
  vgs     379.7115m   348.9115m   352.0000m
  vds      71.0811m   579.4372m    92.2885m
  vbs     -92.2885m   -92.2885m    0.
  vth     348.1452m   346.3042m   339.9633m
  vdsat    72.3168m    59.9533m    63.3025m
  vod      31.5663m     2.6073m    12.0367m
  beta      8.0094m     7.9979m    18.0797m
  gam eff 509.9690m   509.9690m   507.4459m
  gm      244.2460u   180.4157u   445.9237u
  gds      55.4170u   992.3078n    29.6805u
  gmb      48.5001u    35.5882u    93.4274u
  cdtot   385.9337f   156.9172f   343.5657f
  cgtot     2.9250p     2.2164p     2.6480p
  cstot     2.7150p     1.9403p     2.4186p
  cbtot     1.1274p     1.0886p     1.3503p
  cgs       2.4072p     1.6326p     2.0343p
  cgd     128.3230f    41.0091f    90.8615f

****     small-signal transfer characteristics

     v(voutp,voutn)/vip                   =     5.6656
     input resistance at            vip   =  1.000e+20
     output resistance at v(voutp,voutn)  =   67.5182k
```

input range = 472(mV) - 433(mV) = 39(mV)

Acm:

```
****** ac analysis tnom=  25.000 temp=  25.000 ******
acm_in_db=   1.0104
```

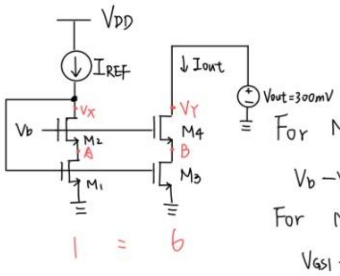
$\mathrm{CMRR_{simulated}} = 20\log(20.0001) - 1.0104 = 25.0103\ \mathrm{dB} > 25\ \mathrm{dB}$

FoM:

$$\mathrm{FoM} = \frac{total\ current(uA)}{input\ range(mV) * -3dB\ bandwidth\ (MHz)} = \frac{21.5346\ (uA)}{39\ (mV) * 1.5355 (MHz)} = 0.35960223$$

PART2

(a)



For M_2 to be sat.

$$V_b - V_{TH2} \le V_X \ (= V_{GS1})$$

For M_1 to be sat

$$V_{GS1} - V_{TH1} \le V_A \ (= V_b - V_{GS2})$$

Thus

$$V_{GS2} + (V_{GS1} - V_{TH1}) \le V_b \le V_{GS1} + V_{TH2}$$

it exists if

$$V_{GS2} + (V_{GS1} - V_{TH1}) \le V_{GS1} + V_{TH2}$$

$$\Rightarrow V_{GS2} - V_{TH2} \le V_{TH1}$$

Let

$$V_{GS2} = V_{GS4} \Rightarrow V_b = V_{GS2} + (V_{GS1} - V_{TH1}) = V_{GS4} + (V_{GS3} - V_{TH3})$$

if $V_{GS2} = V_{GS4}$ $\quad V_A = V_B$

$$\boxed{\Rightarrow V_{out,min} = V_B + V_{ov}}$$

$$I_{REF} = \cancel{\tfrac{1}{2}} \cancel{\mu_n C_{ox}} \cancel{\left(\tfrac{W}{L}\right)_1} (V_X - V_{TH})^2 = \cancel{\tfrac{1}{2}} \cancel{\mu_n C_{ox}} \cancel{\left(\tfrac{W}{L}\right)_2} (V_b - V_A - V_{TH})^2$$

Let $\left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2$ $\quad L_1 = L_2 \Rightarrow V_{TH1} \approx V_{TH2}$

$$\Rightarrow V_X - \cancel{V_{TH}} = V_b - V_A - \cancel{V_{TH}}$$

$$\Rightarrow V_b = V_X + V_A = V_{GS2} + (V_{GS1} - V_{TH}) = V_b - V_A + V_X - V_{TH}$$

if we assume $V_X = 410\,mV$, $V_{TH} = 360\,mV$

$$\Rightarrow V_A = 50\,mV \Rightarrow \boxed{V_b = 460\,mV}$$

For M_2 to be on, sat.

$V_b - V_A \ge V_{TH2}$ $\Rightarrow 460mV - 50mV \ge 360mV$

$V_b - V_{TH2} \le V_X \ (= V_{GS1})$ $\Rightarrow 460mV - 360mV \le 410mV$

For M_1 to be on, sat

$V_X \ge V_{TH}$ $\Rightarrow 410mV \ge 360mV$

$V_{GS1} - V_{TH1} \le V_A \ (= V_b - V_{GS2})$ $\Rightarrow 410mV - 360mV \le 50mV$

I. 在 cascode stage 出現的電阻公式 R_{on} 可以推得 Rout ≅ $(g_{m4}+g_{mb4})r_{o3}r_{o4}$ 但因為 r_o 不好去計算，所以換個方式觀察。

II. 因為此題的 spec 電阻要大於 700k，觀察 Rout = $\frac{Vout}{Iout}$ (at Vout 300mV)，可以推斷如果 current mirror 越快達到 120 uA，也就是如果能讓 M4 在越小的 V_{out} 下達到飽和區，$\frac{Iout}{Vout}$ 更小 ⟹ Rout 更大。所以讓 M4 的 Vov 越小越好 ⟹ $\frac{W_{M4}}{L_{M4}}$ 取大，$\frac{W_{M4}}{L_{M4}} = \frac{60um}{1um}$ (m=5)。

III. 因為 V_B 也會隨著 V_{out} 增加(sweep from 0V to 1.8V)升高到穩定值，如果能讓 M3 的 Vov 越小⟹M3 越快進入飽和 ⟹ $\frac{W_{M3}}{L_{M3}}$ 取大，$\frac{W_{M3}}{L_{M3}} = \frac{60um}{1um}$ (m=5)。

IV. 根據 current mirror 和電流公式①，M1 與 M3 的 size 需成比例 1:6，且如果電流要準確，也需讓 V_A、V_B 同電壓，因此 M2 和 M4 的 size 也需成比例 1:6，所以 $\frac{W_{M1}}{L_{M1}} = \frac{50um}{1um}$、$\frac{W_{M2}}{L_{M2}} = \frac{50um}{1um}$。

第 1 次測試:

Design parameter: $\frac{W_{M4}}{L_{M4}} = \frac{W_{M3}}{L_{M3}} = \frac{60um}{1um}$ (m=5)，$\frac{W_{M1}}{L_{M1}} = \frac{W_{M2}}{L_{M2}} = \frac{50um}{1um}$ (m=1)。

V_b=460mV

Result:

```
**** mosfets


subckt
element  0:mm1      0:mm2      0:mm3      0:mm4
model    0:n_18.1   0:n_18.1   0:n_18.1   0:n_18.1
region   Saturation Saturation Saturation Saturation
  id       20.0000u   20.0000u  119.3503u  119.3503u
  ibs    -2.994e-21 -198.6946a -1.781e-20   -1.1701f
  ibd    -198.6886a   -1.4643f   -1.1701f   -6.4560f
  vgs     406.8730m  404.7895m  406.8730m  405.6272m
  vds      55.2105m  351.6625m   54.3728m  245.6272m
  vbs       0.       -55.2105m    0.        -54.3728m
  vth     387.8947m  397.2736m  387.8340m  397.8590m
  vdsat    72.0633m   67.4882m   72.1362m   67.6291m
  vod      18.9783m    7.5159m   19.0390m    7.7681m
  beta     15.5384m   15.5552m   93.2333m   93.3391m
  gam eff 507.4459m  508.9703m  507.4459m  508.9475m
  gm      362.7761u  422.2649u    2.1578m    2.5181m
  gds     165.8061u    5.1183u    1.0252m   35.0710u
  gmb      76.6425u   85.2646u  455.9953u  509.8988u
  cdtot    98.7943f   70.0477f  597.1579f  432.0074f
  cgtot   278.9623f  244.9956f    1.6758p    1.4747p
  cstot   282.2926f  251.8300f    1.6945p    1.5144p
  cbtot   197.8028f  185.7752f    1.1869p    1.1253p
  cgs     210.4376f  178.8797f    1.2633p    1.0789p
  cgd      27.2568f   18.4535f  165.3625f  111.2860f
```

```
****** dc transfer curves tnom=  25.000 temp=  25.000 ******
deriv=   8.8870u
rout= 112.5239k
```

```
****** operating point information tnom=  25.000 temp=  25.000 ******
****** operating point status is all       simulation time is     0.
   node    =voltage       node    =voltage       node    =voltage

+0:a       =  55.2105m 0:b       =  54.3728m 0:vb      = 460.0000m
+0:vdd     =   1.8000  0:vout    = 300.0000m 0:vss     =   0.
+0:vx      = 406.8730m
```

觀察與微調:

發現 Rout 還不夠大，猜測或許是因為 V_B 小使得 M3 隨著 V_B 增加，進入飽和區時的 V_{out} 值較接近 300mV，所以電流還未完全穩定，所以如果能稍微提高 V_b 的值，電流變化不大的情況下，V_B 的值就會提高，或許能讓 M3 早一些進入飽和區，電流更快在 V_{out} = 300mV 時達到 120 uA。

取 V_b=500mV。

第 2 次測試:

Design parameter: $\frac{W_{M4}}{L_{M4}} = \frac{W_{M3}}{L_{M3}} = \frac{60um}{1um}$ (m=5)，$\frac{W_{M1}}{L_{M1}} = \frac{W_{M2}}{L_{M2}} = \frac{50um}{1um}$ (m=1)。

V_b=500mV

Result:

```
subckt
element  0:mm1      0:mm2      0:mm3      0:mm4
model    0:n_18.1   0:n_18.1   0:n_18.1   0:n_18.1
region   Saturation Saturation Saturation Saturation
  id       20.0000u   20.0000u  119.9688u  119.9688u
  ibs    -2.994e-21 -317.1477a -1.790e-20   -1.8741f
  ibd    -317.1417a   -1.4370f   -1.8741f   -6.4560f
  vgs     399.2933m  411.8749m  399.2933m  412.9127m
  vds      88.1251m  311.1682m   87.0873m  212.9127m
  vbs       0.       -88.1251m    0.        -87.0873m
  vth     387.6459m  404.3293m  387.5867m  404.8305m
  vdsat    68.8065m   67.7421m   68.8751m   68.0019m
  vod      11.6474m    7.5456m   11.7066m    8.0822m
  beta     15.5349m   15.5697m   93.2119m   93.4253m
  gam eff 507.4459m  509.8578m  507.4459m  509.8301m
  gm      404.0178u  422.6658u    2.4199m    2.5297m
  gds      41.3507u    5.3682u  258.0580u   39.1752u
  gmb      84.9775u   83.7456u  509.0688u  502.6171u
  cdtot    82.1234f   70.2192f  493.9052f  433.1188f
  cgtot   258.4317f  244.4387f    1.5517p    1.4756p
  cstot   263.9911f  250.5623f    1.5846p    1.5112p
  cbtot   195.9553f  183.8771f    1.1757p    1.1134p
  cgs     191.6758f  178.9967f    1.1509p    1.0844p
  cgd      20.9109f   18.4549f  125.9415f  111.5889f
```

```
****** dc transfer curves tnom=  25.000 temp=  25.000 ******
deriv=   3.0858u
rout= 324.0639k
```

```
****** operating point information tnom=  25.000 temp=  25.000 ******
****** operating point status is all       simulation time is     0.
   node    =voltage       node    =voltage       node    =voltage

+0:a       =  88.1251m 0:b       =  87.0873m 0:vb      = 500.0000m
+0:vdd     =   1.8000  0:vout    = 300.0000m 0:vss     =   0.
+0:vx      = 399.2933m
```

觀察與微調:

與預期一樣，如果提高 V_B 的值，就可讓 M3 更早進入飽和區，電流在 V_{out}=300mV 時提高了，代表電流更早穩定，Rout 更大，但和標準還差了一點，於是繼續加大 V_b。

取 V_b=550mV。

第 3 次測試:

Design parameter: $\frac{W_{M4}}{L_{M4}} = \frac{W_{M3}}{L_{M3}} = \frac{60um}{1um}$ (m=5)，$\frac{W_{M1}}{L_{M1}} = \frac{W_{M2}}{L_{M2}} = \frac{50um}{1um}$(m=1)。

V_b=550mV

Result:

subckt				
element	0:mm1	0:mm2	0:mm3	0:mm4
model	0:n_18.1	0:n_18.1	0:n_18.1	0:n_18.1
region	Saturation	Saturation	Saturation	Saturation
id	20.0000u	20.0000u	120.1429u	120.1429u
ibs	-2.994e-21	-465.8930a	-1.793e-20	-2.7595f
ibd	-465.8870a	-1.4286f	-2.7595f	-6.4560f
vgs	396.9599m	420.5430m	396.9599m	421.7696m
vds	129.4570m	267.5028m	128.2304m	171.7696m
vbs	0.	-129.4570m	0.	-128.2304m
vth	387.3335m	412.9554m	387.2757m	413.4209m
vdsat	67.9430m	68.0482m	68.0100m	68.4034m
vod	9.6264m	7.5875m	9.6841m	8.3488m
beta	15.5335m	15.5868m	93.2040m	93.5274m
gam eff	507.4459m	510.9513m	507.4459m	510.9192m
gm	416.0795u	423.0616u	2.4983m	2.5304m
gds	13.3167u	5.7736u	81.8960u	49.4711u
gmb	87.3477u	81.8969u	524.5171u	491.2481u
cdtot	77.1896f	70.3553f	463.5270f	435.8225f
cgtot	252.2189f	243.8267f	1.5141p	1.4765p
cstot	259.2146f	249.0509f	1.5560p	1.5051p
cbtot	194.6846f	181.5247f	1.1681p	1.0993p
cgs	185.6486f	179.1737f	1.1147p	1.0899p
cgd	19.3128f	18.4694f	116.0239f	112.5260f

```
****** dc transfer curves tnom=  25.000 temp=  25.000 ******
deriv=   1.3197u
rout= 757.7500k
```

```
****** operating point information tnom=  25.000 temp=  25.000 ******
****** operating point status is all       simulation time is     0.
   node    =voltage      node    =voltage      node    =voltage

+0:a       = 129.4570m 0:b       = 128.2304m 0:vb      = 550.0000m
+0:vdd     =   1.8000  0:vout    = 300.0000m 0:vss     =   0.
+0:vx      = 396.9599m
```

觀察與微調:

達到題目要求，Rout > 700kΩ。

計算驗證:

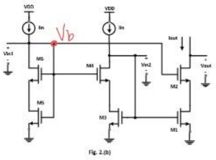
$R_{out} \cong (g_{m4}+g_{mb4})r_{o3}r_{o4}$

$$= (2.5304 \cdot 10^{-3} + 491.2481 \cdot 10^{-6}) \cdot \left(\frac{1}{81.8960 \cdot 10^{-6}}\right) \cdot \left(\frac{1}{49.4711 \cdot 10^{-6}}\right)$$

$$= 745813.9558\ \Omega$$

$$Rout_{\ error} = \left|\frac{757750 - 745813.9558}{745813.9558}\right| \cdot 100\% = 1.6\%$$

(b)



Assume $(\frac{W}{L})_{M1} = (\frac{W}{L})_{M2} = 6(\frac{W}{L})_{M3} = 6(\frac{W}{L})_{M4}$

$(\frac{W}{L})_{M1,M2} = \frac{1}{4}(\frac{W}{L})_{M56}$

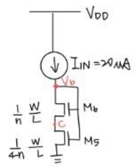
$(V_{GS}-V_{TH})_{M56} = 2(V_{GS}-V_{TH})_{M3}$

$\Rightarrow V_{in1} = V_b = V_{GS,M56} = 2V_{ov,M3} + V_{TH,M56} = 2V_{ov} + V_{TH}$

$V_{in2} = V_{GS3} = V_{TH,M3} + V_{ov,M3} = V_{TH} + V_{ov}$

$V_{out} = V_{DS1} + V_{DS2} = V_{ov,M1} + V_{ov,M2}$

$\Rightarrow V_{out} = 2V_{ov,M2} = 2V_{ov}$



by (a) we can get $V_{ov} \approx 9.6m$ and $V_{GS,M2} \approx 420mV$

$V_b = 2V_{ov} + V_{TH} = V_{ov} + V_{GS,M2} \approx 430mV$ ...... ①

$I_{IN} = \cancel{\frac{1}{2}\mu_n C_{ox}}(\frac{W}{(4-n)L})(V_b - V_{TH,M5})^2 = \cancel{\frac{1}{2}\mu_n C_{ox}}(\frac{W}{nL})(V_b - V_c - V_{TH,M6})^2$

$\frac{1}{4-n}(V_b - V_{TH,M5})^2 = \frac{1}{n}(V_b - V_c - V_{TH,M6})^2$ ...... ②

$\begin{cases} V_c \geq V_b - V_{TH,M5} & \text{(for } M_5 \text{ sat)} \\ V_b \geq V_{TH,M5} & \text{(for } M_5 \text{ on)} \end{cases}$

$\begin{cases} V_b - V_c \geq V_b - V_c & \text{(for } M_6 \text{ sat)} \\ V_b - V_c \geq V_{TH,M6} & \text{(for } M_6 \text{ on)} \end{cases}$ ...... ③

by ① ② ③ and (a) 取 $V_b = 470mV$ $V_c = 30mV$

假設 $L_{,M5} < 1 < L_{,M6} \Rightarrow V_{TH,M5} = 440mV$, $V_{TH,M6} = 340mV$

$\Rightarrow \frac{1}{4-n}(470mV - 440mV)^2 = \frac{1}{n}(470mV - 30mV - 340mV)^2$

$\Rightarrow n \approx 3.5$ (驗證符合一開始假設 $L_{,M5} < 1 < L_{,M6}$)

by $(\frac{W}{L})_{M1,M2} = \frac{1}{4}(\frac{W}{L})_{M56}$, $\Rightarrow W = 50\mu m$

take $W_{,M5} = 50\mu m$ $L_{,M5} = 3.5\mu m$

$W_{,M6} = 50\mu m$ $L_{,M6} = 0.5\mu m$

第 1 次測試:

Design parameter: $\frac{W_{M4}}{L_{M4}} = \frac{W_{M3}}{L_{M3}} = \frac{60um}{1um}$ (m=5)，$\frac{W_{M1}}{L_{M1}} = \frac{W_{M2}}{L_{M2}} = \frac{50um}{1um}$ (m=1)。

$\frac{W_{M5}}{L_{M5}} = \frac{50um}{3.5um}$ (m=1)，$\frac{W_{M6}}{L_{M6}} = \frac{50um}{0.5um}$ (m=1)。

Result:

subckt						
element	0:mm1	0:mm2	0:mm3	0:mm4	0:mm5	0:mm6
model	0:n_18.1	0:n_18.1	0:n_18.1	0:n_18.1	0:n_18.1	0:n_18.1
region	Saturation	Saturation	Saturation	Saturation	Subth	Saturation
id	118.9567u	118.9567u	20.0000u	20.0000u	20.0000u	20.0000u
ibs	-1.775e-20	-940.5868a	-2.994e-21	-160.0061a	-2.994e-21	-95.3333a
ibd	-940.5513a	-6.4560f	-160.0001a	-1.4878f	-95.3273a	-1.6082f
vgs	413.4145m	403.1574m	413.4145m	402.4041m	446.8642m	420.3747m
vds	43.7067m	256.2933m	44.4601m	368.9544m	26.4895m	420.3747m
vbs	0.	-43.7067m	0.	-44.4601m	0.	-26.4895m
vth	387.9146m	395.5579m	387.9760m	394.9095m	447.5388m	337.1020m
vdsat	75.1679m	67.4777m	75.0914m	67.3980m	70.2272m	101.1466m
vod	25.4999m	7.5995m	25.4385m	7.4946m	-674.5779u	83.2728m
beta	93.2515m	93.3099m	15.5415m	15.5503m	32.3998m	4.3011m
gam eff	507.4459m	508.6564m	507.4459m	508.6771m	507.4459m	508.1831m
gm	1.9916m	2.5107m	335.9836u	422.1281u	348.1071u	309.5045u
gds	1.6363m	34.0540u	266.6457u	5.0378u	575.1132u	2.0863u
gmb	421.7711u	511.6096u	71.1344u	85.7675u	70.1558u	62.1721u
cdtot	710.9319f	431.7491f	117.2022f	69.9311f	96.7734f	70.7775f
cgtot	1.7814p	1.4737p	296.4363f	245.1614f	135.7262f	1.1223p
cstot	1.7891p	1.5144p	298.0257f	252.2242f	149.5446f	1.1642p
cbtot	1.1945p	1.1293p	199.0480f	186.3564f	155.1980f	404.7903f
cgs	1.3472p	1.0761p	224.4894f	178.8085f	85.0151f	998.9430f
cgd	210.4275f	111.2369f	34.5368f	18.4537f	26.6011f	17.0002f

```
****** operating point information tnom=  25.000 temp=  25.000 ******
****** operating point status is all       simulation time is     0.
   node    =voltage      node    =voltage      node    =voltage

+0:a       =  44.4601m 0:b       =  43.7067m 0:c       =  26.4895m
+0:vb      = 446.8642m 0:vdd     =   1.8000  0:vout    = 300.0000m
+0:vss     =   0.      0:vx      = 413.4145m
```

觀察與微調:

發現 Vb 比預估的值還要小，所以造成 M5 進入 cut off，要讓 Vb 的值大一些，可以發現 Vb 其實就是 M5 的 V_G，觀察公式①如果要增加 Vb 的值，可以減少 $\frac{W_{M5}}{L_{M5}}$，因為變動到 L 就會同時影響到 Vth，所以我打算調小 W 來降低$\frac{W_{M5}}{L_{M5}}$。

試取 $\frac{W_{M5}}{L_{M5}} = \frac{40um}{3.5um}$，因為 M5 和 M6 要 W 要相同，所以$\frac{W_{M6}}{L_{M6}} = \frac{40um}{0.5um}$。

第 2 次測試:

Design parameter: $\frac{W_{M4}}{L_{M4}} = \frac{W_{M3}}{L_{M3}} = \frac{60um}{1um}$ (m=5)，$\frac{W_{M1}}{L_{M1}} = \frac{W_{M2}}{L_{M2}} = \frac{50um}{1um}$ (m=1)。

$\frac{W_{M5}}{L_{M5}} = \frac{40um}{3.5um}$ (m=1)，$\frac{W_{M6}}{L_{M6}} = \frac{40um}{0.5um}$ (m=1)。

Result:

subckt						
element	0:mm1	0:mm2	0:mm3	0:mm4	0:mm5	0:mm6
model	0:n_18.1	0:n_18.1	0:n_18.1	0:n_18.1	0:n_18.1	0:n_18.1
region	Saturation	Saturation	Saturation	Saturation	Saturation	Saturation
id	119.3818u	119.3818u	20.0000u	20.0000u	20.0000u	20.0000u
ibs	-1.781e-20	-1.1920f	-2.994e-21	-202.3884a	-3.009e-21	-75.1287a
ibd	-1.1920f	-6.4560f	-202.3825a	-1.4626f	-75.1227a	-1.3347f
vgs	406.4158m	405.8601m	406.4158m	405.0147m	461.2516m	435.2889m
vds	55.3915m	244.6085m	56.2369m	350.1788m	25.9627m	435.2889m
vbs	0.	-55.3915m	0.	-56.2369m	0.	-25.9627m
vth	387.8263m	398.0781m	387.8870m	397.4973m	447.6499m	337.0325m
vdsat	71.9307m	67.6426m	71.8580m	67.4966m	76.3698m	110.4754m
vod	18.5895m	7.7820m	18.5288m	7.5174m	13.6017m	98.2564m
beta	93.2321m	93.3419m	15.5382m	15.5557m	25.9232m	3.4432m
gam eff	507.4459m	508.9752m	507.4459m	508.9982m	507.4459m	508.1685m
gm	2.1712m	2.5187m	364.9194u	422.2780u	321.3872u	287.2352u
gds	980.3291u	35.1740u	158.4394u	5.1258u	606.9979u	1.9690u
gmb	458.7350u	509.7144u	77.0801u	85.2167u	64.7027u	57.4187u
cdtot	590.2004f	432.0342f	97.6706f	70.0563f	86.0848f	56.6172f
cgtot	1.6684p	1.4748p	277.7324f	244.9788f	122.1807f	910.0527f
cstot	1.6878p	1.5144p	281.1760f	251.7912f	134.7305f	947.7303f
cbtot	1.1864p	1.1249p	197.7124f	185.7178f	124.7011f	323.3170f
cgs	1.2570p	1.0791p	209.3749f	178.8849f	81.1418f	814.4899f
cgd	162.6341f	111.2918f	26.8170f	18.4535f	24.6231f	13.6124f

```
****** operating point information tnom=  25.000 temp=  25.000 ******
****** operating point status is all       simulation time is    0.
   node      =voltage      node      =voltage      node      =voltage

+0:a        =  56.2369m 0:b        =  55.3915m 0:c        =  25.9627m
+0:vb       = 461.2516m 0:vdd      =   1.8000  0:vout     = 300.0000m
+0:vss      =   0.      0:vx       = 406.4158m
```

觀察與微調:

V_b 的值與預估的一樣提高了，所有 mos 都進入飽和區。

I_{out}=119.33818 uA 約等於 6 ‧I_{in} = 120uA。